Lecture III
Cosmological Simulations

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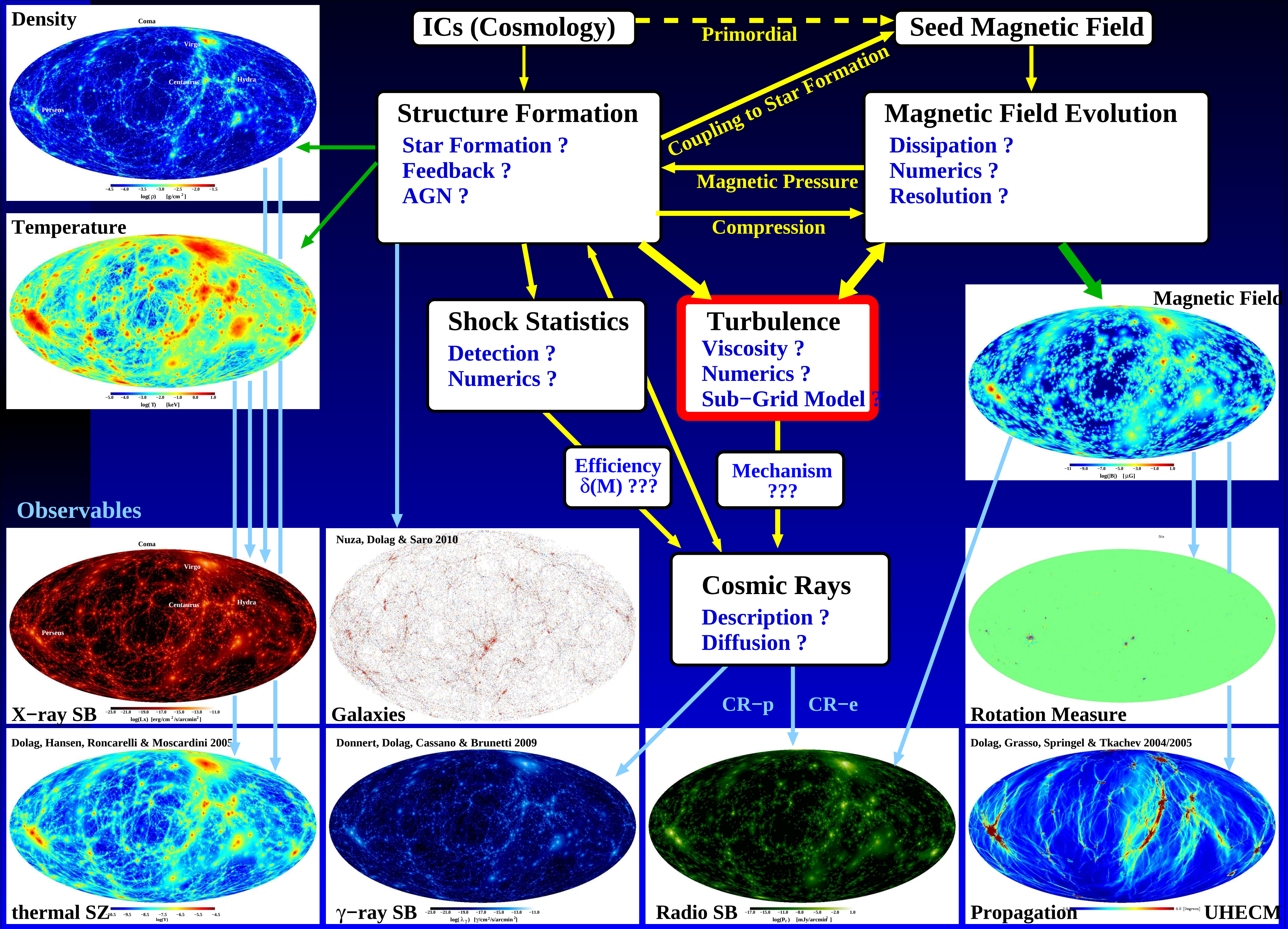
Process Network



ICs (Cosmology)

Primordial

Seed Magnetic Field

Coupling to Star Formation

Structure Formation

Star Formation ?
Feedback ?
AGN ?

Magnetic Pressure

Compression

Magnetic Field Evolution

Dissipation ?
Numerics ?
Resolution ?

Shock Statistics

Detection ?
Numerics ?

Turbulence

Viscosity ?
Numerics ?
Sub−Grid Model ?

Efficiency
δ(M) ???

Mechanism
???

Cosmic Rays

Description ?
Diffusion ?

CR−p

CR−e

Observables

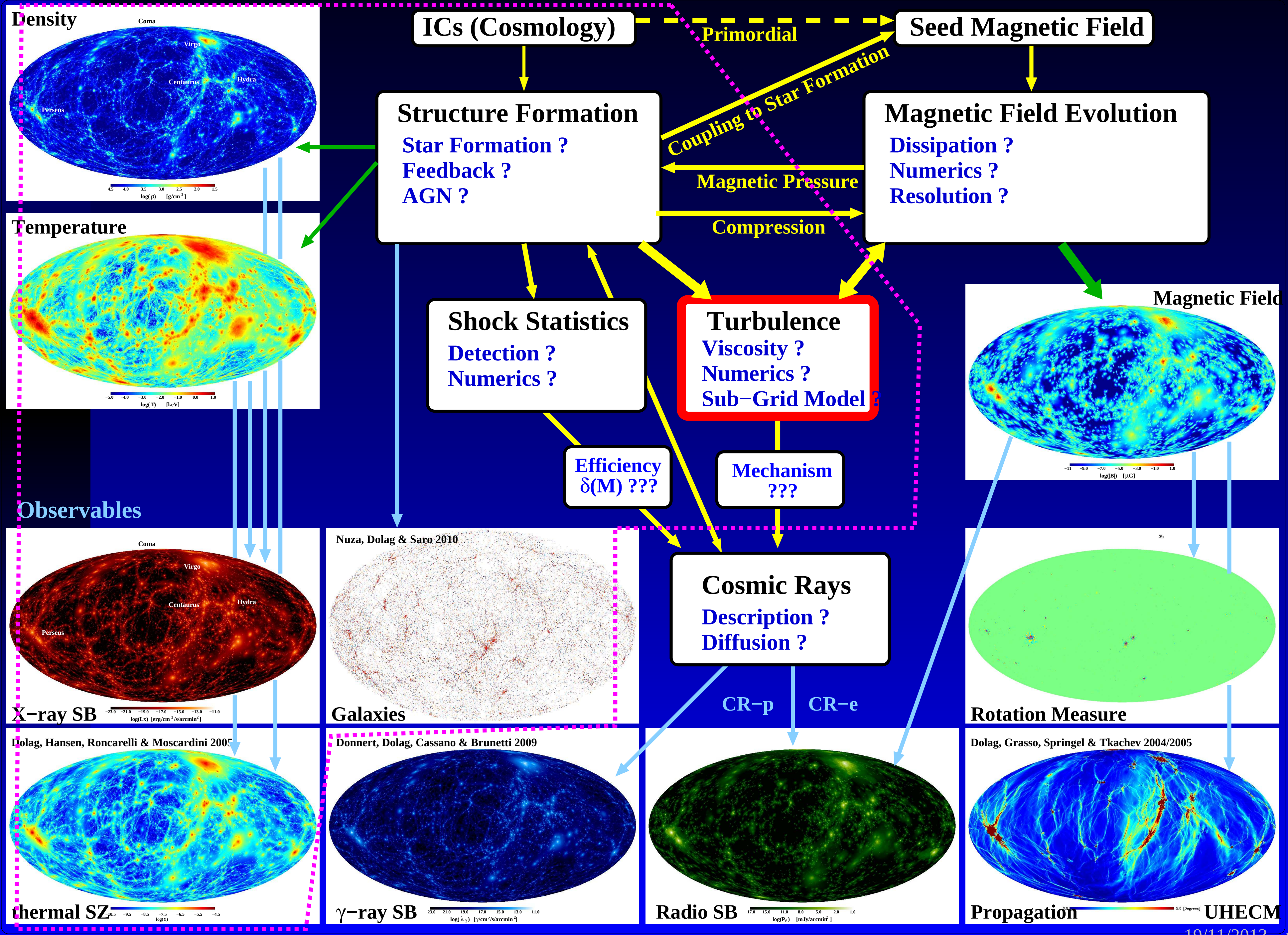
Nuza, Dolag & Saro 2010

Dolag, Hansen, Roncarelli & Moscardini 2005

Donnert, Dolag, Cassano & Brunetti 2009

Dolag, Grasso, Springel & Tkachev 2004/2005

Process Network



ICs (Cosmology)

Seed Magnetic Field

Structure Formation
- Star Formation ?
- Feedback ?
- AGN ?

Magnetic Field Evolution
- Dissipation ?
- Numerics ?
- Resolution ?

Shock Statistics
- Detection ?
- Numerics ?

Turbulence
- Viscosity ?
- Numerics ?
- Sub−Grid Model ?

Cosmic Rays
- Description ?
- Diffusion ?

Efficiency δ(M) ???

Mechanism ???

Nuza, Dolag & Saro 2010

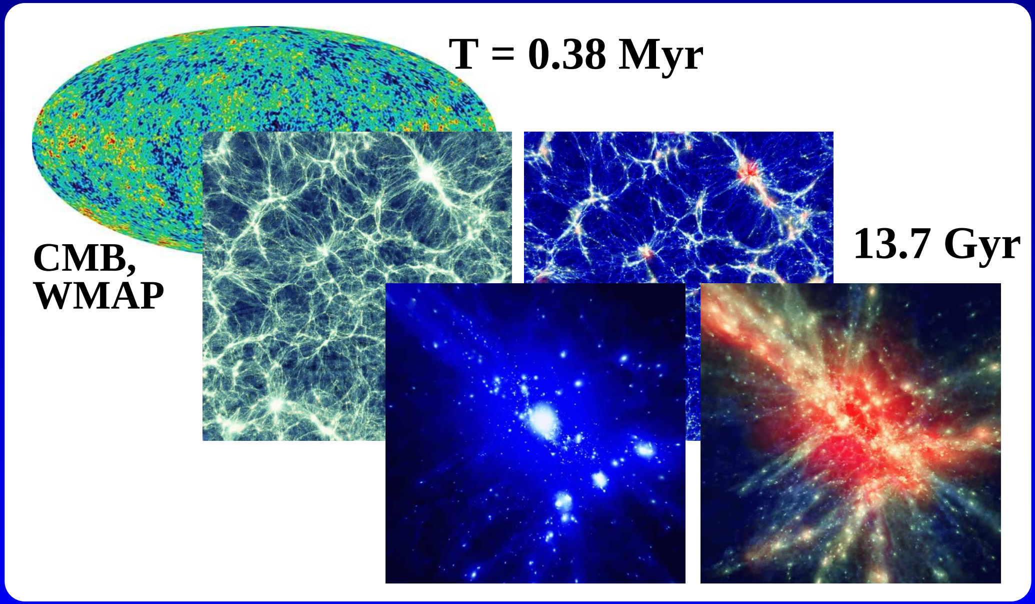
Dolag, Hansen, Roncarelli & Moscardini 2005

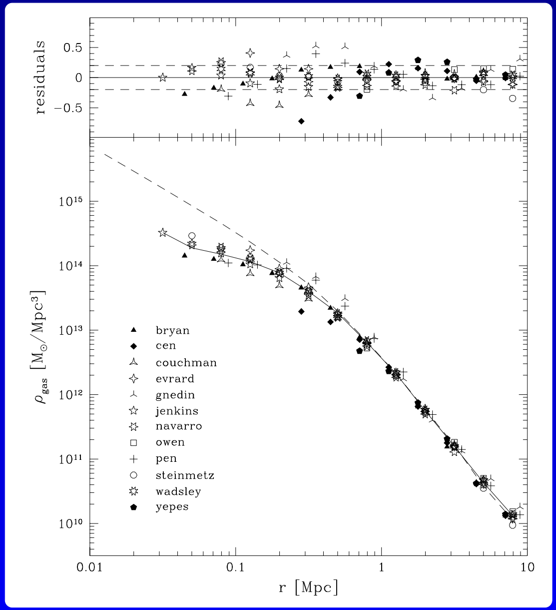
Donnert, Dolag, Cassano & Brunetti 2009

Dolag, Grasso, Springel & Tkachev 2004/2005

Cosmological Simulations

- Gravity (N-Body system)
  Direct sum, Tree, Particle-Mesh, ...
- Hydrodynamics (including shocks)
  Mesh, Adaptive-Mesh, Shock capturing schemes, SPH, ...
- Cooling (radiative losses)
  primordial mixture, metals, ...





Frenk et al. 1999

Cosmological Simulations

- Star-formation (not resolved)
  simplified description, sub-grid models, ...
- Feedback (poorly understood)
  energetics, kinetics, ...
- AGN, Radiolopes, Bubbles
- Cosmic Rays
  from shocks, Feedback (SN), AGN, ...
- **Magnetic Fields**
- Thermal Conduction
- ...

Movie stars,gas

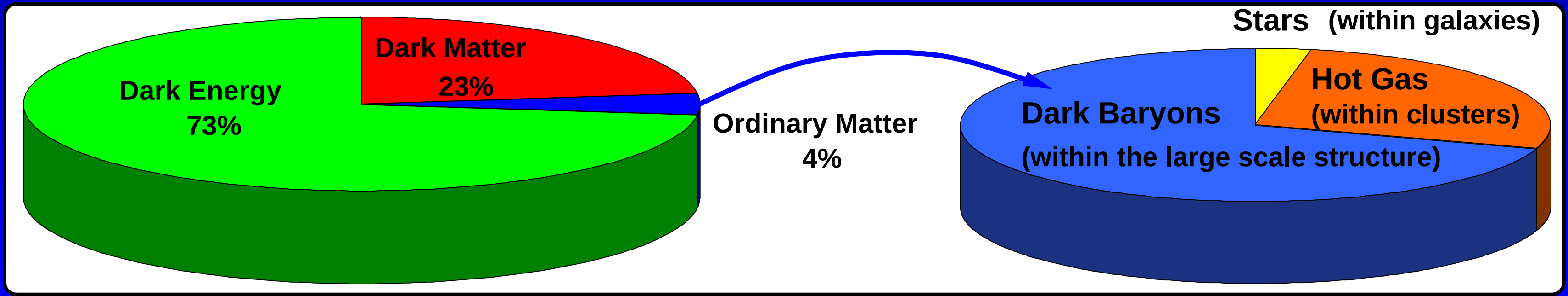
Galaxy Clusters in Numbers

Galaxy clusters are the largest, gravitational bound objects in the Universe and represent an almost fair sample of the cosmological composition.

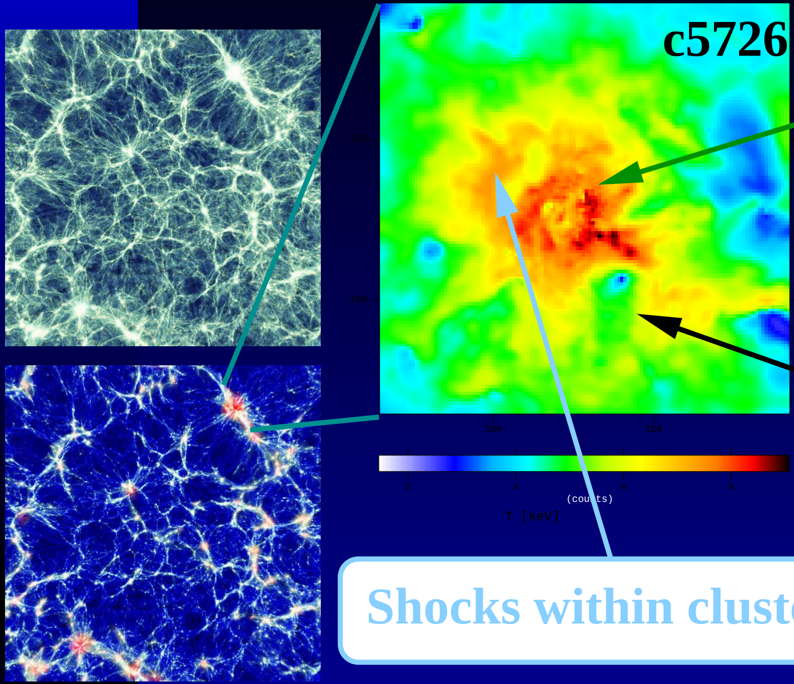
- Up to thousands of galaxies with $\sigma_{\rm gal}$ up to $1000{\rm km/s}$
- Size ($R_{\rm cluster}$) of several Mpc
- Total mass ($M_{\rm tot}$) up to several $10^{15}M_\odot$ ($\Rightarrow$ dark matter)
- Nearly cosmic baryon fraction ($\approx 95\%$)
- ICM temperatures ($T_{\rm ICM}$) larger than 10^8K

Observed to be virialized:

$$3\sigma^2_{\rm galaxies} \approx \frac{GM_{\rm tot}}{R_{\rm cluster}} \approx \frac{3kT_{\rm ICM}}{2\mu m_p}$$



The Big Picture

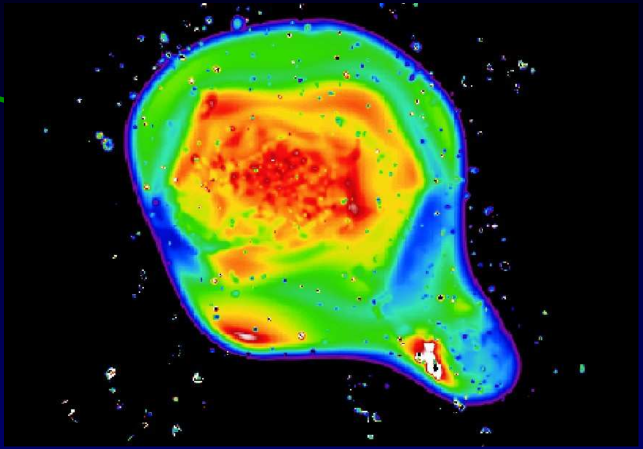


Simulation (Borgani et al. 2004)

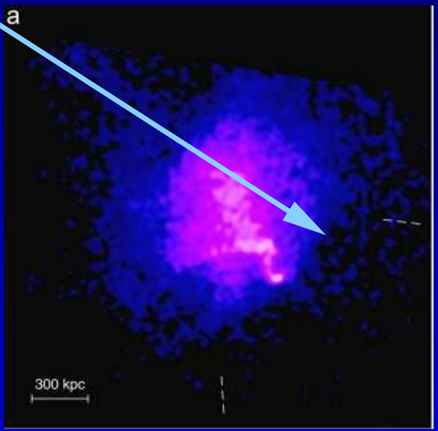
"Turbulence" within clusters

"Ram Pressure Stripping" of galaxies within clusters ?

