Radiation processes and models

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- Lesson 1: Introduction to compact object physics
- Lesson 2: Radiation processes
- Lesson 3: Models for accreting black hole binaries: accretion flows
- Lesson 4: Models for accreting black hole binaries: compact jets

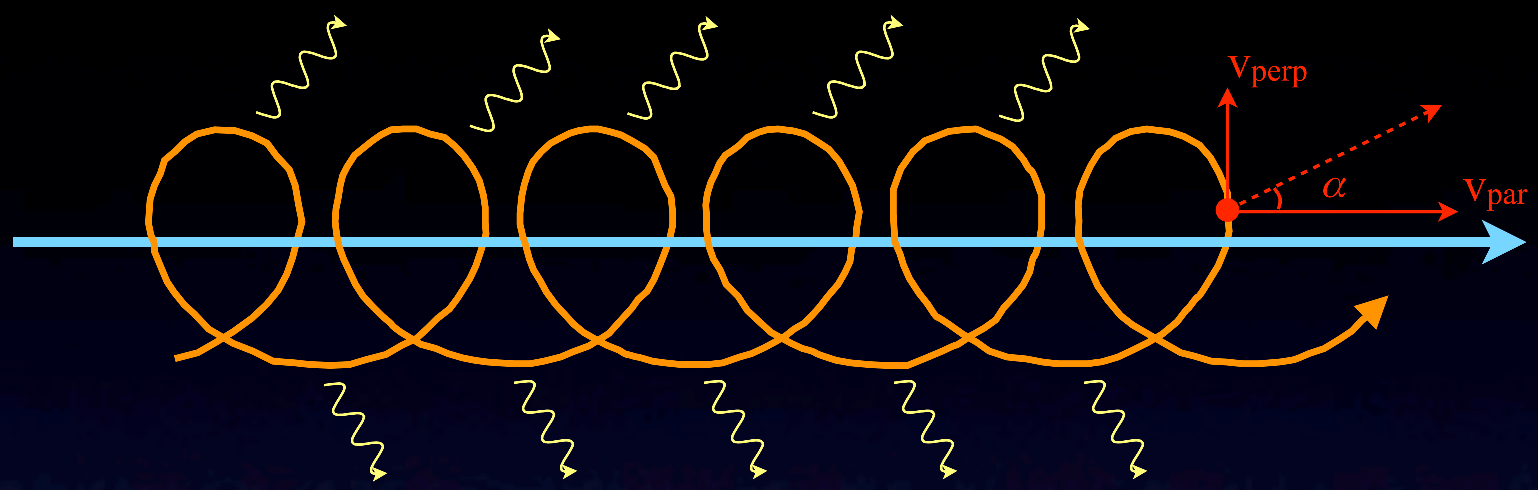
CHAOS

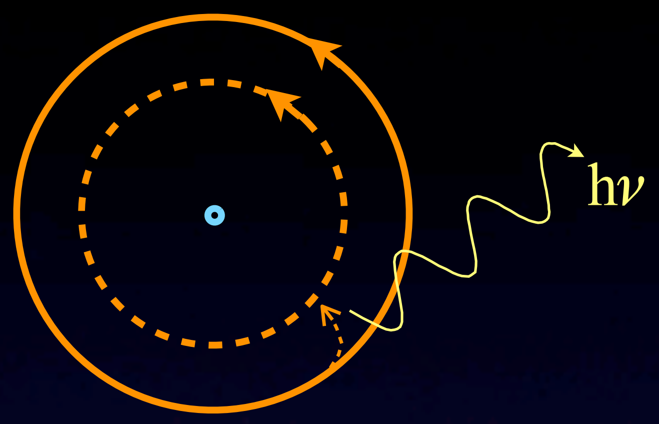
Radiation Processes

- **Synchrotron/curvature radiation**
- **Bremsstrahlung**
- **Compton scattering**
- **Photon-photon e+-e- pair production**

➡ **Special thanks to Renaud Belmont**

Cyclo-synchrotron radiation

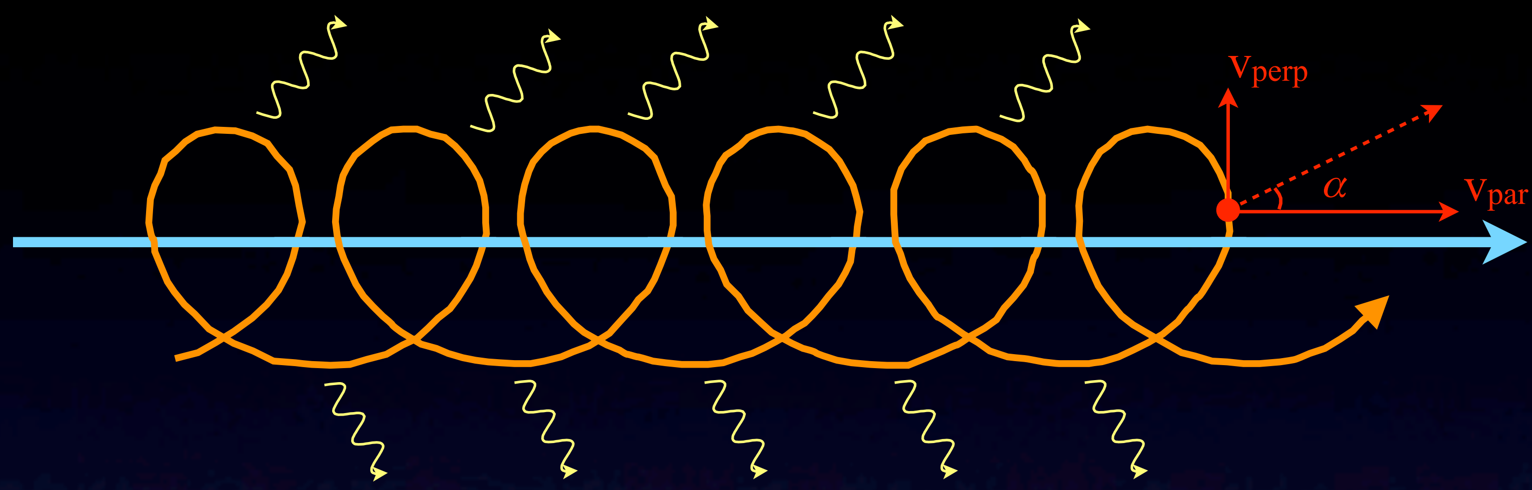


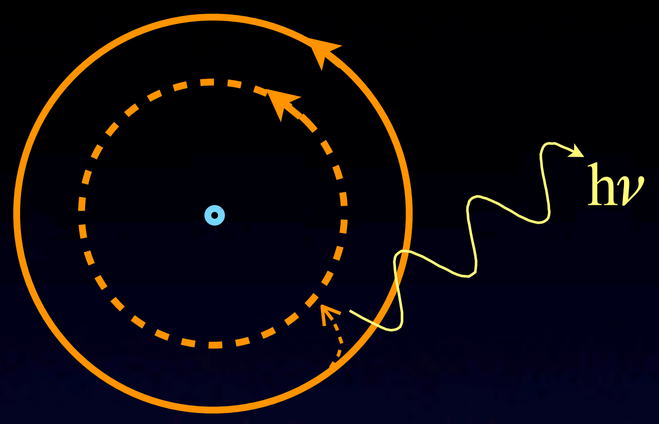


- ✓ Radiation from particles gyrating the magnetic field lines

$$\nu_L = \frac{qB}{2\pi mc} \qquad \nu_B = \frac{1}{\gamma}\frac{qB}{2\pi mc} = 2.80 \times 10^6 \frac{B(\mathrm{G})}{\gamma} \ \mathrm{Hz}$$

Cyclo-synchrotron radiation





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- Assumptions:

$$p = \gamma\beta = \gamma v/c$$

  - B uniform at the Larmor scale (parallel and perp)
    - ! Strong B curvature (pulsar and rapidly rotating neutron stars)
    - ! Small scale turbulence (at the Larmor scale)

$$r_L = p_\perp \frac{mc^2}{qB} = 1.70 \times 10^3 \,\frac{p_\perp}{B(\mathrm{G})}\ \mathrm{cm}$$

  - Small losses ($t_{cool} >> 1/\nu_B$)
  - Classical limit:
    - Otherwise: quantization of energies, Larmor radii...
    - Observable cyclotron lines in accreting neutron stars...

$$B < B_c = \frac{m^2c^3}{\hbar q} = 4.4 \times 10^{13}\ \mathrm{G}$$

- Emission/Absorption

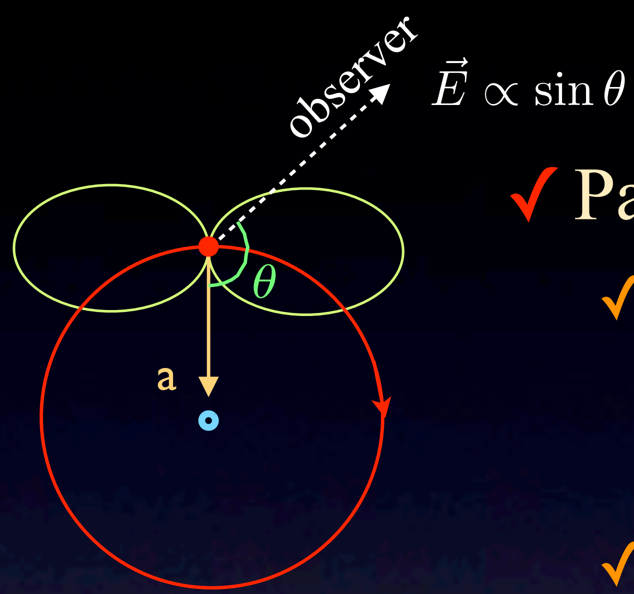
Synchrotron Emitted Power

- Emission of an accelerated particle (erg/s):
  - Non-relativistic: $P = \frac{2q^2}{3c^3} a^2$
  - Relativistic: $P = \frac{2q^2}{3c^3} \gamma^4 \left(a_\perp^2 + \gamma^2 a_\parallel^2 \right)$
- Circular motion: $a = a_\perp = \frac{\nu_B}{2\pi} v_\perp$ $\qquad P = 2c\sigma_T U_B p_\perp^2$
- Isotropic distribution of pitch angles: $p_\perp = p \sin\alpha$

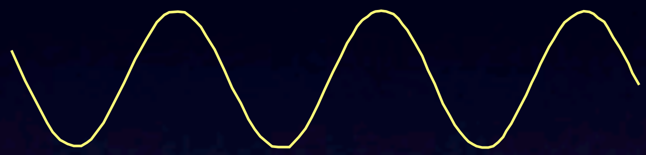
$$\boxed{P = \frac{4}{3} c\sigma_T U_B p^2}$$

- Maximal loss limit: $\gamma mc^2/P > 1/\nu_B$ => $\gamma^2 B < \frac{2q}{r_0^2} = 1.2 \times 10^{16}$ G

Synchrotron Emission Spectrum

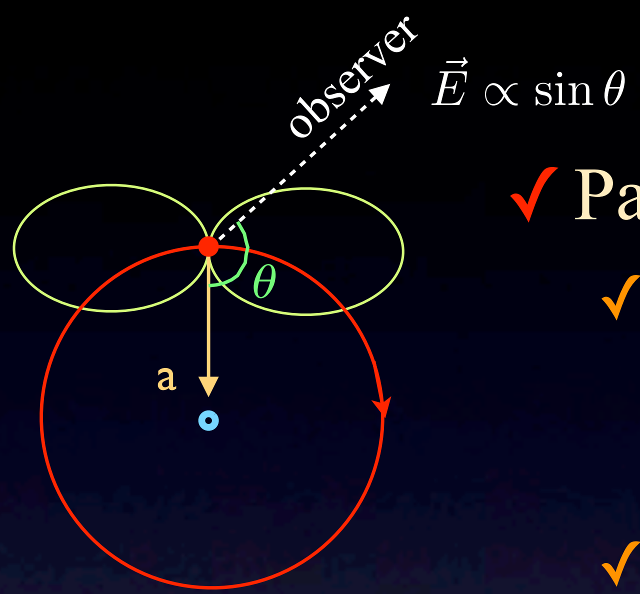


- ✓ Particles nearly at rest:
  - ✓ Simple modulation of the electric field at ν_B: $E(t) = \sin(2\pi\nu_B t)$



  - ✓ Spectrum = one line at ν_B

Synchrotron Emission Spectrum



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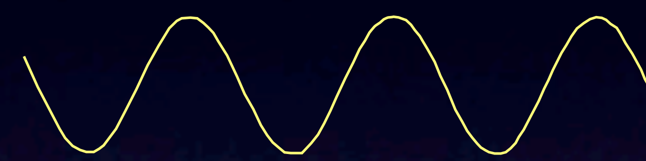
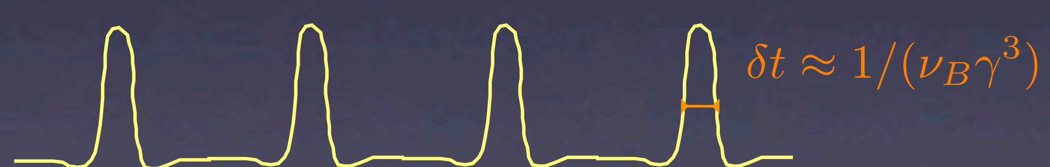


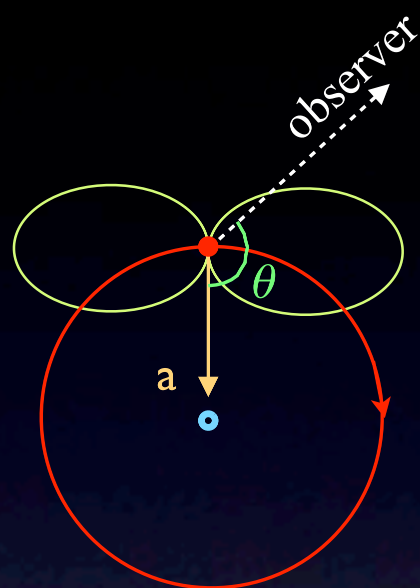
  - Spectrum = one line at ν_B

- Relativistic particles:
  - Relativistic beaming: $\delta\theta = 1/\gamma$
  - Pulsed modulation of the electric field at ν_B:



  - Multiple harmonics of ν_B up to a critical frequency: $\nu_c = \frac{3}{2}\gamma^3 \nu_B \sin\alpha$
  - Spectrum = many lines at $k\nu_B$
  - For $\gamma >> 1$: continuum

Synchrotron Emission Spectrum



$\vec{E} \propto \sin\theta$

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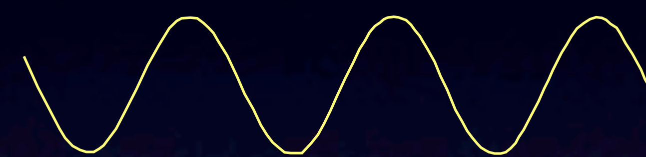
  - Simple modulation of the electric field at ν_B: $E(t) = \sin(2\pi\nu_B t)$



