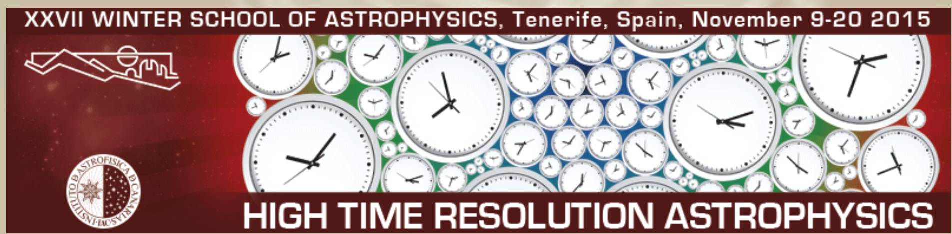
3.a

Pulsar Timing Concepts

(isolated pulsars)



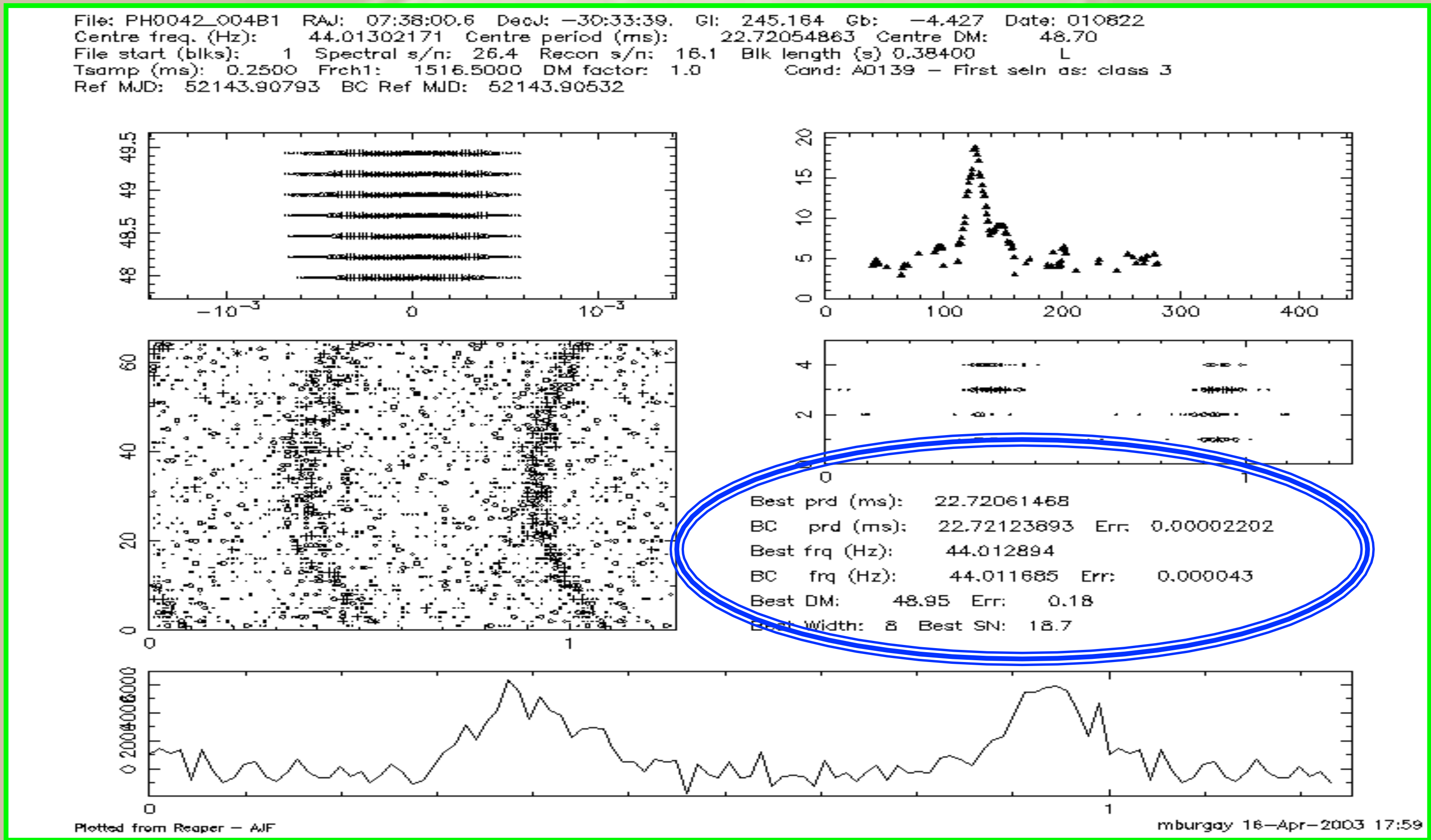
Starting parameters of a just discovered radio pulsar

Approximate Celestial Coordinates RAJ, DECJ with typical ≈ few arcmin uncertainty

Approximate Spin Period P
with typical ≈ 1-100 nsec uncertainty

Approximate Dispersion Measure DM
with typical ≈ $10^{-4} - 10^{-3}$ uncertainty

A famous case: *The discovery plot of the double pulsar PSR J0737-3039A/B*



Timing idea: observations

Performing repeated observations of the times of occurrence (often referred as Times of Arrival, ToAs) of a given repetitive event with respect to an assigned system of reference

and

searching the Times of Arrival for systematic trends on many different timescales, from minutes to decades

Timing idea: modeling

if a physical model adequately describes the systematic trends, it is applied with the smallest number of parameters

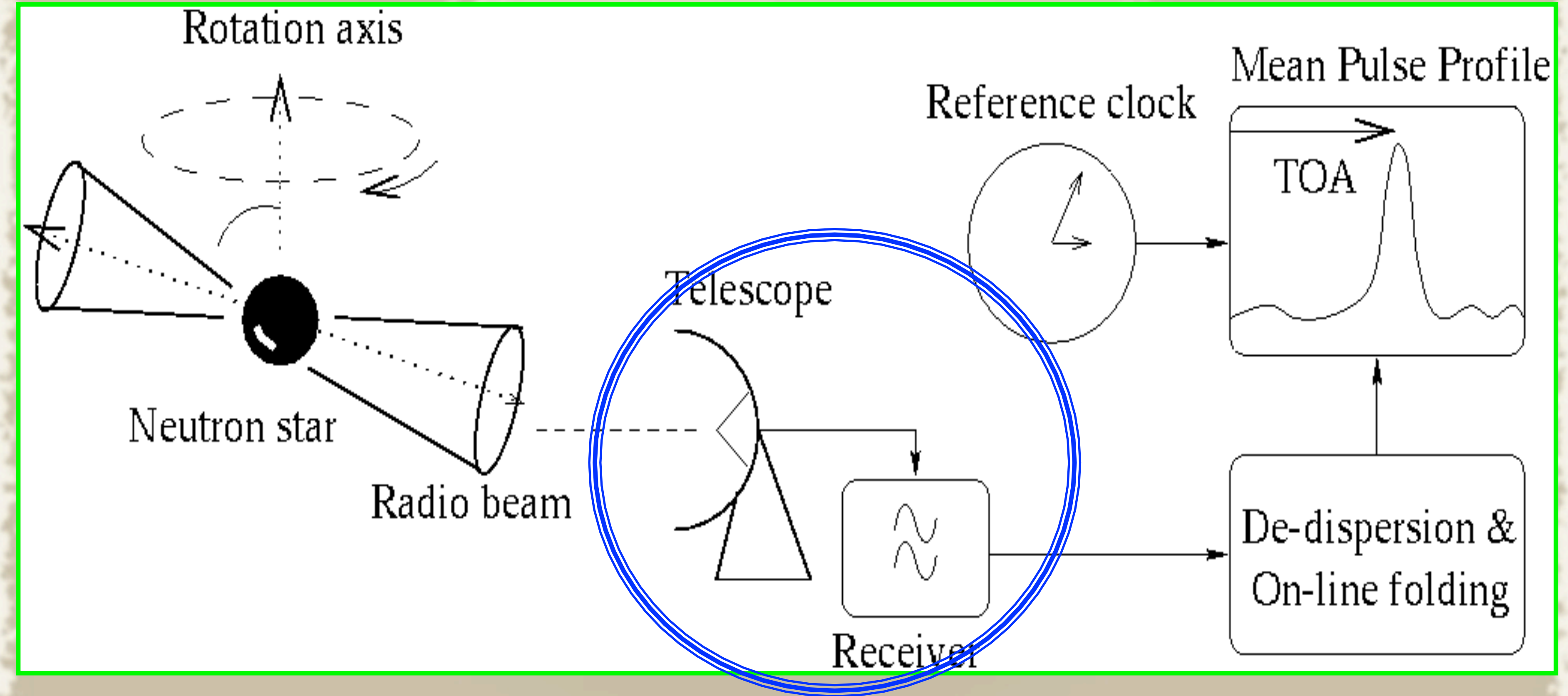
otherwise

if a physical model is not adequate, it is extended (adding parameters) or rejected in favor of another model

↓

when a model finally describes accurately the observed ToAs, the values of the **model's parameters shed light onto the physical properties** of the pulsar and/or of its environment

