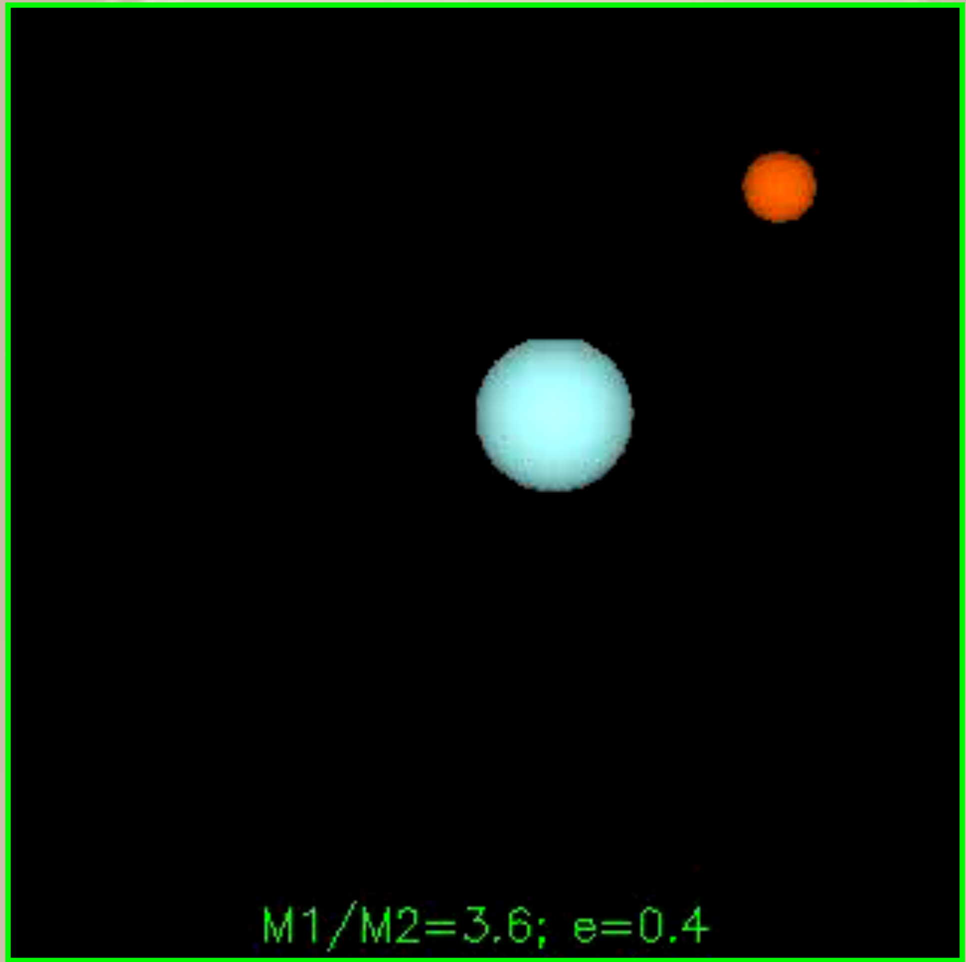
3.b

Pulsar Timing Concepts

(binary pulsars)

Since 1974 pulsars in binary systems are known



Timing idea: modeling

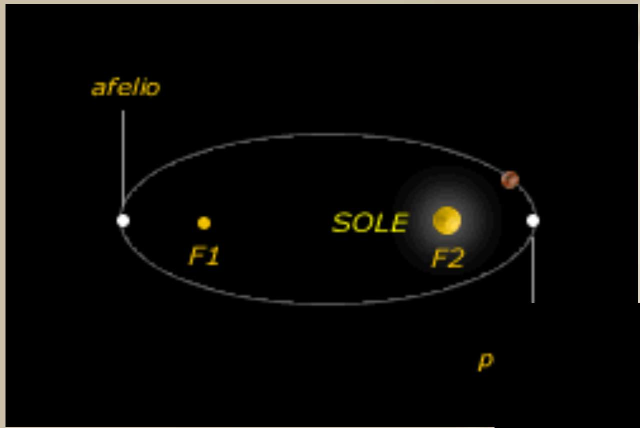
if a physical model adequately describes the systematic trends in the ToAs, it is applied with the smallest number of parameters

otherwise

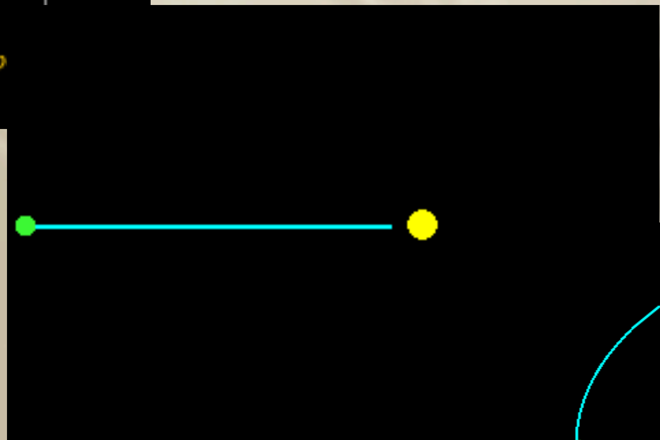
if a physical model is not adequate, it is extended (adding parameters) or rejected in favour of another model

when a model finally describes accurately the observed ToAs, the values of the model's parameters shed light onto the physical properties of the pulsar and/or of its environment

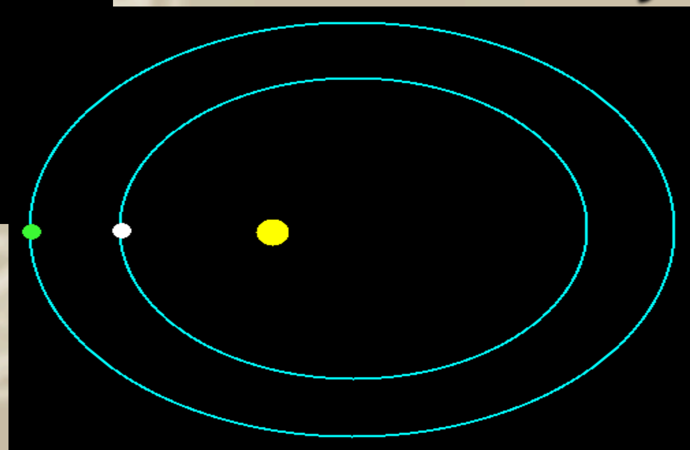
Binary systems: the classic laws



1] Elliptical and planar orbit



2] Constant areolar velocity



3] $a^3 = (G/4\pi^2)\, M_{tot}\, P^2$

Keplero (1609, 1609, 1619)

Correcting ToAs to the binary barycenter

The PULSARCENTRIC ToAs (i.e. ToAs expressed in pulsar proper time) must be corrected, calculating them **at the Pulsar System Barycenter** (PSB)

