Relativistic Magnetohydrodynamics
Governing equations, wave behavior in special relativity

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including work with Z. Meliani, O. Porth, et al.

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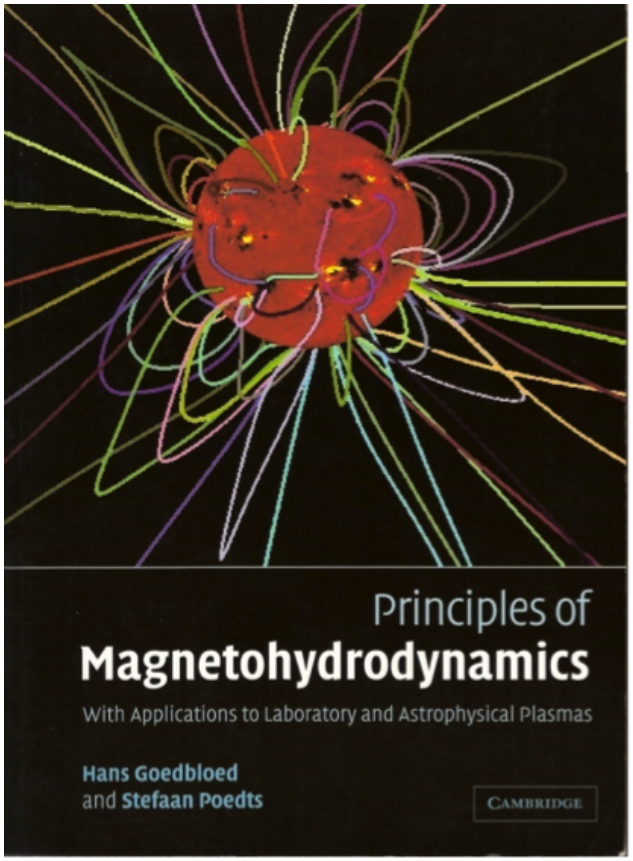
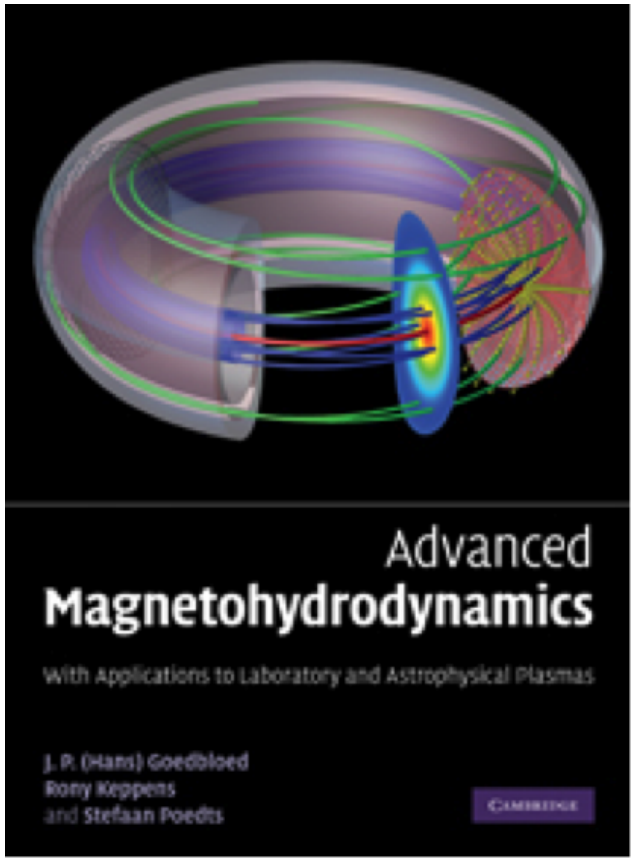
Outline

- Special relativistic MHD introduction
  - ⇒ **SRMHD** equations
  - ⇒ linear waves in homogeneous media
  - ⇒ RMHD shock relations
- **Relativistic MHD simulations**: `MPI-AMRVAC`
  - ⇒ relativistic (M)HD two-component jet simulations
  - ⇒ helically magnetized, relativistic jets
- Outlook

- lecture material from modern (2004 & 2010) textbooks
  - ⇒ **Goedbloed** et al., Cambridge University Press
  - ⇒ chapter 21 on relativistic MHD . . .



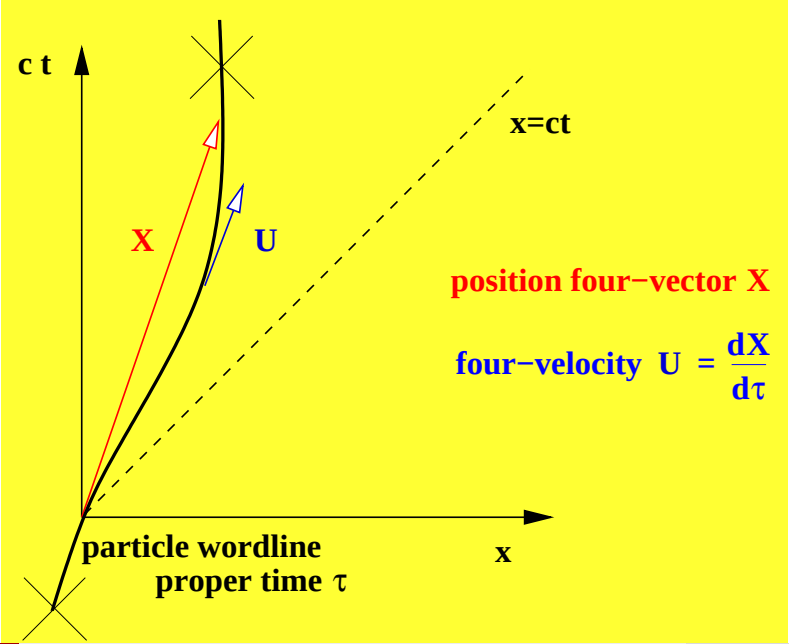


Special Relativity I

- 4D flat space-time, with c as maximal propagation speed
  - $\Rightarrow$ four-vector $\mathbf{X} = (ct, \mathbf{x})^T$ squared length invariant

$$\mathbf{X} \cdot \mathbf{X} = -c^2 t^2 + x_1^2 + x_2^2 + x_3^2$$

  - $\Rightarrow$ Minkowski metric $g_{\alpha\beta} = g^{\alpha\beta} = \mathrm{diag}\,(-1, 1, 1, 1)$
  - $\Rightarrow$ contra- & covariant components $X^\alpha = g^{\alpha\beta} X_\beta$: only reverse $X^0 = -X_0$
- particle wordline: ideal clock for proper time τ



Special Relativity II

- tangent fourvector to worldline
  - $\Rightarrow$ four-velocity $\mathbf{U} = \mathbf{dX}/d\tau$, components

$$U^\alpha = \left(c \underbrace{\frac{dt}{d\tau}}_{\text{dilation}}, \underbrace{\frac{dx_i}{dt}}_{v_i} \frac{dt}{d\tau} \right) = (c\Gamma, \Gamma\mathbf{v})^T$$

  - $\Rightarrow$ spatial three-velocity $\mathbf{v}$ in chosen Lorentzian lab frame
  - $\Rightarrow$ Lorentz factor $\Gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

Special Relativity III

- inertial frames Lorentz transform $\mathbf{X}' = L_{\alpha}^{\alpha'}\mathbf{X}$
  $\Rightarrow$ lost simultaneity, length contracts, time dilates
- proper density: $\rho = m_0 n_0$ with n_0 rest frame number density
  $\Rightarrow$ lab 'density' $D = \Gamma\rho$: volume change by length contraction
- Particle conservation is $\partial_\alpha(\rho U^\alpha) = 0$ or

$$\frac{\partial D}{\partial t} + \nabla\cdot(D\mathbf{v}) = 0$$

- stress-energy tensor:

$$\begin{pmatrix} T^{00} & T^{0i} \\ T^{i0} & T^{ij} \end{pmatrix} = \begin{pmatrix} \text{energy density} & \text{energy flux} \\ \text{momentum flux} & \text{stresses} \end{pmatrix}$$

Special Relativity IV

- gas stress-energy contribution from expression in rest frame:

$$\begin{pmatrix} \underbrace{\rho c^2 + \rho\epsilon}_{\text{rest mass} + \text{internal energy}} & \mathbf{0} \\ \mathbf{0} & \underbrace{p\mathbf{I}}_{\text{isotropic pressure}} \end{pmatrix}$$

$\Rightarrow$ to lab frame by inverse Lorentz $T^{\alpha\beta} = L^{-1,\alpha}_{\alpha'} L^{-1,\beta}_{\beta'} T^{\alpha'\beta'}$

$$\begin{pmatrix} T^{00} & T^{0i} \\ T^{i0} & T^{ij} \end{pmatrix} = \begin{pmatrix} \tau_{\rm g} + Dc^2 & \frac{\mathbf{S}_{\rm g}}{c} \\ \frac{\mathbf{S}_{\rm g}}{c} & \frac{\mathbf{S}_{\rm g}\mathbf{v}}{c^2} + p\mathbf{I} \end{pmatrix}$$

$\Rightarrow$ $\mathbf{S}_{\rm g} = (\rho c^2 + \rho\epsilon + p)\Gamma^2\mathbf{v}$ and $\tau_{\rm g} + Dc^2 = (\rho c^2 + \rho\epsilon + p)\Gamma^2 - p$

Special Relativity V

- when also allowing for electromagnetic fields: EM stress-energy

$$T^{\alpha\beta}_{\rm em} = \begin{pmatrix} \underbrace{\frac{B^2}{2\mu_0} + \epsilon_0 \frac{E^2}{2}}_{\text{EM energy density}} & \frac{\mathbf{S}_{\rm em}}{c} \\ \frac{\mathbf{S}_{\rm em}}{c} & \underbrace{\left(\frac{B^2}{2\mu_0} + \epsilon_0 \frac{E^2}{2}\right)\mathbf{I} - \epsilon_0 \mathbf{E}\mathbf{E} - \frac{\mathbf{B}\mathbf{B}}{\mu_0}}_{\text{Maxwell stress tensor}} \end{pmatrix}$$

$\Rightarrow$ EM energy flux is Poynting flux $\mathbf{S}_{\rm em} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0}$

$\Rightarrow$ use $\mathbf{E} = -\mathbf{v}\times\mathbf{B}$: perfect conductivity

Special Relativity VI

- energy-momentum conservation

$$\partial_\beta \left(T^{\alpha\beta} + T^{\alpha\beta}_{\rm em} \right) = 0$$

- introduce energy density minus rest mass and total energy flux

$$\begin{aligned} \tau &= \tau_{\rm g} + \frac{B^2}{2\mu_0} + \epsilon_0 \frac{B^2 v^2 - (\mathbf{v}\cdot\mathbf{B})^2}{2} \\ \mathbf{S}_{\rm tot} &= \mathbf{S}_{\rm g} + \mathbf{S}_{\rm em} \end{aligned}$$

$\Rightarrow$ temporal part gives

$$\frac{\partial \tau}{\partial t} + \nabla \cdot \left((\tau + p_{\rm tot})\mathbf{v} - (\mathbf{v}\cdot\mathbf{B})\frac{\mathbf{B}}{\mu_0} \right) = 0$$

$\Rightarrow$ spatial part:

$$\frac{\partial \mathbf{S}_{\rm tot}}{\partial t} + \nabla \cdot \left(\mathbf{S}_{\rm tot}\mathbf{v} + p_{\rm tot} c^2 \mathbf{I} - \frac{c^2}{\mu_0}\frac{\mathbf{B}\mathbf{B}}{\Gamma^2} - \frac{1}{\mu_0}(\mathbf{v}\cdot\mathbf{B})\mathbf{v}\mathbf{B} \right) = \mathbf{0}$$

Special Relativity VII

- total pressure $p_{\rm tot} = p + \frac{(\mathbf{v}\cdot\mathbf{B})^2}{2c^2} + \frac{B^2}{2\Gamma^2}$
- close system with homogeneous Maxwell equations:

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = \mathbf{0}$$

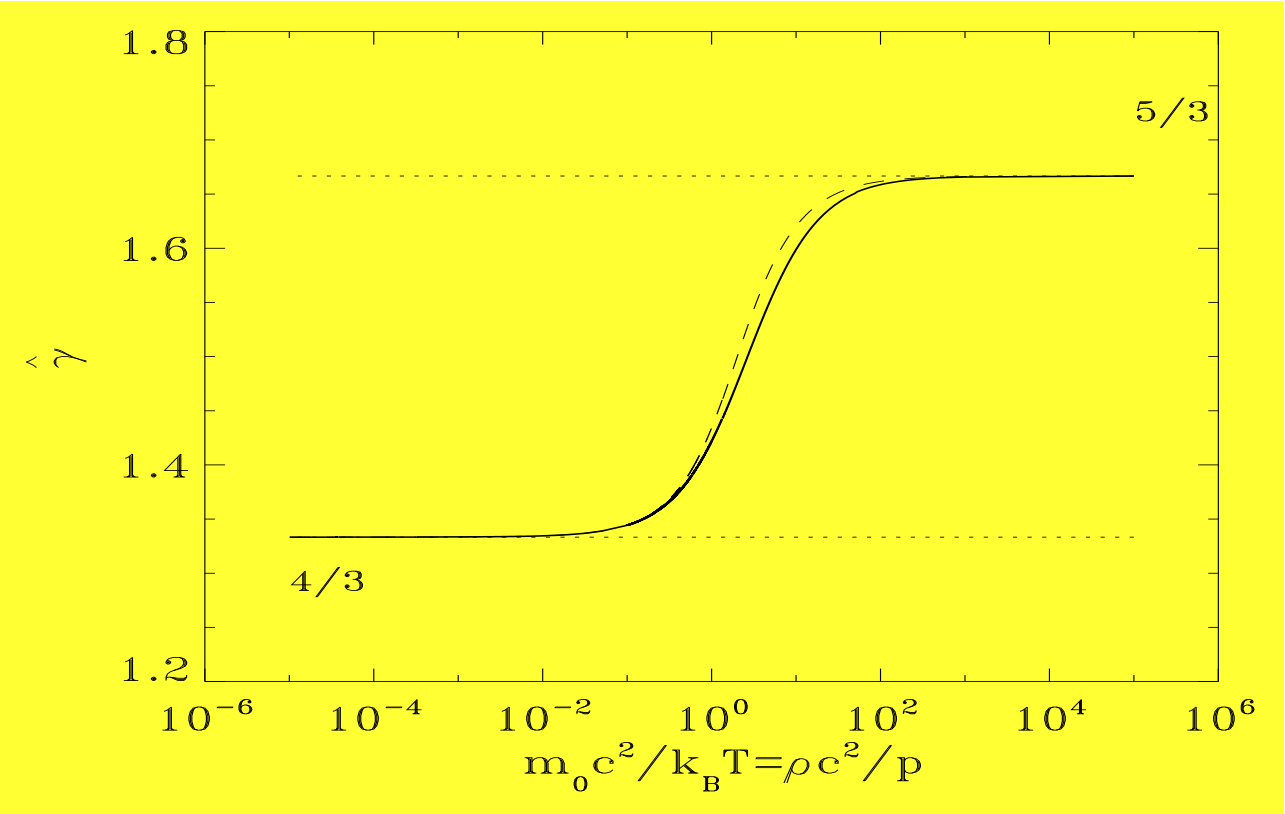
$\Rightarrow$ together with equation of state, e.g. polytropic relation

$$\rho\epsilon = \frac{p}{\gamma - 1}$$

$\Rightarrow$ enters specific enthalpy h where $\rho h = \rho c^2 + \rho\epsilon + p$

Special Relativity VIII

- **Equation of state in relativistic MHD**
  - ⇒ specific internal energy $\epsilon = p/(\gamma - 1)\rho$
  - ⇒ assumes constant polytropic index γ
- **Relativistically correct ideal gas: effective $\hat{\gamma}(T)$**
  - ⇒ compare Synge with Mathews proxy (no Bessel functions)



- **special relativistic magnetofluids** $\rightarrow$ flat Minkowski space-time; particle, tensorial energy-momentum conservation, full Maxwell
- **ideal magnetohydrodynamic: vanishing electric field in comoving frame**

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

$\Rightarrow$ fix Lorentz frame, use $1+3$ split (time/space), obtain

$$\partial_t \mathbf{U} + \partial_i \mathbf{F}^i = 0$$

$\Rightarrow$ conserved variables $\mathbf{U} = (D, \mathbf{S}_{\rm tot}, \tau, \mathbf{B})$

$\Rightarrow$ primitives $(\rho, \mathbf{v}, p, \mathbf{B})$

Newtonian limit: Ideal MHD and conservation laws

- $\Gamma \to 1$: **conservation laws** for density ρ, momentum density $\mathbf{m} = \rho\mathbf{v}$, $\mathcal{H}$ and $\mathbf{B}$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{v}\rho) = 0$$

- $D \to \rho$ and $\mathbf{S}_{\rm tot} \to c^2 \rho \mathbf{v}$ and $p_{tot} \equiv$ thermal + magnetic pressure

Newtonian limit: Ideal MHD and conservation laws

- $\Gamma \to 1$: **conservation laws** for density ρ, momentum density $\mathbf{m} = \rho\mathbf{v}$, $\mathcal{H}$ and $\mathbf{B}$

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot (\mathbf{v}\rho\mathbf{v} - \mathbf{B}\mathbf{B}) + \nabla p_{tot} = \mathbf{0}$$

- $D \to \rho$ and $\mathbf{S}_{\rm tot} \to c^2\rho\mathbf{v}$ and $p_{tot} \equiv$ thermal + magnetic pressure

Newtonian limit: Ideal MHD and conservation laws

- $\Gamma \to 1$: **conservation laws** for density ρ, momentum density $\mathbf{m} = \rho\mathbf{v}$, $\mathcal{H}$ and $\mathbf{B}$

$$\frac{\partial \mathcal{H}}{\partial t} + \nabla \cdot (\mathbf{v}\mathcal{H} + \mathbf{v}p_{tot} - \mathbf{B}\mathbf{B} \cdot \mathbf{v}) = 0$$

- total energy density $\tau \to \mathcal{H}$ has 3 contributions

$$\mathcal{H} = \underbrace{\rho\epsilon}_{\text{internal}} + \underbrace{\frac{\rho v^2}{2}}_{\text{kinetic}} + \underbrace{\frac{1}{2}B^2}_{\text{magnetic}}$$

Newtonian limit: Ideal MHD and conservation laws

- $\Gamma \to 1$: **conservation laws** for density ρ, momentum density $\mathbf{m} = \rho\mathbf{v}$, $\mathcal{H}$ and $\mathbf{B}$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{vB} - \mathbf{Bv}) = \mathbf{0}$$

- idem in relativistic/Newtonian setting

Newtonian intermezzo: wave diagrams

- **linearize (ideal) MHD equations about uniform, static state, uniform field $\mathbf{B}_0$**

  $\Rightarrow$ Lagrangian displacement ξ, normal mode analysis $e^{-\mathrm{i}\omega t}$

  $\Rightarrow$ algebraic eigenvalue problem

  $\Rightarrow$ analytic expressions for dispersion relation $\omega^2(\mathbf{k})$, when perturbations assume plane wave form

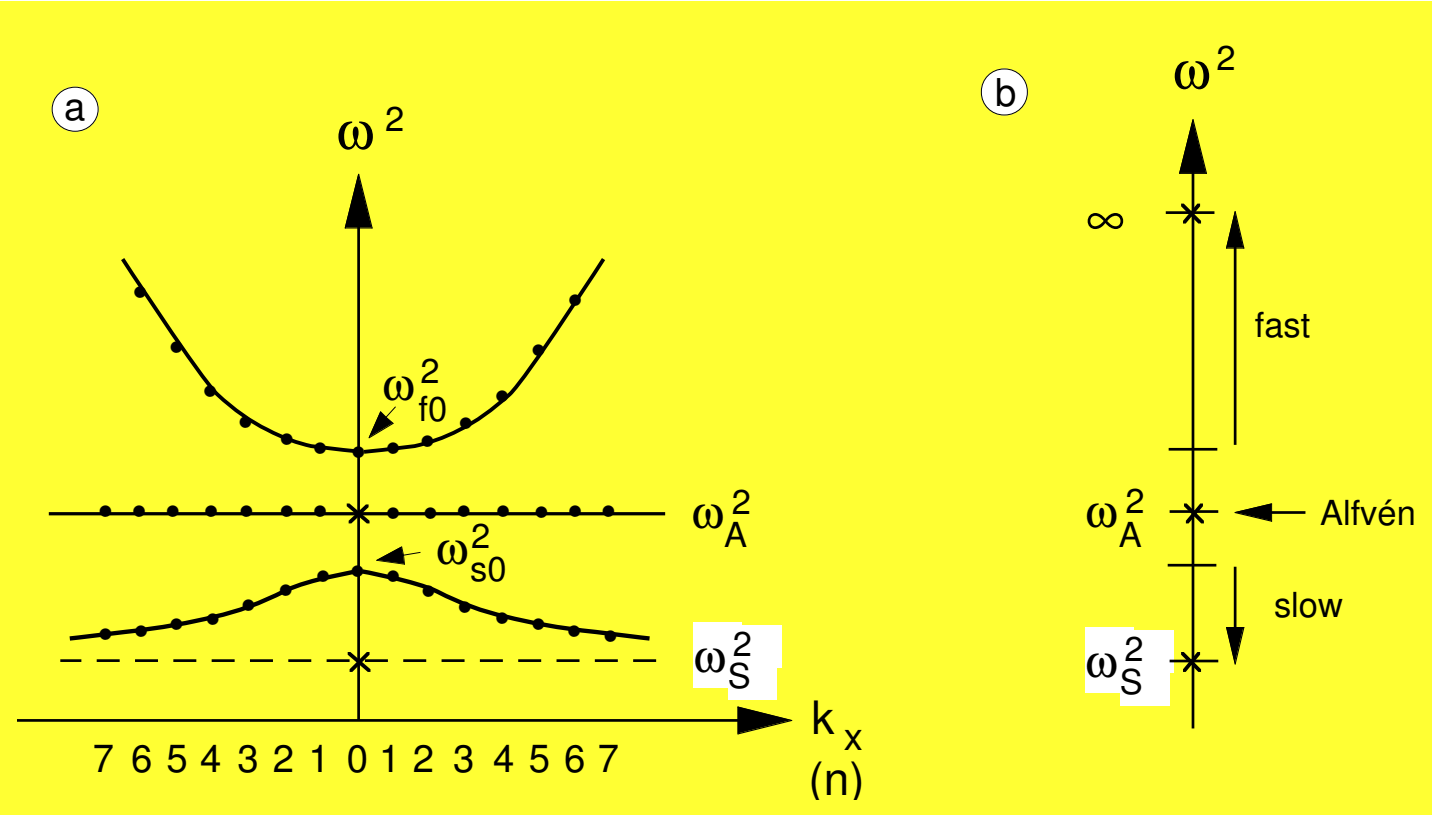
$$\hat{\xi}(\mathbf{k};\omega)\exp\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

  $\Rightarrow$ Alfvén modes then e.g. $\omega_A^2 = (\mathbf{k}\cdot\mathbf{B}_0)^2/\mu_0\rho_0$

