

4.C

Constraining the Equation of State for the Nuclear Matter

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HIGH TIME RESOLUTION ASTROPHYSICS

The equilibrium of a NS

Tolman-Oppenheimer-Volkoff equation of Hydrostatic equilibrium in GR...

$$\frac{dP(r)}{dr} = -\frac{G}{r^2} \left[\rho(r) + \frac{P(r)}{c^2} \right] \left[M(r) + 4\pi r^3 \frac{P(r)}{c^2} \right] \left[1 - \frac{2GM(r)}{c^2 r} \right]^{-1}$$

...complemented with the mass continuity equation...

$$\frac{dM(r)}{dr} = 4\pi \rho(r) r^2$$

... and with the Equation of State (EoS) for the matter composing the star...

$$P(r) = f(\rho(r))$$

...completely determines the structure of a spherically symmetric non rotating body of isotropic material in equilibrium

Given the BOUNDARY conditions:

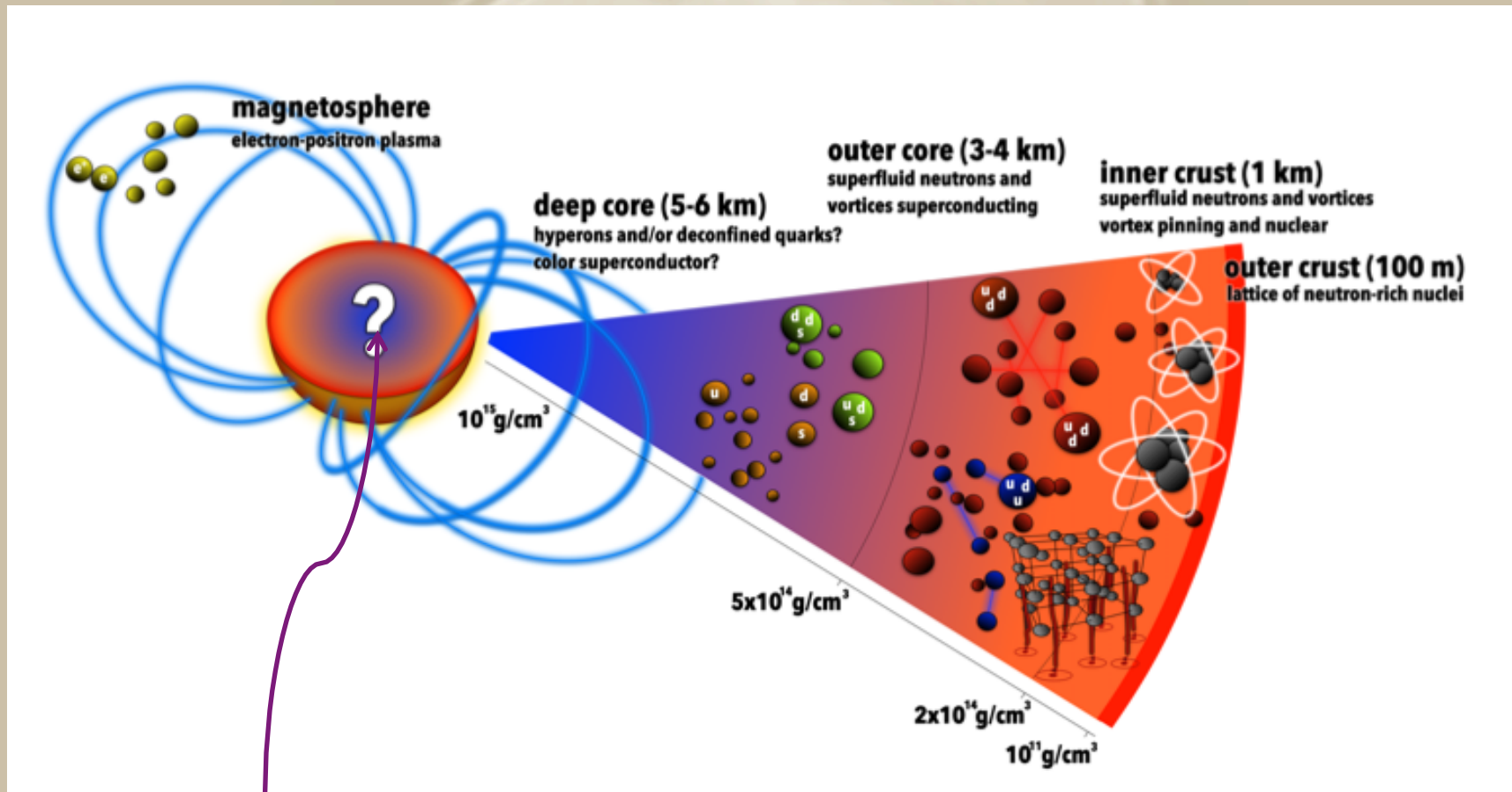
$$M(0) = 0; \quad \rho(0) = \rho_c; \quad P(0) = f(\rho_c); \quad P(R) = 0$$