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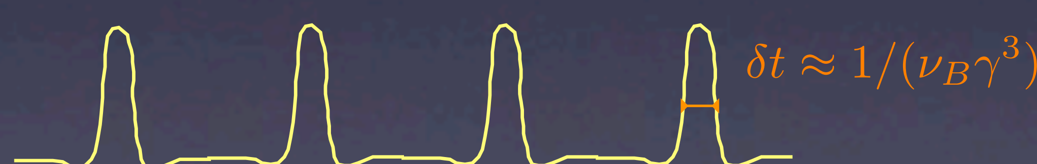

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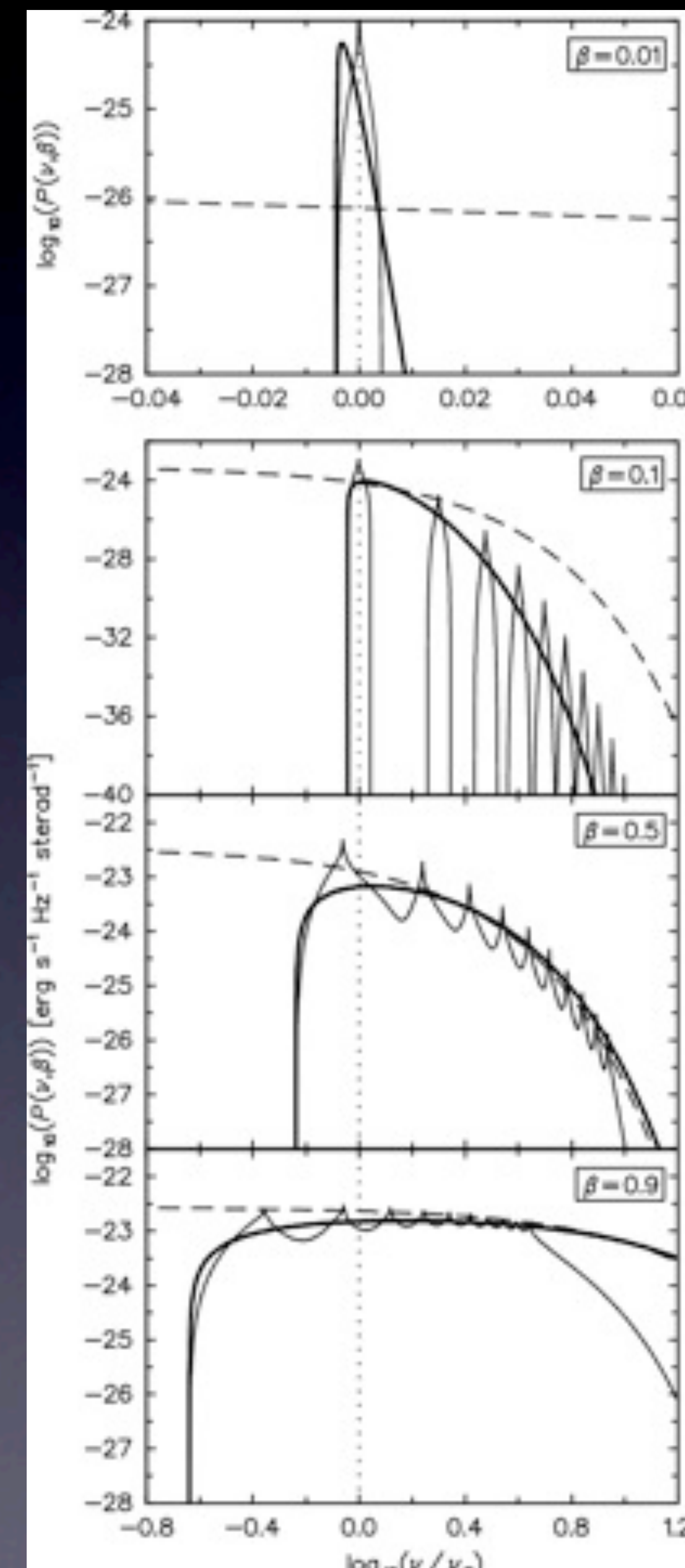
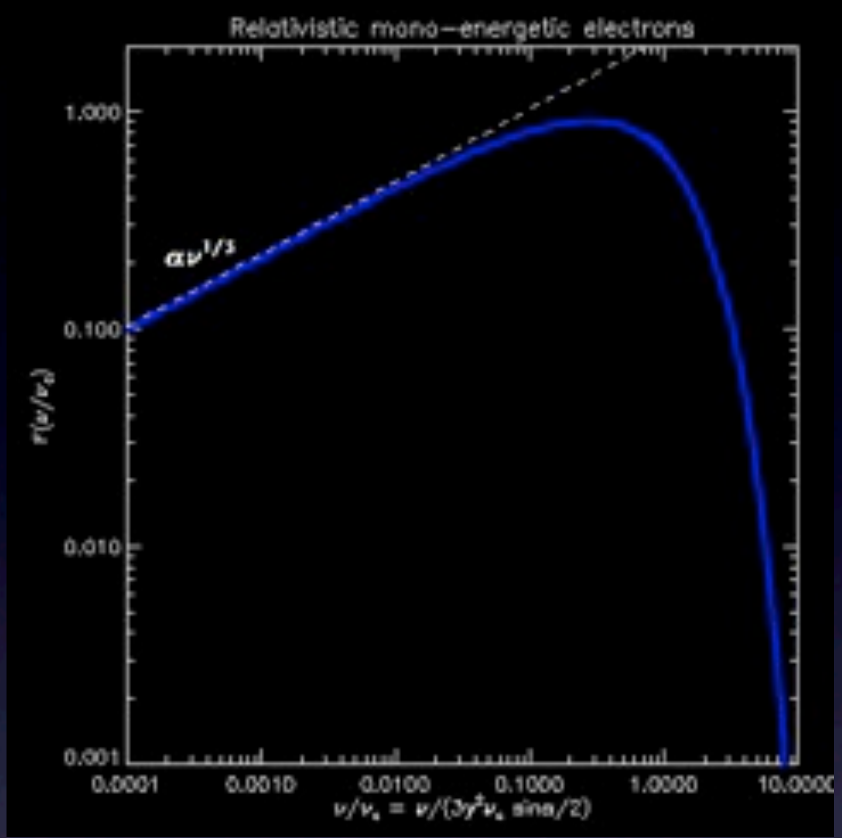
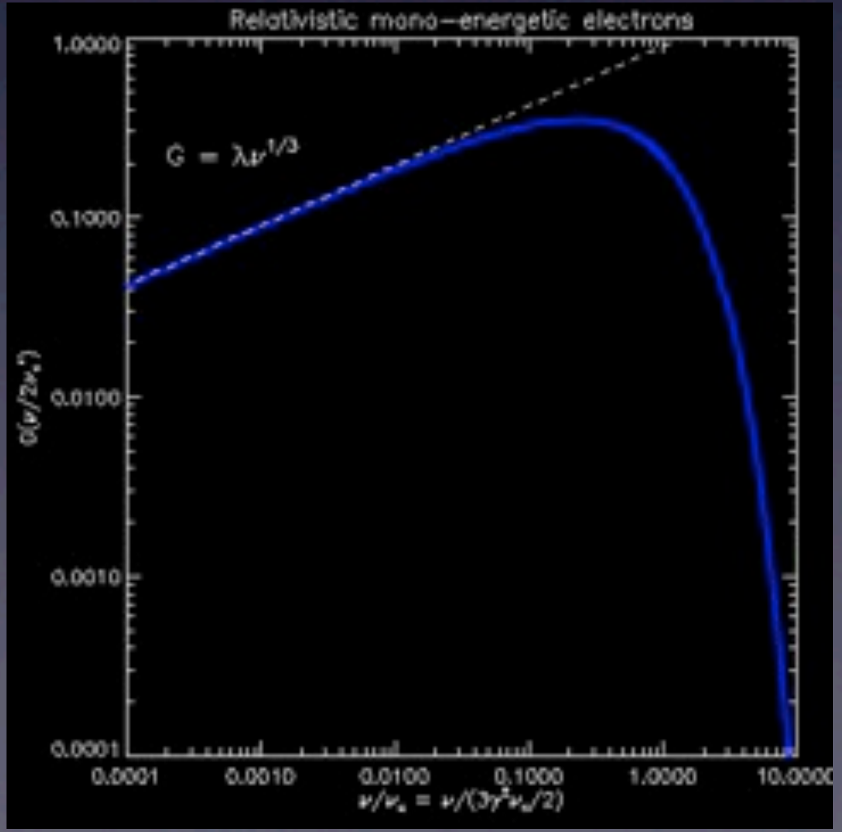


Spectrum of Relativistic Particles





- ✓ Relativistic particles have a continuous spectrum (erg/s/Hz)
  - ✓ Pitch-angle dependent spectrum:

$$\frac{\partial P}{\partial \nu}(\alpha, p, \nu) = \frac{\sqrt{3} q^3 B}{mc^2} \sin\alpha F(\nu/\nu_c)$$

$$F(x) = x \int_x^\infty K_{5/3}(z) dz \qquad \nu_c = \frac{3}{2}\nu_B \gamma^3 \sin\alpha$$

  - ✓ Pitch-angle averaged spectrum:

$$\frac{\partial P}{\partial \nu}(\gamma, \nu) = \frac{12\sqrt{3}\sigma_T c U_B}{\nu_L} G\left(\frac{\nu}{2\nu_c^*}\right)$$

$$G(x) = x^2 \left[K_{4/3}(x) K_{1/3}(x) - \frac{3x}{5}\left(K_{4/3}^2(x) - K_{1/3}^2(x)\right)\right]$$

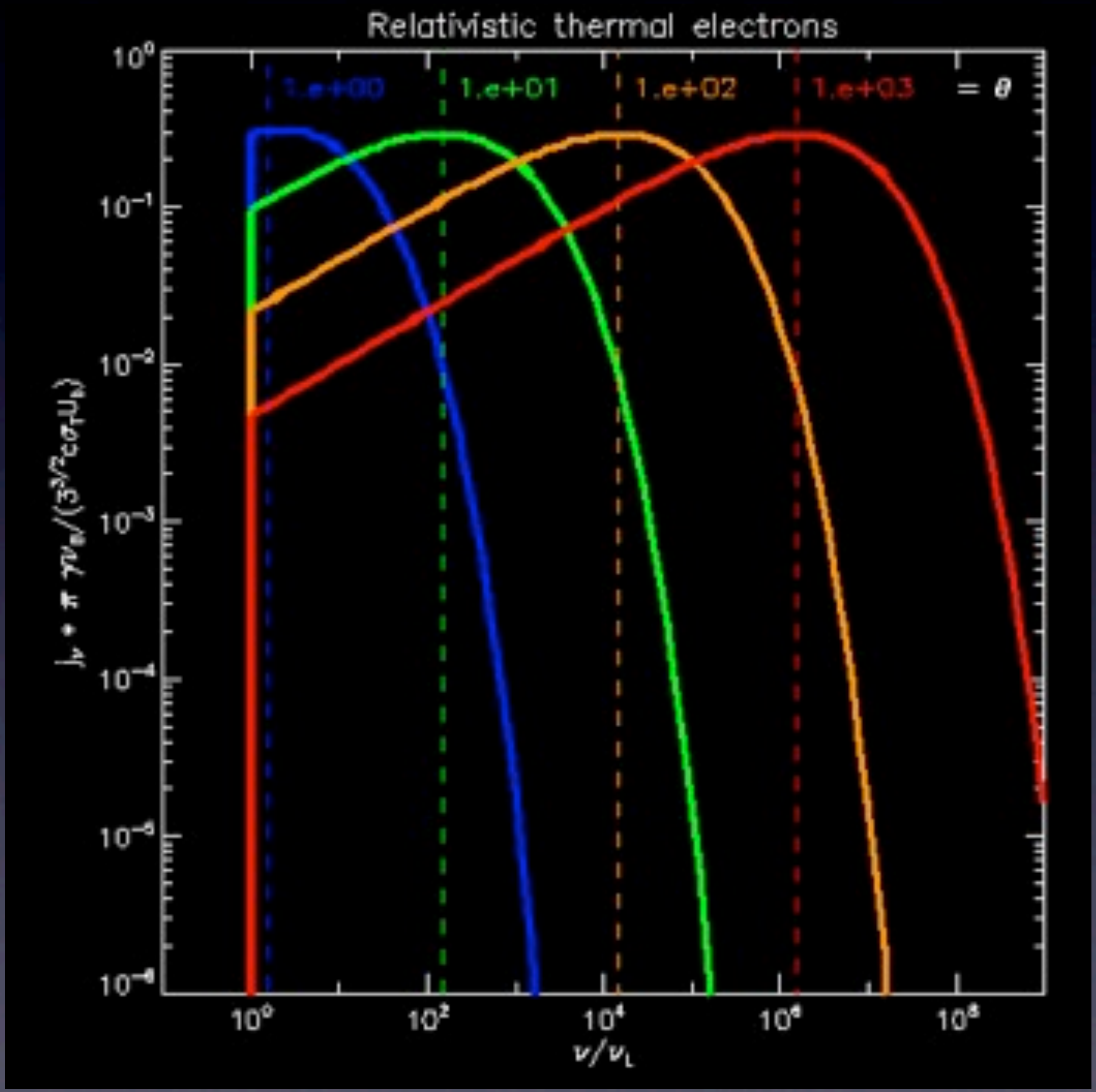
- ✓ Emission peaked at: $\boldsymbol{\nu \approx \nu_c^* = \frac{3}{2}\nu_L \gamma^2}$ $\quad \nu \propto B\gamma^2$

- ✓ Maximal energy

$$\gamma^2 B < \frac{2q}{r_0^2} \qquad h\nu = 3\ mc^2/\alpha_f = 70\ \text{MeV}$$

Spectrum of Relativistic Particles

$N_\gamma \propto \gamma^2 e^{-\gamma/\theta} \quad \text{for } \theta = k_B T/m_e c^2 > 1$