Timing of a radio pulsar



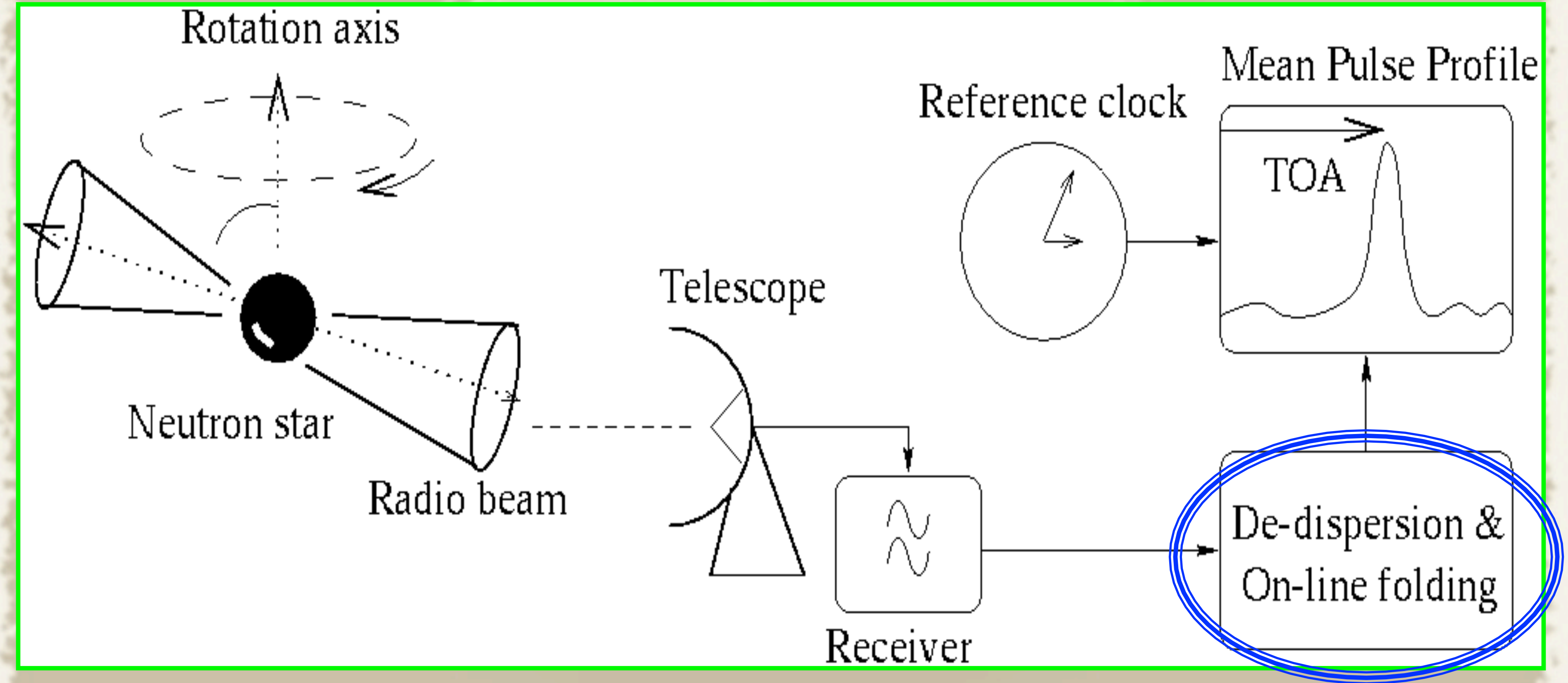
@ Lorimer

Acquisition of a time series

digitization @ 1 or 2 or 4 or 8 or 16 bits

07346100374221775320153201532110233030367162

Timing of a radio pulsar



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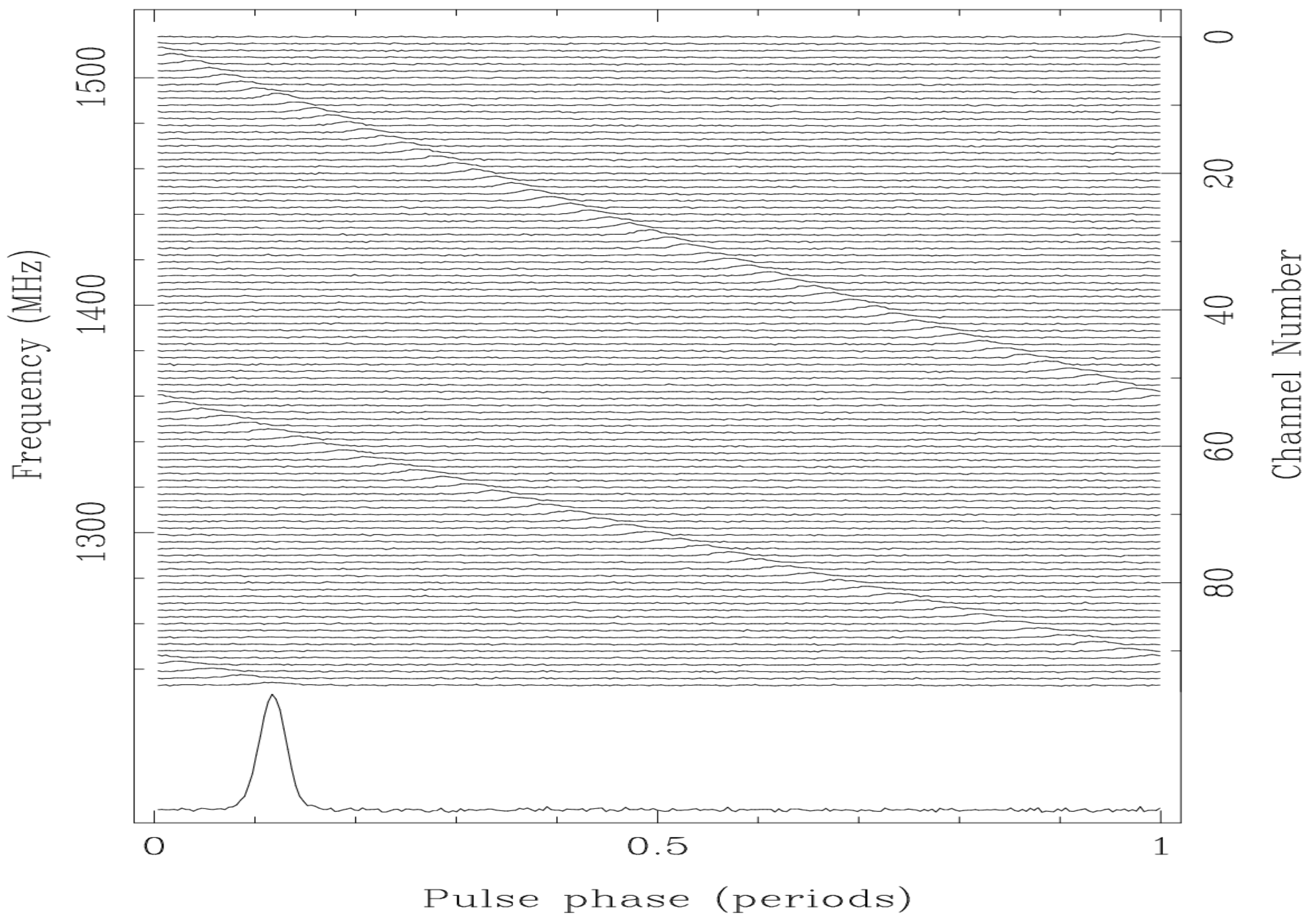
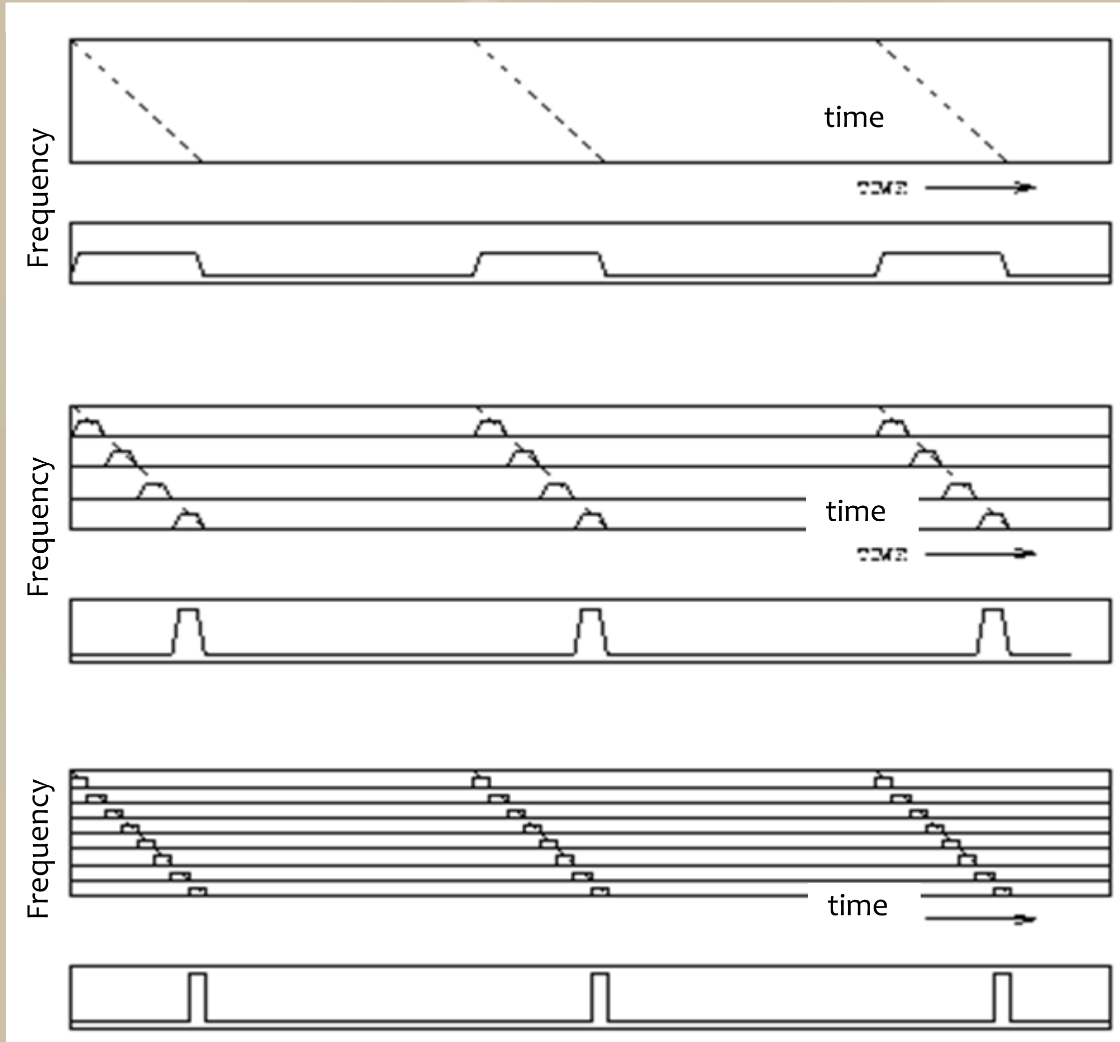
The dedispersion