Newtonian intermezzo: wave diagrams

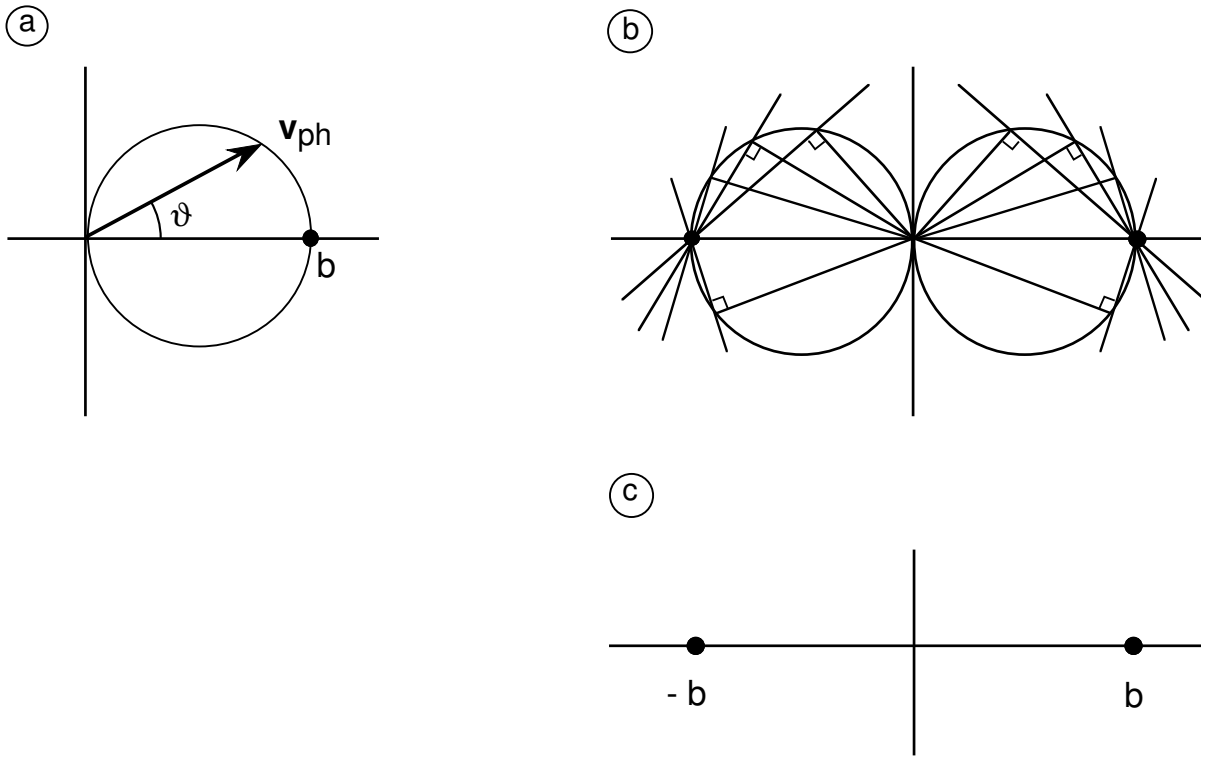
- **linearize (ideal) MHD equations about uniform, static state, uniform field $\mathbf{B}_0$**

  $\Rightarrow$ dispersion diagram $\omega^2 = \omega^2(k_x)$ for k_y and $k_z = k_{||}$ fixed



- continuous curves to quantized modes: $k_x = n\pi/a$ if $x \in [0, a]$

- phase diagram: endpoint of **k** vector as angle between **k** and $\mathbf{B}_0$ varies: for Alfvén yields two spheres left/right of origin ($\mathbf{b} = \mathbf{B}_0/\sqrt{\mu_0\rho_0}$)

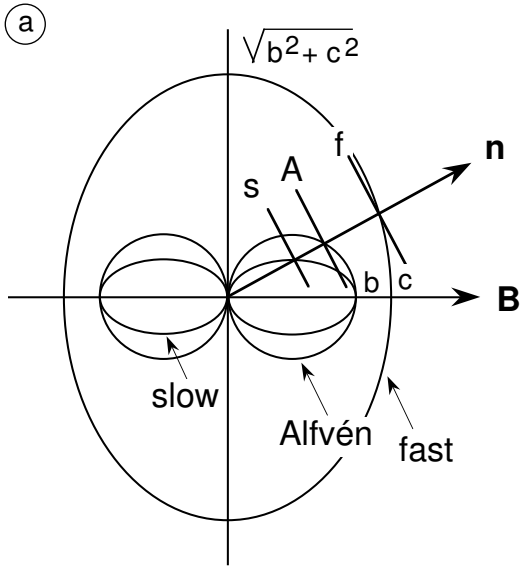


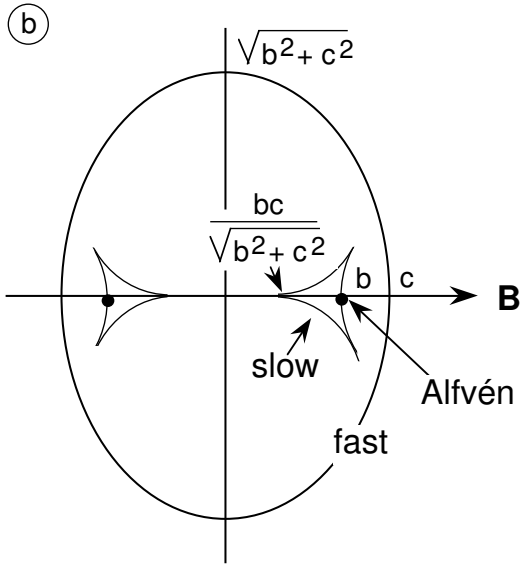
(a) Phase diagram for Alfvén waves is circle
$\Rightarrow$(b) wavefronts pass through **points** $\pm b$
$\Rightarrow$(c) those points are the group diagram.

Phase and group diagrams

Friedrichs diagrams (schematic)

parameter $c/b = \frac{1}{2}\gamma\beta\,,\ \beta \equiv 2p/B^2$



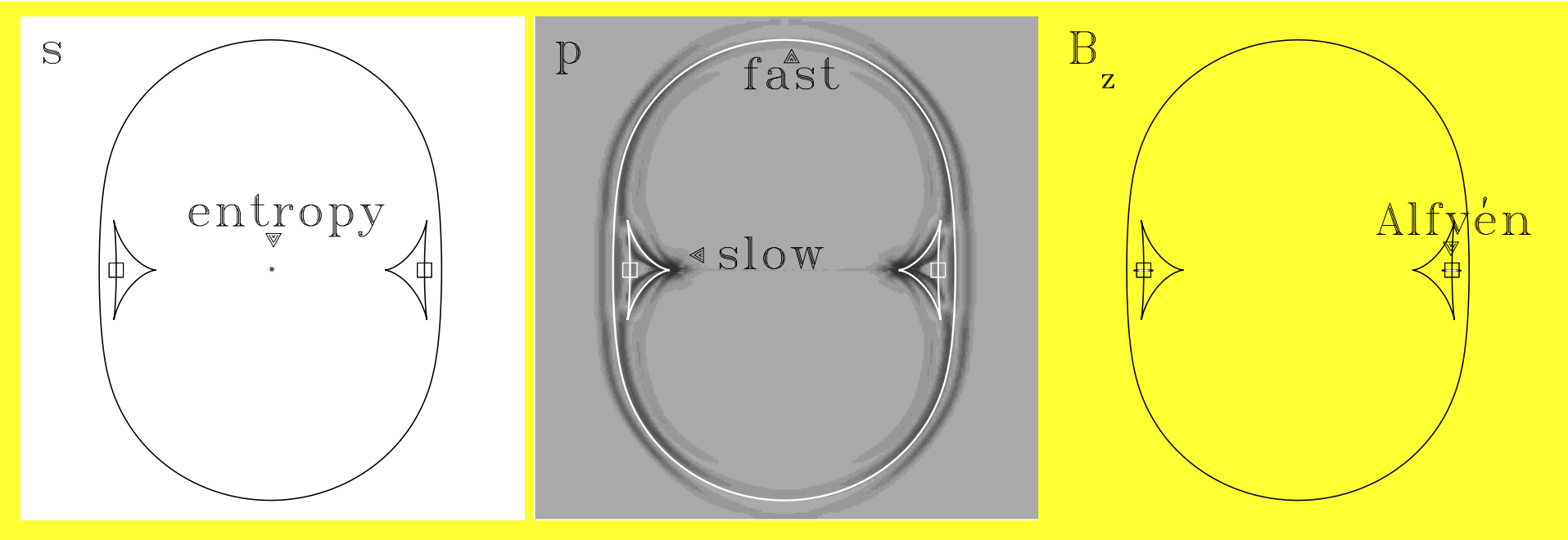


Phase diagram
(plane waves)

Group diagram
(point disturbances)

MHD waves

- 7 wavespeeds *entropy,* $\pm$ *slow,* $\pm$ *Alfvén,* $\pm$ *fast* [anisotropic!]
  - $\Rightarrow$ speeds $v, v \pm c_s, v \pm b, v \pm c_f$
  - $\Rightarrow$ **7 characteristic speeds of the hyperbolic PDE system**
- **MHD waves in uniform medium**



Special Relativistic HD

- **relativistic hydro in $3+1$ form** reads:

$$
\begin{aligned}
\frac{\partial S}{\partial t} &+ \mathbf{v}\cdot\nabla S = 0\,,\\
\frac{\partial \rho}{\partial t} &+ \mathbf{v}\cdot\nabla\rho + \frac{\rho h}{u}\nabla\cdot\mathbf{v}\\
&- \frac{1}{u\Gamma^2}\mathbf{v}\cdot\nabla\left(S\rho^\gamma\right) = 0\,,\\
\frac{\partial \mathbf{v}}{\partial t} &+ \left(\mathbf{v}\cdot\nabla\right)\mathbf{v} + \frac{c^2}{\rho h\Gamma^2}\nabla\left(S\rho^\gamma\right)\\
&- \mathbf{v}\left(\nabla\cdot\mathbf{v}\right)\left[1-\frac{yc^2}{u}\right] - \mathbf{v}\frac{yc^2}{u\rho h\Gamma^2}\mathbf{v}\cdot\nabla\left(S\rho^\gamma\right) = 0\,.
\end{aligned}
$$

$\Rightarrow$ using entropy $S = p\rho^{-\gamma}$, rest frame density ρ, 3-velocity $\mathbf{v}$

Linear waves in RHD I

- linearize about **static $\mathbf{v} = 0$, uniform gas (constant S, ρ)**
  $\Rightarrow$ assume plane wave variation of linear quantities $S_1, \rho_1, \mathbf{v}_1$

$$\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{x})$$

- obtain **in chosen (rest) Lorentz frame**

$$\begin{aligned}
\omega S_1 &= 0\,, \\
\omega \rho_1 &= \rho \mathbf{k}\cdot\mathbf{v}_1\,, \\
\omega \mathbf{v}_1 &= \frac{c^2}{\rho h}\mathbf{k}\left(S\gamma\rho^{\gamma-1}\rho_1 + \rho^\gamma S_1\right)\,.
\end{aligned}$$

$\Rightarrow$ five solutions, **entropy + shear waves at $\omega = 0$, two sound waves**

Linear waves in RHD II

- sound waves **dispersion relation**

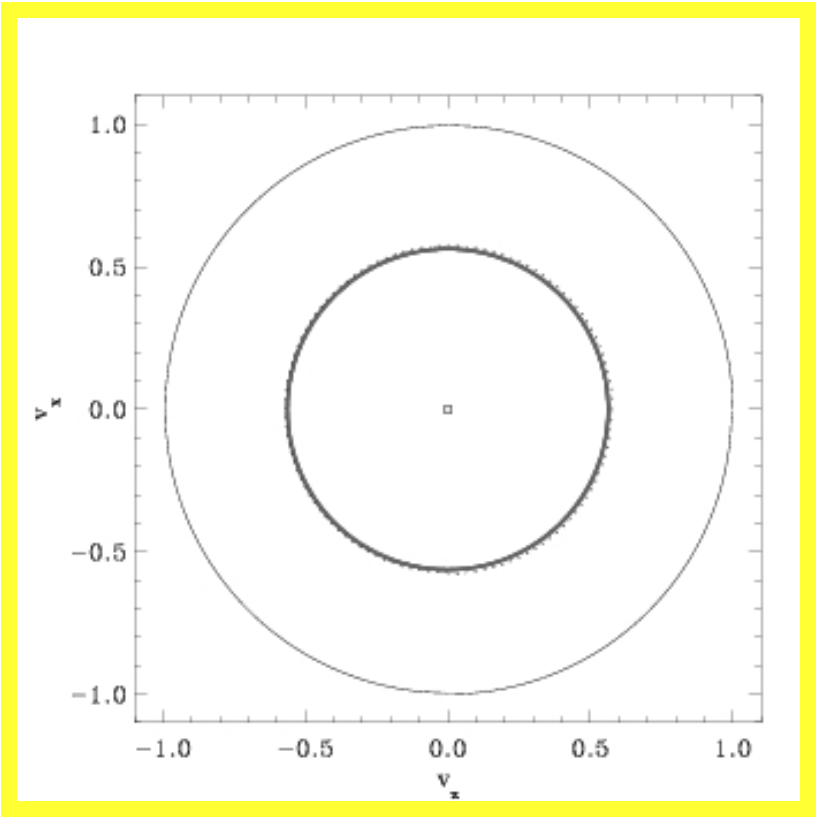
$$\frac{\omega^2}{k^2c^2} = \frac{\gamma S \rho^{\gamma-1}}{h} = \frac{\gamma p}{\rho h} = \frac{c_g^2}{c^2}$$

$\Rightarrow$ phase speed for plane wave with wavevector $\mathbf{k} = k\mathbf{n}$ from

$$\frac{\mathbf{v}_{\rm ph}}{c} = \frac{c_g}{c}\mathbf{n}$$

Linear waves in RHD III

- vary direction of wavevector over 2π, obtain phase diagram



$\Rightarrow$ isotropic propagation at sound speed

$\Rightarrow$ **group** (energy propagation) and phase speed coincide

$$\frac{\mathbf{v}_{\rm gr}}{c} = \frac{\partial \omega}{\partial \mathbf{k}} = \frac{c_g}{c}\mathbf{n}$$

Linear waves in RHD IV

- in frame L' where source moves at velocity $\mathbf{v}$
  - $\Rightarrow$ Lorentz transform: L' coordinates $(ct', \mathbf{x}')$ and L with $(ct, \mathbf{x})$
- **plane wave in L' with $\exp(-i\omega' t' + i\mathbf{k}' \cdot \mathbf{x}')$ still plane wave in L with $\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{x})$**
  - $\Rightarrow$ changed frequency: **relativistic Doppler effect**
  - $\Rightarrow$ altered wave vector direction: **Relativistic wave aberration**

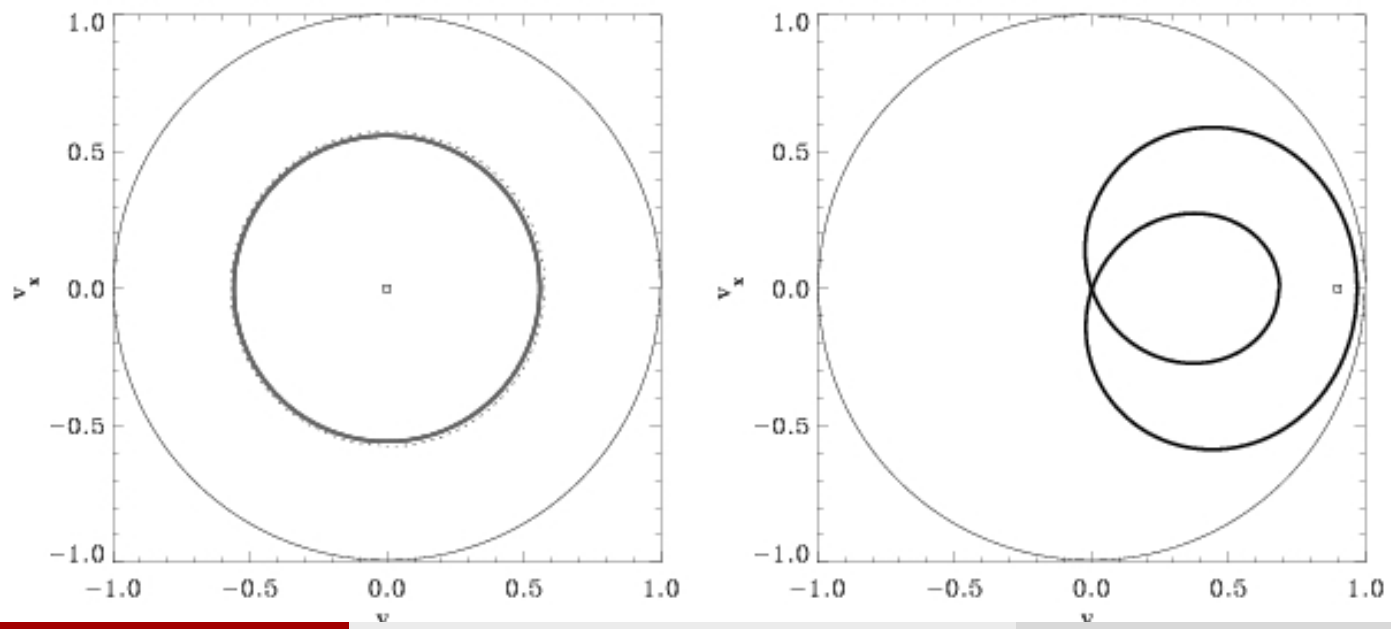
$$\begin{aligned} \omega &= \Gamma\left(\omega' + \mathbf{k}' \cdot \mathbf{v}\right) , \\ \mathbf{k} &= \mathbf{k}' + \mathbf{v}\left[\frac{\omega'\Gamma}{c^2} + \left(\mathbf{k}' \cdot \mathbf{v}\right)\frac{\Gamma - 1}{v^2}\right] \end{aligned}$$

Linear waves in RHD V

- phase speed relation is then

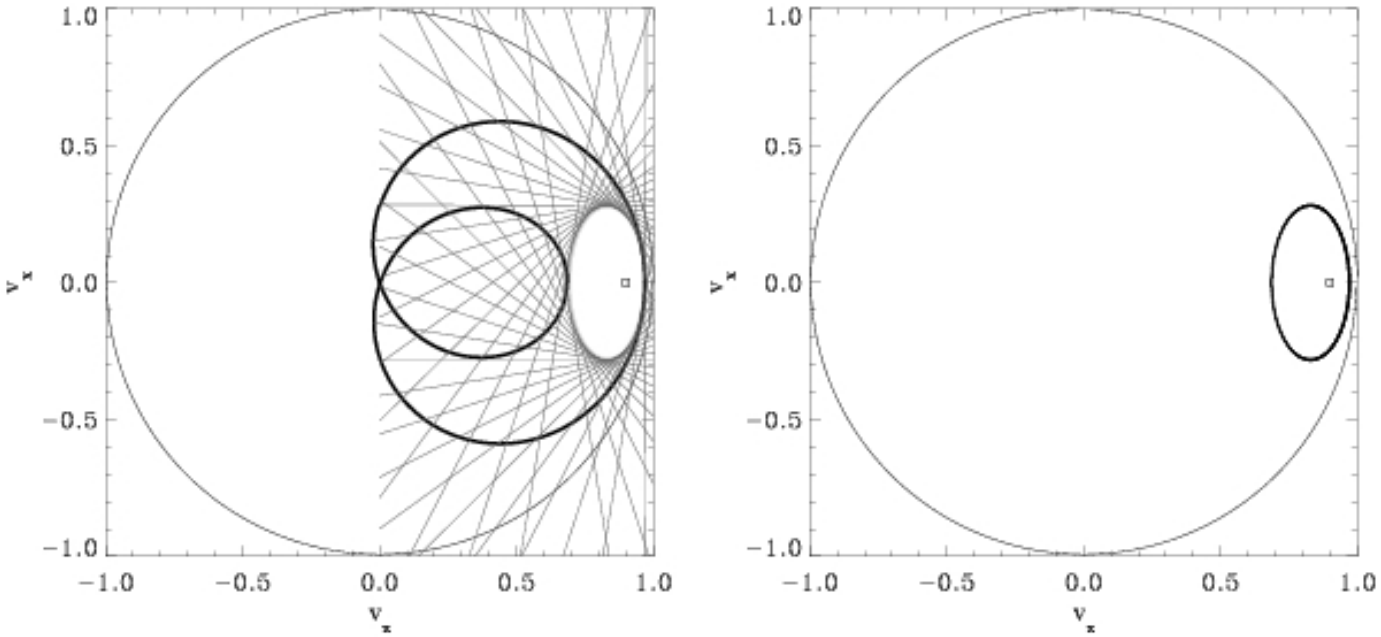
$$\frac{v_{\rm ph}'^2}{c^2} = \frac{\Gamma^2\left(v_{\rm ph} - \mathbf{n}\cdot\mathbf{v}\right)^2}{c^2 + \Gamma^2\left(v_{\rm ph} - \mathbf{n}\cdot\mathbf{v}\right)^2 - v_{\rm ph}^2}$$

$\Rightarrow$ graphically: **phase diagram for moving source (wave aberration)**



Linear waves in RHD VI

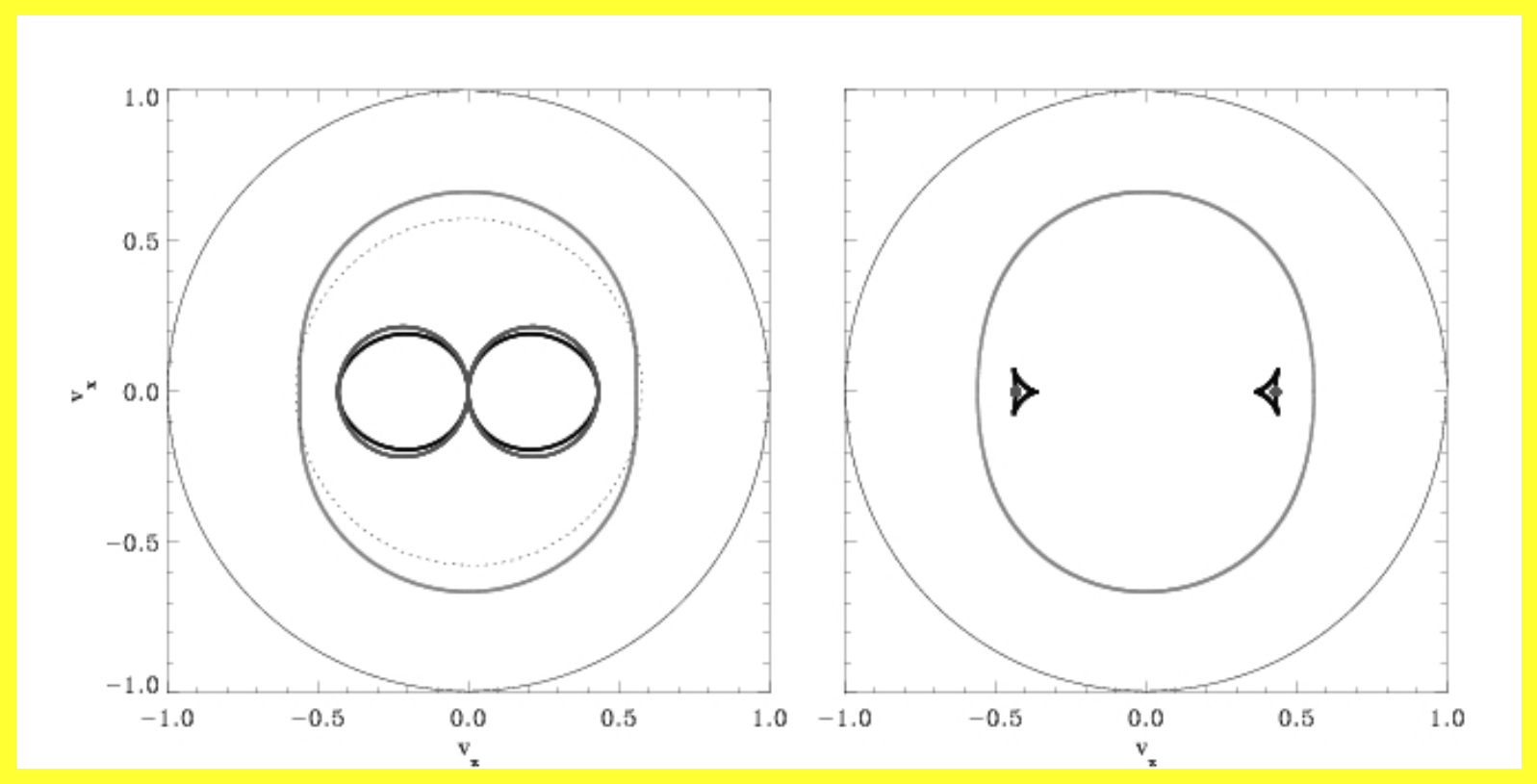
- **Group diagram** in same Lorentz frame: use **Huygens construction**



