Constraints on a stochastic non-helical background of primordial magnetic fields with compensated initial conditions from CMB anisotropies power spectrum

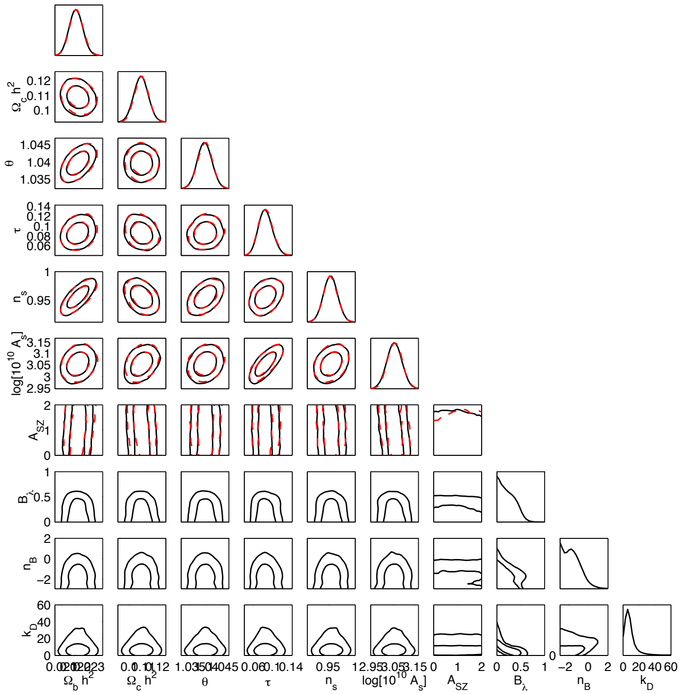
WMAP 7 + ACBAR/QUAD

Paoletti, FF, PRD 2010

- We perform an Markov Chain Monte Carlo exploration of 9-dimensional parameters space

$$\{\Omega_b h^2 \,, \Omega_c h^2 \,, \tau \,, \theta \,, \ln(10^{10} A_S) \,, n_S \,, A_{\rm SZ} \,, B_\lambda \,, n_B\}$$

- We use an improved combination of WMAP 7 yr, ACBAR, QUAD and BICEP data (improved since we minimize the cross-correlation among different data sets) up to l = 2000.
- We use an approximation for the PMF EMT correlators valid for any n_B ("validated" since anchored to the exact results for integer and semi-integer values of n_B obtained in the previous papers)
- We use scalar compensated and vector contribution (the tensor compensated is subleading). Lewis 2004
- We take into account the cross-correlation between Ω_B and L_B
- We use $[0\,,10]$ and $[-2.9\,,3]$ as priors for $B_{|1{\rm Mpc}}/(10{\rm nG})$ and n_B, respectively.



B_λ and n_B are not correlated with the other parameters

No evidence for PMF: B_λ < 5 nG at 95%CL

Λ CDM model (without PMF)

CMB constraints are dominated by the vector contribution (in agreement with Yamazaki et al. 2006)

Results consistent with Shaw & Lewis 2009, 2010

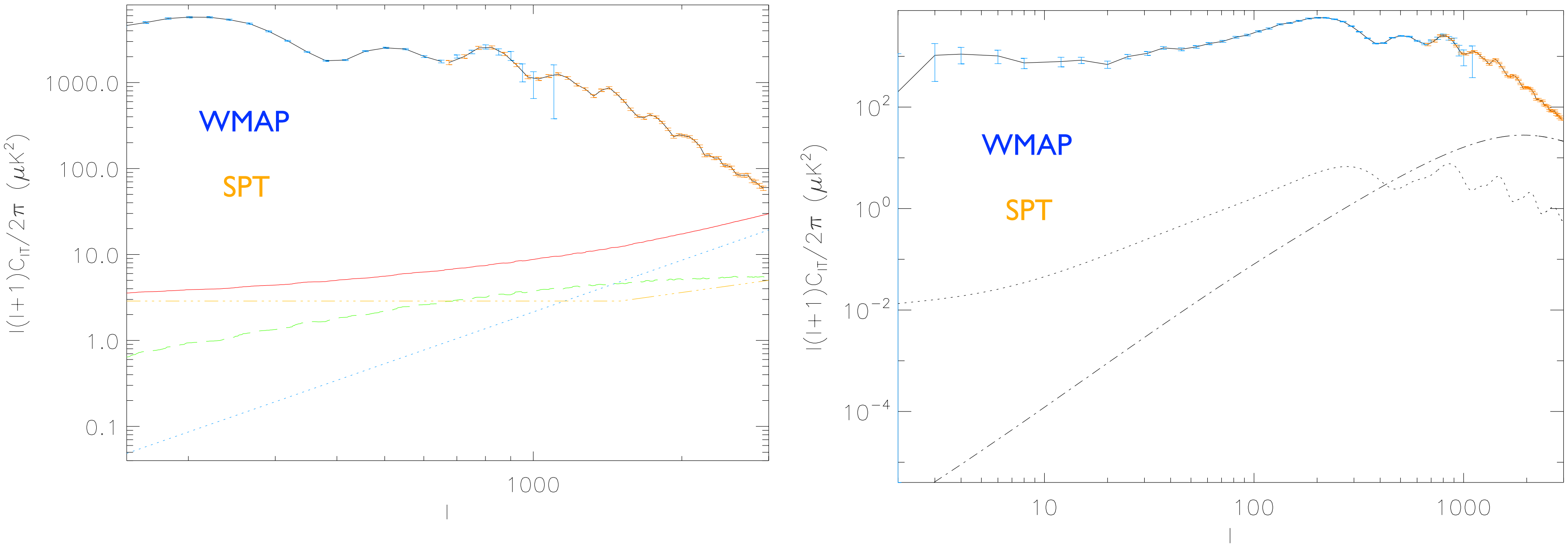
CMB constraints depend on n_B: for $n_B=2$ $B_{1{\rm Mpc}} < 23$ pG.

WMAP 7 + SPT

Paoletti, FF, 2012

- Available CMB data at higher multipoles: Atacama Cosmology Telescope, South Pole Telescope, Planck ...

