

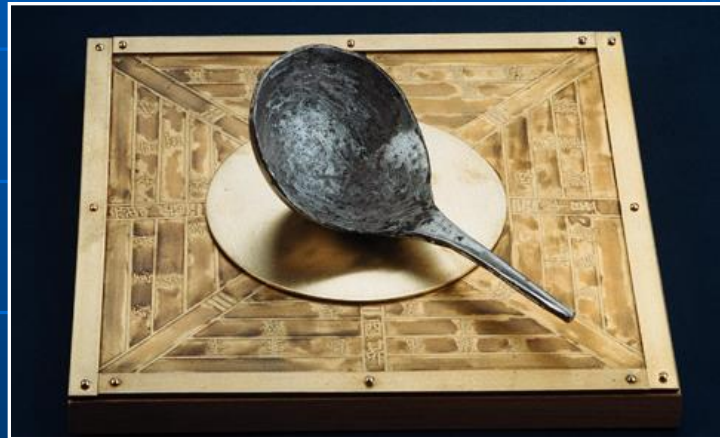
## Lecture 3:

# Measuring Cosmic Magnetic Fields

Rainer Beck, MPIfR Bonn

# History

- Magnetit ( $\text{Fe}_3\text{O}_4$ ): known since  $> 2000$  years
- Magnetism: probably named after *Magnesia* in Thessalia/Greece
- Compass: invented in China about 200 BC,  
first used by Chinese sailors about 1000 AD



James Clerk Maxwell (1864)



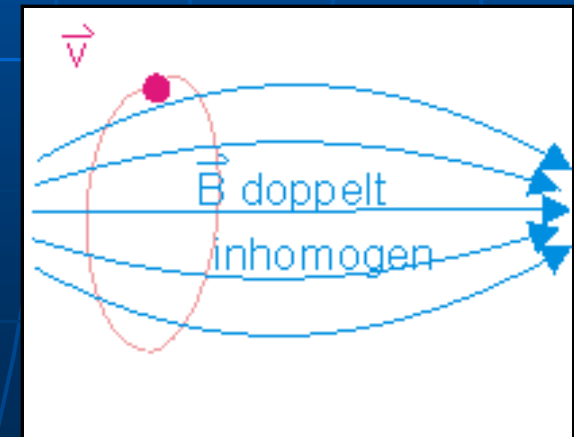
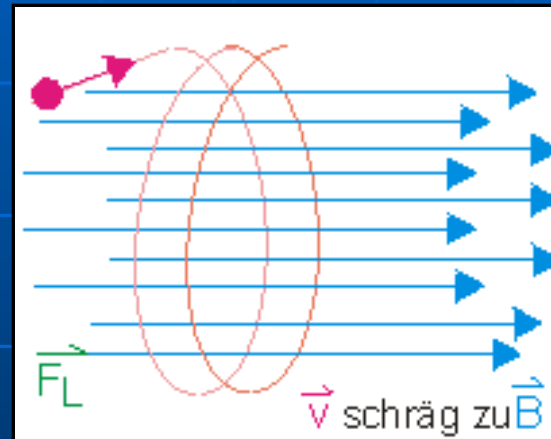
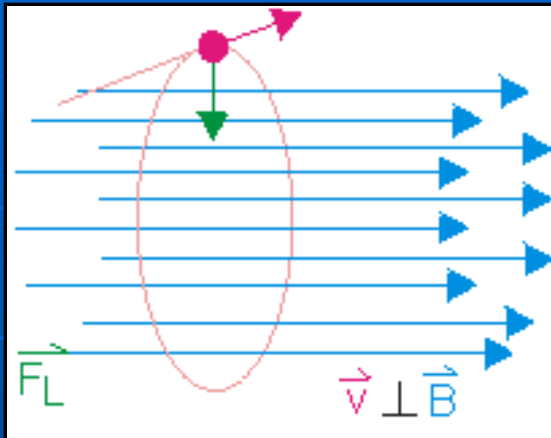
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

# Charged particles moving in magnetic fields: Lorentz force $F = q (v \times B)$

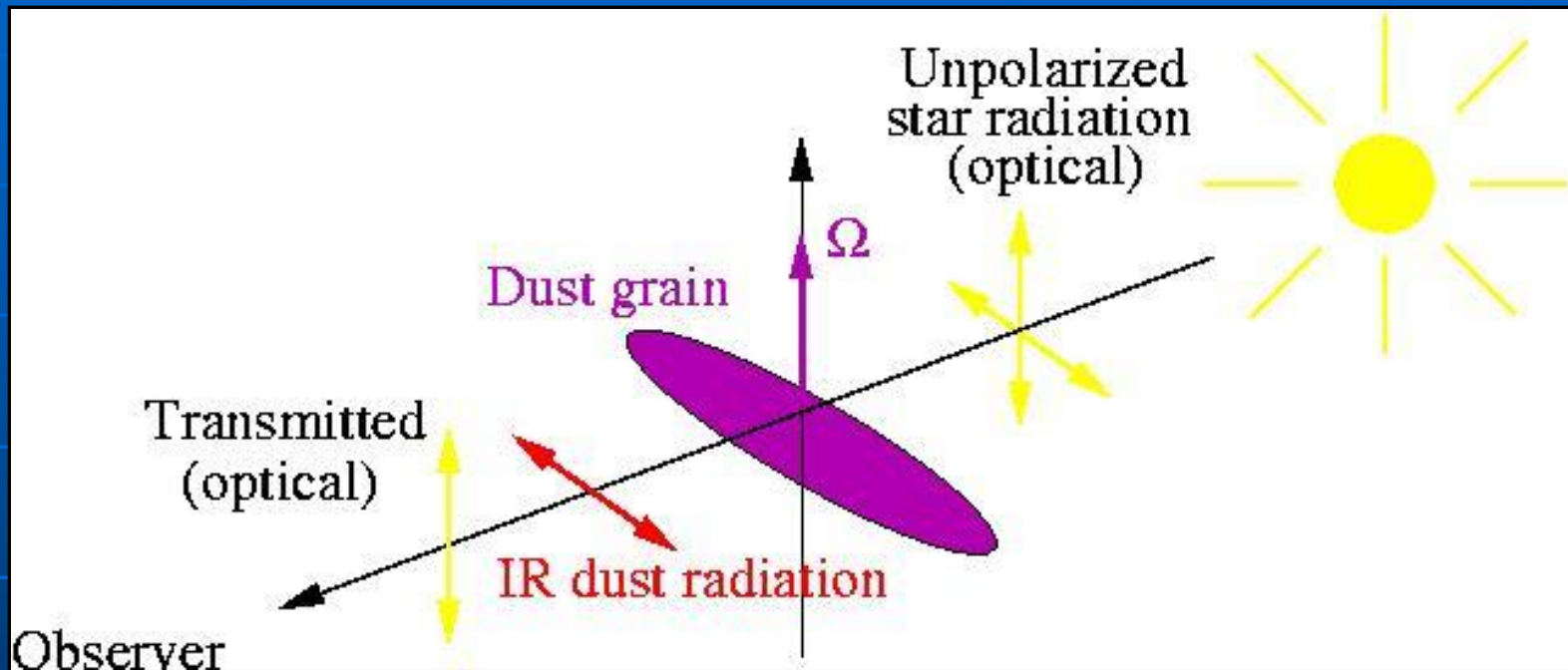


"magnetic bottle"

# Tools to study cosmic magnetic fields

- Polarization by dust grains (optical or infrared or submm continuum emission)
- Zeeman effect (line emission)
- Goldreich-Kylafis effect (line emission)
- Total synchrotron intensity (radio or optical continuum)
- Polarized synchrotron intensity (radio or optical)
- Faraday rotation of polarization plane (radio)
- Faraday depolarization (radio)

# Dust grain alignment in magnetic fields



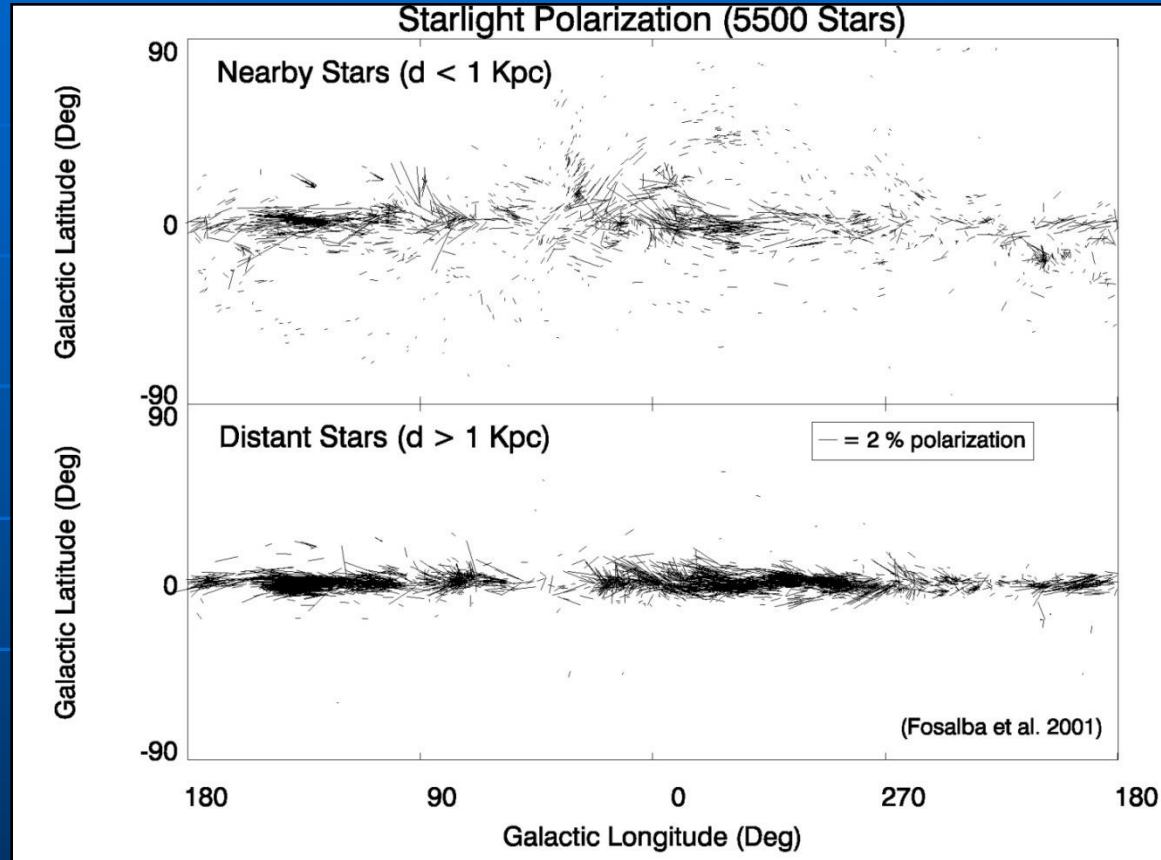
- Optical polarization:  $\parallel$  magnetic field line
- IR polarization:  $\perp$  magnetic field line

# Dust grain alignment in magnetic fields

## Physics of alignment:

- Paramagnetic dissipation of energy (very slow)  
(Davis & Greenstein 1951)
- Radiative torque alignment  
(Hoang & Lazarian 2008)

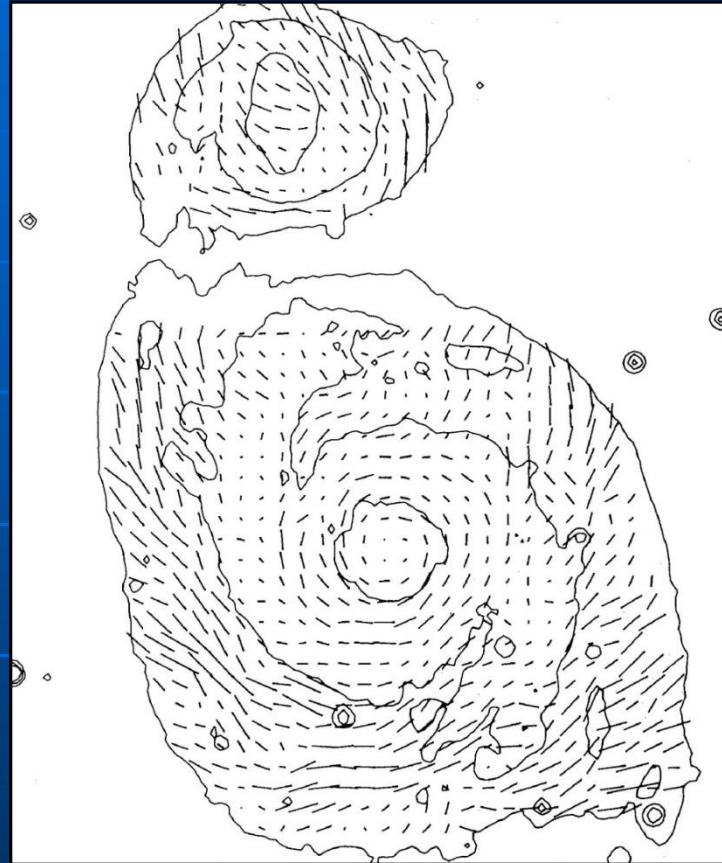
# Starlight polarization: Ordered fields ( $B_{\perp}$ )



Large-scale ordered field along the Galactic plane



# Starlight polarization in the galaxy M 51

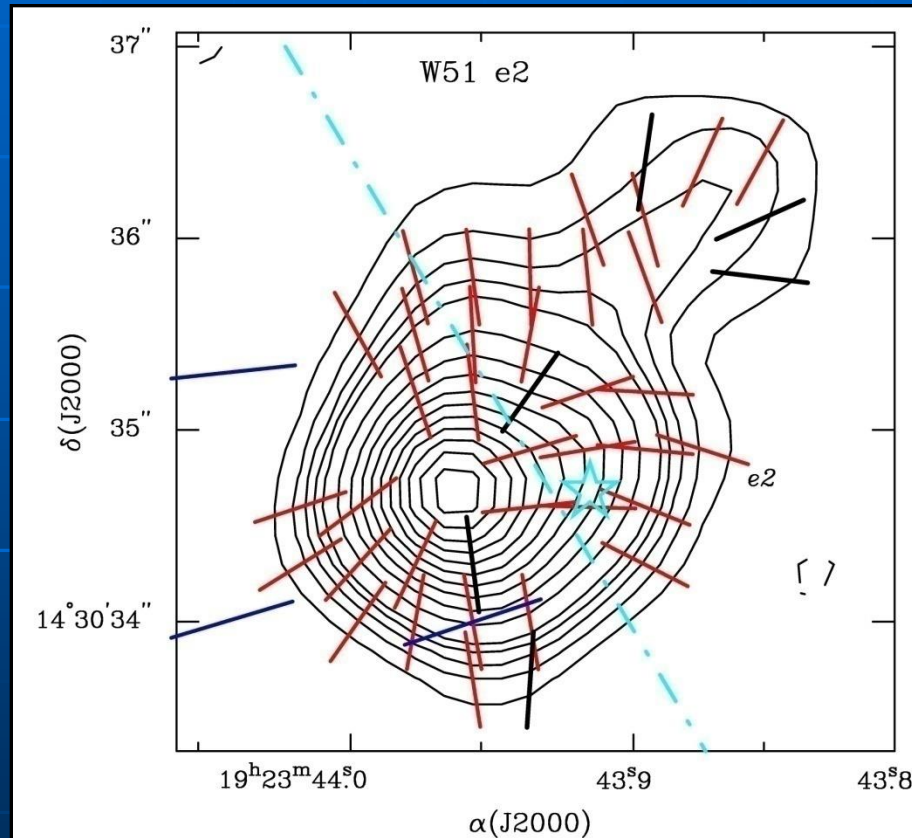


Large-scale spiral field - or scattered light ?