$\Rightarrow$ group diagram: **observed wavefront for moving point source**

Relativistic MHD waves I

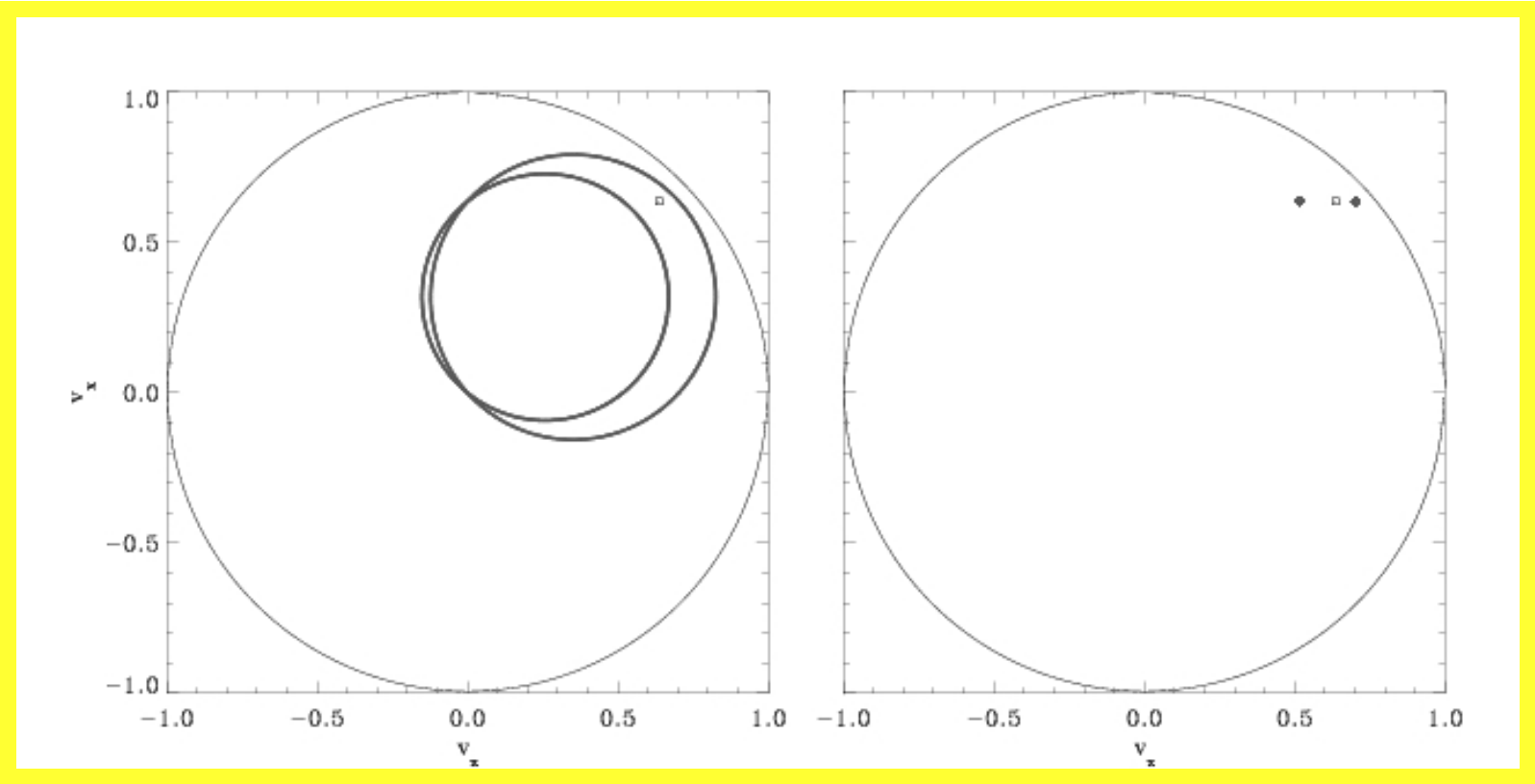
- in MHD: anisotropic wave behavior **in rest frame**
  - $\Rightarrow$ phase & group (Friedrich) diagrams for slow, Alfvén, fast



  - $\Rightarrow$ horizontal **B**, uniform plasma
  - $\Rightarrow$ δ-perturbation yields group diagram, also Huygens construction

Relativistic MHD waves II

- Alfvén waves as source moves at
  $\mathbf{v} = 0.9\,[\sin(\pi/4)\mathbf{e}_x + \cos(\pi/4)\mathbf{e}_z]$
  $\Rightarrow$ circular phase diagrams get displaced

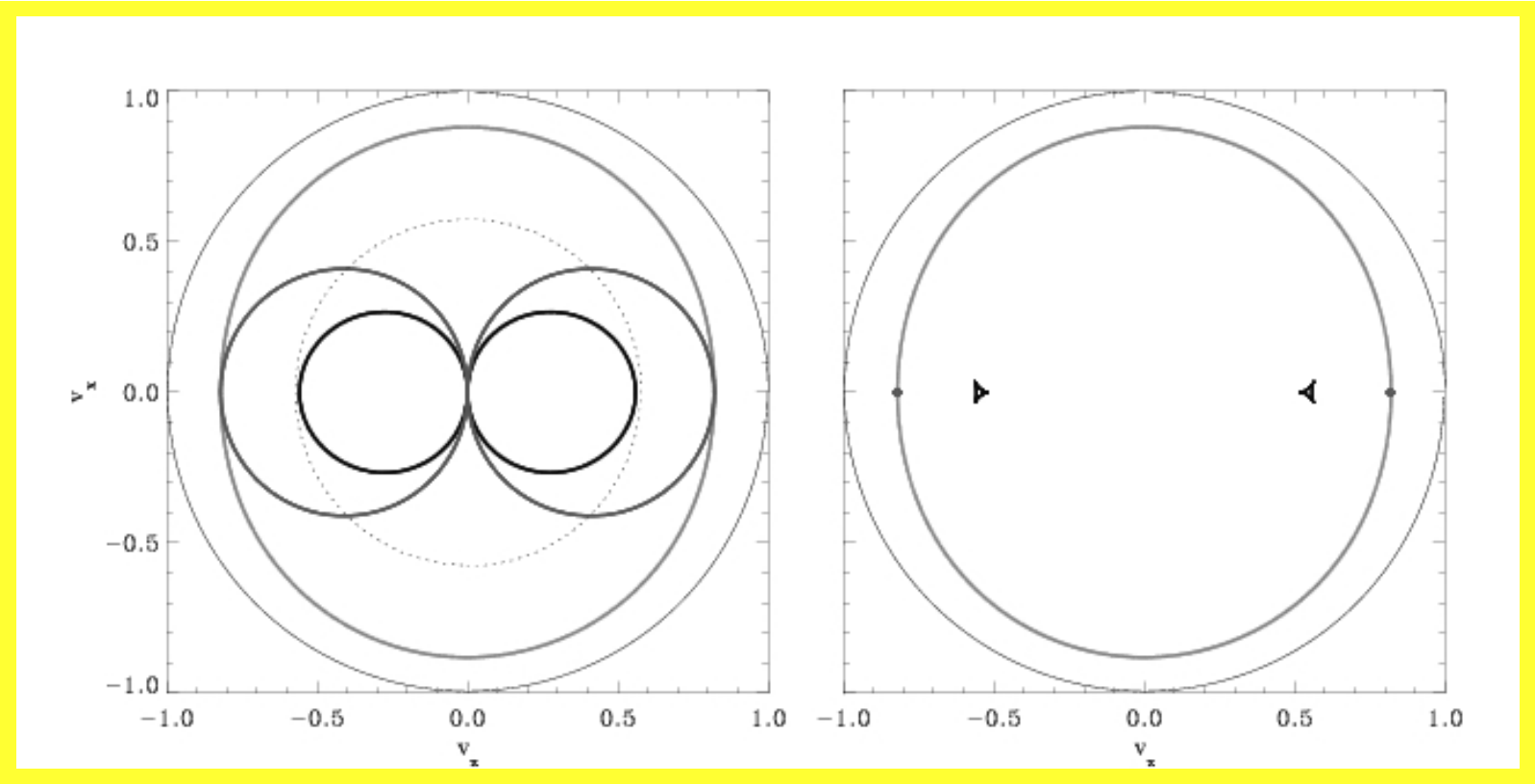


$\Rightarrow$ group diagram: **Alfvén pulse traveling along perturbed fieldline**

Relativistic MHD waves III

- Depending on uniform medium: **Alfvén & fast speeds may approach *c***

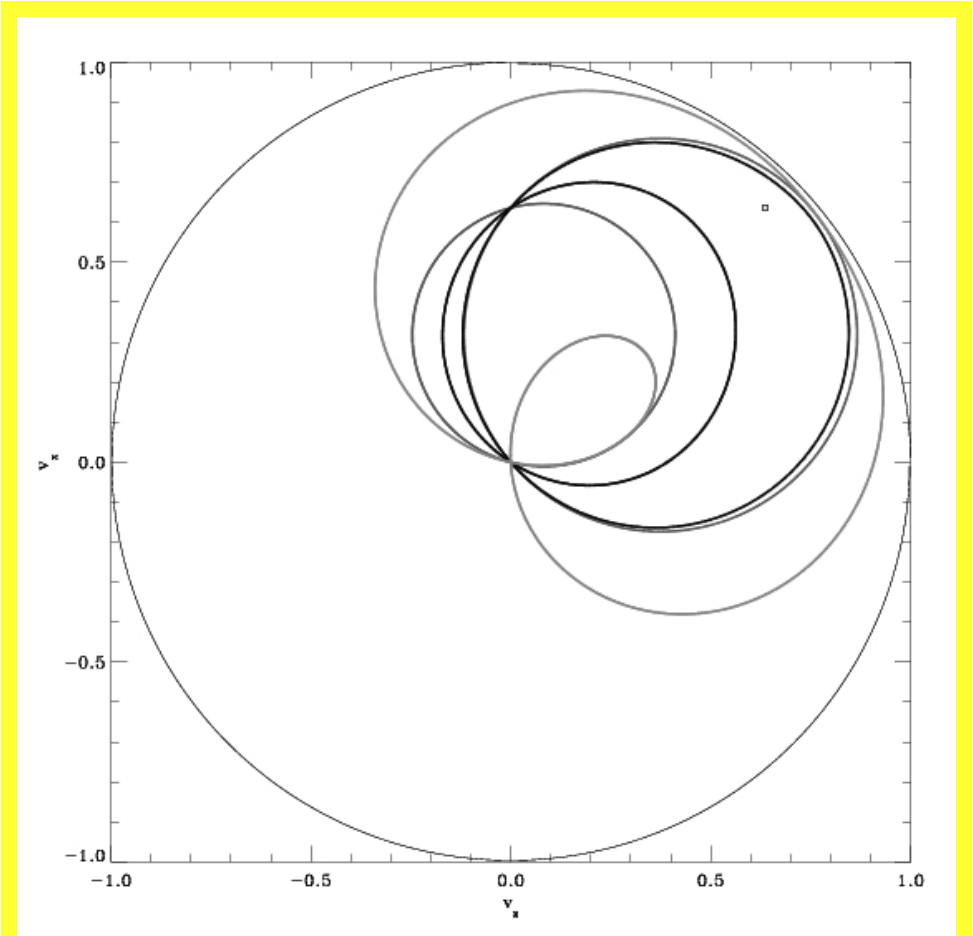
  $\Rightarrow$ phase and group diagrams for slow, Alfvén, fast modes **in rest frame**



Relativistic MHD waves IV

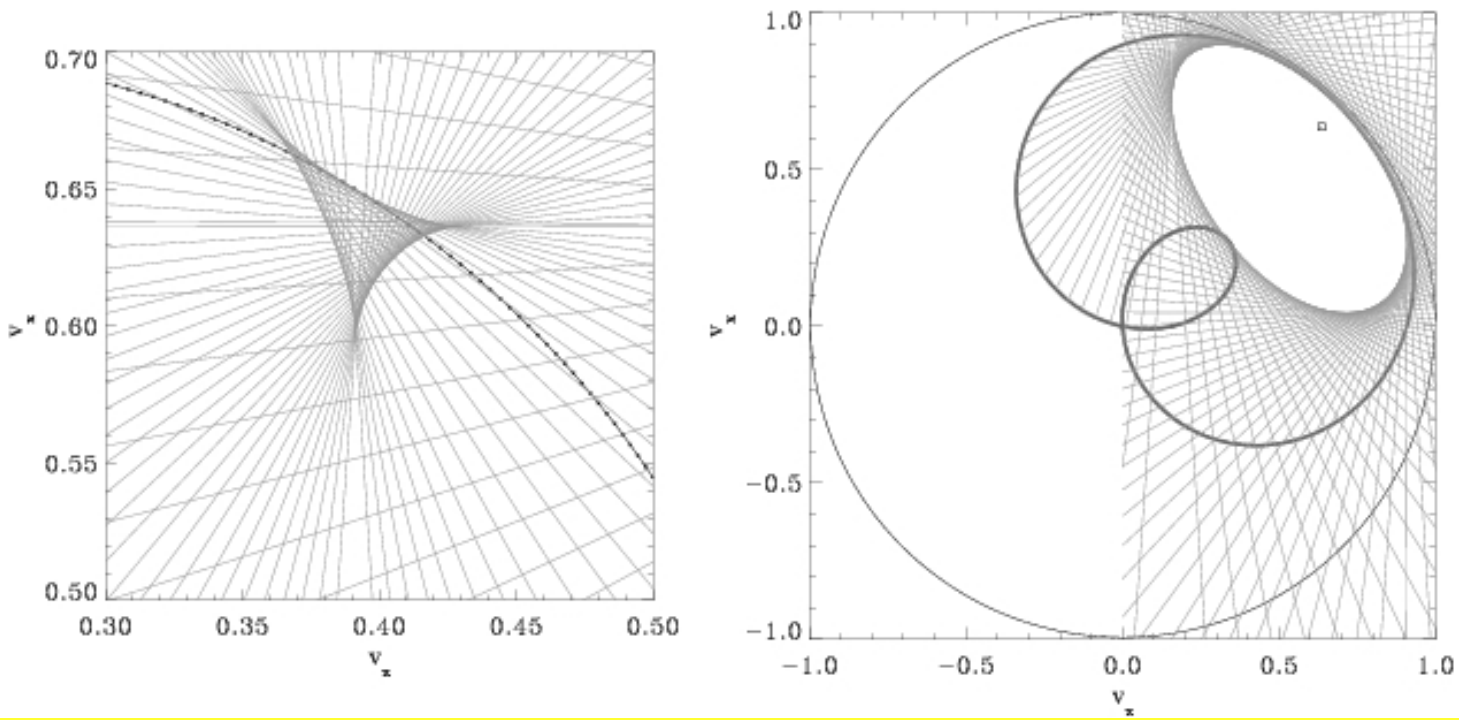
- same case: draw **phase diagram when source moves at**

$$\mathbf{v} = 0.9\,[\sin(\pi/4)\mathbf{e}_x + \cos(\pi/4)\mathbf{e}_z]$$



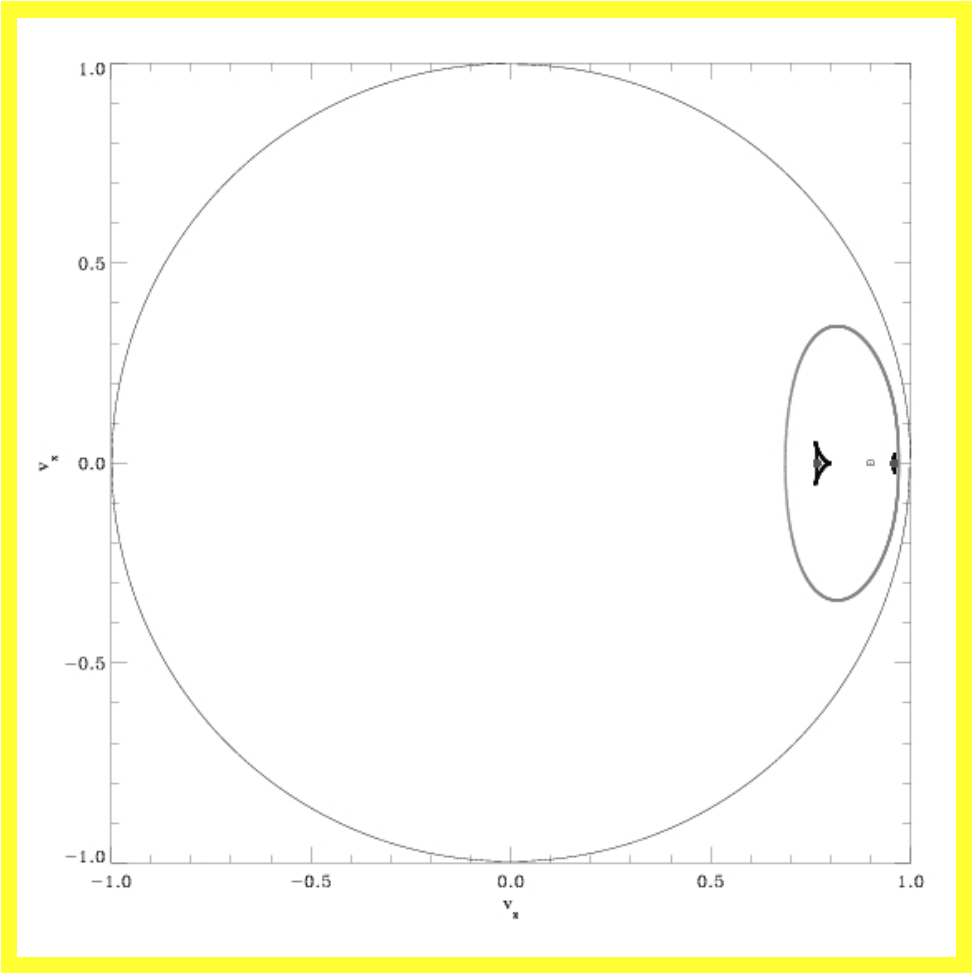
Relativistic MHD waves V

- group speed diagram then fully 3D objects, no more symmetry
  $\Rightarrow$ use **Huygens constructions: slow and fast fronts**



Relativistic MHD waves VI

- when speed $\mathbf{v} = 0.9c\mathbf{e}_z$ aligned with $\mathbf{B}$, still up-down symmetry
  $\Rightarrow$ from Lorentz transform get group diagram



- see **Physics of Plasmas 15, 102103, 2008**

- MHD wave speed expressions: analytic expressions for entropy and Alfvén phase speeds, while fast and slow pairs from quartic polynomial (can use e.g. Laguerre iteration to locate roots)

  $\Rightarrow$ needed for solvers like TVDLF, HLL(C), ... (characteristic wave speeds)

Relativistic MHD shocks

- shockfront: discontinuity across 4-manifold $\phi(ct, \mathbf{x}) = 0$

  $\Rightarrow$ normal to shockfront: space-like 4-vector $\mathbf{l}$, components $l_\alpha = \partial_\alpha \phi$

  $\Rightarrow$ Rankine-Hugoniot express conservation across manifold

$$\begin{aligned} [\![\rho U^\alpha]\!] l_\alpha &= 0 \\ [\![T^{\alpha\beta}]\!] l_\alpha &= 0 \end{aligned}$$

  $\Rightarrow$ directly follow from laws $\partial_\alpha(\rho U^\alpha) = 0$ and $\partial_\alpha(T^{\alpha\beta}) = 0$

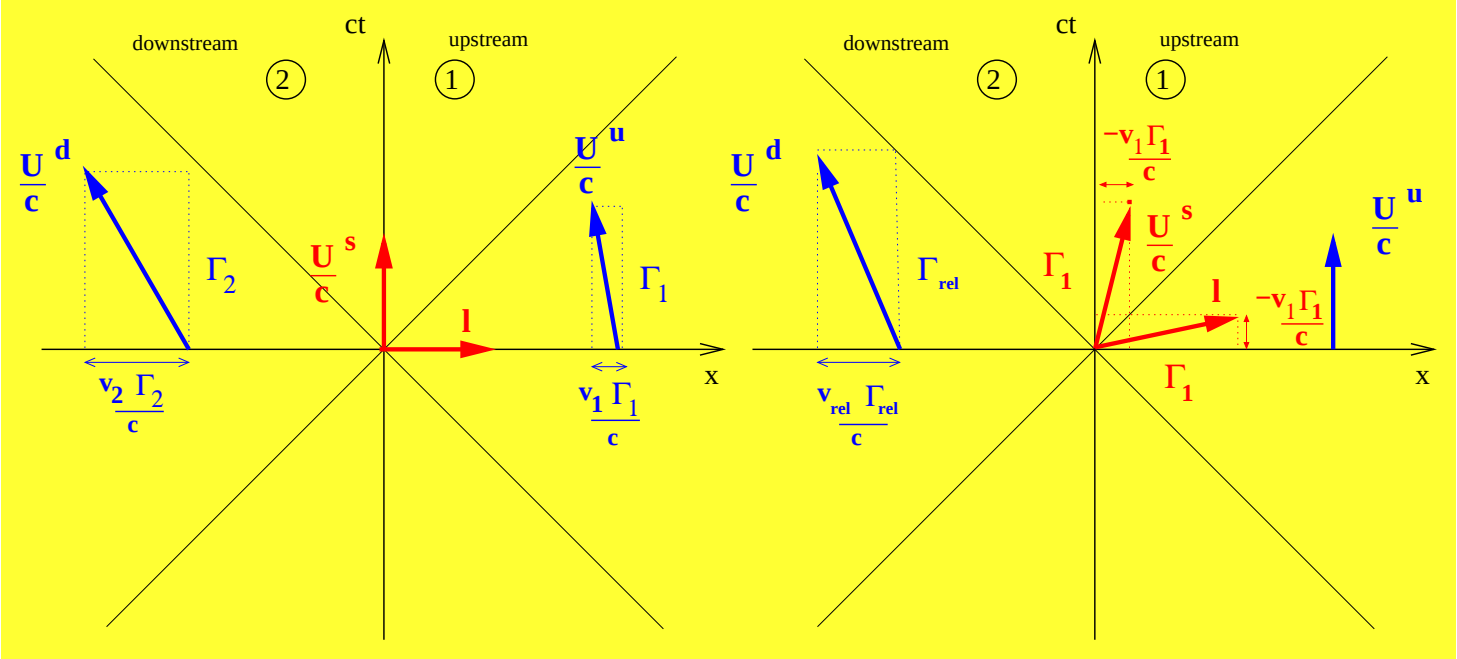
- four-vector for magnetic field (ideal MHD)

$$b^\alpha = \left[\Gamma \frac{\mathbf{v}\cdot\mathbf{B}}{c}, \frac{\mathbf{B}}{\Gamma} + \Gamma(\mathbf{v}\cdot\mathbf{B})\mathbf{v}/c^2\right]^{\mathrm{T}}$$