$$dM(0)/dr = d\rho(0)/dr = dP(0)/dr = 0$$

for each EoS f , it results a mono-parametric family of Neutron Stars
 $M = M_f(\rho_c)$ or equivalently $M = M_f(R)$

The internal structure of a NS

...up to some $\times 10^{14} \text{ g/cm}^3$, the behavior of the nuclear matter is almost understood and thus the first few kms inside a NS can be somehow described...



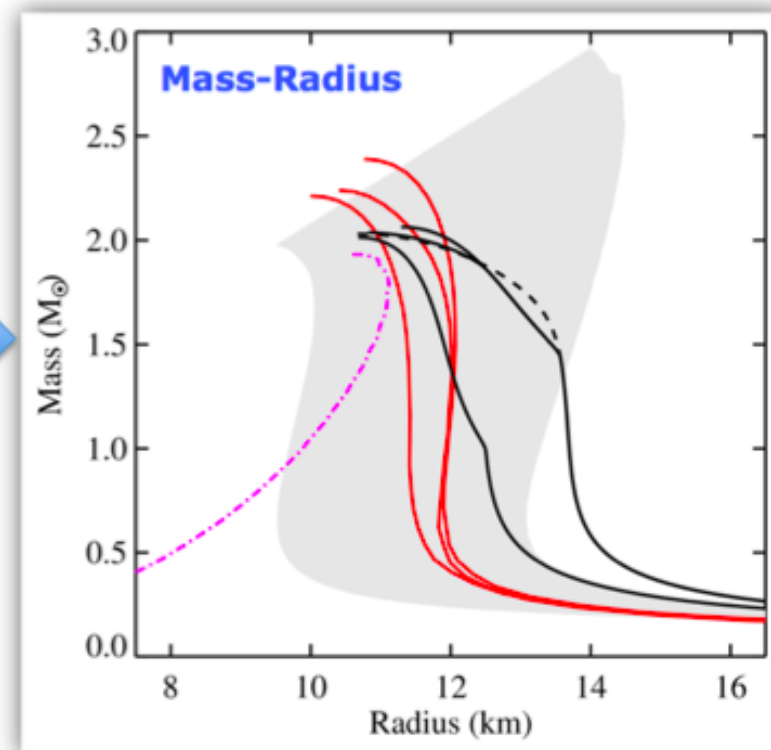
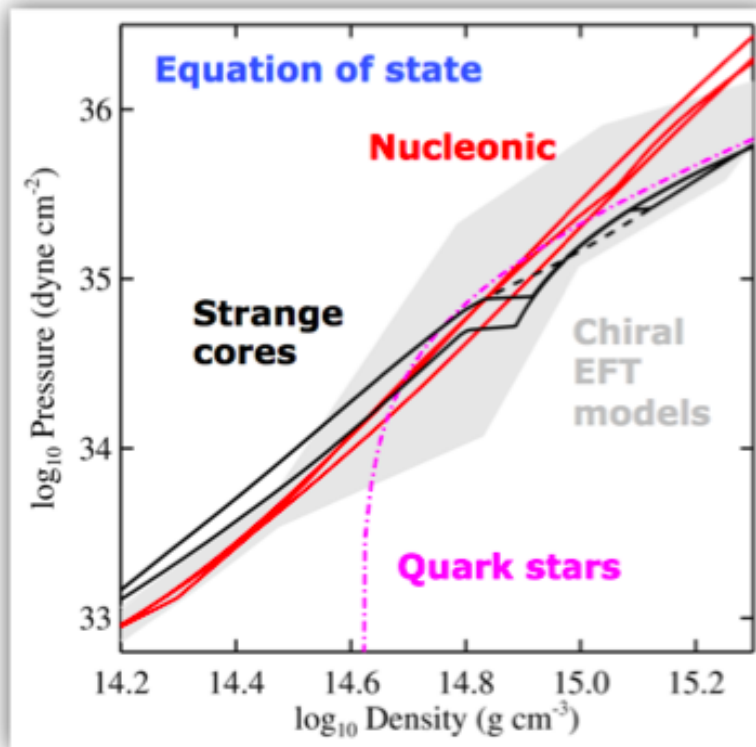
The unknown composition of the innermost part