Dispersion smearing



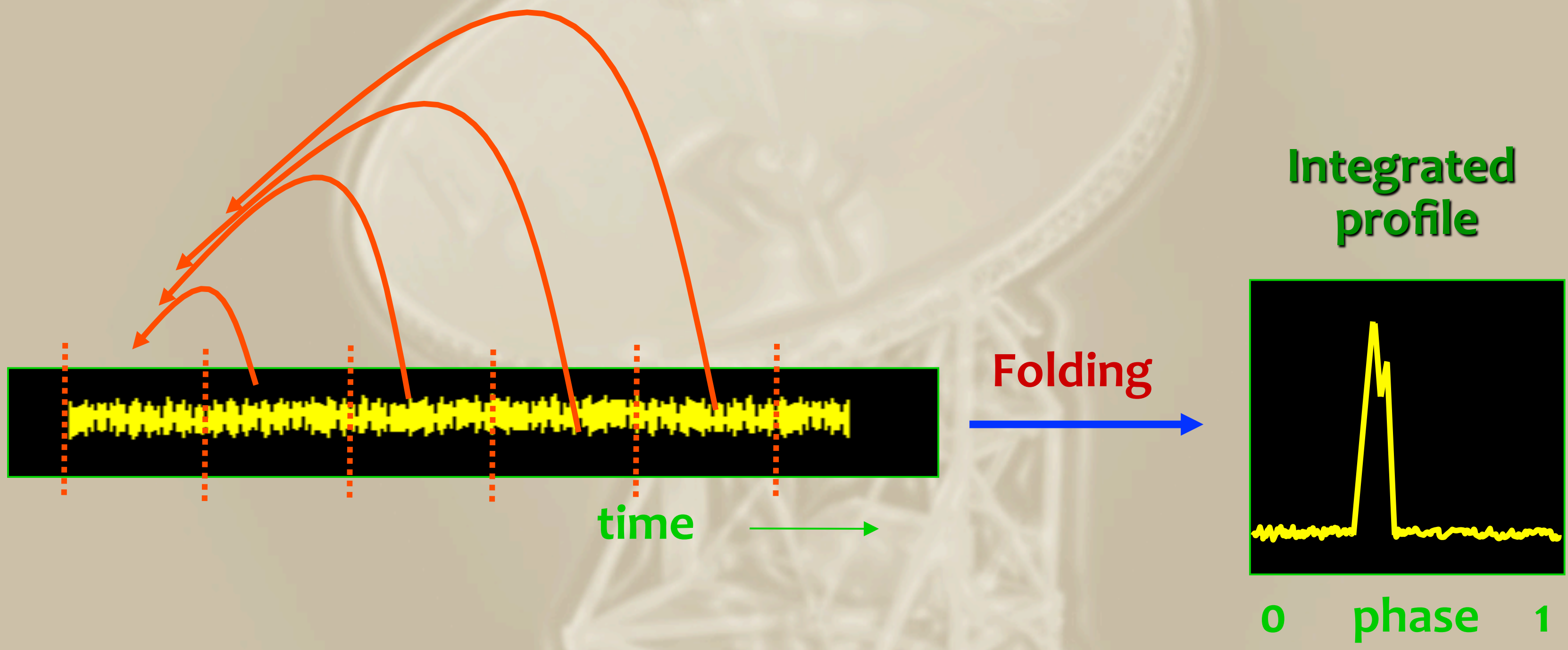
DM in pc/cm^3

$$\delta t_{DM} = \frac{DM}{1.2\ 10^{-4}} \frac{\delta\nu}{\nu^3}$$

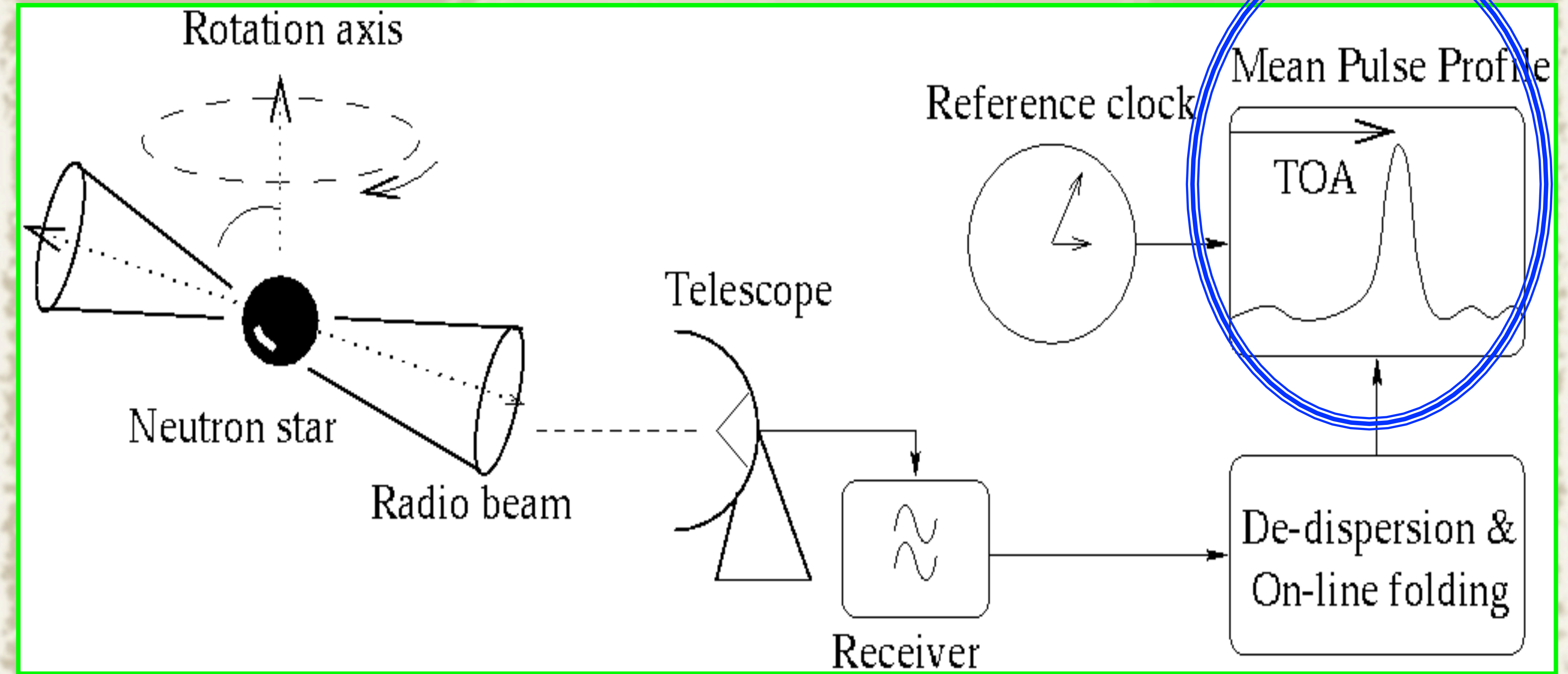
430 MHz → 100 μs / DM / MHz

1400 MHz → 3 μs / DM / MHz

The folding



Timing of a radio pulsar



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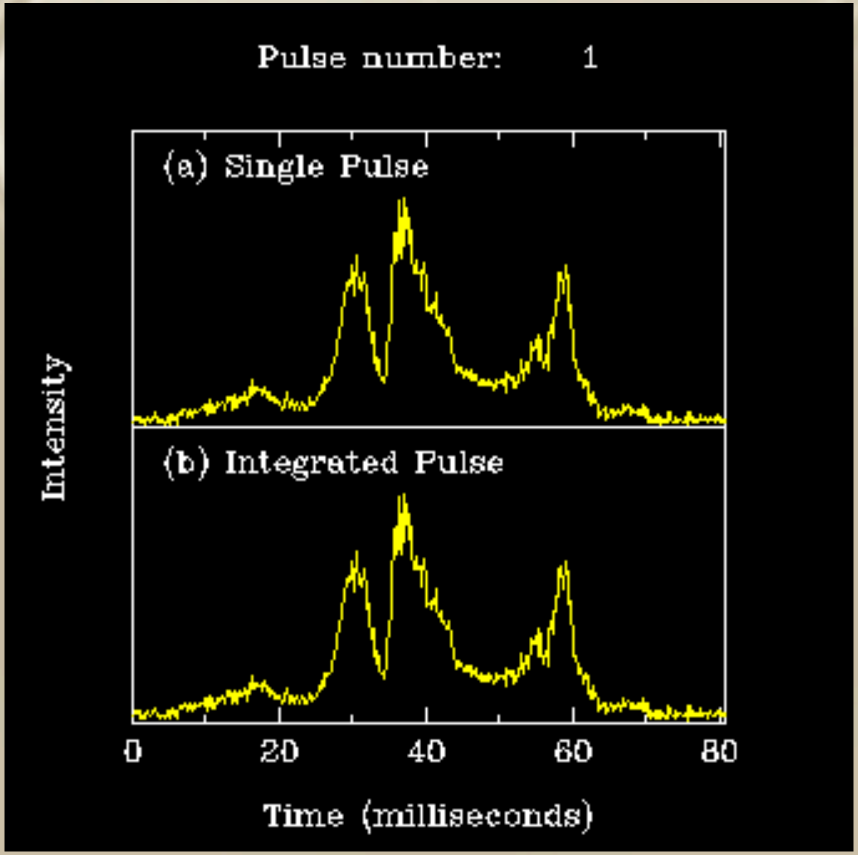
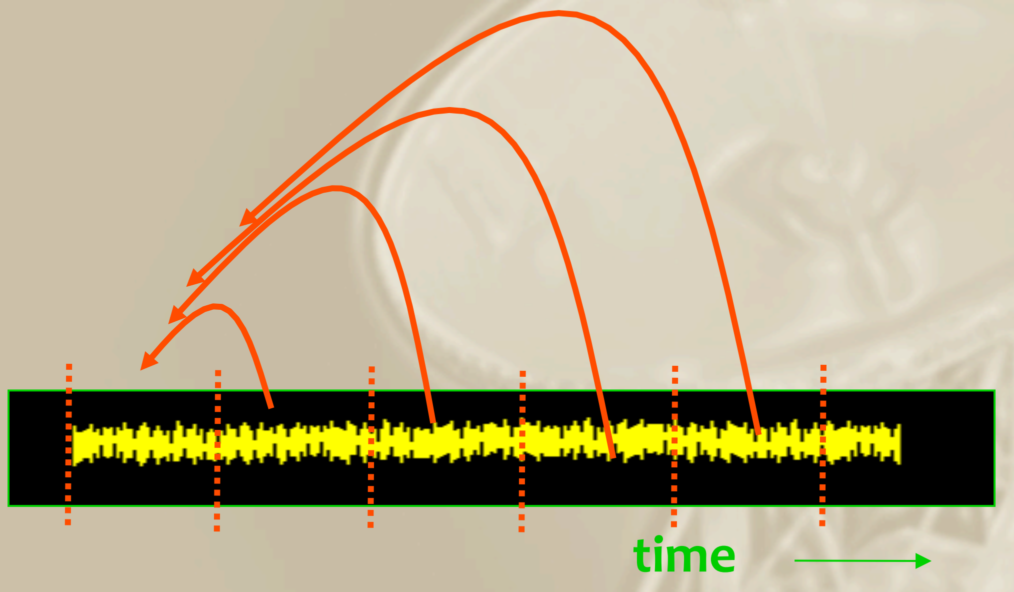
Single pulse profile vs integrated profile