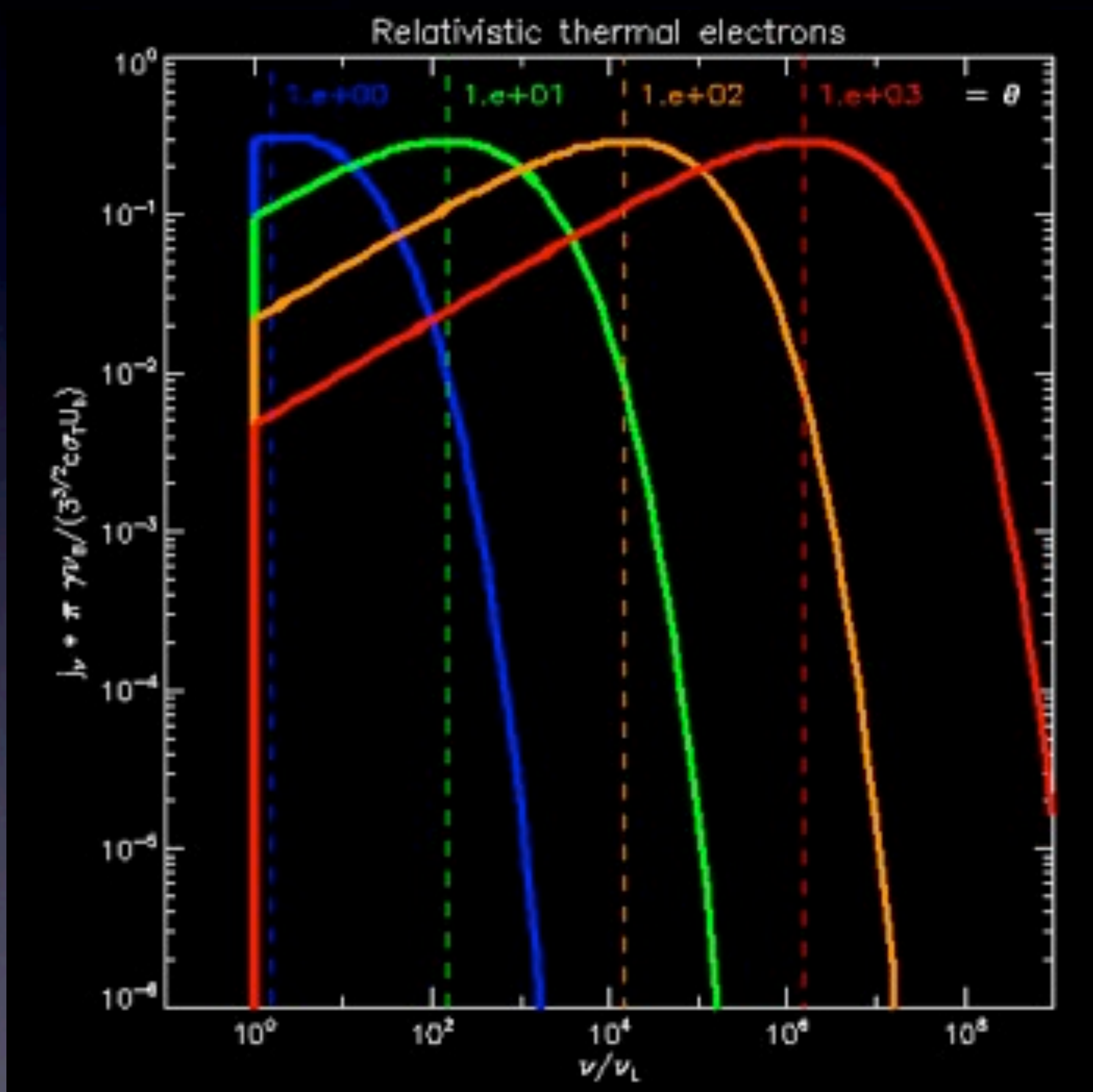


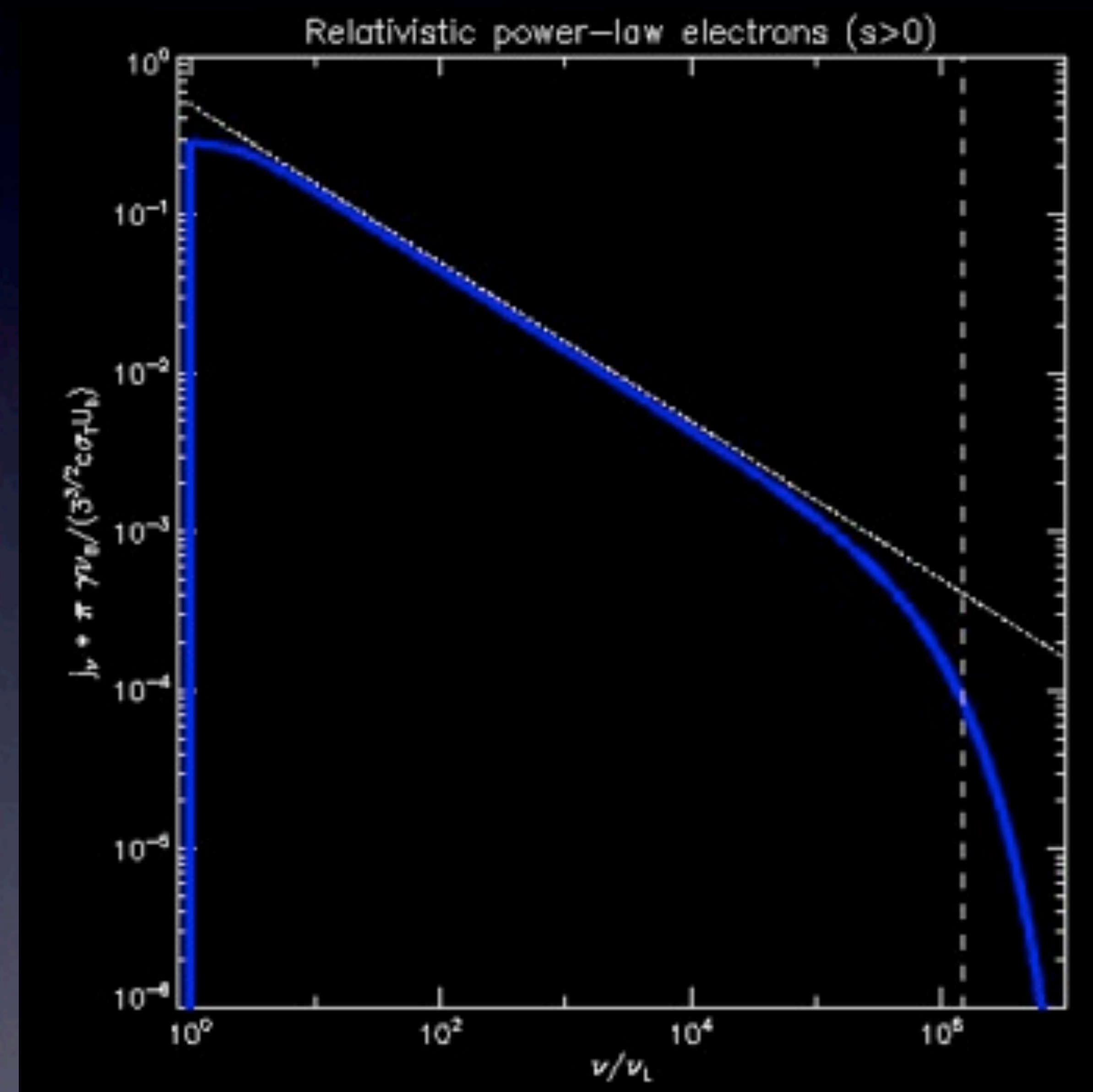
Spectrum peaking at $h\nu = h\nu_c^*(\theta) = 3/2\ h\nu_L\ \theta^2$

Spectrum of Relativistic Particles

$N_\gamma \propto \gamma^2 e^{-\gamma/\theta}$ for $\theta = k_B T/m_e c^2 > 1$

$N_\gamma \propto \gamma^{-s}$ for $1 < \gamma_{\rm min} < \gamma < \gamma_{\rm max}$

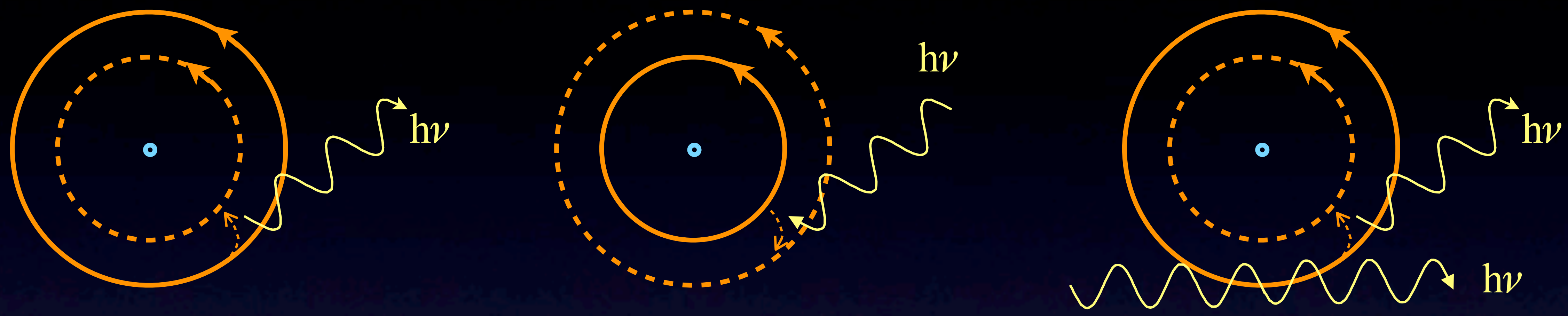




Spectrum peaking at $h\nu = h\nu_c^*(\theta) = 3/2\ h\nu_L\ \theta^2$

Power-law spectrum $P = \nu^{-\alpha}$
with slope: $\alpha=(s-1)/2$
cutoff: $h\nu = h\nu_c^*(\gamma_{\rm max}) = 3/2\ h\nu_L\ \gamma_{\rm max}^2$

Synchrotron self-absorption

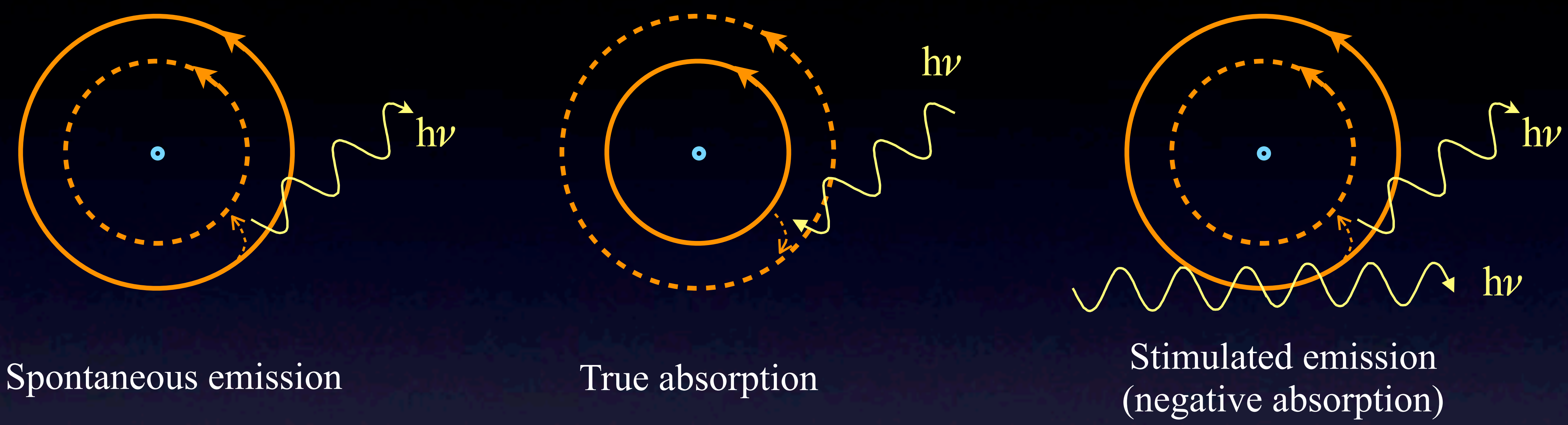


Spontaneous emission

True absorption

Stimulated emission
(negative absorption)

Synchrotron self-absorption



- Transition probabilities described by Einstein coefficients
- Absorption (cm^{-1}Hz^{-1}) can be expressed as a function of the emissivity:

$$\alpha_\nu(p,\nu) = \frac{c^2}{2mh\nu^3}\frac{1}{p\gamma}\left[\gamma p j_\nu\right]_\gamma^{\gamma+h\nu/mc^2}$$

True absorption

Stimulated emission
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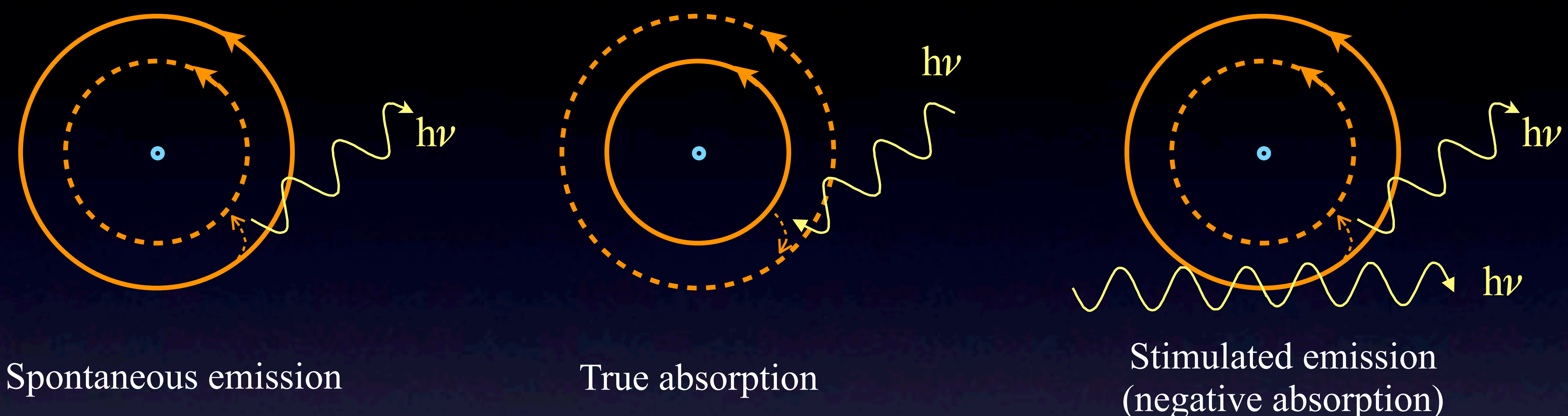
$$\alpha_\nu(p,\nu) \approx \frac{1}{2m\nu^2}\frac{1}{p\gamma}\partial_\gamma\left(\gamma p j_\nu\right)$$