The internal structure of a NS

... but above some $\times 10^{14}$ g/cm³, the behavior of the nuclear matter is much more debated and cannot be tested in laboratory. Hence a few tens of Equations of State $P(r) = f(\rho(r))$ have been proposed

... that reflects in a variety of $M=M_f(R)$ which have been proposed

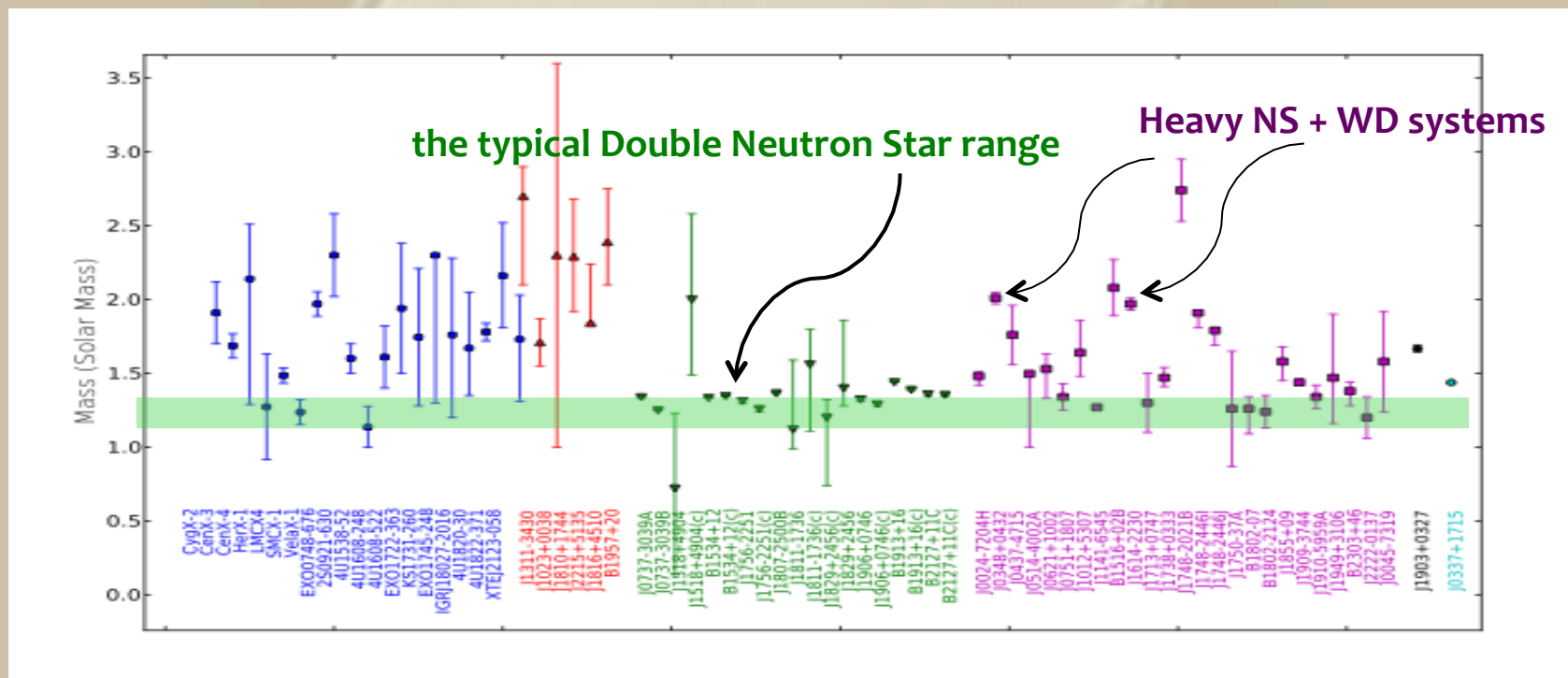
A high precision determination of M and R for a given NS would exclude the vast majority of the EoS