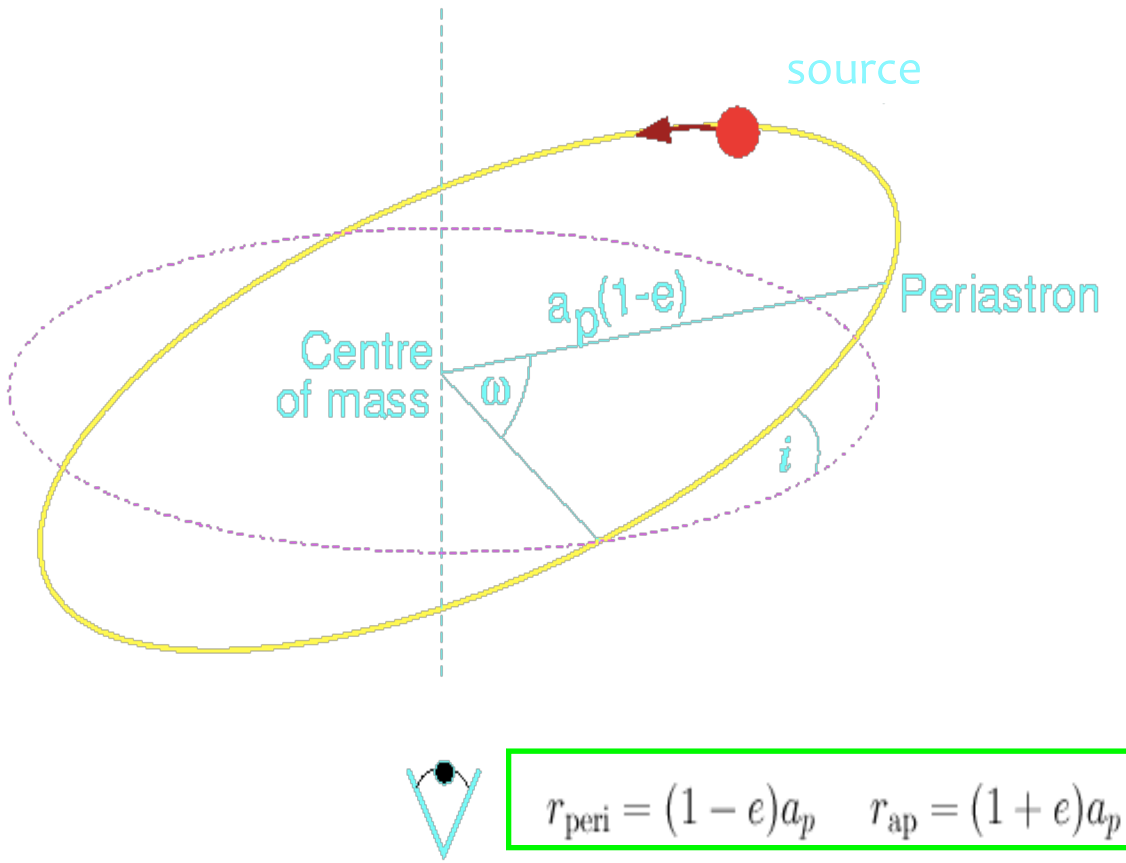
$$t_{PSR\text{-}BARY} = T_{psr} + \Delta_{R,b} + \Delta_{E,b} + \Delta_{S,b} + \Delta_{A}$$

$t_{PSR\text{-}BARY}$: Time at pulsar system barycenter
T_{psr} : Time in pulsar proper time (measured as at pulsar surface)
$\Delta_{R,b}$: Roemer delay (propagation delay) from pulsar to PSB
$\Delta_{S,b}$: Shapiro delay in pulsar binary
$\Delta_{E,b}$: Einstein delay in pulsar binary
Δ_{A} : Aberration delay due to pulsar rotation

Those terms contain various parameters of the binary system and thus a least-square fit to the residuals of a model including those parameters can allow to measure them…

Keplerian parameters

- P_b: Orbital period
- $x = a_p \sin i$: Projected semi-major axis
- ω: Longitude of periastron
- e: Eccentricity of orbit
- T_o: Time of periastron

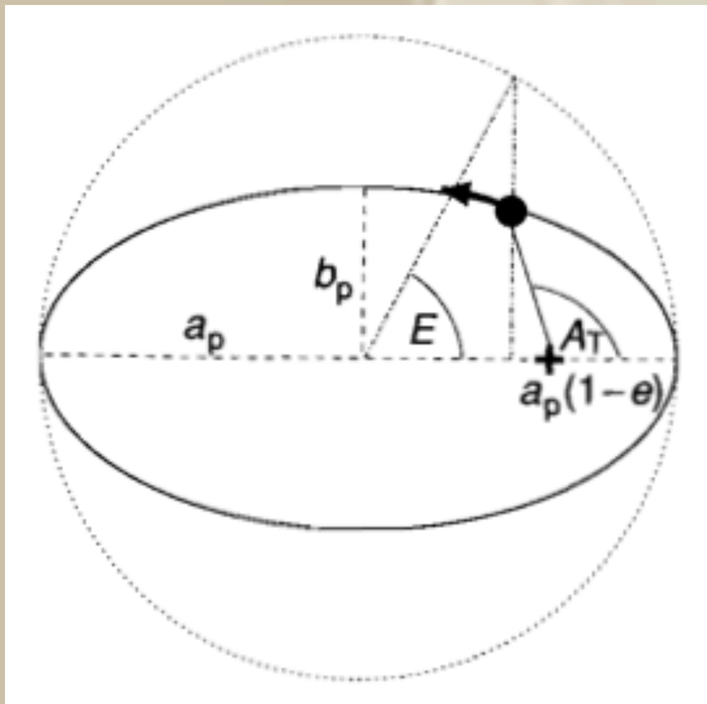


$$r_{\rm peri} = (1-e)a_p \qquad r_{\rm ap} = (1+e)a_p$$

Kepler equation (orbital time evolution in the simplest classical approach)

- **E: Eccentric Anomaly**
- **A_T: True Anomaly**

$$A_T(E) = 2\arctan\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right]$$



$$E - e\sin E = \frac{2\pi}{P_b}(t - T_0)$$

The binarycentering terms (Binary Roemer)