Synchrotron self-absorption



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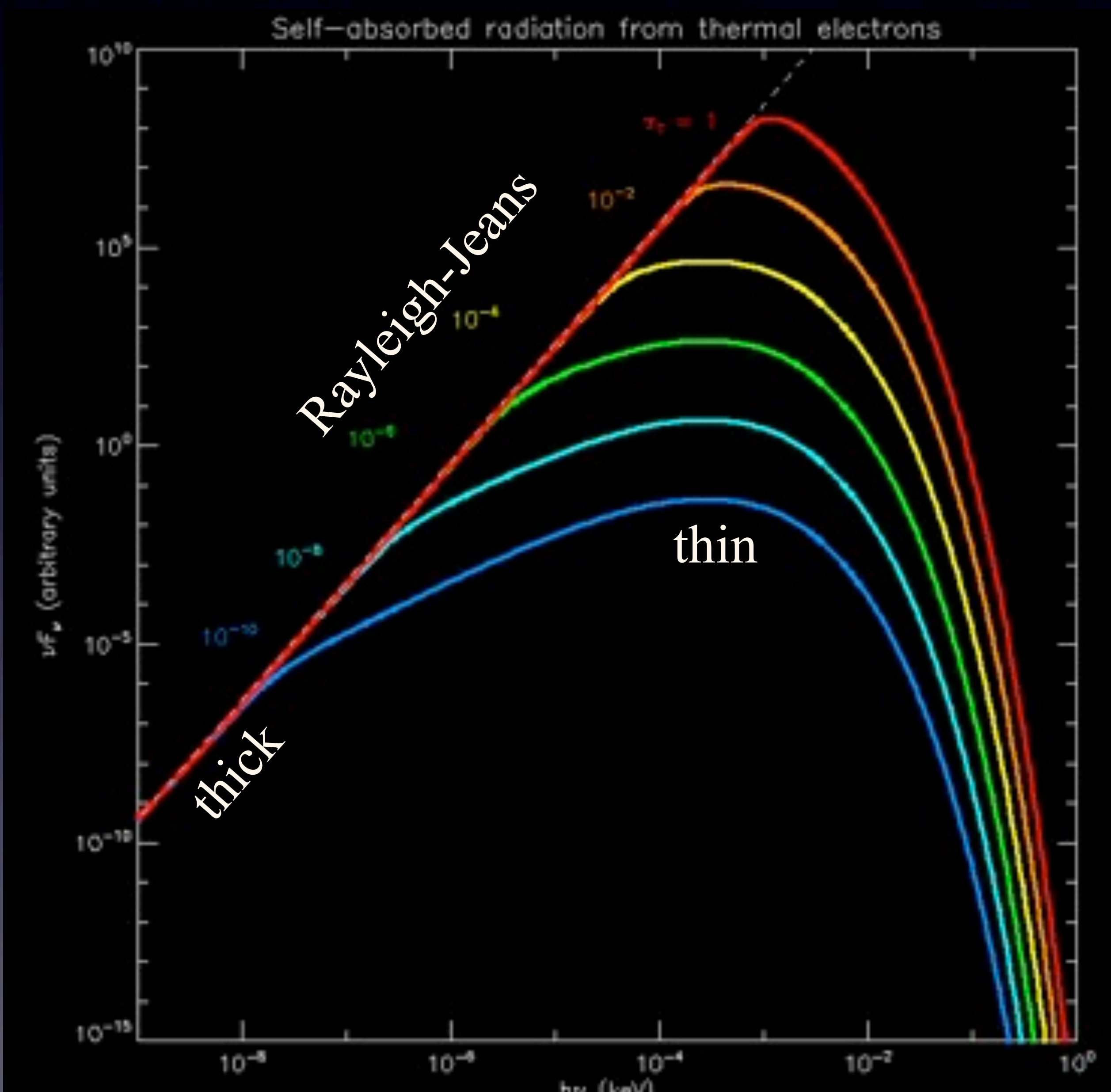
Stimulated emission
(negative absorption)

$$\alpha_\nu(p,\nu) \approx \frac{1}{2m\nu^2}\frac{1}{p\gamma}\partial_\gamma\left(\gamma p j_\nu\right)$$

- Radiative transfer problem: $I_\nu \approx \frac{j_\nu}{\alpha_\nu}\left(1-e^{-\alpha_\nu L}\right)$
- Transition thick/thin at the turnover frequency defined by: $\alpha_\nu(\nu_t)L \approx 1$

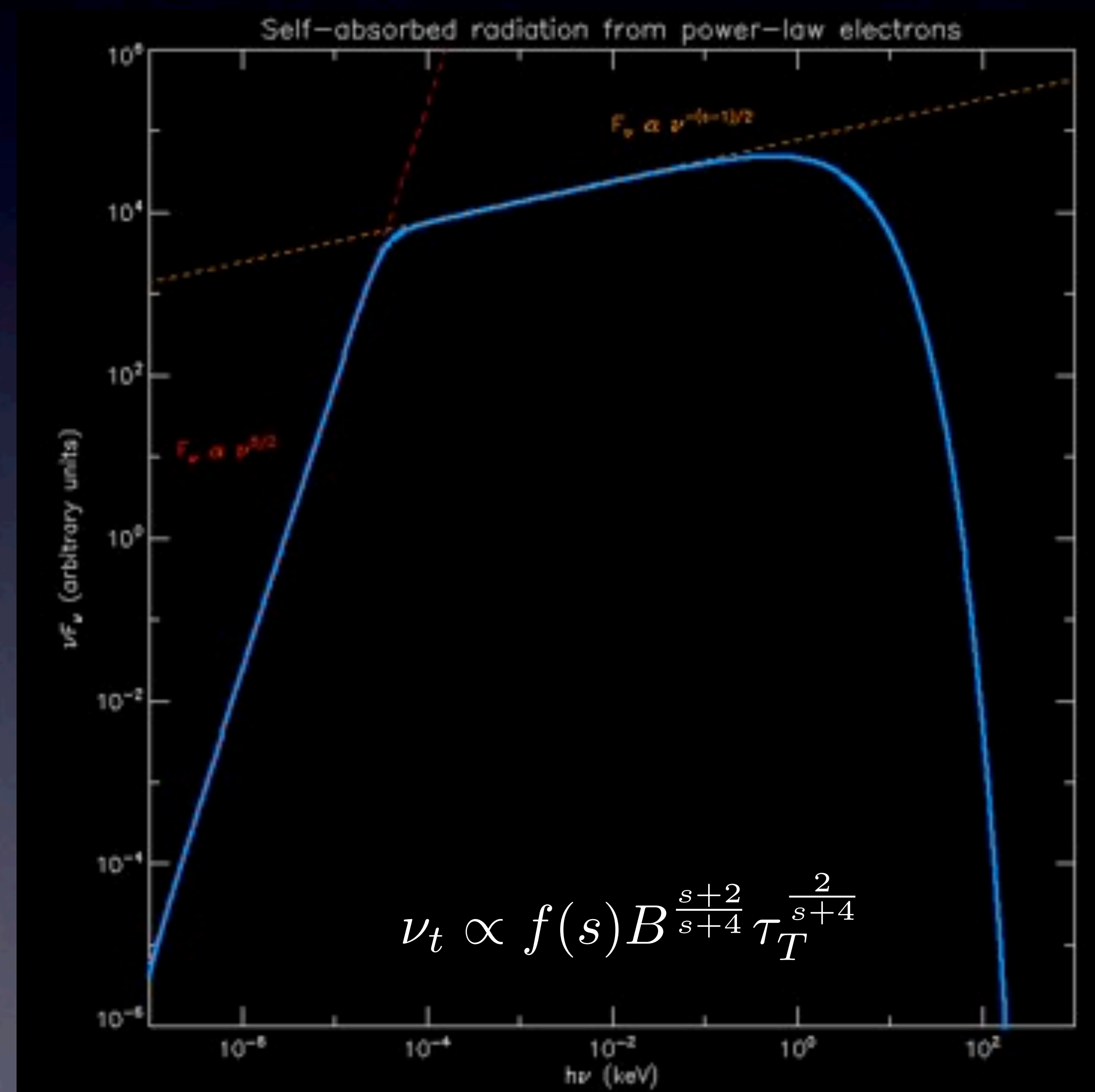
Self-absorbed Spectra

✓ Thermal distribution



ν_t **depends on** $\tau_T = n_e \sigma_T L$

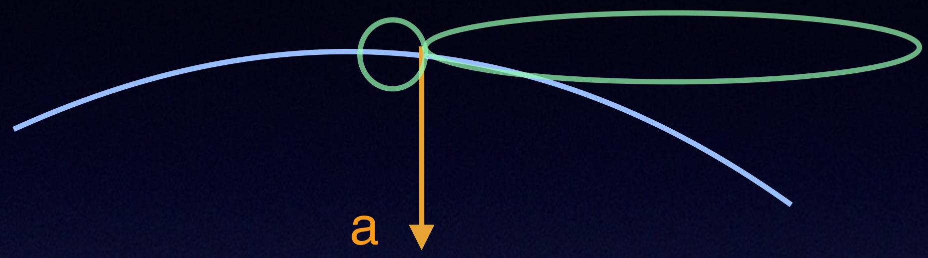
✓ Power-law distribution



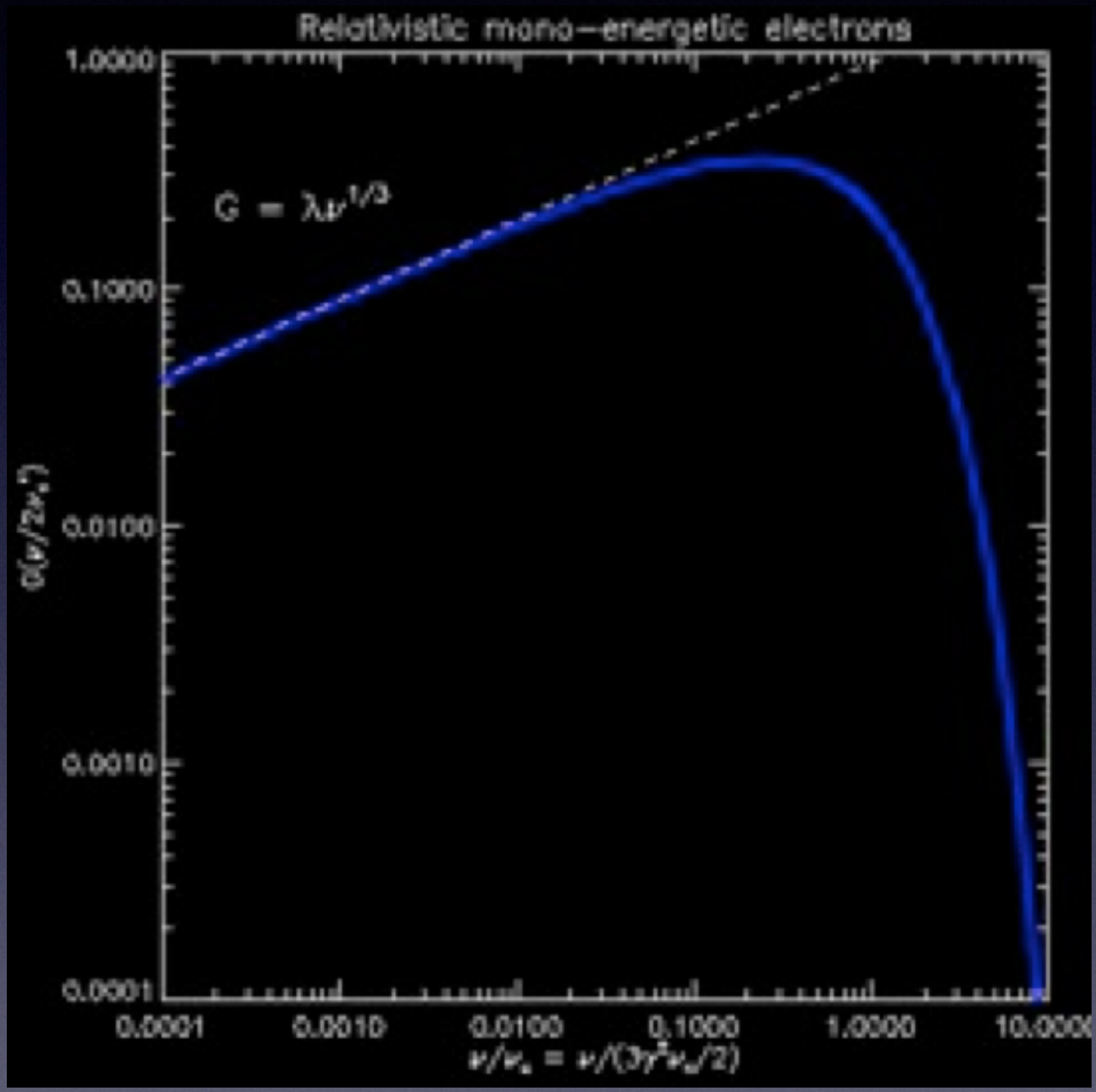
$$\nu_t \propto f(s) B^{\frac{s+2}{s+4}} \tau_T^{\frac{2}{s+4}}$$

Curvature Radiation

✓ Relativistic particles moving along curved magnetic field lines



curvature radius R



✓ Similar to synchrotron with: $\nu_{\rm L} = \dfrac{c}{2\pi R}$

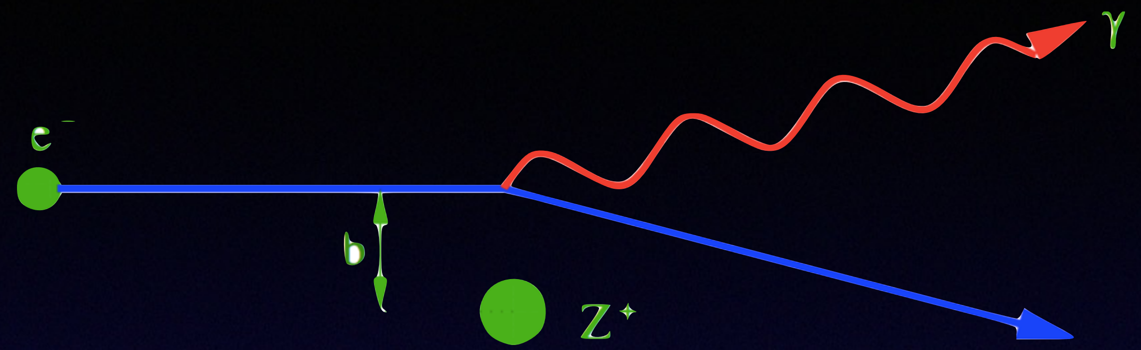
✓ Peak mono-energetic particle emission

$$\simeq \frac{3}{2}\gamma^2\nu_L$$

✓ Observed spectrum depends on energy and spatial distribution of particles and field lines in magnetosphere

✓ Possibility of coherent emission amplification

Bremsstrahlung



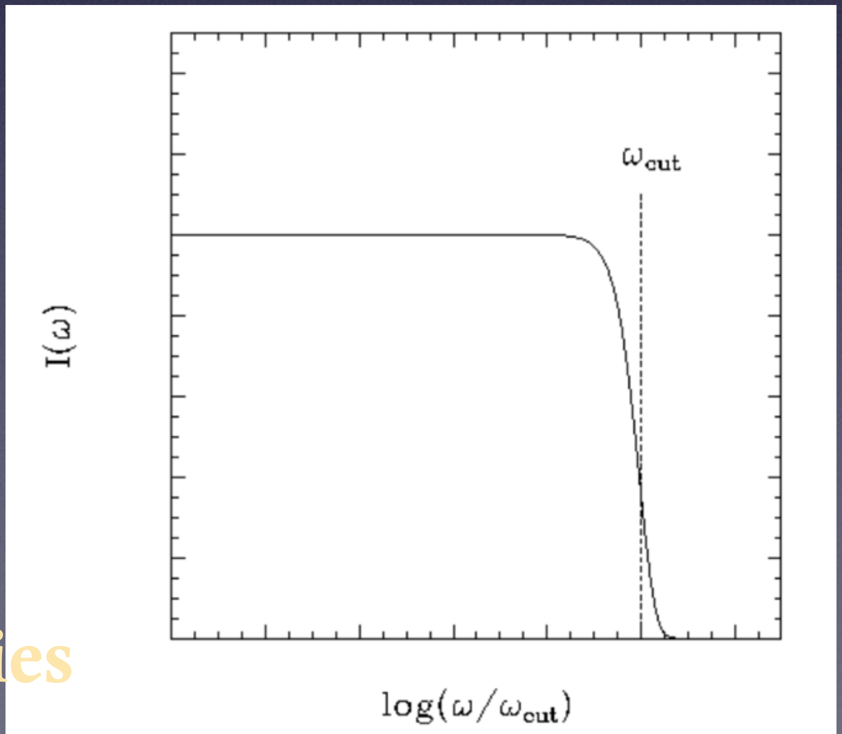
✓ Ion at rest, moving electron is deflected

✓ Transverse acceleration: distant observer sees a pulse of electric field.