Pulsars as laboratories for the nuclear matter EoS: the maximum mass

...since measuring both M and R with good enough precision is difficult, a easiest way for discriminating EoS is to rely on the measurement of M ...

[Watts et al 2015, from Antoniadis catalogue]



...and aiming for values of M high enough for excluding few equations...

Pulsars as laboratories for the nuclear matter EoS: the accurately measured maximum masses

PSR J1614-2230

mass = $1.97 \pm 0.04 M_{\odot}$

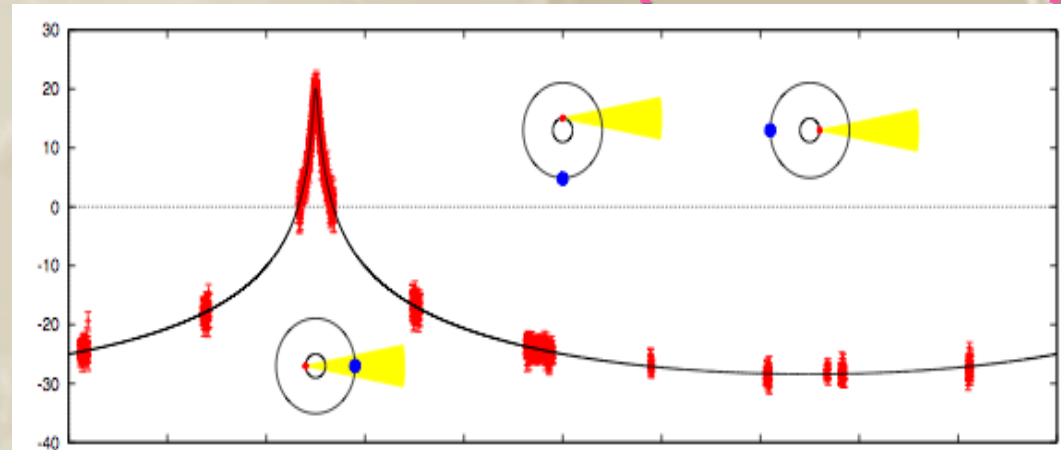
$P_{\text{spin}} = 3.2$ ms ; circular orb; $P_{\text{b}} = 8.69$ d

$M_{\text{comp}} = 0.500 \pm 0.006 M_{\odot}$

orbital Inclination $i = 89.17 \pm 0.02$

seen the two parameters
(i and M_{comp}) of the Shapiro Delay

[Demorest et al. 2010]