  $\Rightarrow$ induction equations yields

$$[\![U^\alpha b^\beta - b^\alpha U^\beta]\!] l_\alpha = 0$$

- involved relations, add complication of different reference frames

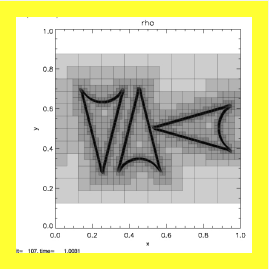


$\Rightarrow$ in SRF (left): $\mathbf{l} = (0, \mathbf{e}_x)$, with four-velocities up/downstream

$$\begin{aligned} \mathbf{U}^{\mathrm{u}} &= (c\Gamma_1, \Gamma_1 \mathbf{v}_1) \\ \mathbf{U}^{\mathrm{d}} &= (c\Gamma_2, \Gamma_2 \mathbf{v}_2) \end{aligned}$$

- many relativistic MHD shock invariants known, e.g. Lichnerowicz adiabat (like Hugoniot/Taub adiabat)

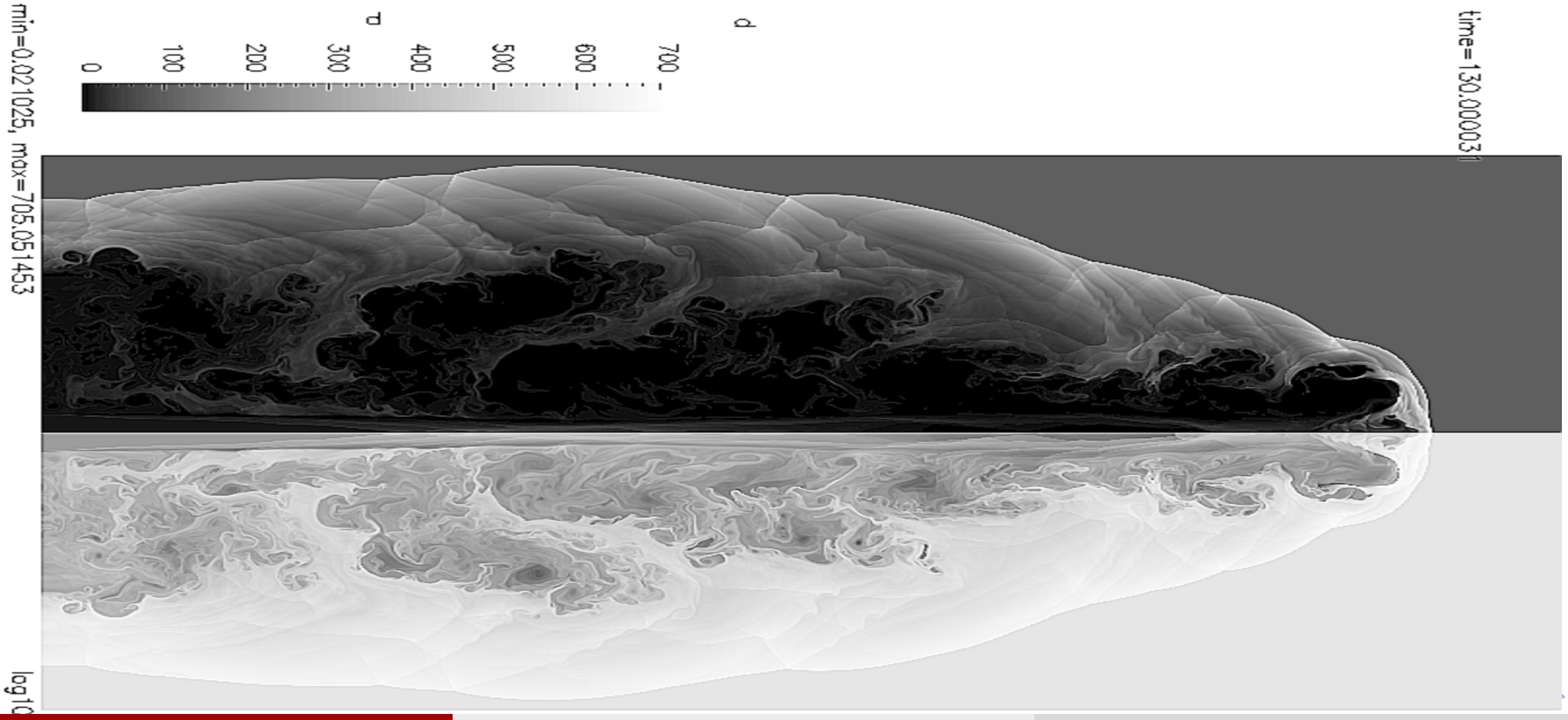
Adaptive Mesh Refinement & MPI-AMRVAC



- extreme contrasts, positive p, ρ, τ, $v < 1$, $\Gamma \geq 1$, solenoidal **B**
  - $\Rightarrow$ stringent demands on numerics and accuracy: **AMR vital**
- **Special relativistic HD and MHD: 'modules' in MPI-AMRVAC**
  - $\Rightarrow$ *CPC* **179** 2008, 617; *JCP* **226** 2007, 925; *MNRAS* **376** 2007, 1189; *JCP* **231** 2012, 718
  - $\Rightarrow$ advection, hydro, MHD, relativistic (M)HD modules
  - $\Rightarrow$ **different EOS implemented for relativistic modules**
  - $\Rightarrow$ any-D, explicit **grid adaptive framework**
  - $\Rightarrow$ **full MPI octree variant, cartesian/cylindrical/spherical**
- shock-capturing schemes (TVDLF/HLL/HLLC/Roe), 2nd to higher order reconstructions

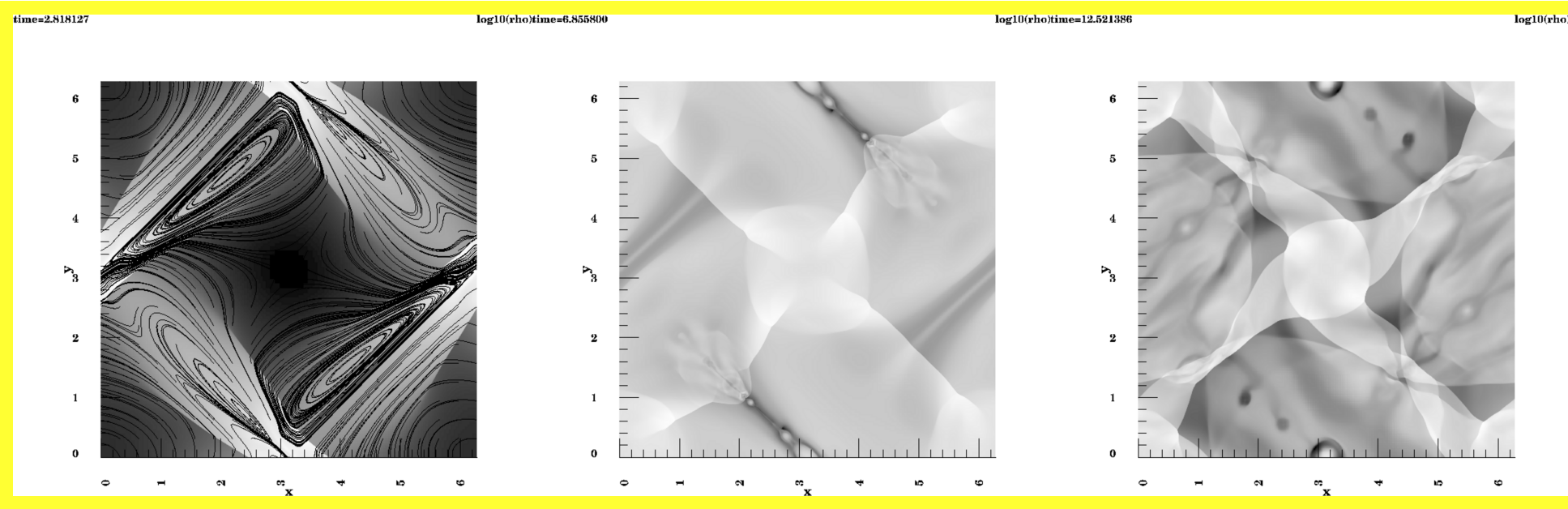
Relativistic Hydro jets: standard test case

- Relativistic HD Jet: classical Mach $v_{\rm jet}/c_s \simeq 11$
  - $\Rightarrow$ uniform pressure, density $\rho_{\rm jet}/\rho_{\rm ext} = 0.01$ (light jet)
  - $\Rightarrow$ $\Gamma \simeq 7$, **effective resolution** 1440×4800 **(5 levels)**
  - $\Rightarrow$ morphology **as in Marti et al 1997, Mignone et al 2005**



RMHD Orszag-Tang test

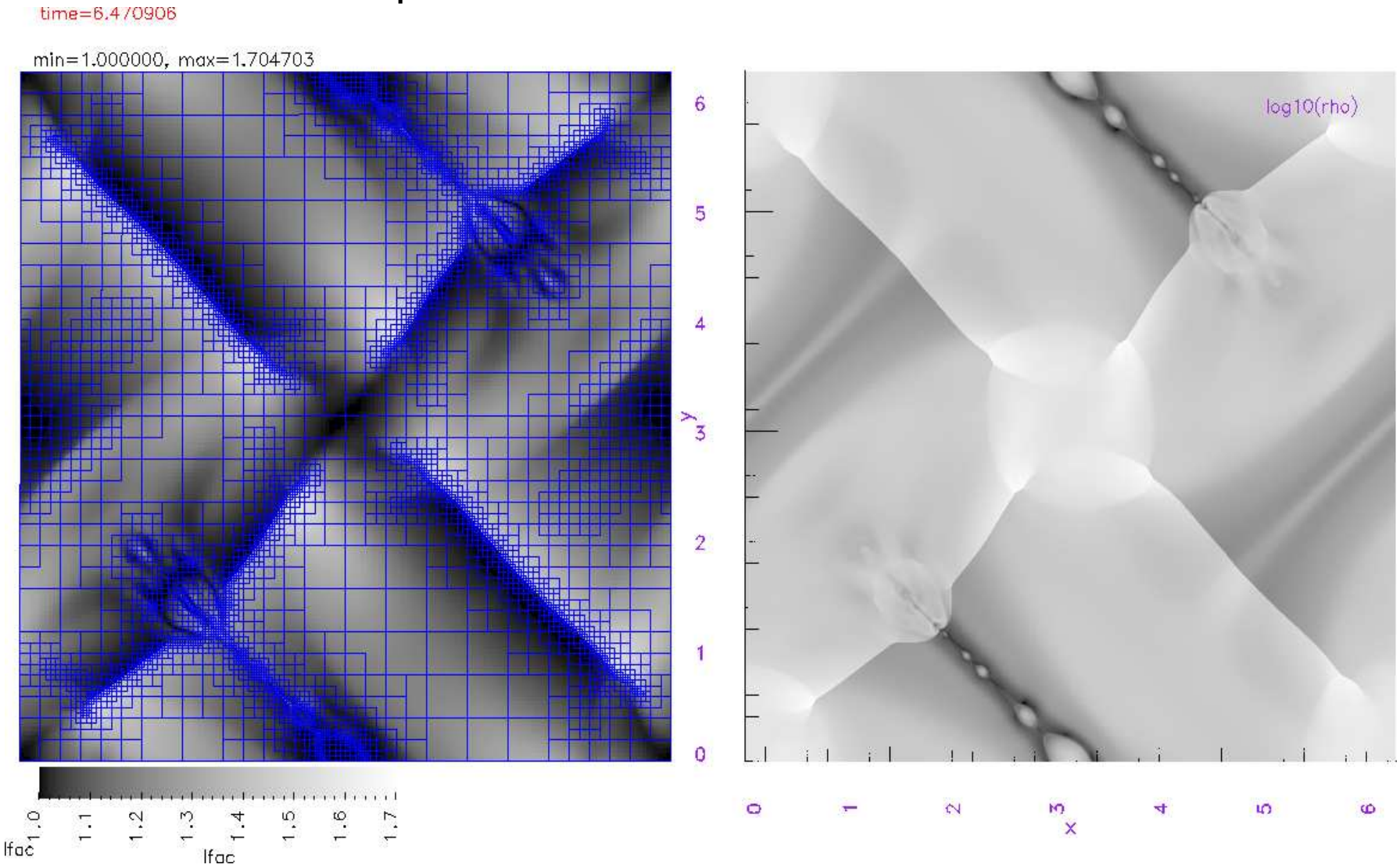
- relativistic analogue of 2D MHD Orszag-Tang test
  - ⇒ double periodic, supersonic relativistic vortex rotation
  - ⇒ initial field configuration: double island structure



⇒

current sheets form, shock interactions, reconnections

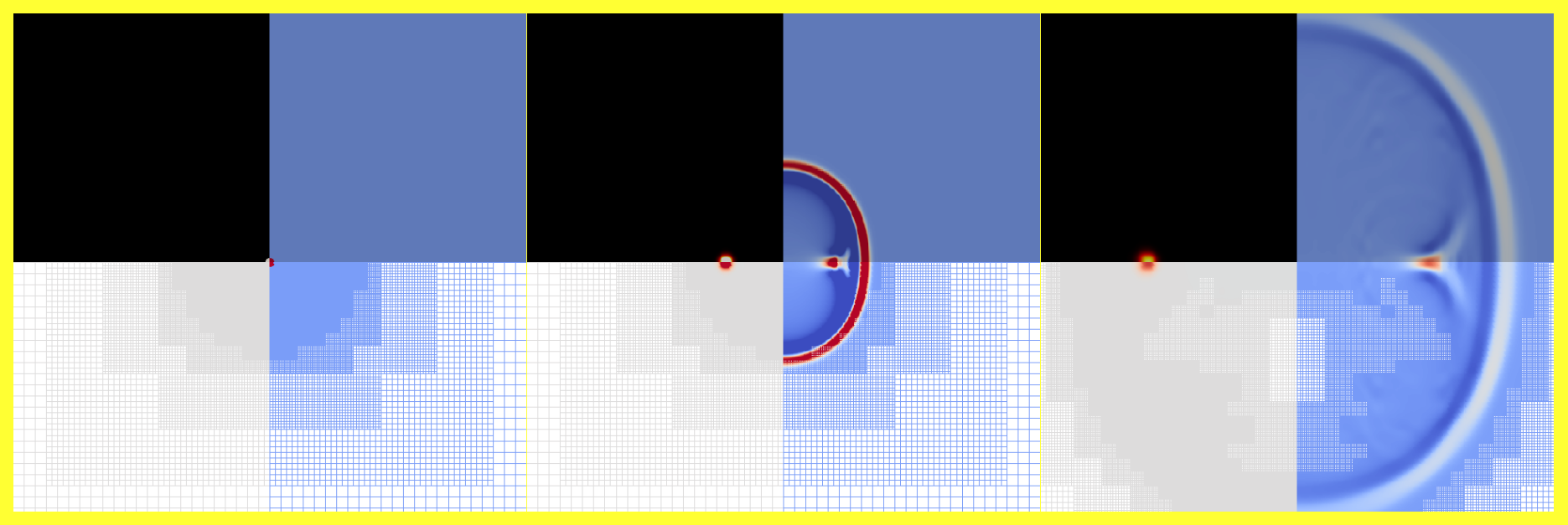
- AMR vital: captures small-scale ‘reconnection’ effects



$\Rightarrow$ to revisit in true resistive RMHD!

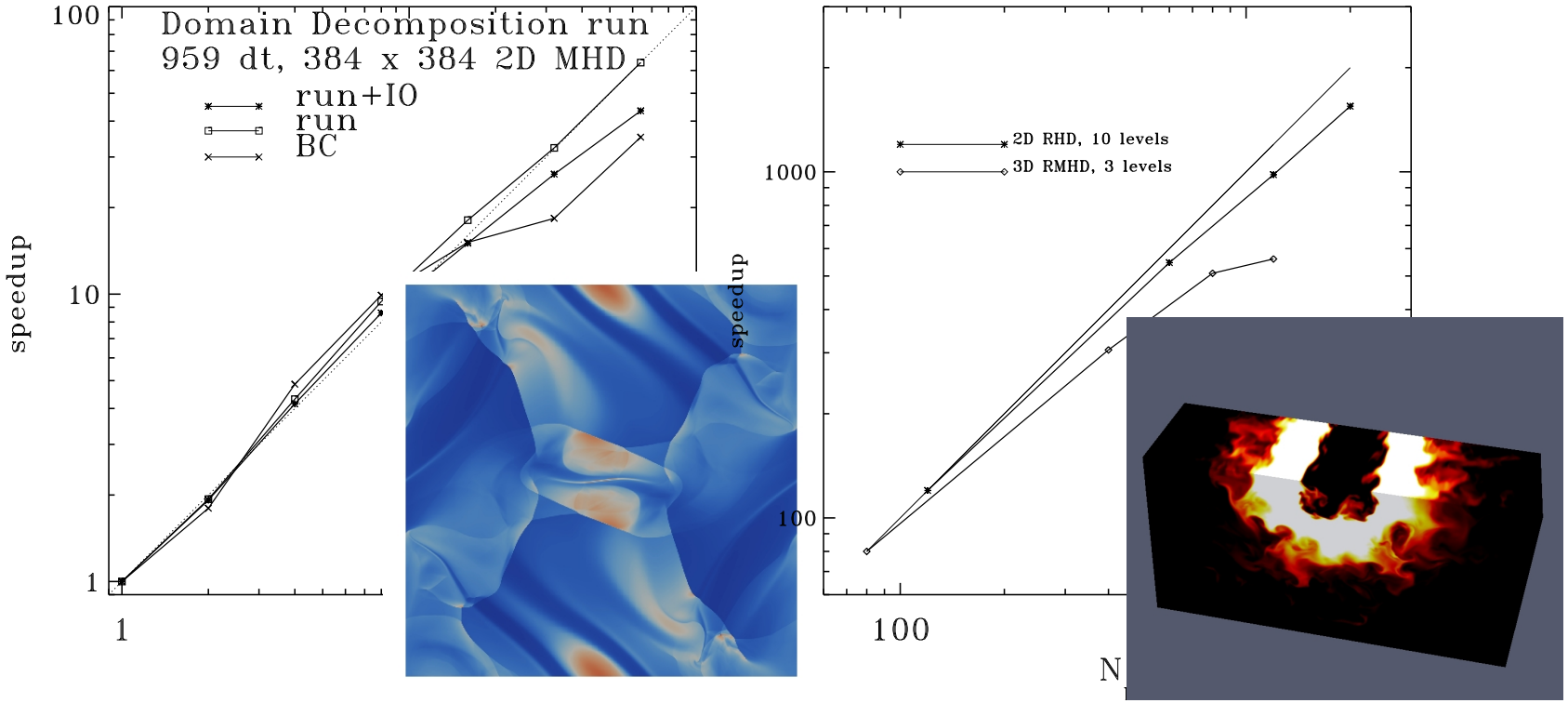
RMHD wave test

- linear waves in homogeneous plasma with $\rho = 1 = p$
  - $\Rightarrow$ uniform $\mathbf{B} = 0.3\mathbf{e}_x$, perturb with $\delta p = 0.1 = \delta v_z$



  - $\Rightarrow$ **triggers all 3 wave signals**, reproduces group diagram
  - $\Rightarrow$ note: AMR only active late: typical easy to detect shocks!