✓ Spectrum: $$I_\omega = \frac{8Z^2e^6}{3\pi c^3 m_e^2 v_e^2 b^2} = cst$$

for $$\omega < \omega_{\rm cut} \simeq (2\Delta t_{\rm int})^{-1} \sim \frac{v_e}{2b}$$

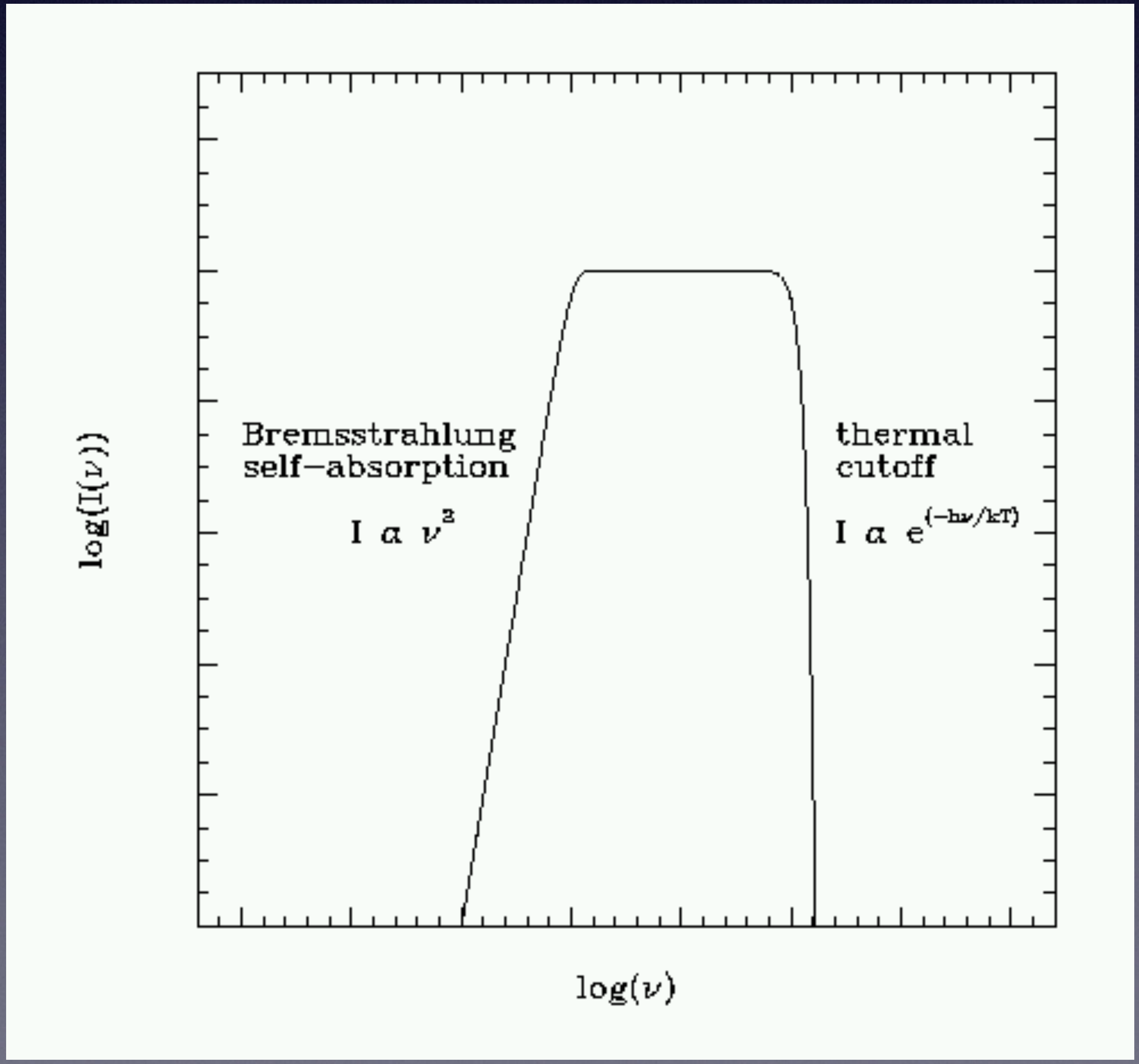
exponential cut off above that



✓ Distribution of electron: integration over velocities and impact factors

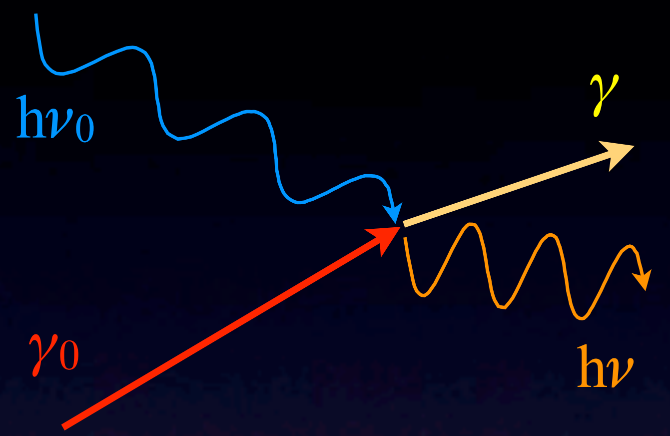
Thermal Bremsstrahlung

- ✓ **Emissivity:** $$j_\nu \propto n_e n_p T^{-\frac{1}{2}} \exp(-\frac{h\nu}{kT})$$
- ✓ **Total power:** $$J(T) \simeq 2.4 \times 10^{-27} \bar{g}_{ff}(T)\, n_e n_p T^{\frac{1}{2}} \quad \mathrm{erg\, s^{-1} cm^{-3}}$$
- ✓ **Self absorption**

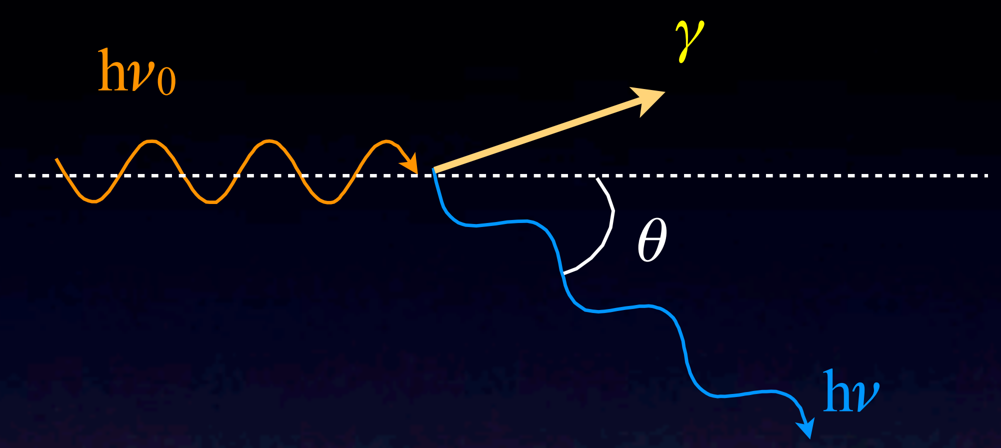


Compton Scattering

In the lab frame:



In the electron rest frame:



✓ Energy and momentum conservation: $$\frac{h\nu}{h\nu_0} = \frac{1}{1 + \frac{h\nu_0}{m_e c^2}(1 - \cos\theta)}$$

Compton Scattering

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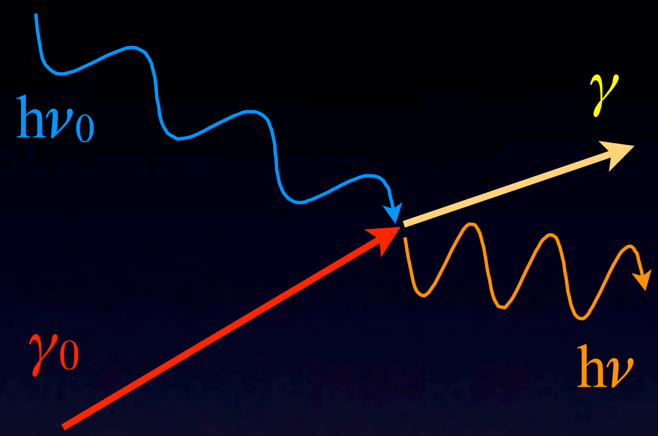

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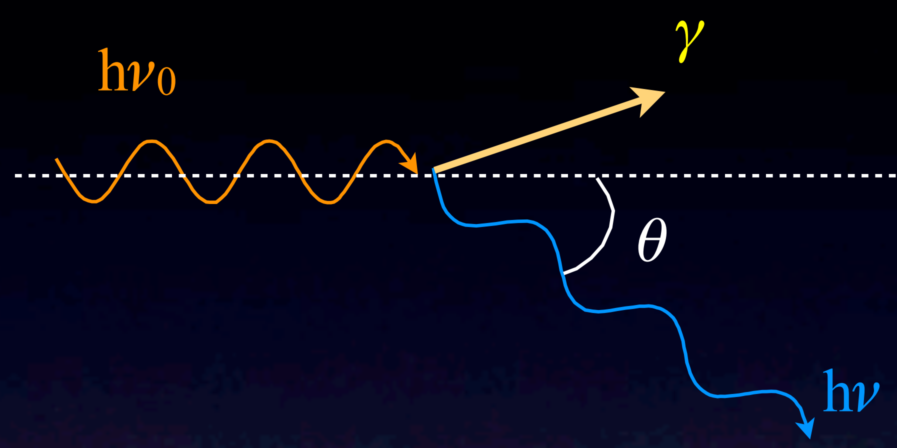

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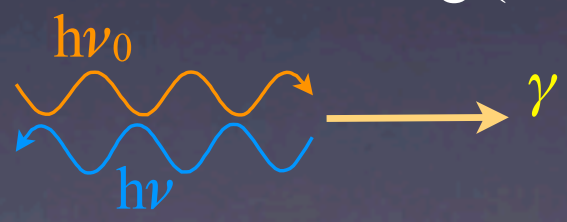
$$\frac{h\nu_0}{1 + 2h\nu_0/m_e c^2} \leq h\nu \leq h\nu_0$$

backward scattering (θ=π)



forward scattering (θ=0)

Compton Scattering

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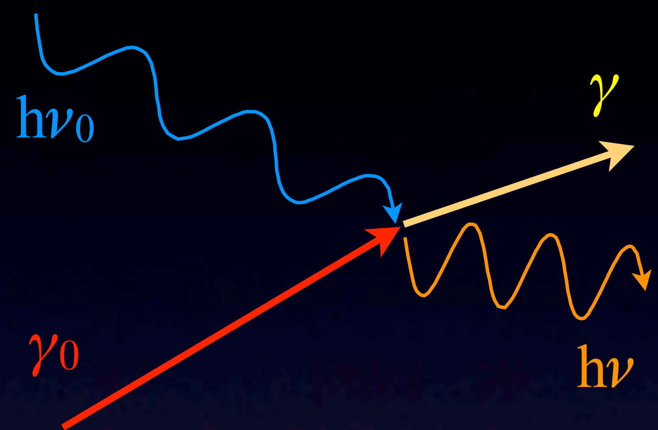

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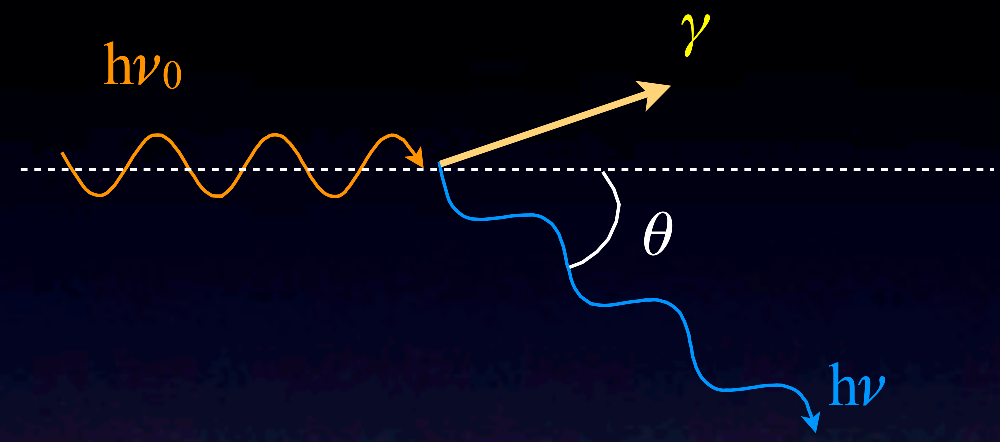

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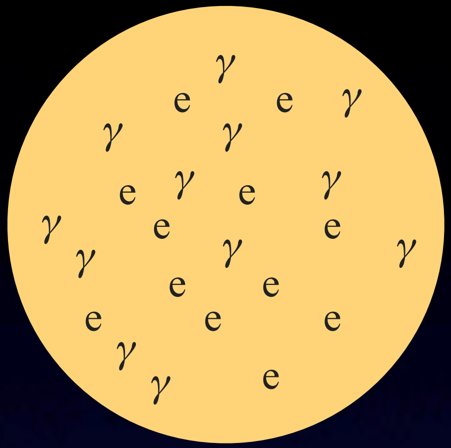
$$\frac{h\nu_0}{1 + 2h\nu_0/m_e c^2} \quad \leq \quad h\nu \quad \leq \quad h\nu_0$$

backward scattering (θ=π)

forward scattering (θ=0)

✓ For low energy photons ($h\nu_0 << mc^2$): Coherent scattering $h\nu_0 = h\nu$

Total Cross Section



- Source with 2 interacting species:
- The simplest case:
  - one species at rest, with number density n_1
  - one species with one single velocity v_2, number density n_2
  - otherwise: change of frame...