The folding



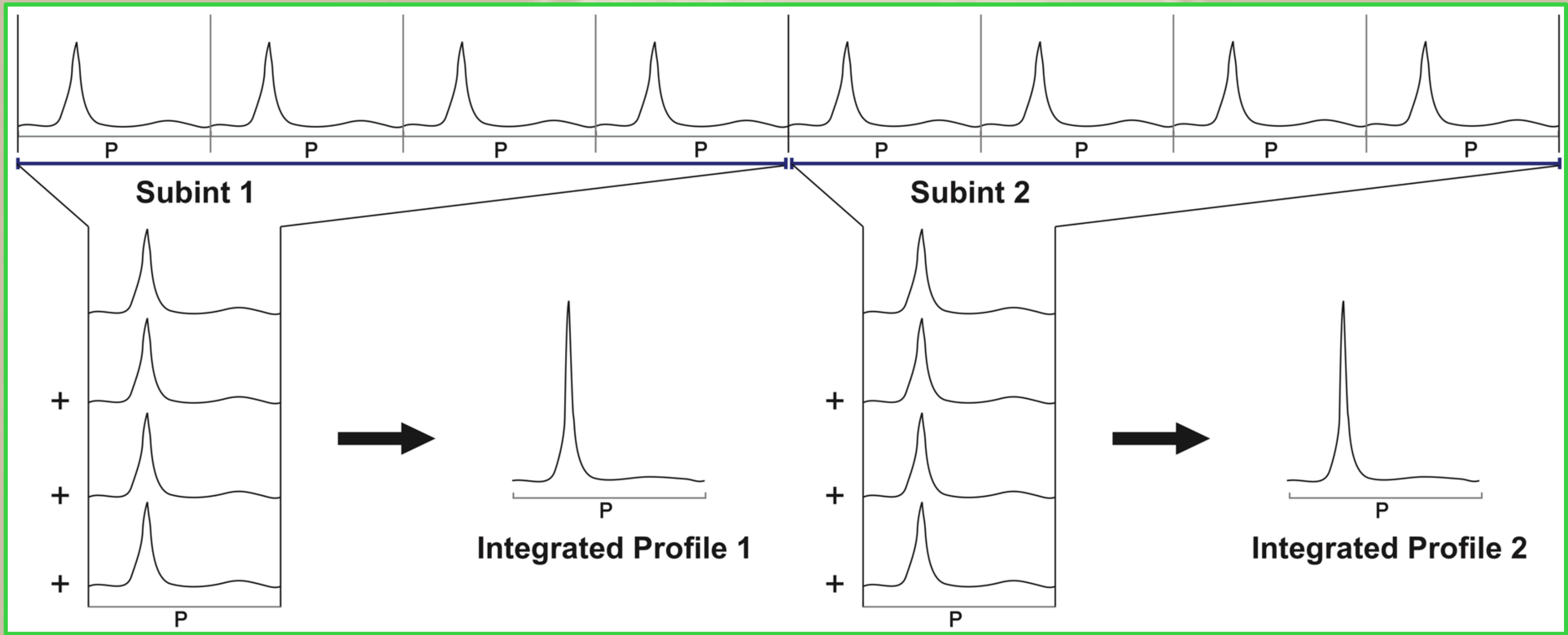
Folding

if particularly stable

and having a high S/N

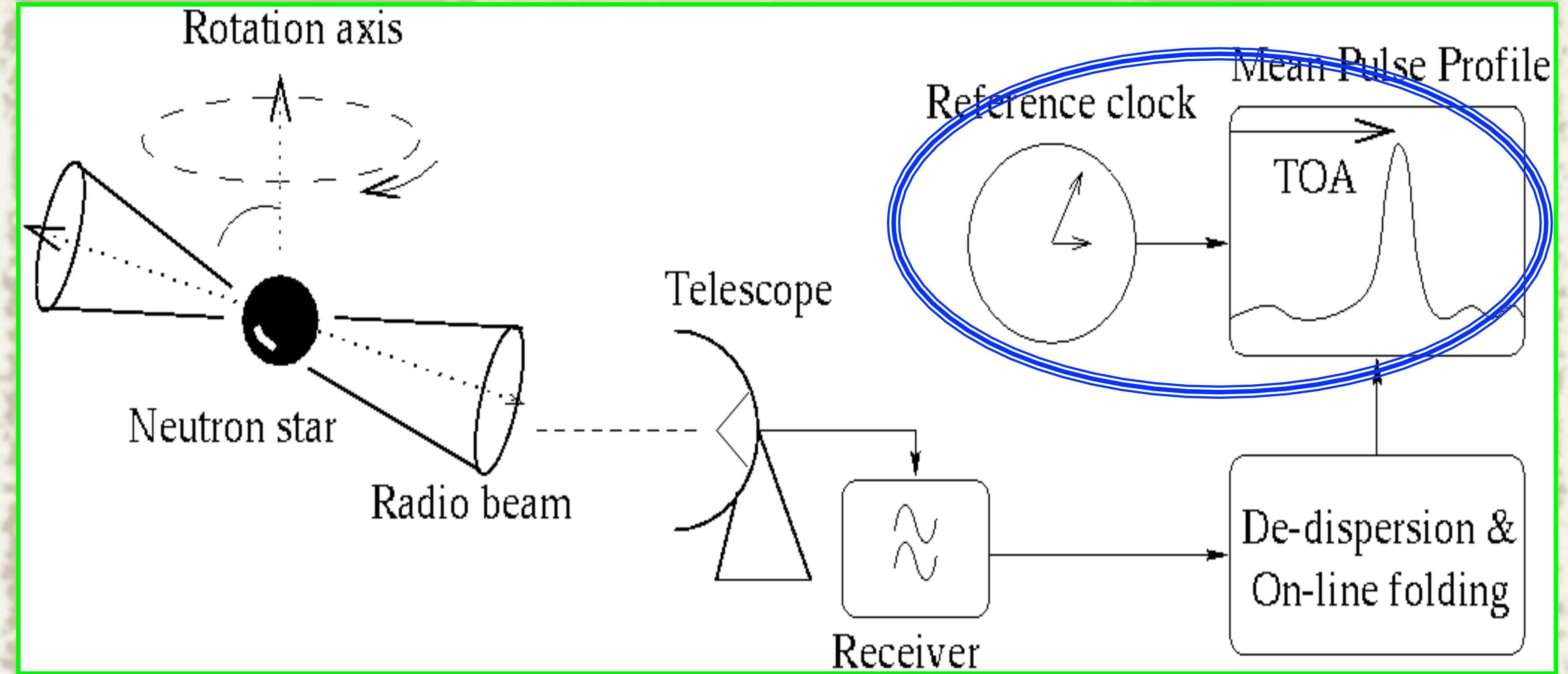
Standard profile = template for calculating ToAs

Folding in Sub-integrations



@ Ridolfi

Timing of a radio pulsar



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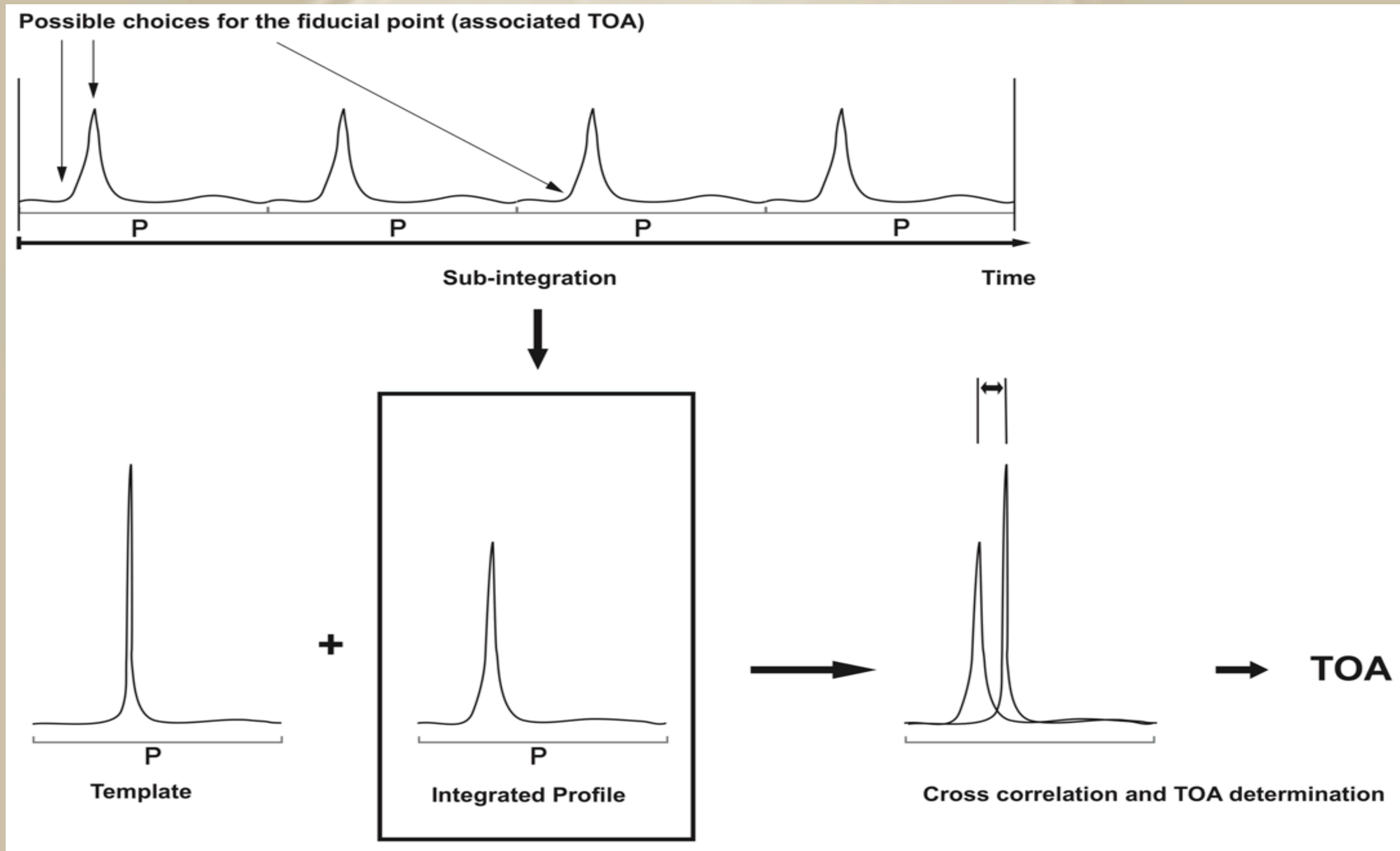
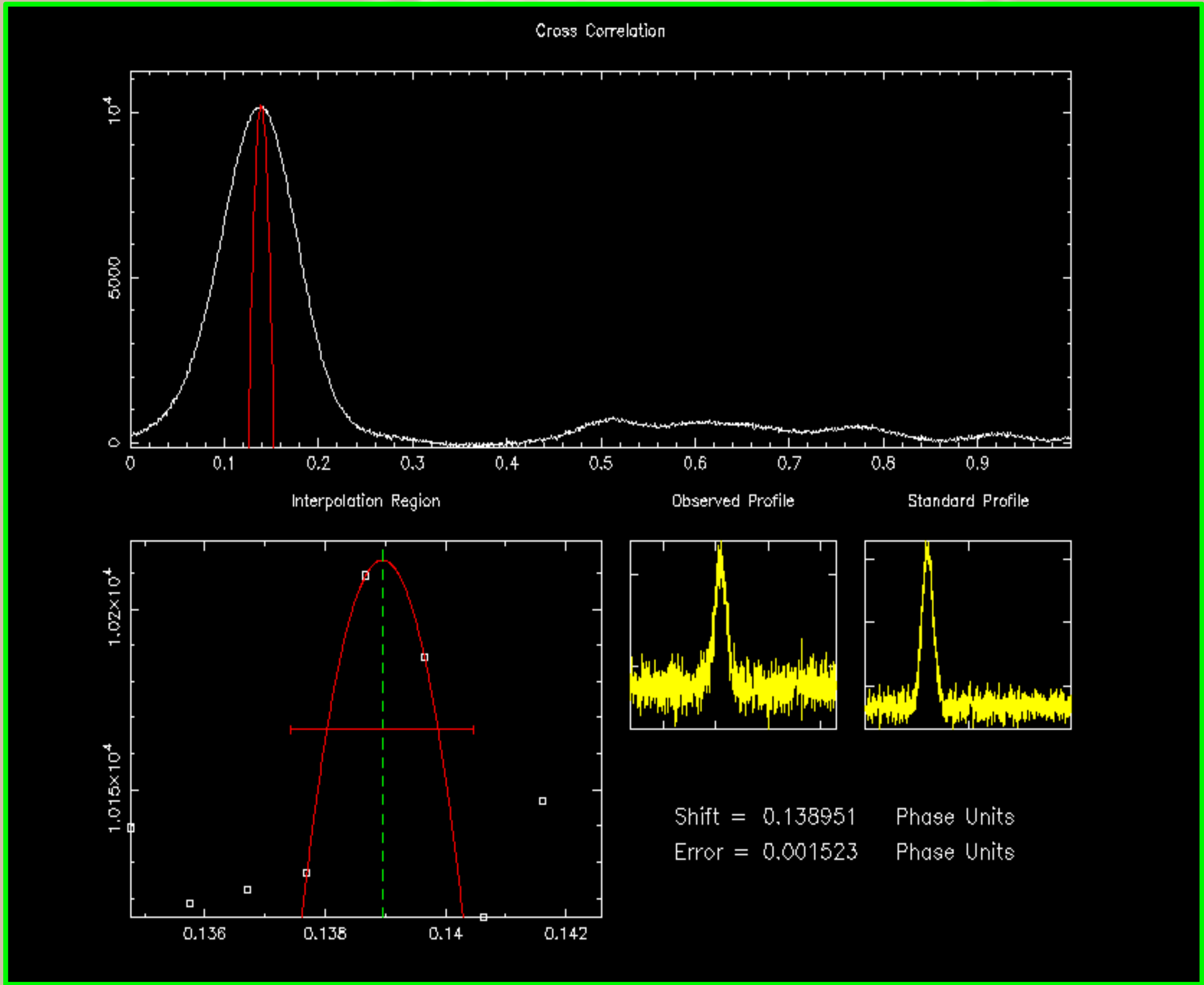
Determination of the TOPOCENTRIC Times of Arrival (ToAs)