- SPT measurements reach high multipoles where the Silk damping suppress the CMB primary contribution and estimates the contribution of radio sources, CIB and SZ. SPT data up to l = 3000 from Keisler et al. (2011) Reichardt et al. (2012) @150 GHz



adiabatic bestfit
SZ
Radio sources
Clustering
Total FG contribution

- adiabatic best-fit
.... scalar magnetic
.-.- vector magnetic

- PMF contribution to CMB anisotropies is not suppressed by Silk damping as the adiabatic primary contribution. Unfortunately other foreground residuals and secondary anisotropies are relevant at those multipoles.

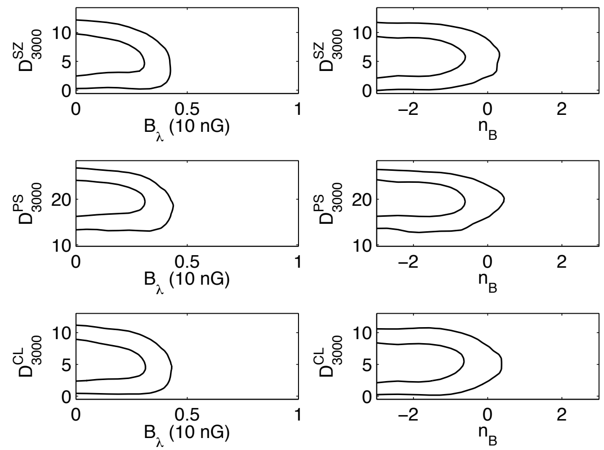
- High l foregrounds modelled as $D^{SZ}_{3000}\,,D^{PS}_{3000}\,,D^{CL}_{3000}$

- Constraints on the amplitude marginalized on n_B $\quad B_{1\mathrm{Mpc}} < 3.5\,\mathrm{nG\,at\,95\%\,CL}$

- Fixing n_B

$$n_B = 0 \quad B_{1\mathrm{Mpc}} < 0.56\,\mathrm{nG\,at\,95\%\,CL}$$
$$n_B = 2 \quad B_{1\mathrm{Mpc}} < 6.6\,\mathrm{pG\,at\,95\%\,CL}$$
$$n_B = 3 \quad B_{1\mathrm{Mpc}} < 0.7\,\mathrm{pG\,at\,95\%\,CL}$$

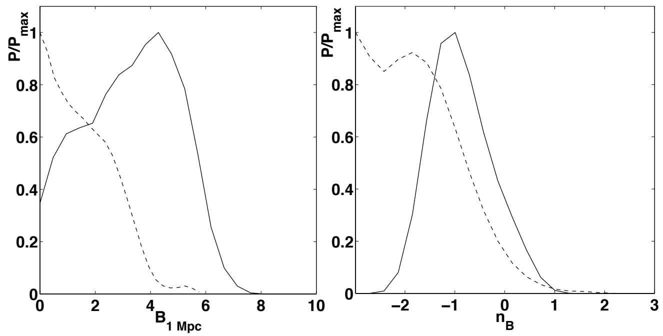
These foreground parameters are not degenerated with the magnetic ones.



- Be aware: not modeling high-l foregrounds at all leads to surprising results

solid: no FG

dashed: FG included



Planck Forecast

- We use Planck simulated data with nominal mission sensitivities and angular resolution as from pre-flight characterization.

- We combine in inverse noise weighting the 70, 100, 143, 217, 353 GHz channels.

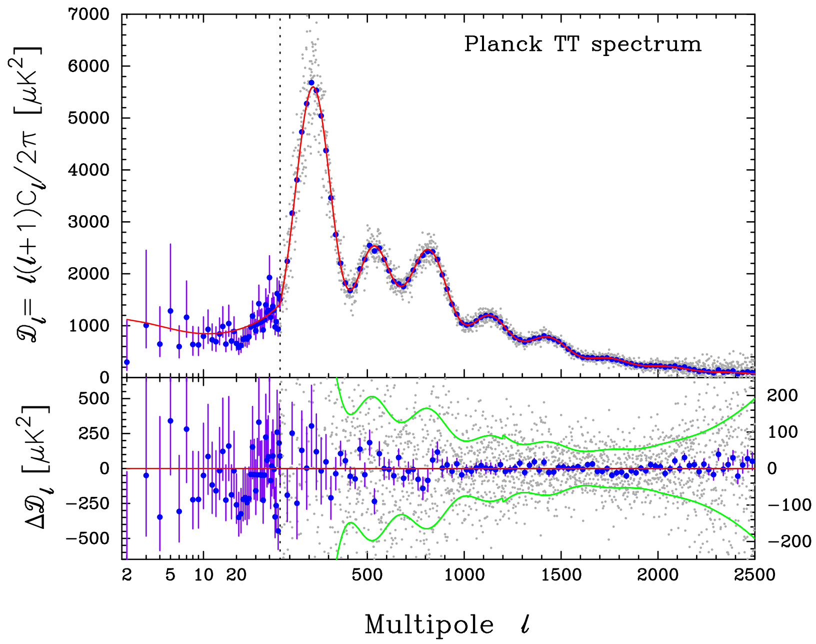
- We assume a fiducial model without PMF with input cosmological parameters in agreement with current data

- When taking into account foreground residuals and secondary anisotropies in temperature at high multipoles following the model by Paoletti et al. 2011, arXiv:1112.3260 at high multipoles the constraint on B degrades.