- Number of interactions per unit time and volume:

$$\frac{dn}{dt} = \sigma n_1 (v_2 n_2)$$

total cross section

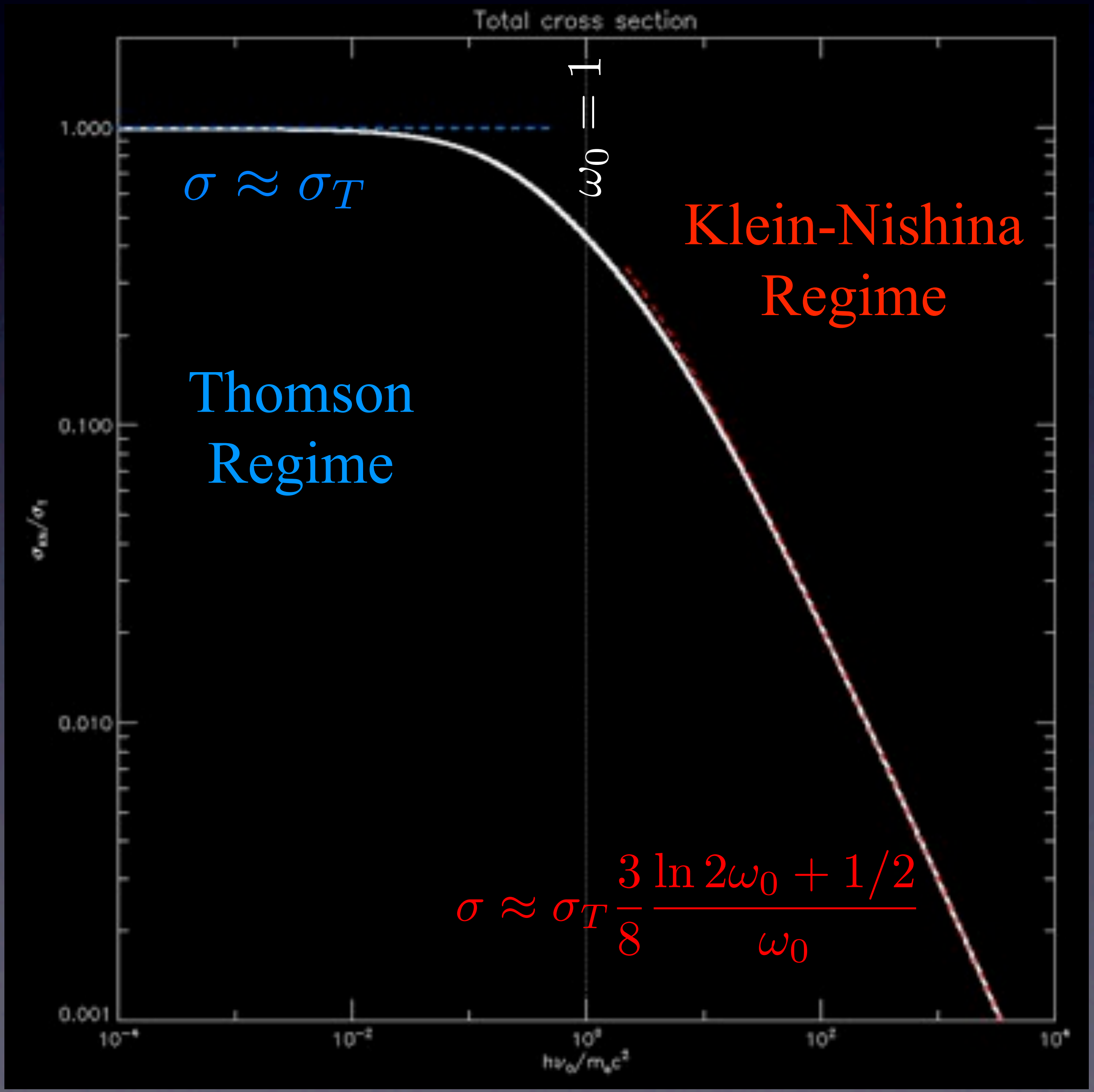
target density

incoming flux

Compton Total cross section

$$\sigma = \sigma_T \frac{3}{4}\left[\frac{1+\omega_0}{\omega_0^3}\left(\frac{2\omega_0(1+\omega_0)}{1+2\omega_0} - \ln(1+2\omega_0)\right) + \frac{\ln(1+2\omega_0)}{2\omega_0} - \frac{1+3\omega_0}{(1+2\omega_0)^2}\right]$$

$$\omega_0 = \frac{h\nu_0}{m_e c^2}$$



(in the particle rest frame)

The Total cross section

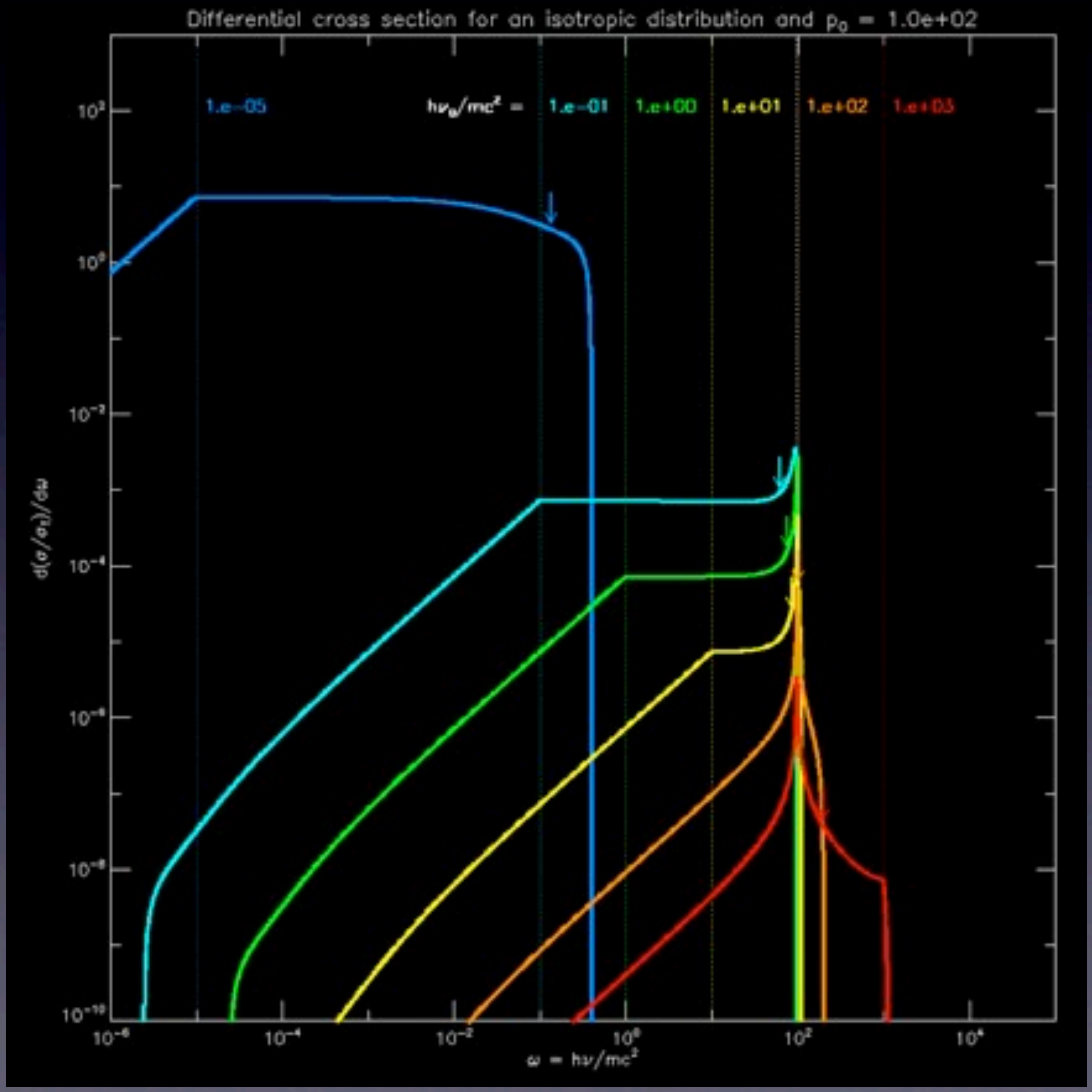
(in the plasma frame)

$$\sigma(\omega_0, p_0) \approx \sigma_T \frac{3}{4}\left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right) + \frac{\ln(1+2x)}{2x} - \frac{1+3x}{(1+2x)^2}\right]$$

$$x = \gamma\omega = \gamma\frac{h\nu}{m_e c^2}$$

➡ **Transition to KN regime for** $\gamma\omega > 1$

Spectrum of a single scattering



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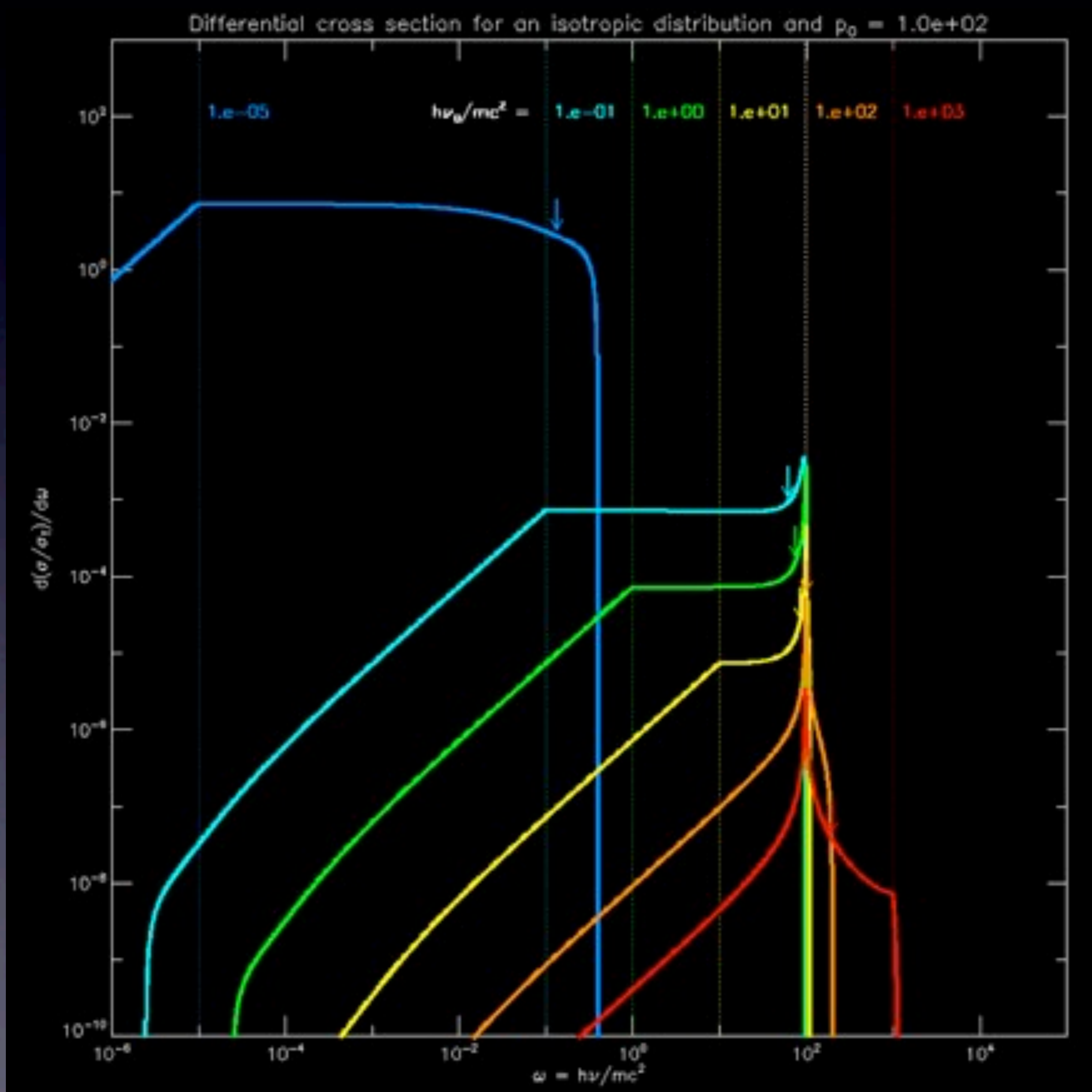


✓ γ^2-1 << ω: down-scattering

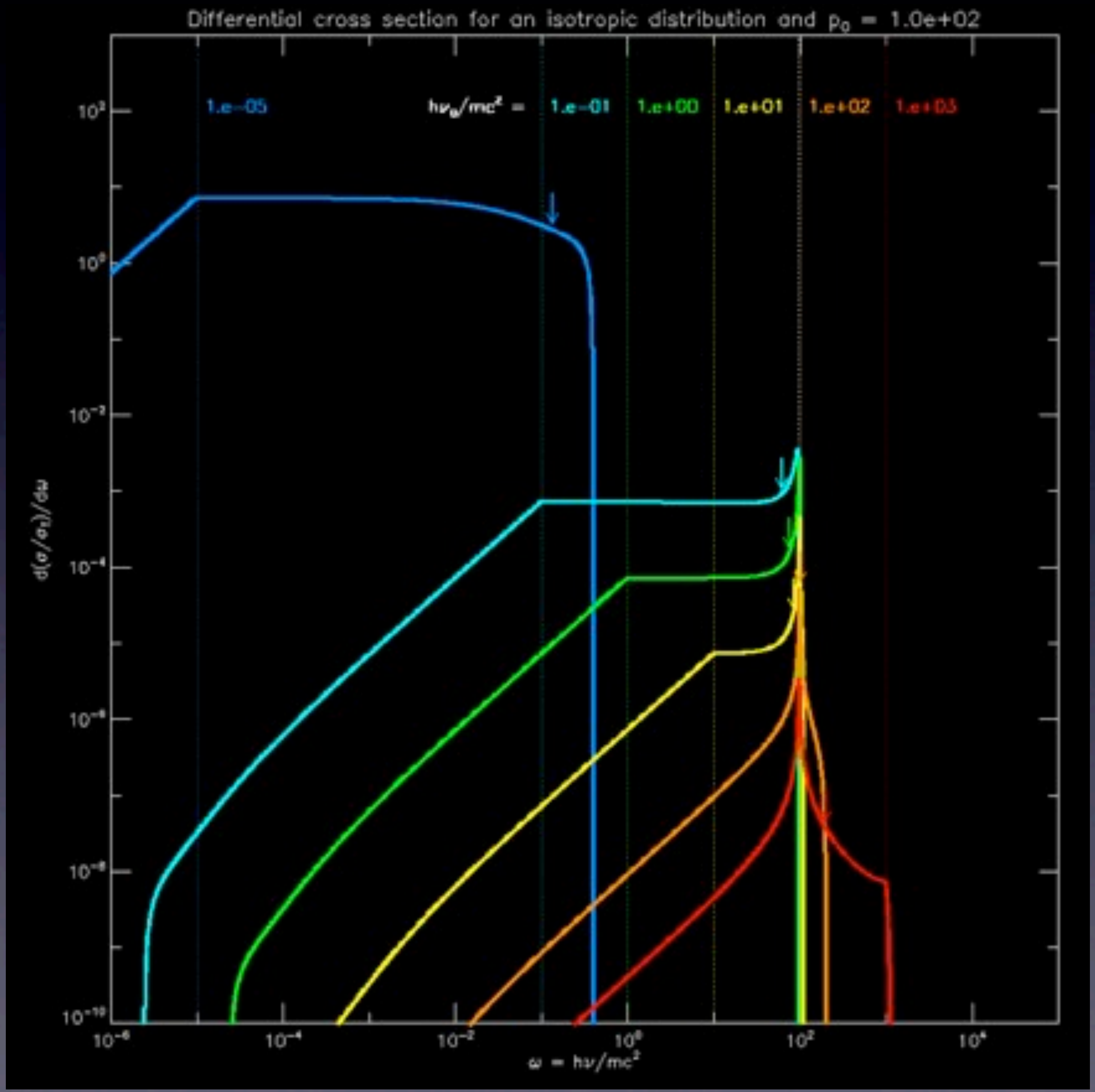
✓ γ^2-1 >> ω: up-scattering

- Amplification factor:

$$A = \frac{< \omega >}{\omega_0}$$

- In the Thomson regime ($\gamma\omega$<<1):