PSR J0348+0432

mass = $2.01 \pm 0.04 M_{\odot}$

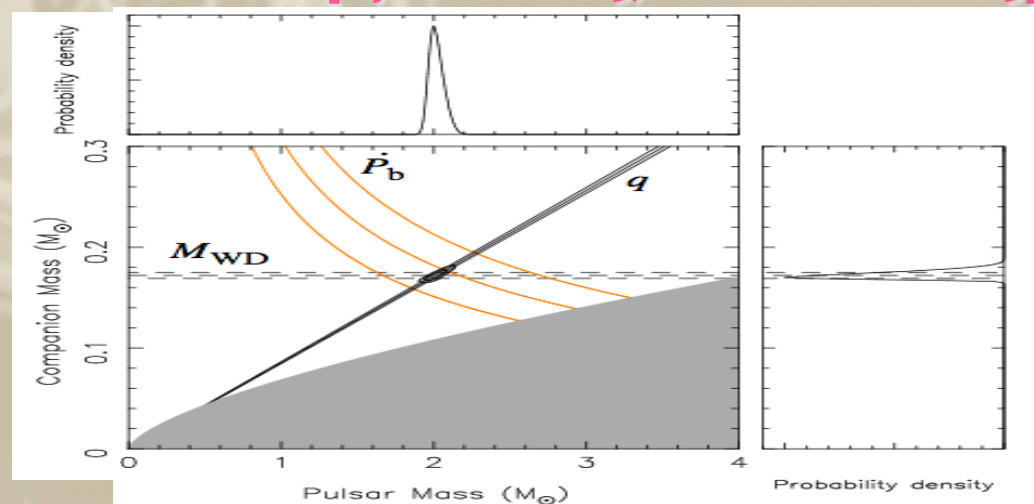
$P_{\text{spin}} = 39.1$ ms; circular orb; $P_{\text{b}} = 2.46$ h

$M_{\text{comp}} = 0.172 \pm 0.003 M_{\odot}$

orbital Inclination $i = 40.2 \pm 0.6$

seen the Orb period derivative
and M_{comp} from optical data

[Lynch et al 2013; Antoniadis et al 2013]



Pulsars as laboratories for the nuclear matter EoS: even higher maximum mass ?