@ Ridolfi

Determination of the TOPOCENTRIC Times of Arrival (ToAs)



Summary of Steps for getting a ToA

- Need telescope, receiver, spectrometer (filterbank, digital correlator, digital filterbank or baseband system), data acquisition system
- Start observation at known time and synchronously average 1000 or more pulses (typically 5 - 10 min), dedisperse and sum orthogonal polarizations to get mean total intensity (Stokes I) pulse profile
- Cross-correlate this with a standard template to obtain the arrival time at the telescope of a fiducial point on profile, usually the pulse peak – that is the pulse time-of-arrival (ToA)
- Measure a series of ToAs (t_{obs}) over days–weeks–months–years
- ToA r.m.s. uncertainty (ω = width of the pulse, P=pulsar period):

$$\sigma_{TOA} \approx w/(S/N) \approx \frac{S_{\rm sys}}{S_{\rm psr}\,(t_{\rm obs}\Delta\nu)^{1/2}}\,P\delta^{3/2}, \quad \delta = w/P$$

Courtesy D.Manchester

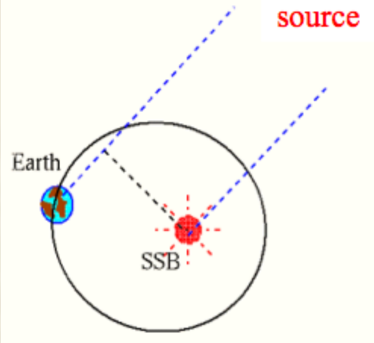
Collection of the Times of Arrival

The Times of Arrival (ToAs) of a selected event are "observed" at a detector by using a local "clock"

These "times" are usually referred as **topocentric ToAs** (for a ground based observatory) or **on-board ToAs** (for a probe in the space)

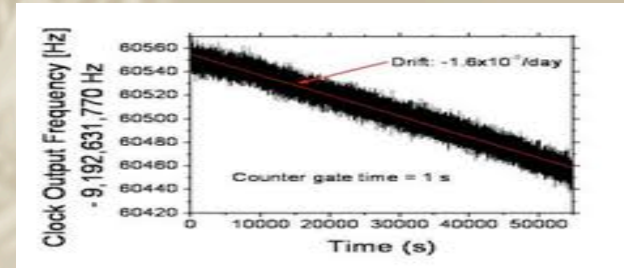
Why topocentric (or on board) ToAs are inadequate?

We aim to "clean" the observed ToAs, by getting rid of all the effects which are not intrinsic to the source of the events



Topocentric / On board ToAs are:

Dependent on the varying position of the detector wrt to the source



Dependent on the putative instability (or drift) of the local or "onboard" clock

Dependent on additional effects which are related to the medium which the signal went across

Which reference system?

The TOPOCENTRIC (or the ON BOARD) ToAs of the events must be converted to a common and well defined system of reference in both space and time

Usually to infinite frequency at Solar System Barycenter (SSB) thus obtaining the so called BARYCENTERED ToAs

The time scale is (in Tempo2 convention) the Barycentric Coordinate Time (TCB), i.e. the proper time of an observer at SSB, were the gravity field of Sun and Planets absent

Getting barycentered ToAs

The TOPOCENTRIC ToAs must be corrected, calculating them **to infinite frequency at Solar System Barycentre** (SSB) thus obtaining the BARYCENTERED ToAs: the time scale is (Tempo2) the Barycentric Coordinate Time (TCB), i.e. the proper time of an observer at SSB were the gravity field of Sun and Planets absent

$$t_{\rm SSB} = t_{\rm obs} + t_{\rm clk} - D/f^2 + \Delta_R + \Delta_S + \Delta_E$$

t_{SSB} : Calculated BARYCENTERED ToA at INFINITE frequency
t_{obs} : Observed TOPOCENTRIC ToA
t_{clk} : Observatory clock correction to TAI (= UTC + leap sec), via GPS
D/f^2 : Dispersion term
Δ_R : Roemer delay (propagation delay) to SSB (need SS ephemeris, e.g. DE405)
Δ_S : Shapiro delay in Solar-System
Δ_E : Einstein delay at Earth

The clock corrections

$$t_{\mathrm{SSB}} = t_{\mathrm{obs}} + t_{\mathrm{clk}} - D/f^2 + \Delta_R + \Delta_S + \Delta_E$$

$$t_{clk} = t_{gps\text{-}obs} + t_{utc\text{-}gps} + t_{tai\text{-}utc} + t_{tt(tai)\text{-}tai}$$

$$t_{clk} = t_{utc(ita)\text{-}obs} + t_{utc\text{-}utc(ita)} + t_{tai\text{-}utc} + t_{tt(tai)\text{-}tai}$$

t_{obs} : ToAs recorded against local observatory clock (often an H-maser) or "onboard" clock

$t_{gps\text{-}obs}$: difference btw obs clock and GPS time

GPS time is the clock signal broadcast from the ensemble of GPS satellites

$t_{utc(ita)\text{-}obs}$: difference btw obs clock and national time-scale Universal Time Coordinate, e.g. UTC(ITA)

UTC(ITA) is obtained weighting data from an ensemble of atomic clocks around Italy

$t_{utc\text{-}gps}$: difference bwt GPS time and Coordinate Universal Time (UTC), provided by Circular T of Bureau International des Poids at Mesures (BIPM)

UTC is obtained from weighting of data from an ensemble of atomic clocks around the world

$t_{utc\text{-}utc(ita)}$: difference btw UTC(ITA) and UTC, provided by Circular T of BIPM

$t_{tai\text{-}utc}$: difference btw UTC and Temps Atomique International (TAI), provided by Circular C of BIPM. It consists of integer "leap" s, to maintain approx synchrony bwt UTC and irregular rotation of the Earth

TAI is the most stable long-term time-scale available in near real-time

$t_{tt(tai)\text{-}tai}$: difference btw the Terrestrial Time TT(TAI) realization and TAI, differing (since 1971) for a constant offset

TT is an ideal time-scale whose units corresponds, on the surface of the geoid, to the SI second in the Geocentric Coordinate time-scale TCG.

Due to instability of TAI, may be better TT(BIPM0X), available only retro-actively

The barycentering terms (Roemer)

$$t_{\mathrm{SSB}} = t_{\mathrm{obs}} + t_{\mathrm{clk}} - D/f^2 + \Delta_R + \Delta_S + \Delta_E$$

$$\Delta_R = \frac{(\vec{r} \cdot \vec{n})}{c} + \frac{(\vec{r} \cdot \vec{n})^2 - |\vec{r}|^2}{2\,c\,d}$$

vect r : Vector from SSB to the phase center of the telescope [observatory positions are usually referred to International Terrestrial Reference Frame (ITRF)]

vect n : Vector from SSB to the source. If DE405 planetary ephemeris, source positions are usually in the International Celestial Referenc System (ICRS)]

d : Distance from the SSB to the source

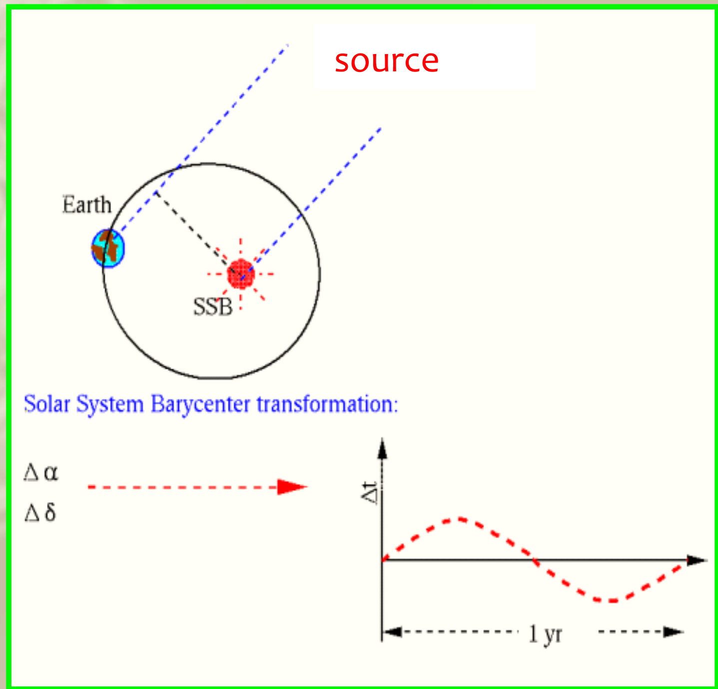
Transformation btw ITRF and ICRS implies knowledge of

- Polar motion : obtained from C04 series of Earth Orientation Parameters of International Earth Rotation Service (IERS)
- Precession+nutation : from IAU 2000B precession-nutation model (accurate at 0.1 ns)
- Earth rotation : linear function of the UT1 time-scale. Offset UTC-UT1 is given in C04 series

The barycentering terms (R)

$$\Delta_R = \frac{(\vec{r}\cdot\vec{n})}{c} + \frac{(\vec{r}\cdot\vec{n})^2 - |\vec{r}|^2}{2\,c\,d}$$

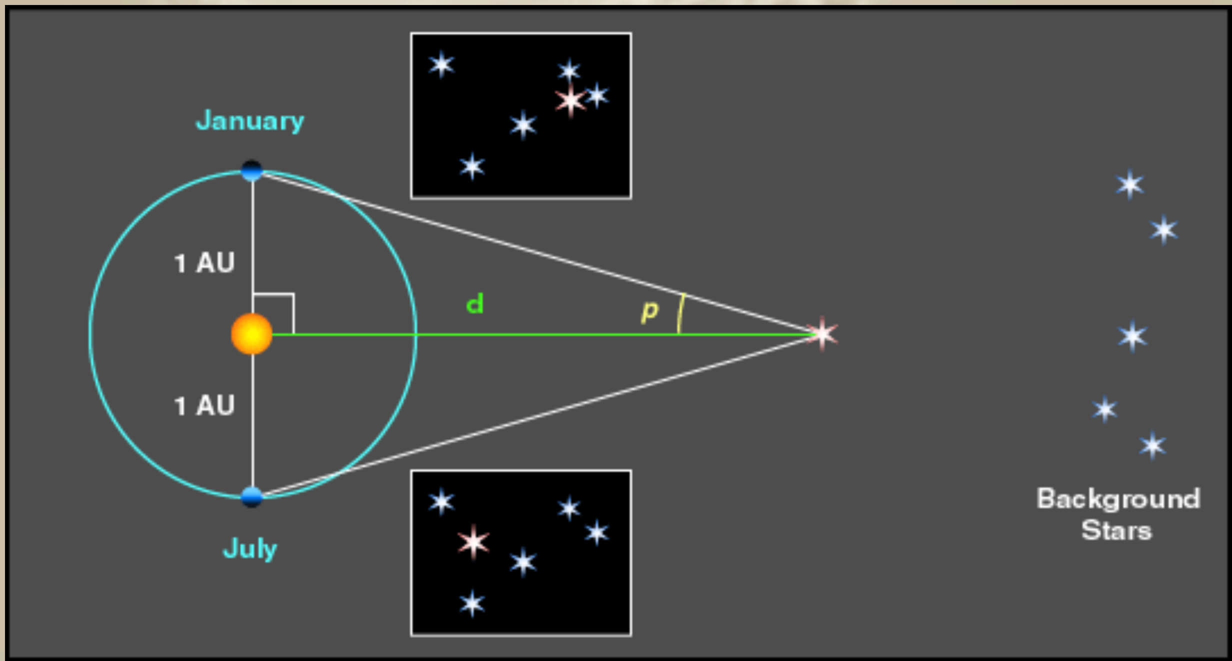
1[st] term: Earth motion on annual basis : depend on **source position** and **proper motion**