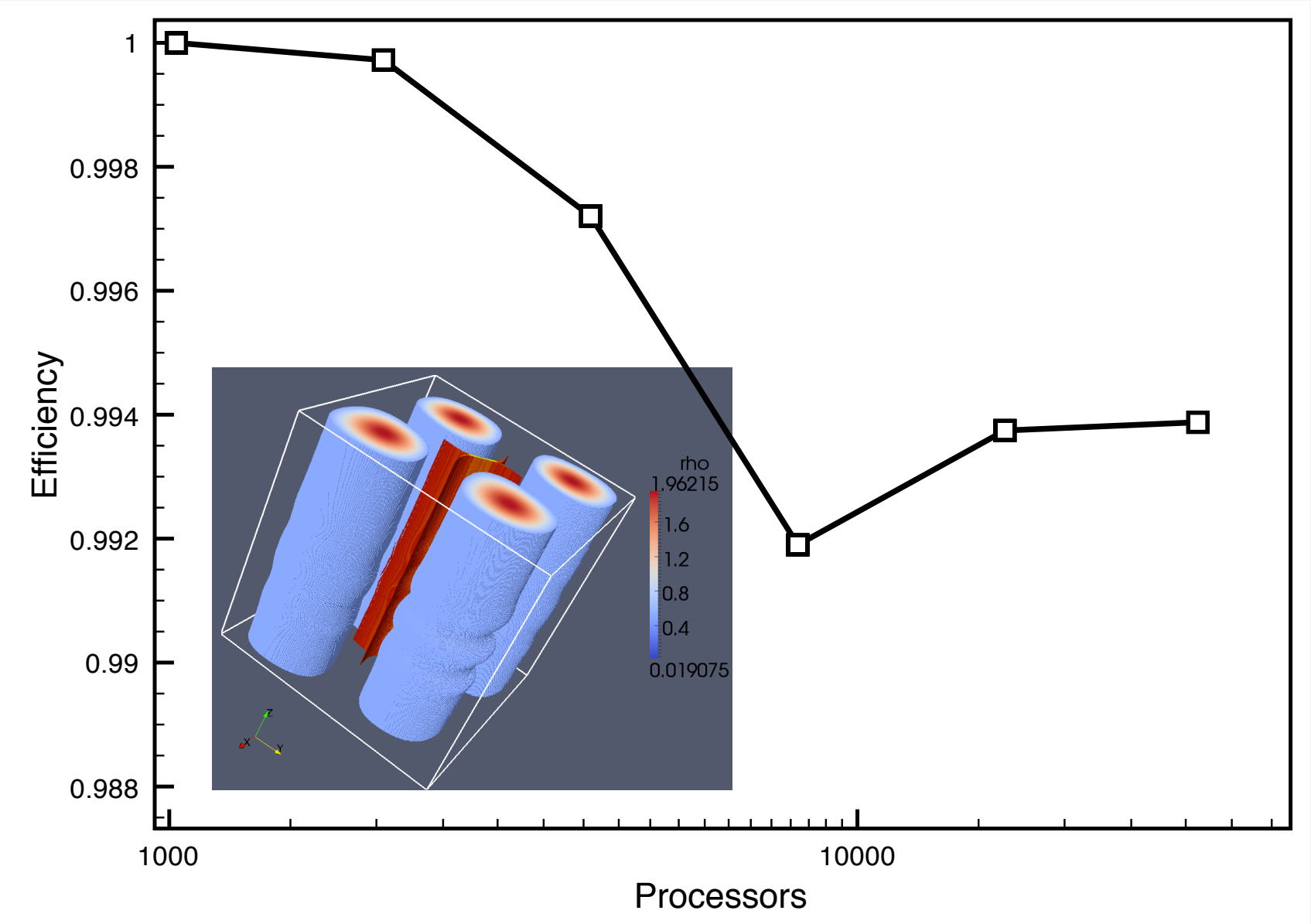
MPI-AMRVAC and HPC-Europa2



- **excellent scaling**: domain decomposition and **multi-level AMR**

  $\Rightarrow$ 2D MHD at $\simeq 400^2$, 1000 Δt in $<$ 5 seconds (include IO)

  $\Rightarrow$ **10 level AMR special relativistic HD sustained 80% efficiency on 2000 CPUs!**



- 3D MHD weak scaling to >31000 CPUs (Fermi)

- conservative to primitive transformation: no longer purely algebraic as in Newtonian MHD

  $\Rightarrow$ in each grid point, local rootfinding required

  $\Rightarrow$ many equivalent formulations exist: accuracy/speed crucial in selection

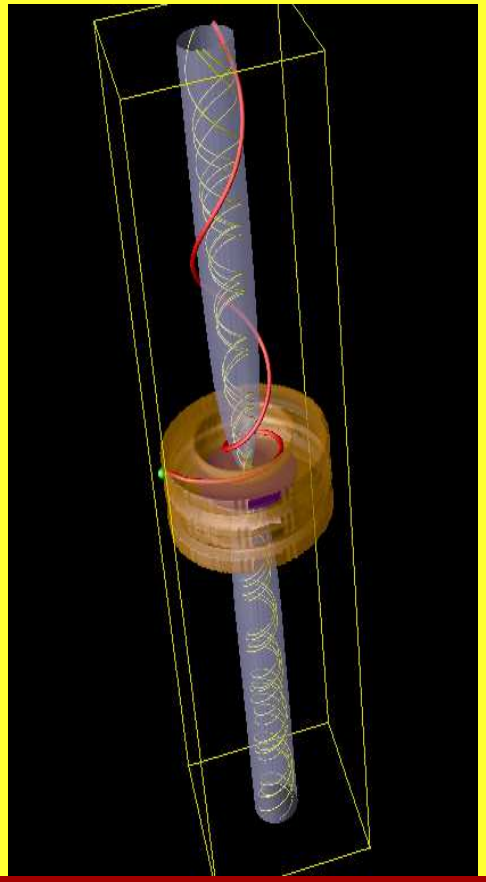
- MPI-AMRVAC: use auxiliary variables ($\xi = \rho h \Gamma^2, \Gamma$)

  $\Rightarrow$ nonlinear transcendental equation solves ξ from

$$0 = \xi - p - \tau - D + B^2 - \frac{1}{2}\left[\frac{B^2}{\Gamma^2} + \frac{(\mathbf{S}_{\rm tot}\cdot\mathbf{B})^2}{\xi^2}\right]$$

Internal stratification effects and jet deceleration

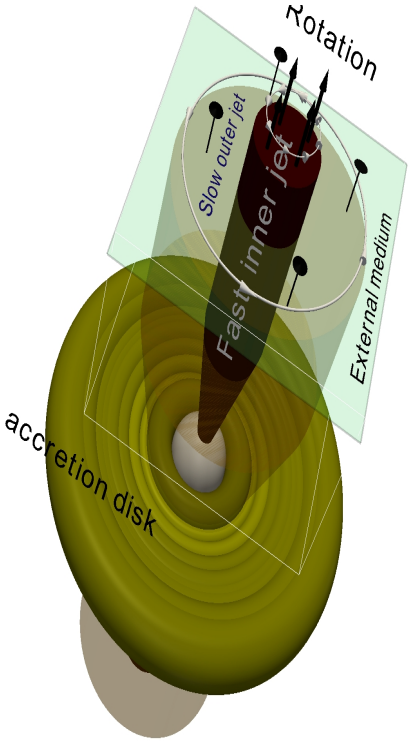
- AGN jets **radial stratification: fast inner, slow outer jet**
  - $\Rightarrow$ different launch mechanism $\rightarrow$ different rotation
- **outer 'disk' jet launched magnetocentrifugally**
  - $\Rightarrow$ Magnetized Accretion-Ejection Structure (MAES)



- generic mechanism for jet launch
  - $\Rightarrow$ magnetic torque brakes disk matter
  - $\Rightarrow$ magnetic torque spins up jet matter
  - $\Rightarrow$ mass source for jet: disk
  - $\Rightarrow$ **B** collimates, accelerates
  - $\Rightarrow$ **Jet formation** & **Escaping accretion**
- accretor can be compact object, AGN

Two-component jet model

- **close to engine: GR mechanisms launch inner jet**
  - $\Rightarrow$ efficient extraction AM from inner disk + black hole (Blandford-Znajek mechanism)
  - $\Rightarrow$ fast rotating inner jet, introduce **radially layered jet**
  - $\Rightarrow$ inner $\Gamma \sim 30$, outer $\Gamma \sim 3$

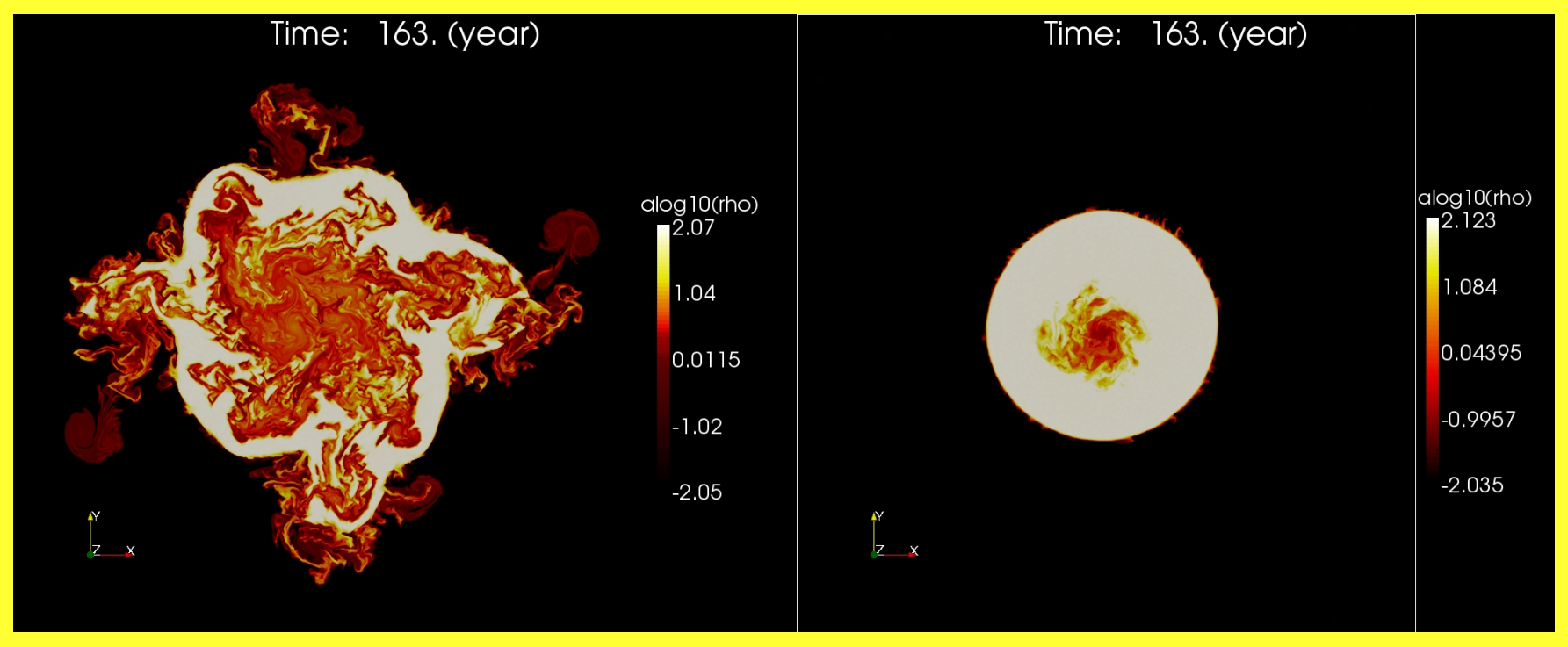


- perform 2.5D runs in cross-section
  - $\Rightarrow$ both HD and MHD runs
  - $\Rightarrow$ explore differences in effective inertia
- study jet integrity for axisymmetric runs
  - $\Rightarrow$ vary precise spine-sheath structure

Meliani & Keppens, ApJ 705, 1594-1606 (2009)

- vary relative contribution inner jet to total $L_{\mathrm{Jet,Kin}} \sim 10^{46} ergs/s$

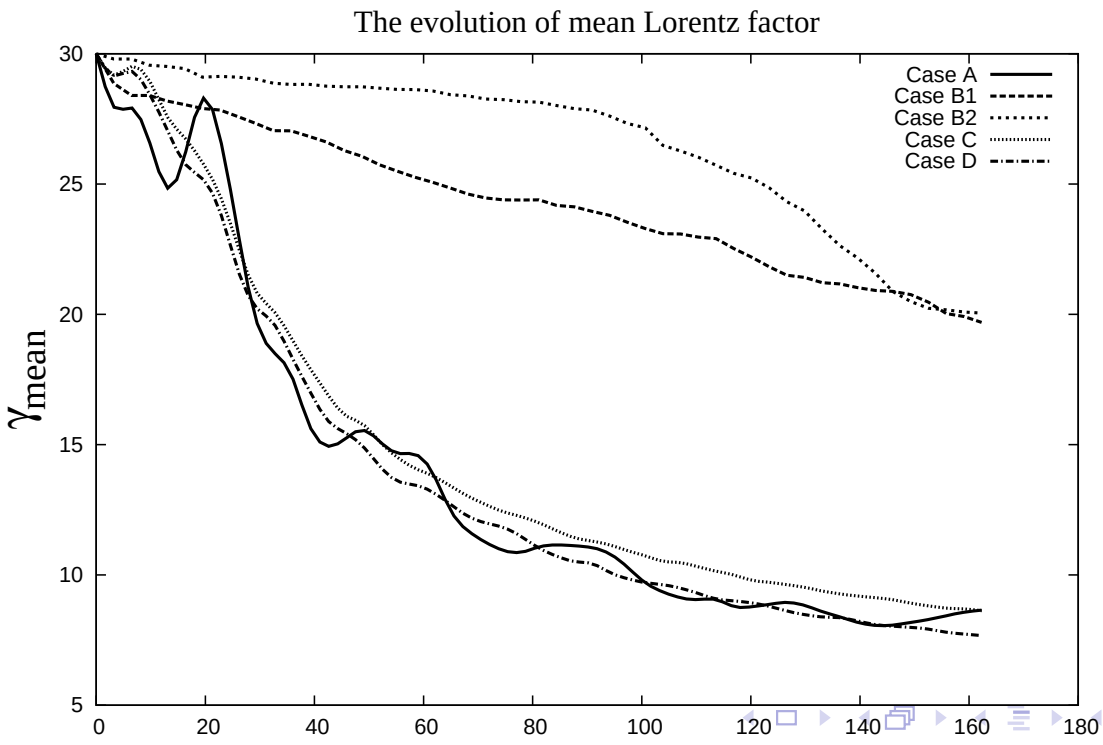
  $\Rightarrow$ discover **new relativistic, centrifugal Rayleigh-Taylor**



$\Rightarrow$

efficient AM redistribution, enhance inner/outer jet mixing

- novel **relativistically enhanced Rayleigh-Taylor mode**
  - ⇒ **Stable: effective inertia outer > inner jet**
  - ⇒ **No classical counterpart** (relativistic flow essential)!
  - ⇒ $\Gamma^2 h$ effect with h specific enthalpy
- stable versus unstable jets: **design initial conditions with varying contribution inner/outer jet to total kinetic energy**
  - ⇒ criterion predicts cases A, C, D stable; B1, B2 unstable
  - ⇒ **evolution of inner jet mean Lorentz factor**



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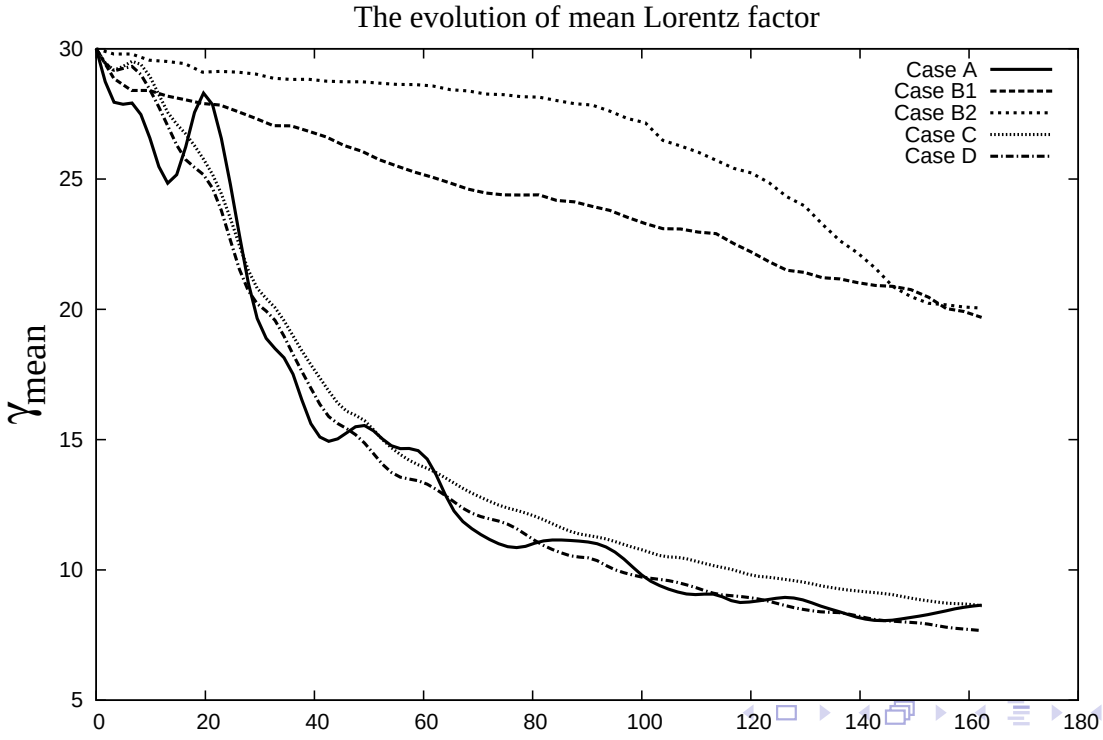

- novel **relativistically enhanced Rayleigh-Taylor mode**
  - $\Rightarrow$ approximate **dispersion relation**
  - $\Rightarrow$ insert spatio-temporal dependence $\exp(\lambda\, t - k\ |\ \zeta\ |)$ with displacement ζ

$$\lambda^2 \propto k \left[\left(\Gamma^2 \rho h + B_z^2 \right)_{\rm in} - \left(\Gamma^2 \rho h + B_z^2 \right)_{\rm out} \right]$$

- **Stability: effective inertia outer jet $>$ inner jet**
  - $\Rightarrow$ works for both HD and MHD relativistic jets
  - $\Rightarrow$ purely poloidal **B** effect incorporated
  - $\Rightarrow$ **relativistic EOS crucial**: cold/hot outer/inner jet

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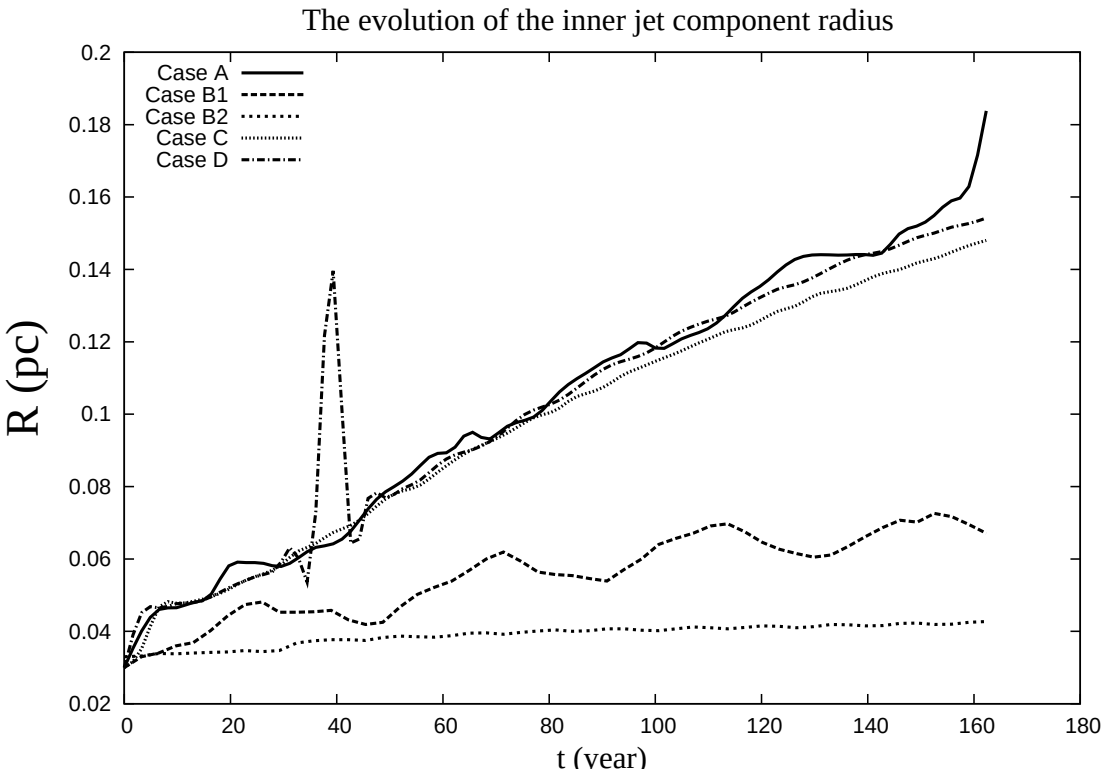
- can quantify jet de-collimation due to mode development



  ⇒ due to non-axisymmetric mode development

  ⇒ relativistic RT decelerates inner, decollimates total jet

- **FR II/FR I transition thereby related to central engine**

  ⇒ depends on distribution kinetic energy over two-component jet

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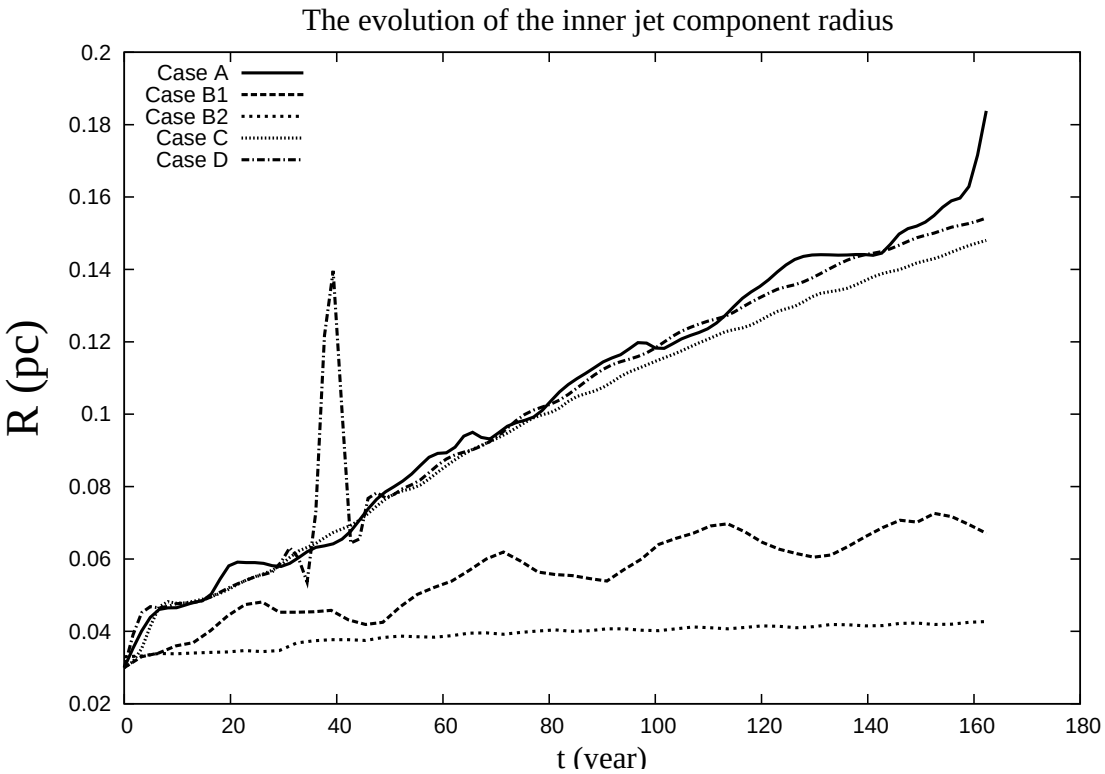

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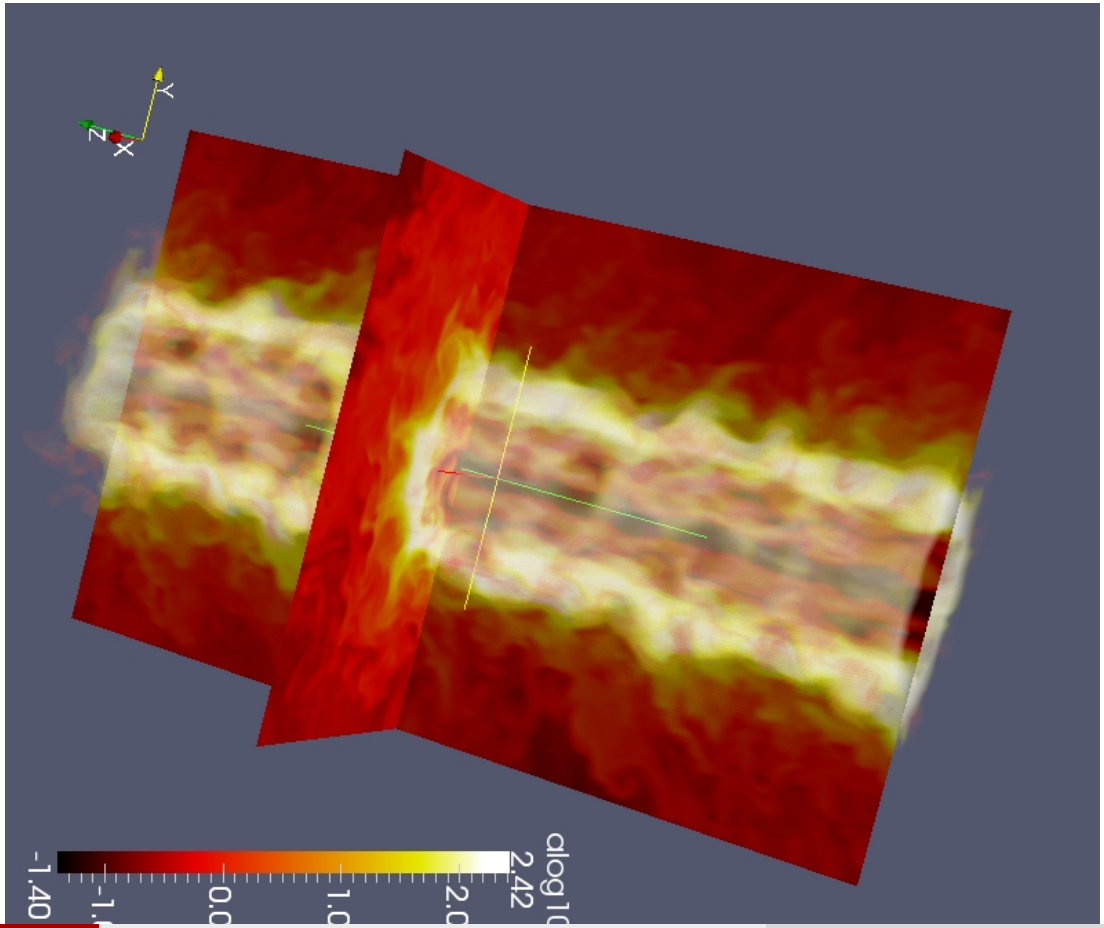
3D two-component scenarios