Taking into account foregrounds	$B_{1\mathrm{Mpc}} < 3.6\,\mathrm{nG\,at\,95\%\,CL}$	Paoletti, FF 2012
Taking into account only instrumental specifications (sensitivity and angular resolution)	$B_{1\mathrm{Mpc}} < 2.7\,\mathrm{nG\,at\,95\%\,CL}$	Paoletti, FF 2010

Planck 2013 results for PMF

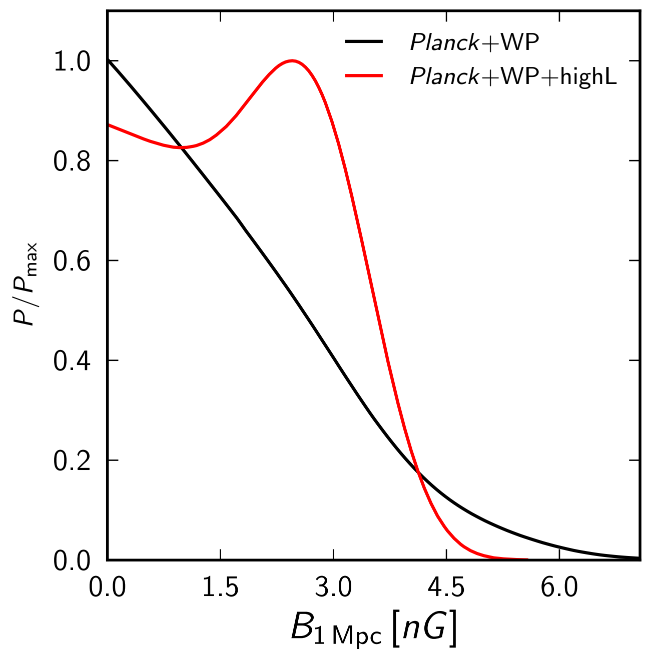
Planck 2013 results XXVI: Cosmological parameters



- High l foregrounds model in the Planck likelihood which includes 11 foreground nuisance parameters
- When Planck complemented with high-l 25 foreground nuisance parameters

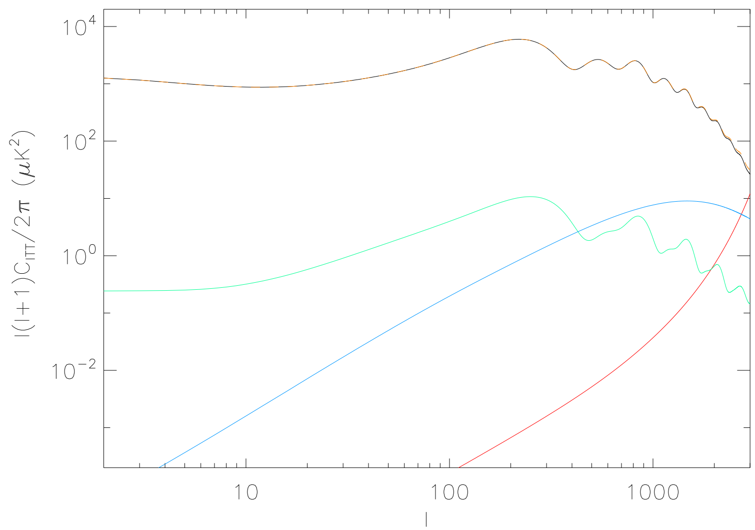
Planck + WP $\quad B_{1\mathrm{Mpc}} < 4.1\,\mathrm{nG}$ at 95% CL

Planck + WP + high-l $\quad B_{1\mathrm{Mpc}} < 3.4\,\mathrm{nG}$ at 95% CL



COrE Forecast

- We use COrE simulated data with sensitivities at http://www.core-mission.org.
- We combine in inverse noise weighting the 75, 105, 135, 165, 195, 225 GHz channels.
- We assume a fiducial model without PMF with input cosmological parameters in agreement with current data