The barycentering terms (R)

$$\Delta_R = \frac{(\vec{r}\cdot\vec{n})}{c} + \frac{(\vec{r}\cdot\vec{n})^2 - |\vec{r}|^2}{2\,c\,d}$$

2nd term corresponds to the curvature of the wave-front : depend on source annual parallax



The barycentering terms (E)

$$t_{\mathrm{SSB}} = t_{\mathrm{obs}} + t_{\mathrm{clk}} - D/f^2 + \Delta_R + \Delta_S + \Delta_E$$

$$\frac{d\Delta_E}{dt} = \Sigma_i \left(\frac{G\, m_i}{c^2\, r_i}\right) + \frac{(v_{\mathrm{Earth\text{-}SSB}})^2}{2c^2}$$

r_i : Distance between the Earth and the i-th body of the SS

m_i : Mass of i-th body in the SS

$V_{\mathrm{earth\text{-}SSB}}$: Velocity of the Earth with respect to the SSB

1st + 2nd term: gravitational redshift and time dilation due to the motion of Earth and the presence of other massive bodies in SS: can in principle be used for measuring masses of SS bodies

Applying this Einstein correction, we basically transform the time scale from TT to our desidered Barycentric Coordinate Time (TCB)

N.B. Tempo1 uses Barycentric Dynamical Time (TDB), which is not complaint with IAU resolution A4 (1991) since its units are NOT SI seconds

The barycentering terms (S)

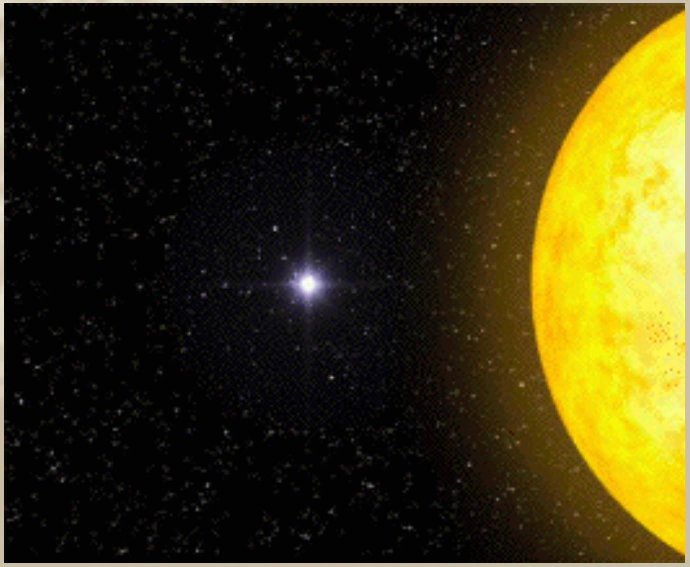
$$t_{\rm SSB} = t_{\rm obs} + t_{\rm clk} - D/f^2 + \Delta_R + \Delta_S + \Delta_E$$

$$\Delta_S = -2\, T_{sun} \log_{10} (1 + \cos\theta)$$

θ : Angle at Pulsar subtended by SUN-PULSAR-EARTH

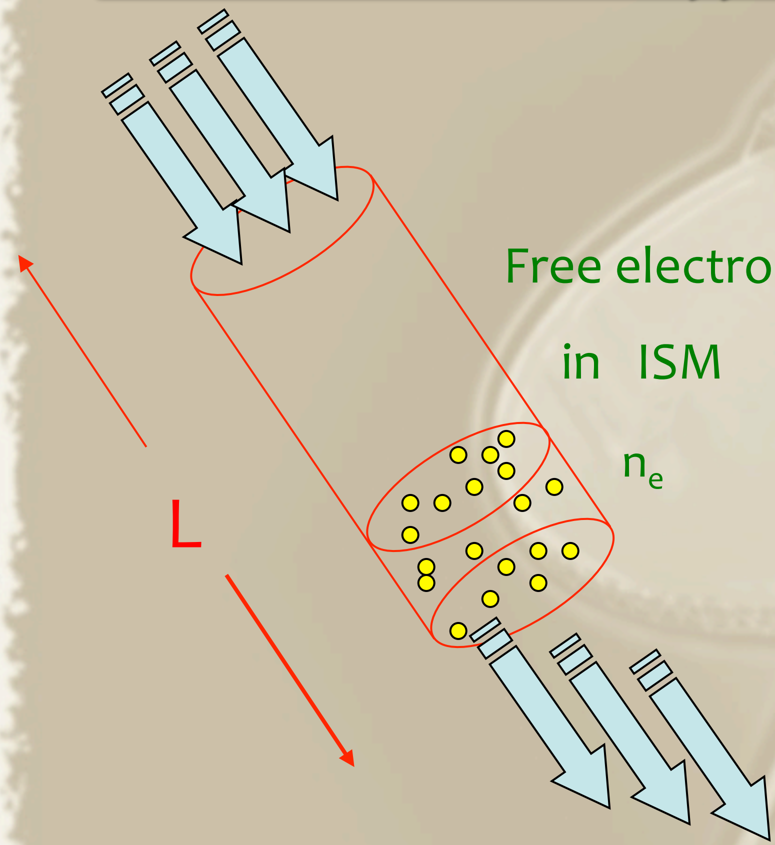
T_{sun} : constant $GM_{sun}/c^3 = 4.92549\ \mu sec$

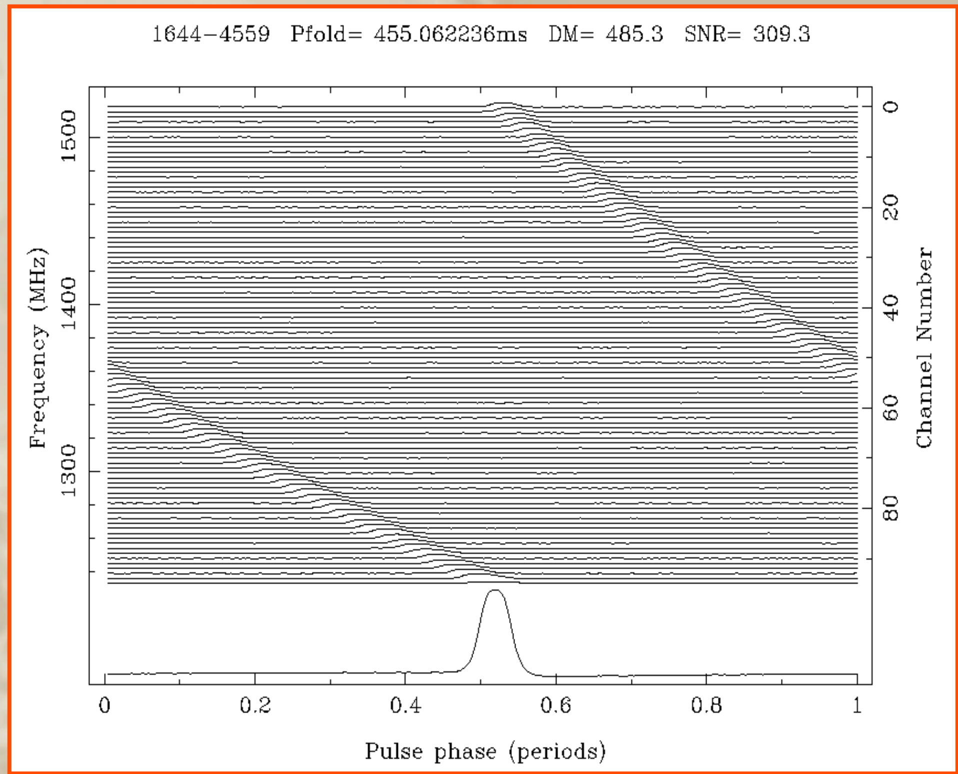
This term is due to the optical path of the e.m. signal in the solar gravitational well