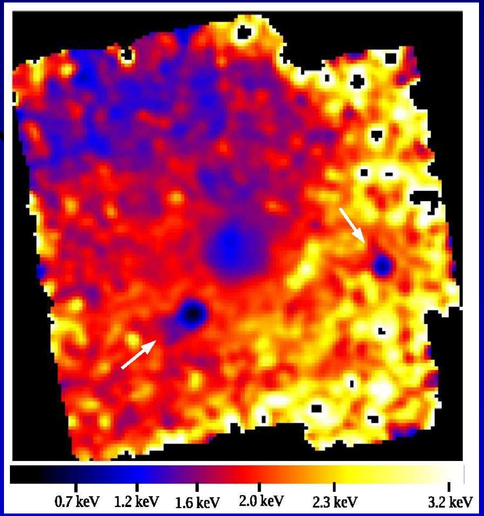
Shocks within clusters



Coma (Schuecker et al. 2004)



A520 (Markevitch et al. 2005)



Fornax (Scharf et al. 2004)

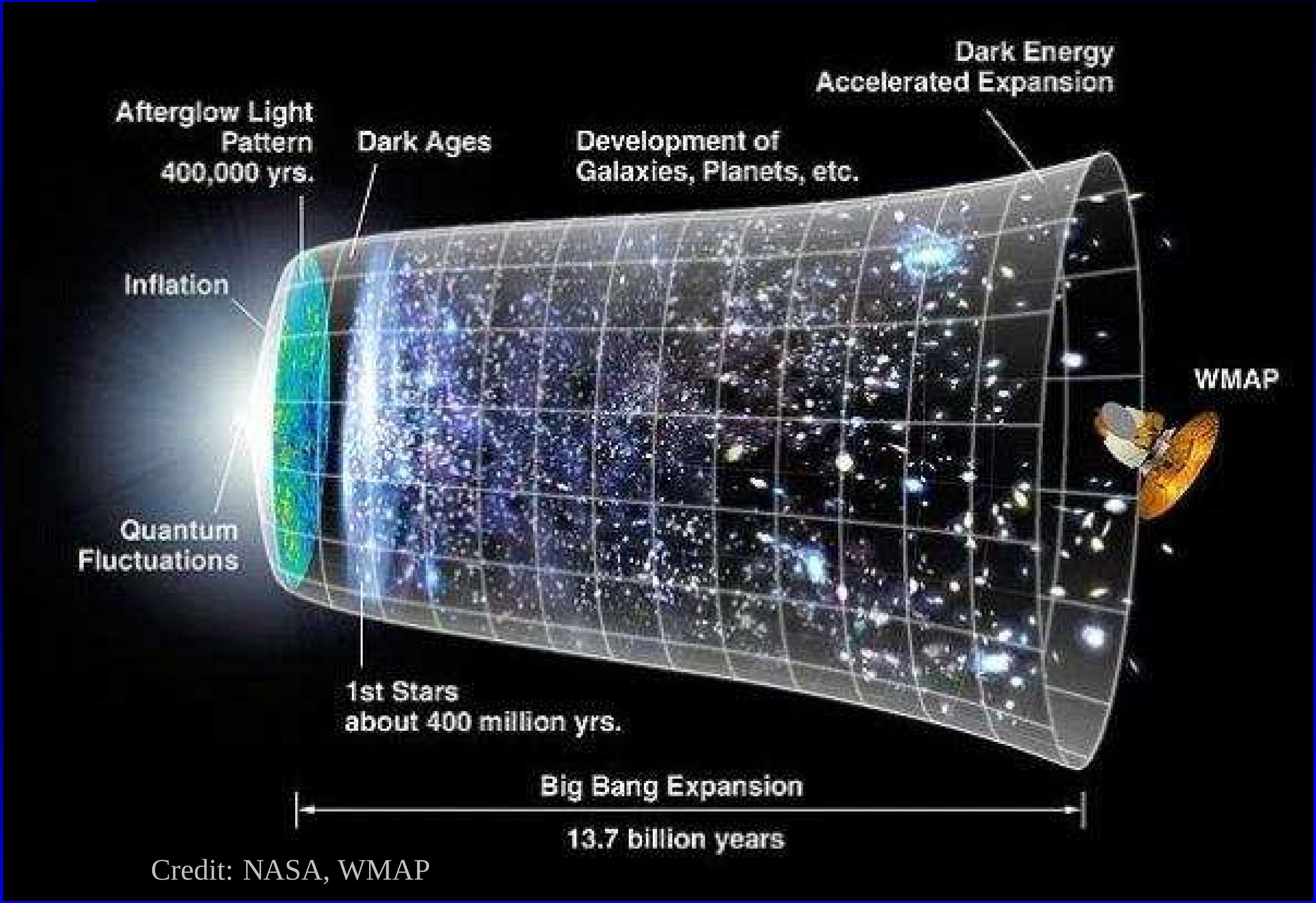
But:
Thermal conduction ?
Viscosity / Turbulence ?
none thermal components ?

Complex thermal structures. Depend on many physical processes !

Detailed comparison with observation traces physical processes.

$$c \approx 1500 > \sigma \approx 1000 \approx v_{th} \approx 1000 > v_{turb} \approx 330 > v_a \approx 120$$

Structure Formation

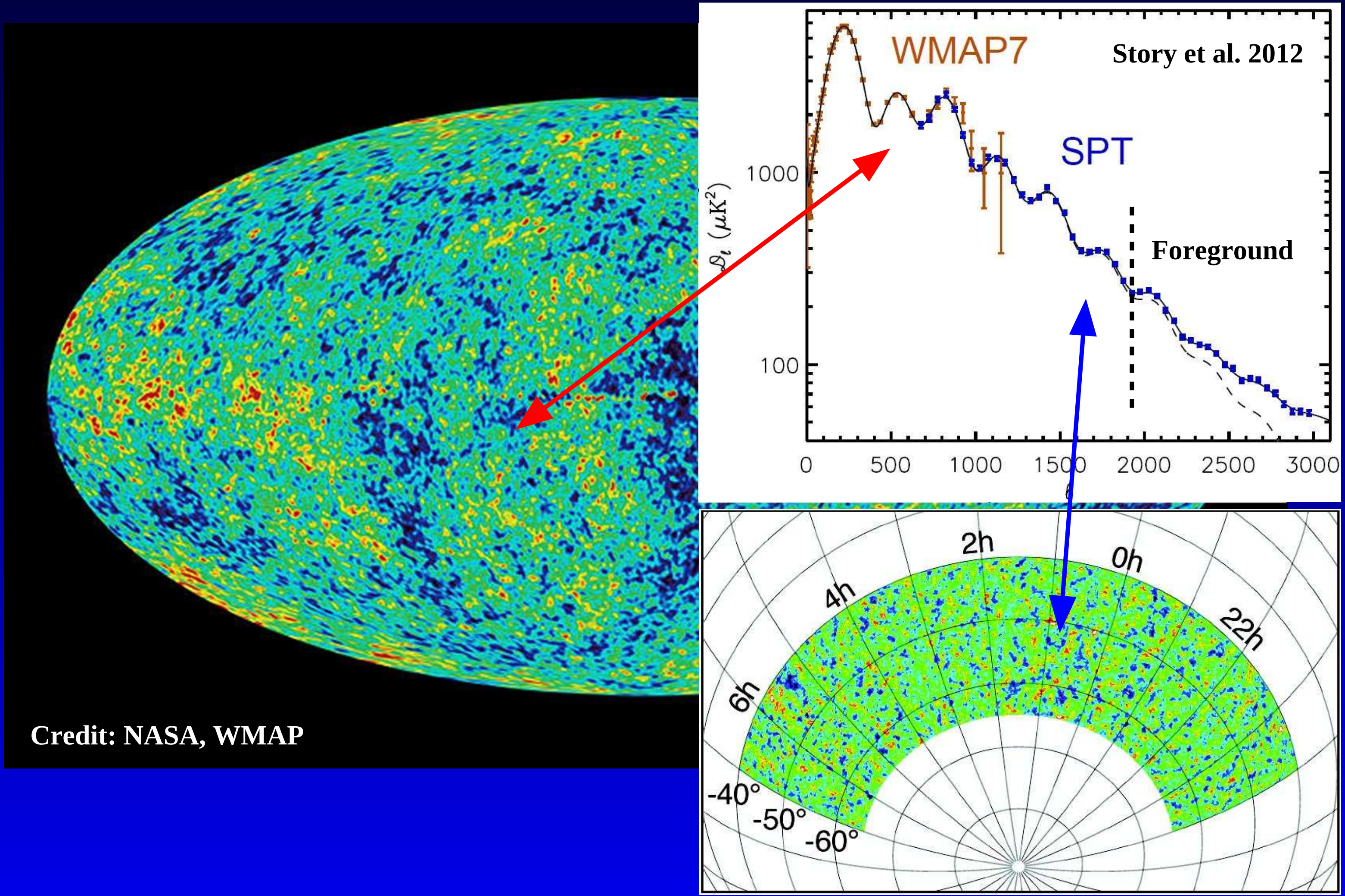


Credit: NASA, WMAP

Interplay between background cosmology and structure formation.

Structure Formation

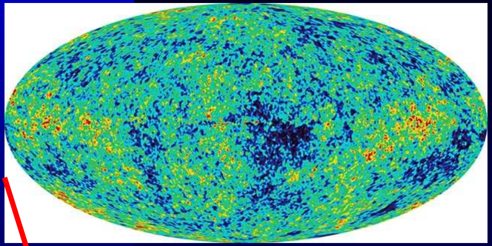
Measuring the angular size corresponding to the first acoustic peak in the CMB power spectrum, one can establish the measured angular diameter distance.



Credit: NASA, WMAP

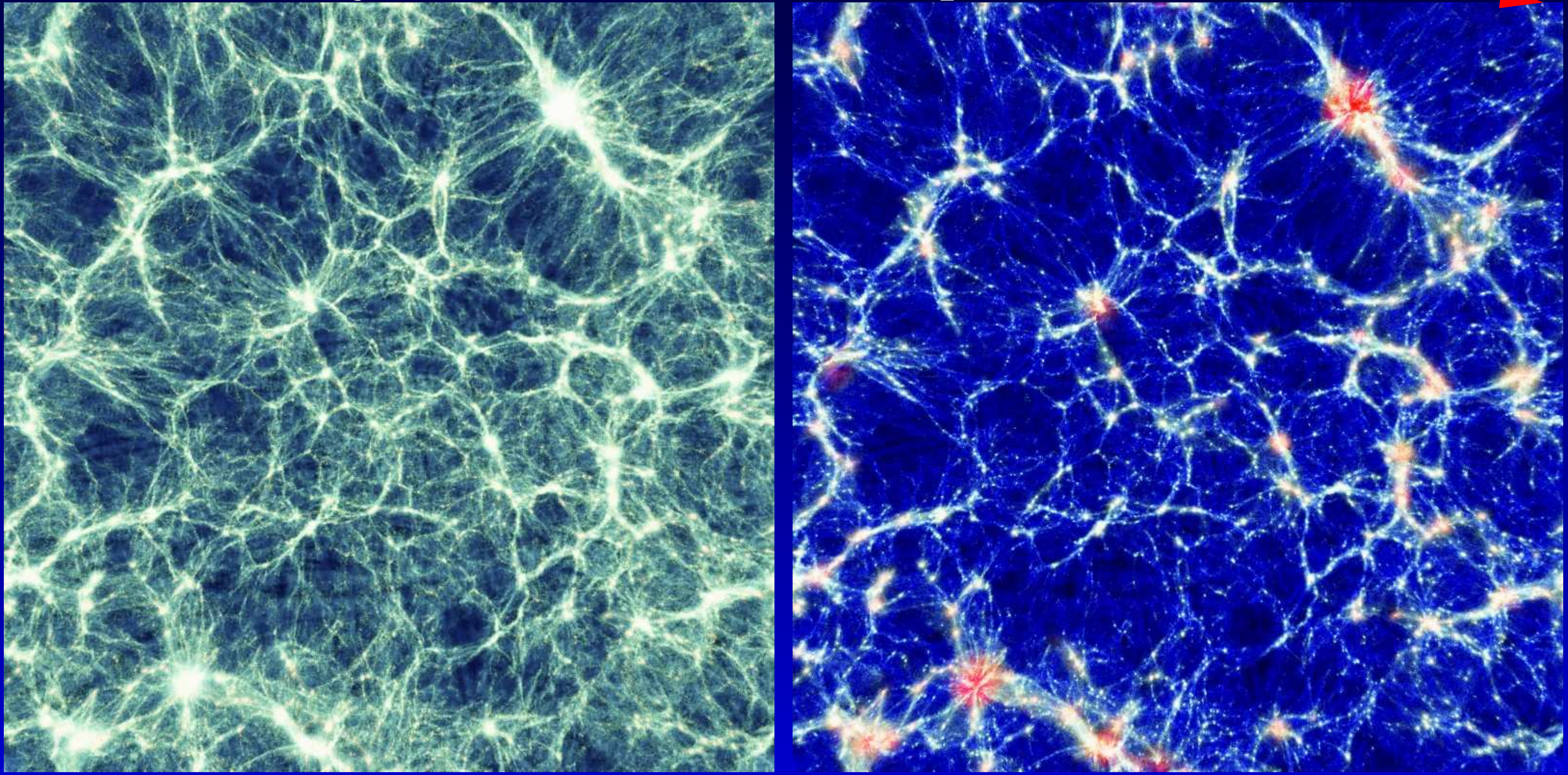
$\Omega_\Lambda = 0.750 \pm 0.02$ (e.g. $\Omega_K = 0$), $\sigma_8 = 0.750$.

Structure Formation



CMB (t = 0.38 Myr)

Cosmic structure today (t = 13.7 Gyr)



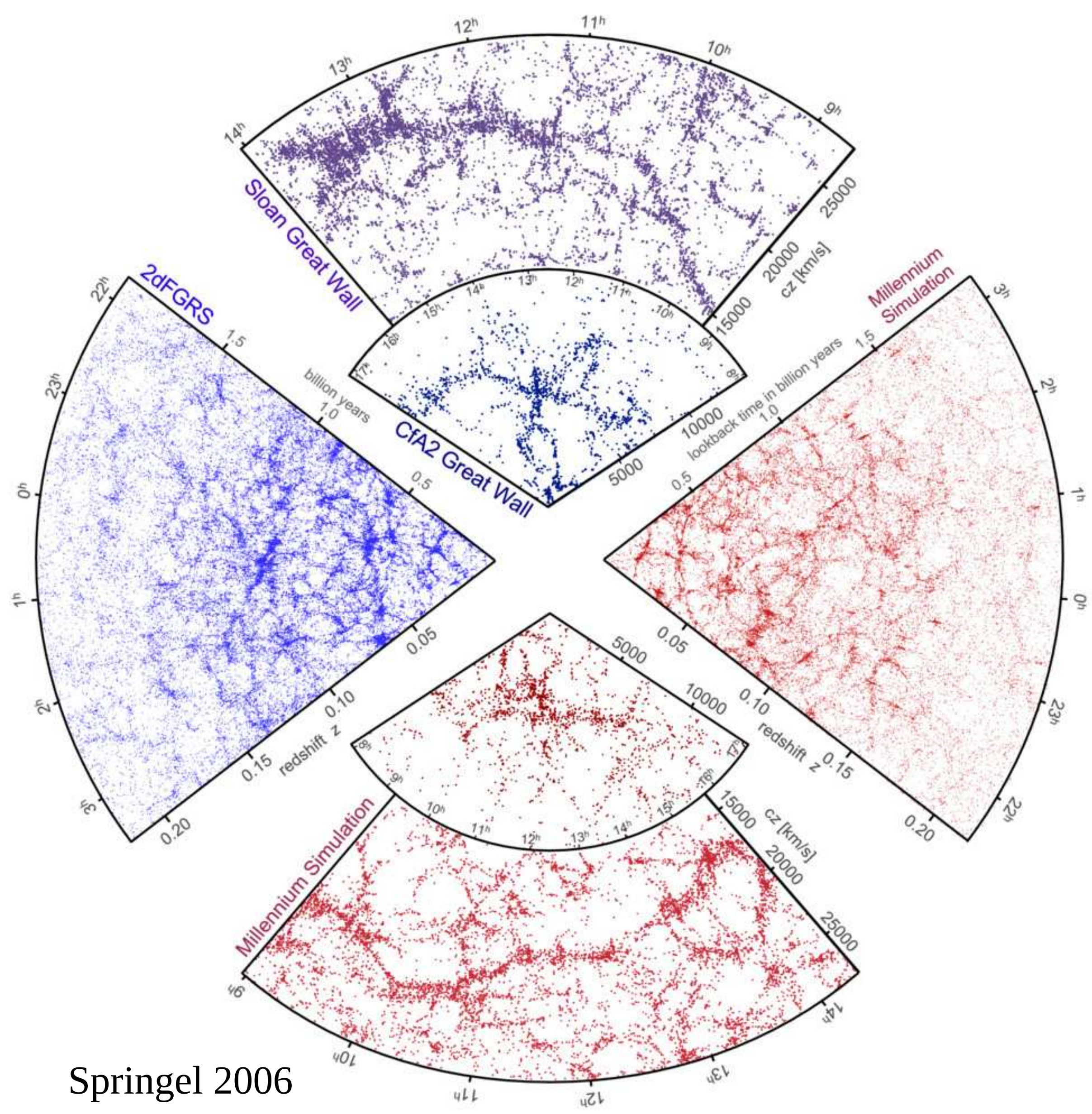
Density **Temperature**

Borgani, Murante, Springel, Diaferio, Dolag et al. 2004

275 Mpc

The cosmic web today ($z = 0$) is mainly accessible through simulations (warm, thin). Simulations important to predict the non linear formation of cosmological structures.

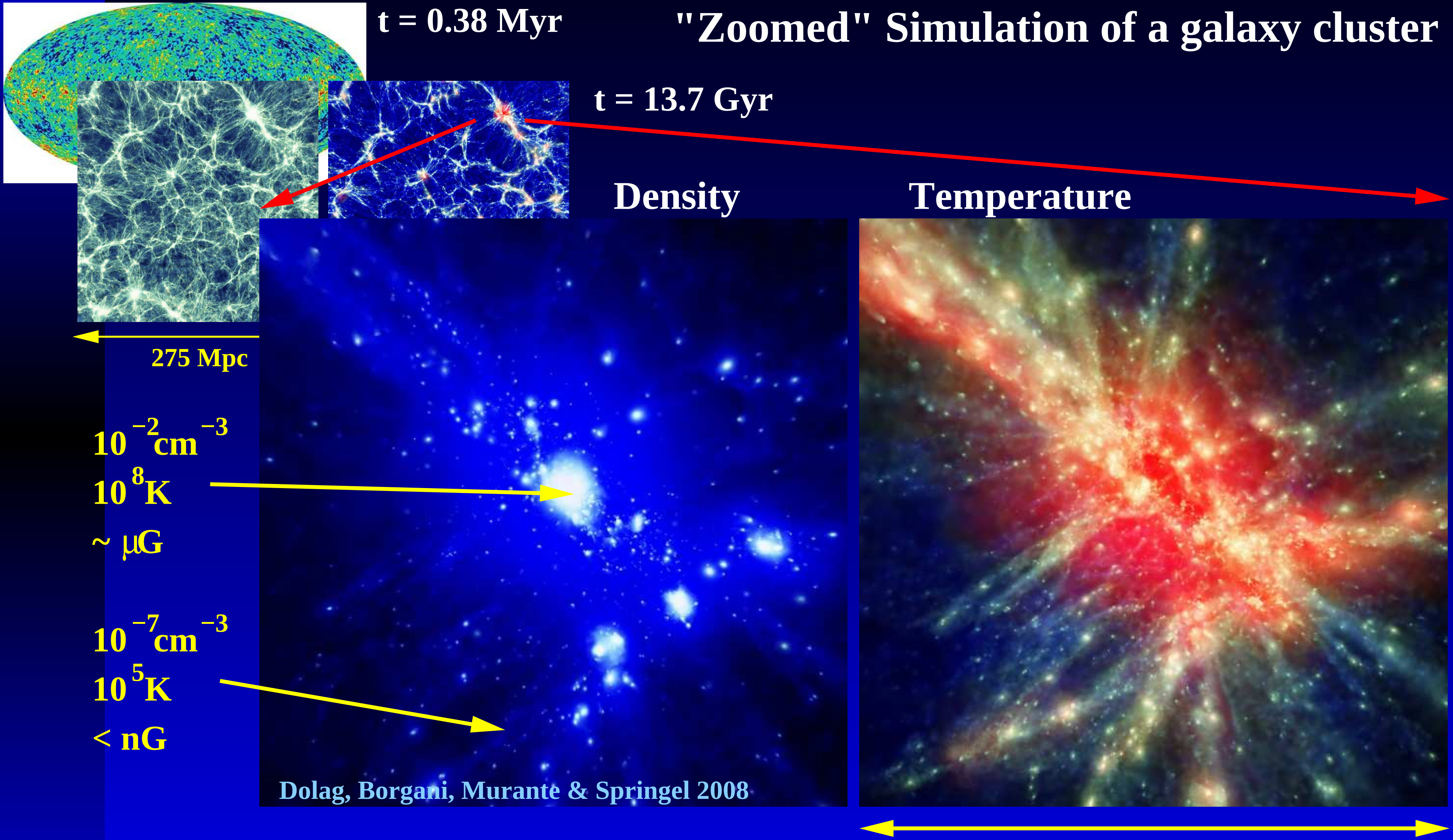
Structure Formation



Springel 2006

Large scale structures traced by galaxies
DM simulations ⇒ halos ⇒ Galaxies

Structure Formation



t = 0.38 Myr

"Zoomed" Simulation of a galaxy cluster

t = 13.7 Gyr

Density

Temperature

275 Mpc

10^{-2} cm^{-3}
10^{8} K
~ µG

10^{-7} cm^{-3}
10^{5} K
< nG

Dolag, Borgani, Murante & Springel 2008

40 Mpc

Clusters form at the nodes of the cosmic web and trace the high density environments. The gas falls into the potential, cools and form stars.