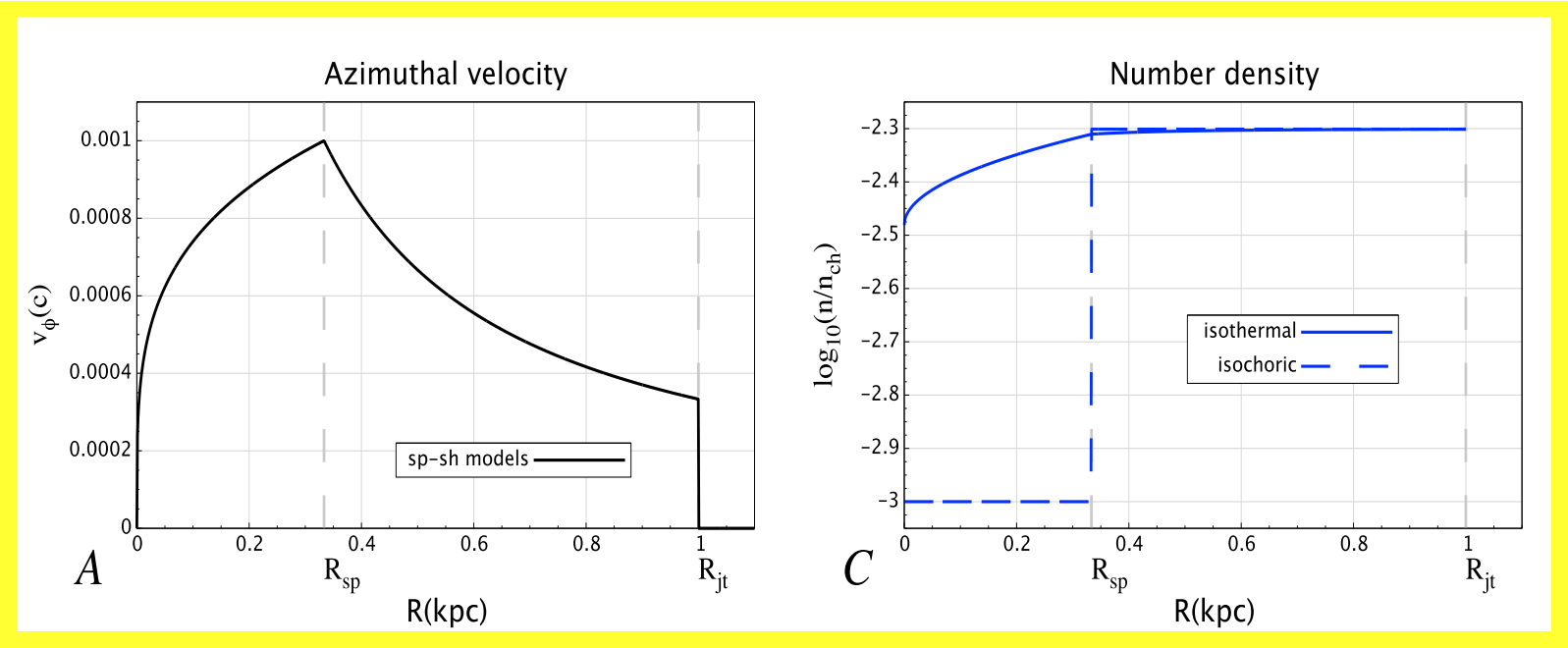
- set up cylindrical, two-component jet models
  $\Rightarrow$ assume periodic segment, ignore jet opening angle
- **3D case liable to RT mode** versus **stable to RT**



Summary two-component jet evolutions in 3D

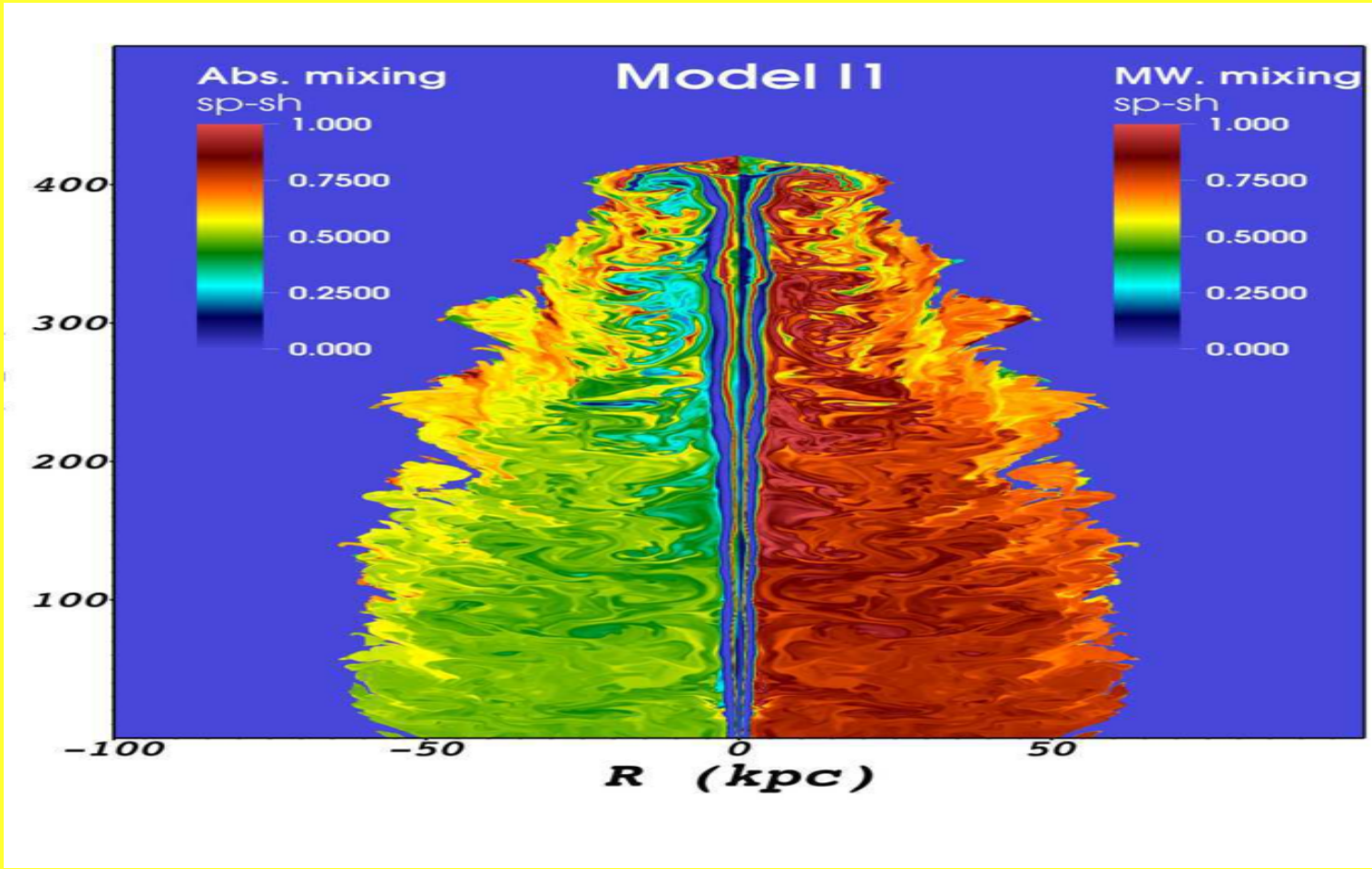
- despite additional Kelvin-Helmholtz modes with axial variation
  - $\Rightarrow$ **main evolution driven by newly discovered RT mode**
- further work
  - $\Rightarrow$ observational consequences: synthetic radio maps!
  - $\Rightarrow$ integrity two-components over large propagation distances

- two-component model and mixing during propagation
  - ⇒ axisymmetric, rotating jets (suppress enhanced RT mode)
  - ⇒ investigating spine-sheath integrity at kpc scales!
- radial jet stratification: rotation & isothermal/isochoric



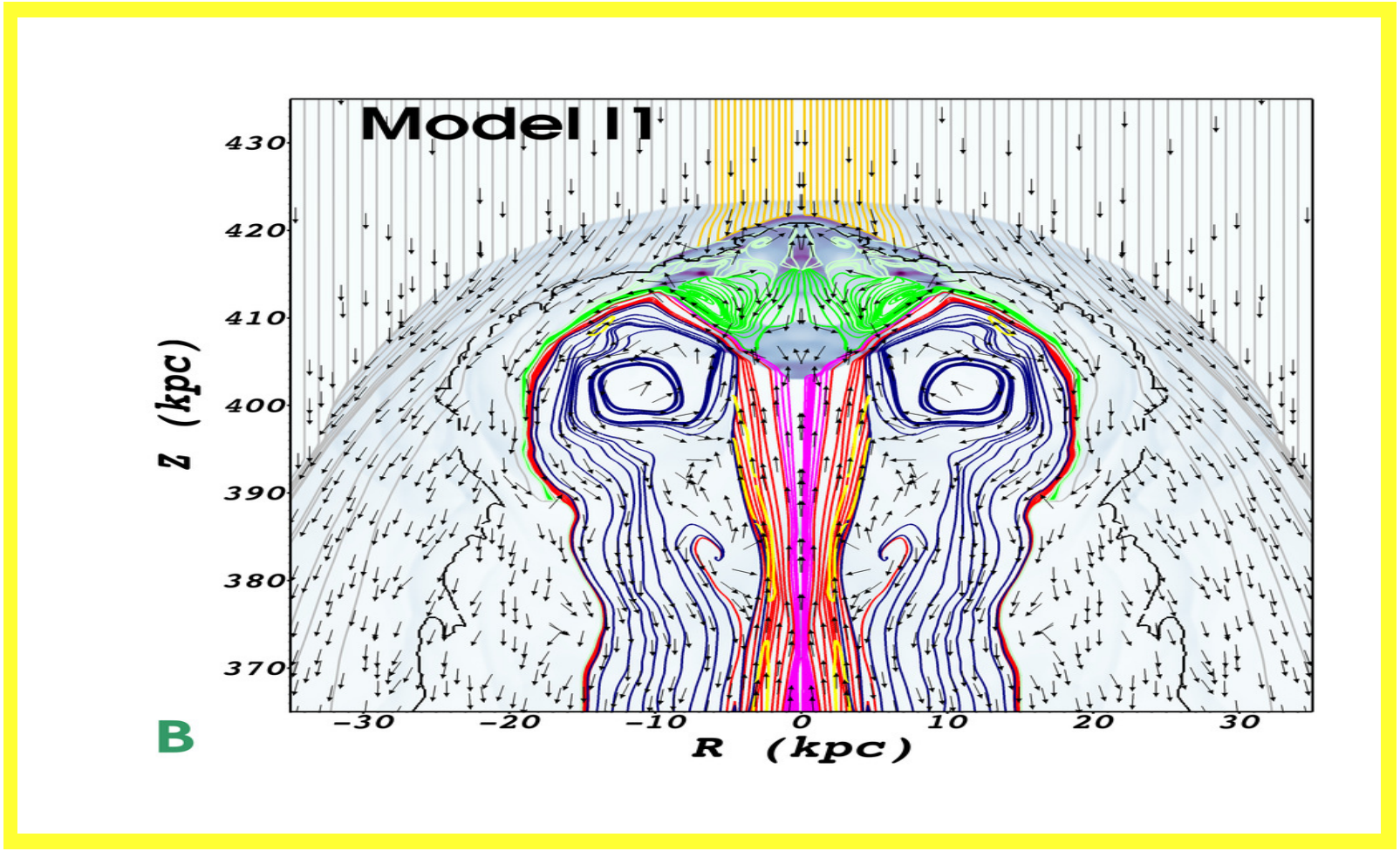
- isothermal spine-sheath model: layered structure persists on long scales! (Walg et al, 2013)

  ⇒ **view on density and mixing**



- identified effective impact area of radially structured jet (important to quantify jet propagation!)

  $\Rightarrow$ yellow streamlines, all plotted in Mach disk rest frame



- even after about 9 internal jet beam shocks . . .

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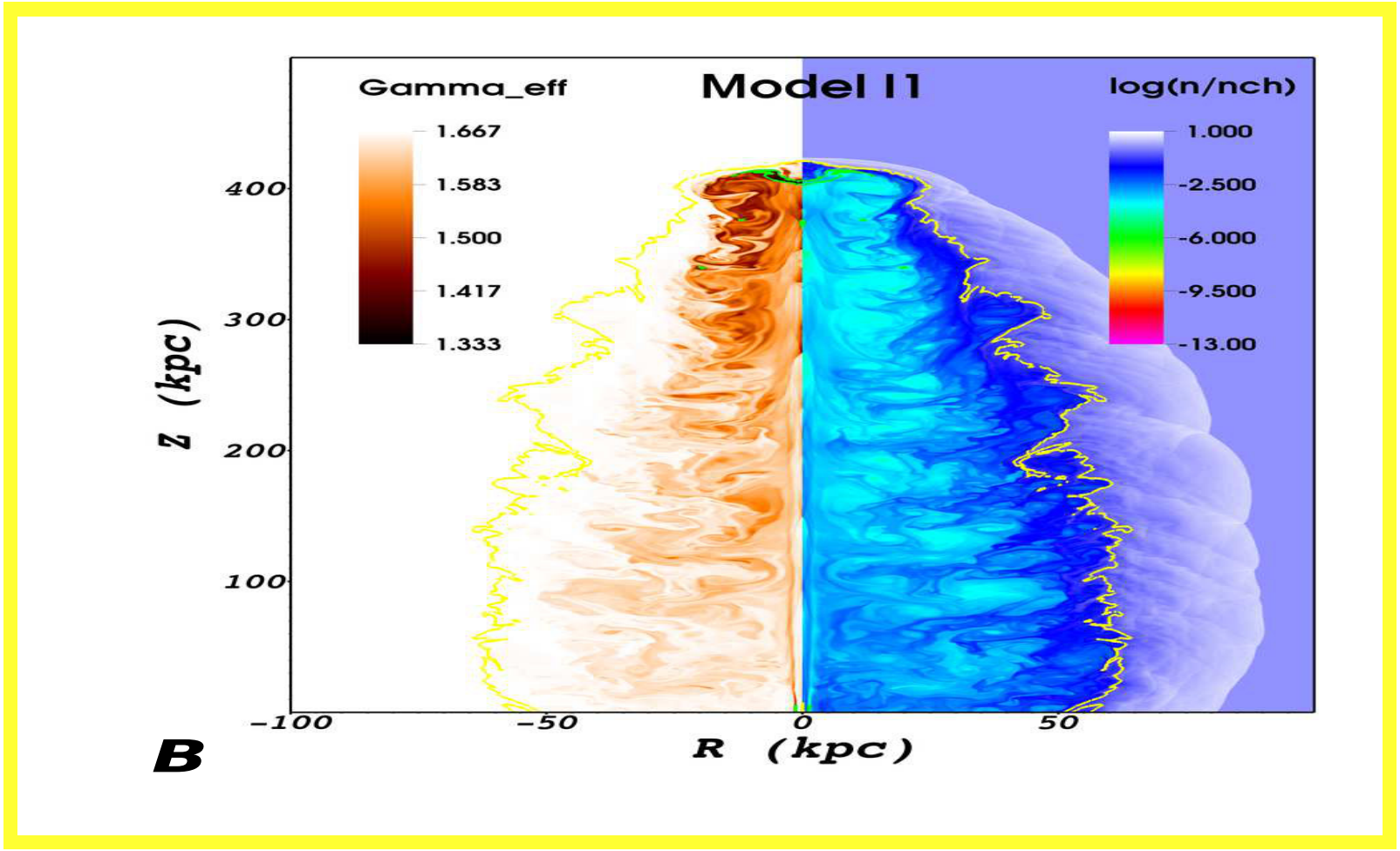
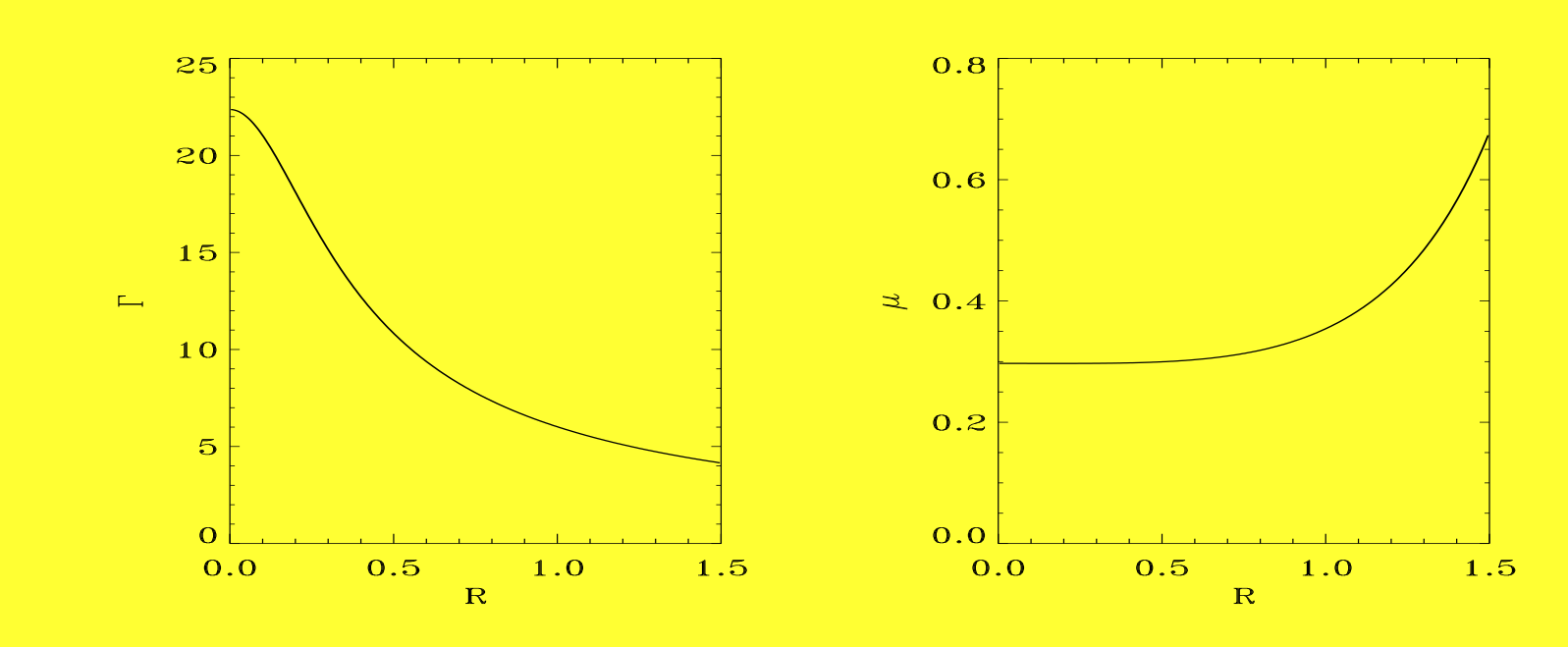


- even after about 9 internal jet beam shocks . . .