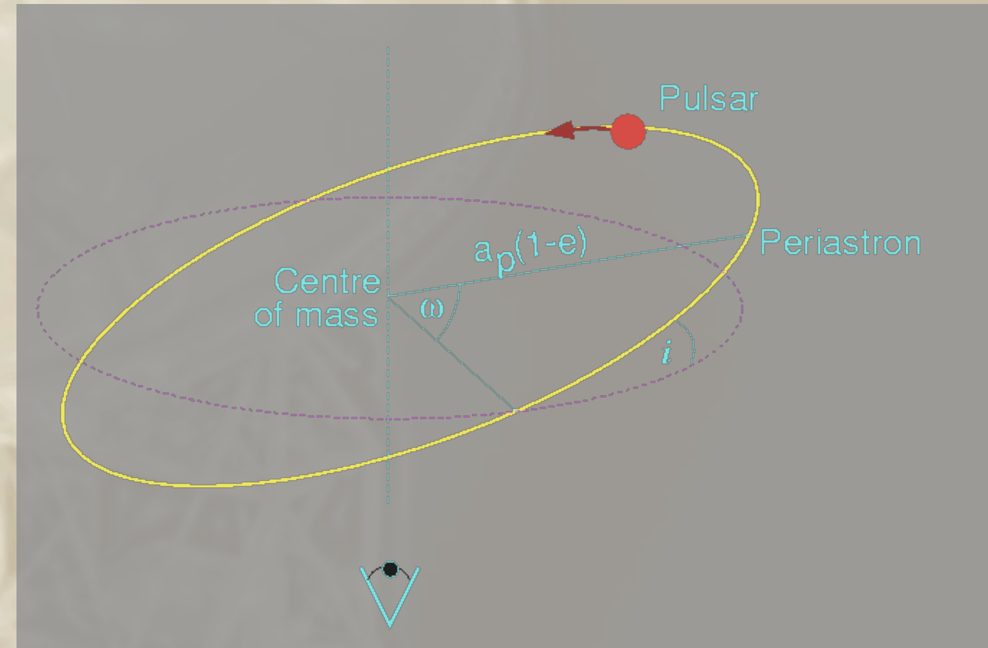
$$t_{BSB} = T_{source} + \Delta_{RB} + \Delta_{EB} + \Delta_{SB} + \Delta_{AB}$$

In classical approach

$$\Delta_{\rm RB} = x\,(\cos E - e)\sin\omega + x\sin E\,\sqrt{1-e^2}\,\cos\omega,$$

For most binaries, 5 keplerian parameters are measured and they are (well) enough to satisfactorily describe all the data

- P_b : Orbital period
- $x = a_p \sin i$: Projected semi-major axis
- ω : Longitude of periastron
- e : Eccentricity of orbit
- T_0 : Time of periastron



- Whence the pulsar mass function:

$$f(m_p, m_c) = \frac{4\pi^2}{G}\frac{(a_p \sin i)^3}{P_{orb}^2} = \frac{(m_c \sin i)^3}{(m_p + m_c)^2}$$

for $i = 90°$ Minimum companion mass

for $i = 60°$ Median companion mass

From the measurement of the orbital period P and of the projected orbital separation $x = a (\sin i)$ one can determine the minimum TOTAL MASS $M_{min} = M_{tot} (\sin i)^3$ of the binary system

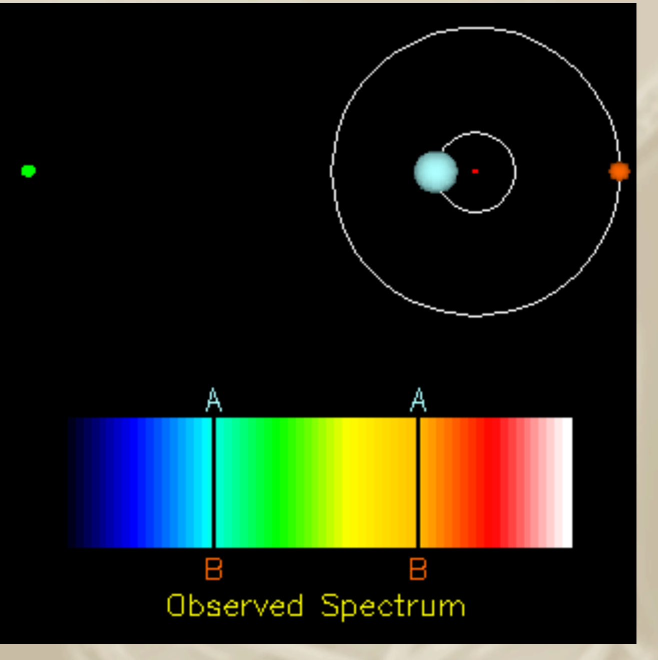
$$x^3 = (a \sin i)^3 = (G/4\pi^2)\, M_{tot}\, (\sin i)^3\, P^2 =$$

$$= (G/4\pi^2)\, M_{min}\, P^2$$

The measured minimum TOTAL MASS being obviously related to the sum of the masses of the two involved stars

$$M_{min} = M_{tot} (\sin i)^3 = (M_1 + M_2)(\sin i)^3$$

… if the companion can be detected in the optical band the spectroscopic observation of the cyclic motion of its spectral lines allows one to determine the MASS FUNCTION OF THE COMPANION