# Practical problems with measuring magnetic fields from optical polarization

- Contamination by polarization from scattered light
- Degrees of polarization are very small:  
bright sources are needed
- Physics of grain alignment not understood:  
field strengths can hardly be measured

# Submm polarization: Ordered magnetic fields in a molecular gas disk (SMA 870 $\mu\text{m}$ , 0.02 pc resolution)



X-shaped field:  
Ambipolar diffusion?

# Practical problems with measuring magnetic fields from infrared/submm polarization

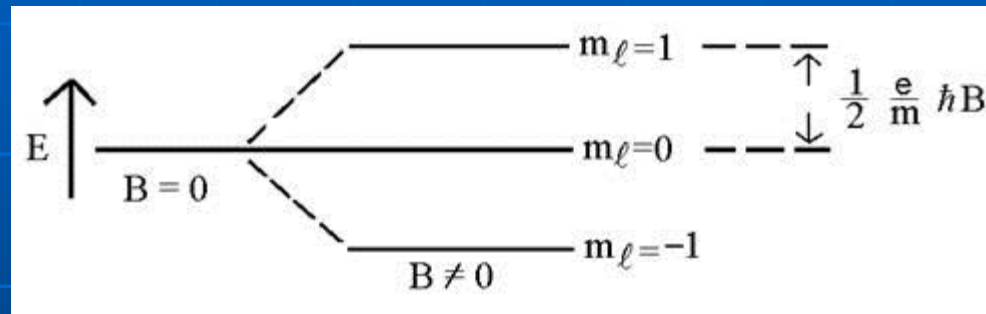
- Degrees of polarization are low: bright sources are needed
- Molecular clouds are small: high angular resolution needed
- Physics of grain alignment not understood: field strengths can hardly be measured

# Chandrasekhar-Fermi method (1953)

- Competition between magnetic force and turbulent gas motions in the interstellar medium ( $v$ ) with density  $\rho$
- Strong field: straight field lines, small dispersion of polarization vectors ( $\alpha$ )
- The dispersion  $\alpha$  allows to estimate the field strength  $B$ :  
$$B \approx (4/3 \pi \rho)^{1/2} v / \alpha$$
- Improved method: correct for errors in polarization angle, signal integration (Hildebrand et al. 2009, Houde et al. 2009, 2011)
- Applied also to radio polarization vectors (Houde et al. 2013)

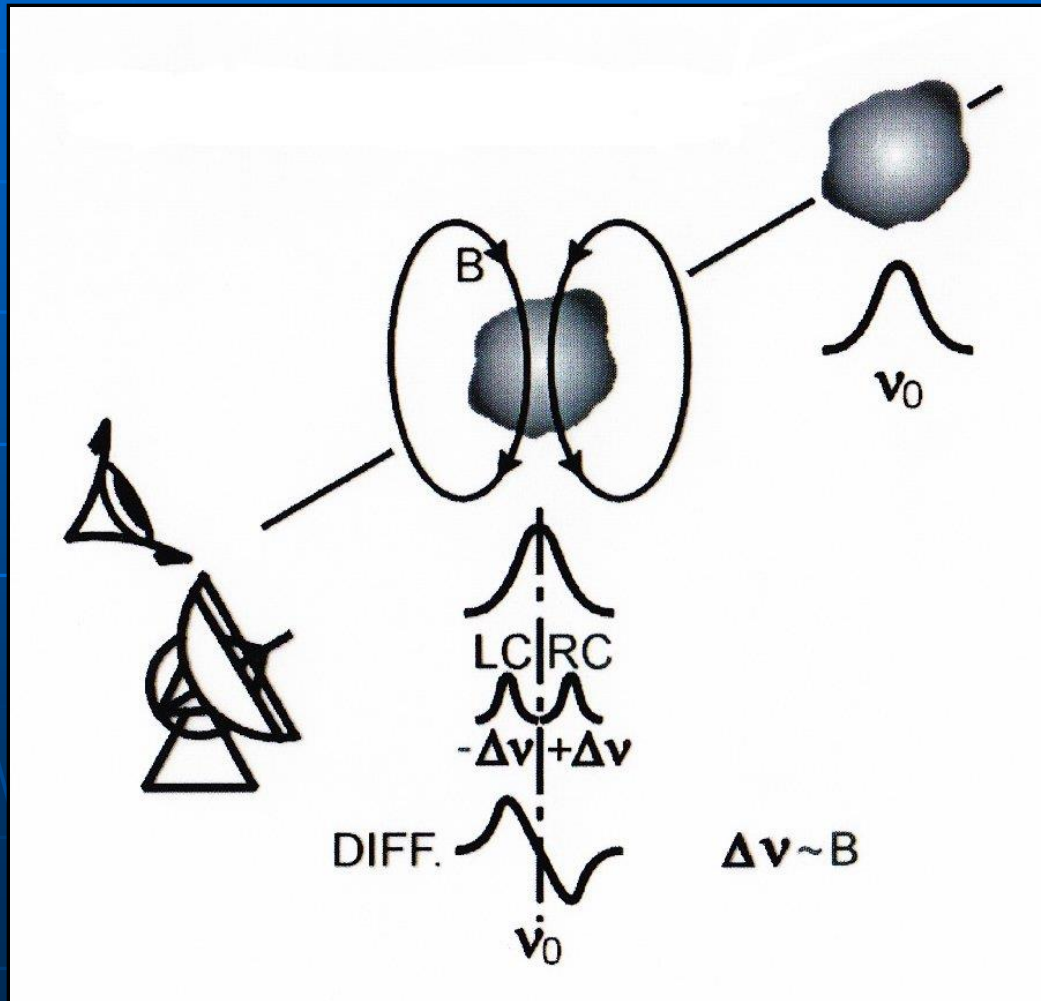
# Zeeman effect

- Splitting of spectral lines by a magnetic field into three components ( $-\sigma$ ,  $\pi$ ,  $+\sigma$ ):

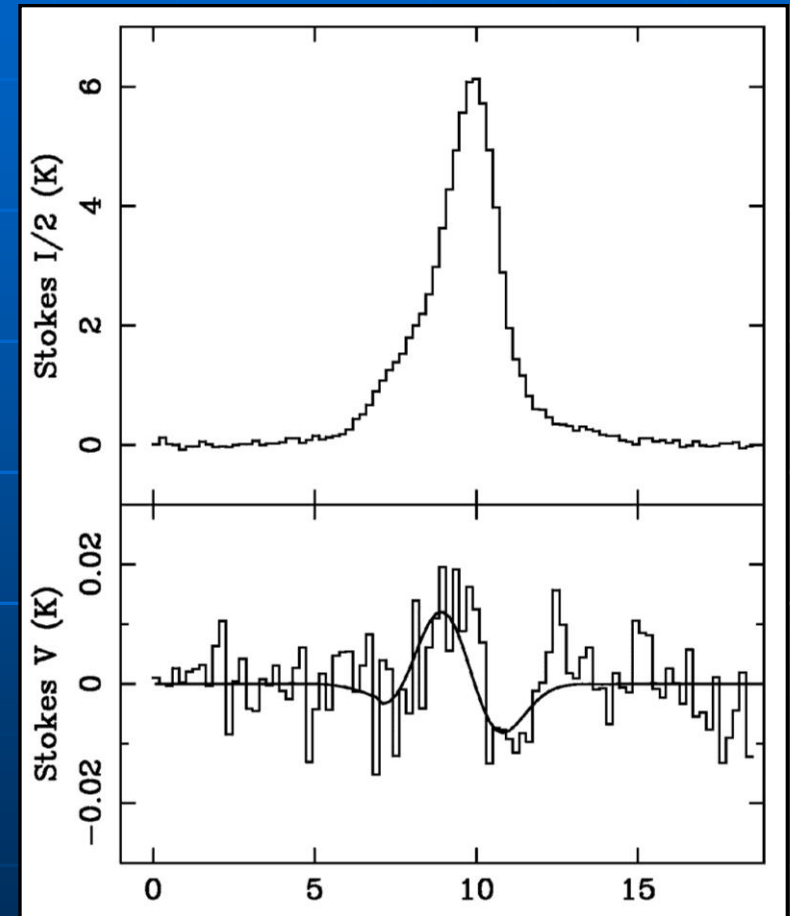
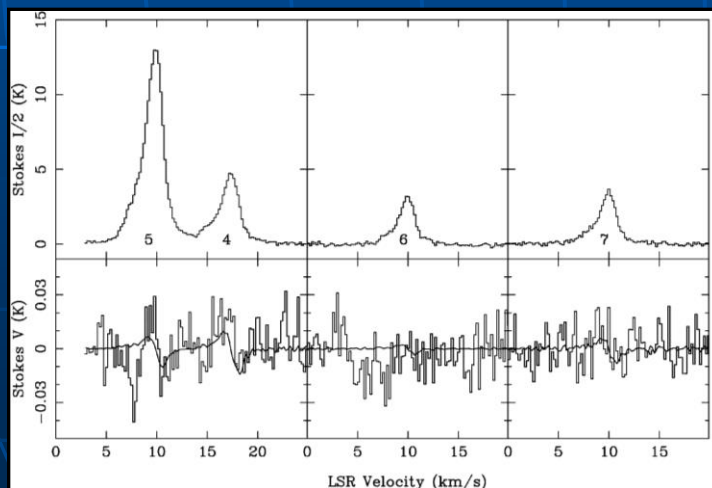
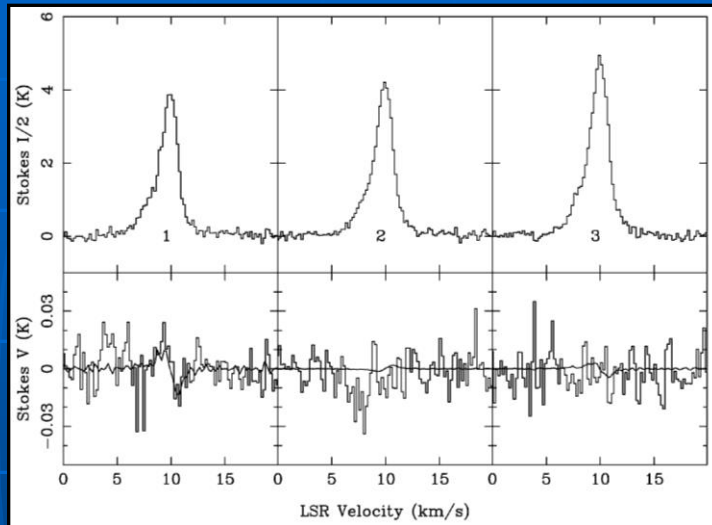


- Line of sight parallel to  $B$  (longitudinal Zeeman effect): two  $\sigma$  lines (circularly polarized)
- Line of sight perpendicular to  $B$  (transversal Zeeman effect): two  $\sigma$  lines + one  $\pi$  line (all linearly polarized)

# Zeeman effect (longitudinal): Ordered fields ( $B_{\parallel}$ ) in gas clouds



# Orion MC1: CN(1-0) Zeeman (113 GHz, 7 hyperfine lines)

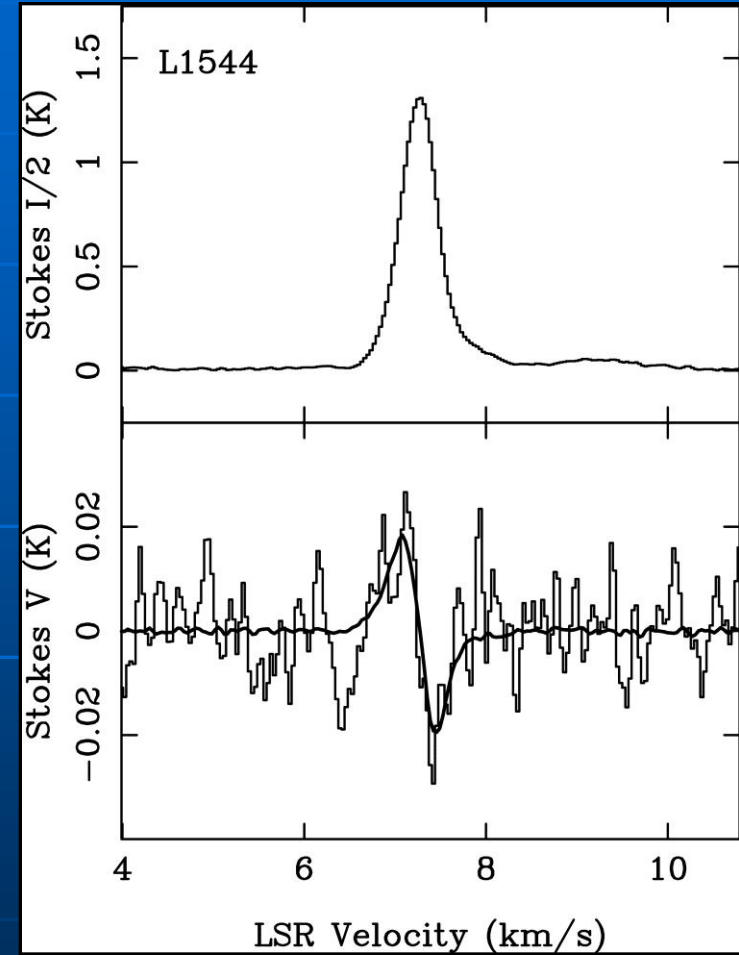
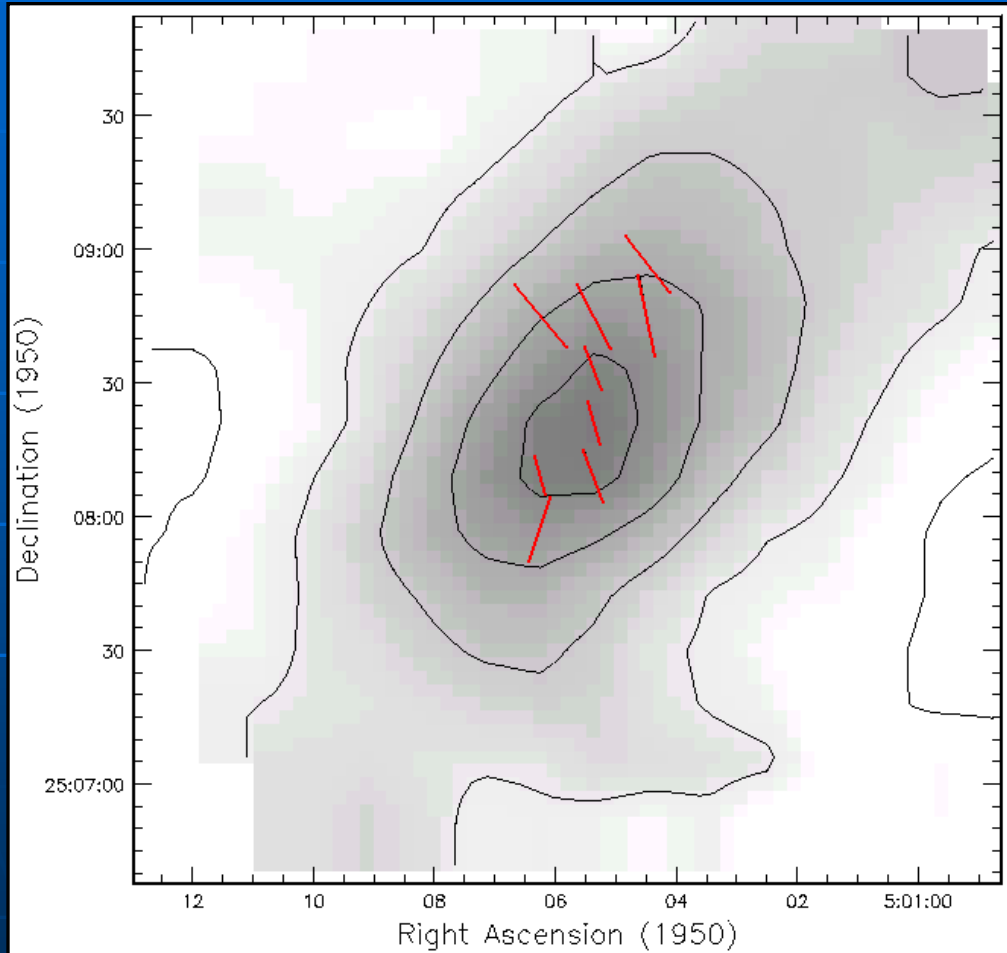


$$B_{\parallel} = -0.36 \pm 0.08 \text{ mG}$$

(Crutcher et al. 1999)