PSR J1748-2021B

in NGC 6440

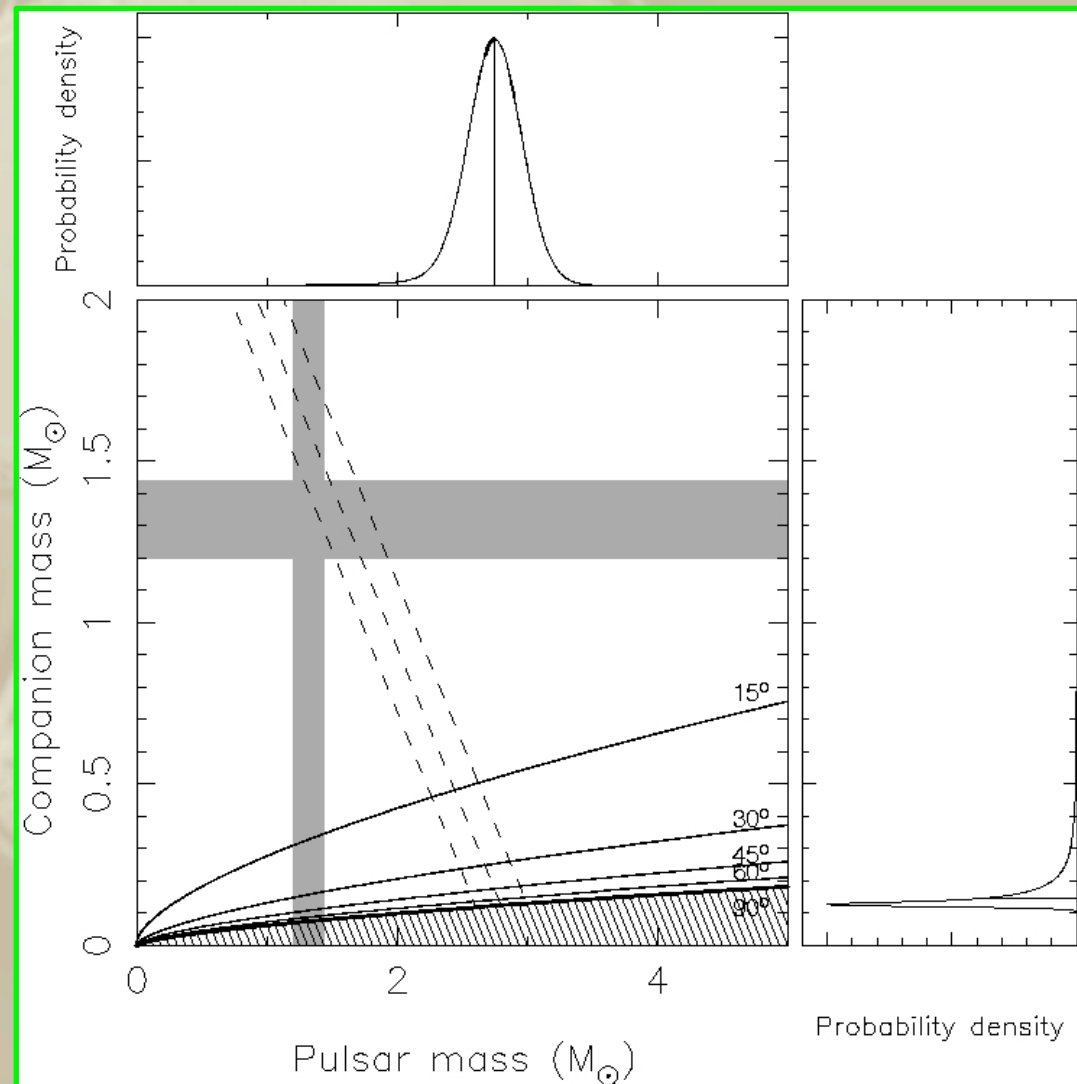
$P_{\text{spin}} \sim 16$ ms

$P_{\text{orb}} \sim 20$ day

most probable mass

$2.74 \pm 0.21 M_{\text{sun}}$

1% probability for
 $M < 2 M_{\text{sun}}$ due
to a low orbital
inclination



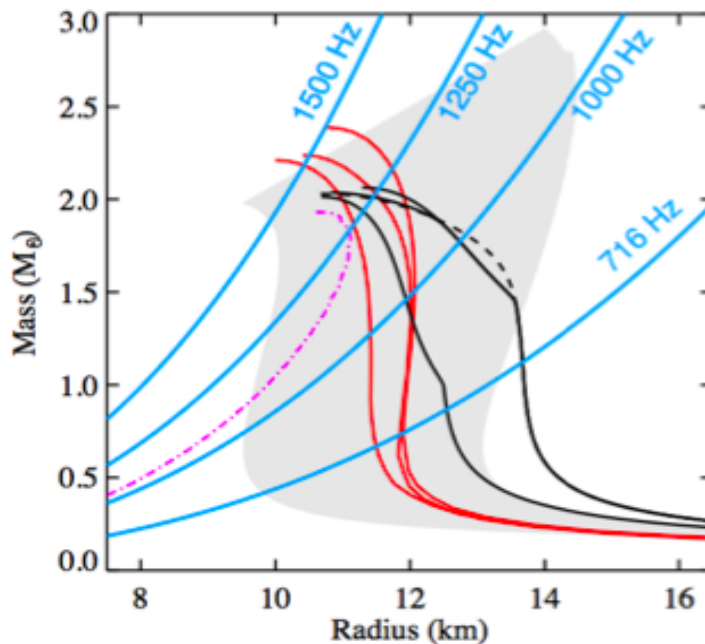
[Freire et al. 2008]

Pulsars as laboratories for the nuclear matter EoS: the maximum spin rate

... a NS cannot rotate at a too large spin rate, because it will be centrifugally disrupted...