Simulating the universe

Assuming that the matter filling the universe is collision-less and non-relativistic (e.g. cold dark matter, CDM), the evolution of its phase-space distribution function $f(\vec{x}, \vec{p}, t)$ can be described by the collision-less *Boltzmann* (e.g. *Vlasov*) equation.

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{ma(t)^2}\vec{\nabla} f - m\vec{\nabla}\Phi\frac{\partial f}{\partial \vec{p}} = 0$$

coupled with the *Poisson* equation

$$\vec{\nabla}^2\Phi(\vec{x}, t) = 4\pi G a(t)^2\left[\rho(\vec{x}, t) - \bar{\rho}(t)\right]$$

Φ: gravitational potential; $\bar{\rho}(t)$: background density.

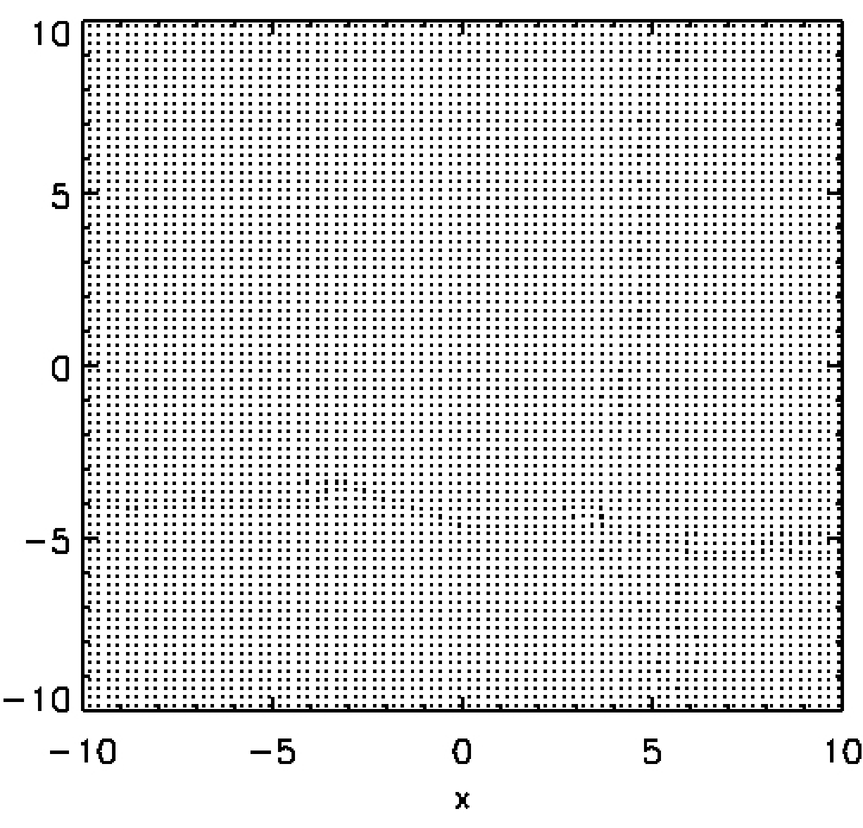
$$\rho(\vec{x}, t) = \int f(\vec{x}, \vec{p}, t)\mathrm{d}^3 p$$

$\Rightarrow$ high-dimensional problem !

Simulating the universe

One method to solve these equations is to sample the phase space density using tracing particles and to solve their equation of motion (e.g. n-body simulation).

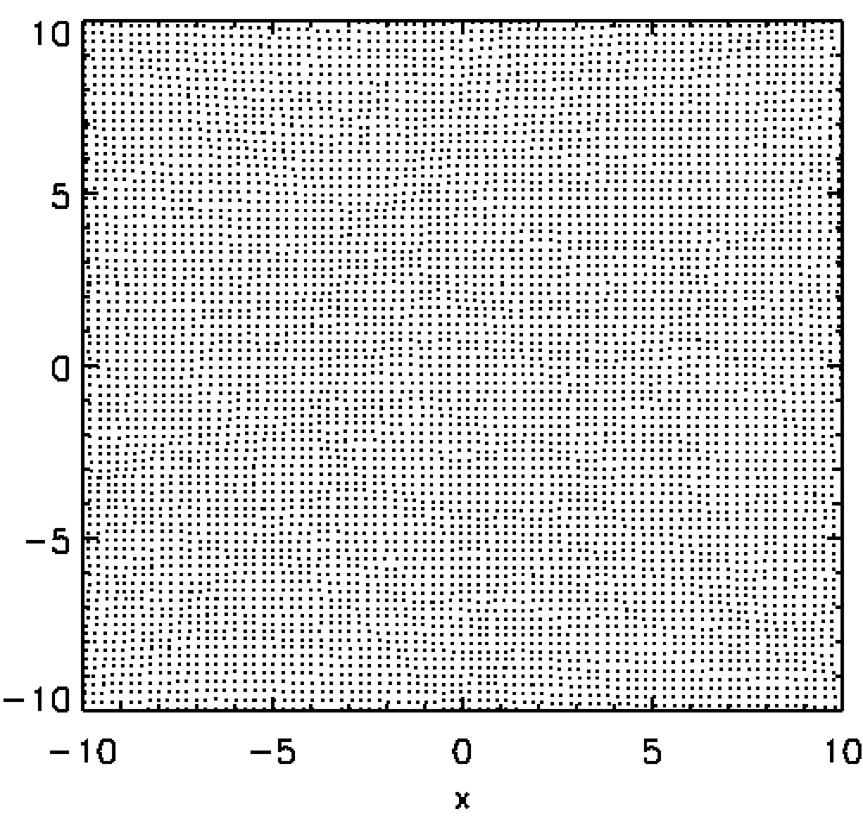
$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \vec{v}H(a) = -\frac{\vec{\nabla}\Phi}{a(t)}.$$



Simulating the universe

Assuming initial Gaussian density perturbation corresponding to a given density power spectrum, $P(|\vec{k}|)$, one can compute the potential $\Phi(\vec{q})$ on the grid $\vec{q}$. Using *Zel'dovich* approximation), one obtains the perturbed particle positions as initial conditions.

$$\vec{x}(a_{ini}) = \vec{q} - D_+(a_{ini})\Phi(\vec{q}), \qquad \vec{v}(a_{ini}) = \dot{D}_+(a_{ini})\vec{\nabla}\Phi(\vec{q})$$



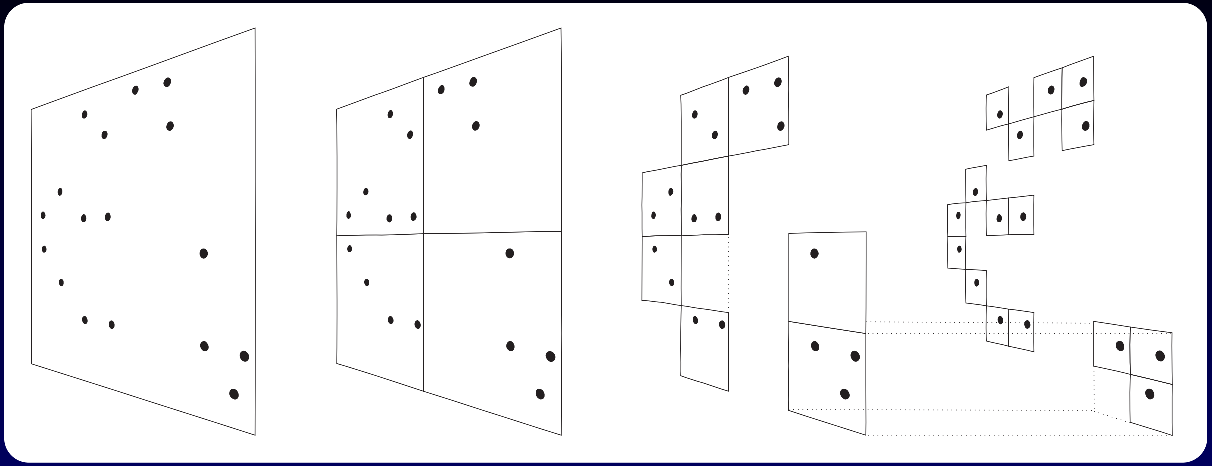
Simulating the universe

Integrate the equation of motion, within the framework of the expanding universe.

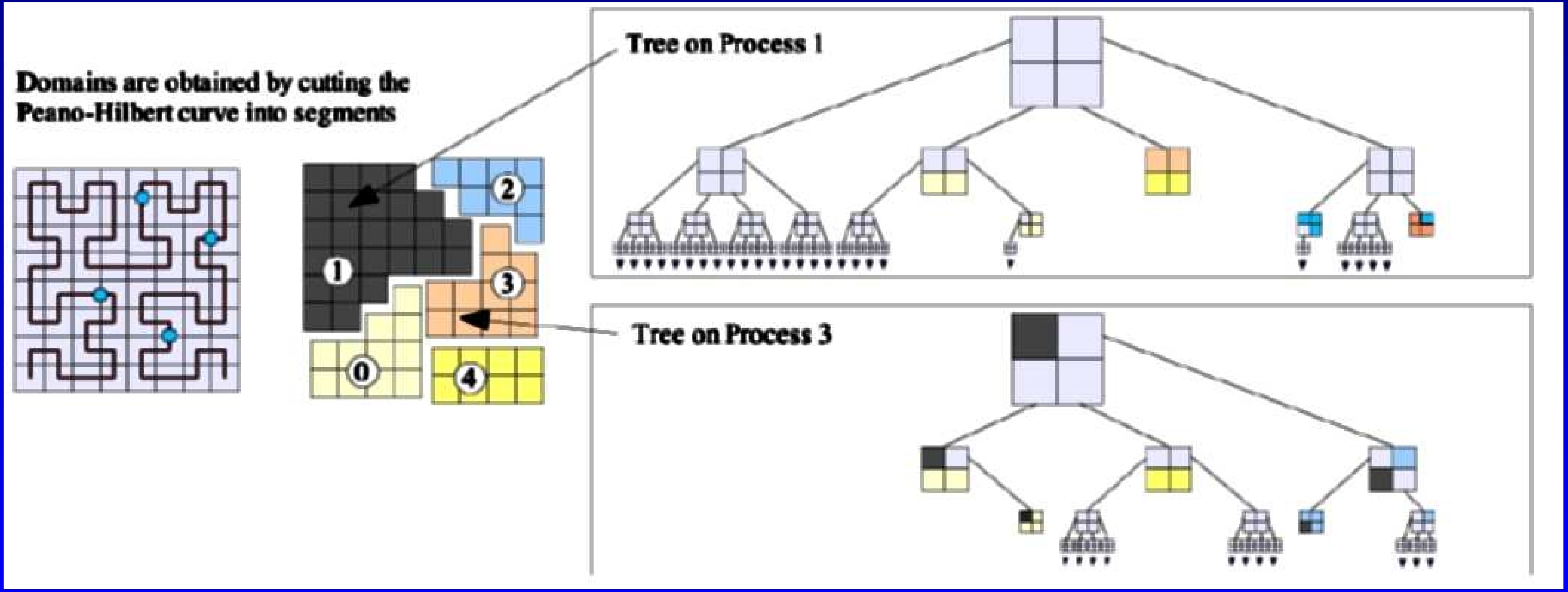
$\Rightarrow$ formation of typical, cosmic structures like voids, filaments and collapsed objects (e.g. galaxies and galaxy clusters)

Gravity

Tree-PM



$$\Phi(\vec{r}) = -G\sum_j \frac{m_j}{(|\vec{r}-\vec{r}_j|^2+\epsilon^2)^{\frac{1}{2}}}.$$



Gravity

Tree-PM

- density on the grid

$$\rho_m = \frac{1}{h^3} \sum_i m_i W(\vec{x}_i - \vec{x}_m).$$

- solve for Φ using FFT methods FFTW

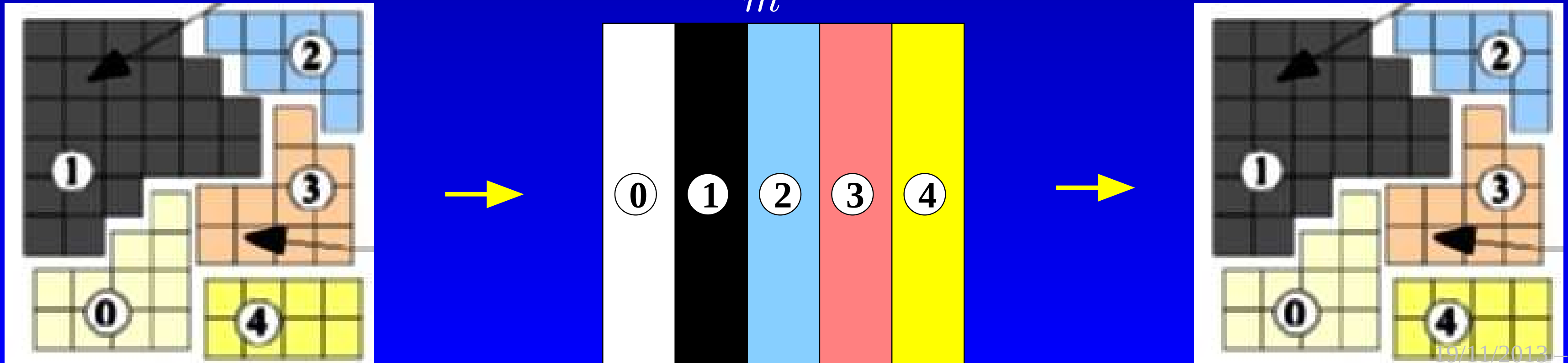
$$\Phi(\vec{x}) = \int g(\vec{x} - \vec{x}')\rho(\vec{x}')\mathrm{d}\vec{x}'$$

- calculate force using finite differences

$$f^{(x)}_{i,j,k} = -\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h}.$$

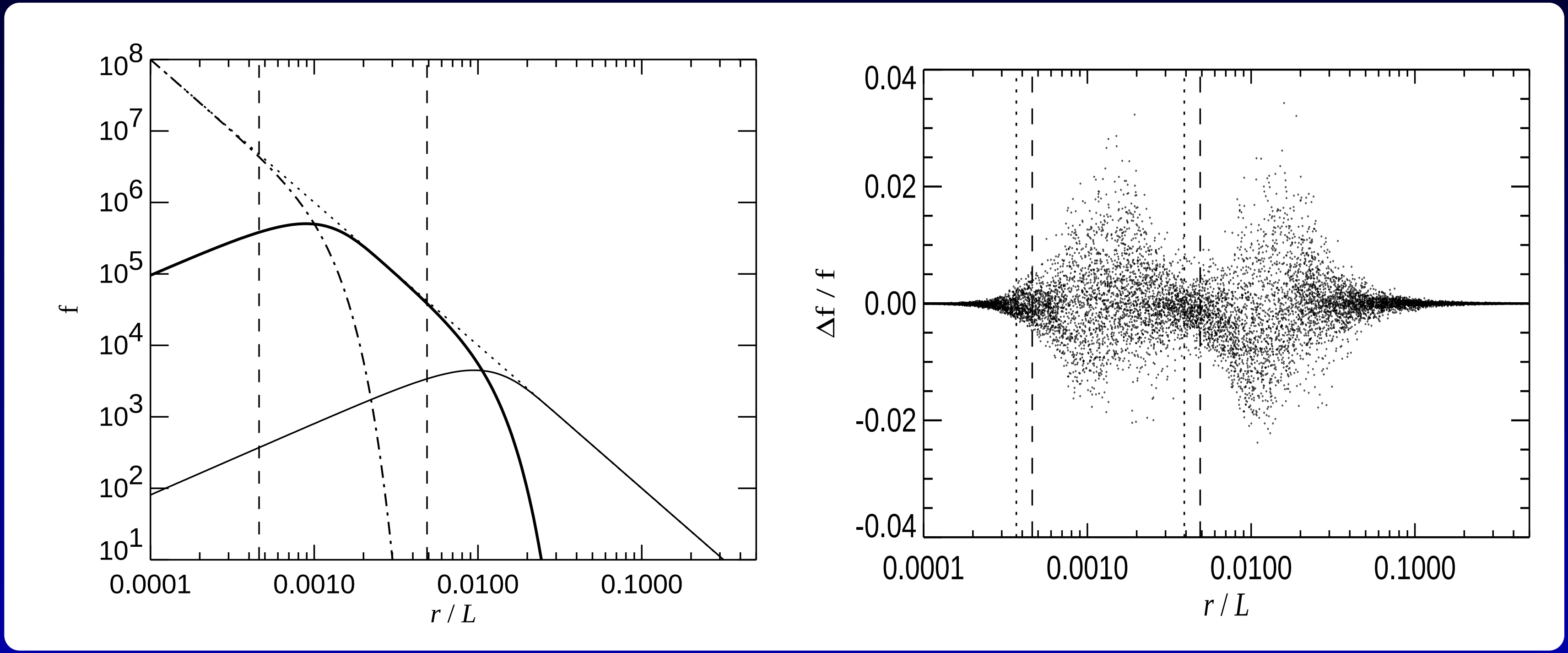
- interpolate force back to particles

$$\vec{f}(\vec{x}_i) = \sum_m W(\vec{x}_i - \vec{x}_m)\vec{f}_m,$$



Gravity

Tree-PM: $\Phi_{\vec{k}} = \Phi^{\mathrm{long}}_{\vec{k}} + \Phi^{\mathrm{short}}_{\vec{k}}$



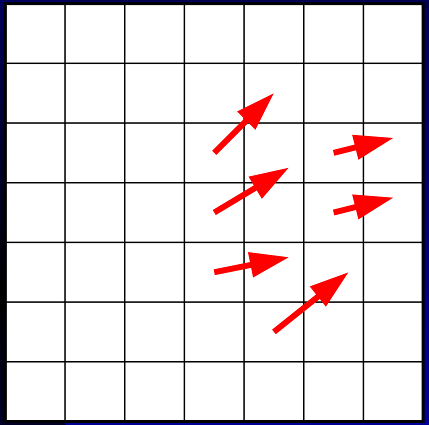
$$\Phi^{\mathrm{long}}_{\vec{k}} = \Phi_{\vec{k}} \exp(-\vec{k}^2 r_s^2)$$

$$\Phi^{\mathrm{short}}(\vec{x}) = -G \sum_i \frac{m_i}{\vec{r}_i} \mathrm{erfc}\left(\frac{\vec{r}_i}{2r_s}\right)$$

From dark to light (SPH)

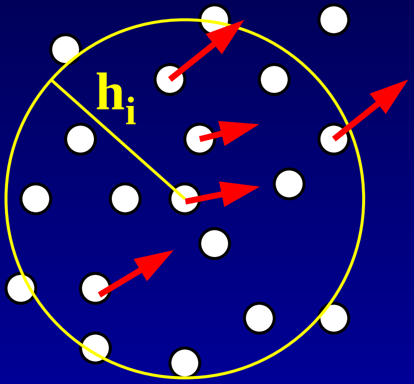
Eulerian

discretized space



Lagrangian

discretized mass



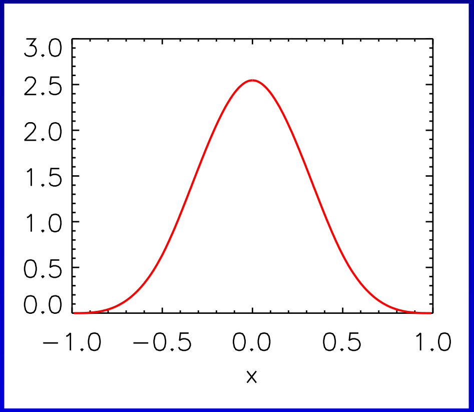
SPH

kernel estimate

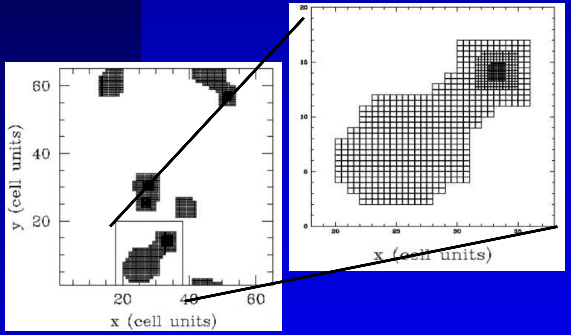
$$\langle A(\mathbf{r})\rangle = \int W(\mathbf{r}-\mathbf{r}',h)\,A(\mathbf{r}')\,\mathrm{d}^3r'$$

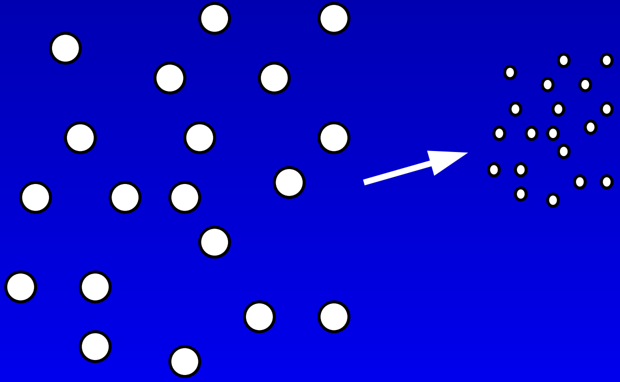
$$\mathrm{d}^3r' \mapsto \frac{m_j}{\rho_j}$$

$$\langle A_i\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} A_j\, W(\mathbf{r}_{ij};h_i)$$



Collapse:





From dark to light (SPH)

Add a baryonic component into n-body simulations (e.g. additional tracing particles) which has also hydrodynamic interactions (e.g. fluid equations) where continuous fluid quantities are based on kernel estimates

$$\langle A(\vec{x})\rangle = \int W(\vec{x}-\vec{x}', h)A(\vec{x}')\mathrm{d}\vec{x}' = \sum_j \frac{m_j}{\rho_j} A_j W(\vec{x}_i - \vec{x}_j, h)$$

Lagrangian for the fluid (represented by the tracer particles)

$$L(\vec{q}, \dot{\vec{q}}) = \frac{1}{2}\sum_i m_i \dot{\vec{x}}_i^2 - \sum_i m_i u_i, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\vec{q}}_i} - \frac{\partial L}{\partial \vec{q}_i} = 0$$

$\Rightarrow$ equation of motion (e.g. *Euler* equation)

$$\frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = -\sum_j m_j \left(\frac{P_j}{\rho_j^2} \vec{\nabla}_i W(\vec{x}_i - \vec{x}_j, h_j) + \frac{P_i}{\rho_i^2} \vec{\nabla}_i W(\vec{x}_i - \vec{x}_j, h_i) \right)$$

with $P_i = (\gamma - 1)\rho u_i$ (EoS) and artificial viscosity (shocks).