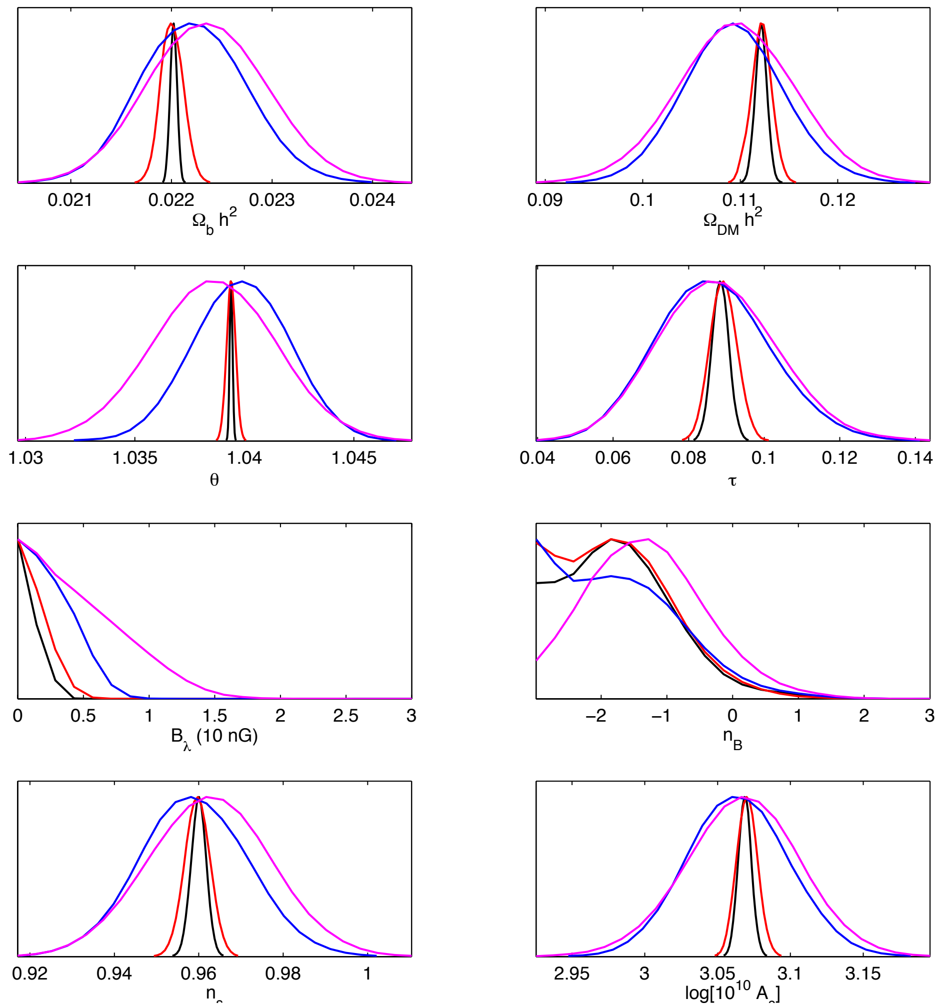


WMAP 7 yr

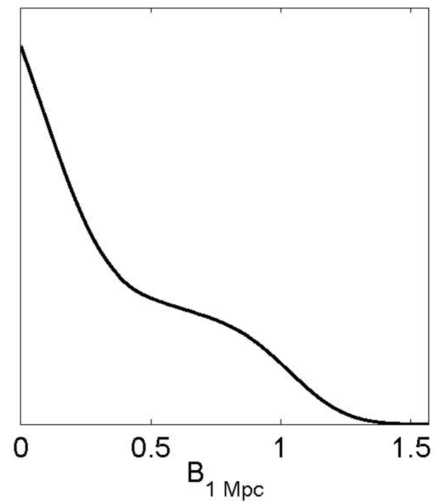
WMAP 7 yr, ACBAR, QUAD and BICEP data

Planck Forecast

COrE forecast: constraints on B around 1 nG, as see also COrE white paper.



Prism Forecast

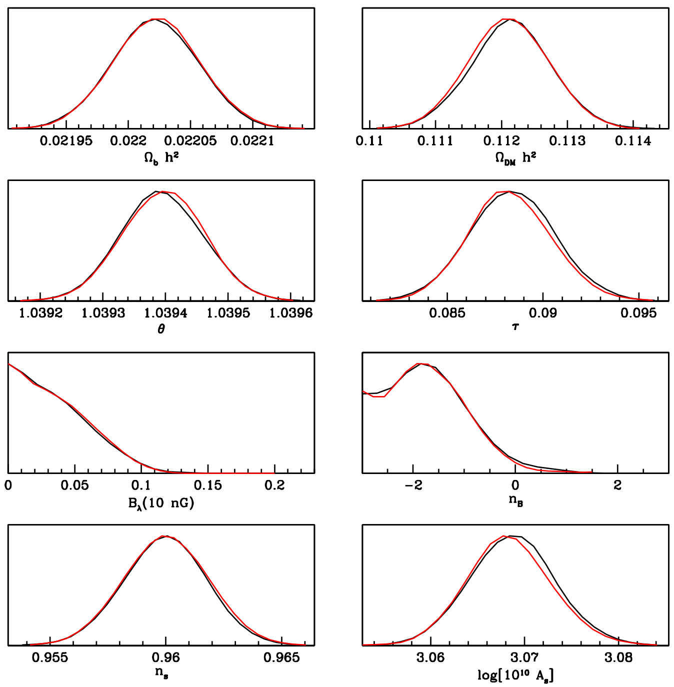


Prism forecast constraints on B below 1 nG

Faraday Rotation

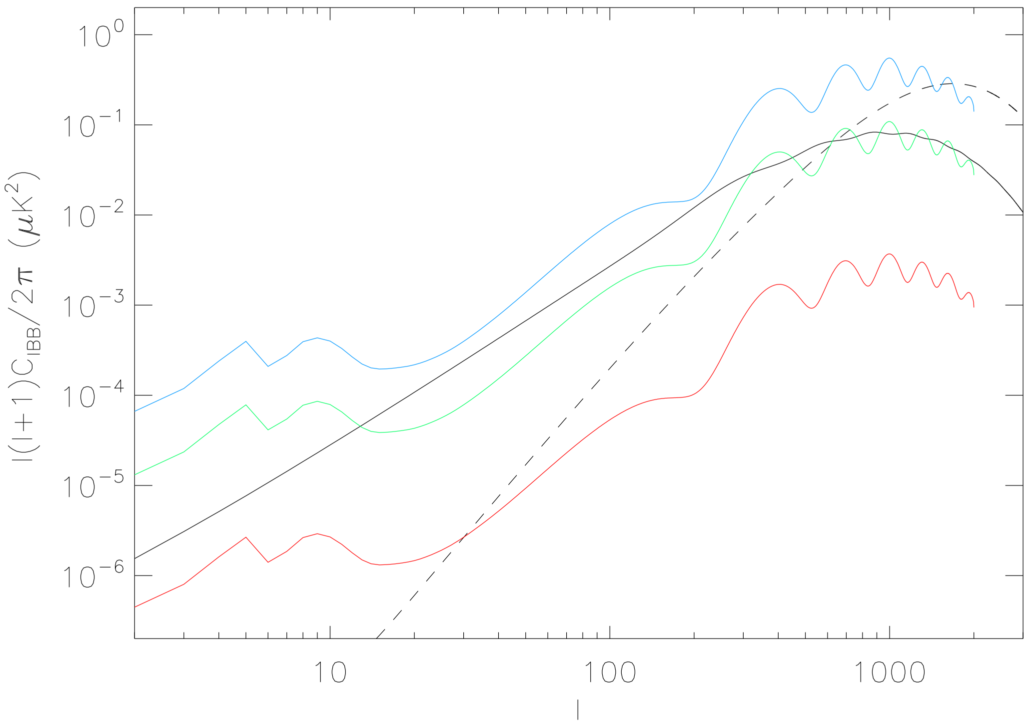
- For this simplest COrE simulation the constraint on B is not dominated by B polarization. This might not be completely true when taking into account foreground residuals and secondary ansiotropies at high multipoles, but it requires more work than what we have done for this simple forecast.

- Faraday rotation induced by a SB of PMF (Kosowsky et al., 2005) might be an additional handle on constraining the hypothesis for a SB of PMF. Due to its frequency dependence is a useful tool to distinguish this cosmological scenario from others which have signatures at high multipoles (as topological defects, for instance).



Including BB polarization

Not including BB polarization



BB from lensing (solid) and vector (dashed)

BB from Faraday rotation (B=2.5 nG) @ 25 GHz, 45 GHz, 75 GHz

BB from Faraday computation by Ruiz-Granados, Rubino-Martin

CMB non-gaussianity

- Inhomogeneous PMF EMT components are non-gaussian (with distributions close to χ^2, Brown & Crittenden 2005). Linear fluctuations in matter and gravity sourced by PMF are therefore non-gaussian.

- It is known that PMF can be targeted by refined non-gaussian signatures, as correlation between a_{lm}.