[Burderi & D'Amico 1997]

$$P_{shedd} = 1.75 P_K = 0.957 \left(\frac{R_{static}}{10 \text{ km}} \right)^{3/2} \left(\frac{M_{\odot}}{M_{G,static}} \right)^{1/2} \text{ ms}$$



[Watts et al 2015]

Table 3.1: Physical characteristics of static and max-rotating models

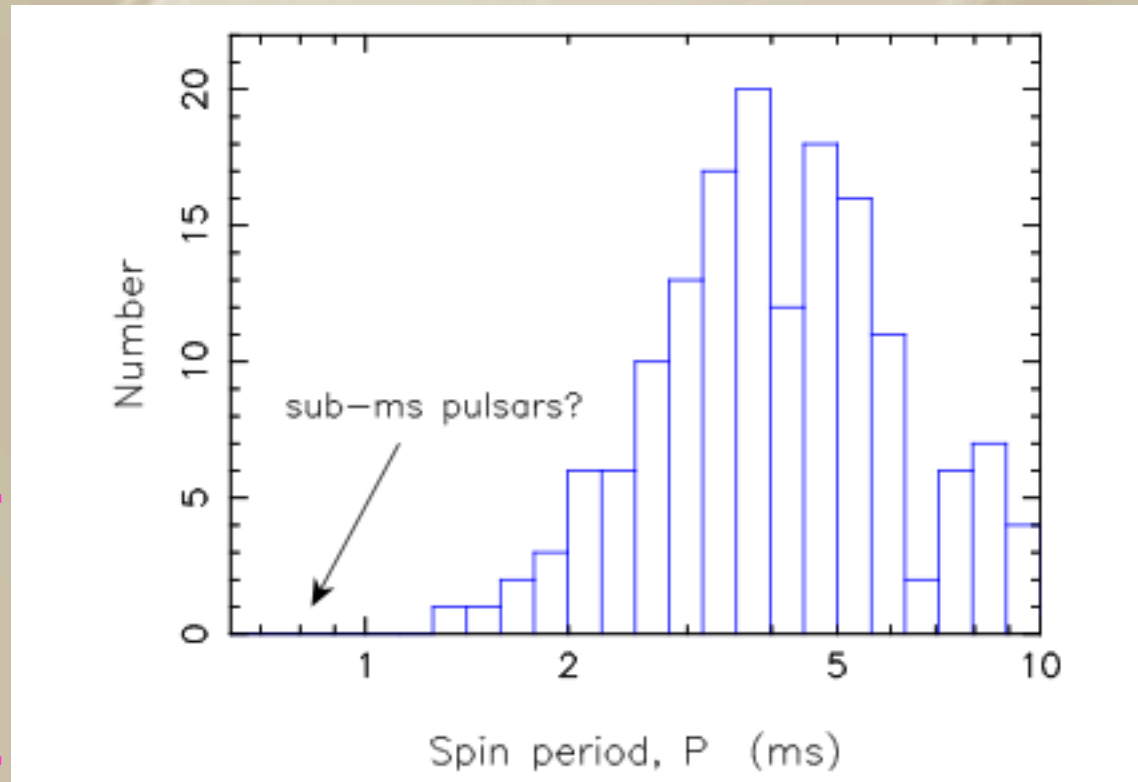
EoS	max static		max rotating		1.4 M _⊙ -star		
	M _{G,max}	R _{static}	M _{G,max}	P _{min} ^{abs}	R _{static}	R _{Ω,max}	P _{min} ^{1.4 M_⊙-star}
F	1.46	7.9	1.67	0.47	9.2	14.6	0.80
A	1.66	8.4	1.95	0.47	9.6	13.7	0.73
E	1.75	8.6	2.05	0.48	10.0	14.3	0.78
AU	2.13	9.4	2.55	0.47	10.4	14.4	0.80
FP	1.90	9.4	n.a.	n.a.	10.6	n.a.	0.86
D	1.65	9.3	1.95	0.57	10.7	15.3	0.86
FPS	1.80	9.3	2.12	0.53	10.9	15.5	0.87
UT	1.84	9.5	2.19	0.54	10.9	15.5	0.87
BBB1†	1.79	9.6	2.14	0.57	11.0	15.8	0.89
BBB2†	1.92	9.5	2.27	0.52	11.0	15.9	0.90
UU	2.20	9.8	2.61	0.50	11.2	15.7	0.89
BPAL21†	1.69	9.3	1.97	0.56	11.4	n.a.	n.a.
C	1.86	9.9	2.17	0.59	12.1	17.6	1.06
N*	2.64	12.7	3.22	0.68	13.6	19.1	1.20
BPAL32†	1.95	10.5	2.30	0.63	13.8	n.a.	n.a.
L	2.70	13.7	3.27	0.76	15.0	21.3	1.40
PS	2.28	16.0	n.a.	n.a.	16.4	n.a.	1.67
M	1.80	11.6	2.10	0.81	16.7	24.6	1.75