From dark to light (SPH)

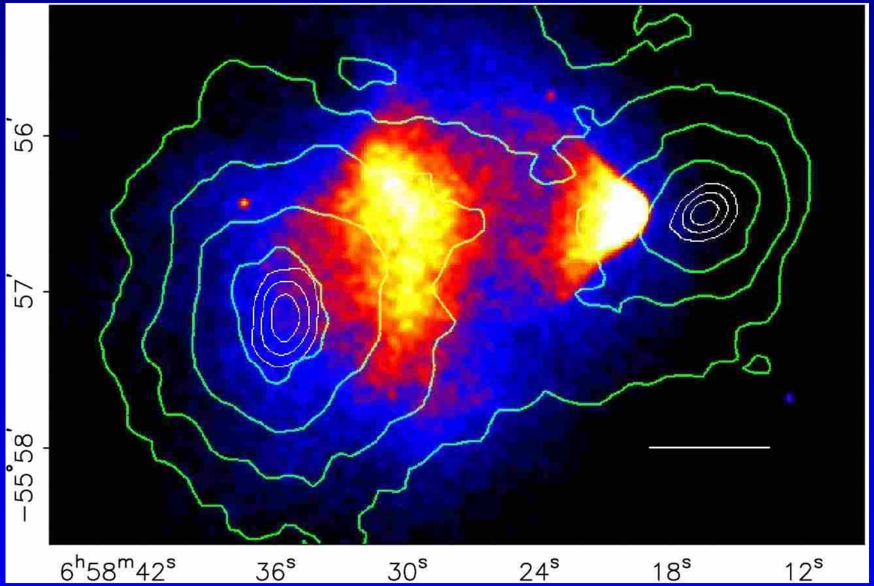
Now we can follow the equation of motion of two species of particle, but for the gas additional terms appear.

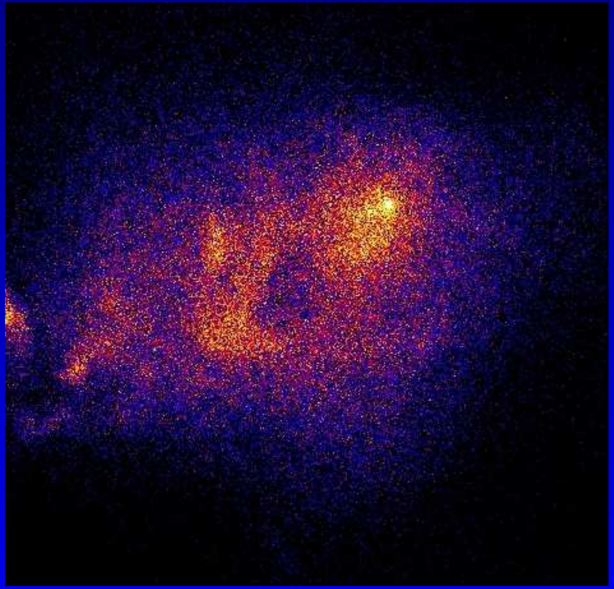
$$\left(\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\right)_{\mathrm{D}M} = -\vec{\nabla}\Phi, \quad \left(\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\right)_{gas} = -\frac{\vec{\nabla}P}{\rho} - \rho\vec{\nabla}\Pi_{ij} - \vec{\nabla}\Phi$$

From dark to light (SPH)