# L1544 Starless Core: Dust pol (850 $\mu\text{m}$ ) and OH Zeeman (1.67 GHz)



$n(\text{H}_2) \approx 5 \times 10^5 \text{ cm}^{-3}$ ,  $\mathbf{B}_\perp \approx 140 \mu\text{G}$   
(Crutcher et al. 2004)

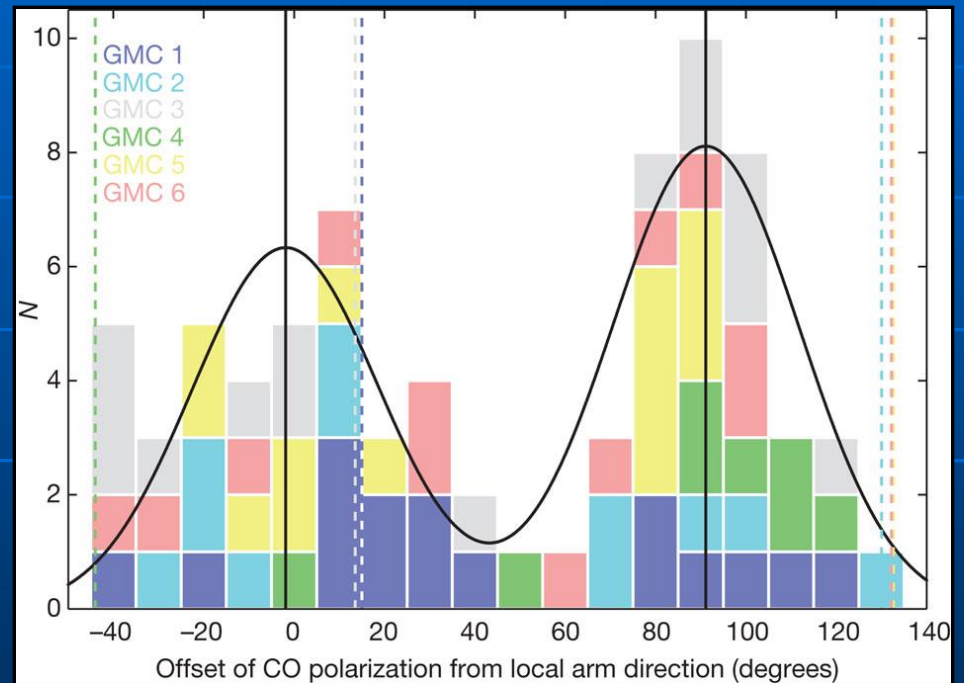
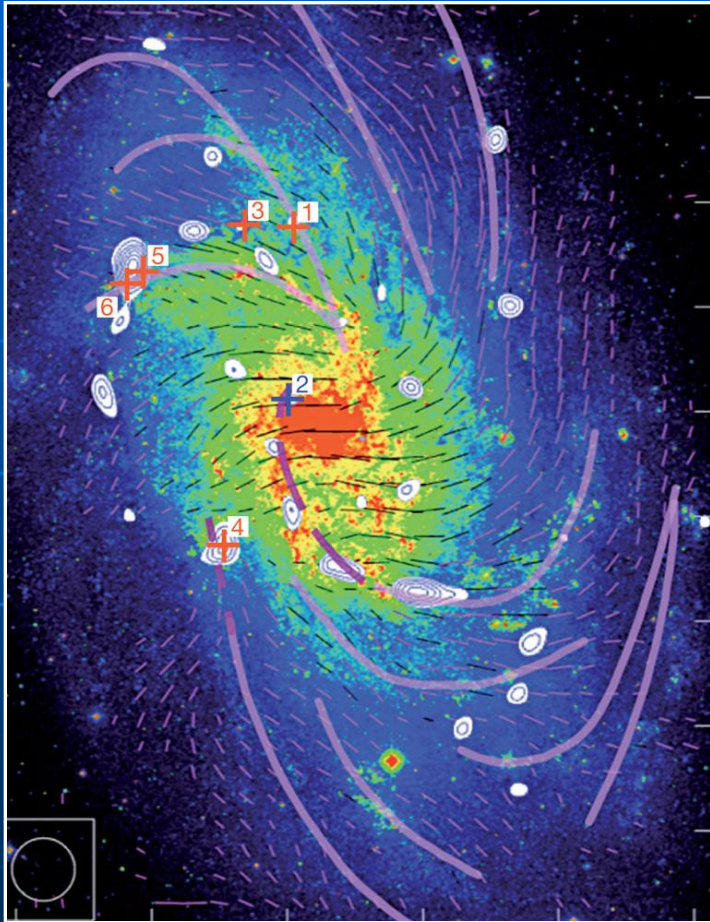
$n(\text{H}_2) \approx 1 \times 10^4 \text{ cm}^{-3}$ ,  $\mathbf{B}_\parallel = +11 \mu\text{G}$   
(Crutcher & Troland 2000)

# Goldreich-Kylafis effect (1981)

- Line of sight perpendicular to  $B$  (transversal Zeeman effect): two  $\sigma$  lines + one  $\pi$  line (linearly polarized)
- Symmetric conditions: no polarization detectable
- **Detection of linearly polarized lines becomes possible for:**
  - *Unequal populations of the different sublevels*
  - *Gradient in optical depth or velocity*
  - *Anisotropic velocity field*
- **Linear polarization can be parallel or perpendicular to the magnetic field orientation**
- Detections: molecular clouds, star-forming regions, outflows of young stellar systems, supernova remnants

# Goldreich-Kylafis effect: CO lines + synchrotron polarization in M 33

(Li & Henning 2011)

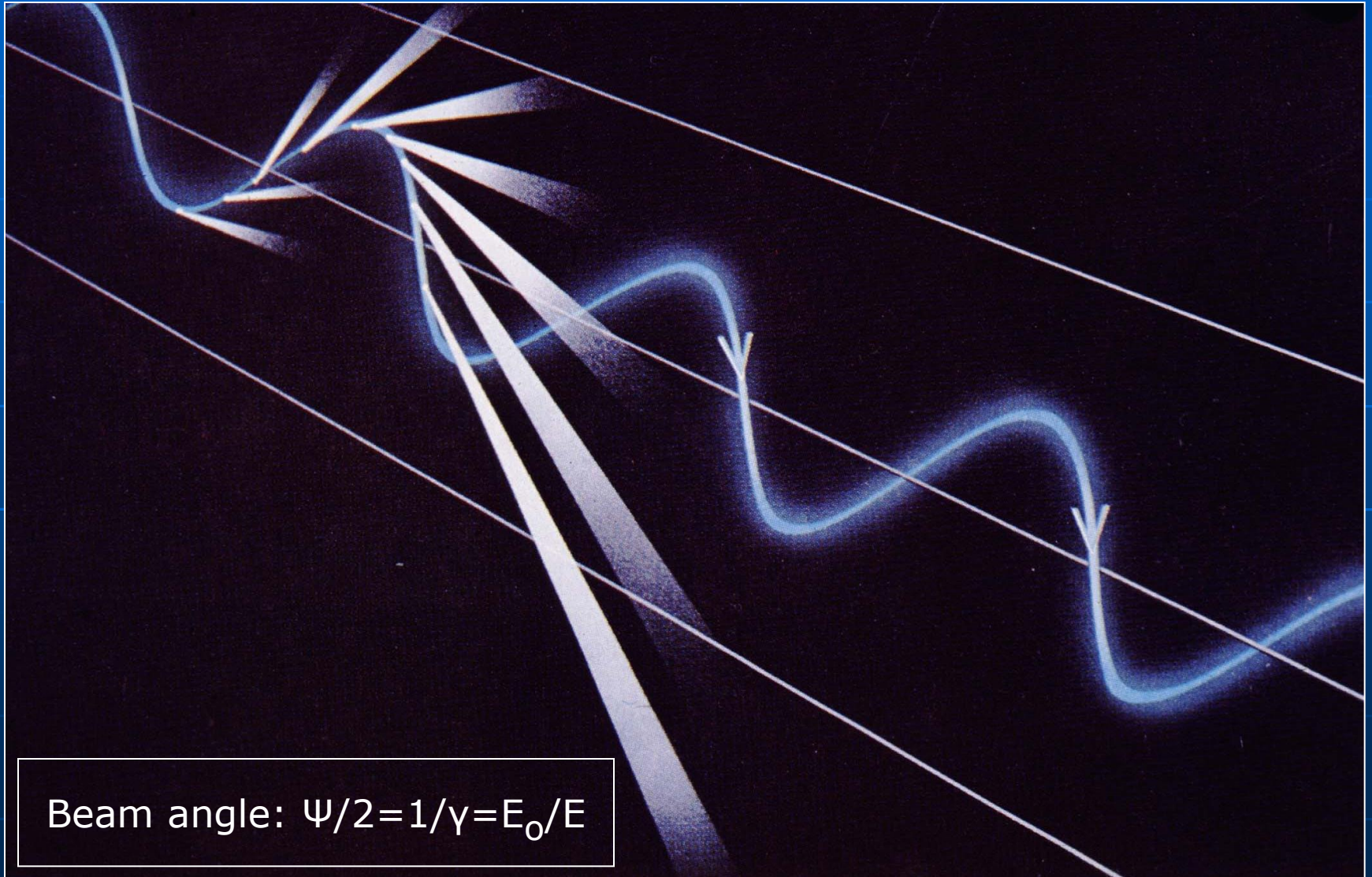


Magnetic fields in the molecular clouds  
are aligned with the spiral arms

# Practical problems with measuring Zeeman splitting

- Weak effect: low instrumental polarization required
- HI line traces only diffuse (warm) gas
- Other suitable lines (e.g. OH, CN) are much weaker
- Goldreich-Kylafis effect: magnetic field orientation is ambiguous by  $\pm 90^\circ$

# Synchrotron emission: Magnetic fields illuminated by cosmic-ray electrons

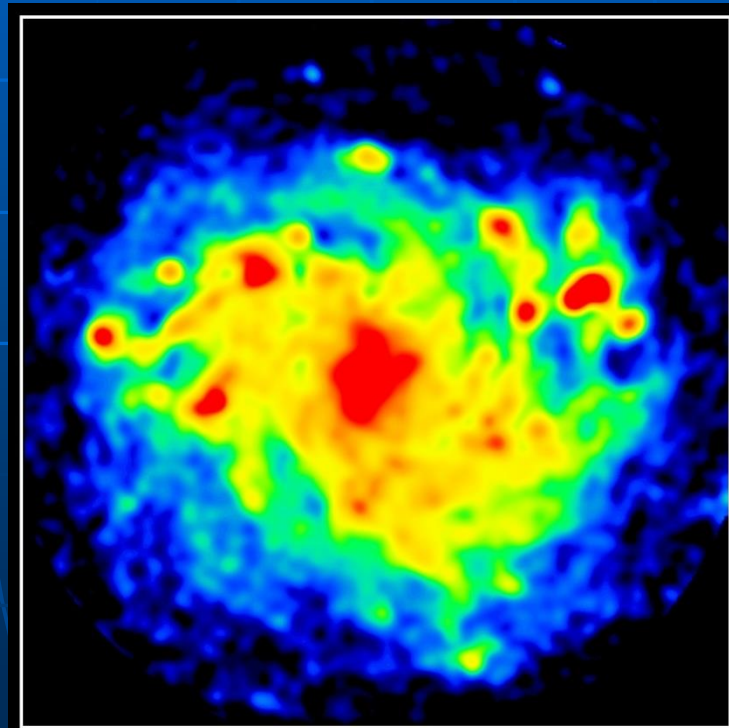


# Practical problems with measuring synchrotron emission

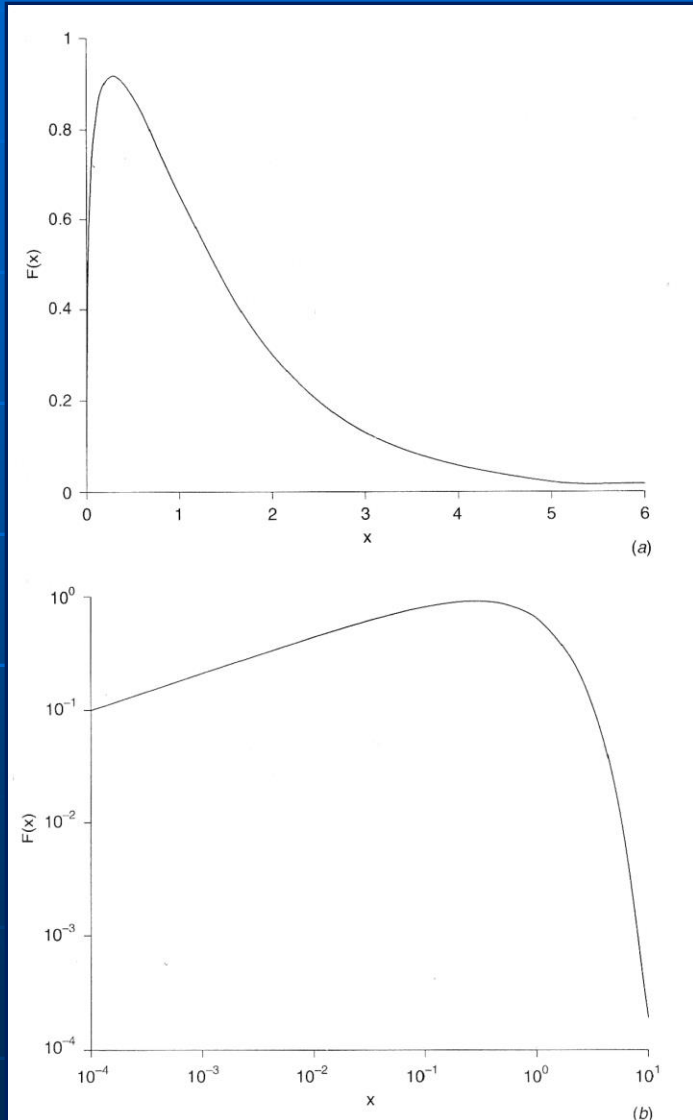
- High-energy cosmic-ray electrons needed
- Thermal emission needs to be subtracted

Total synchrotron emission (NGC 6946):

*Tracer of total magnetic fields*



# Synchrotron emission



Intensity spectrum of  
a single cosmic-ray electron  
(linear and logarithmic scale)

Maximum emission:

$$\nu_{\max} \approx 5 \text{ MHz } E [\text{GeV}]^2 B_{\perp} [\mu\text{G}]$$



# Synchrotron emission

- Cosmic-ray electrons:

Power-law energy spectrum with spectral index  $\epsilon_e$

$$N(E) dE = N_0 E^{-\epsilon_e} dE$$

- Intensity of synchrotron spectrum:

$$I_\nu = c_5(\epsilon_e) \int N_0 B_\perp^{(\epsilon_e+1)/2} (\nu/2c_1)^{-(\epsilon_e-1)/2} dL$$