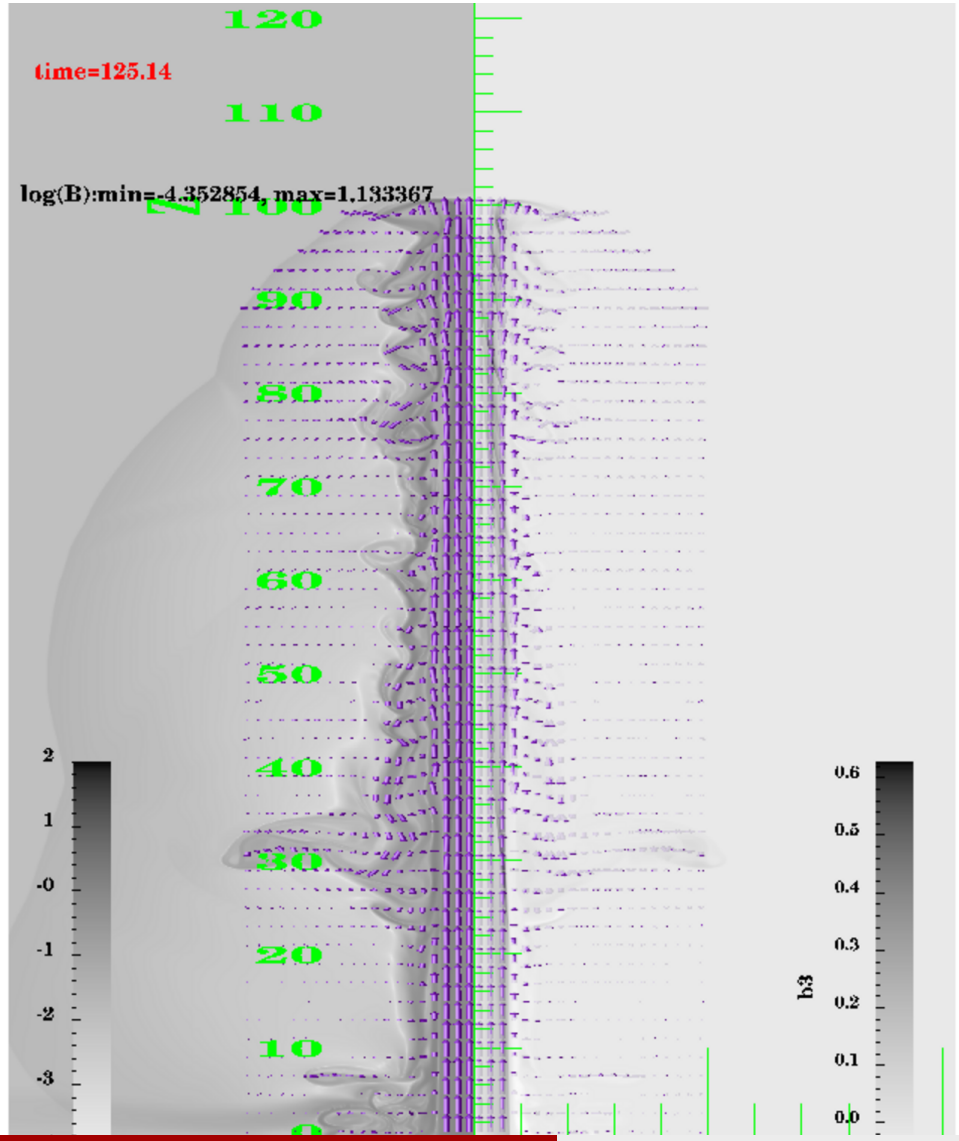
Helically magnetized jets

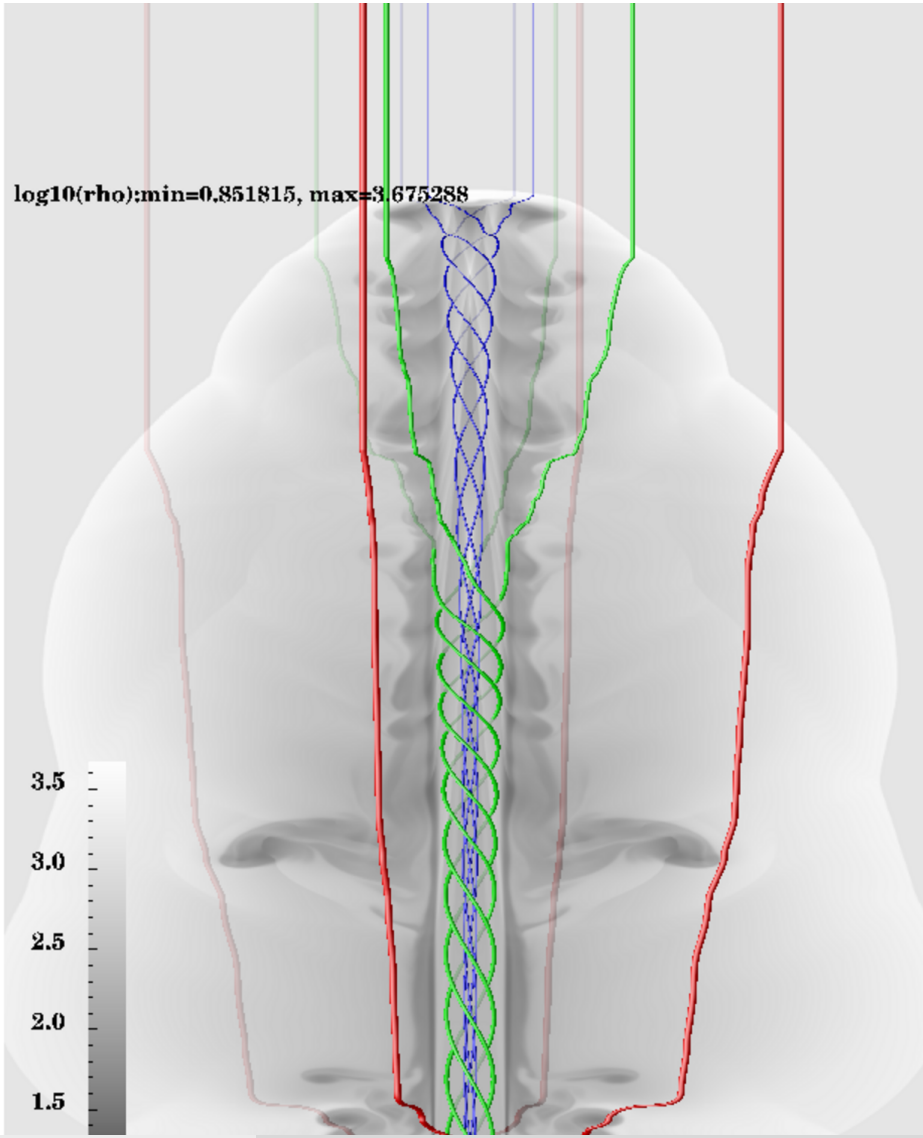
- Axisymmetric **helical fields** (Keppens et al. 2008)
  - $\Rightarrow$ again 2.5D, density contrast $1/10$: light jet
  - $\Rightarrow$ inlet profile of Γ and $\mu = \frac{R_j B_\varphi}{R B_Z}$



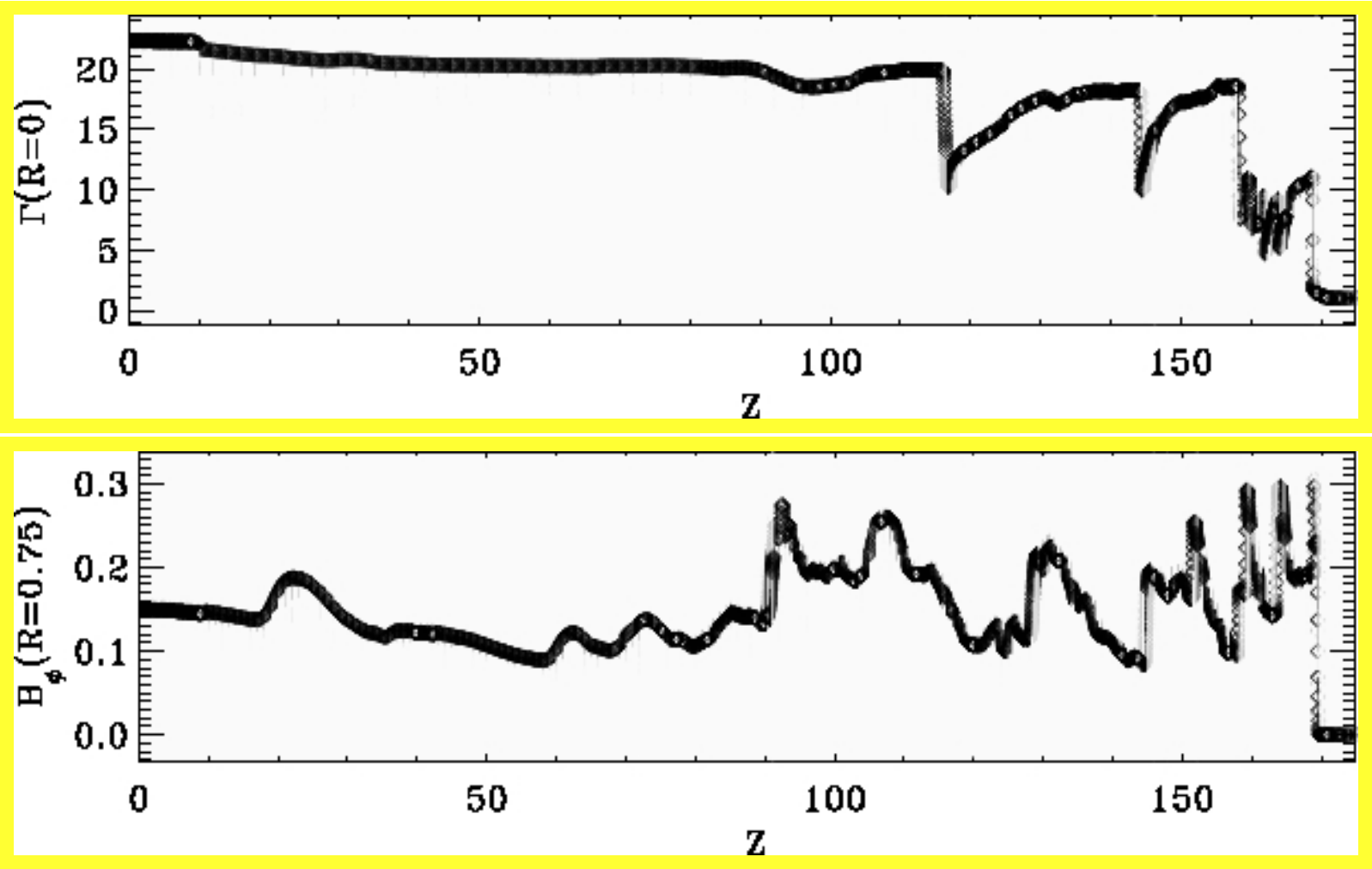
- average $\bar{\Gamma} \simeq 7$, $\beta_I = 0.3$ and $\sigma = 0.006$
  - $\Rightarrow$ **kinetic energy dominated, near equipartition**
- both helical field and rotation within jet!

- magnetic field: **helicity throughout the jet beam**
  - ⇒ changes at internal cross-shocks
  - ⇒ localized mainly toroidal field within vortical backflows

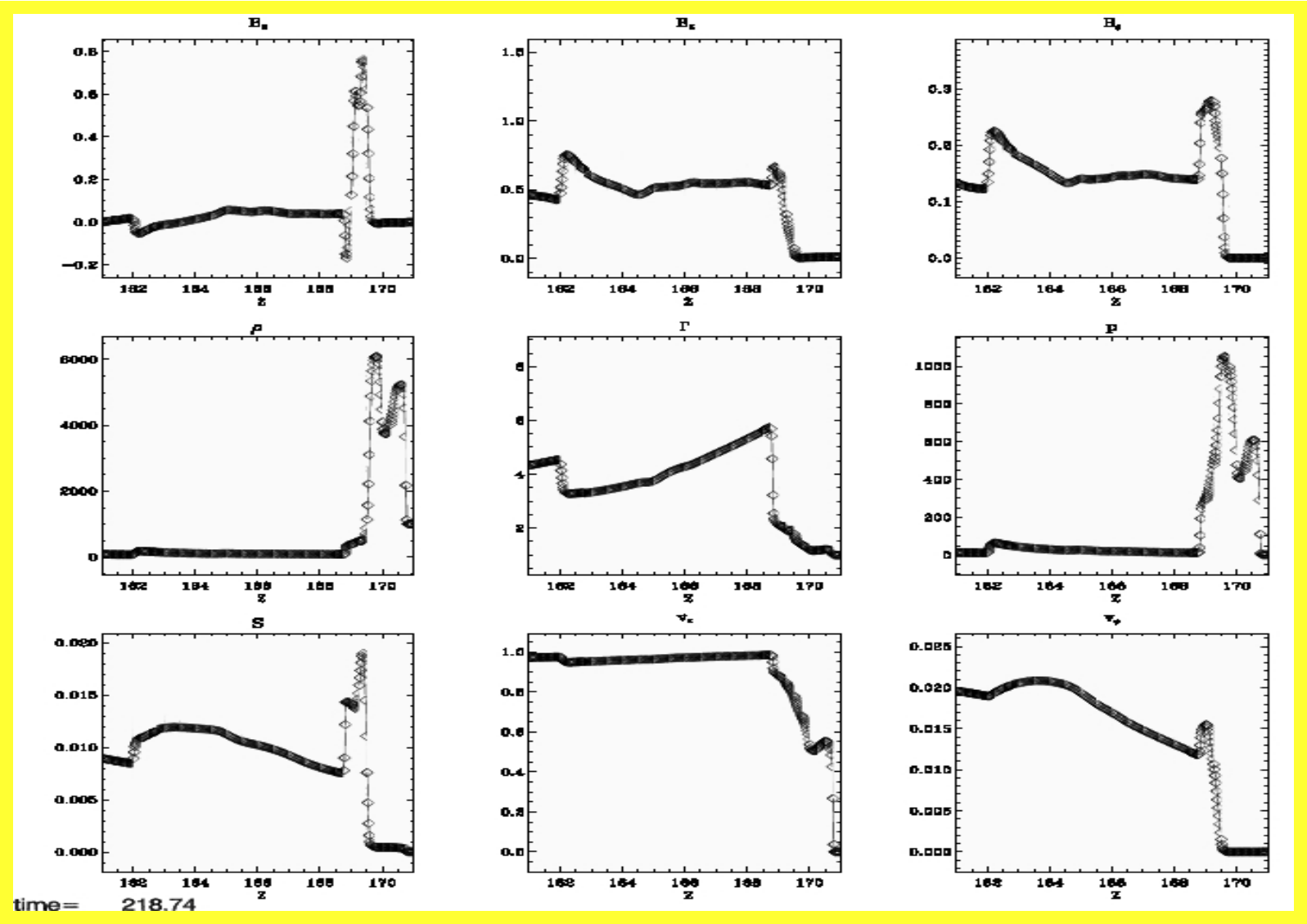




- beam cross-shocks: **helical field pinches flow**
  - ⇒ matter reaccelerates up to next cross-shock
  - ⇒ deceleration jet with equipartition **B**: extreme lengths

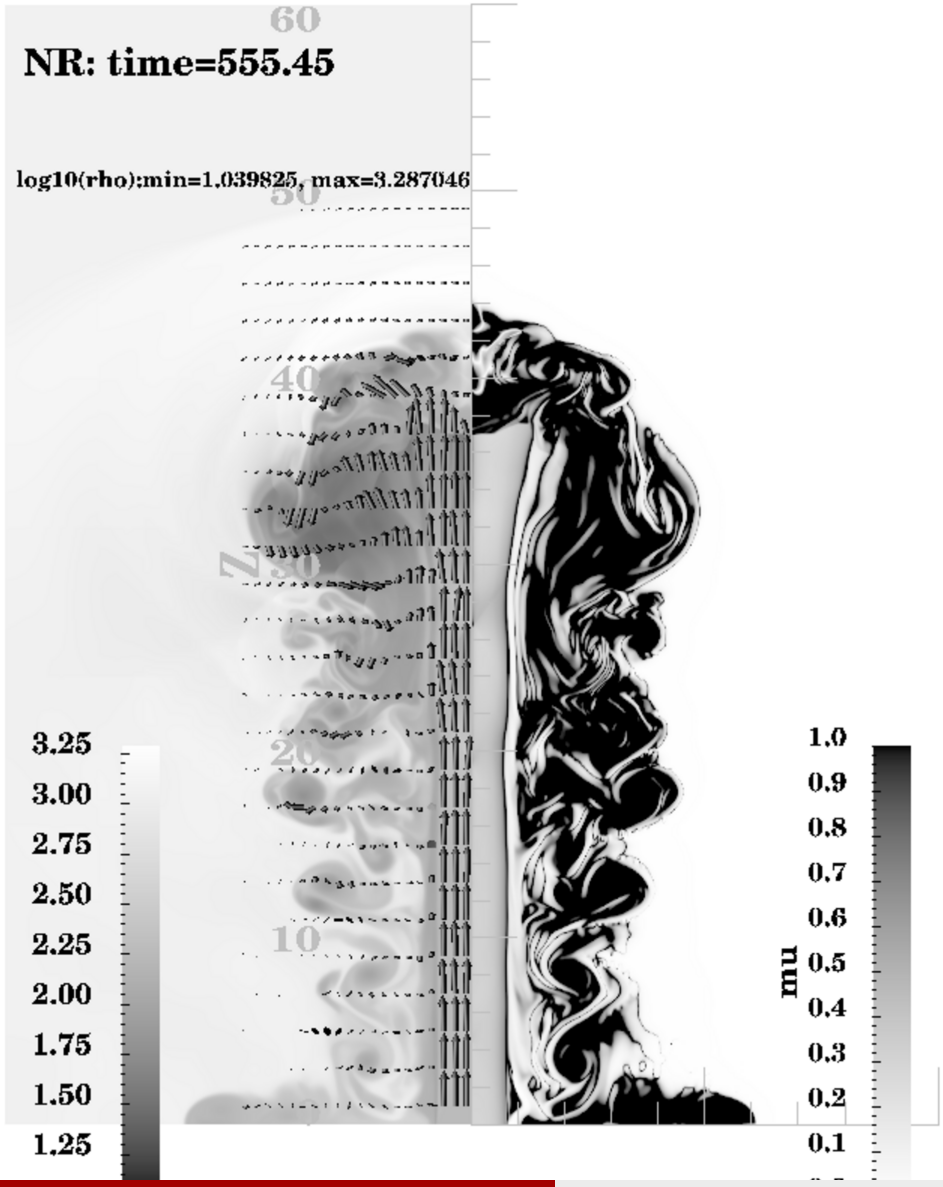


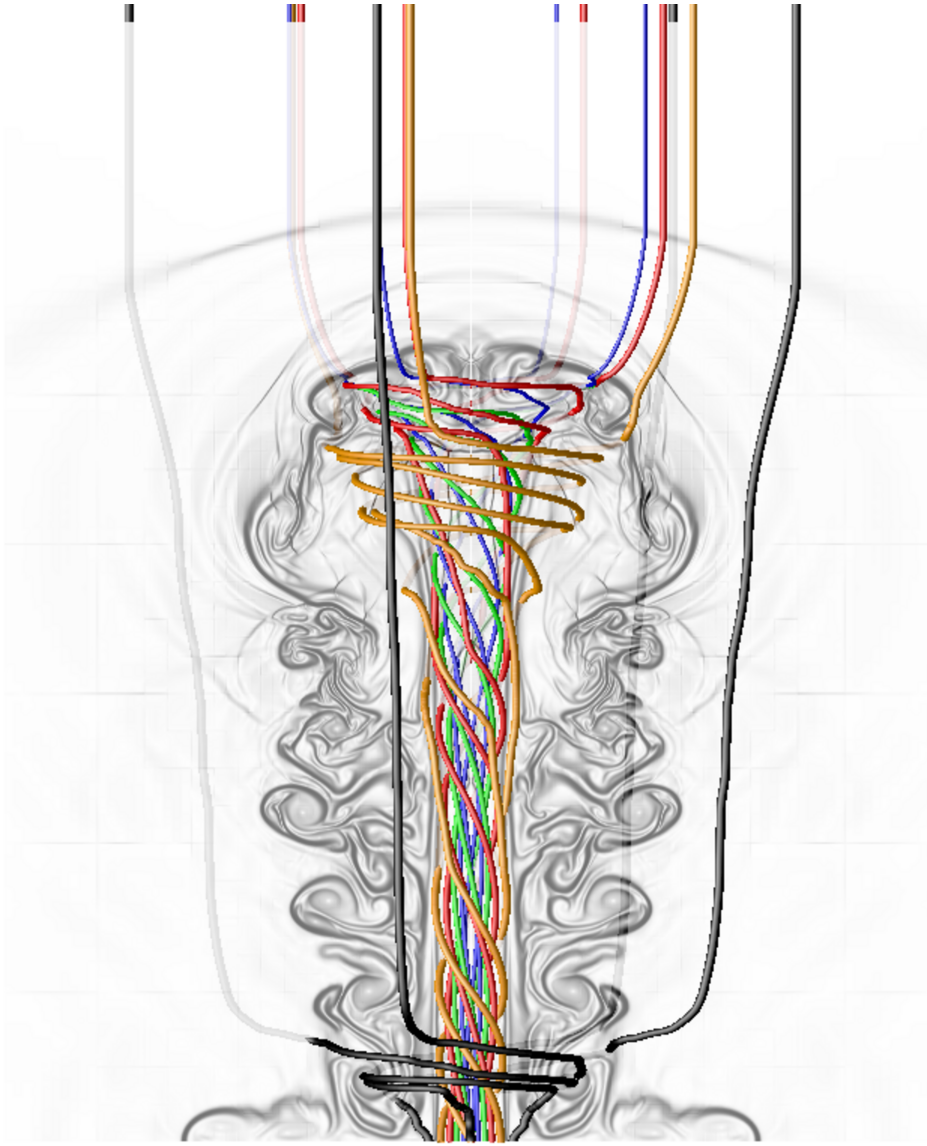
- detailed variation of field quantities at jet head
  $\Rightarrow$ significant 2D effects compared to 1D Riemann problems



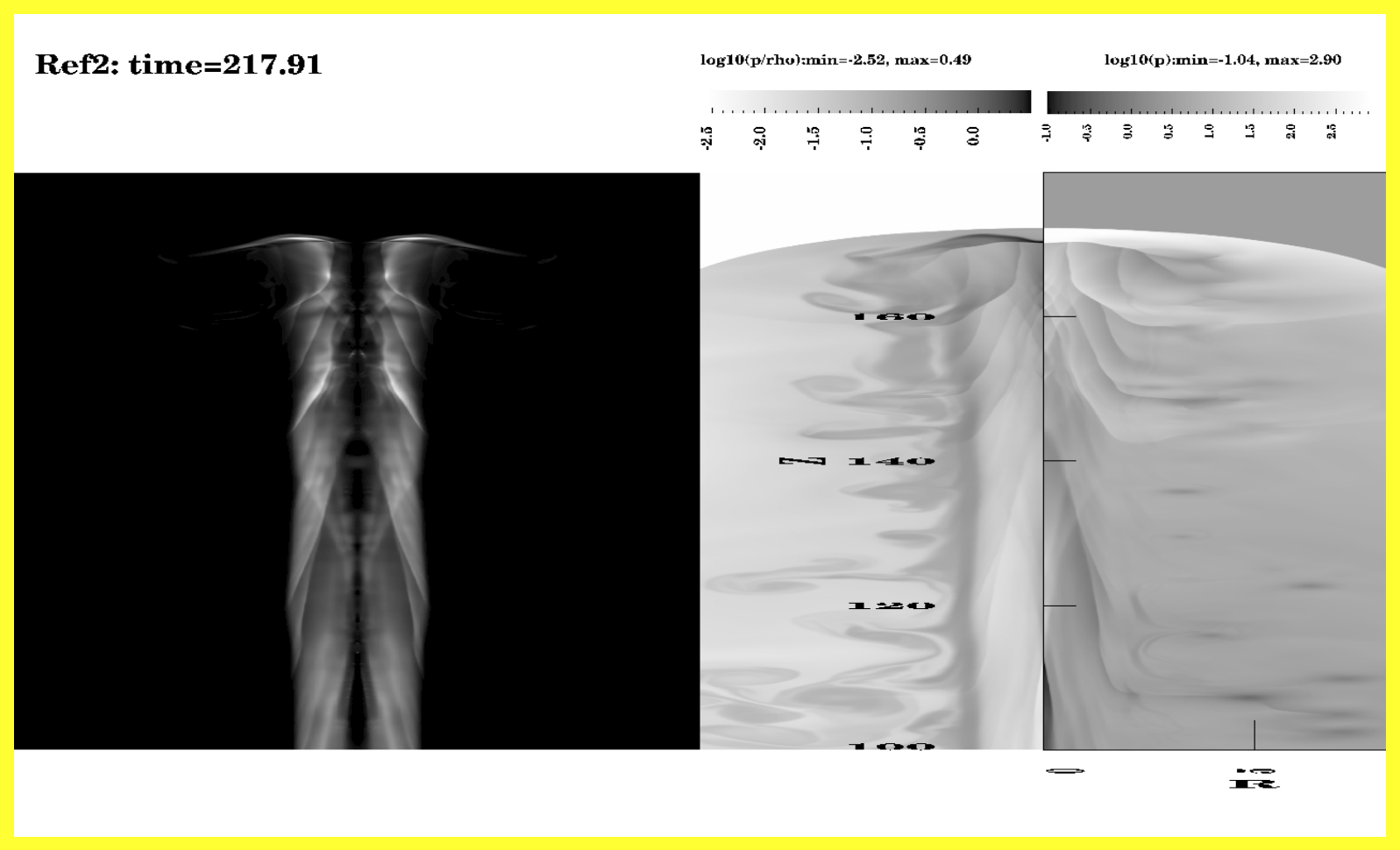
- quantified propagation characteristics

- explored transition $\bar{\Gamma} = 1.15 \rightarrow 7$
  $\Rightarrow$ non-relativistic: **strong toroidal field in cocoon**