  Durrer, Kahniashvili & Yates 1998
  Kahnishvili & Lavrelashvili 2010

- Several recent studies of bispectrum of CMB temperature anisotropies from scalar, vector and tensor perturbations induced by PMF.

- Bispectrum evaluations available for:

  scalar Seshadri & Subramanian 2009, Caprini, FF, Paoletti & Riotto 2009
  vector Shiraishi et al. 2010; Shiraishi et al. 2011
  tensor Shiraishi et al. 2010; Shiraishi et al. 2011
  scalar (passive) Trivedi et al. 2010

- All configurations are comparable in magnitude (for scalar compensated modes)

- Bispectrum constraints from current CMB data and forecasted for Planck are above the nG, not quantitatively very different from those just presented at the power spectrum level.

- Trispectrum calculation available Trivedi et al. 2011

CMB temperature bispectrum

- We model the temperature anisotropy on large angular scales as

$$\frac{\delta T^{\,\rm mag}}{T}(\theta,\phi) = \alpha\frac{\Omega_B(\theta,\phi)}{4} = \sum_{\ell m} a^{\rm mag}_{\ell m} Y_{\ell m}(\theta,\phi)$$

Paoletti, FF, Paci 2009

with $\alpha \sim \mathcal{O}(0.1)$ which gives a good fit to CMB spectra.

- We then need to compute

$$\begin{aligned}\langle a^{\rm mag}_{\ell_1 m_1} a^{\rm mag}_{\ell_2 m_2} a^{\rm mag}_{\ell_3 m_3}\rangle &= \frac{(4\pi)^3(-i)^{\ell_1+\ell_2+\ell_3}}{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}\int \frac{d^3k\, d^3q\, d^3p}{(2\pi)^9} Y^*_{\ell_1 m_1}(\hat{k}) Y^*_{\ell_2 m_2}(\hat{q}) Y^*_{\ell_3 m_3}(\hat{p}) \\ &\quad \times\langle \Theta^{\rm mag}_{\ell_1}(\mathbf{k})\Theta^{\rm mag}_{\ell_2}(\mathbf{q})\Theta^{\rm mag}_{\ell_3}(\mathbf{p})\rangle \\ &= \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b_{\ell_1\ell_2\ell_3}\end{aligned}$$

where $\frac{\Theta^{\rm mag}_\ell(\mathbf{k})}{2\ell+1} = \frac{\alpha}{4}\Omega_B(\mathbf{k}) j_\ell(k(\eta_0-\eta_{\rm dec}))$, $\mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}$ is the Gaunt integral and $b_{\ell_1\ell_2\ell_3}$ is the reduced bispectrum.

- We need to compute the three-point function

$$\begin{aligned}\langle \rho_B(\mathbf{k})\rho_B(\mathbf{q})\rho_B(\mathbf{p})\rangle = \frac{\delta(\mathbf{k}+\mathbf{p}+\mathbf{q})}{384\pi^3} \quad & \left\{\int d\tilde{\mathbf{k}}\, P_{ij}(\tilde{\mathbf{k}}) P_{jl}(\mathbf{k}-\tilde{\mathbf{k}})[P_{il}(\mathbf{q}+\tilde{\mathbf{k}}) + P_{il}(\mathbf{p}+\tilde{\mathbf{k}})]\right. \\ & + \int d\tilde{\mathbf{k}}\, P_{ij}(\tilde{\mathbf{k}}) P_{jl}(\mathbf{q}-\tilde{\mathbf{k}})[P_{il}(\mathbf{k}+\tilde{\mathbf{k}}) + P_{il}(\mathbf{p}+\tilde{\mathbf{k}})] \\ & \left. + \int d\tilde{\mathbf{k}}\, P_{ij}(\tilde{\mathbf{k}}) P_{jl}(\mathbf{p}-\tilde{\mathbf{k}})[P_{il}(\mathbf{q}+\tilde{\mathbf{k}}) + P_{il}(\mathbf{k}+\tilde{\mathbf{k}})]\right\} .\end{aligned}$$

where $P_{ij}(\mathbf{k}) = P_B(k)\,(\delta_{ij} - \hat{k}_i\hat{k}_j)$

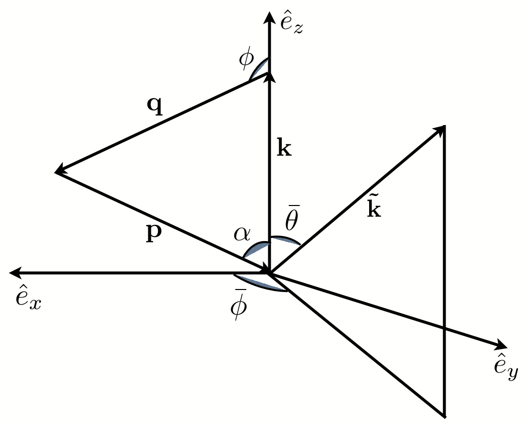
Support for $0<k/k_D<2$, threshold value $n_B=-1$

- Much more complex than the two point function

Support for $0<k/k_D<2$, threshold value $n_B=-3/2$

$$\langle \rho_B(\mathbf{k})\rho_B^*(\mathbf{q})\rangle \equiv (2\pi)^3\delta(\mathbf{k}-\mathbf{q})|\rho_B(k)|^2 = \frac{2}{(8\pi)^2}\delta(\mathbf{k}-\mathbf{q})\int d^3p\, P_B(p)P_B(|\mathbf{k}-\mathbf{p}|)(1+\mu^2)$$

- Some insight can be gained by focusing on particular configurations



equilateral: the length of the three vectors are equal

squeezed: the length of a vector is much smaller compared to the other two (which have almost opposite direction)

collinear: the length of a vector is twice the other two (which have almost same direction)

- Most of the inflationary predictions are dominated by the squeezed configuration, although there are models with equilateral contribution as well (DBI inflation). Collinear contribution can be turned on by setting non-vacuum initial conditions for fluctuations during inflation. Holman & Tolley, 2008