… and that leads to establish the ratio of the masses

M_1 / M_2

By combining the two constraints

$$M_{min} = (M_1 + M_2)\,(\sin i)^3$$

$$q = M_1 / M_2$$

one can infer M_1 and M_2

only modulo the uncertainty on sin i

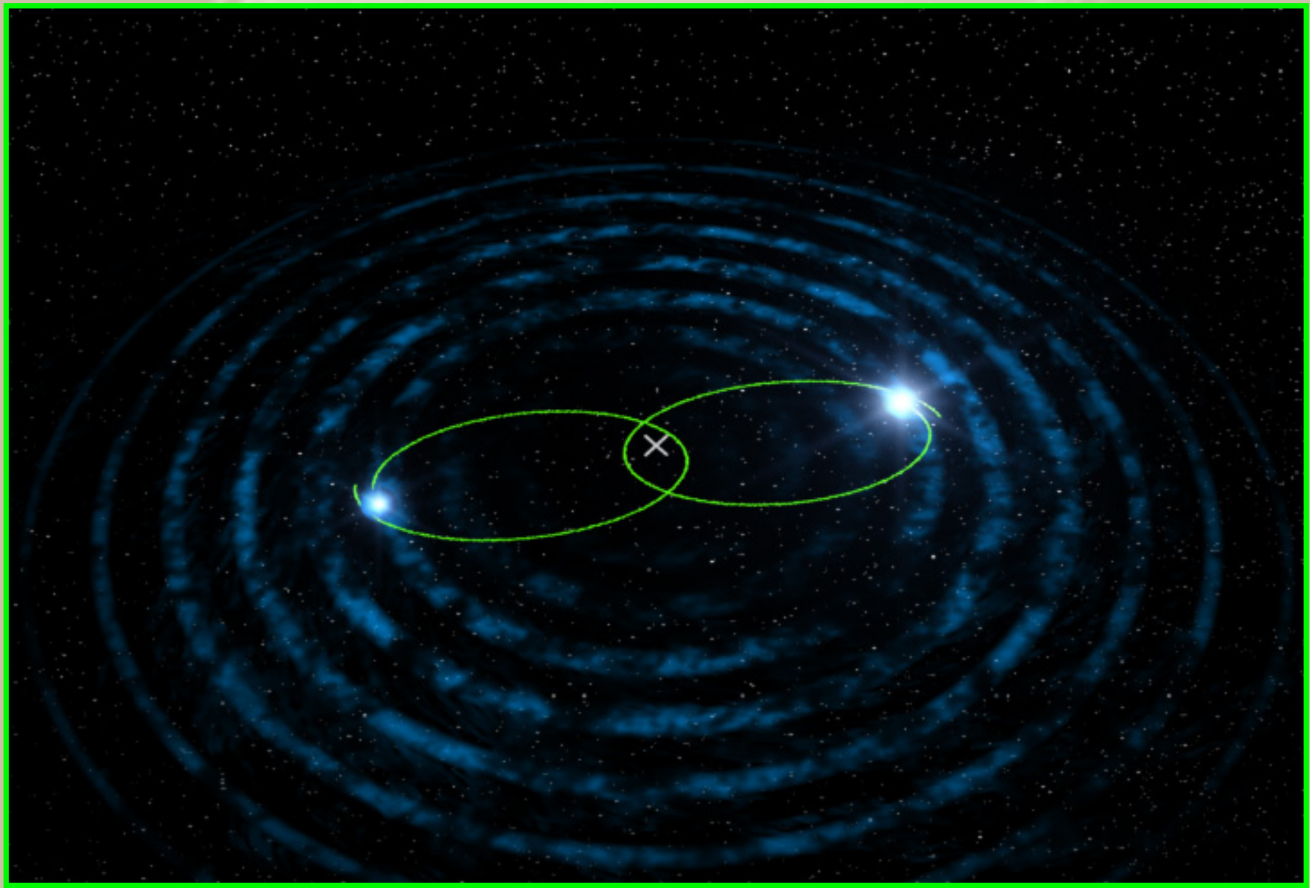
Pulsar Timing: relativistic pulsars

periastron precession

Relativistic effect

The modification in the shape of the orbit

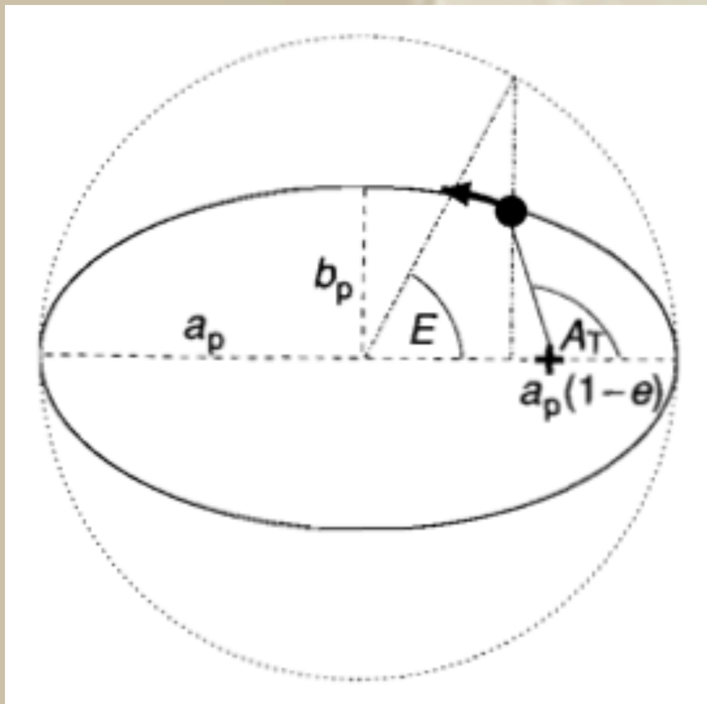
orbital decay



Kepler equation (orbital time evolution with two post-Keplerian parameters)

- **E**: Eccentric Anomaly
- **A_T**: True Anomaly

$$A_T(E) = 2\arctan\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right]$$



- **dP_b/dt**: Orbital period derivative
- **$d\omega/dt$**: Longitude of periastron time deriv

(may be classical or relativistic effects)

$$E - e\sin E = \frac{2\pi}{P_b}\left[(t-T_0) - \frac{1}{2}\frac{\dot{P}_b}{P_b}(t-T_0)^2\right]$$

$$\omega = \omega_0 + \frac{\dot{\omega}P_b}{2\pi}A_T(E)$$

The binarycentering terms
(Binary Roemer, relativistic correction)

$$t_{BSB} = T_{source} + \Delta_{RB} + \Delta_{EB} + \Delta_{SB} + \Delta_{AB}$$

In classical approach

$$\Delta_{RB} = x\,(\cos E - e)\sin\omega + x\sin E\,\sqrt{1-e^2}\,\cos\omega,$$

In a relativistic approach

$$\Delta_{RB} = x\,(\cos E - e_r)\sin\omega + x\sin E\,\sqrt{1-e_\theta^2}\,\cos\omega,$$

where the new eccentricities

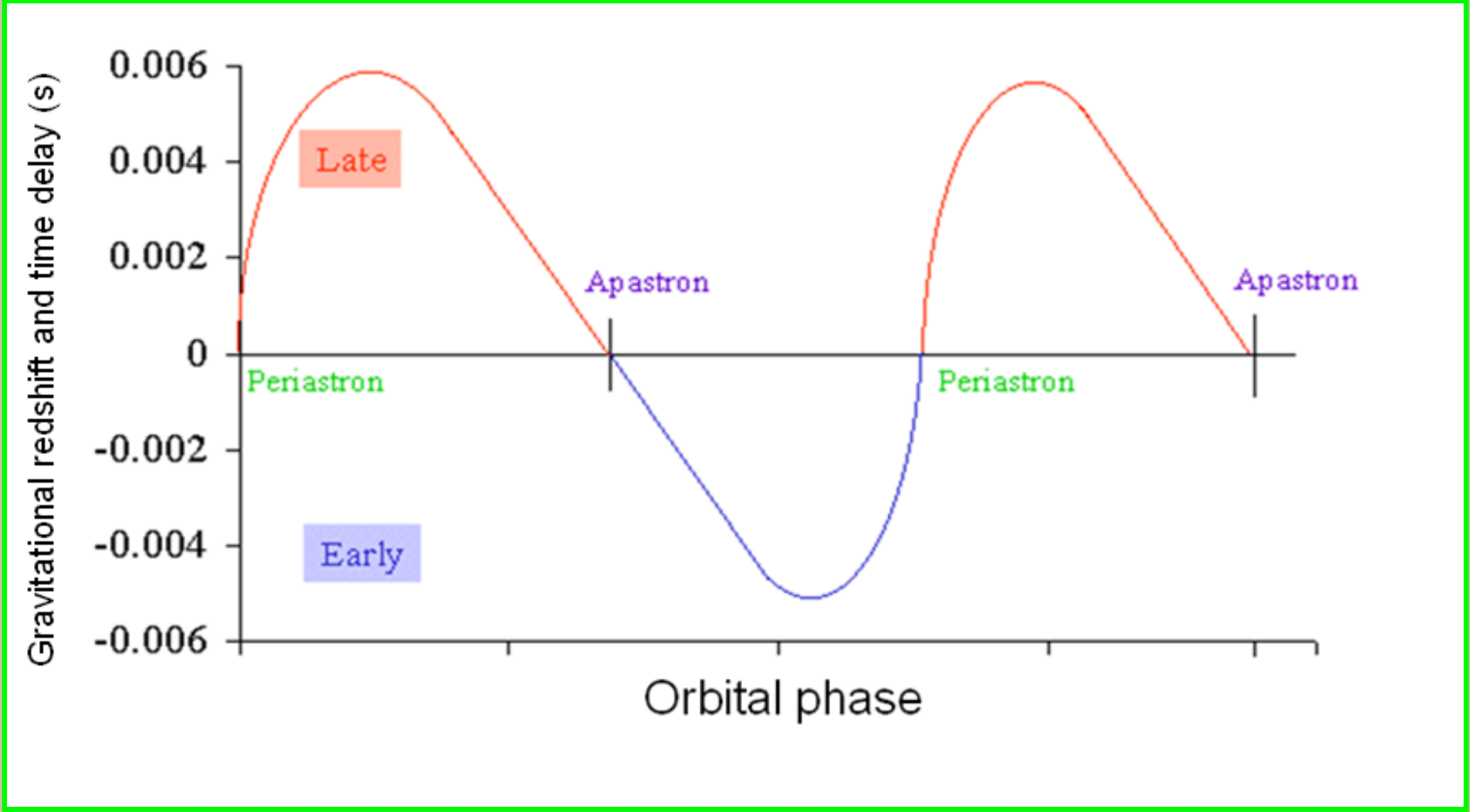
$$e_r \equiv e\,(1+\delta_r) \qquad (8.37)$$