Spectrum of a single scattering



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✓ γ^2-1 >> ω: up-scattering

- Amplification factor:

$$A = \frac{<\omega>}{\omega_0}$$

- In the Thomson regime ($\gamma\omega$<<1):

$$A \approx 1 + \frac{4}{3}p^2$$

$$A \approx \frac{4}{3}p^2 \quad \text{for} \quad p >> 1$$

Electron losses in Thomson regime

✓ Average energy radiated by an electron in one interaction (Thomson limit):

$$\frac{\Delta E}{m_e c^2} = (A-1)\omega = \frac{4}{3}p^2\omega$$

$$\gamma\omega << 1$$

$$p^2 >> \omega$$

✓ Electron scattering rate

$$\frac{dn}{dt} = \sigma_{\rm T} c n_{ph}$$

✓ Radiated power: $P = \frac{dn}{dt}\Delta E$

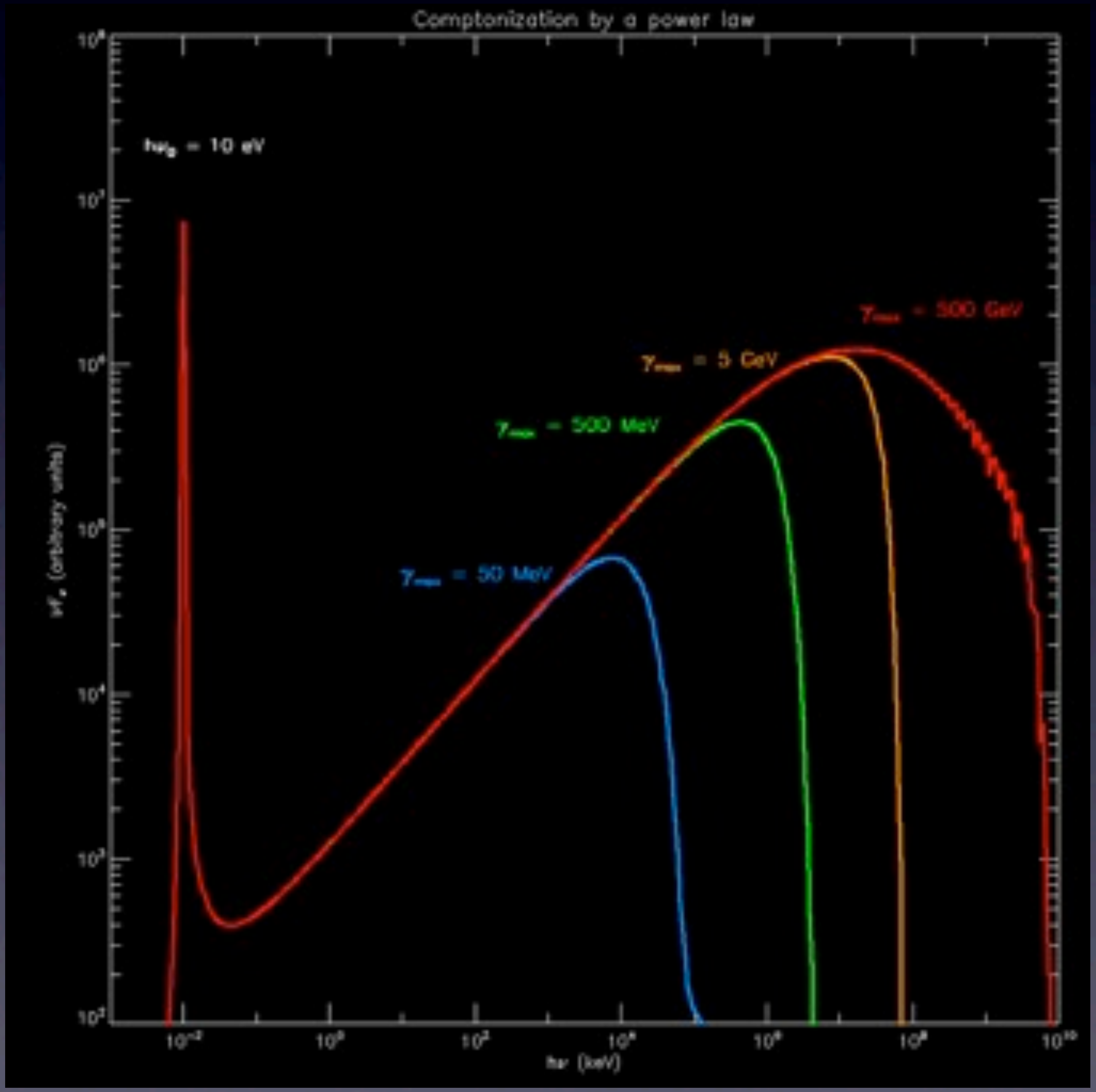
$$P = \frac{4}{3}c\sigma_T p^2 U_{\rm ph}$$

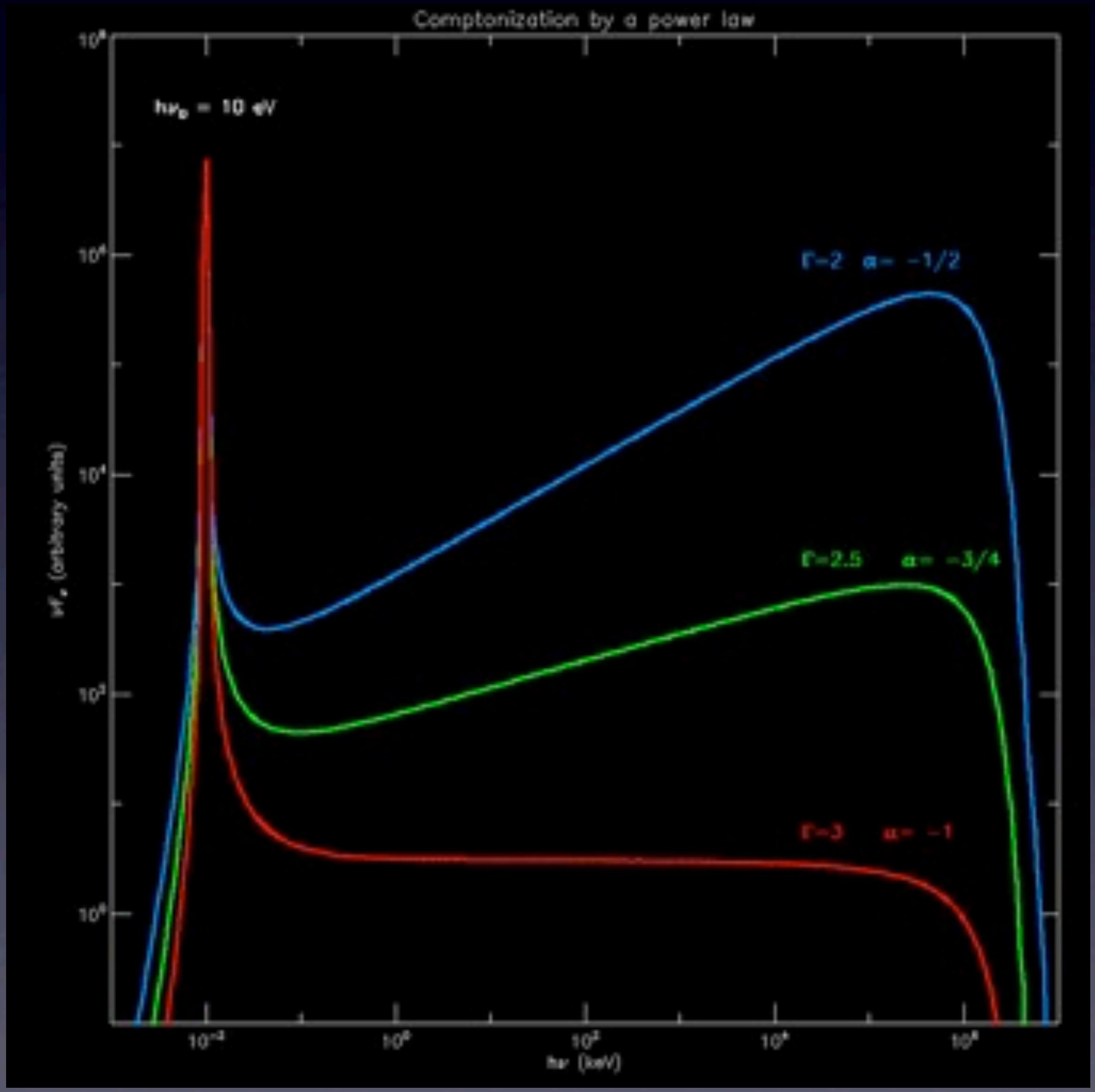
✓ Independent of energy distribution of target photons

✓ Similar to Synchrotron losses

Single scattering off a power-law

- ✓ Power-law particle distribution $N(\gamma) = \gamma^{-s}$ for $\gamma_{min} < \gamma < \gamma_{max}$
- ✓ Spectrum: power-law: $F_\nu = \nu^{-\alpha}$





$h\nu_{max} = \gamma_{max} mc^2$

$\alpha = (s-1)/2$ (like synchrotron)

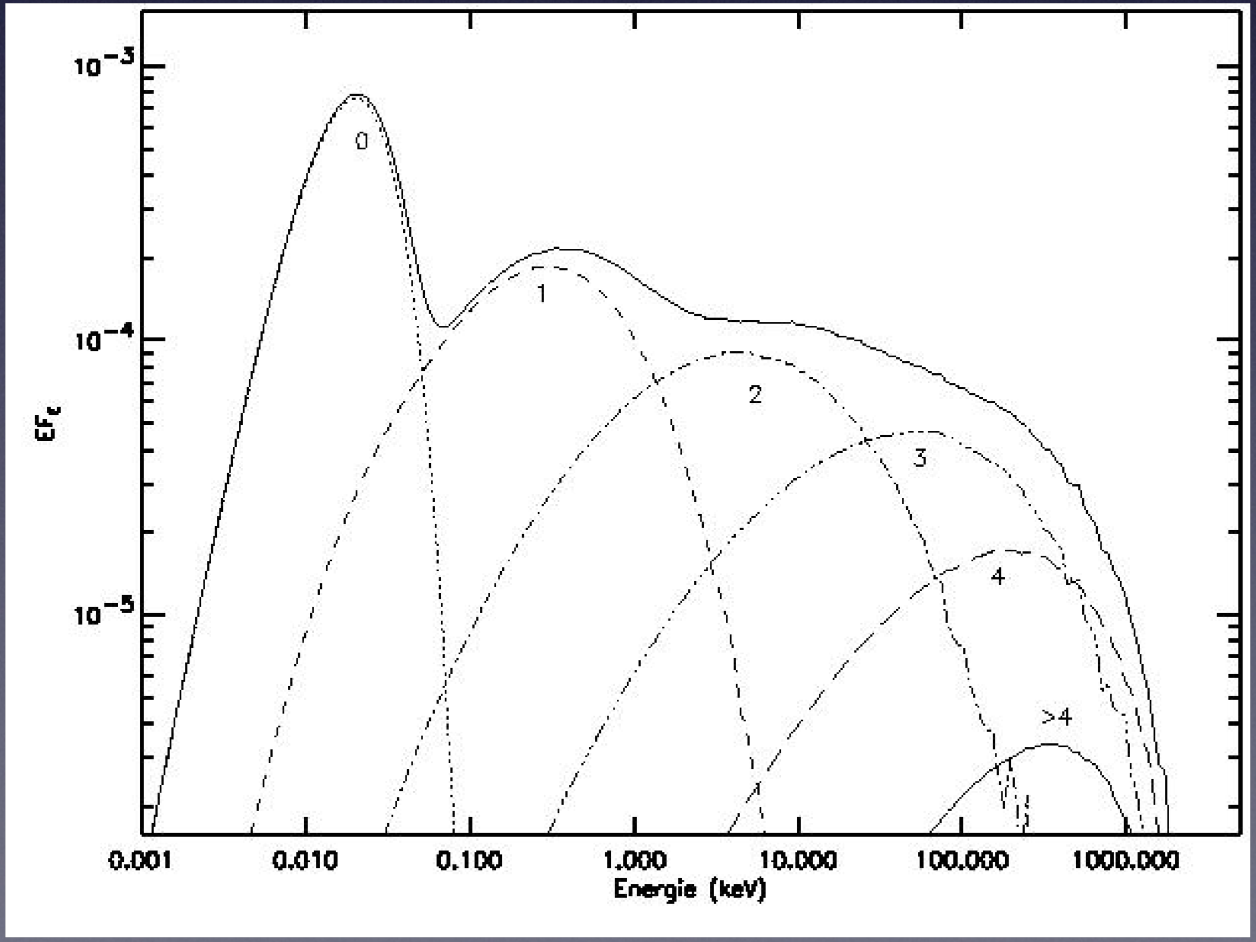
Multiple Scatterings

- ✓ Photons can undergo successive scattering
- ✓ Medium of finite size L: Thomson optical depth: $\tau = \sigma_T N_e L$
- ✓ Competition scattering/escape/absorption:
  - ✓ τ = Mean number of scattering before escape (or τ^2)
    - ✓ small τ: inefficient Compton scattering
    - ✓ large τ: efficient Compton scattering

Thermal Comptonization

- ✓ Comptonization of soft photons on a thermal plasma of electrons (Maxwellian energy distribution)
- ✓ Parametrized by temperature T and Thomson optical depth

$$\tau = n_e \sigma_T R$$



$$F_E \propto E^{-\Gamma(kT,\tau)} \exp\left(-\frac{E}{E_c(kT,\tau)}\right)$$

$$E_c \simeq kT$$

$$\Gamma(kT_{\mathrm{e}}, \tau)$$

Spectral degeneracy: different T_{e} **and** τ **give same** Γ

Pair production/Photon annihilation

- In the center of momentum of the 2 incoming photons
- Conservation of momentum and energy: $\omega = \gamma$
  - 2 photons of energy hν
  - 2 leptons of energy: γ (for all production direction)
  - Production threshold $\omega > 1$

Total cross section

In the lab frame

- ✓ Threshold: $\gamma_{cm} = \omega_1 \omega_2 \dfrac{1 - \cos\theta}{2} \geq 1$
  - ✓ Head-on collisions (θ=π): $\omega_1 \, \omega_2 > 1$
  - ✓ Trailing collisions (θ=0): $\omega_1 \, \omega_2 \rightarrow \infty$