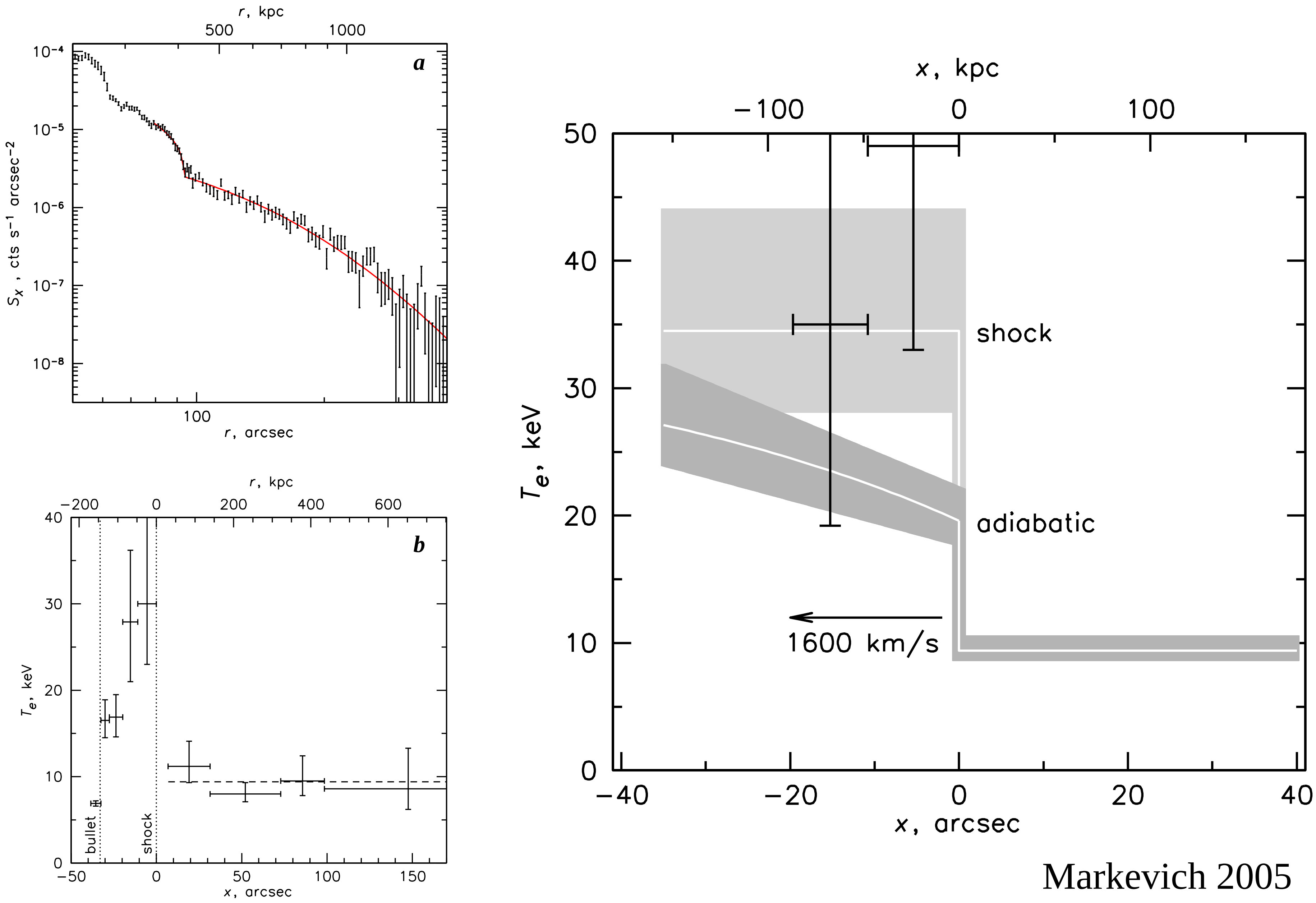
The Bullet Cluster

Example: Bullet Cluster





The Bullet Cluster

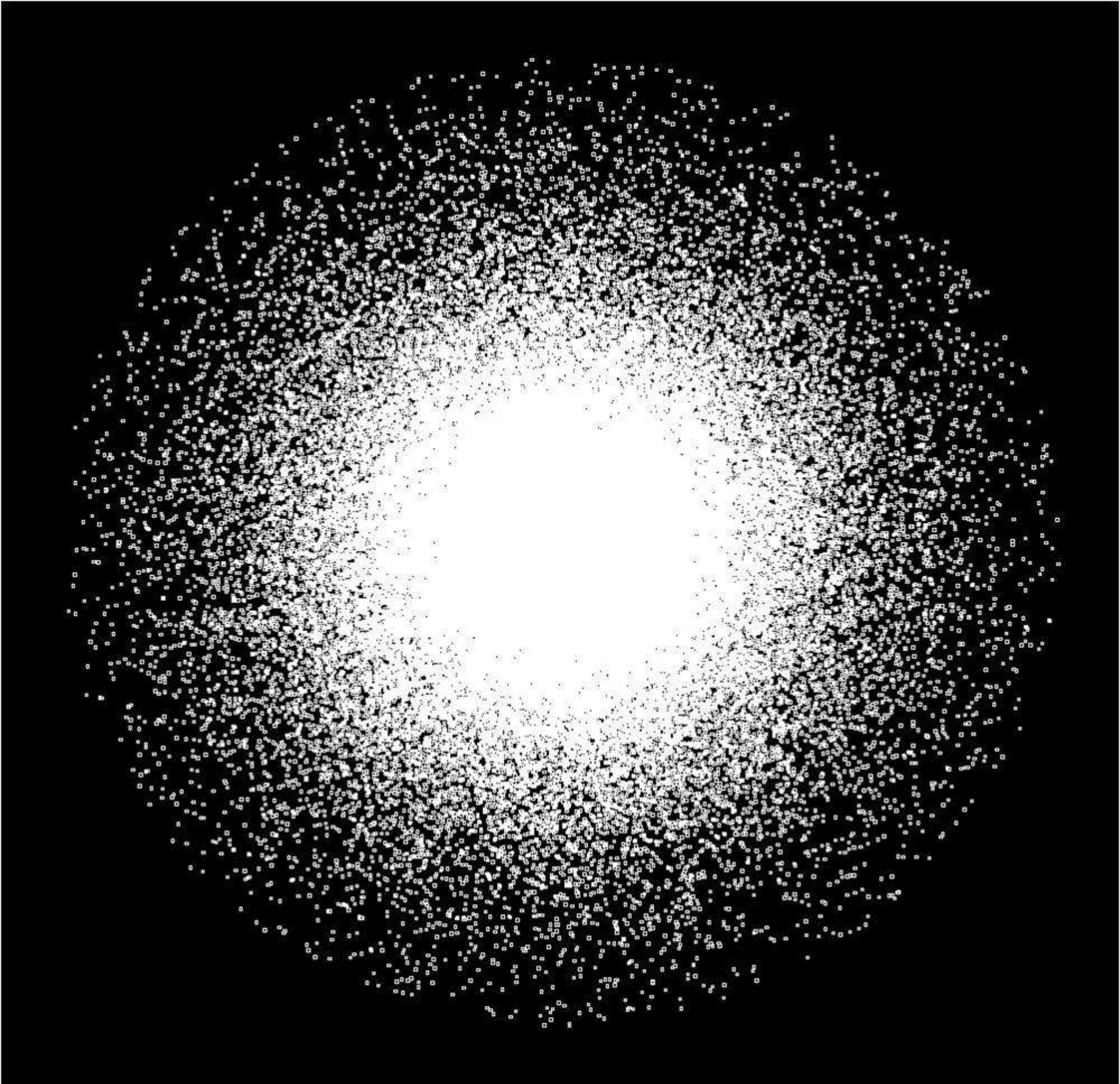


Markevich 2005

Electron-Ion equiblibration much faster than Spitzer timescale
⇒ Hydro/MHD

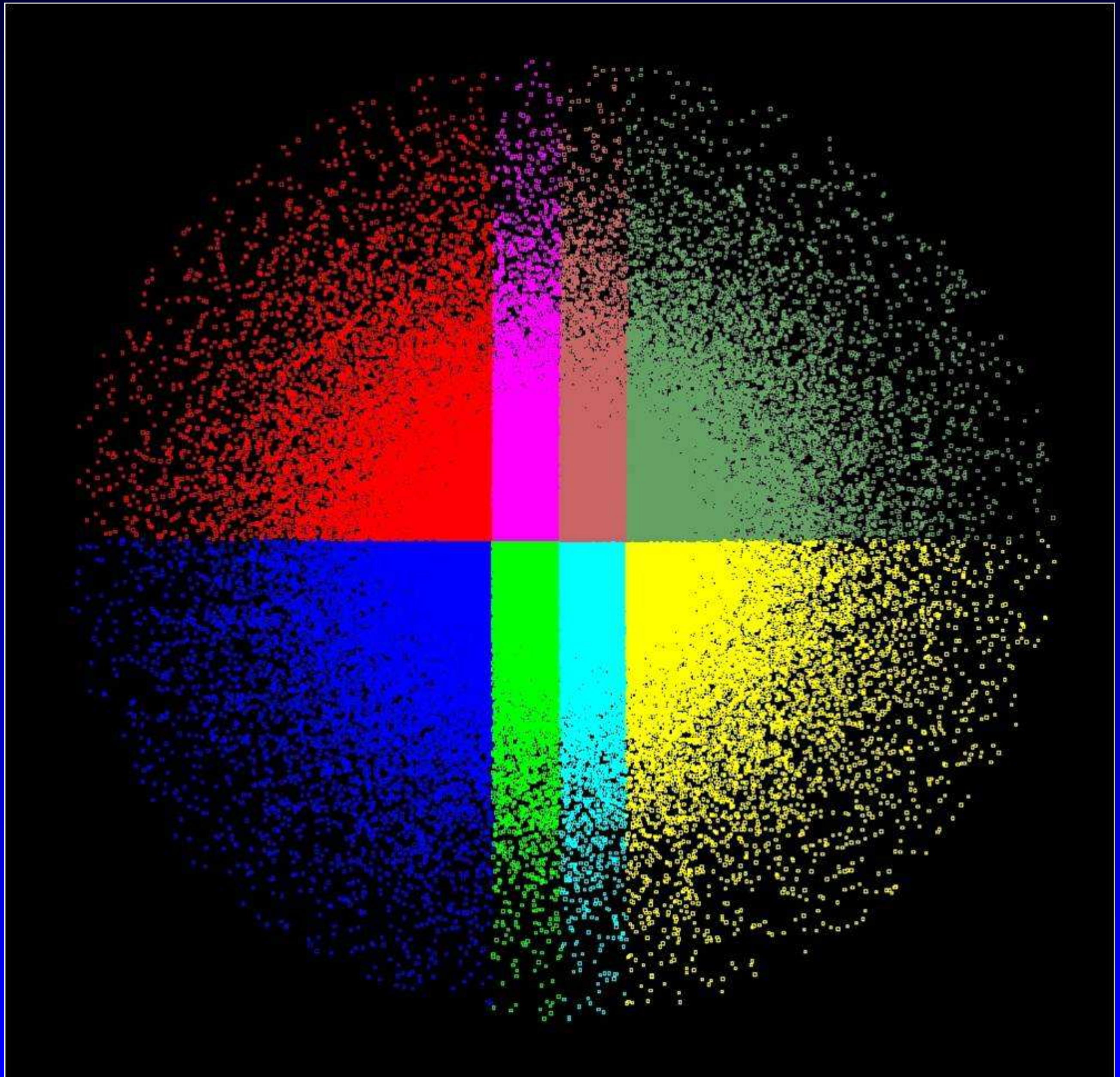
Distributing the Work

Example



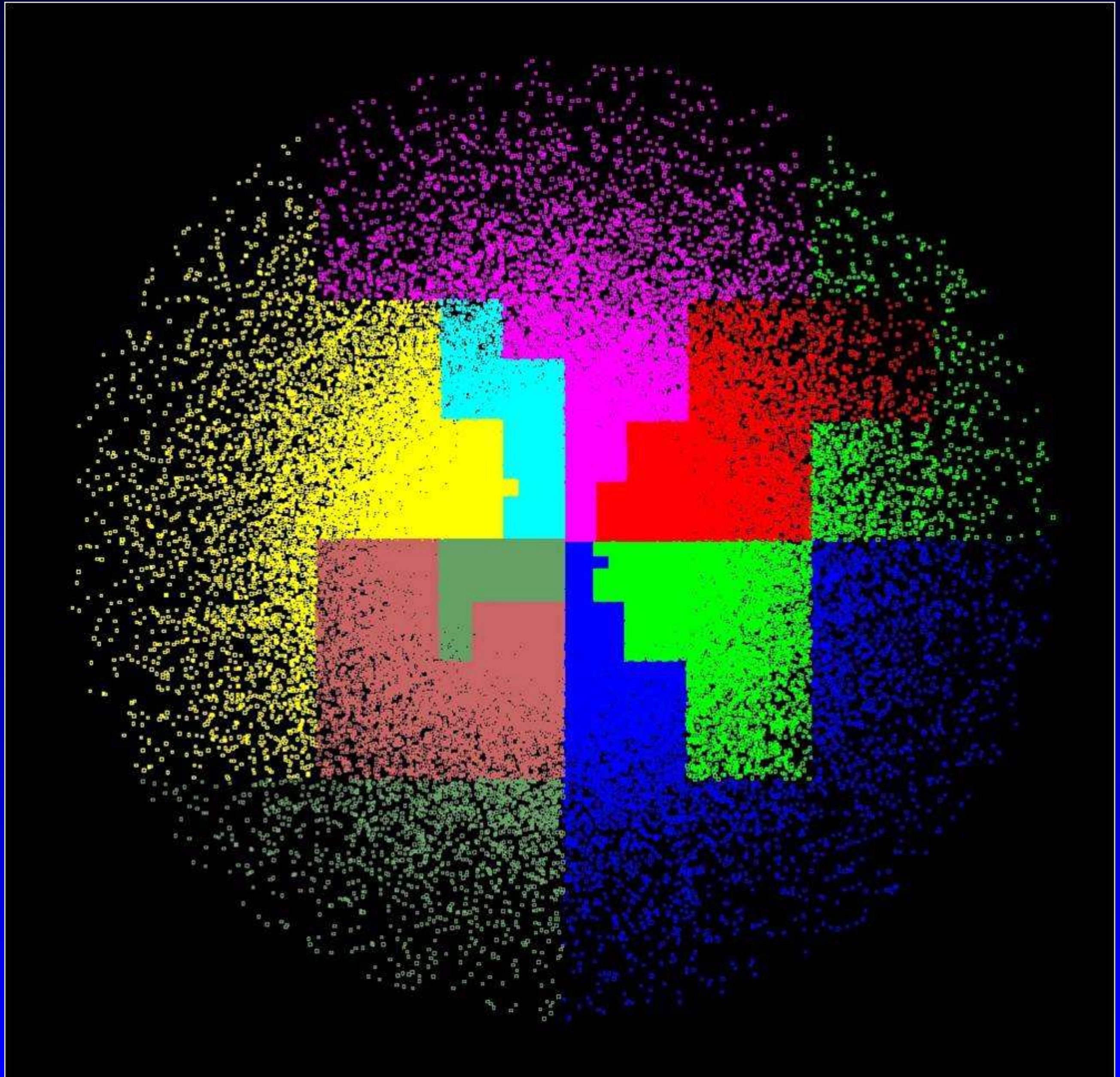
Distributing the Work

Simple



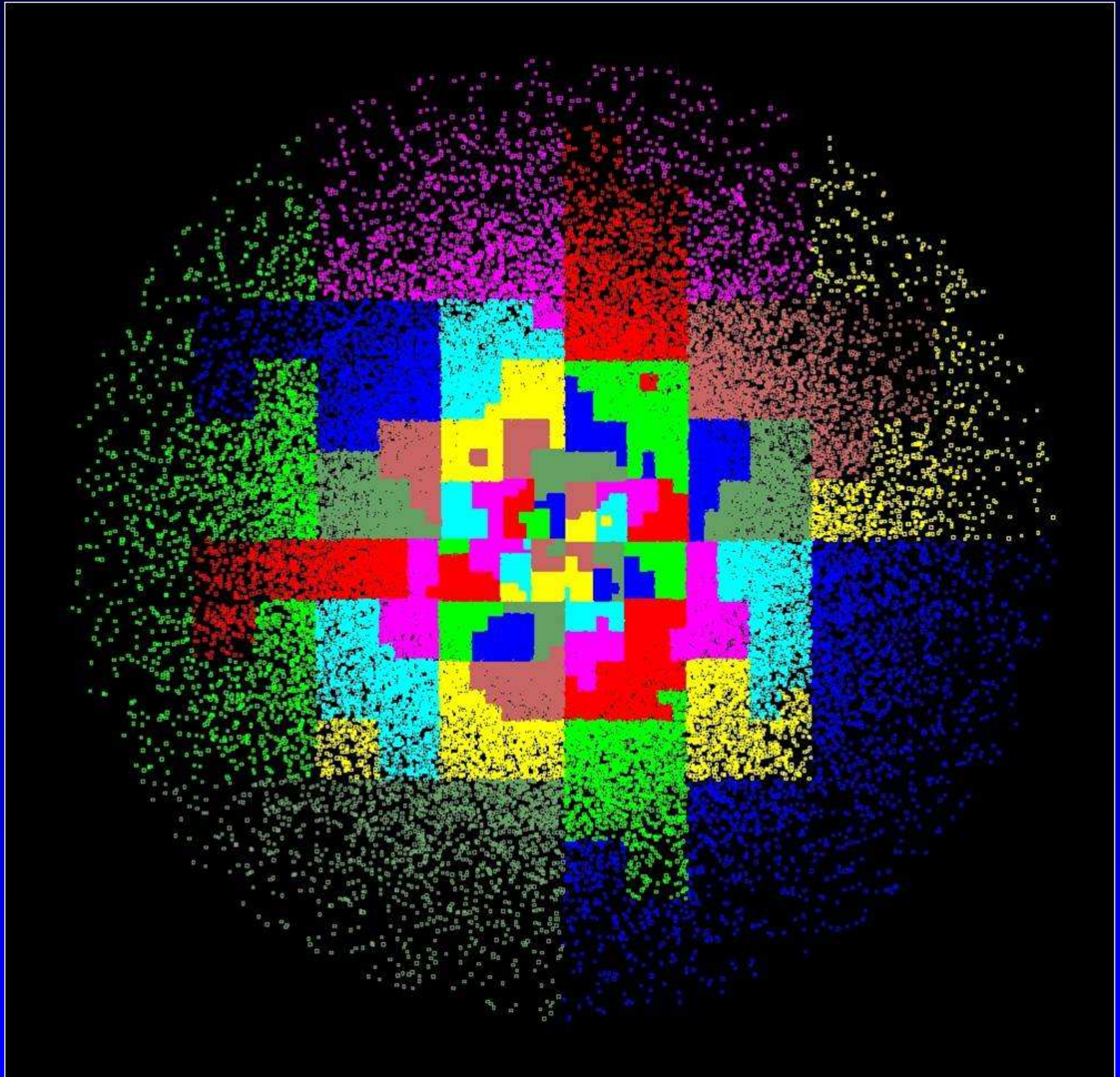
Distributing the Work

Space-filling curve (Peano Hilbert)

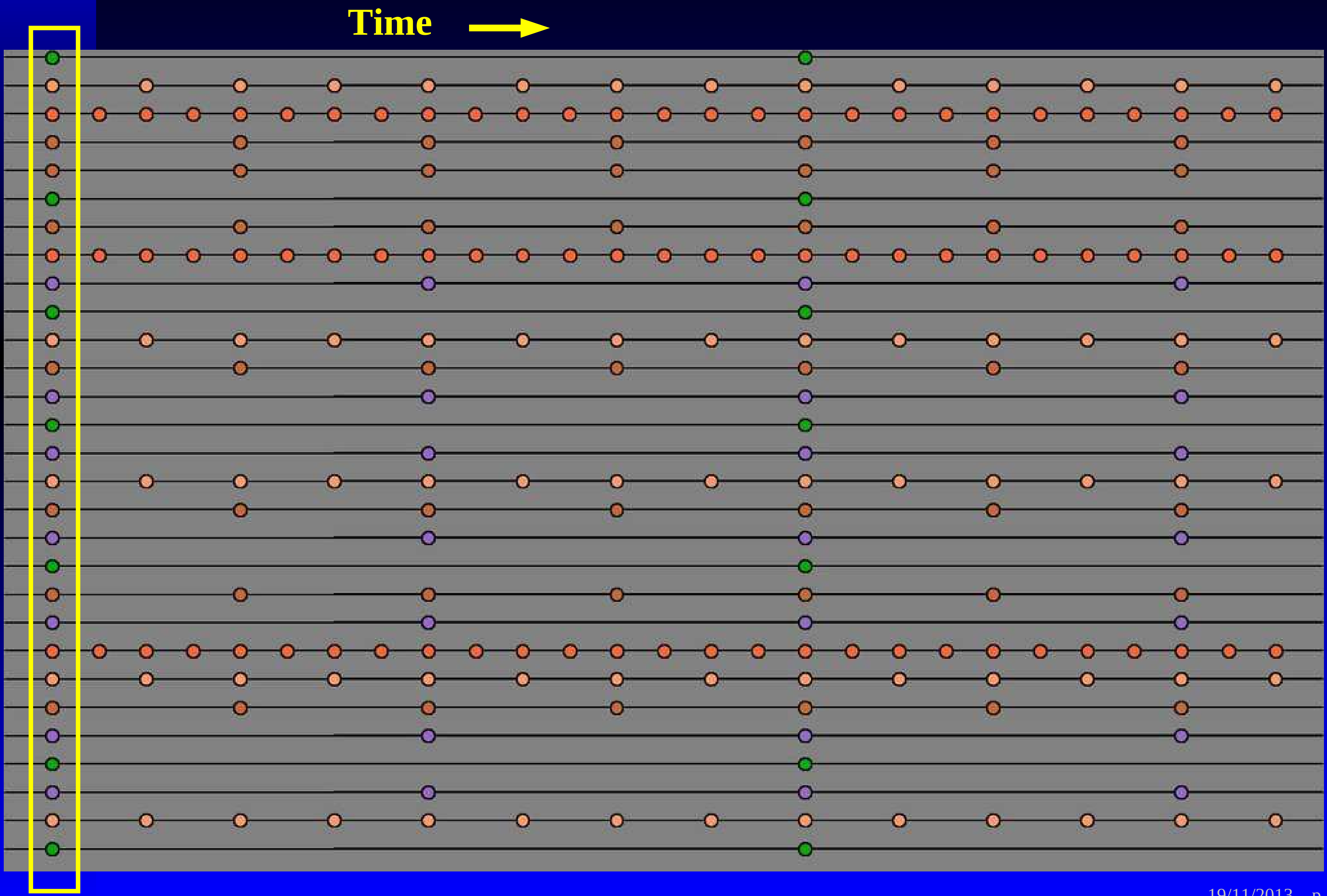


Distributing the Work

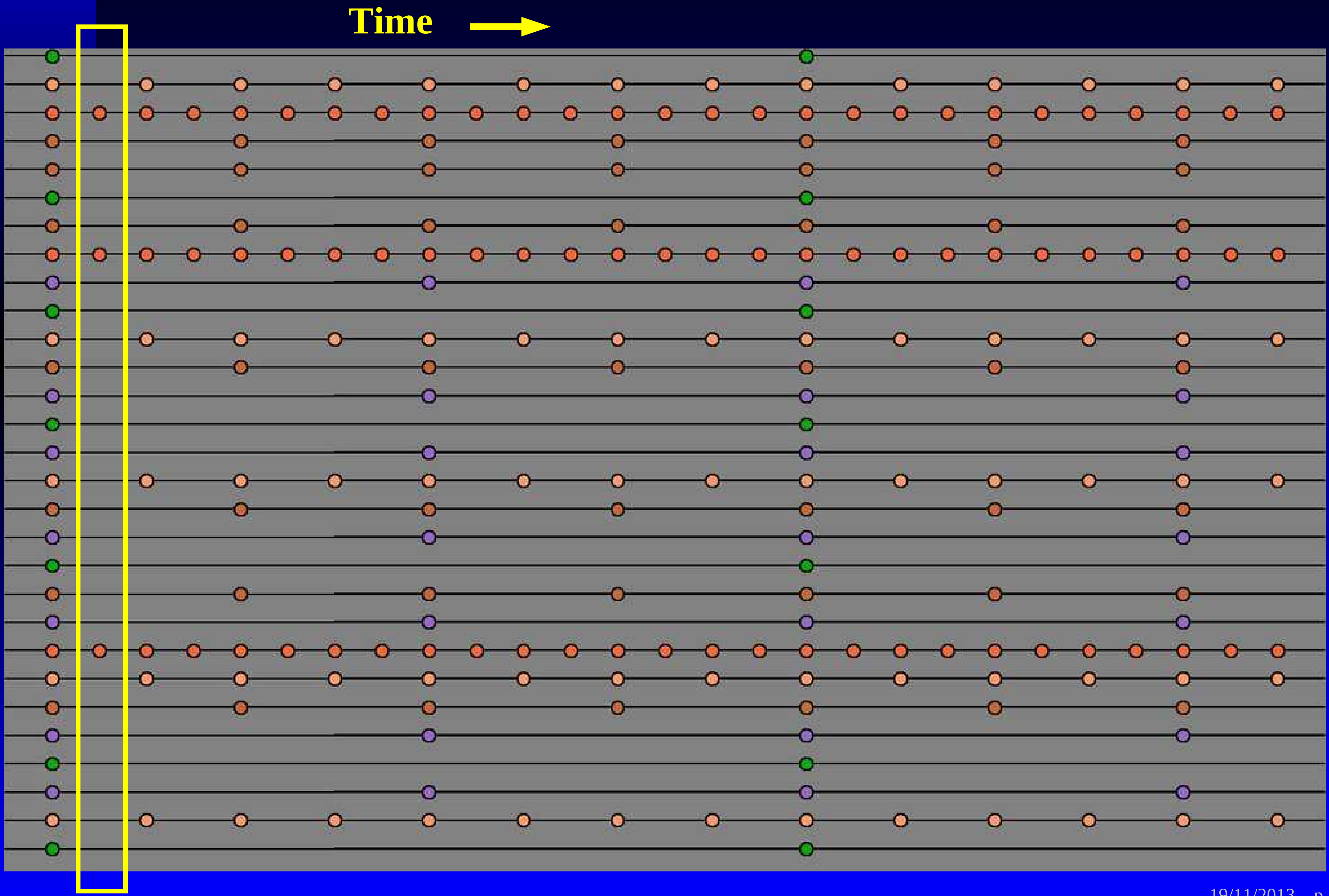
Multiple space-filling curve



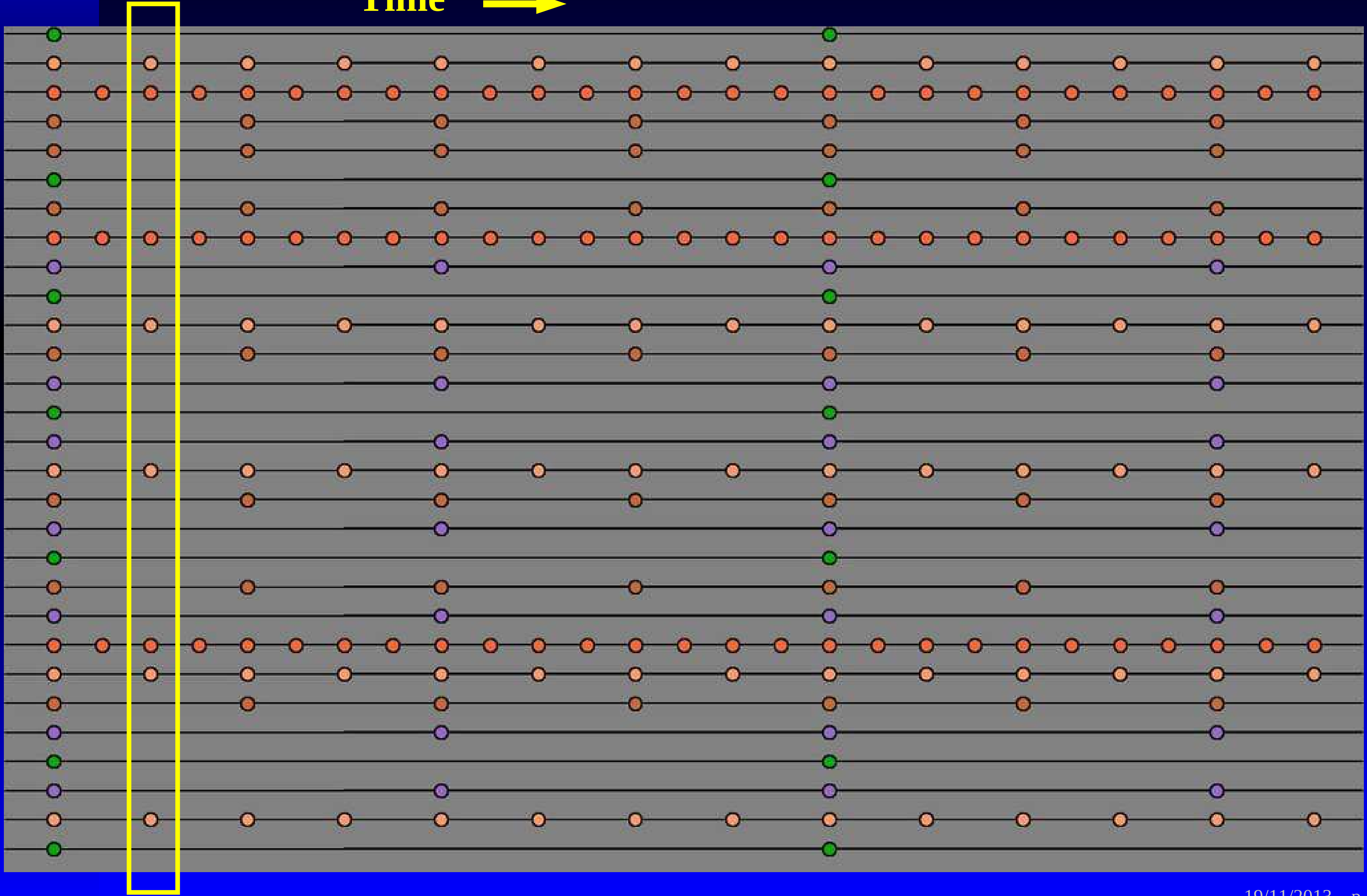
Time Integration



Time Integration

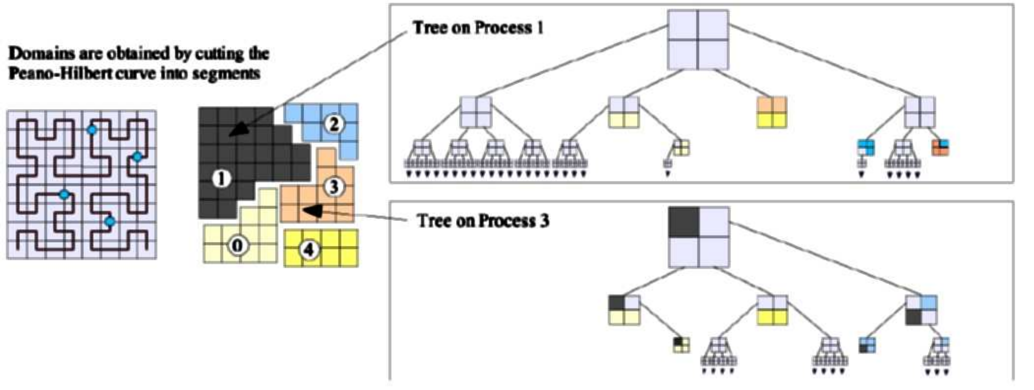


Time Integration



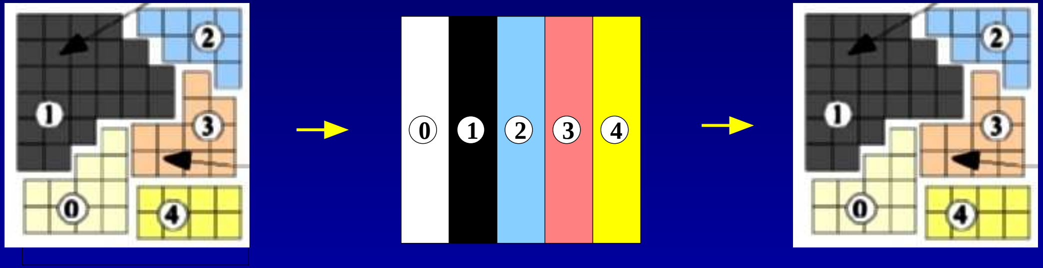
Summary

- **Tree like**



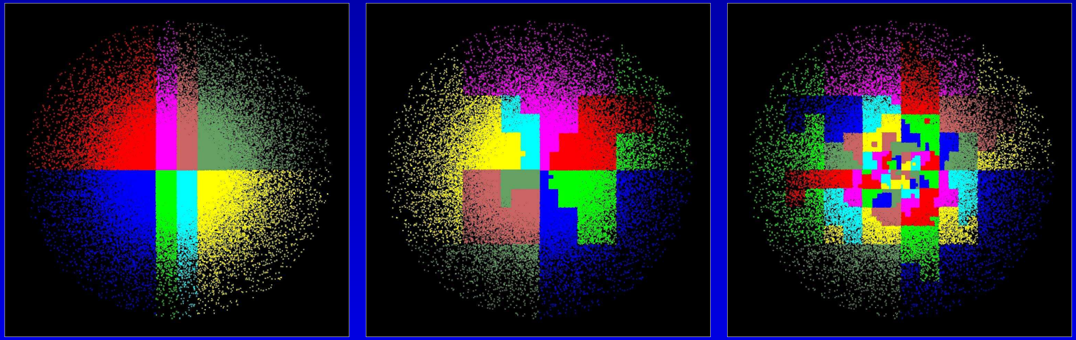
Short range gravity, Hydrodynamics, Transport, Star-formation/AGN feedback

- **Grid like**



Long range gravity (including FFTW)

- **Work load**

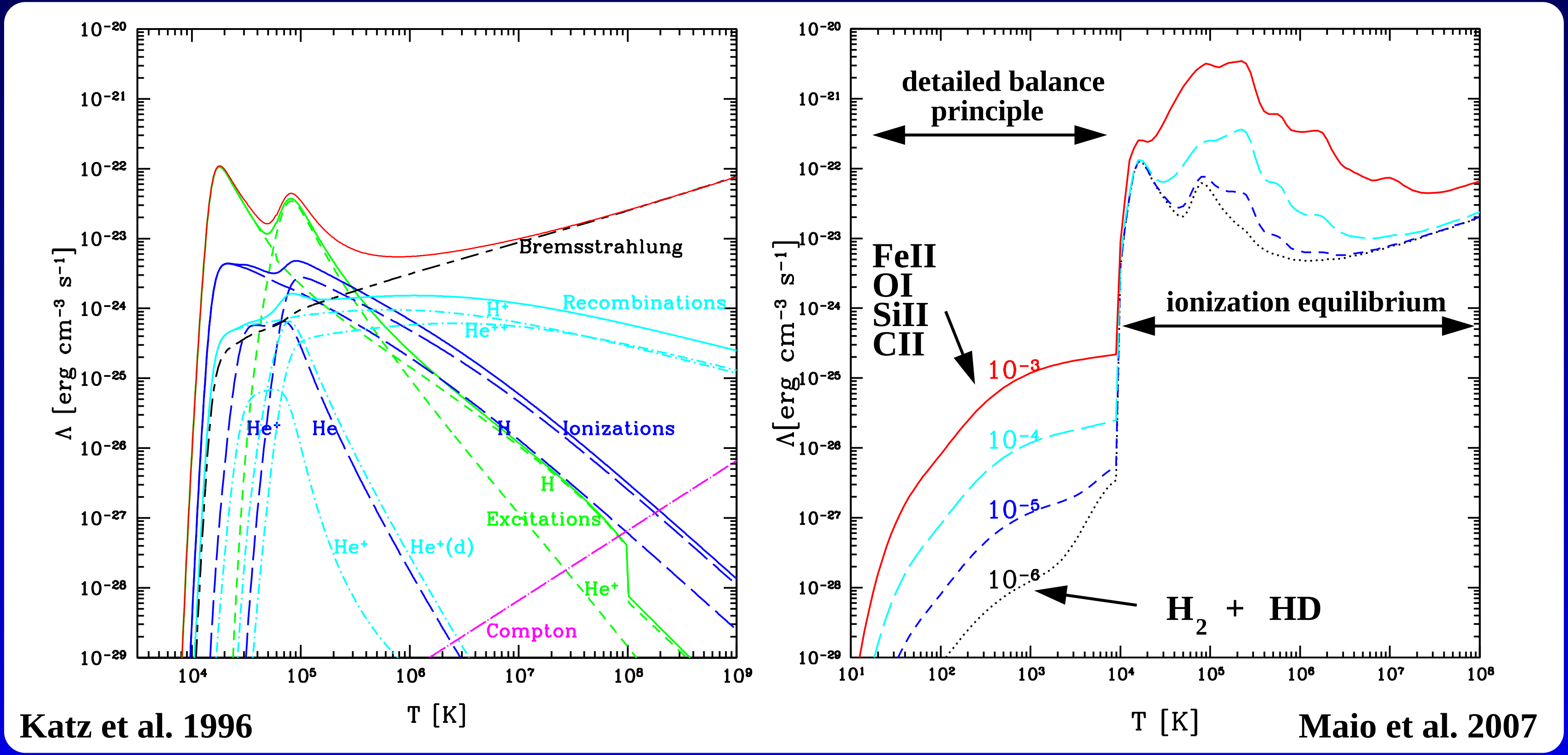


- **Post processing**

Various different algorithms, all distributed.