- Synchrotron spectral index:

$$\alpha = (\epsilon_e - 1)/2$$

# Energy spectra of cosmic rays

Diffusive shock acceleration:

$$\epsilon \geq 2$$

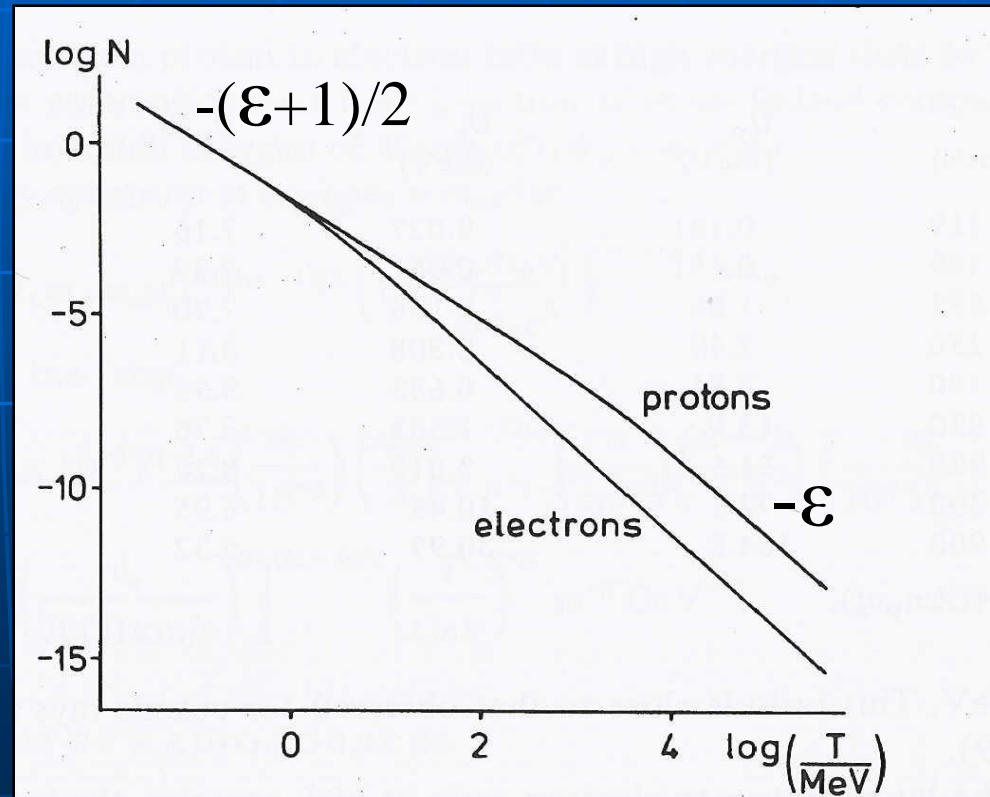
$$\alpha \geq 0.5$$

$E > 1$  GeV:

$$K = (m_p/m_e)^{(\epsilon-1)/2}$$

$$\epsilon \approx 2.2: K \approx 90$$

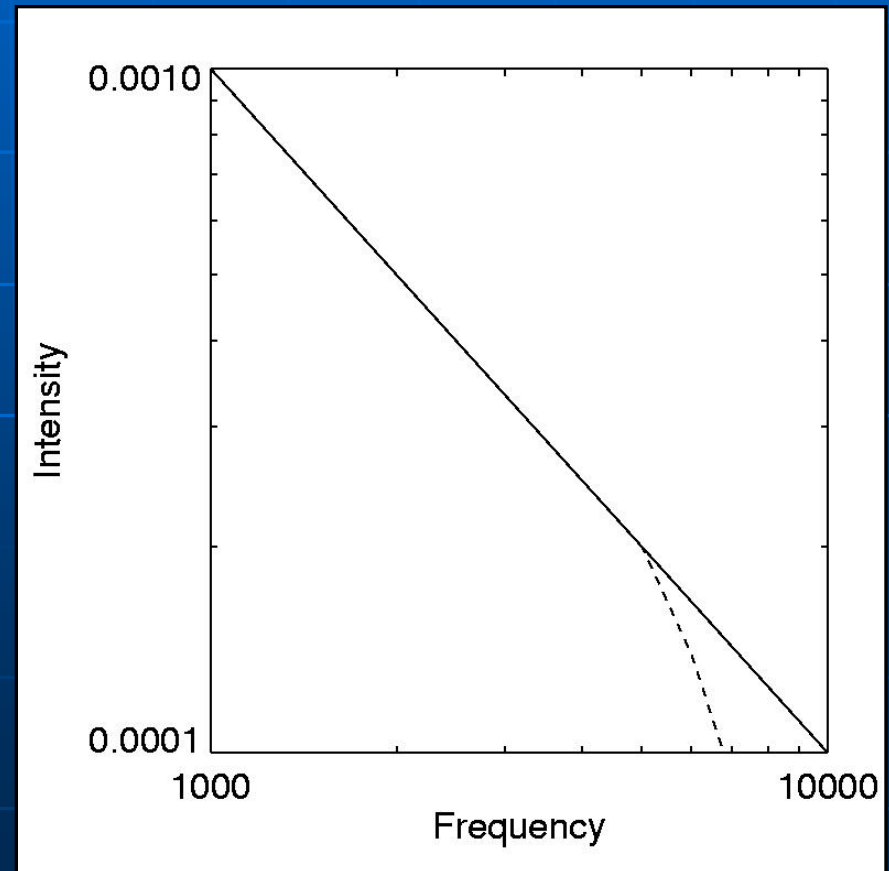
Bell 1978



# Lifetime of synchrotron-emitting electrons

$$t_{\text{syn}} \approx 1 \text{ Gyr } B_{\perp} [\mu\text{G}]^{-1.5} \nu_{\text{syn}} [\text{GHz}]^{-0.5}$$

Synchrotron spectrum  
steepens above a critical  
frequency

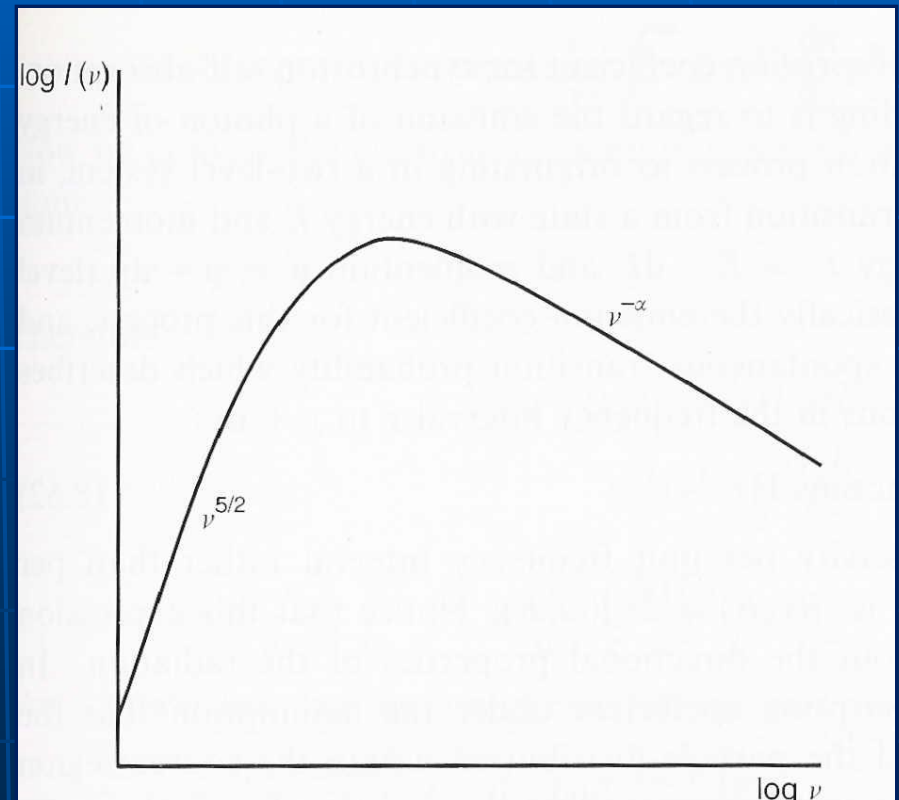


# Synchrotron self-absorption

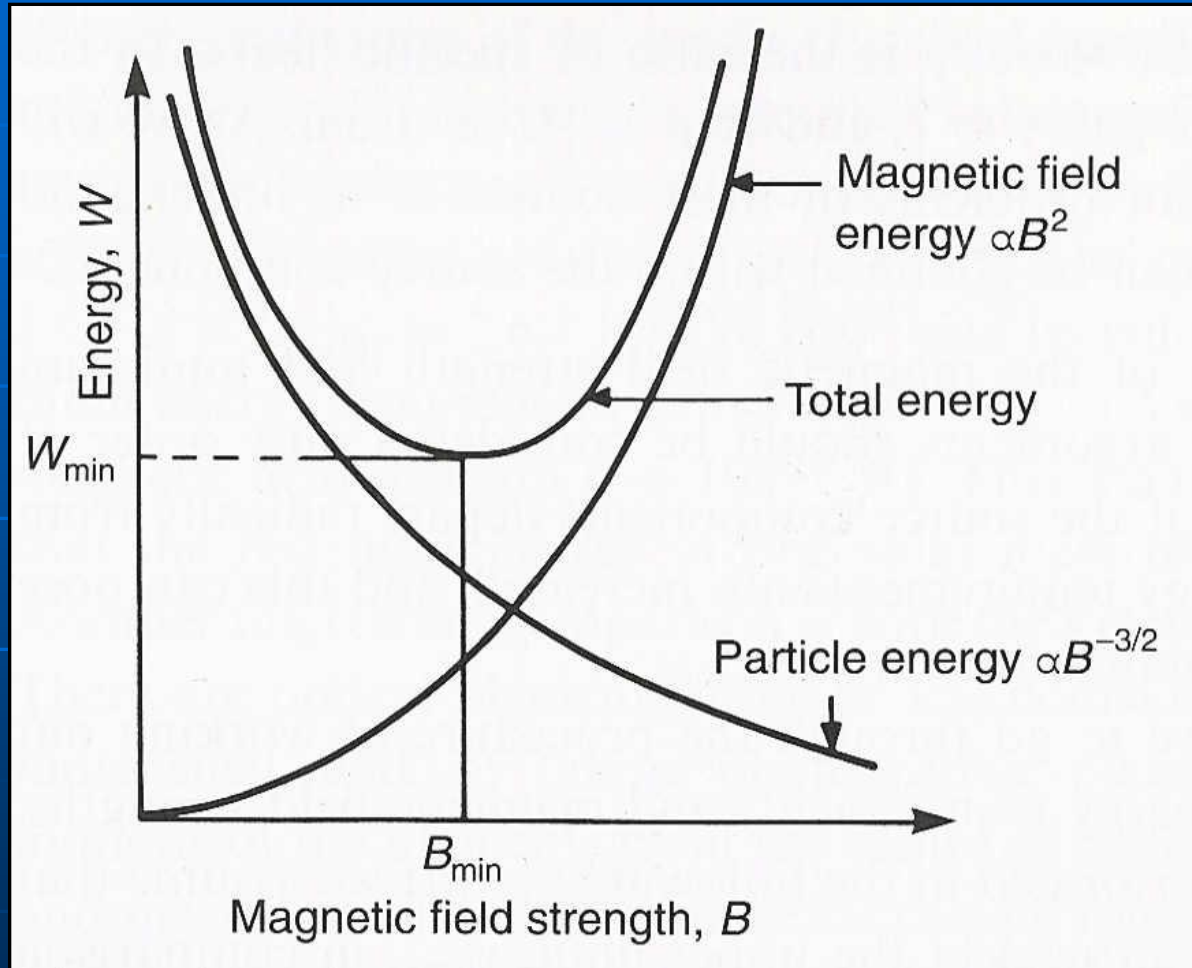
$$\nu_{sa} \approx 34 \text{ MHz } (S[\text{Jy}] / \theta[\text{arcsec}]^2)^{2/5} B_{\perp} [\text{G}]^{1/5}$$

Measurement of field strength

Observable in bright,  
compact sources



# Minimum-energy field strength



# Energy equipartition formula

$$B_{\text{eq},\perp} \propto \left( I_{\text{sync}} (K+1) / L \right)^{1/(3+\alpha)}$$

$B_{\text{eq},\perp}$ : Strength of the equipartition field in the sky plane

$I_{\text{sync}}$ : Synchrotron intensity

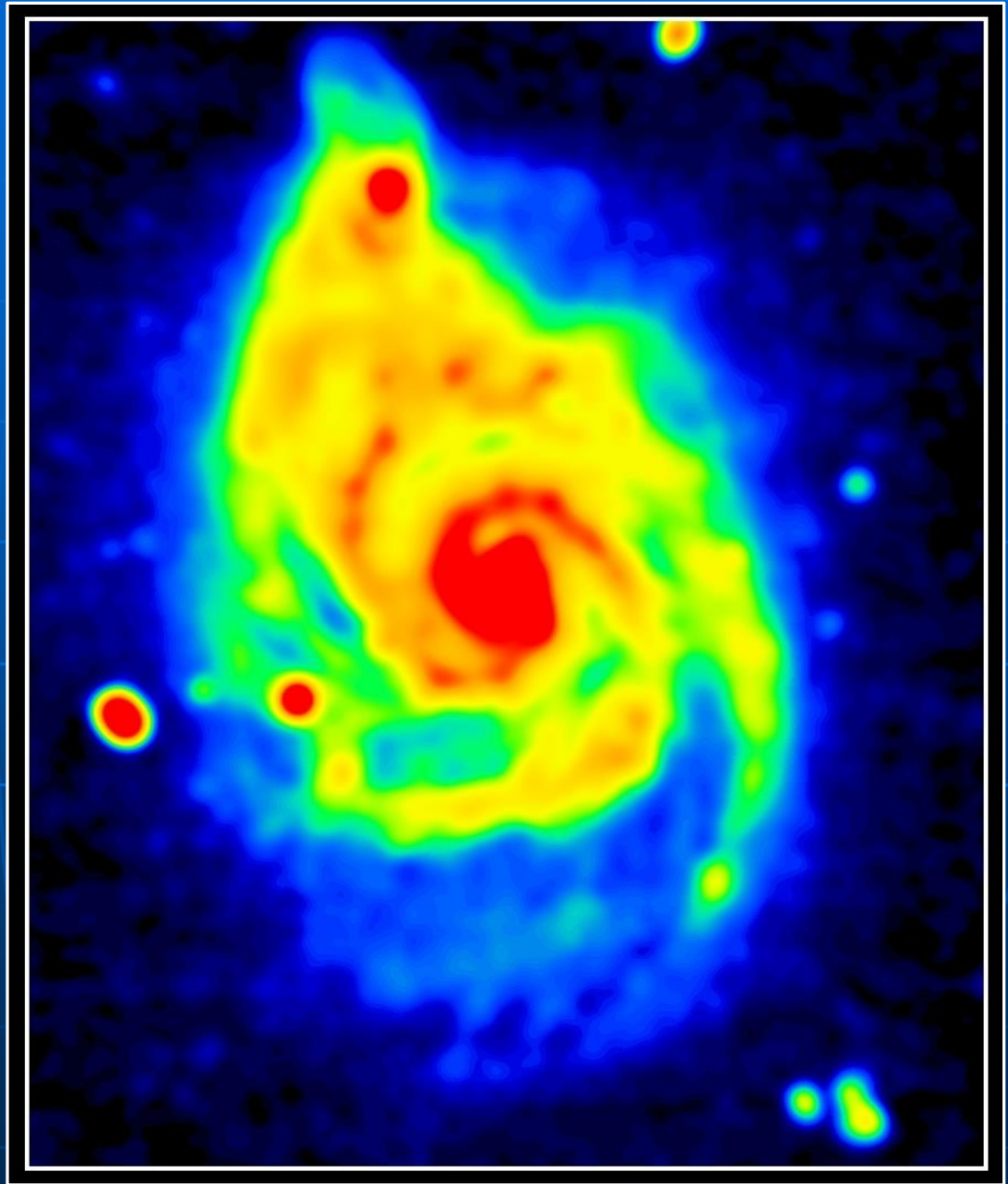
$K$ : Proton/electron ratio

$L$ : Pathlength through source

$\alpha$ : Slope of the synchrotron spectrum (spectral index)