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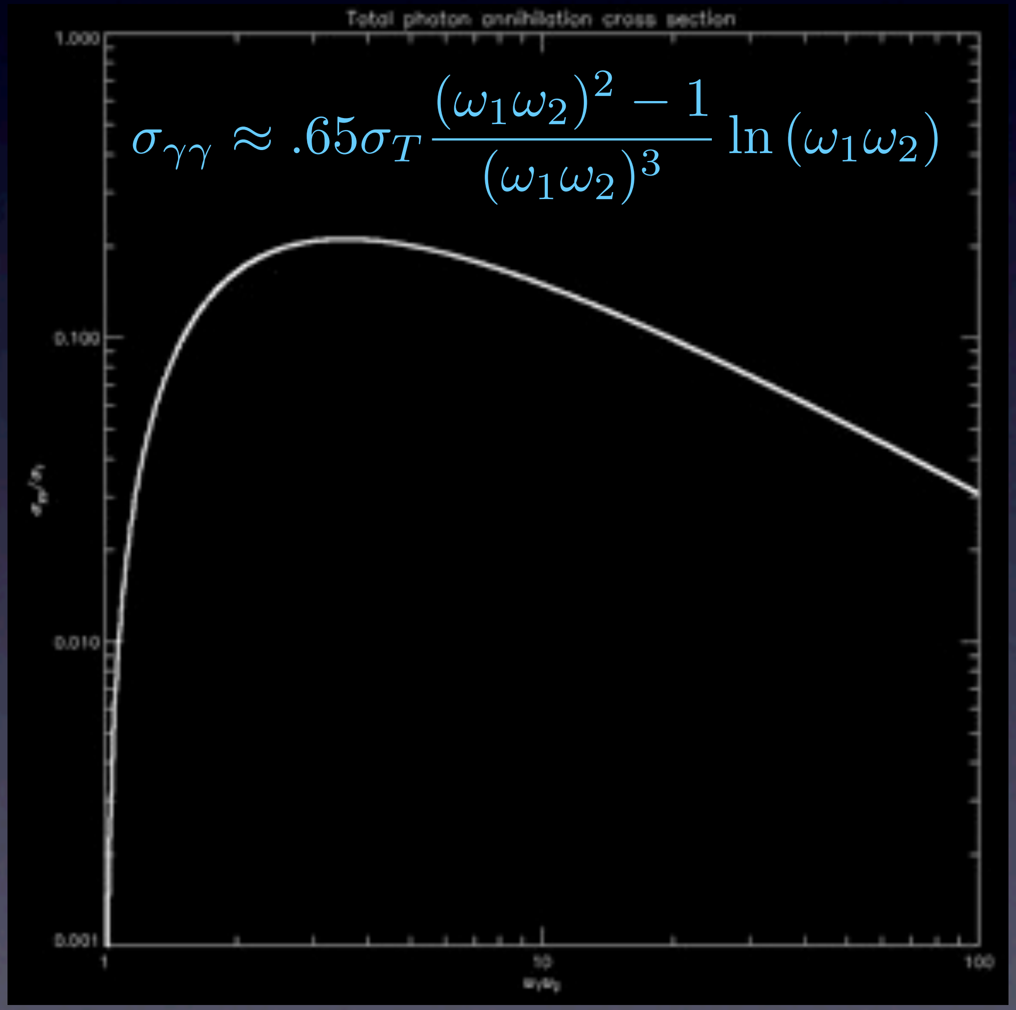
- Total cross section for isotropic photon field
  - Analytical (Gould&Schreder67, approx: Coppi&Blandford90)
  - Maximal absorption for: $\omega_2 \approx 1/\omega_1$
  - TeV $\leftrightarrow$ 0.1 eV
  - GeV $\leftrightarrow$ 100 eV



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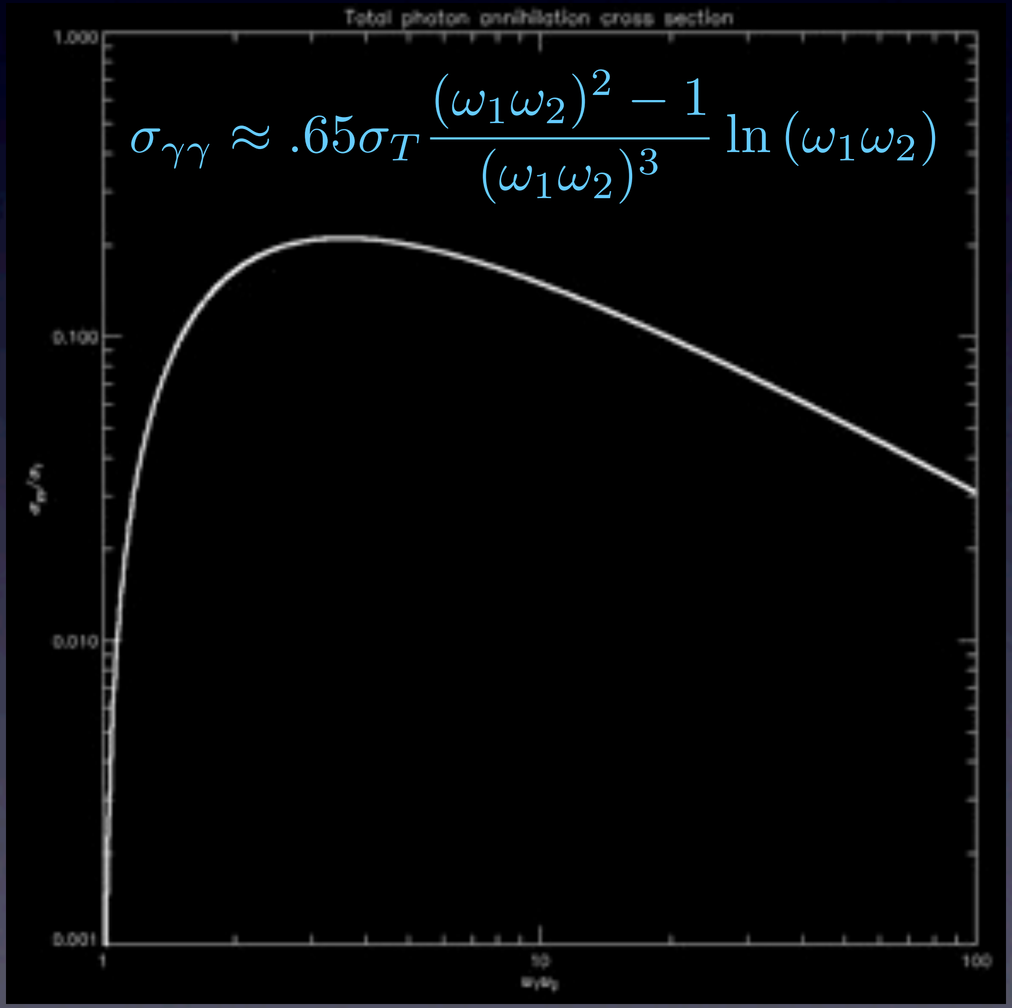
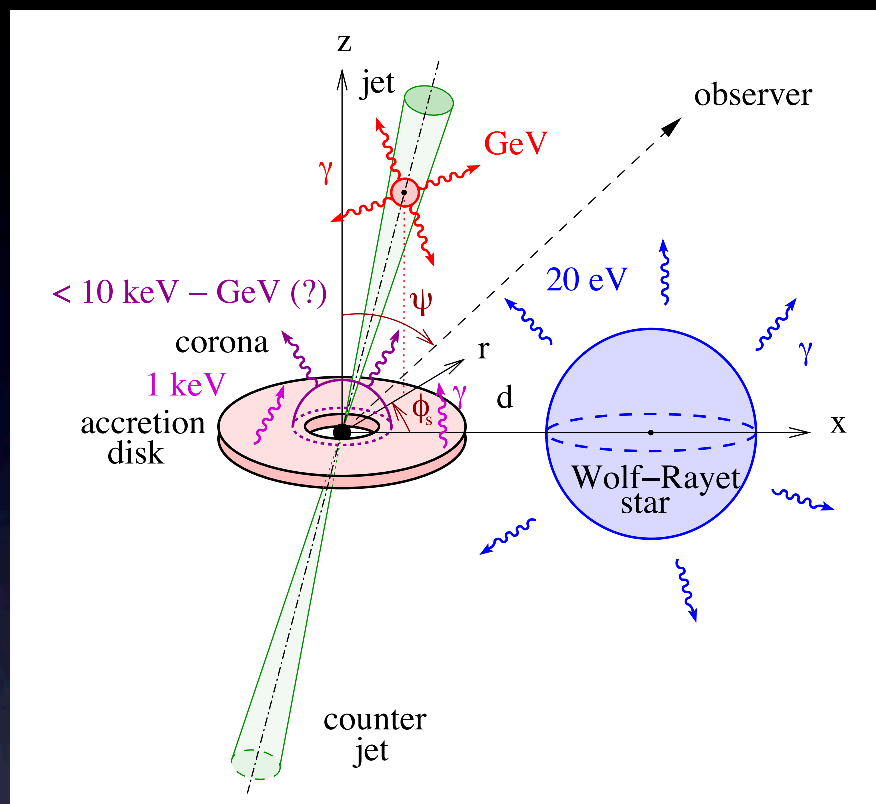
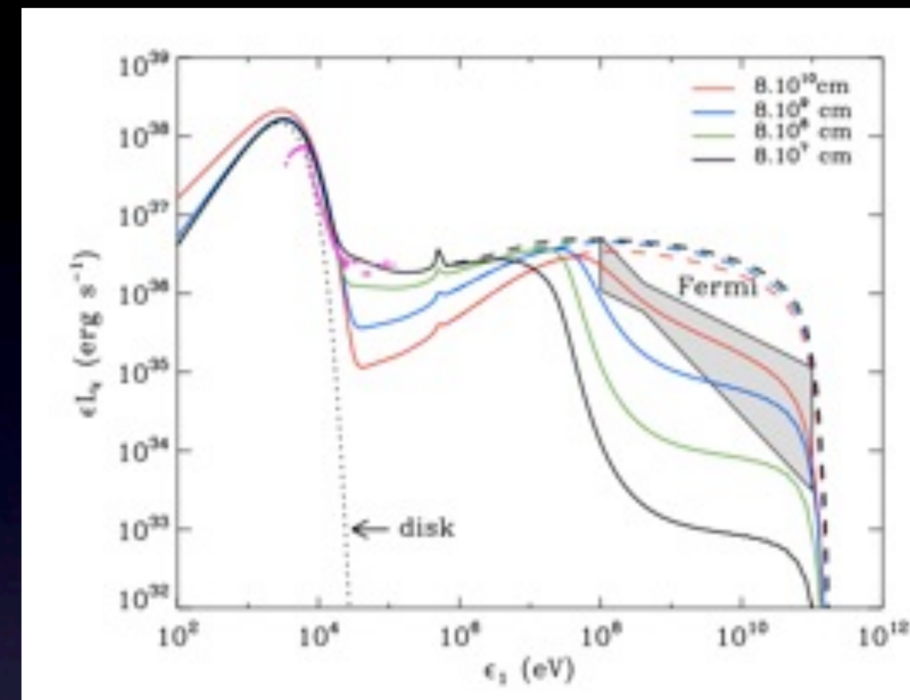


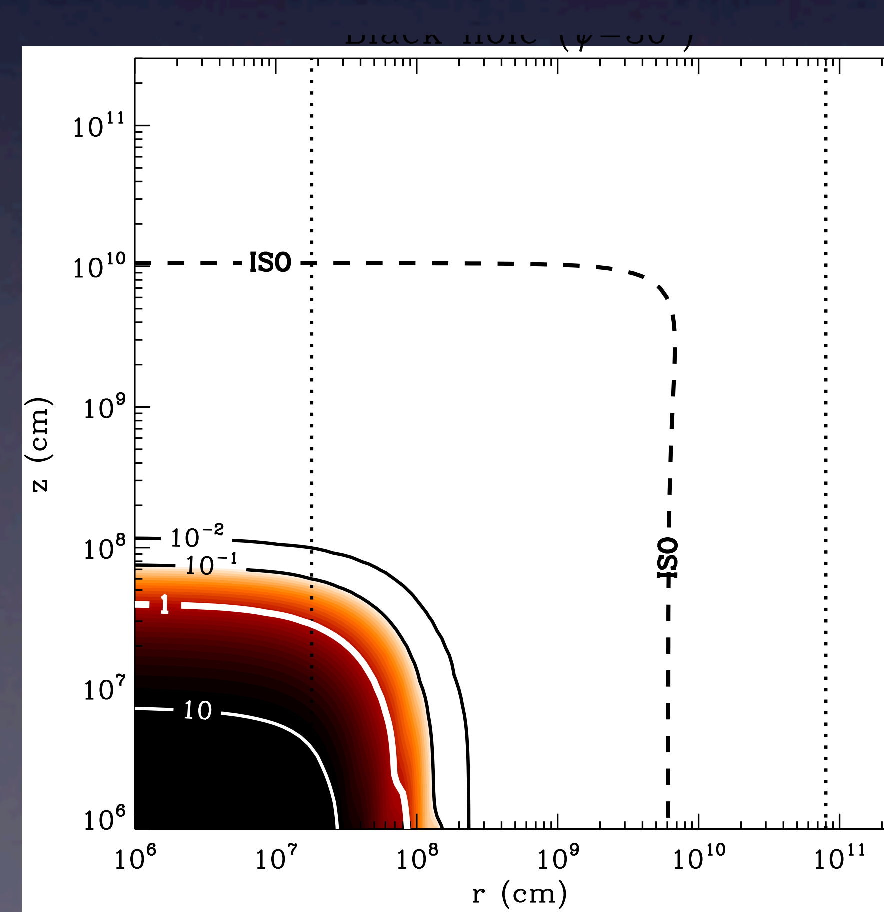
- ✓ Photon-photon absorption

$$\tau_{\gamma\gamma}(\omega) = L \int N_\omega(\omega_0)\sigma_{\gamma\gamma}(\omega_0, \omega)d\omega_0 \approx 0.2\sigma_T L \frac{N_\omega(1/\omega)}{\omega}$$

Photon absorption in gamma binaries







- ✓ Strong photon field
  - ✓ From the companion star
  - ✓ The accretion disk
- ✓ Efficient photon-photon absorption
- ✓ Ex: Cyg -X3:
  - ✓ GeV detection by Fermi (Abdo et al. 2009)
  - ✓ Anisotropic Absorption maps (Cerutti et al. 2011)
  - ✓ => GeV production far from the BH (not coronal)

Conclusions

- At high energy
  - Total cross sections drop off
  - Differential cross sections become highly anisotropic
- Particle cooling:
  - Synchrotron: $P \propto \sigma_T p^2 U_B$
  - Compton in the Thomson regime: $P \propto \sigma_T p^2 U_{ph}$
  - Bremsstrahlung: $P \propto \sigma_T \alpha_f \, p \, U_i$ (with $U_i = n_i \, m_e c^2$)
- Photons:
  - Synchrotron:
    - Thin spectrum of 1 particle peaks at $\nu_c \propto \gamma^2 B$
    - Thin spectrum of a power-law distribution is a power-law
    - Absorption => Thick spectrum at low frequency
  - Compton
    - Amplification factor in the Thomson regime: $A = \gamma^2$
    - Mildly relativistic particles: power-law spectrum
    - Comptonization by a relativistic power-law distribution is a PL spectrum
  - $\gamma - \gamma$ pair production:
    - Threshold at $\omega_1 \omega_2 \approx 1$
    - Most efficient photon absorption for $\omega_1 \omega_2 \approx 1$