Cooling & Star-formation

This diffuse, optically thin gas with primordial composition (e.g. H,He) can cool because of various physical processes (e.g assuming ionization equilibrium). But pollution from metals (e.g. Fe,O,Si,C) will increase the cooling function.



Katz et al. 1996

Maio et al. 2007

But cooling $\propto n^2$ therefore cooling catastrophe in collapsed objects !

Cooling & Star-formation

In nature cooled gas will form stars (on scales much below resolution). Therefore "*suitable*" recipes have to be build in. Simplest form (Katz et al 1996):

- convergent flow, e.g. $\vec{\nabla}\vec{v} < 0$.
- high density region, e.g. $\rho > 0.1$ Atoms/cm^3.
- Jeans instability $\frac{h}{c} > t_{\mathrm{dyn}}$ with $t_{\mathrm{dyn}} = (4\pi G\rho)^{-0.5}$.

⇒ Conversion of cold gas into stars: $\frac{\mathrm{d}\rho_*}{\mathrm{d}t} = -\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{c_*\rho}{t_*}$.
c_*: Star-formation efficiency (e.g. $\approx 10\%$).
t_*: Star-formation time (e.g. max of t_{dyn} and $t_{cool} = u/\dot{u}$).

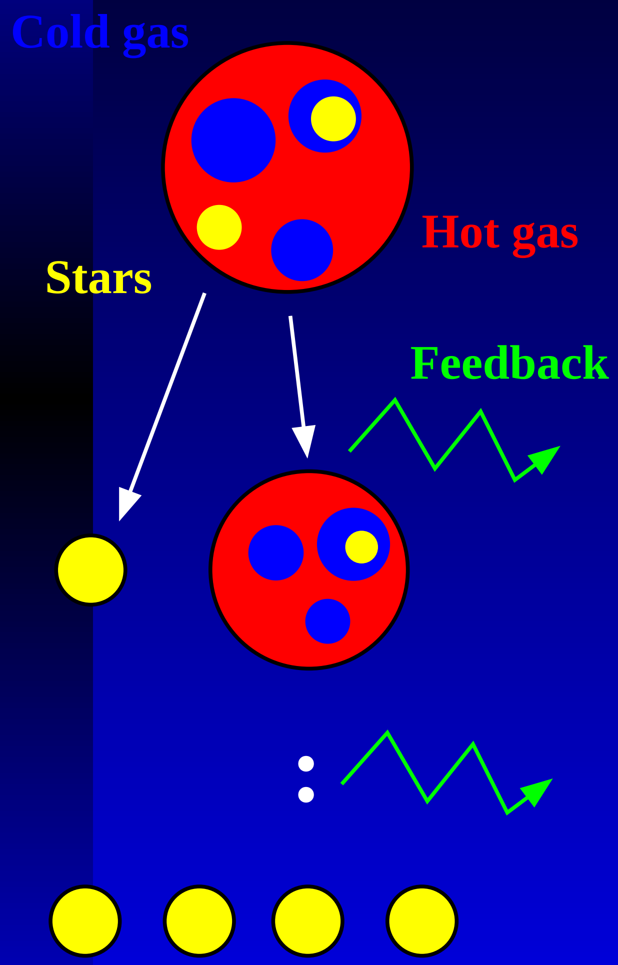
⇒ Heating of the gas by type-II supernovae (e.g. 10^{51} erg/SN).
Livetime of $m_* > 8M_\odot$ typically $<$ than Δt_{sim} (e.g. IRA).
Number of SNII from initial mass function (e.g. *Salpeter*).

⇒ When significant fraction of gas is converted to stars, spawn new star particles (e.g. 2-3).
Such star particles further interact in a collisionless way.

Cooling & Star-formation

Multi phase model (sub−scale)

Springel & Hernquist 2002



Star formation

supernova mass fraction

$$\frac{d\rho_\star}{dt} = (1-\beta)\frac{\rho_c}{t_\star}$$

star formation timescale

Cloud evaporation

$$\left.\frac{d\rho_h}{dt}\right|_{\mathrm{evap}} = A\beta\frac{\rho_c}{t_\star}$$

cloud evaporation parameter

Growth of clouds

cooling function

$$\left.\frac{d\rho_c}{dt}\right|_{\mathrm{TI}} = -\left.\frac{d\rho_h}{dt}\right|_{\mathrm{TI}} = \frac{\Lambda_{\mathrm{net}}(\rho_h, u_h)}{u_h - u_c}$$

Sub-scale model for star-formation:
gas particle ($m = 10^9 M_o$) = star formation region
start particle ($m = 10^8 M_o$) = star cluster

Cooling & Star-formation

Extending star-formation model for a more detailed description of the evolution of the stellar population.
(Tornatore, Borgani, Dolag & Matteucci 2007):

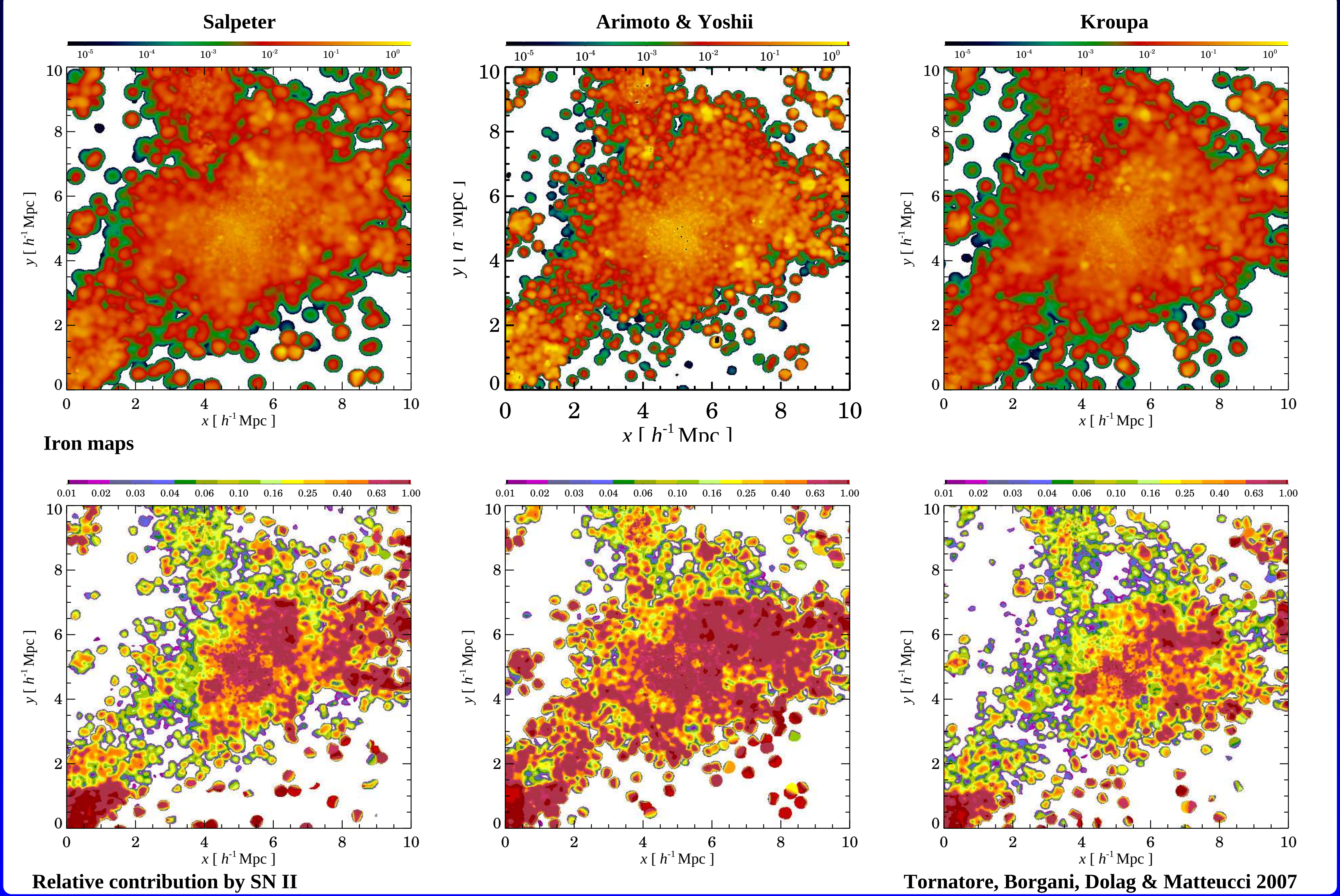
- Model rate of SN Ia (e.g. binary systems $0.8 - 8M_{\odot}$).
- Adopt lifetime function $\tau(m)$
  (Padovani & Matteucci 1993, Maeder & Maynet 1989, ...).
- Adopt yields $p_{Z_i}(m, Z)$
  (Hoek & Groenewegen 1997, Thielemann et al. 2003, Woosley & Weaver 1995, ...).
- The IMF fixes the number of stars at given mass
  (Salpeter 1955, Arimoto & Yoshii 1987, Kroupa 2001, ...).

$\Rightarrow$ Explicitly follow the evolution of rates for SN Ia, SN II and AGB stars along with their respective metal production.

$\Rightarrow$ "Stars" evolve and give back H, He, Fe, O, C, Mg, S to the surrounding medium.

Cooling & Star-formation

Example of the enrichment (e.g. Fe) of the ICM in a galaxy cluster simulation obtained using different IMFs.



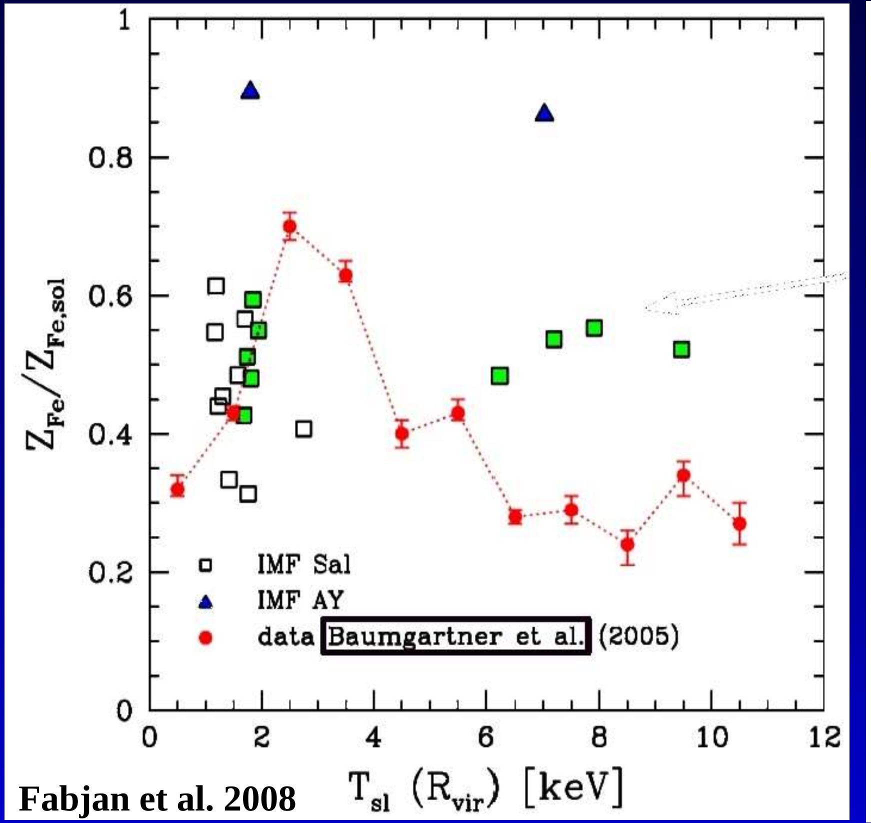
Iron maps

Relative contribution by SN II

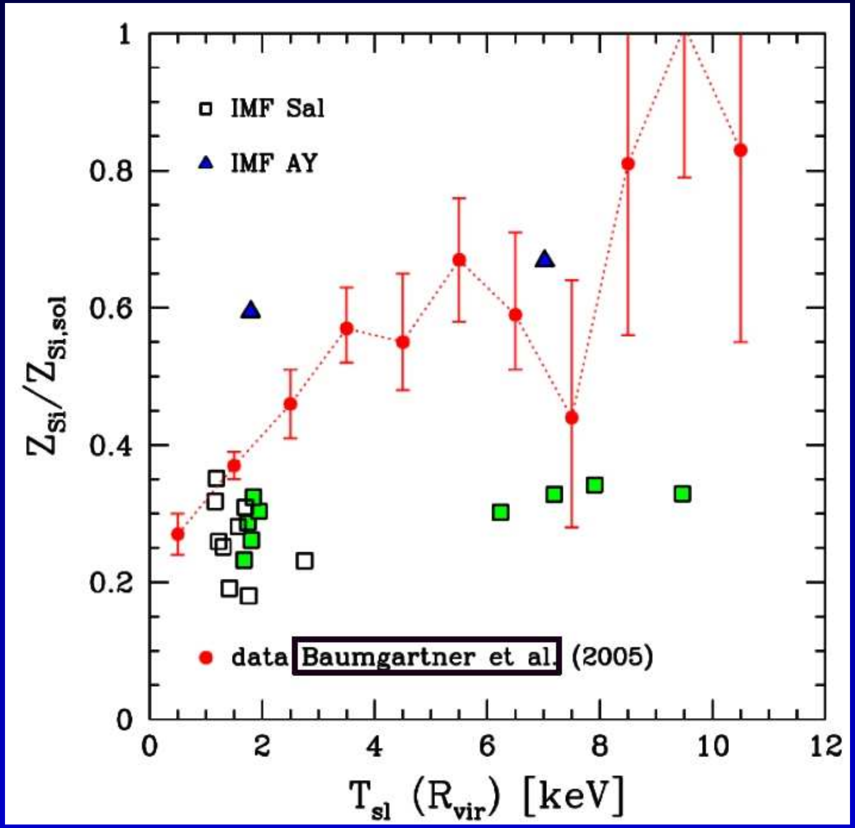
Tornatore, Borgani, Dolag & Matteucci 2007

Cooling & Star-formation

Example of the resulting Si and Fe content of the ICM in cluster simulations using different IMFs.



Fabjan et al. 2008



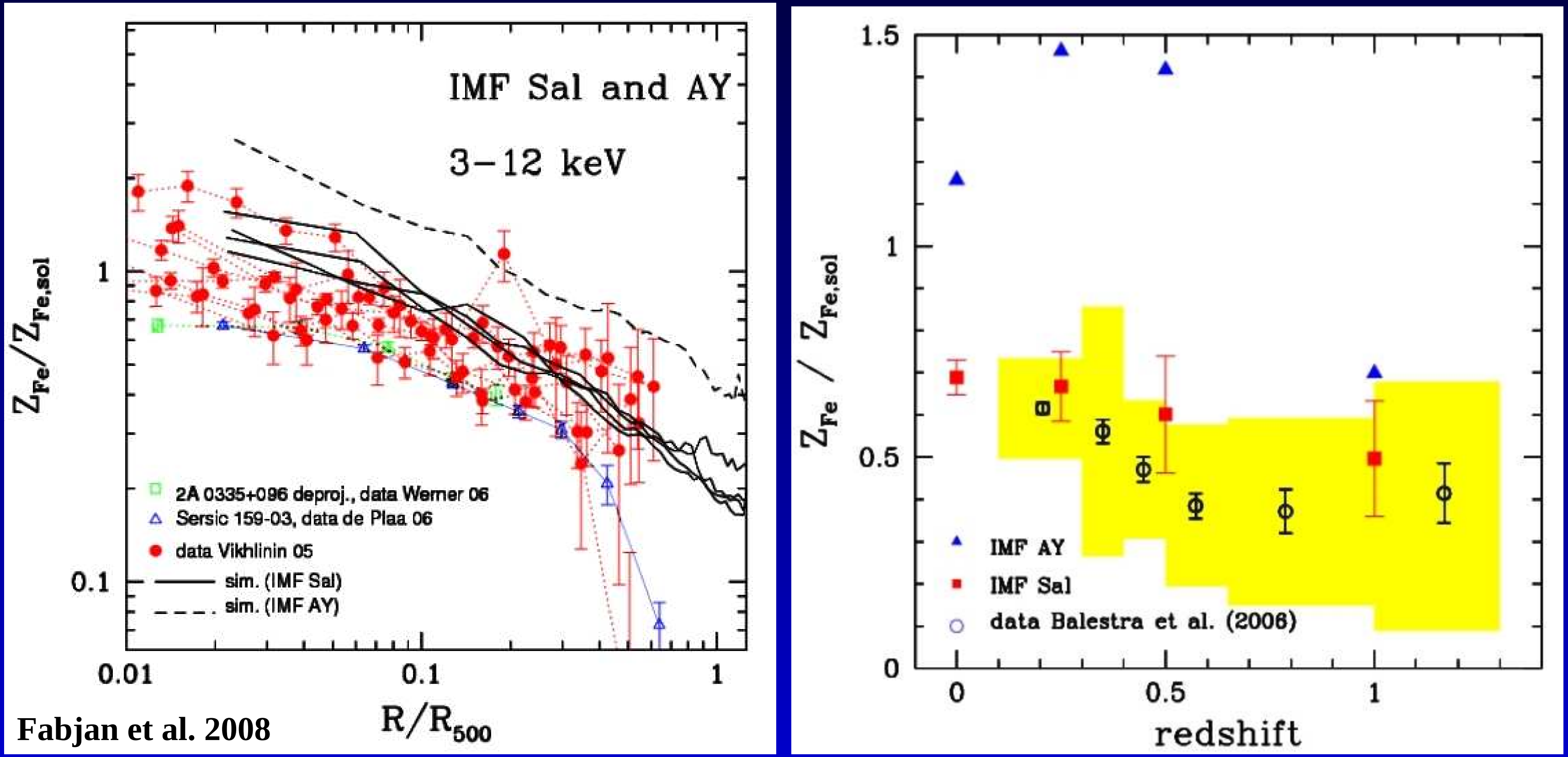
⇒ Salpeter IMF better for group data ?

⇒ Arimoto & Yoshii IMF better for Si in clusters ?