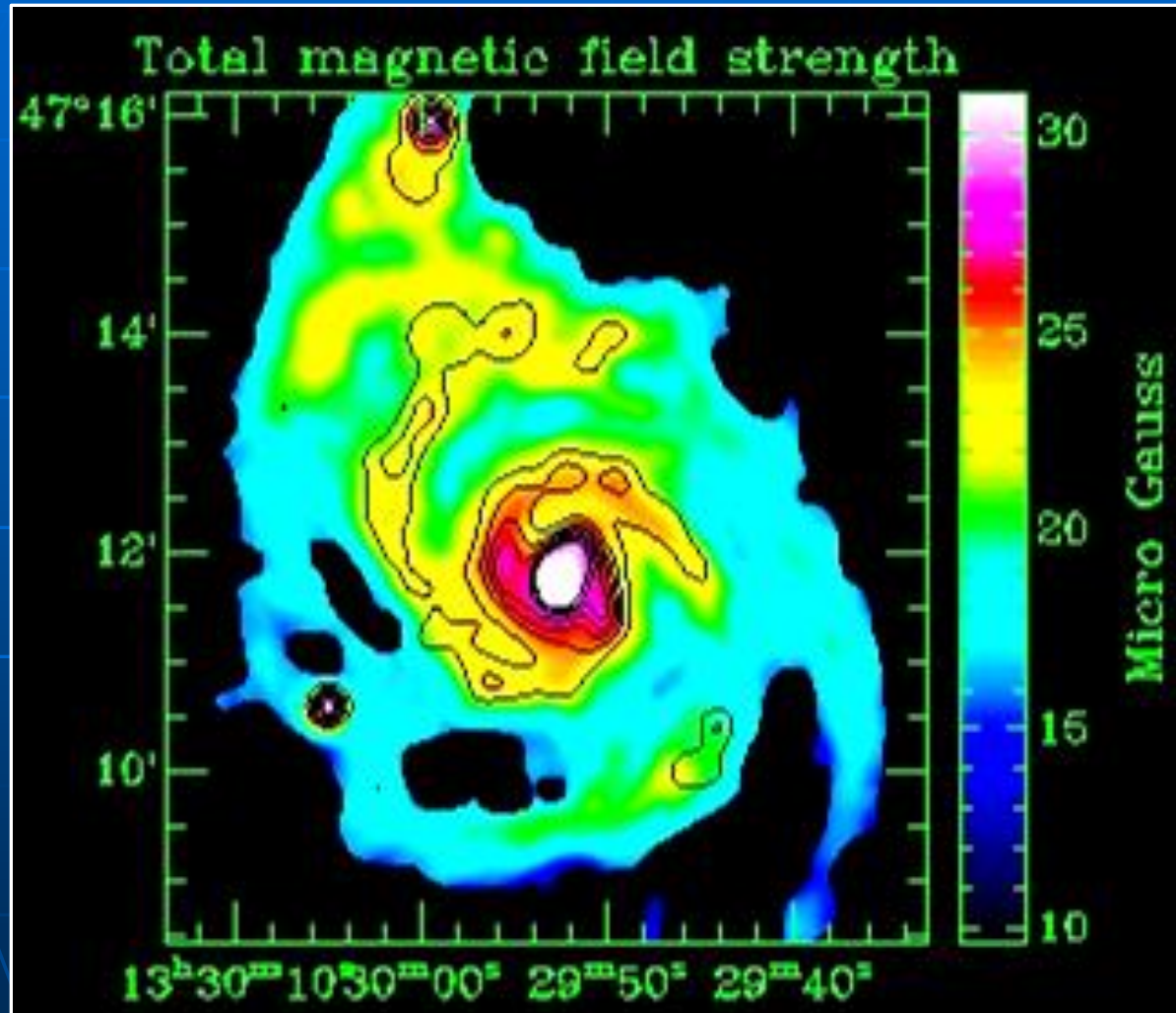
# M 51

VLA 20 cm  
Total intensity



Fletcher et al. 2011

# Equipartition field strengths in M 51



Fletcher et al. 2011



# Equipartition field strengths in galaxies

Average total field in spiral galaxies: **5 – 15  $\mu\text{G}$**

Total field in spiral arms: **20 – 30  $\mu\text{G}$**

Total field in starburst galaxies: **40 – 100  $\mu\text{G}$**

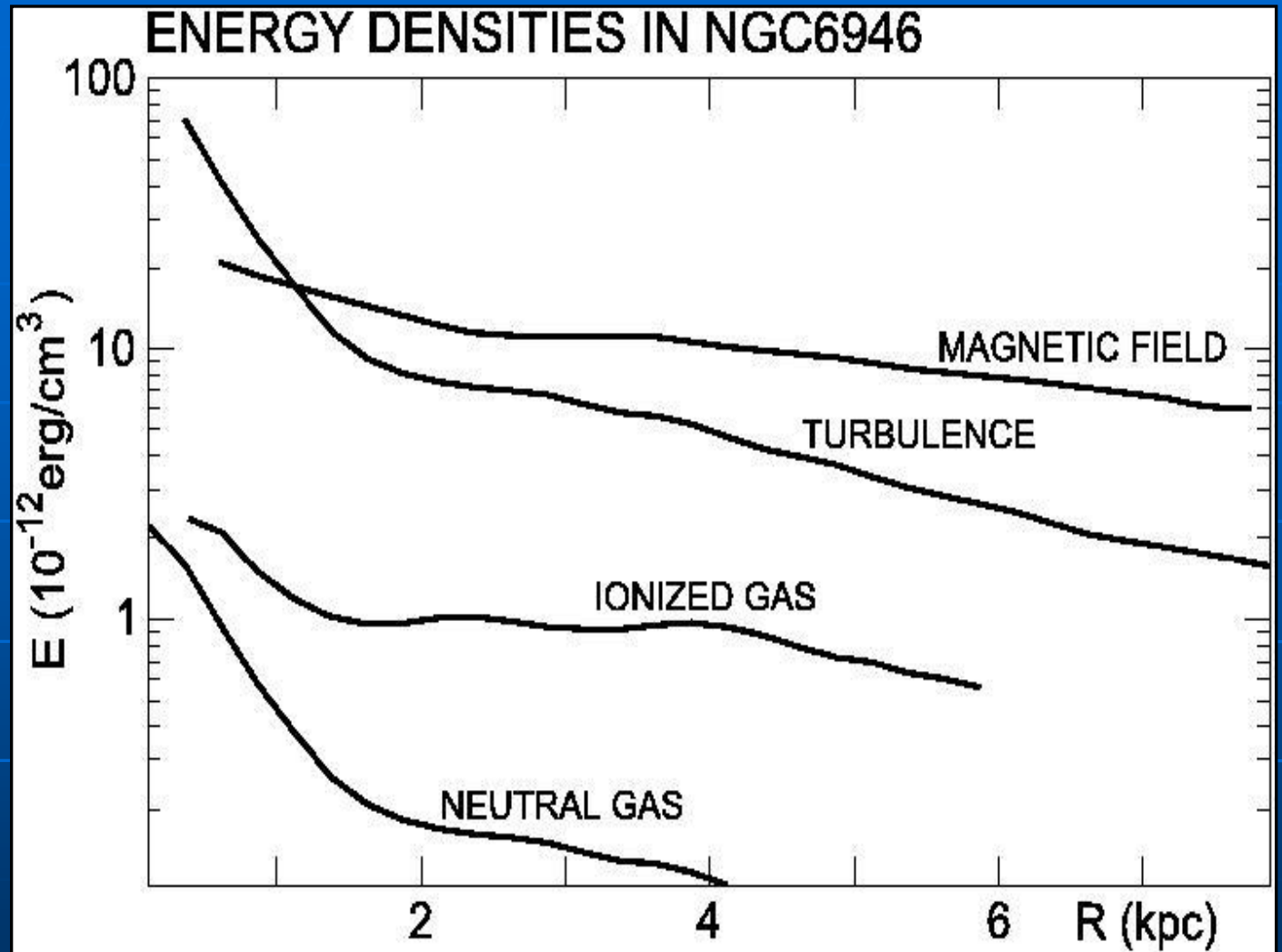
*Note: Field strengths may be underestimated due to energy losses of cosmic-ray electrons*

# Energy densities

- $E_{\text{magn}} = B^2 / 8 \pi$
- $E_{\text{turb}} = 1/2 n v_{\text{turb}}^2$
- $E_{\text{therm}} = 3/2 n k T$

$V_{\text{turb}} \approx 7 \text{ km/s}$   
 $T_e \approx 10^4 \text{ K}$   
 $T_n \approx 50 \text{ K}$

Beck 2007



The energy density of the total magnetic field is roughly in equipartition with that of the turbulent cloud motions

# Practical problems with measuring magnetic field strengths with the equipartition assumption

- Equipartition may be violated on small time scales and/or spatial scales
- Cosmic ray energy dominated by protons, but only electrons are observable in the radio range
- Energy spectra of protons and electrons can be different
- Needed: independent data on cosmic-ray proton spectrum (e.g. from  $\gamma$  rays)

# Linear polarization of synchrotron emission

- Tracer of **ordered magnetic fields** in the sky plane ( $B_{\perp}$ )
- B-vector: oriented along the magnetic field line (if no Faraday rotation occurs)
- **Intrinsic degree of linear polarization** (if no depolarization occurs):

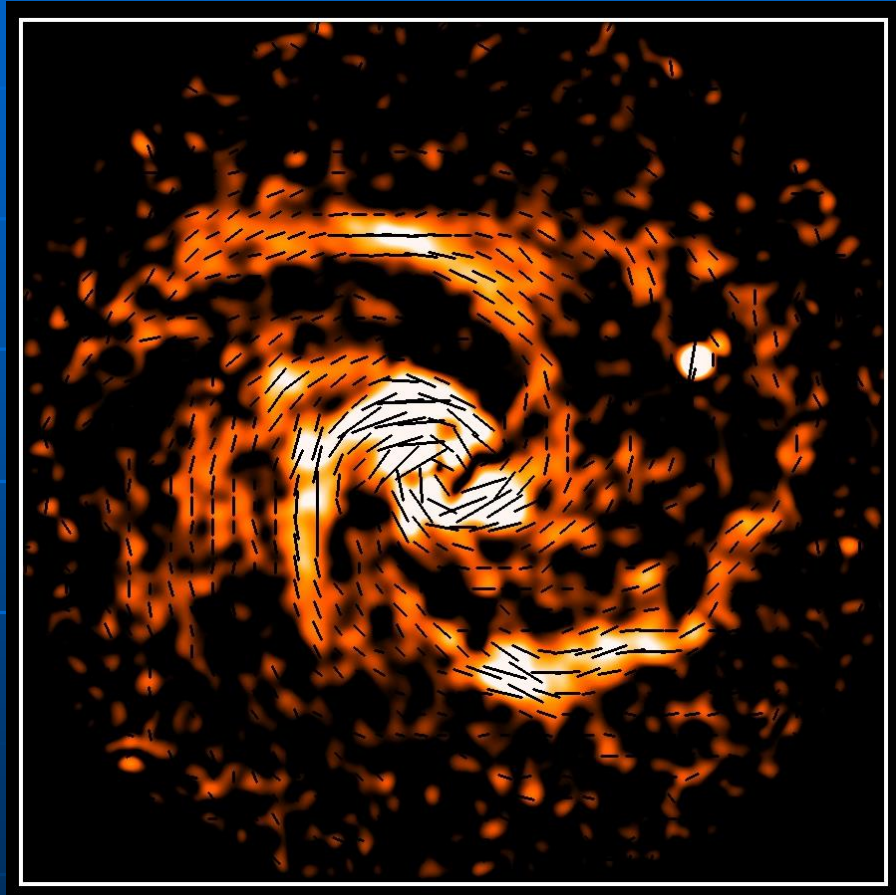
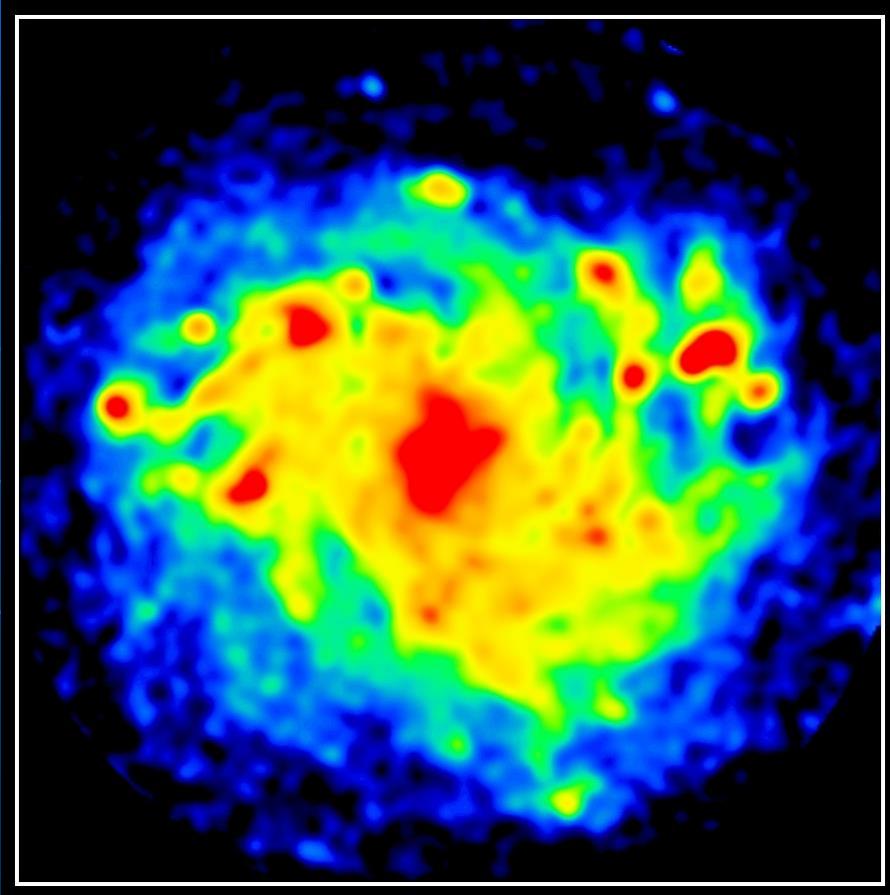
$$\begin{aligned} p_o &= (\epsilon + 1) / (\epsilon + 7/3) \\ &= (\alpha + 1) / (\alpha + 5/3) \end{aligned}$$