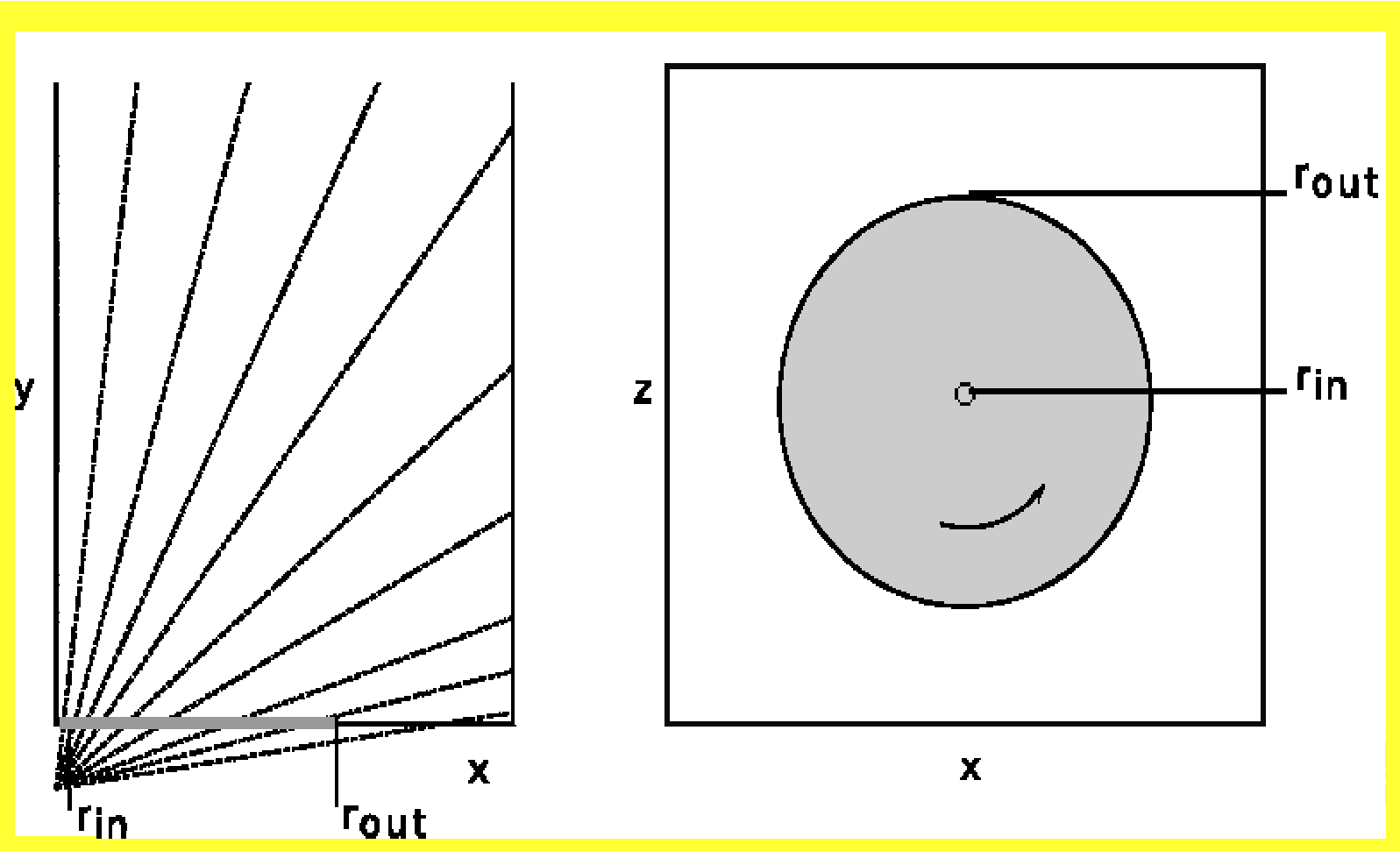


- power maps give **indication of sites of synchrotron emission**
  - ⇒ total radiation emitted is $\propto v^2 \Gamma^2 B^2 \sin^2 \Psi$
  - ⇒ varies significantly from toroidal to poloidal field cases
  - ⇒ simultaneous plots of pressure/temperature at right

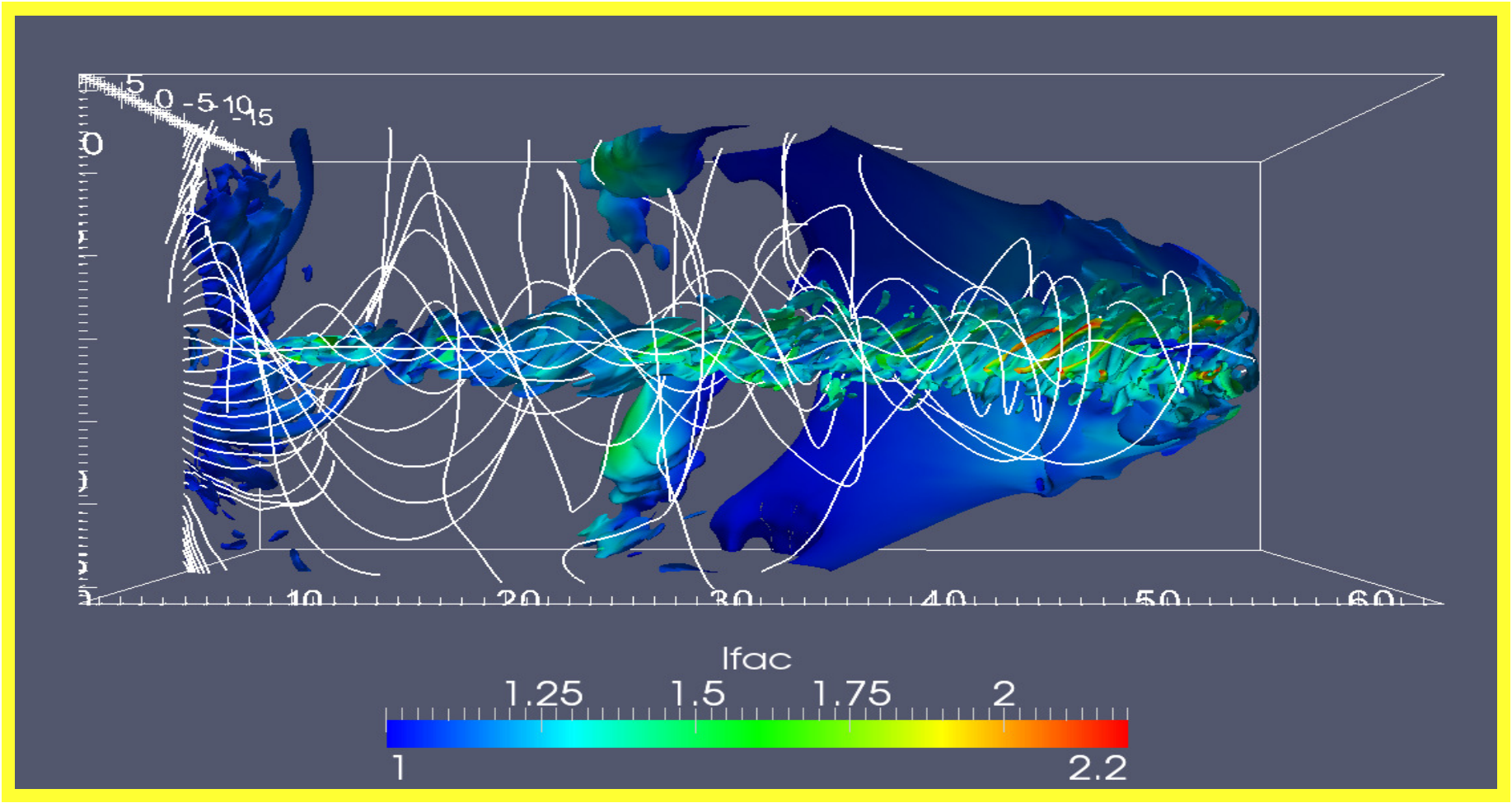


- Porth (2013): look at 3D MHD jet launch issues

  $\Rightarrow$ mimic keplerian disc corona, start from monopole-flared magnetic field

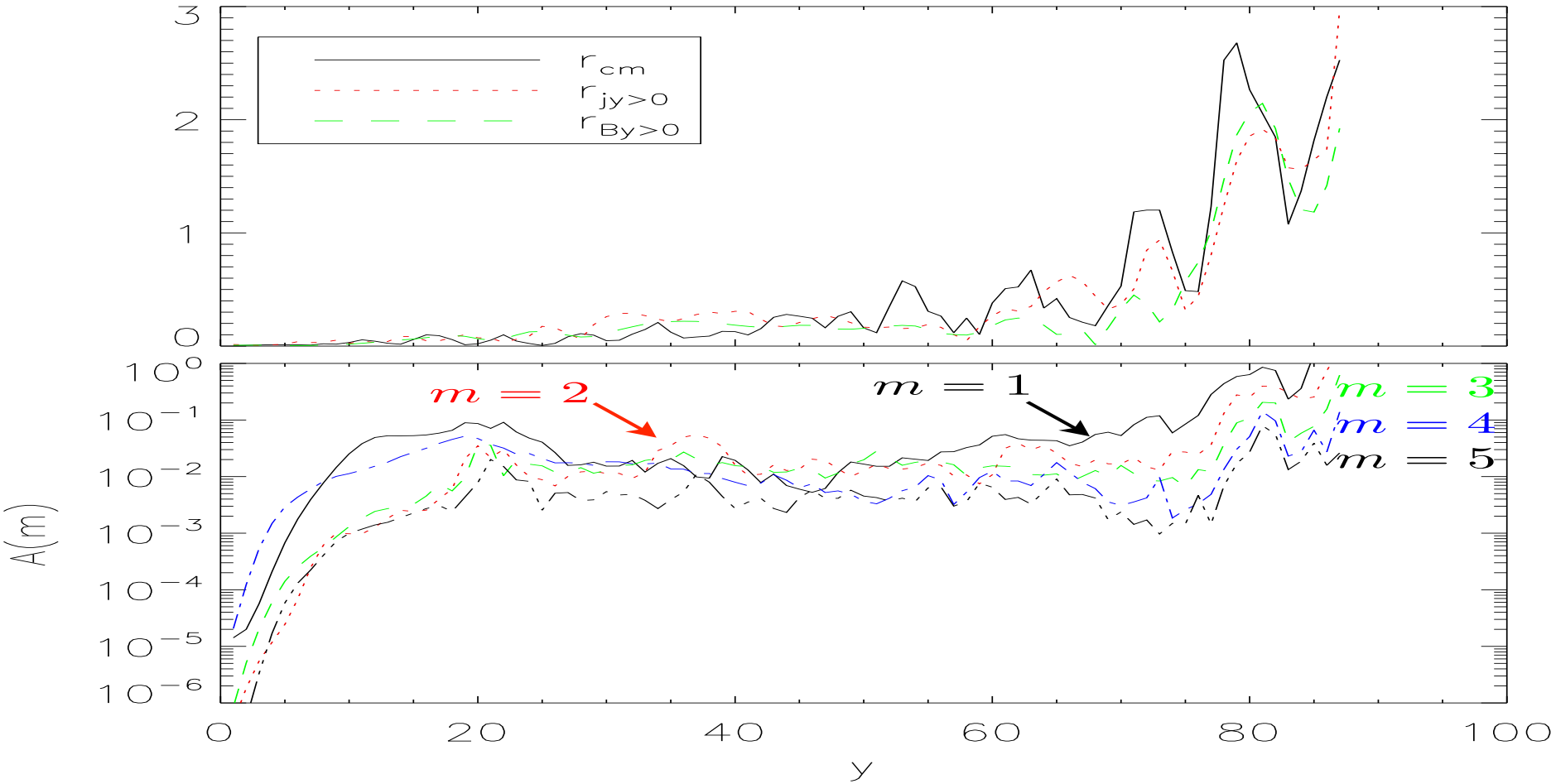


- jet self-collimates, reaches $\Gamma \simeq 2$ speeds

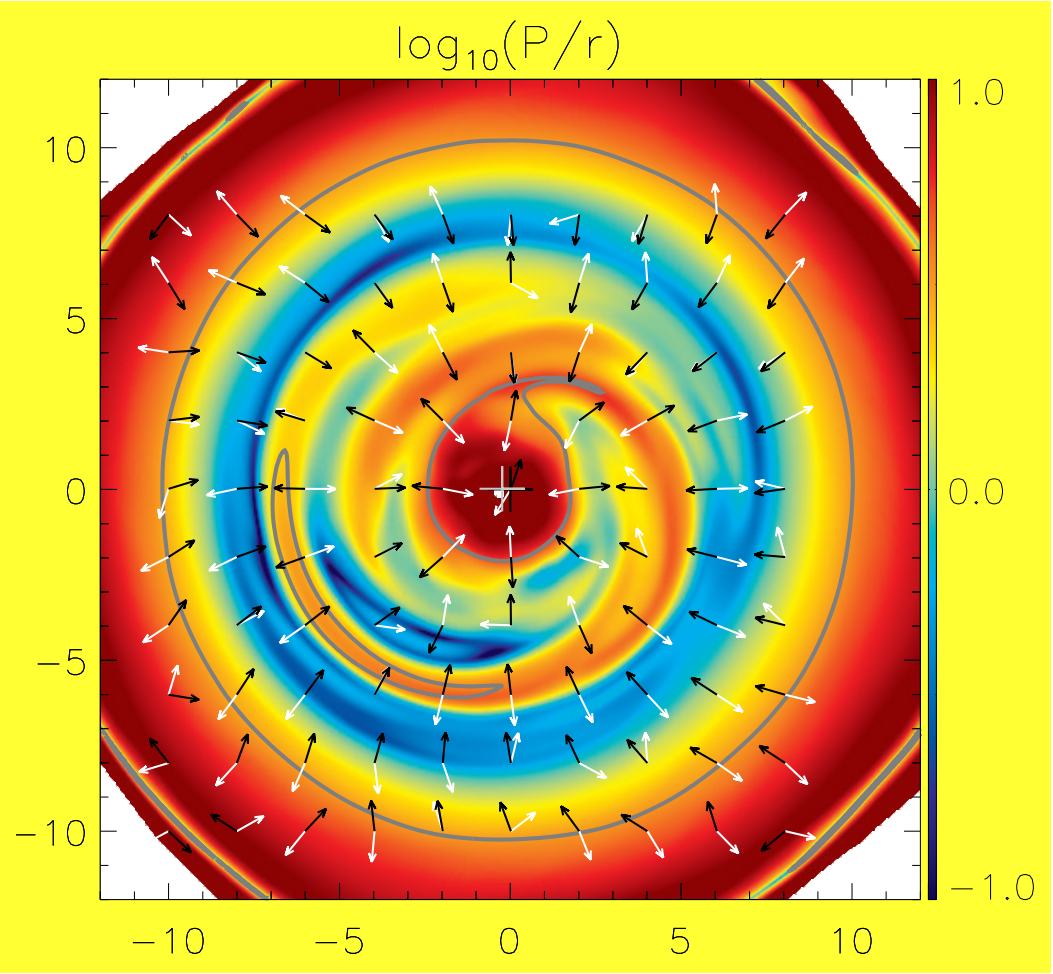


- quantify poloidal mode dominance along the jet

  $\Rightarrow$ barycenter location: axial deviations only beyond 70-80 disc radii: **self-stabilizes to kink**!

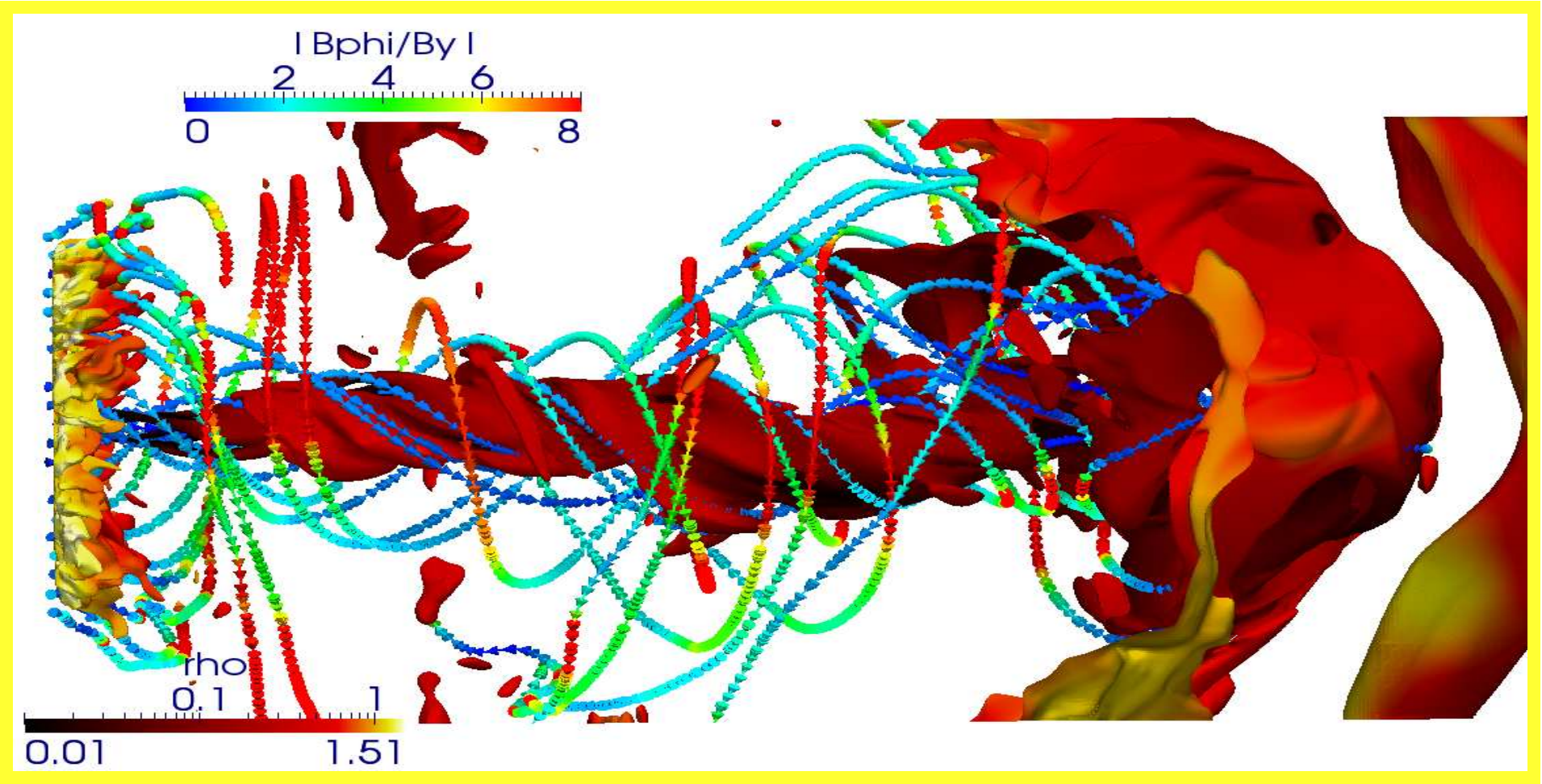


- pitch profile, and electric force (black) and Lorentz force (white)
  $\Rightarrow$ electric forces counteract magnetic contribution!

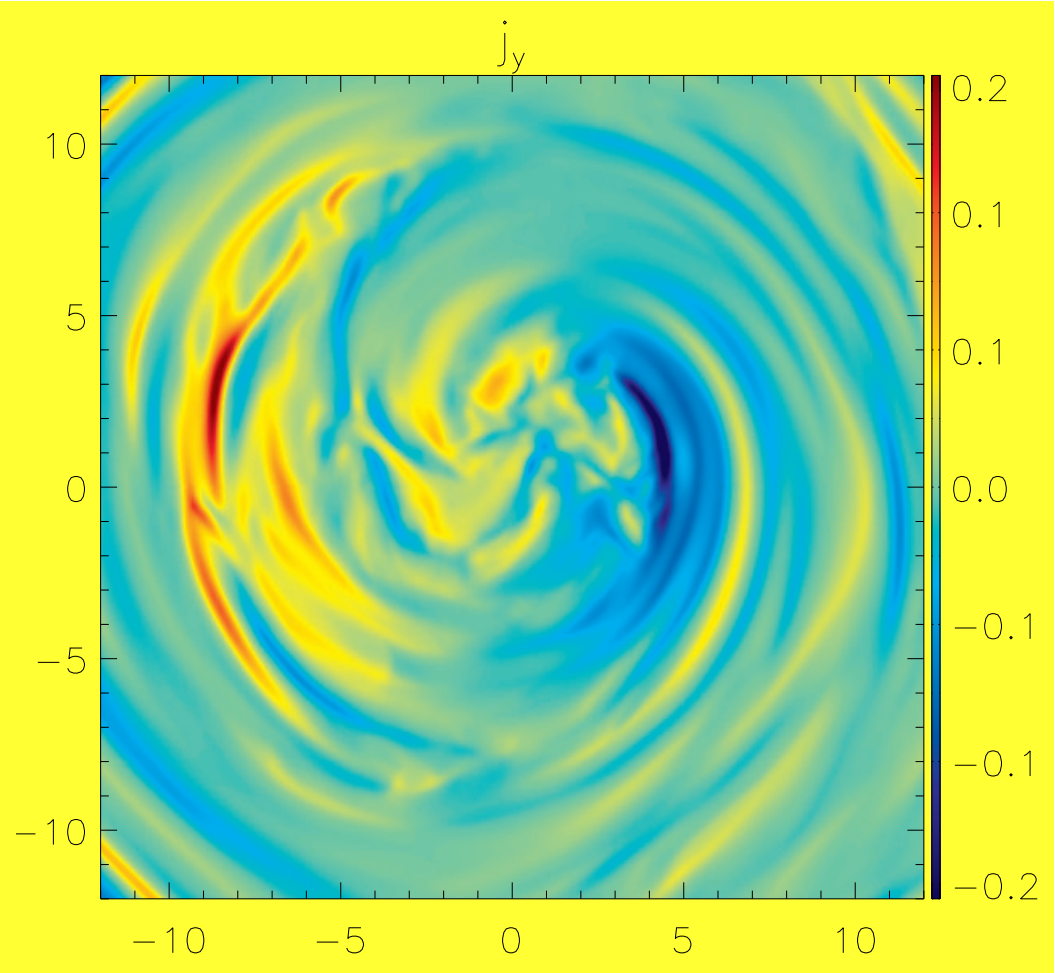


- to get significant non-axial perturbation: clumpy medium

  $\Rightarrow$ toroidal field has decreased: jet seeks path of least resistance, still kink-stable!



- filamentary current layer structure develops: particle acceleration sites & reconnection!



- *summary:* relativistic MHD models for AGN jets
  - ⇒ radial stratification: mixing in 2-component (M)HD jets (spine-sheath) due to relativistic RT
  - ⇒ helical **B**: magnetic reacceleration at cross-shocks
  - ⇒ self-consistent stabilization to kink during launch from disc
- **Related References:**
  - ⇒ Keppens & Meliani, Phys. of Plasmas 15, 102103, (2008)
  - ⇒ Keppens et al., A&A 486, 663 (2008) **A&A Highlight**
  - ⇒ Meliani & Keppens, ApJ 705, 1594 (2009)
  - ⇒ Keppens et al., JCP 231, 718 (2012)
  - ⇒ Porth, MNRAS 429, 2482 (2013)

Outlook

- relativistic MHD: relax the $v \ll c$ assumption
  - $\Rightarrow$ stressed special relativistic, ideal MHD
  - $\Rightarrow$ modern efforts: GRMHD in evolving spacetimes, ideal to resistive RMHD, extremely energetic events (magnetars, GRB engines, ...)
- applications to relativistic jets (microquasar, AGN, GRB, PWN)
  - $\Rightarrow$ synthetic observations confront reality!
- future: cross-scale challenges (reconnection and microphysics, large scale collimated and accelerated flow patterns)

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