Additional effects. For radio band…

Interstellar Dispersion

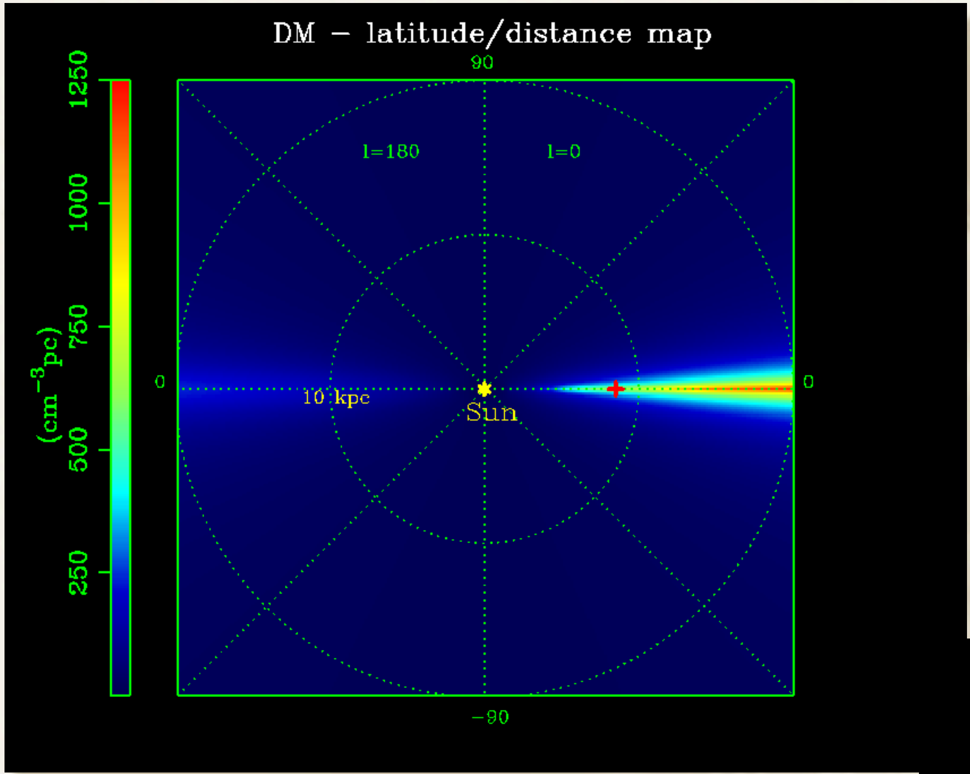


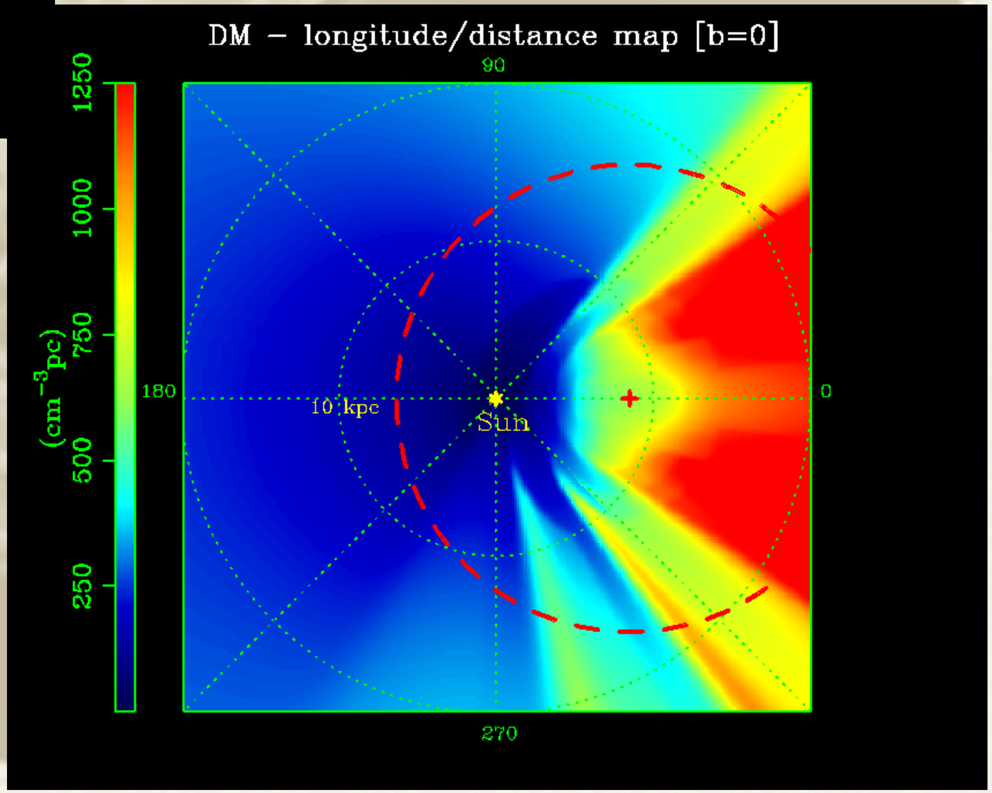


$$t_2 - t_1 \propto (\nu_2^{-2} - \nu_1^{-2})\ DM$$

$$DM = \int_0^L n_e \, dl$$

Ionised gas in the interstellar medium causes lower radio frequencies to arrive at the Earth with a delay compared to higher frequencies


DM – latitude/distance map
90
l=180
l=0
0
10 kpc
Sun
0
-90
(cm⁻³pc)
1250
1000
750
500
250


DM – longitude/distance map [b=0]
90
180
10 kpc
Sun
0
270
(cm⁻³pc)
1250
1000
750
500
250

The barycentering terms (D)

$$t_{\mathrm{SSB}} = t_{\mathrm{obs}} + t_{\mathrm{clk}} - D/f^2 + \Delta_R + \Delta_S + \Delta_E$$

$$D/f^{2} = [\, DM \;/\; (2.41 \cdot 10^{-16})\, s\,] / f^{2}$$

D : Dispersion constant
DM : Dispersion Measure of the pulsar
f : Central frequency of the observing band (Doppler corrected!!)

This term accounts for the dispersion delay, which is time variable due to the changing Doppler term of the telescope wrt the pulsar

Refractive index: $\mu = \left[1 - (\nu_p/\nu)^2\right]^{1/2}$

Plasma frequency: $\nu_p = \left(\frac{e^2 n_e}{\pi m_e}\right)^{1/2}$

Group velocity: $v_g = \mu c$

Delay: $\Delta t = \int_0^d \frac{\mathrm{d}l}{v_g} = \frac{e^2}{2\pi m_e c} \frac{\int_0^d n_e \mathrm{d}l}{\nu^2}$

Dispersion Measure: $\mathrm{DM} = \int_0^d n_e \mathrm{d}l$

Timing model: rotational terms

1. Have series of barycentered ToAs: t_i
2. Model pulsar frequency evolution $\nu(t)$ by Taylor series, and then integrate to get pulse phase evolution ($\phi(t)$ = 1 for t=P)

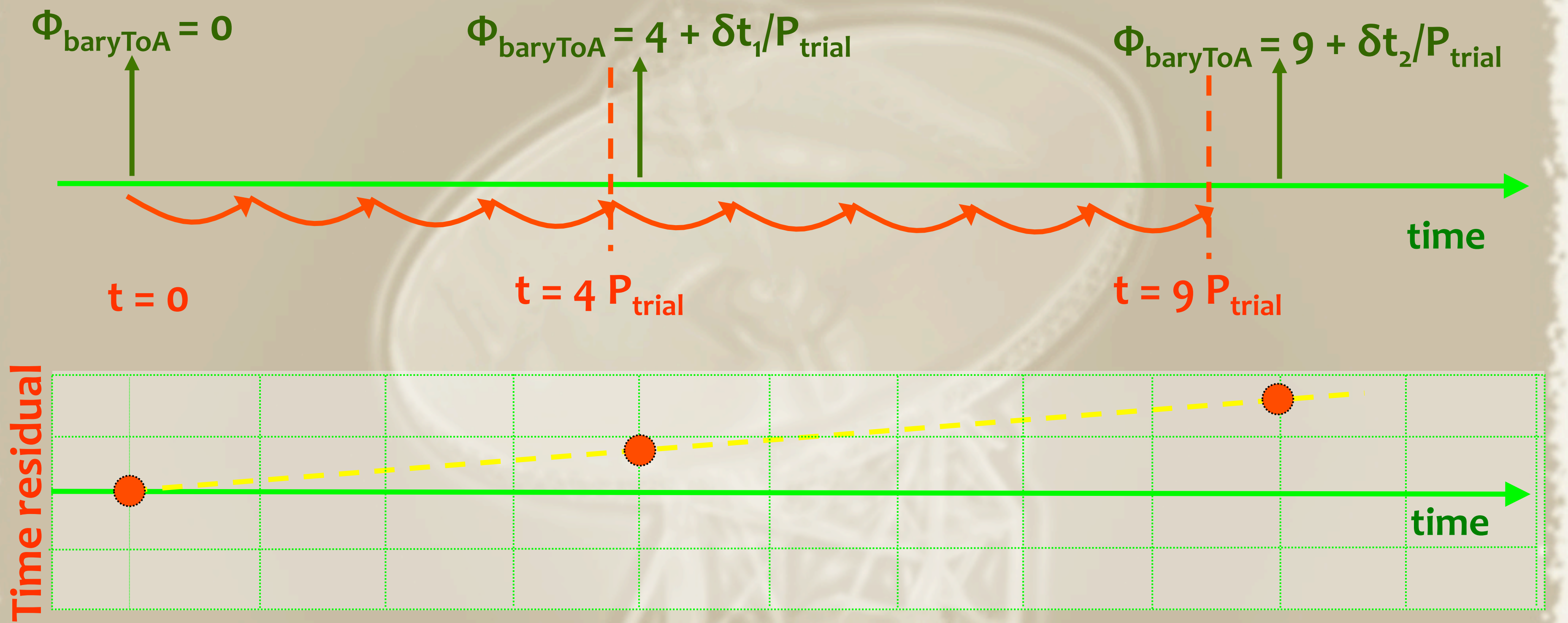
$$\nu(t) = \nu_0 + \dot{\nu}_0 t + \frac{1}{2}\ddot{\nu}_0 t^2 + \ldots$$

$$\phi_i = \phi_0 + \nu_0 t_i + \frac{1}{2}\dot{\nu}_0 t_i^2 + \frac{1}{6}\ddot{\nu}_0 t_i^3 + \ldots$$

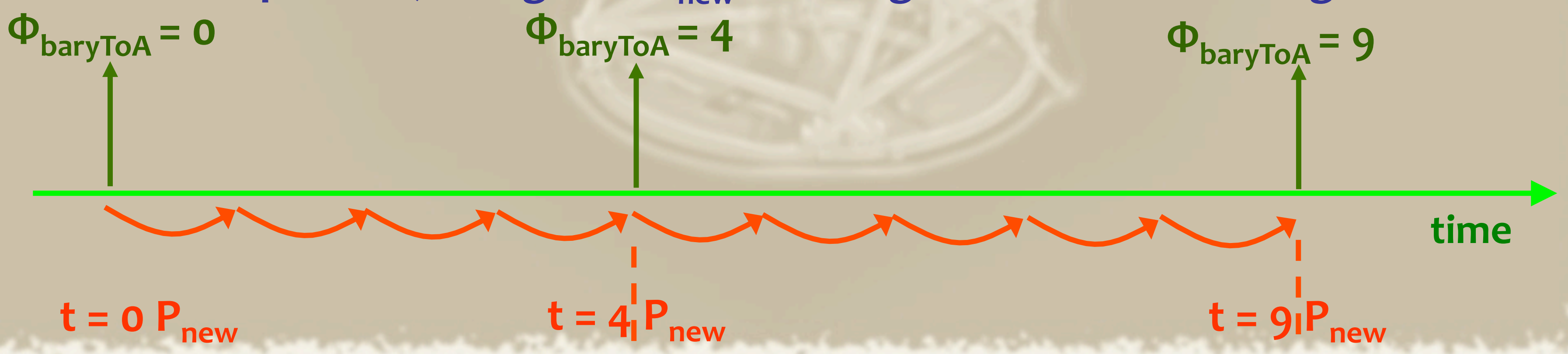
3. Choose t = 0 to be first ToA, t_0
4. Form **residuals** $r_i = \phi_i - n_i$ where n_i is the nearest integer to ϕ_i

5. If pulsar model is accurate, then $r_i \ll 1$
6. Corrections to model parameters are obtained by making **least-squares fit to trends in r_i**

Pulsars as clocks



From a least square fit, one gets a P_{new} allowing the rms residual to go closer to 0