Typical value:  $\alpha=0.9$ ,  $p_o=74\%$

- **Circular polarization:** generally negligible

# Synchrotron polarization

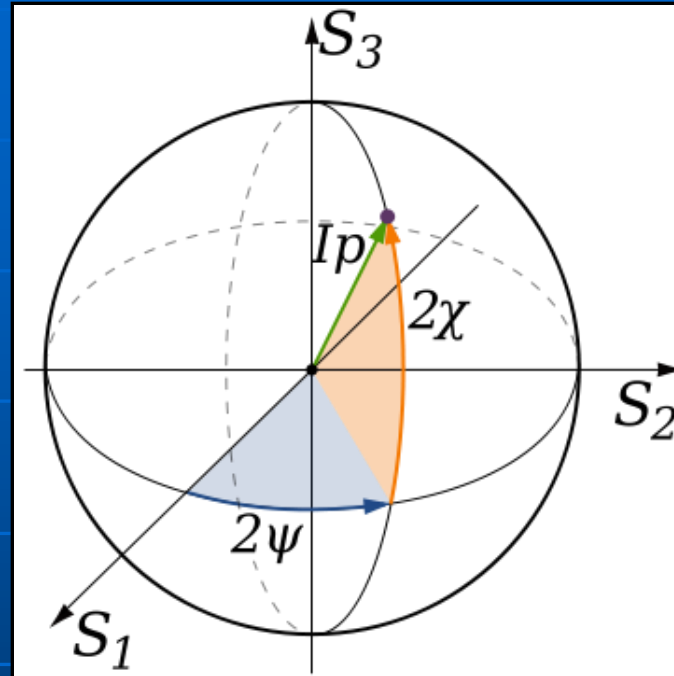
Beck & Hoernes 1996



NGC 6946: Total and polarized intensity at 6cm

# Stokes parameters

Poincaré sphere



$I$

$$Q = I \rho \cos 2\psi \cos 2\chi$$

$$U = I \rho \sin 2\psi \cos 2\chi$$

$$V = I \rho \sin 2\chi$$

$\rho$ : degree of polarization

$\psi$ : Angle of linear polarization

$$\psi = 0.5 \arctan (U/Q)$$

$\chi$ : "Angle" of circular polarization

# Linear polarization in complex notation

$$\mathbf{P} = p \exp ( 2 i \psi )$$

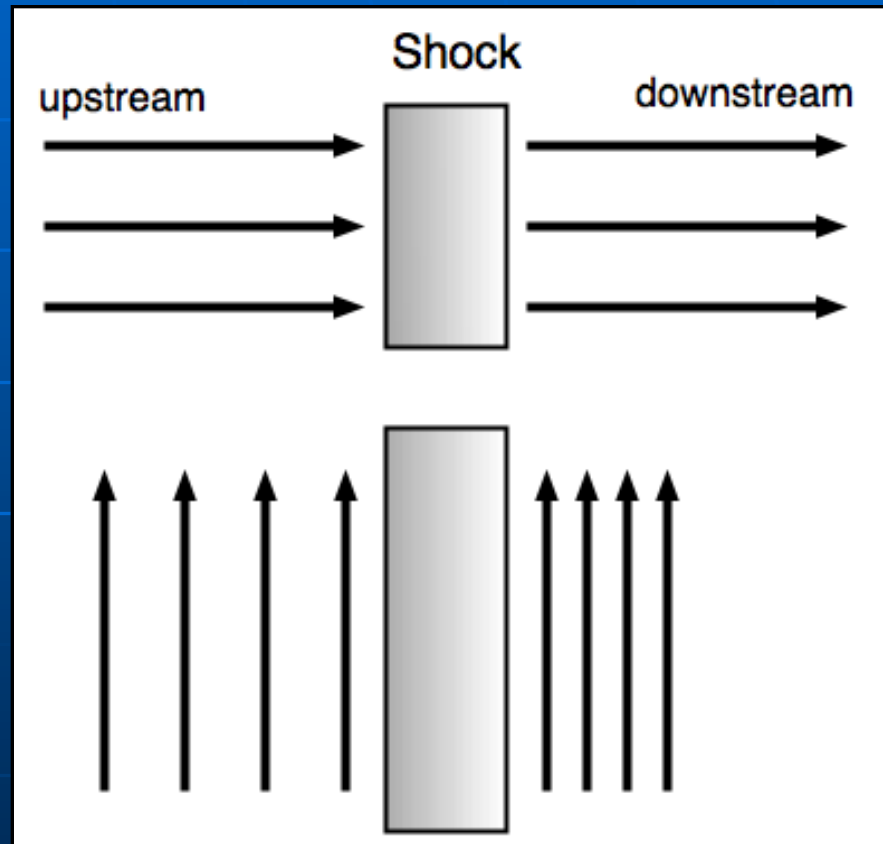


*The projected polarization plane is  
NOT a vector !  
(no direction, only orientation)*

# The effect of shocks on the magnetic field (1)

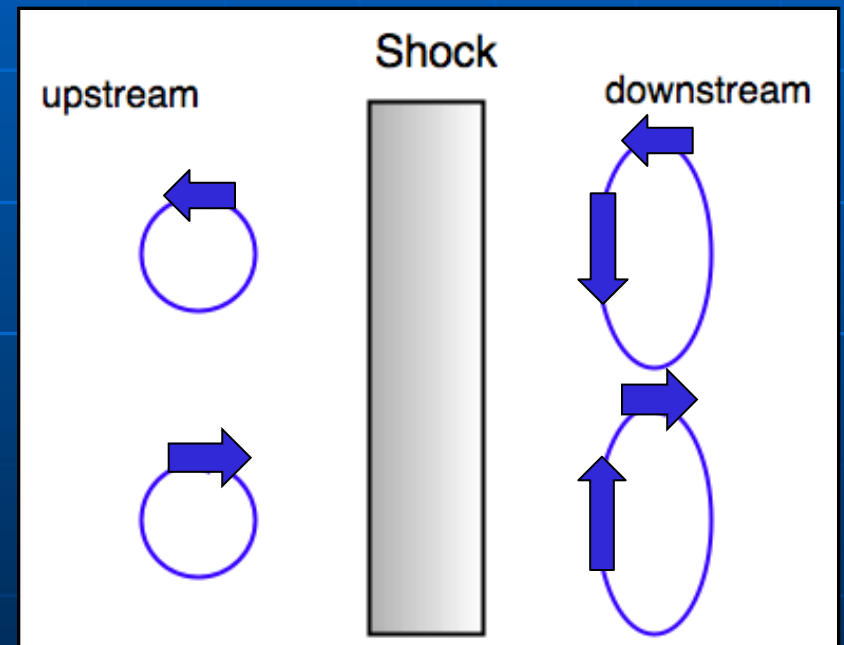
- A shock perpendicular to field has no effect
- A parallel shock increases the magnetic field strength

$$B \propto \rho$$
$$PI \propto \rho^{2 \rightarrow 4}$$



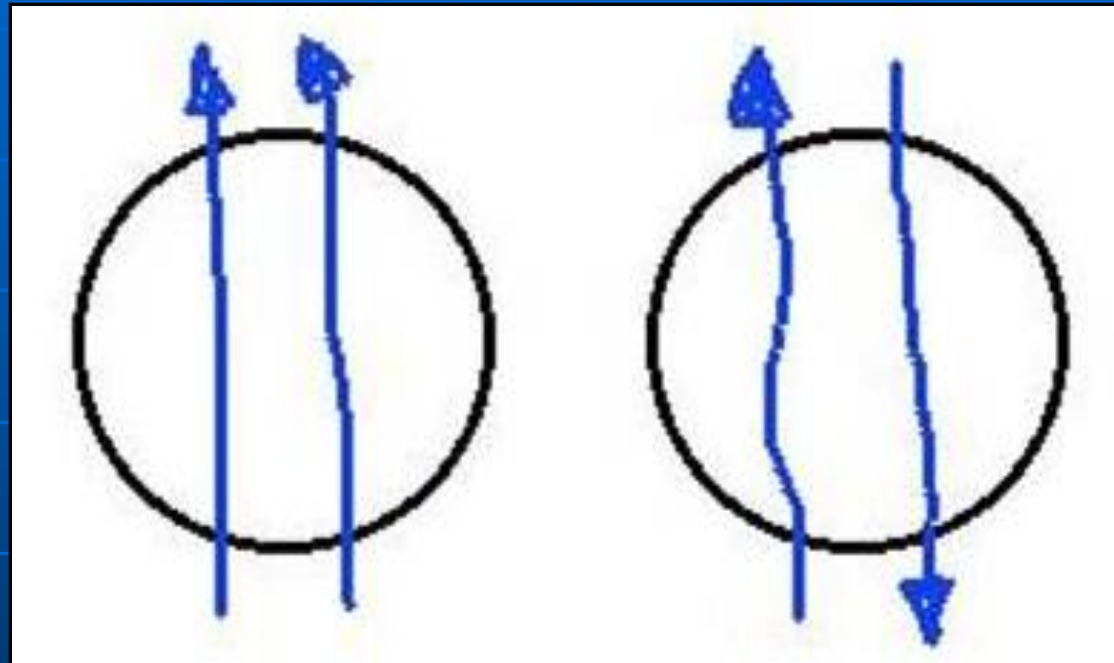
# The effect of shocks on the magnetic field (2)

- An isotropic turbulent magnetic field is made anisotropic
- **Isotropic turbulent field:**  
produces I but no PI
- **Anisotropic turbulent field:**  
produces PI and I



Regular  
(coherent)  
field

Anisotropic turbulent  
(incoherent)  
field



Polarization : **strong**

**strong**

Faraday rotation : **high**

low

# Magnetic field components

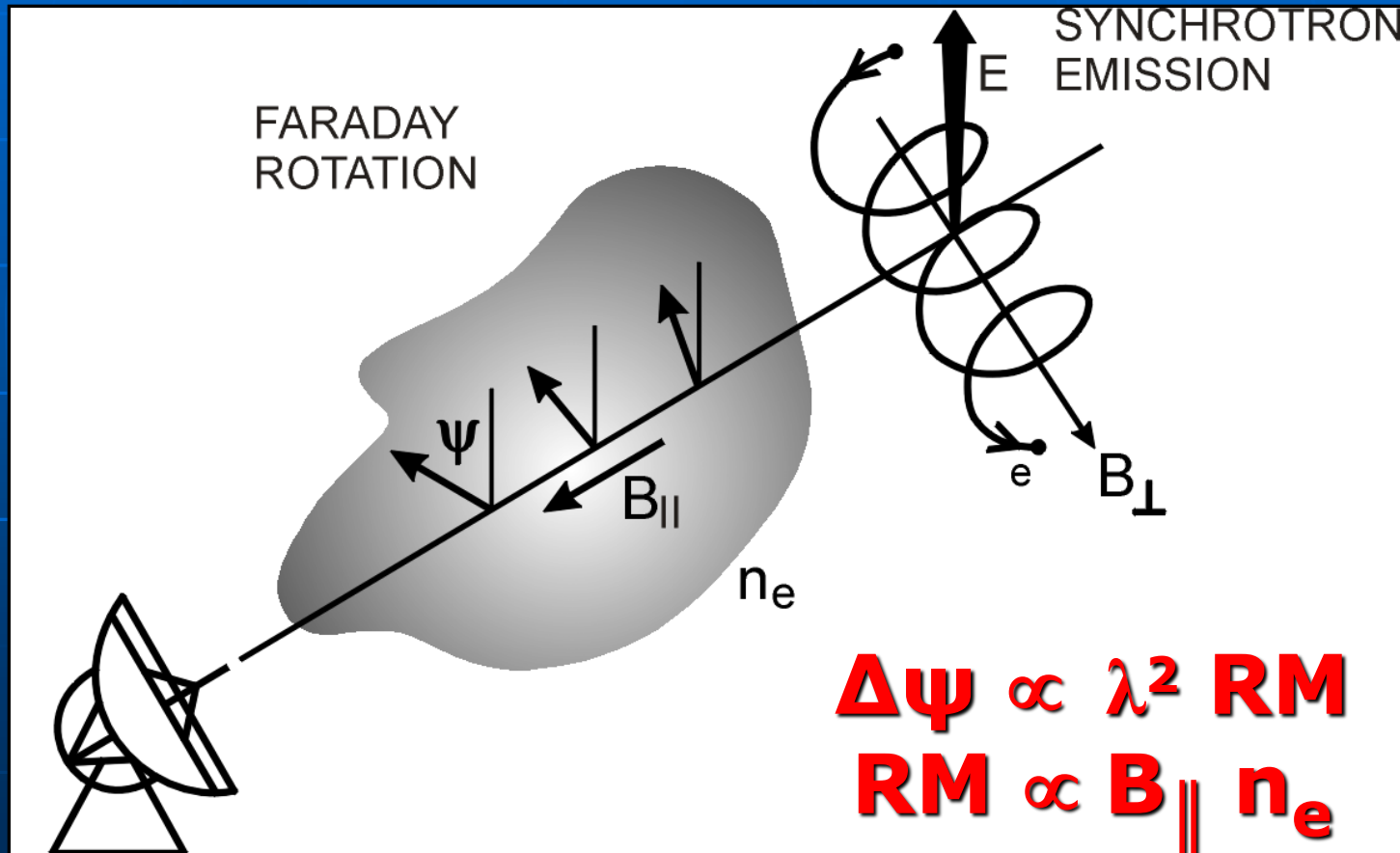
- **Total field  $B_{\text{tot}}$**   
(measured by total synchrotron intensity)
- **Isotropic turbulent field  $B_r$**   
(measured by unpolarized synchrotron intensity)
- **Anisotropic turbulent field  $B_{\text{an}}$**   
(measured by polarized synchrotron intensity)
- **Coherent (regular) field  $B_{\text{reg}}$**   
(measured by polarized synchrotron intensity **and** Faraday rotation)

$$B_{\text{tot}}^2 = B_r^2 + B_{\text{an}}^2 + B_{\text{reg}}^2$$

Observational test:

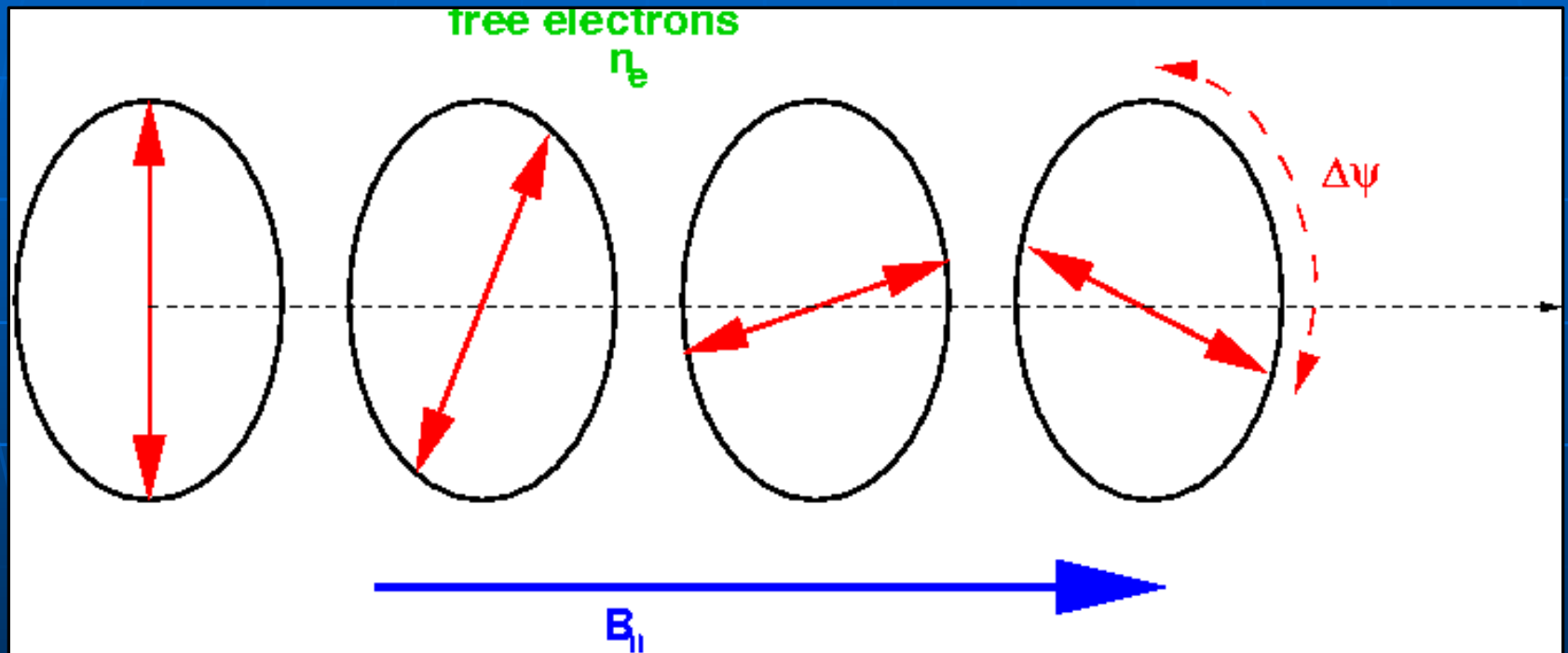
*Regular fields  
should give rise to  
Faraday rotation*