$$e_\theta \equiv e\,(1+\delta_\theta) \qquad (8.38)$$

Einstein correction

The modification in the time of arrival of the pulses

gravitational redshift and time dilation



The binarycentering terms
(Einstein correction)

$$t_{BSB} = T_{source} + \Delta_{RB} + \Delta_{EB} + \Delta_{SB} + \Delta_{AB}$$

The effect (evident in elliptical orbits) is due to the varying distance of the companion (gravitational red-shift) and varying speed of the source along the orbit (time dilation)

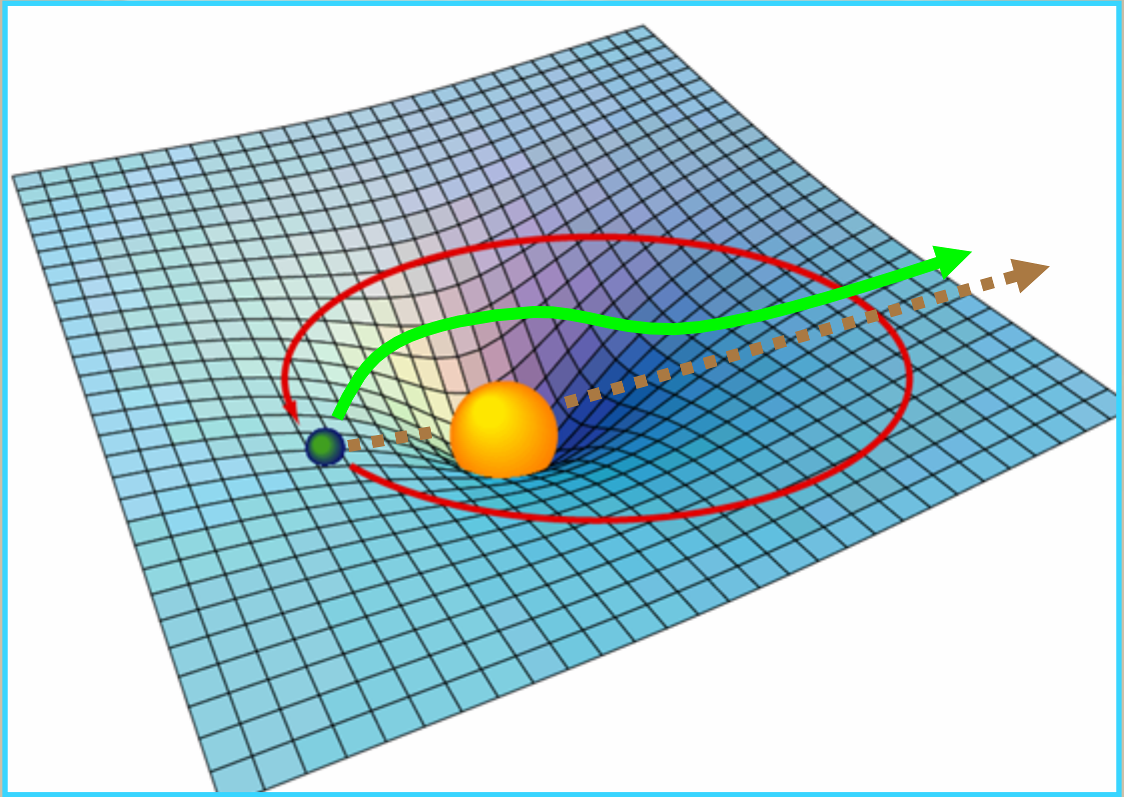
$$\Delta_{EB} = \gamma \sin E,$$

Where : γ post-Keplerian parameter gamma

Relativistic parameters

The modification in the time of arrival of the pulses

Shapiro delay



The binarycentering terms
(Shapiro delay correction)

$$t_{BSB} = T_{source} + \Delta_{RB} + \Delta_{EB} + \Delta_{SB} + \Delta_{AB}$$

The effect (evident in almost edge-on orbits) is due to the curvature of the space time in the surroundings of the companion

$$\Delta_{SB} = -2r \ln\left[1 - e\cos E - s\left(\sin\omega(\cos E - e) + \sqrt{1-e^2}\cos\omega\sin E\right)\right],$$

where the maximum is expected for superior conjunction
$\Phi = \omega + A_T(E) = \pi/2.$

Where : **r** post-Keplerian parameter Range of the Shapiro Delay

s post-Keplerian parameter Shape of the Shapiro Delay

For a small eccentricity binary (with Φ : orbital phase from ascending node)

$$\Delta_{SB} = -2r \ln[1 - s\sin\Phi],$$

The binarycentering terms
(Aberration delay correction)

$$t_{BSB} = T_{source} + \Delta_{RB} + \Delta_{EB} + \Delta_{SB} + \Delta_{AB}$$

The effect is also known in classical physics and is caused by the motion of the source. However, it is almost degenerate with other PK parameters, so very hard to measure

$$\Delta_{AB} = A\left[\sin\left[\omega + A_T(E)\right] + e\sin\omega\right] + B\left[\cos\left[\omega + A_T(E)\right] + e\cos\omega\right].$$

Where : **A** first aberration parameter

B second aberration parameter

High precision pulsar timing: which targets?