⇒ But uncertainties in observational data and biases in the observational process (see Rasia et al. 2007) !

Cooling & Star-formation

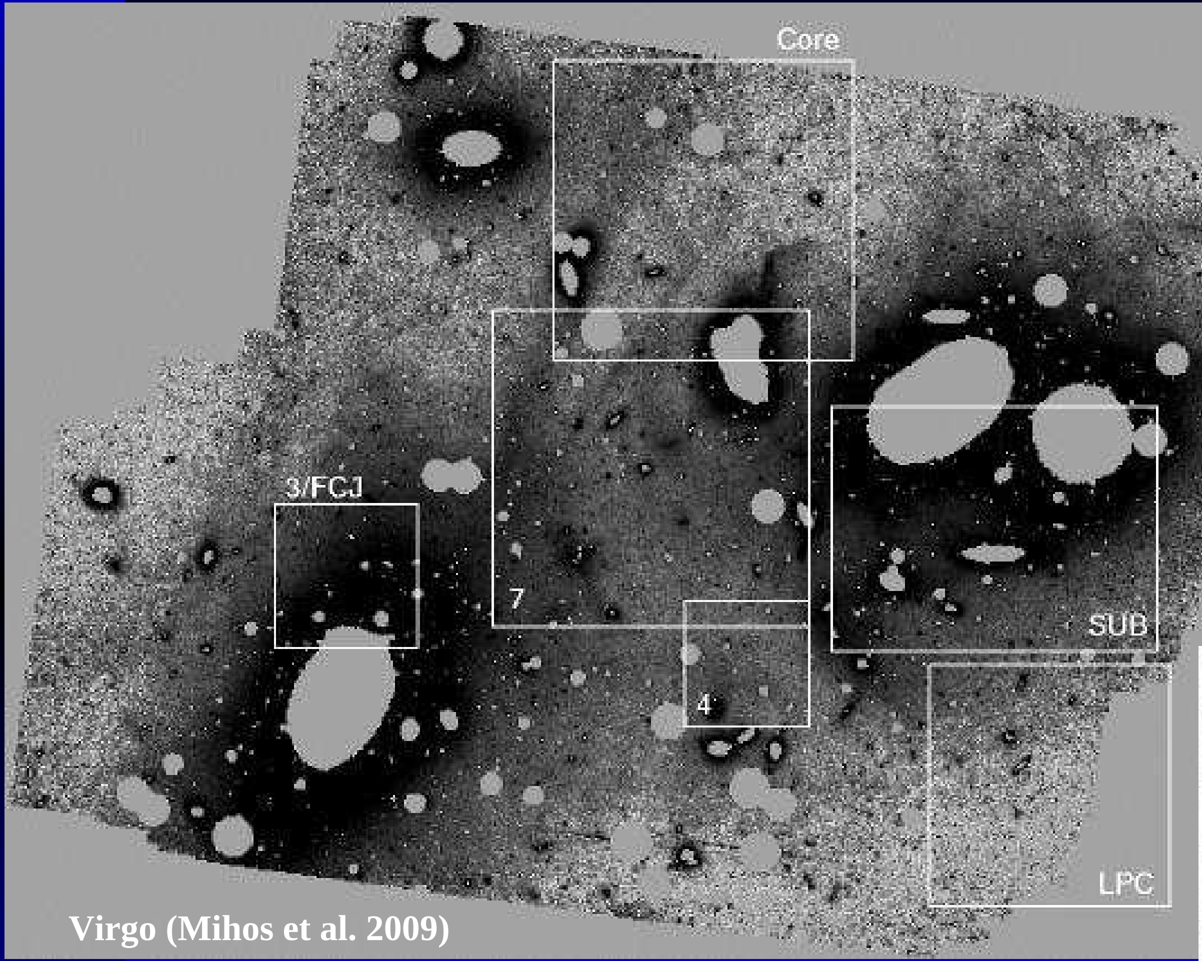
Example of the obtained Iron distribution and evolution in cluster simulations using different IMFs.



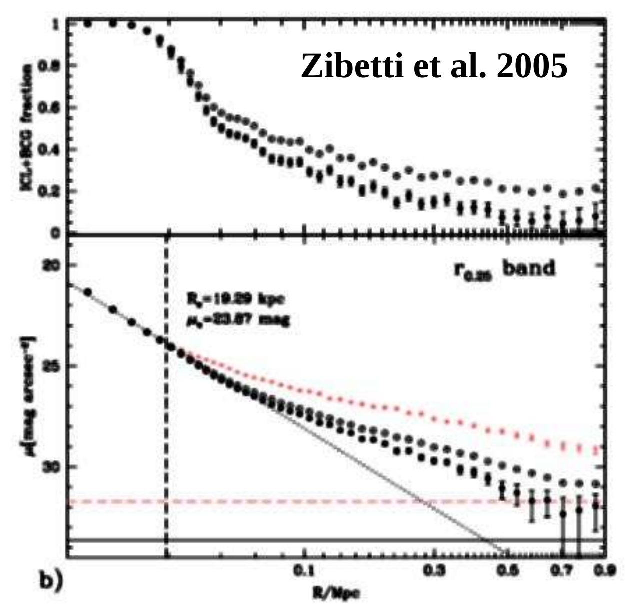
Fabjan et al. 2008

⇒ General trends reproduce observations.
⇒ Simulated metal gradients too steep.
⇒ No needs for non-Salpeter IMF.

The ICL component

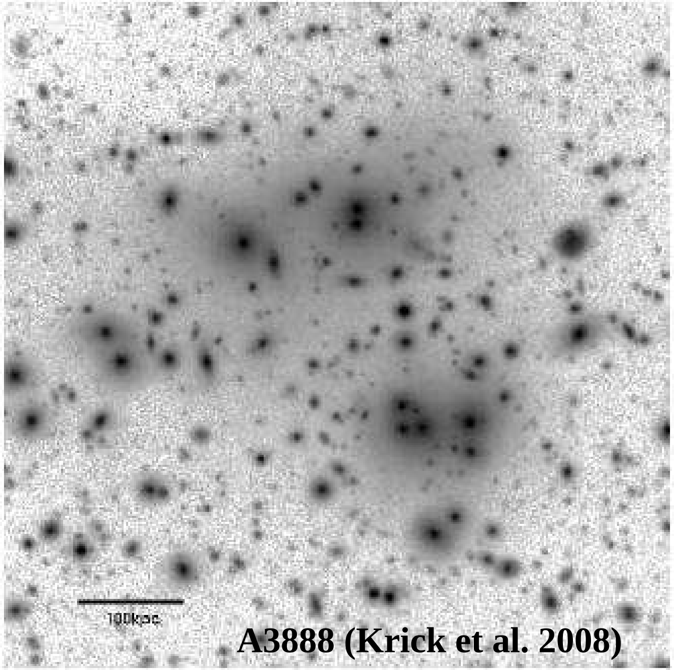


Virgo (Mihos et al. 2009)



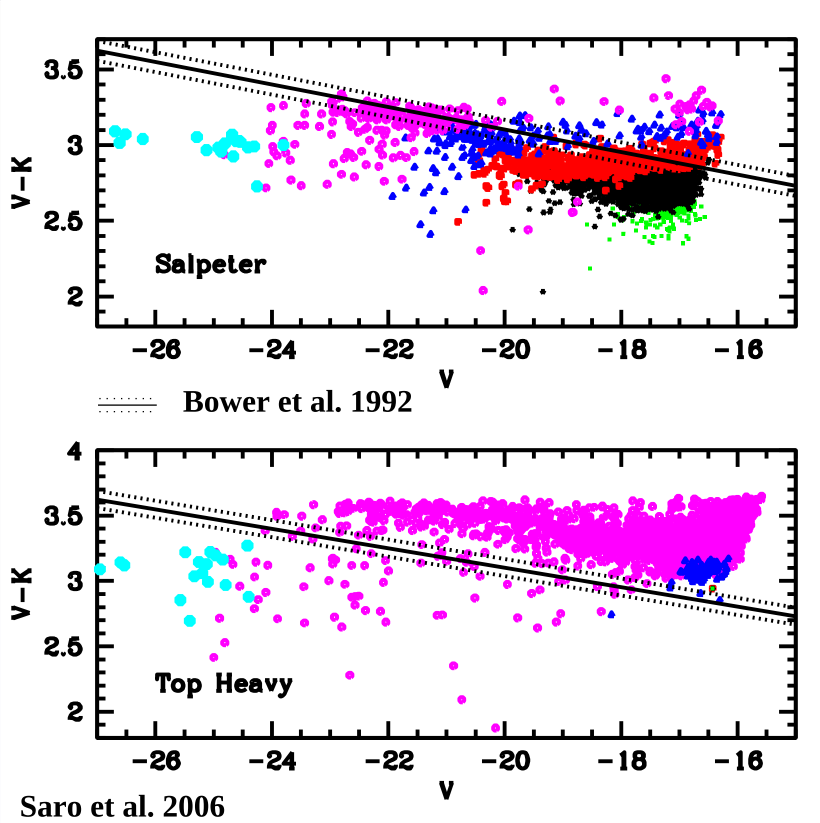
Zibetti et al. 2005

Intra Cluster Ligth can make 10-40% of the stars in a cluster

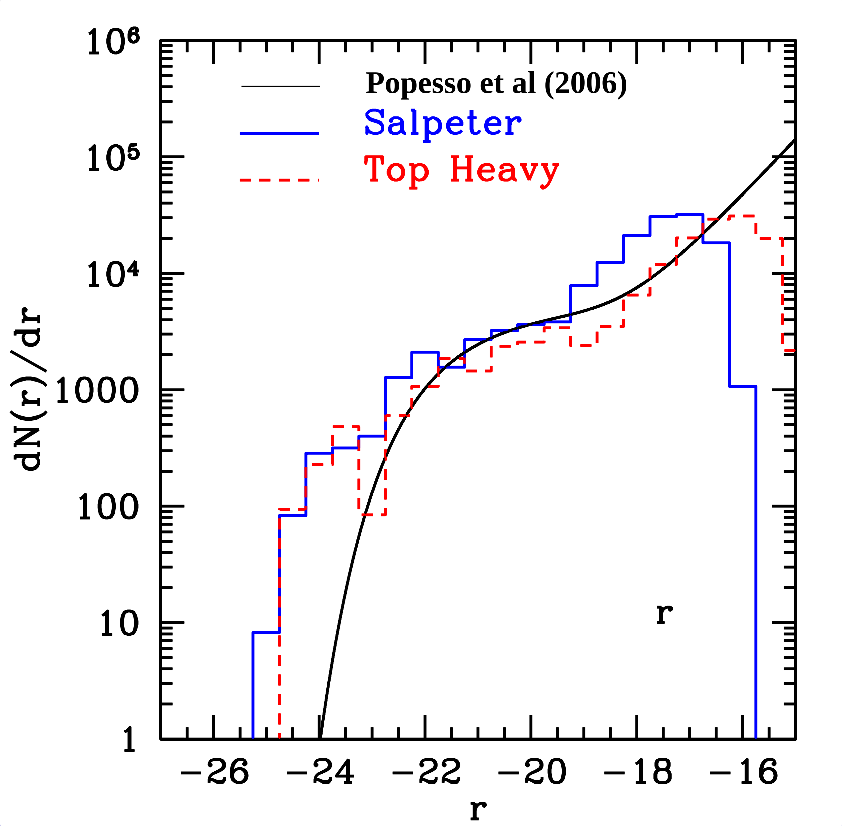


A3888 (Krick et al. 2008)

The ICL component



Saro et al. 2006

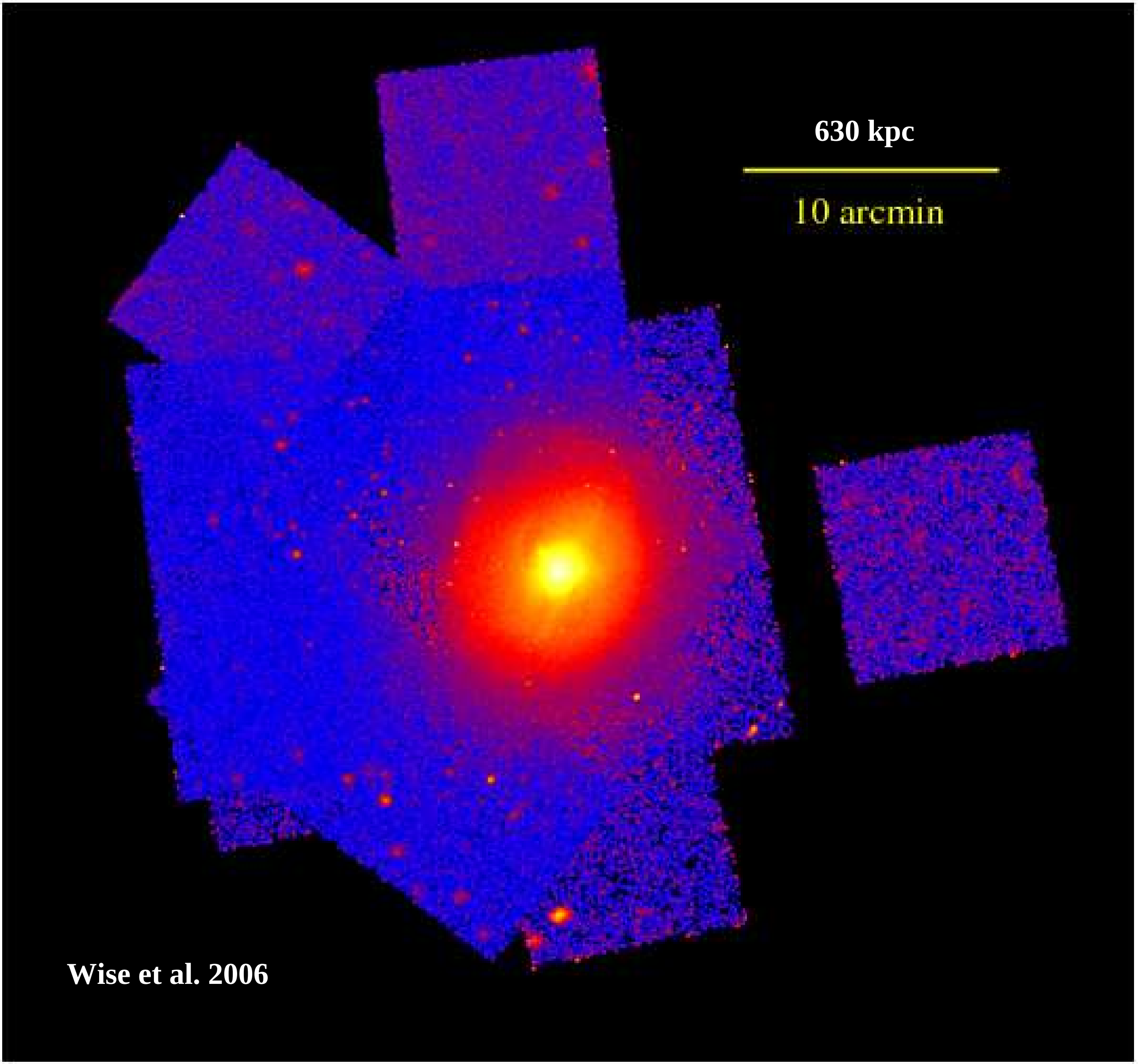


Properties of galaxies from hydrodynamical simulations.

⇒ Colour-Magnitude relation reproduced (depending on IMF) (e.g. using chemical enrichment)

⇒ Luminosity function as well reproduced.

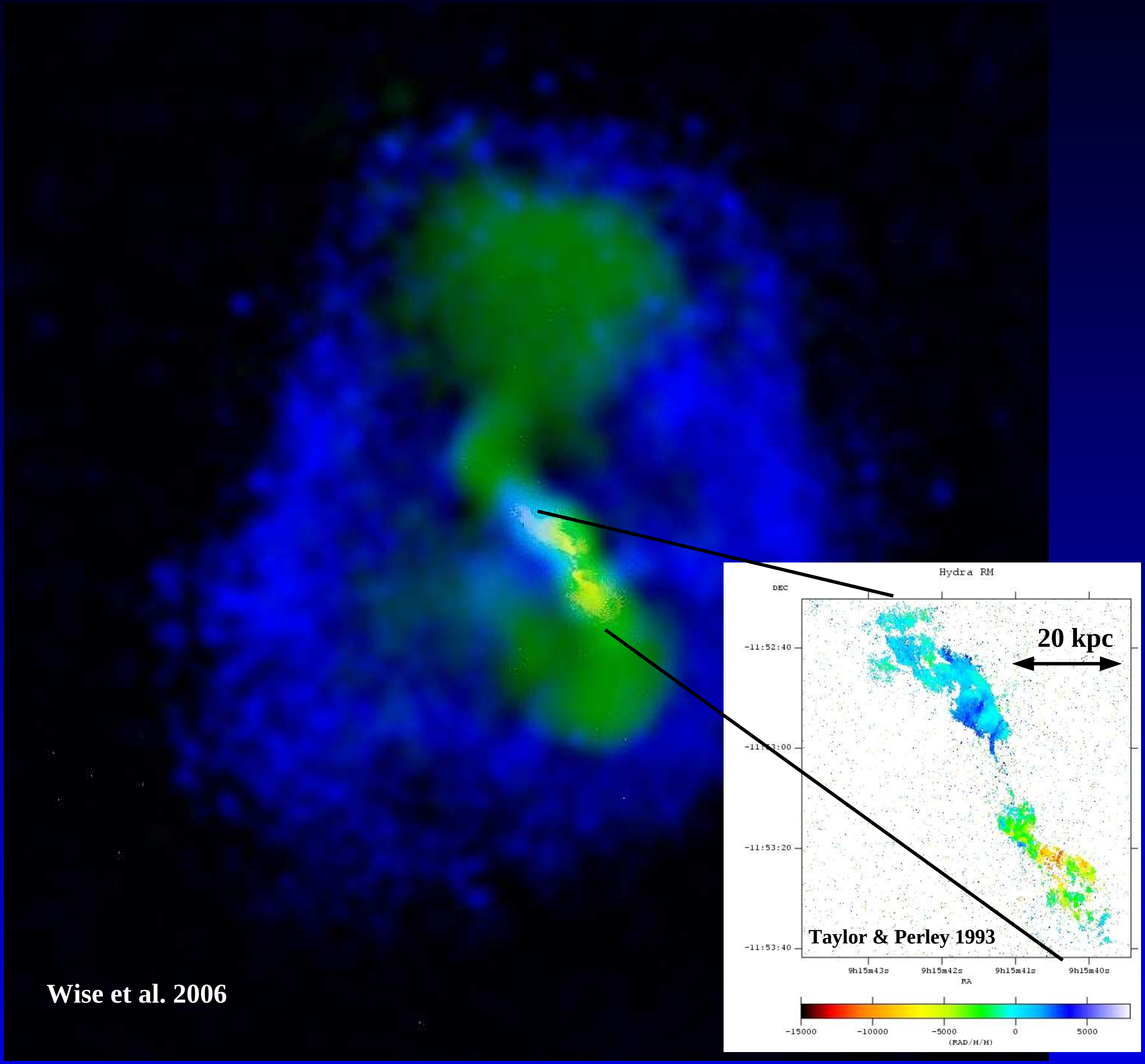
Cool-cores and AGN feedback



Wise et al. 2006

Chandra X-ray image of the Hydra cluster (cool core)
Simulations: Only cool-core clusters, to massive galaxies.

Cool-cores and AGN feedback



Composite image to illustrate the connection between the X-ray cavity (blue) and 330Mhz radio emission (green).

Cool-cores and AGN feedback

Jets in realistic galaxy clusters environment

Mendygral, Jones & Dolag, ApJ, 2012

Cool-cores and AGN feedback

BH model (sub−scale)

Springel & Di Matteo 2006

Seeding

Constant seeding
Seeding on m−sigma

Accretion on BH

α −Bondi (Springl & Di Matteo 06)
β −Bondi (Booth & Schaye 09)
....

Feedback

Thermal (Springel & Di Matteo 06)
Bubbles (Sijacki et al. 07)
....

Merging

Instant merging
Based on velocity
....