# Faraday rotation of polarized emission: Ordered magnetic fields



RM: Faraday rotation measure

# Faraday rotation





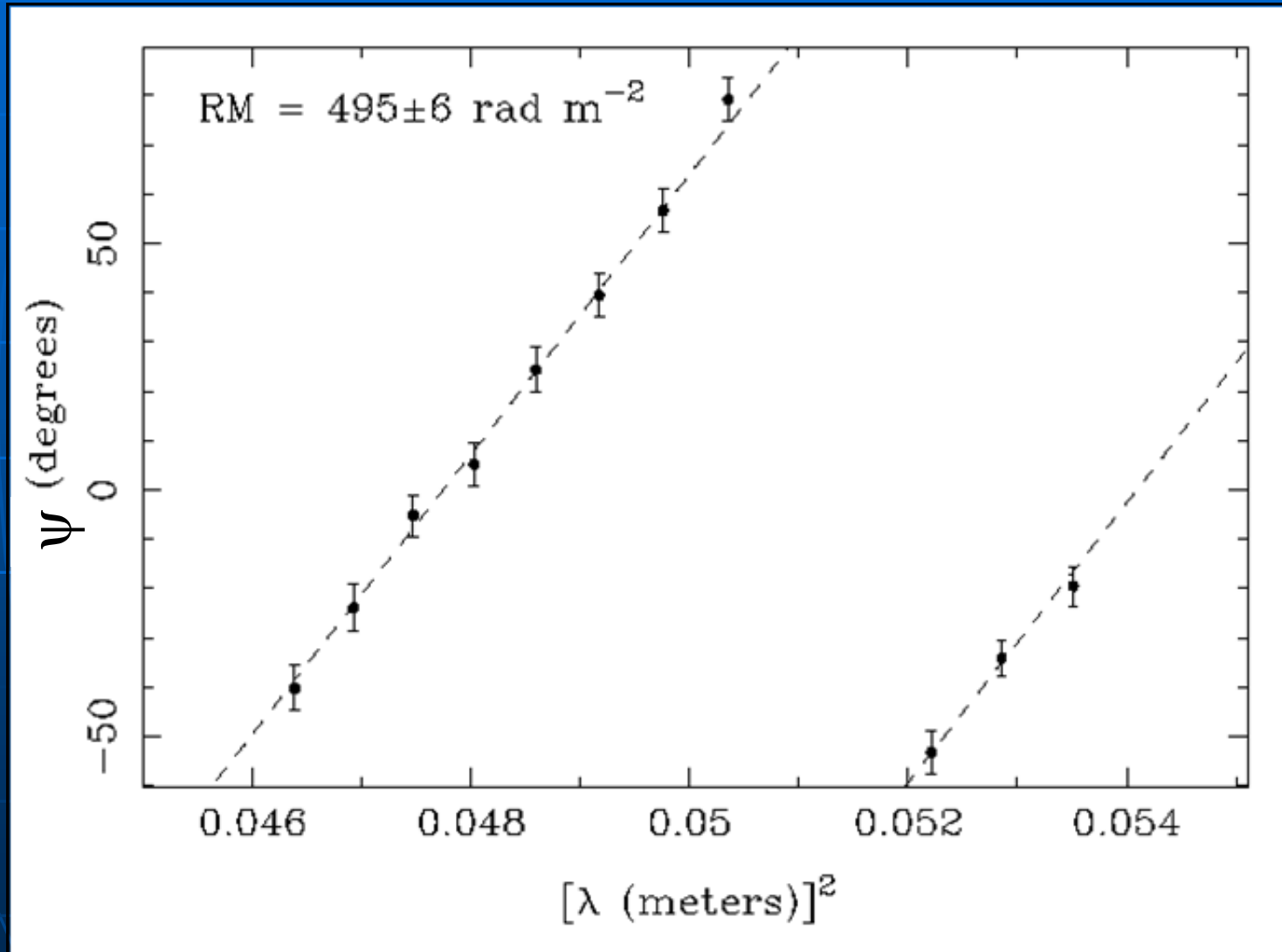
# Faraday rotation

$$\begin{aligned}\Delta\psi &= 0.81 \text{ (rad)} \lambda \text{ (m)}^2 \int n_e \text{ (cm}^{-3}\text{)} B_{\text{reg}\parallel} \text{ (\mu G)} dl \text{ (pc)} \\ &= \mathbf{RM} \lambda \text{ (m)}^2\end{aligned}$$

Classical measurement of RM:

Fitting  $\Delta\chi$  as a function of  $\lambda^2$

# Faraday rotation of a pulsar



# Typical Faraday rotation measures

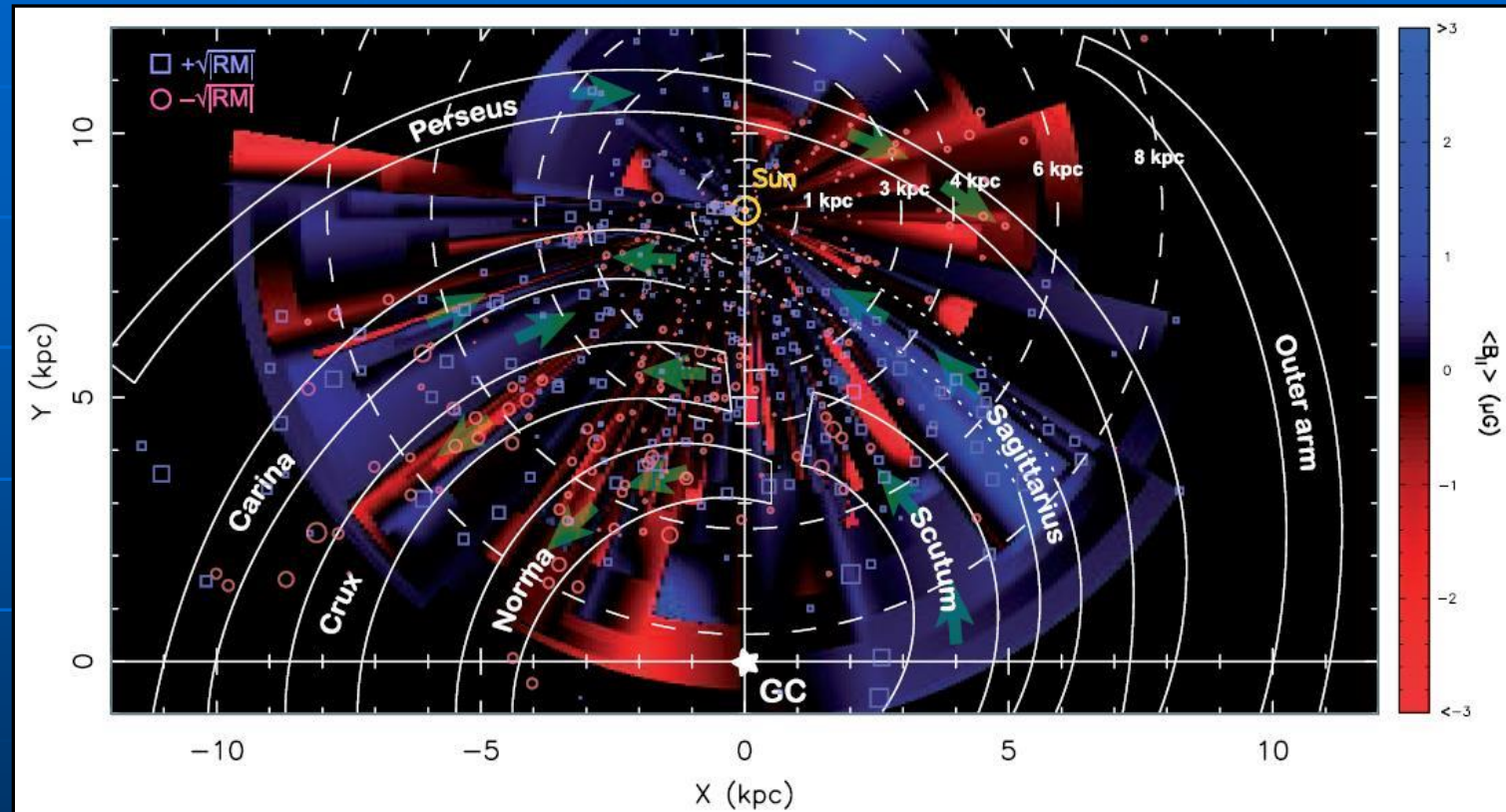
- Intergalactic medium:  $\approx 0.01-0.1 \text{ rad m}^{-2}$
- Galactic halos:  $\approx 0.1-1 \text{ rad m}^{-2}$
- Galactic disks:  $\approx 10-100 \text{ rad m}^{-2}$
- Galaxy clusters:  $\approx 100-1000 \text{ rad m}^{-2}$
- Cooling cores of clusters:  $\approx 1000-10000 \text{ rad m}^{-2}$

# Faraday rotation angles

	<b> RM </b>	=	100	10	1	0.1 rad m <sup>-2</sup>
1400 MHz	$\Delta\chi =$		263°	26°	3°	0.3°
200 MHz	$\Delta\chi =$		12900°	1290°	129°	13°
120 MHz	$\Delta\chi =$		35800°	3580°	358°	36°

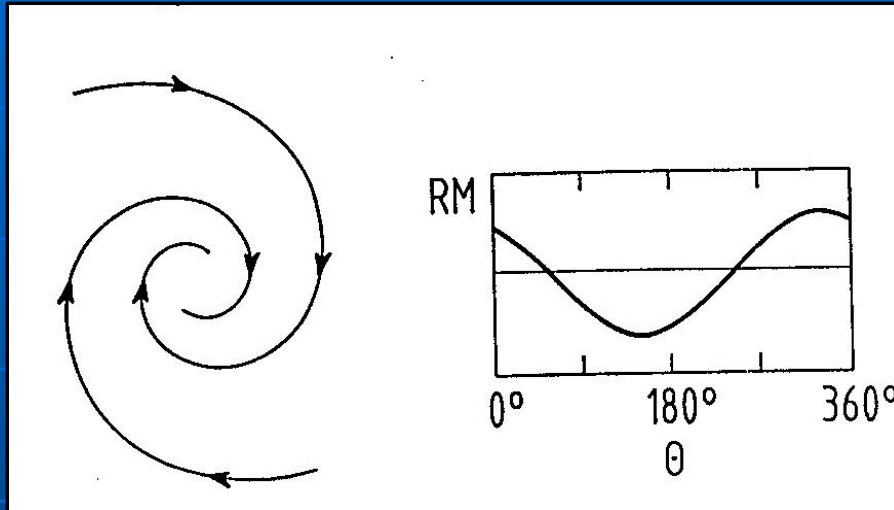
# Faraday rotation of pulsars in the Milky Way

Noutsos 2010



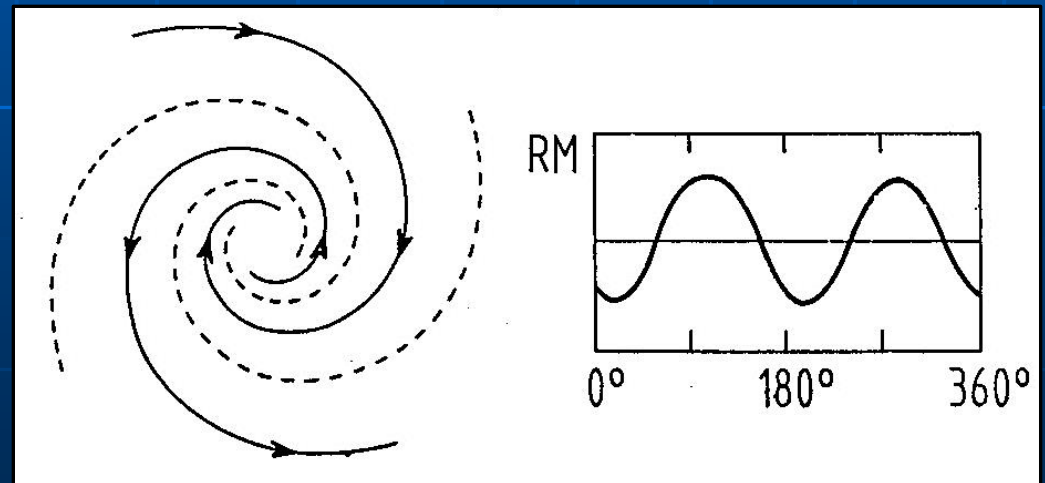
- Pulsars are highly polarized
- RM is well defined
- But distances are not well known

# Azimuthal variation of Faraday rotation in galaxies



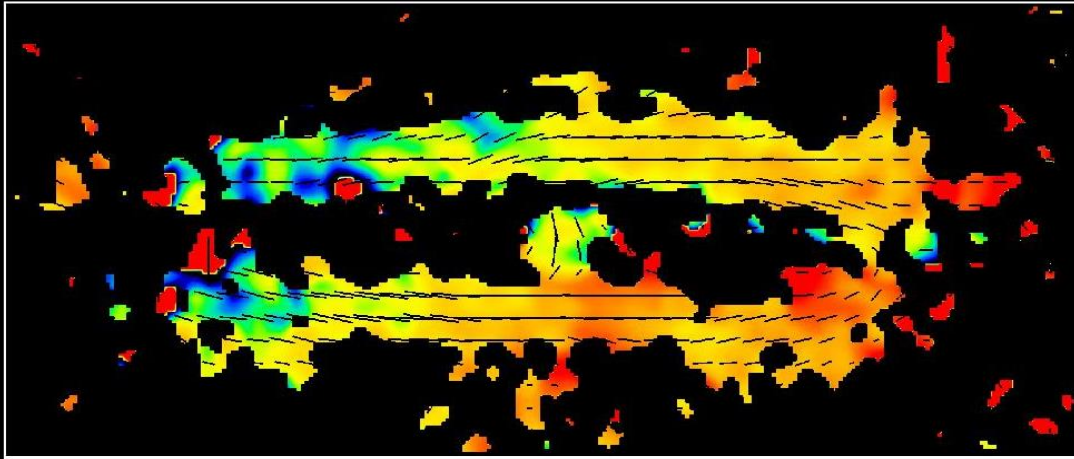
Axisymmetric spiral  
(mode  $m=0$ )

Bisymmetric spiral  
(mode  $m=1$ )



# The dynamo of M31

M31 RM 6/11cm + Magnetic Field (Effelsberg)



Copyright: MPIfR, Bonn (R.Beck, E.M.Berkhuijsen & P.Hoernes)