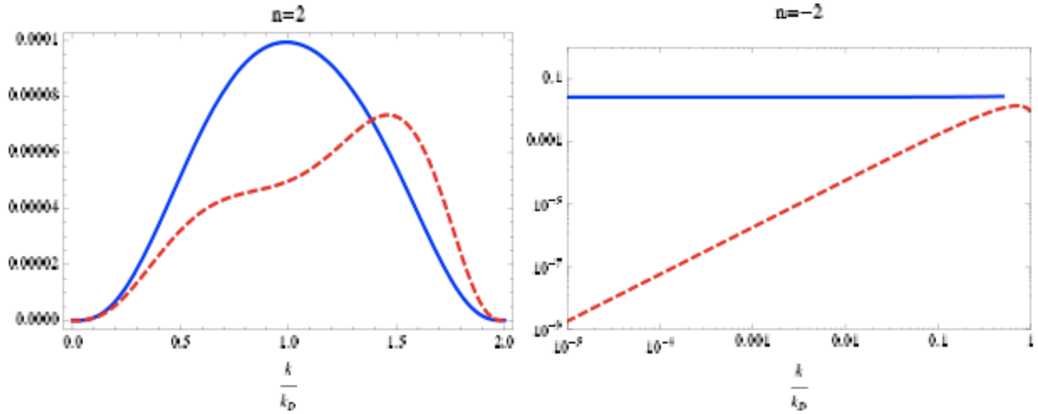
- For a SB of PMFs a particular configuration does not dominate the three point function for the energy density and they are all comparable when the perimeter of the triangle is smaller and smaller.

- As approximation to the three point function:

$$\langle \rho_B(\mathbf{k})\rho_B(\mathbf{q})\rho_B(\mathbf{p})\rangle \simeq \frac{\delta(\mathbf{k}+\mathbf{p}+\mathbf{q})}{48\pi^2}A^3 \times$$
$$\left\{\frac{n}{(n+3)(2n+3)}q^n k^{2n+3} + \frac{n}{(3n+3)(2n+3)}q^{3n+3} + \frac{k_D^{3n+3}}{3n+3}\right.$$
$$+\frac{n}{(n+3)(2n+3)}p^n k^{2n+3} + \frac{n}{(3n+3)(2n+3)}p^{3n+3} + \frac{k_D^{3n+3}}{3n+3}$$
$$\left.+\frac{n}{(n+3)(2n+3)}p^n q^{2n+3} + \frac{n}{(3n+3)(2n+3)}p^{3n+3} + \frac{k_D^{3n+3}}{3n+3}\right\} \qquad \text{for } k \le q \le p \le$$

and so on with all the ordered permutations of the wave-numbers.

- Is larger the bispectrum or $(k^3|\rho_B(k)|^2)^{\frac{3}{2}}$?



spectra, bispectra (colinear)

- We have also tested our approximation against exact cases with satisfactory results. By inserting the approximation in the formula for the reduced bispectrum we have:

$$b_{\ell_1\ell_2\ell_3} \simeq \frac{\pi^5\,\alpha^3}{96}\frac{(n+3)^3}{n+1}\frac{\langle B^2\rangle^3}{\rho_{\rm rel}^3}\frac{1}{(k_D\eta_0)^4}\,, \qquad \text{for } n > -1\,.$$

$$b_{\ell_1\ell_2\ell_3} \simeq \frac{\pi^6\,\alpha^3}{288}\frac{\langle B^2\rangle^3}{\rho_{\rm rel}^3}\frac{1}{(k_D\eta_0)^3}\left\{\frac{1}{\ell_1}\left[\log\left(\frac{k_D\eta_0}{\sqrt{\ell_2}\sqrt{\ell_3}}\right) - \frac{2k_D\eta_0}{3\pi}\frac{1}{\ell_1}\right] + \frac{1}{\ell_2}\left[\frac{1}{2}\log\left(\frac{k_D\eta_0}{\ell_3}\right) - \frac{k_D\eta_0}{3\pi}\frac{1}{\ell_2}\right]\right\} + \text{ permutations}\,, \qquad \text{for } n=-2\,.$$

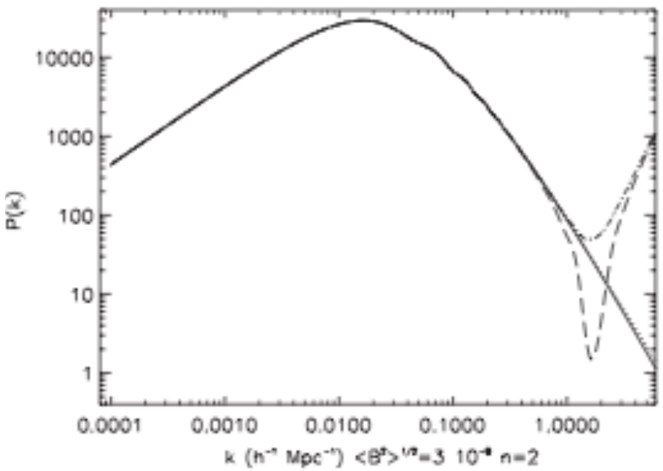
$$b_{\ell_1\ell_2\ell_3} \simeq \frac{\pi^5\,\alpha^3}{288}\frac{n(n+3)^2}{2n+3}\frac{\langle B^2\rangle^3}{\rho_{\rm rel}^3}\left(\frac{1}{\ell_1^2\ell_2^2}+\frac{1}{\ell_1^2\ell_3^2}+\frac{1}{\ell_2^2\ell_3^2}\right)+\quad \text{permutations}\,, \qquad \text{for } n\approx -3\,,$$

- By using WMAP 5 yrs results on f_{NL} (Komatsu et al. [WMAP5] 2008) we obtain:

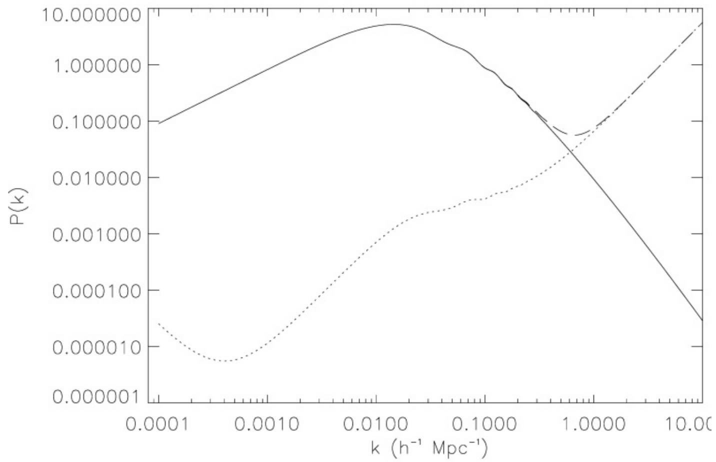
$$\begin{aligned}
\sqrt{\langle B^2\rangle} &\leq 20\text{nG} \quad \text{for } n_B > -1 \quad (B_{0.1\rm Mpc} < 2\mu\text{G})\,\text{for}\, n_B = 2 \\
\sqrt{\langle B^2\rangle} &\leq 25\text{nG} \quad \text{for } n_B = -2 \quad (B_{0.1\rm Mpc} < 26\text{nG}) \\
\sqrt{\langle B^2\rangle} &\leq 9\text{nG} \quad \text{for } n_B \approx -3 \quad (B_{0.1\rm Mpc} < 9\text{nG})
\end{aligned}$$

PMF impact on the matter power spectrum

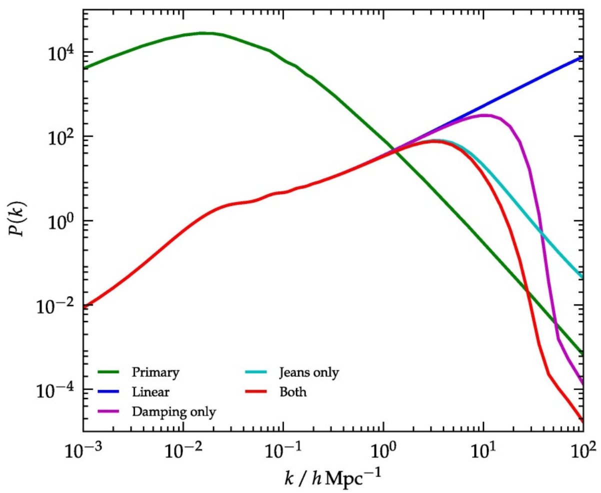
- We have scrutinized in depth the window of wavelength relevant for CMB anisotropies.
- Because of the type of initial conditions and of the peculiar shape of the Fourier spectrum of the PMF EMT components we expect a large impact also on smaller scales, probed by the observable as the matter power spectrum.
- By using the CAMB code to evolve the magnetized linear transfer functions and to predict the matter power spectrum including the PMF contribution, the results are quite large in the regime in which non-linear effects are expected to be important.



FF, Paci, Paoletti 2008

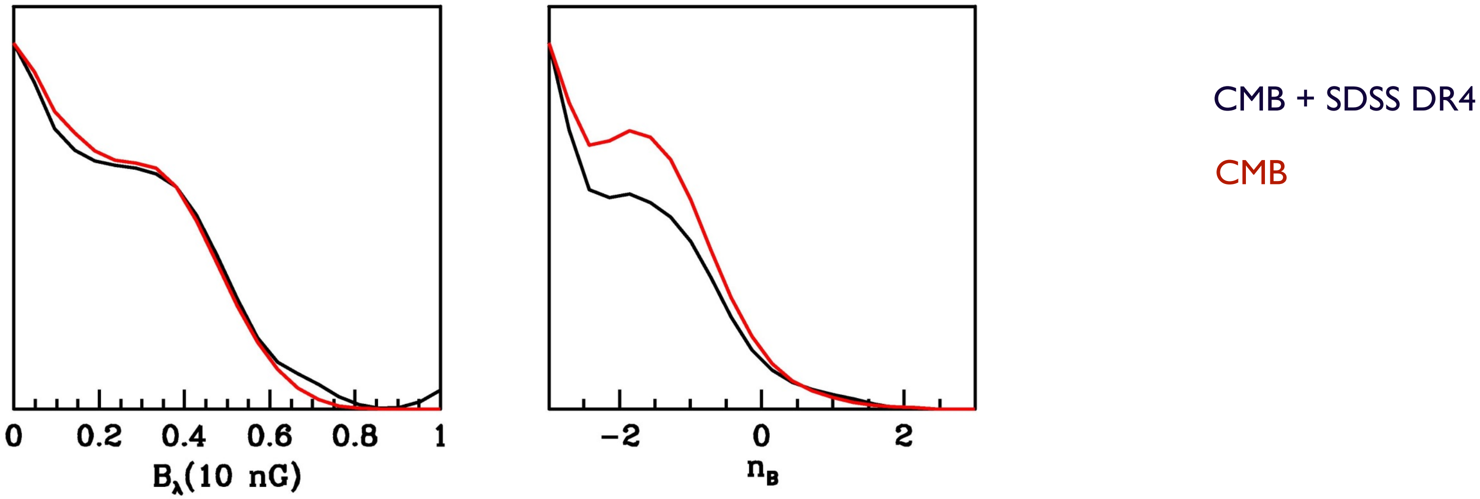


Paoletti, PhD Thesis 2011



Shaw & Lewis 2012

- A non linear estimate of the matter power spectrum will require an extension of the HALOFIT code which still has not been developed for a cosmological model with the contribution of a SB of PMF.
- Linear scales probed by the matter power spectrum are not a significant addition to the constraints derived from the CMB anisotropies power spectrum



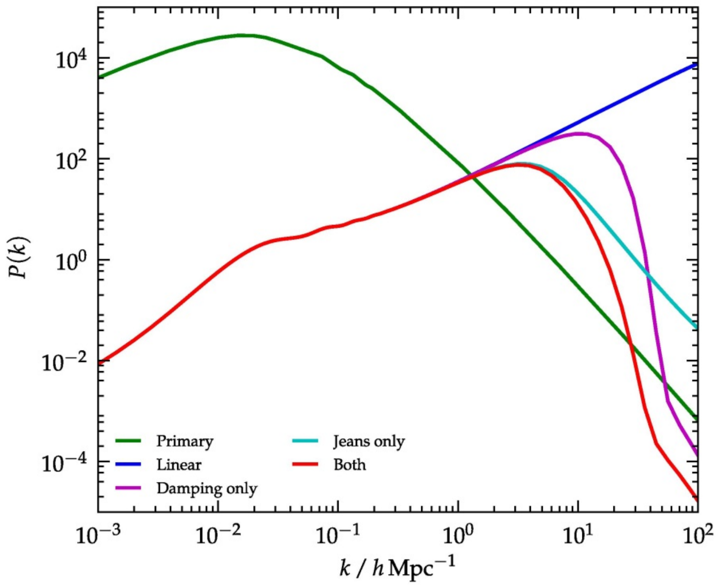
- Shaw & Lewis 2012 considered an alternative non-linear approach based on two ingredients:

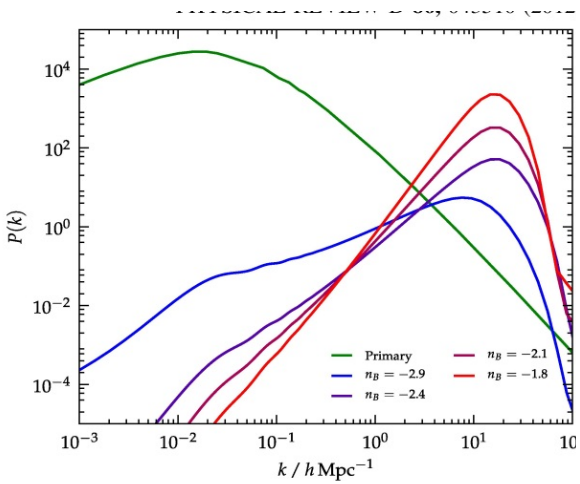
  a. They mimic a magnetic Jeans scale through a modification of the speed of sound

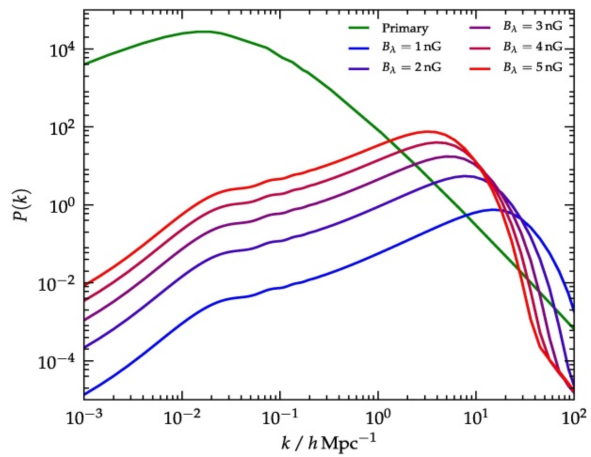
     of baryons due to the Alfven waves

  b. The damping of the PMF EMT is considered as exponential in time up to

     recombination







Shaw & Lewis 2012

- The impact of PMF on LSS may have another consequence which is of interest for current and future CMB measurements: the impact on the SZ power spectrum. Since PMF modify the structure formation they may modify the galaxy cluster abundance and as a consequence the SZ power spectrum.

- Using the model by Komatsu & Seljak it is possible to estimate the total SZ contribution by taking into account PMF. In linear regime was done by Tashiro & Sugiyama 2009 finding a very high effect due to the non-consideration of non linear effects. Shaw & Lewis included non-linear effects, still finding interesting, although smaller, effects.

PMF and CMB black-body distortions

- CMB spectrum is described by an almost perfect black body. This is the result of the almost- equilibrium between matter and radiation at CMB generation. But if some mechanism produce energy injection into the plasma at redshifts between 10^6 and the recombination 10^3, these can leave imprints on the CMB spectrum as distortions.
- Depending on the redshift of the energy injection, the capability of the plasma to riequilibrate with Compton and double Compton scattering, changes, therefore different type of distortion may take place:

  - $z < 10^4$ the distortions are described with a non-vanishing Compton parameter y;
  - $10^4 < z < 10^6$ the photon distribution is described by a Bose-Einstein with non zero chemical potential μ.

- The dissipation of PMF into the cosmological plasma is responsible for possible distortions in the CMB black body spectrum. It is possible, with the use of approximations, to estimate the spectral distortions due to PMF and constrain PMF using COBE FIRAS data on the CMB spectrum.
- The first straightforward mechanism to inject magnetic energy in the plasma was investigated by Jedamzik et al. 2000. PMF exerts a force on the photon-baryon plasma, prior to recombination, accelerating the plasma particles. This process converts magnetic energy into kinetic energy, which in turn is efficiently converted into heat. This energy injection in the plasma can cause both distortions of μ and y type.

- By considering the variation of the chemical potential as related with the variation of the magnetic energy, Jedamzik et al. obtain 30 nG as a constraint on the total amplitude of the PMF stochastic background from COBE FIRAS. Kunze and Komatsu 2013 obtain a similar constraint.

- Kunze and Komatsu 2013 considered also the dissipation after recombination which occurs mainly due to

  ambipolar diffusion

  MHD decaying turbulence

  The first effect dominates at lower redshifts whereas the second at higher redshifts. Constraints expected by PiXie not tight as from early distortion.

Conclusions

- PMF is a hot topic in cosmology, plenty of recent developments in connection with inflationary models and vector fields.
- The scientific case for large scale magnetic fields has been corroborated by HE observations, which set a (non-zero) lower limit for EGMF in voids. These HE observations are not in contradiction with a primordial hypothesis for large scale magnetic fields.
- A SB of PMF has a host of predictions which derive from the non-gaussian contribution to all the types of cosmological perturbations and therefore on the CMB anisotropy pattern and LSS.
- At present CMB power spectrum and bispectrum data provide upper limits to the magnetic amplitude at 1 Mpc scale of non-helical PMF at the level of pG (for blue spectra) or nG (for red spectra). Great expectations from next year Planck release which will include polarization.
- Current and future measurements can squeeze the window for PMF between CMB/LSS and the lower limits from HE.
- Future missions can target other predictions, such as B-mode CMB polarization from Faraday rotation/vector modes with CoRE, CMB black-body spectrum distortions with Pixie/Prism, as those for large scale structure with EUCLID.

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