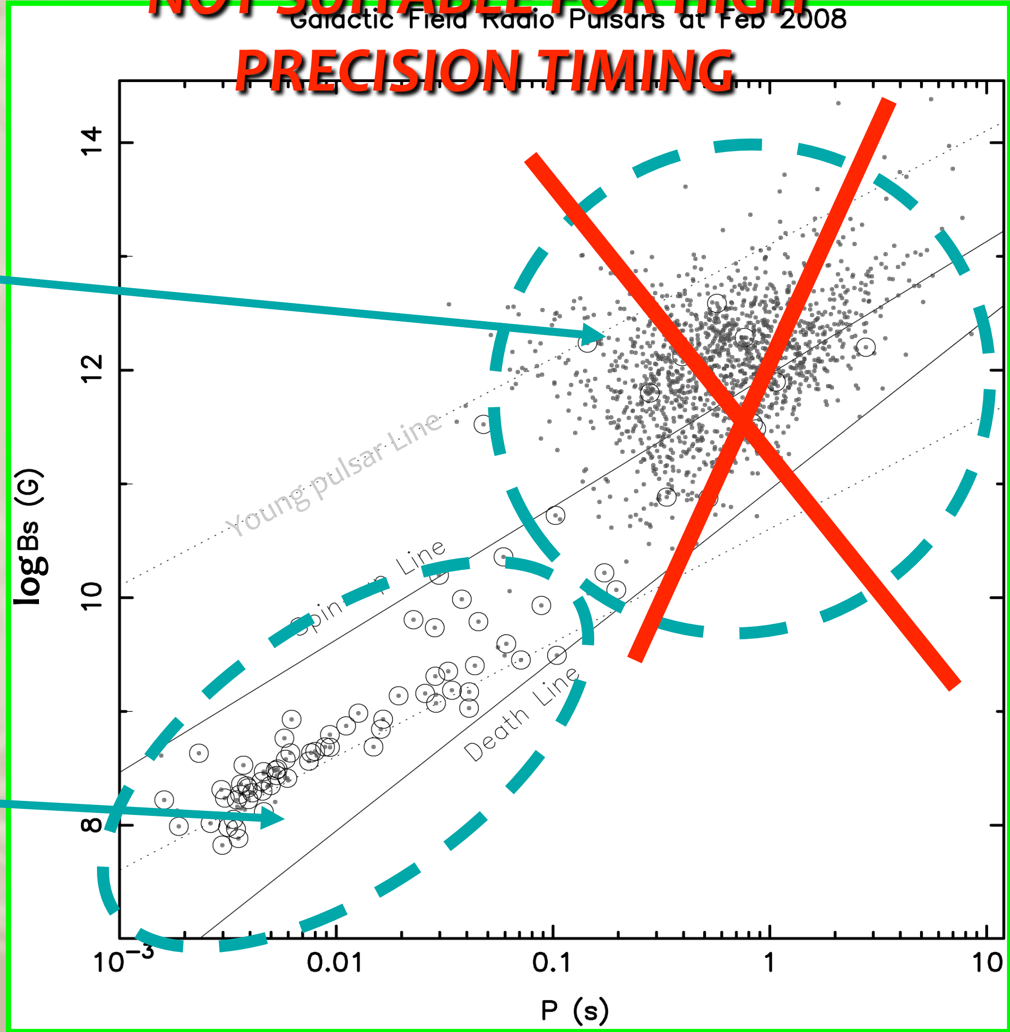
Ordinary pulsars:
~ 1900 known objects;
NS_{age} < few 10^7 yr

relatively long pulses & rotational irregularities

Recycled pulsars:
~ 250 known objects;
NS_{age} > 10^8-10^9 yr

The most rapidly rotating are known as millisecond pulsars

NOT SUITABLE FOR HIGH PRECISION TIMING



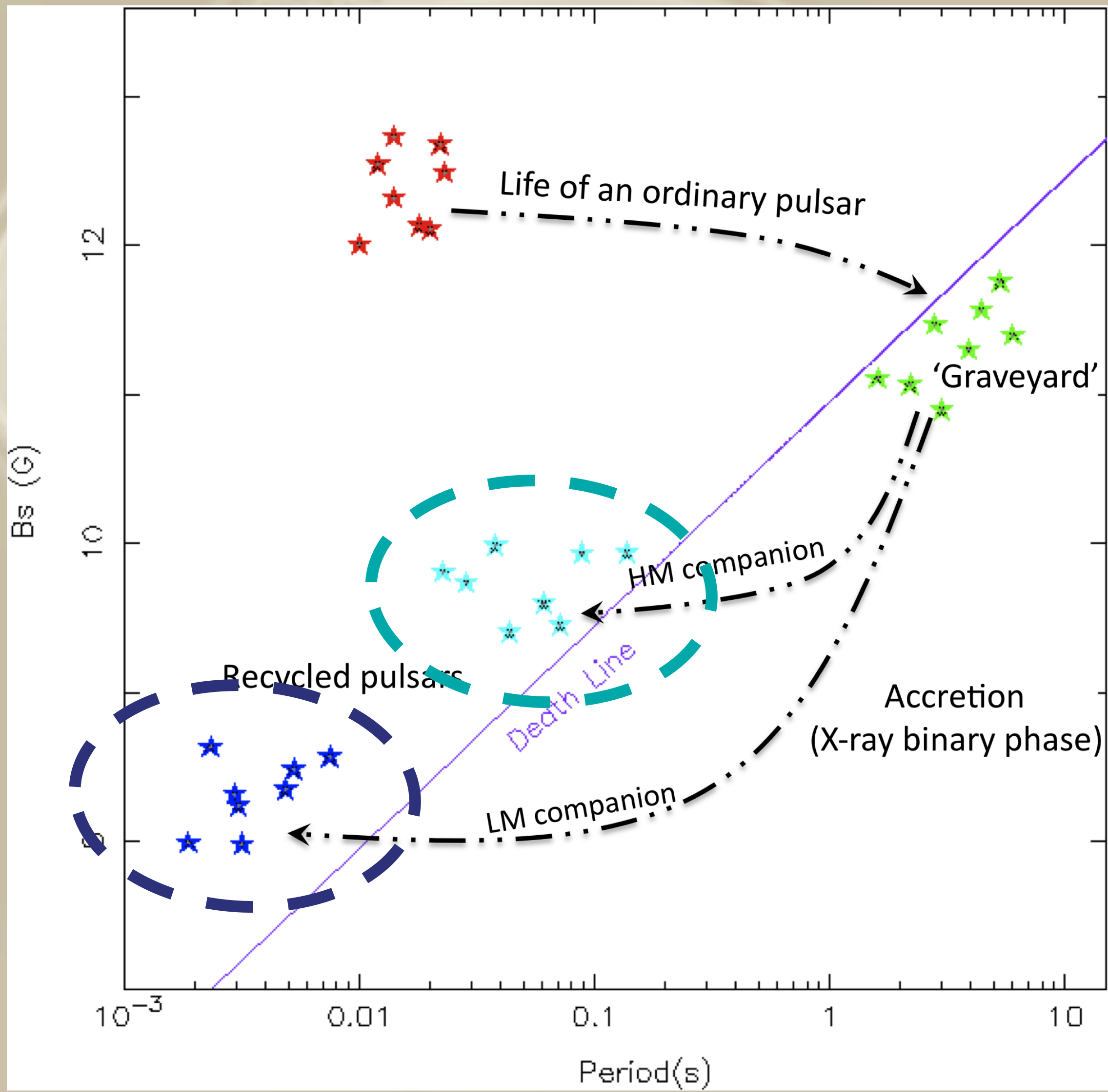
ATNF Pulsar Catalogue

Best timers: which evolution ?