Timing key quantity: the residuals

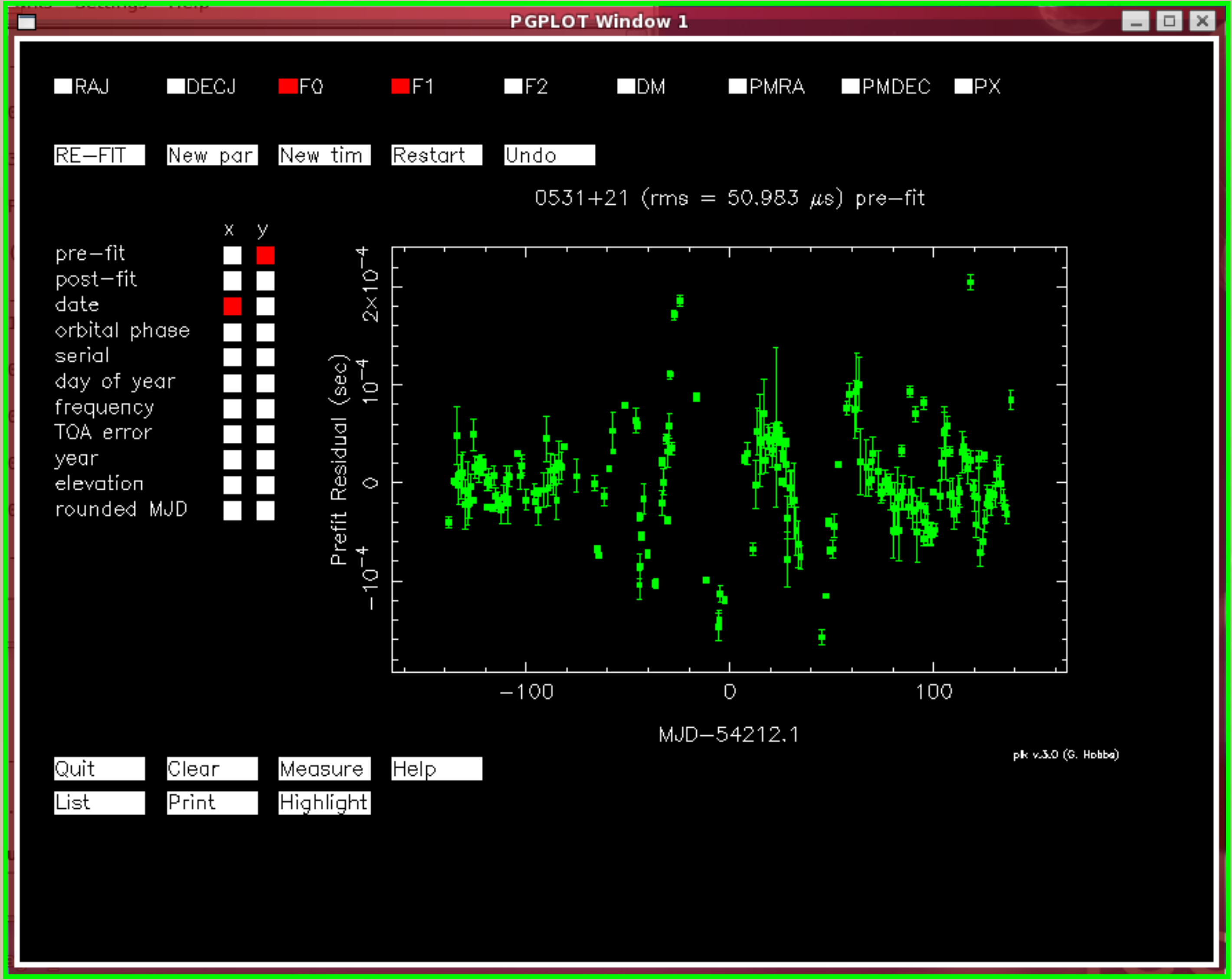
Given the full set of parameters (a_1, a_2, … , a_n) of a model, the i-th **residual r_i is the difference in rotational phase Φ** (with $-0.5 < r_i < +0.5$) between the observed phase of arrival of the i-th pulse and the phase of arrival of that pulse as predicted by the model

$$r_i = \Phi_{observed}(\text{i-th pulse}) - \Phi_{model(a_1, a_2, \ldots, a_n)}(\text{i-th pulse})$$

In an iterative procedure, **one least-square fits** on suitable subsets of the possible parameters (a_1, a_2, … , a_n) of the model, **in the aim** to remove apparent trends and thus eventually **to approach $r_i << 1$**

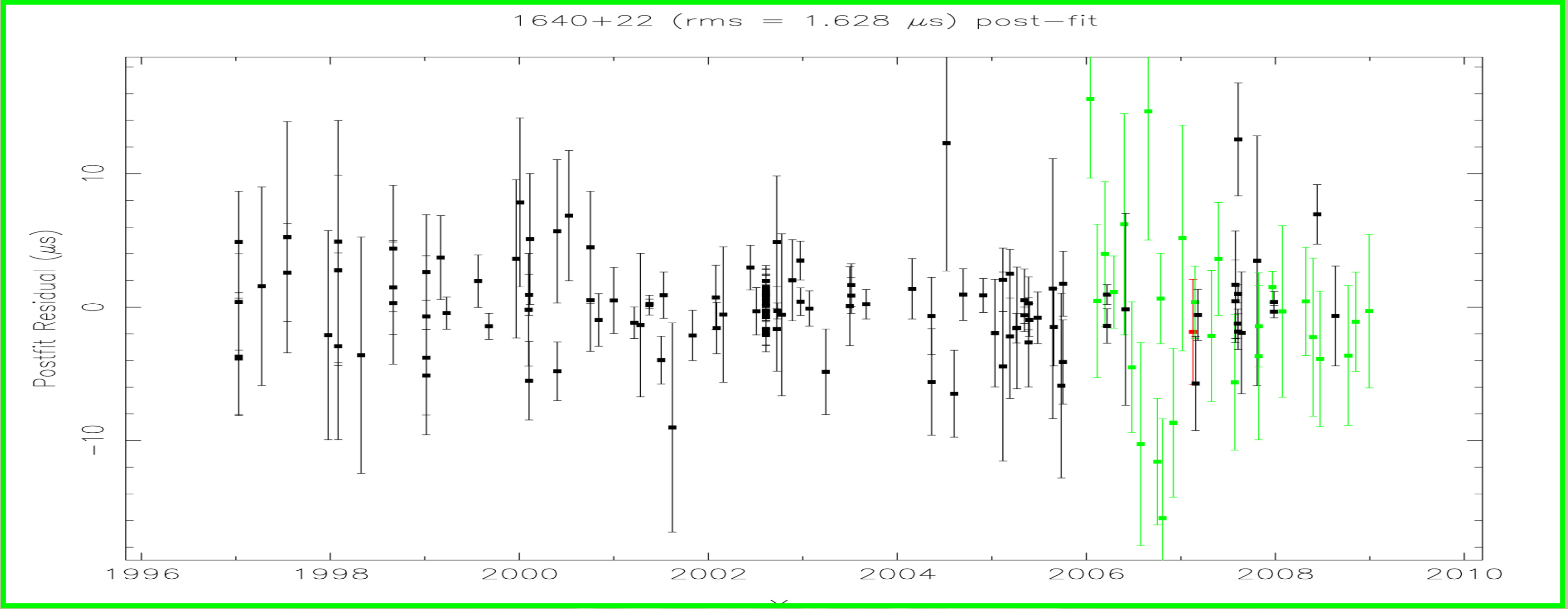
Il fit con TEMPO2

Timing program (e.g. TEMPO or TEMPO2) **does SSB corrections,** computes r_i and improves model parameters



Timing analysis quality: rms

Good timing solution → no evident trend and $r_i << 1$ for all observed pulses

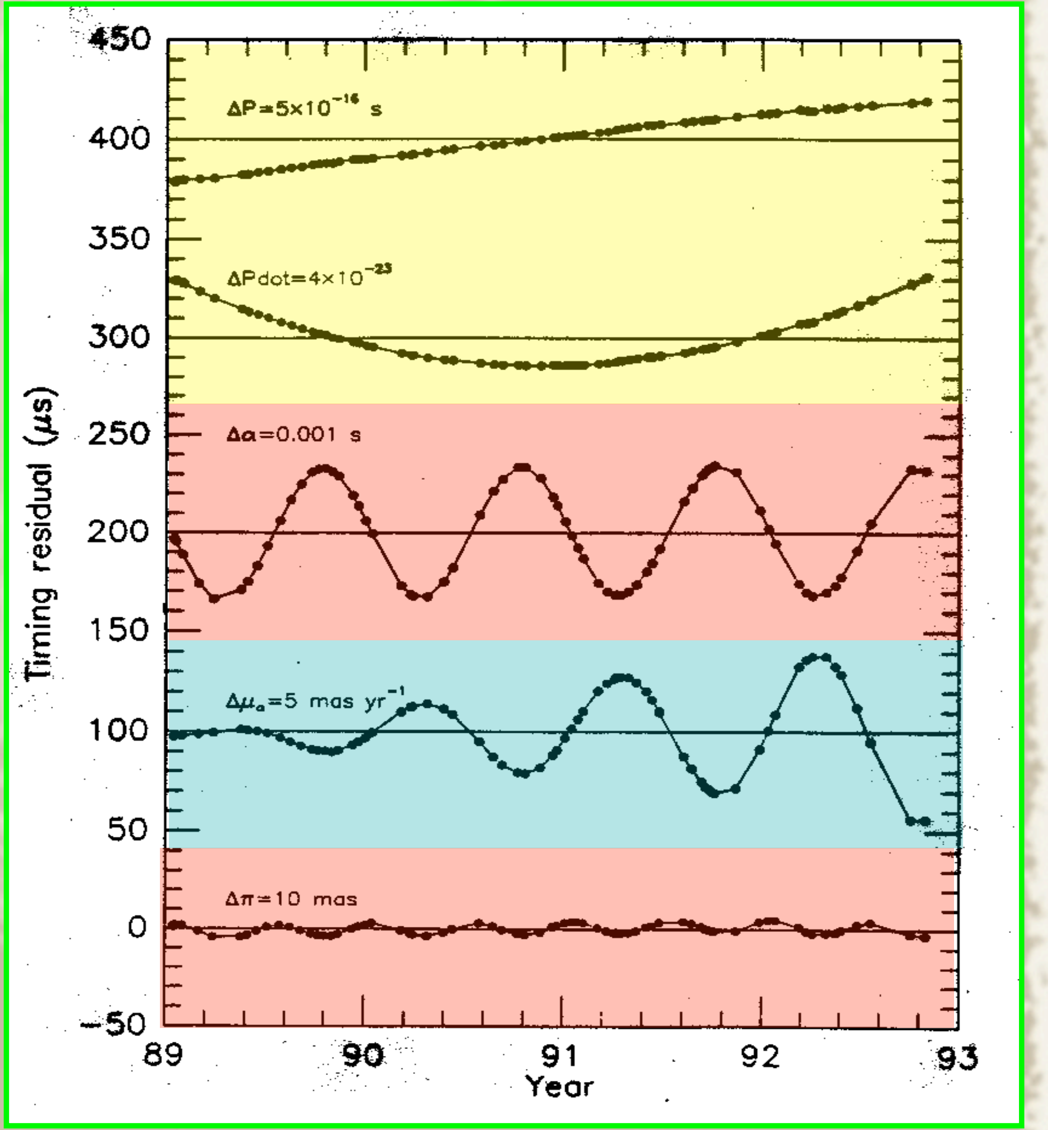


The quality of the timing solution is usually given in term of the root mean square rms of the residuals:

the smaller rms is, the smaller physical effects can be measured

Timing analysis: removing trends

Thanks to the least-square fit procedure, one can iteratively solve for rotational, positional and kinematic parameters, as well as for other parameters, when applicable



Timing model: isolated pulsars

From timing of an isolated pulsar over a long enough time span, one can in principle get

- **RA & DEC: Celestian coordinates**
- **PMRA & PMDEC: Proper Motion**
- **π : Trigonometric Parallax (i.e. Distance)**
- **DM : Accurate Dispersion Measure**
- **DM1 : Time Derivative of Dispersion Measure**
- **P0: Rotational Period**
- **P1: Time derivative of P0**
- **P2: Second time derivative of P0**
- **P3: Third time derivative of P0**
- **…**