Table 3.1: The 2nd and the 3rd columns list the maximum gravitational mass (M_G) and the circumferential radius (in km) for static models. The 4th and the 5th columns report the maximum mass and the minimum period (in ms) for rotating models. The last 3 columns refer to models of constant M_G = 1.4 M_⊙. The EoSs marked with (†) are from [8], all the other ones from [7], except FP-EoS [10] and PS-EoS [11]. Some values are not available (n.a.) in literature. FP and PS's values are partly from the model of Chapter 5.

[Possenti 2000]

... the observation of the occurrence of ultra rapid
sub-millisecond rotational period of a
NEUTRON STAR would lead to exclude wide families of EoS

Such NSs would also be potential continuous GW emitters



[Tauris et al 2015]

The current P-distribution of the population of the recycled pulsars

Pulsars as laboratories for the nuclear matter EoS: the maximum spin rate

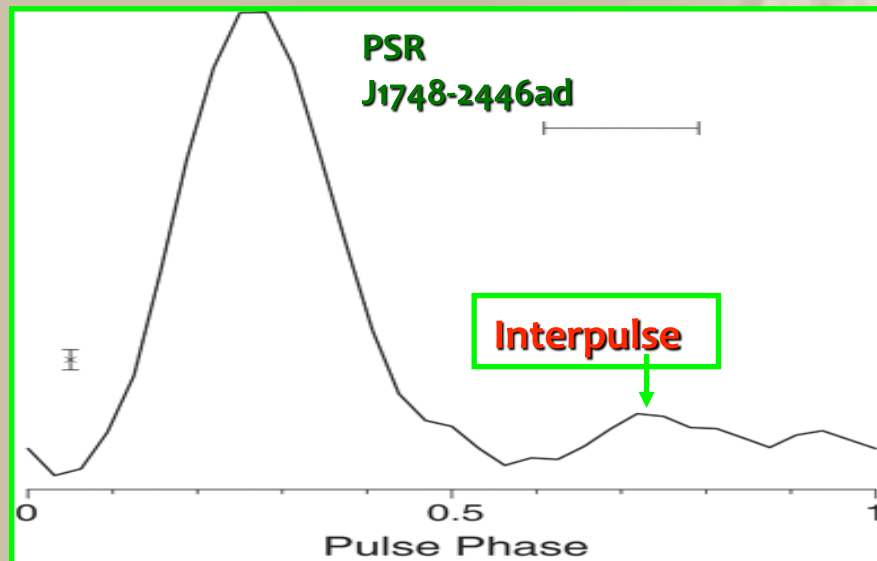
PSR J1748-2446ad
in the GC Terzan 5

$P=1.3959$ ms, $f_0=716.3$ Hz
Binary, circular orbit, $P_b = 1.09$ d
Eclipsed for $\sim 40\%$ of orbit

$m_{\text{comp}} > 0.14 M_{\odot}$



[© HST]



[Hessels et al. 2006]