Mildly recycled: e.g. double neutron stars

Fully recycled: in most cases

MSP+WD



[@ Burgay 2011]

Millisecond pulsars (MSPs) as clocks

Pulsar periods can sometimes be measured with unrivalled precision

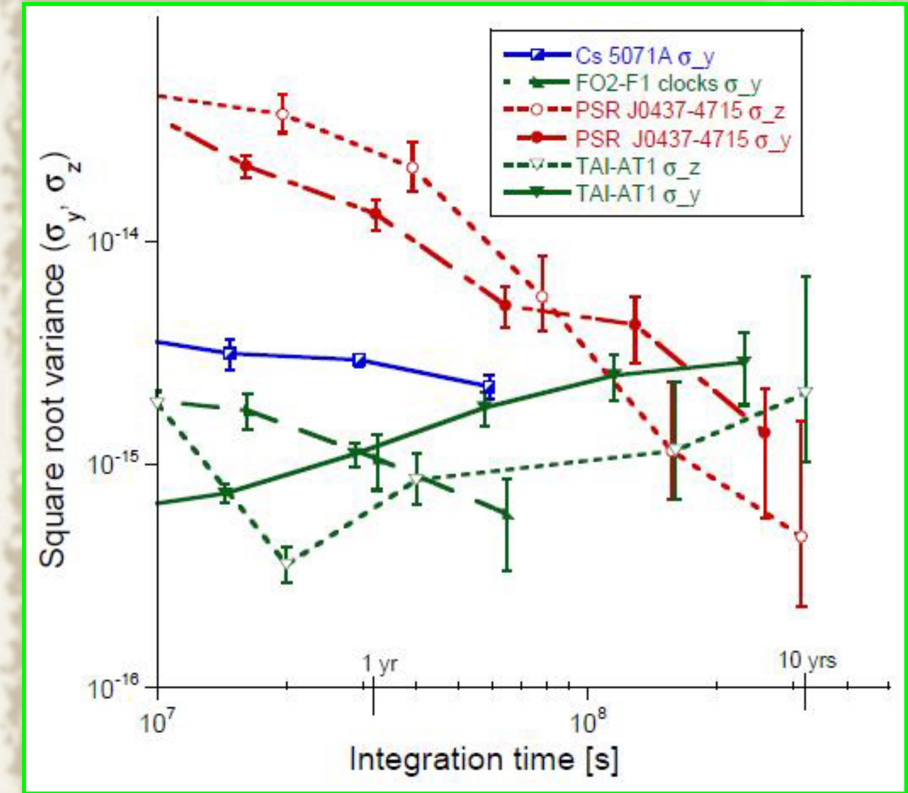
e.g. on Jan 16, 1999 at UT=00:00

PSR J0437-4715 (a MSP+WD) had a period of

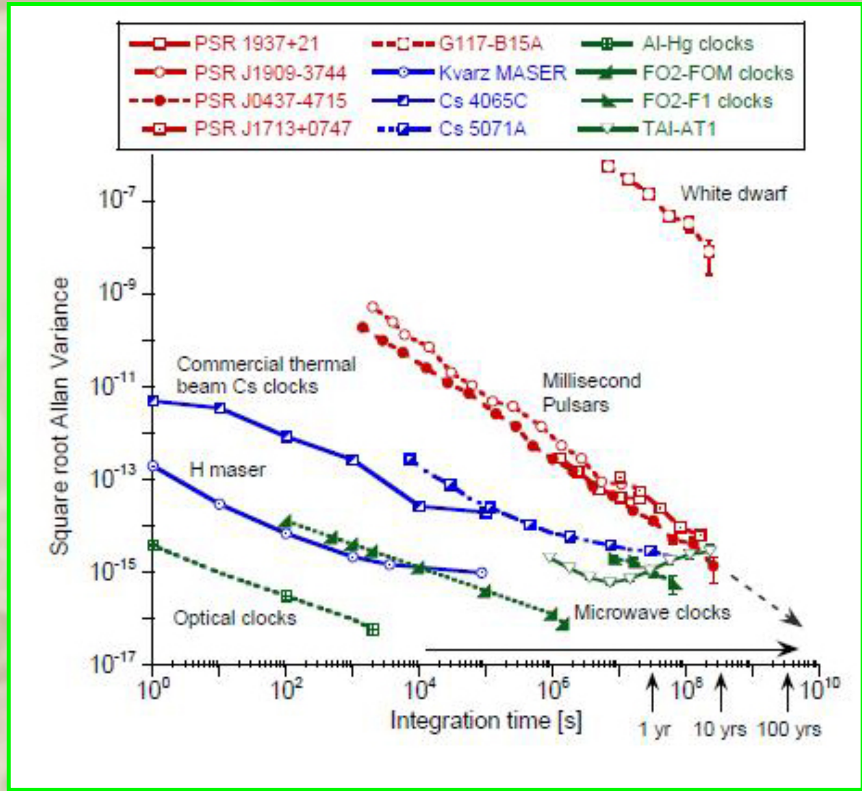
5.757451831072007 ± 0.000000000000008 ms

16 significant digits!