Growth of BH

gas density

$$\dot{M}_{\rm B} = \alpha \times \, 4\pi R_{\rm B}^2 \, \rho \, c_s \simeq \frac{4\pi\alpha G^2 M_\bullet^2 \, \rho}{(c_s^2 + v^2)^{3/2}}$$

sound speed

$$\dot{M}_\bullet = \min(\dot{M}_{\rm B}, \dot{M}_{\rm Edd})$$

Feedback by BH

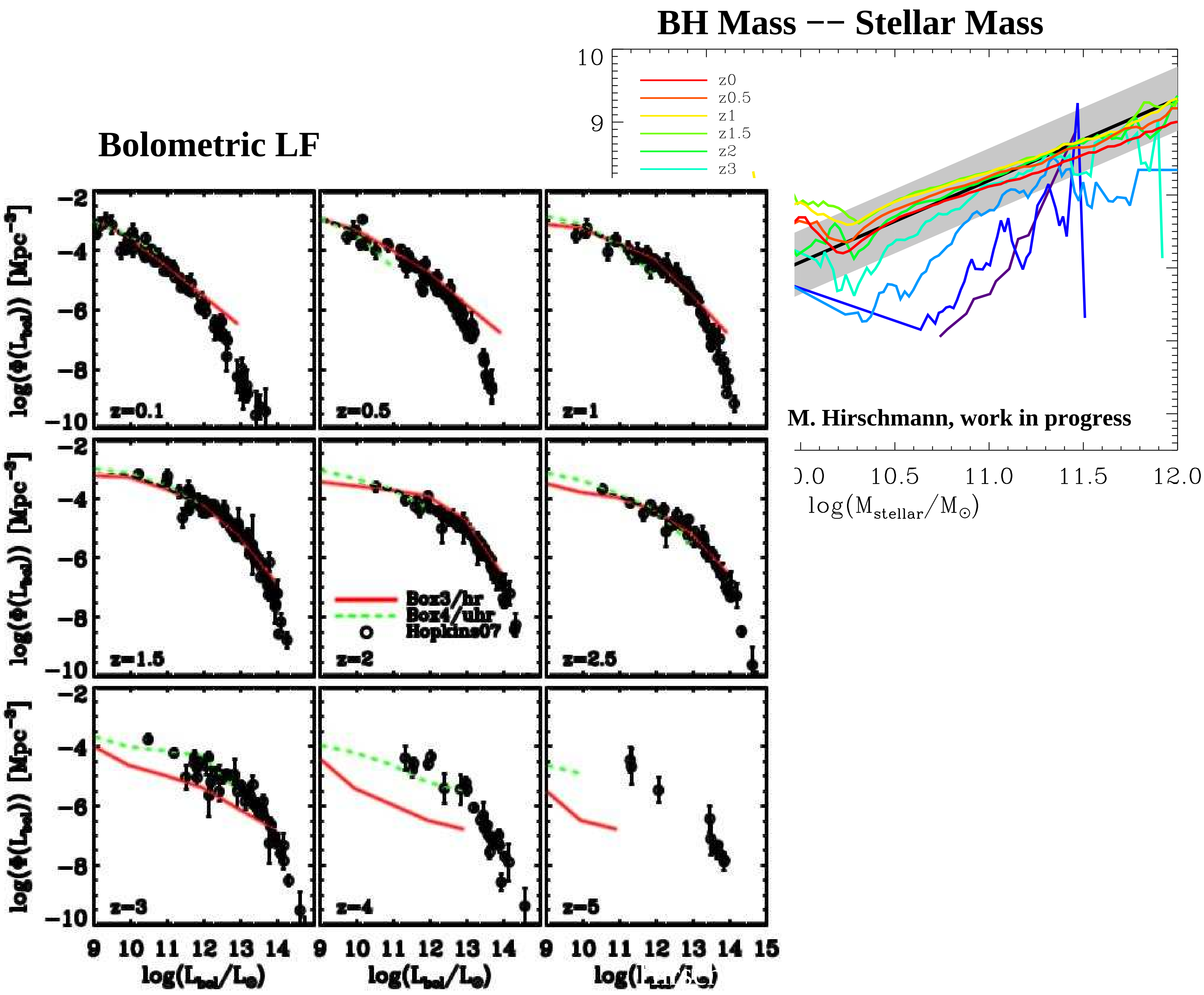
$$L_{\rm bol} = 0.1 \times \dot{M}_\bullet c^2$$

$$\dot{E}_{\rm feedback} = f \times L_{\rm bol}$$

efficiency

Sub-scale model for BH growth:
Resolution dependence ?
Various subtle extensions ...

Cool-cores and AGN feedback

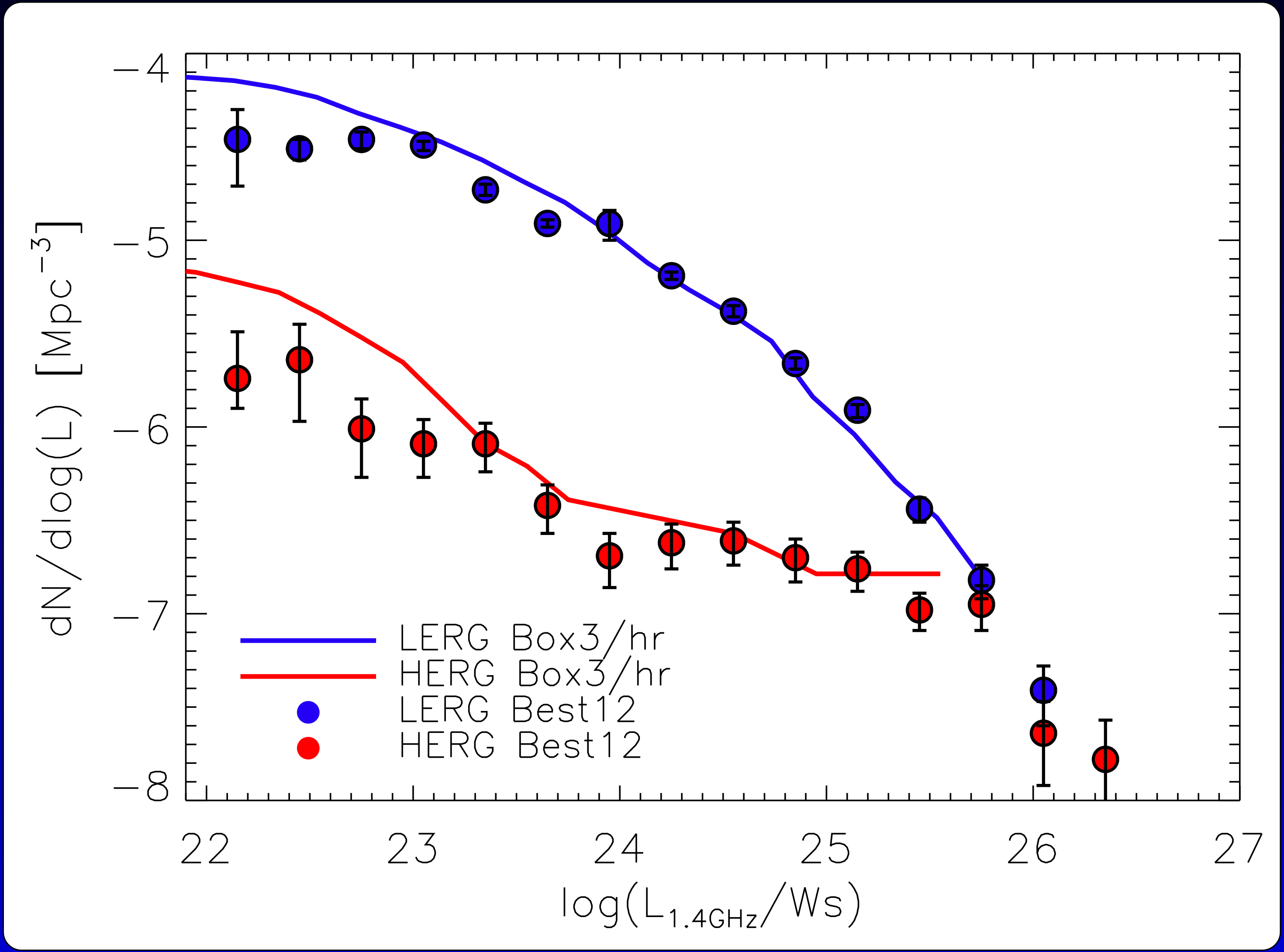


BH Mass -- Stellar Mass

Bolometric LF

M. Hirschmann, work in progress

Cool-cores and AGN feedback

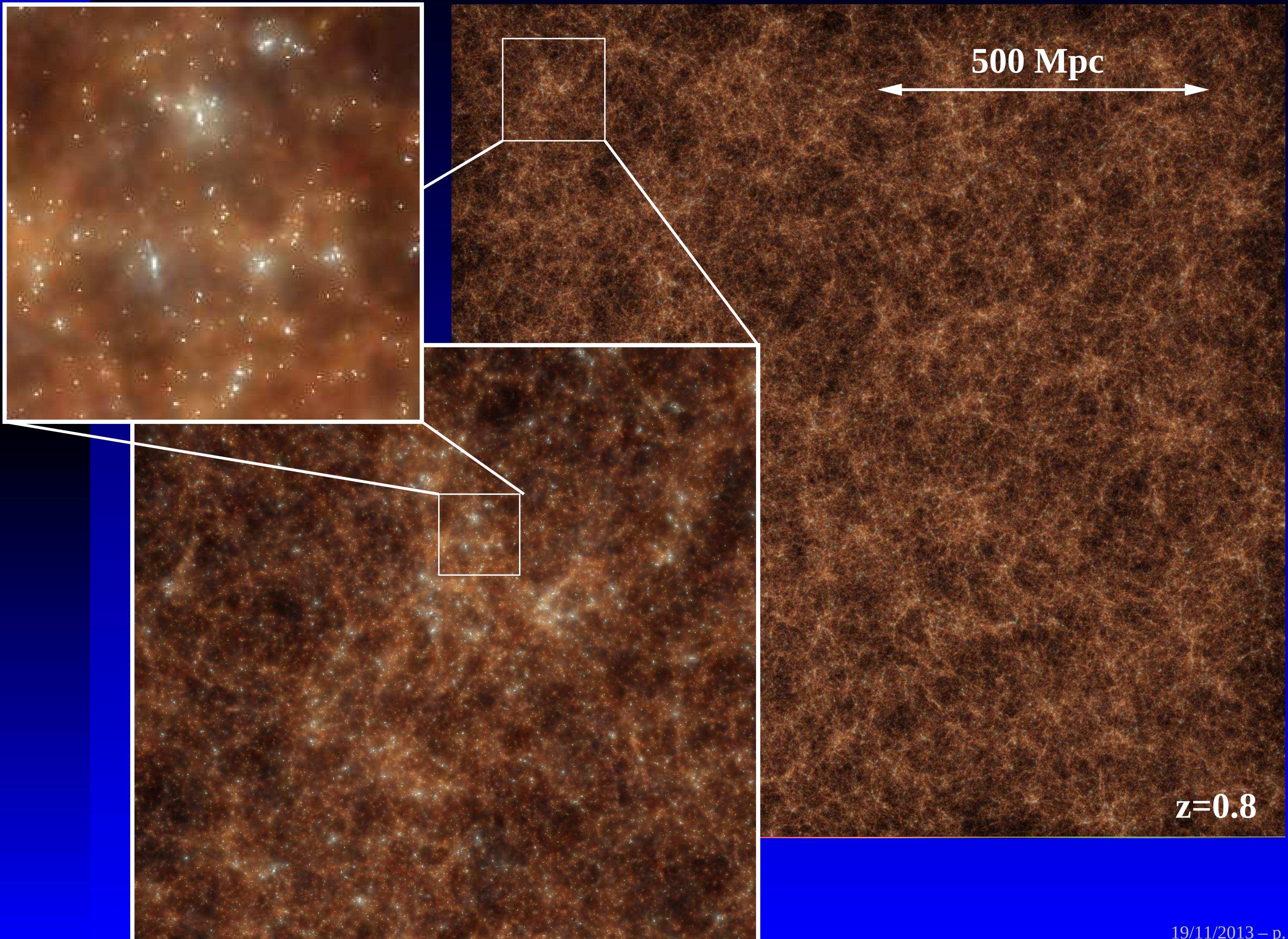


LERG: $f_{\rm edd} < 0.01$, HERG: $f_{\rm edd} > 0.01$

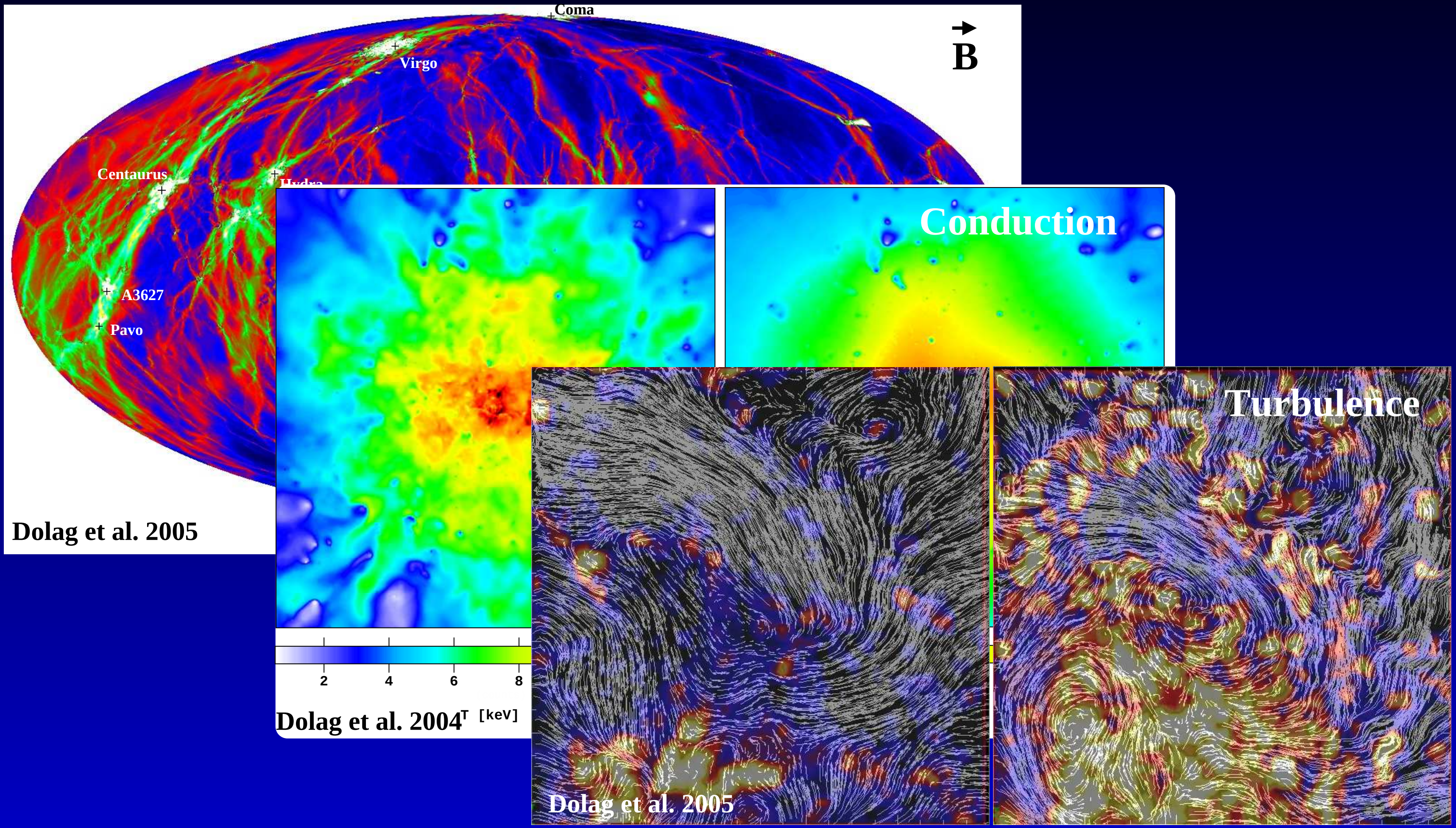
$(L_{\rm rad} + L_{\rm mech})/L_{\rm edd} =$ -1.6 , -3 (Best & Heckman 2012)

$L_{\rm mech} = 10^{36} \times (L_{1.4{\rm GHz}}/10^{24}{\rm WHz}^{-1})^{0.7}{\rm W}$ (Cavagnolo et al. 2010)

Example

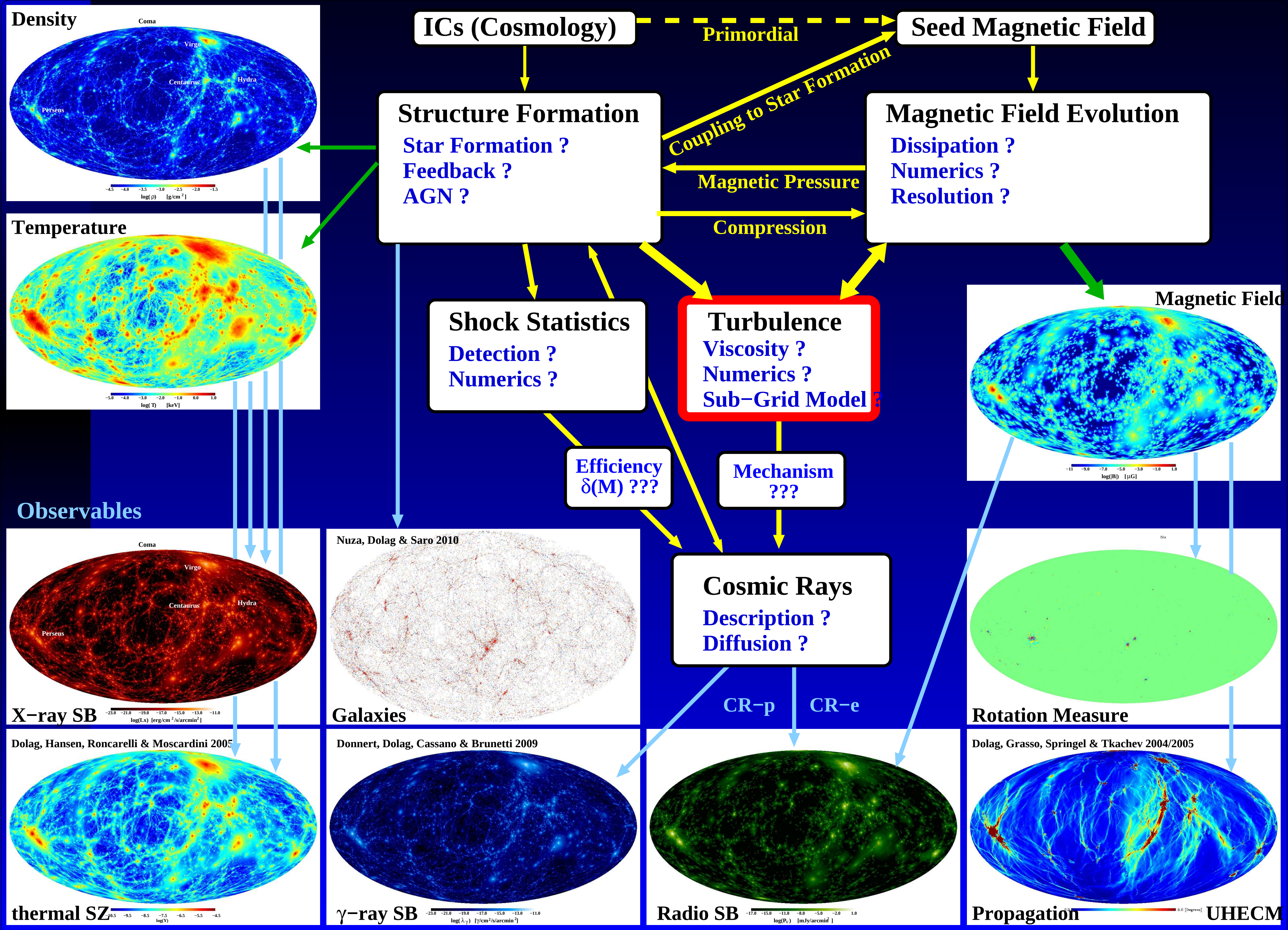


Further Complications



- Turbulence (e.g. distributing metals within ICM)
- Thermal conduction (e.g. thermal structure of ICM)
- $\vec{B}$ (e.g. non thermal pressure, turbulence, conduction)
- Cosmic Rays (e.g. non thermal pressure support, heating)

Process Network



Nuza, Dolag & Saro 2010

Dolag, Hansen, Roncarelli & Moscardini 2005

Donnert, Dolag, Cassano & Brunetti 2009

Dolag, Grasso, Springel & Tkachev 2004/2005