Atomic clocks vs pulsar timing

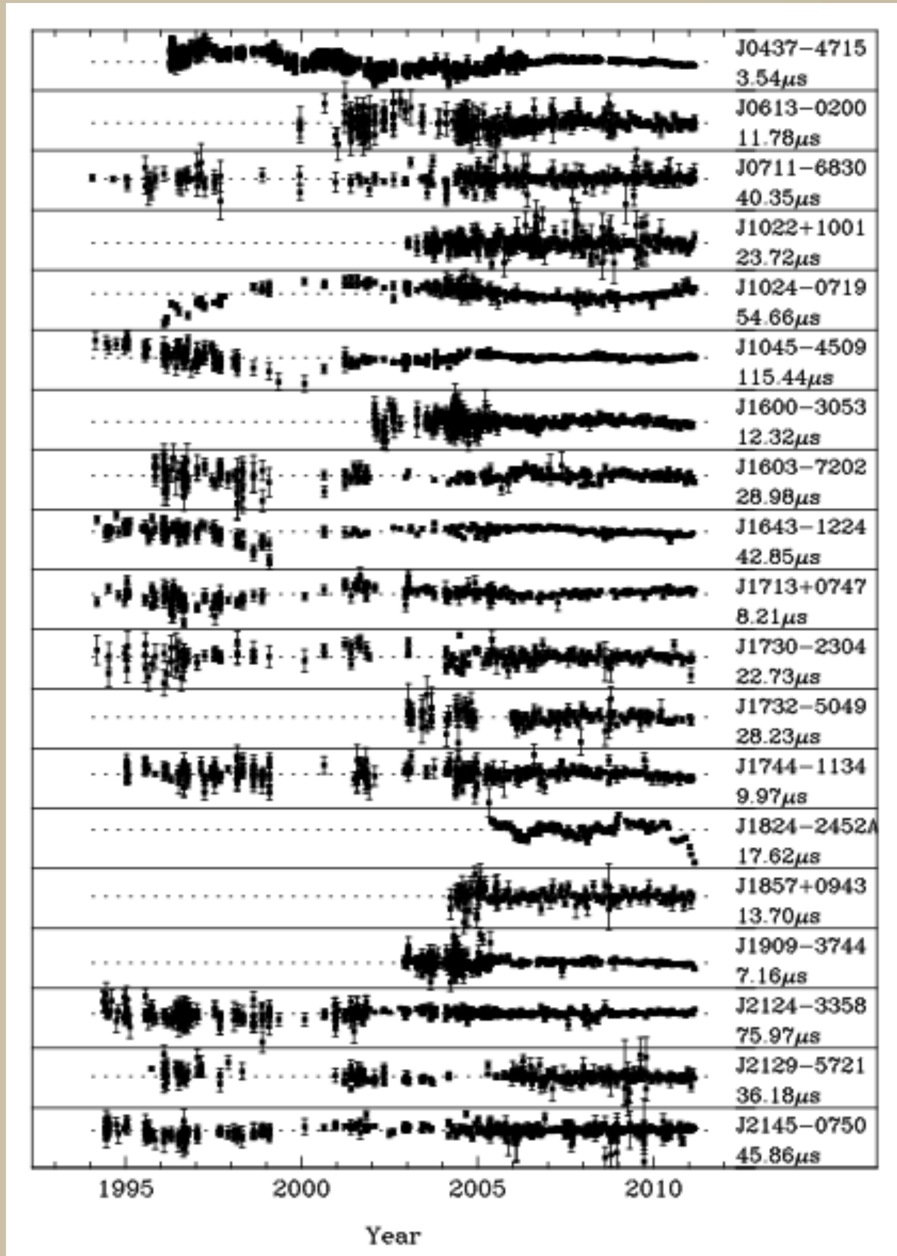


[Hartnett & Luiten 2010]



Unfortunately only a subsample of the recycled pulsars is able to achieve such a rotational stability

The majority of the ordinary pulsars undergo timing irregularities



[Hobbs at al 2012]

A ensemble of recycled pulsars with high rotational stability can be used for establishing a

time scale based on pulsar rotation dubbed e.g.

Ensemble Pulsar Scale

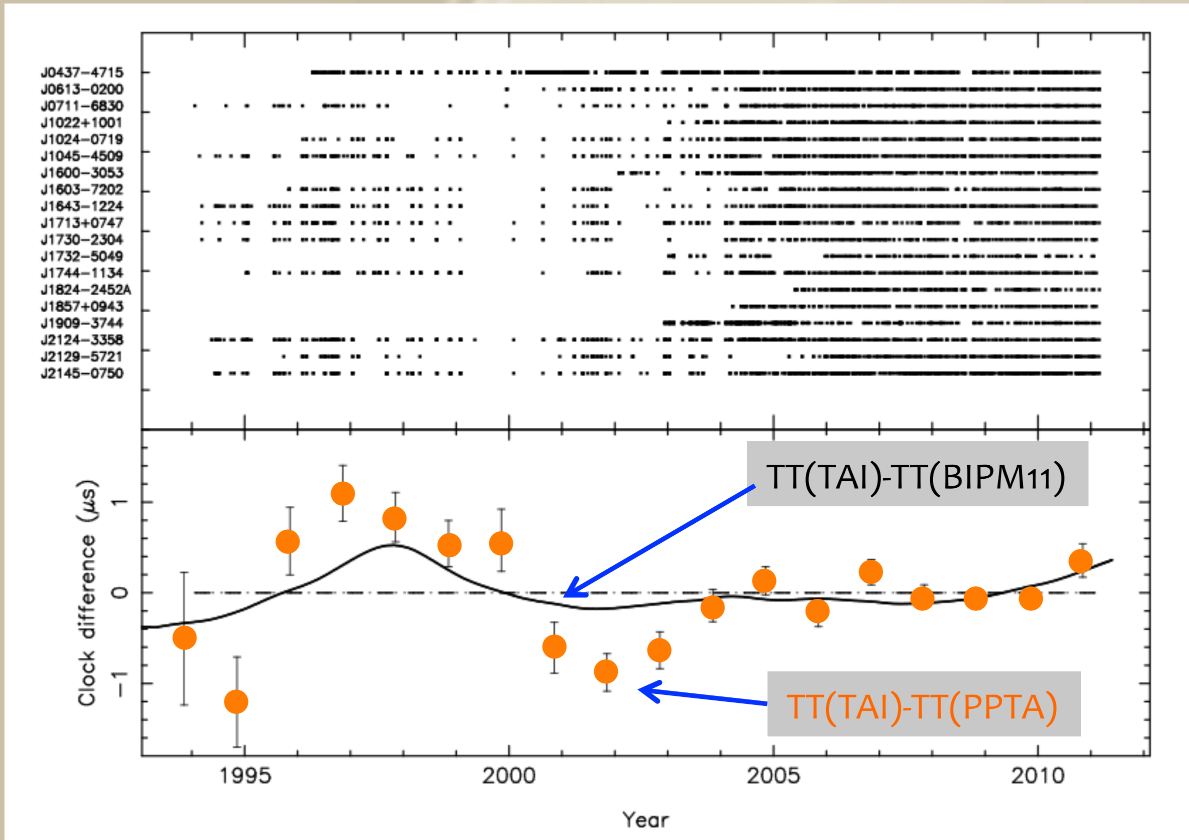
...similar to the Échelle Atomique Libre (EAL)

from which the International Atomic Time (TAI) is built (after applying corrections to satisfy the SI definition of second)

and in turn a realization of the Terrestrial Time is obtained TT(TAI) (by referencing clocks to the Earth geoid)

as well as the post-corrected realization of TT published each year and known as TT(BIPMyy)

The Ensemble Pulsar Scale (ESP) can be linked ("steered") to e.g. TT(TAI) [or to TT(BIPMyy)] and then fluctuations of the reference time scale wrt ESP can be spotted and used for creating a pulsar based realization of the terrestrial time TT(PTAs)



- Independent check on TT(BIPMyy) with a non terrestrial "device"
- Not based on quantum effects on clocks
- Never ending time scale

[Hobbs at al 2012]