Berkhuijsen et al. 2003



Fletcher et al. 2004

The spiral field of M31 is coherent and axisymmetric

# Practical problems with measuring field strengths with Faraday rotation

- Faraday rotation measures the average product of thermal electron density  $n_e$  and field strength:

**Independent data about  $n_e$  needed**, e.g. from thermal emission or from radio recombination lines

- $\langle n_e B_{\parallel} \rangle \neq \langle n_e \rangle \langle B_{\parallel} \rangle$  :

A **correlation or anticorrelation between  $n_e$  and  $B_{\parallel}$**  affects the field strength estimate

- **Several emitting & rotating regions along the line of sight:**  
polarization angle does **not** vary with  $\lambda^2$
- **Faraday depolarization occurs:**  
polarization angle does **not** vary with  $\lambda^2$

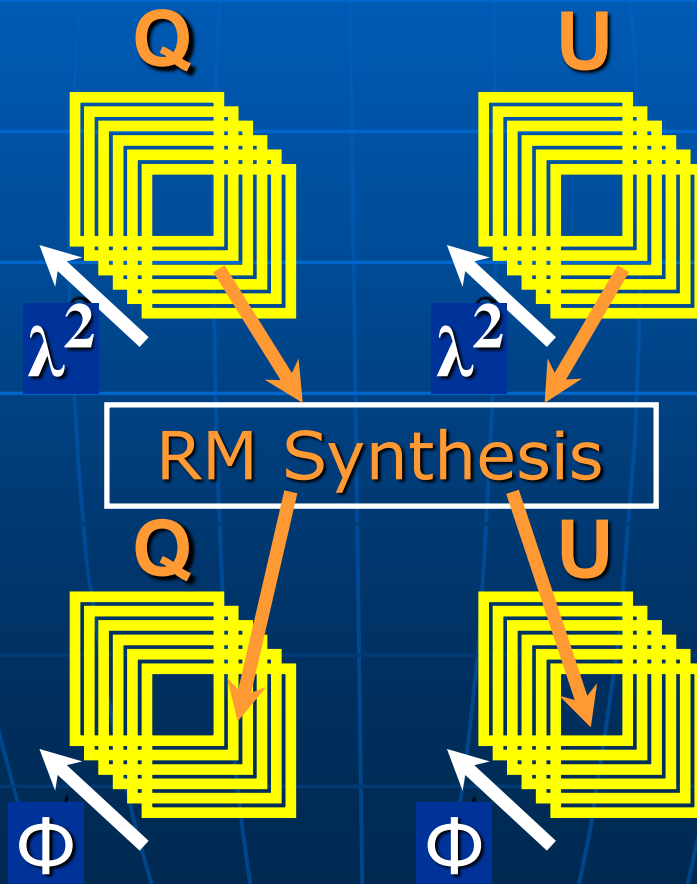


*RM Synthesis  
opens a new dimension  
in the phase space  
of observations:*

***the Faraday Space***

# The future: Spectro-polarimetry (RM Synthesis)

- RM synthesis works on observed Q,U cubes in wavelength to produce **RM-cubes** in Faraday depth:



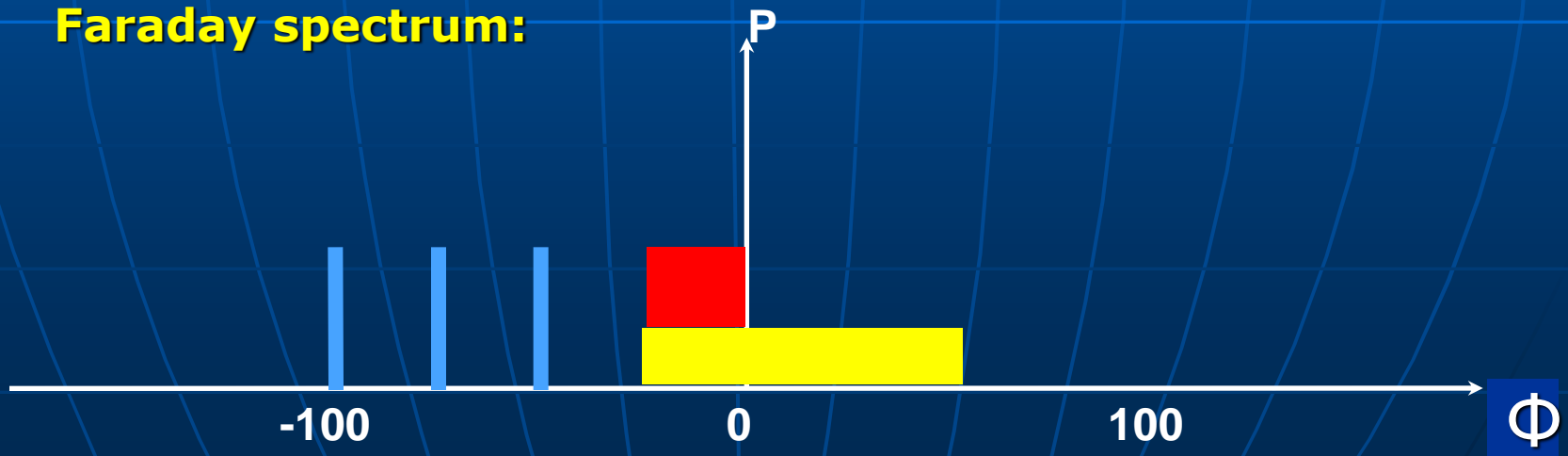
# RM Synthesis: basic facts

- The *Faraday spectrum*  $F(\varphi)$  is the Fourier transform of the observed complex polarized intensity  $P(\lambda^2)$
- *Faraday depth* ( $\varphi \propto \int B_{\parallel} n_e dl$ ) is generally different from classical rotation measure  $RM$
- Foreground *Faraday screen*:  $\varphi = RM$
- Homogeneous mixed emitting & rotating layer, no depolarization:  $\varphi = 2 RM$
- The  $\lambda^2$  coverage of observations determines the resolution in Faraday space (*RM Spread Function, RMSF*)

# Example



**Faraday spectrum:**



## Resolution in Faraday space:

$$\Delta\varphi \approx 2 \sqrt{3} / \Delta\lambda^2 = \sqrt{3} (v_0/\Delta v) / \lambda_0^2$$

Needed: long wavelength **and** large bandwidth

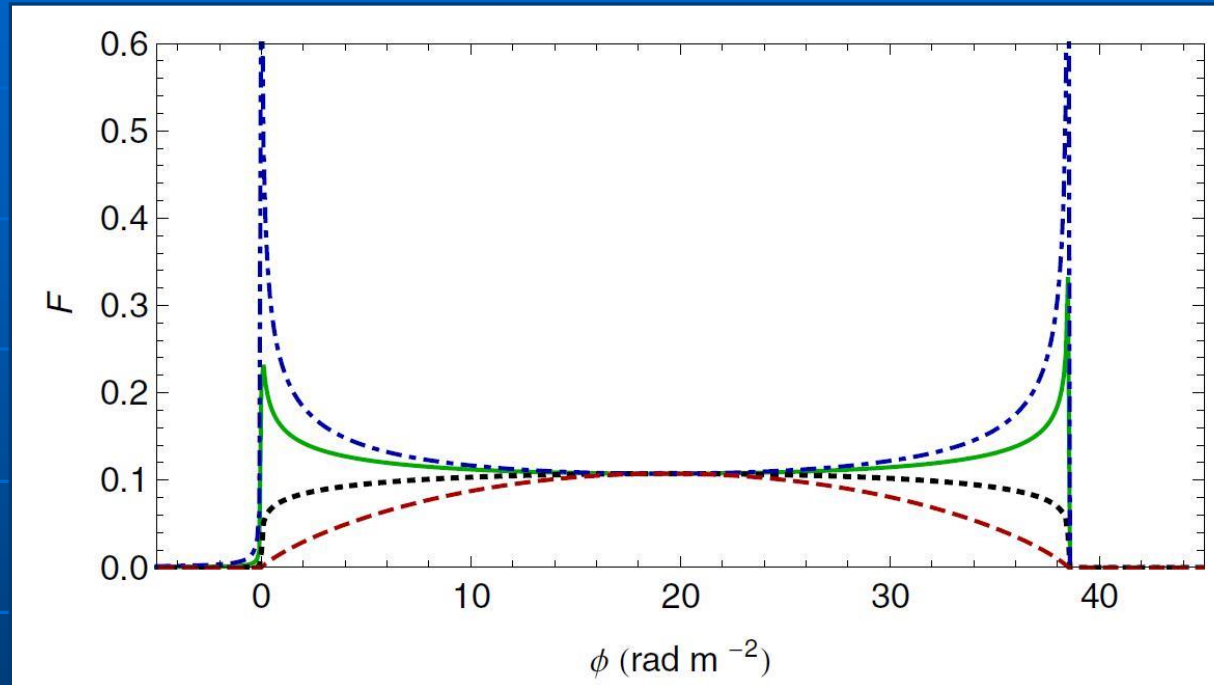
Measurement precision of a narrow FD component:

$$\varphi_i = \Delta\varphi / (2 (S/N))$$

(S/N): Signal-to-noise of the polarized intensity

$\Delta\lambda^2$  is as important as signal-to-noise

# Faraday spectrum of an extended source with a regular field: "Faraday horns"

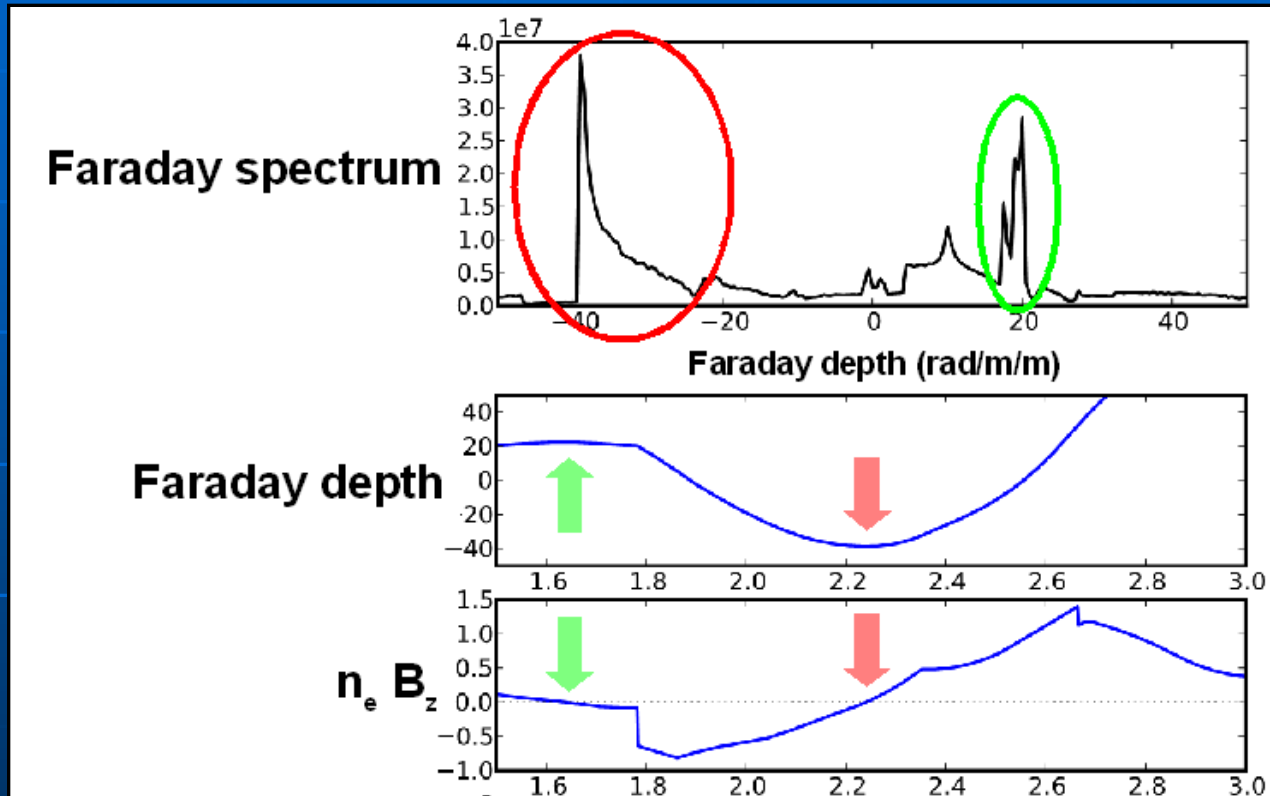


Beck et al. 2012

- Two symmetric Faraday horns indicate an extended source
- Height of the horns shows relative distribution of cosmic-ray and thermal electrons

# Field reversals generate "Faraday caustics"

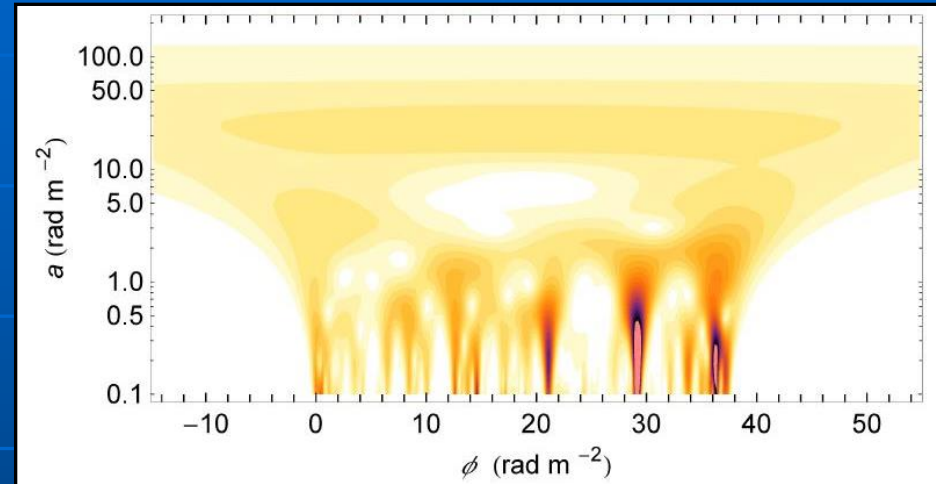
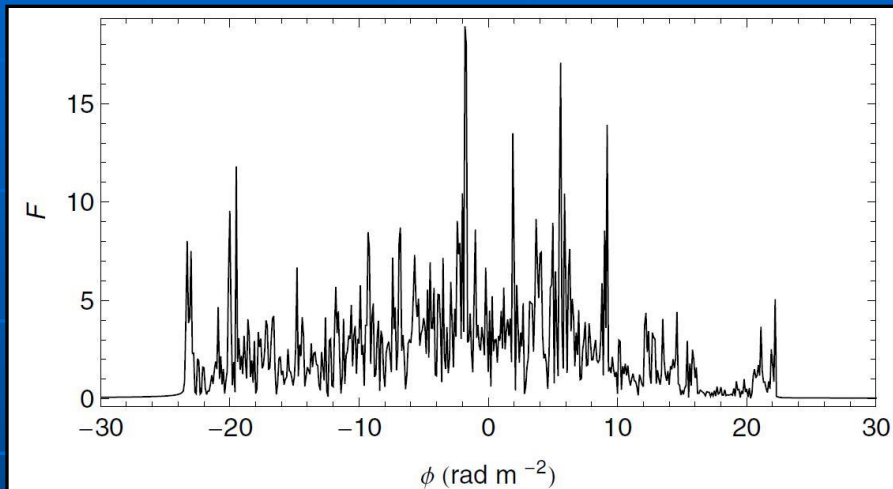
Bell et al. 2011



Asymmetric FD components: high FD resolution & large wavelength ratio required

# Turbulent fields generate a "Faraday forest"

Beck et al. 2012



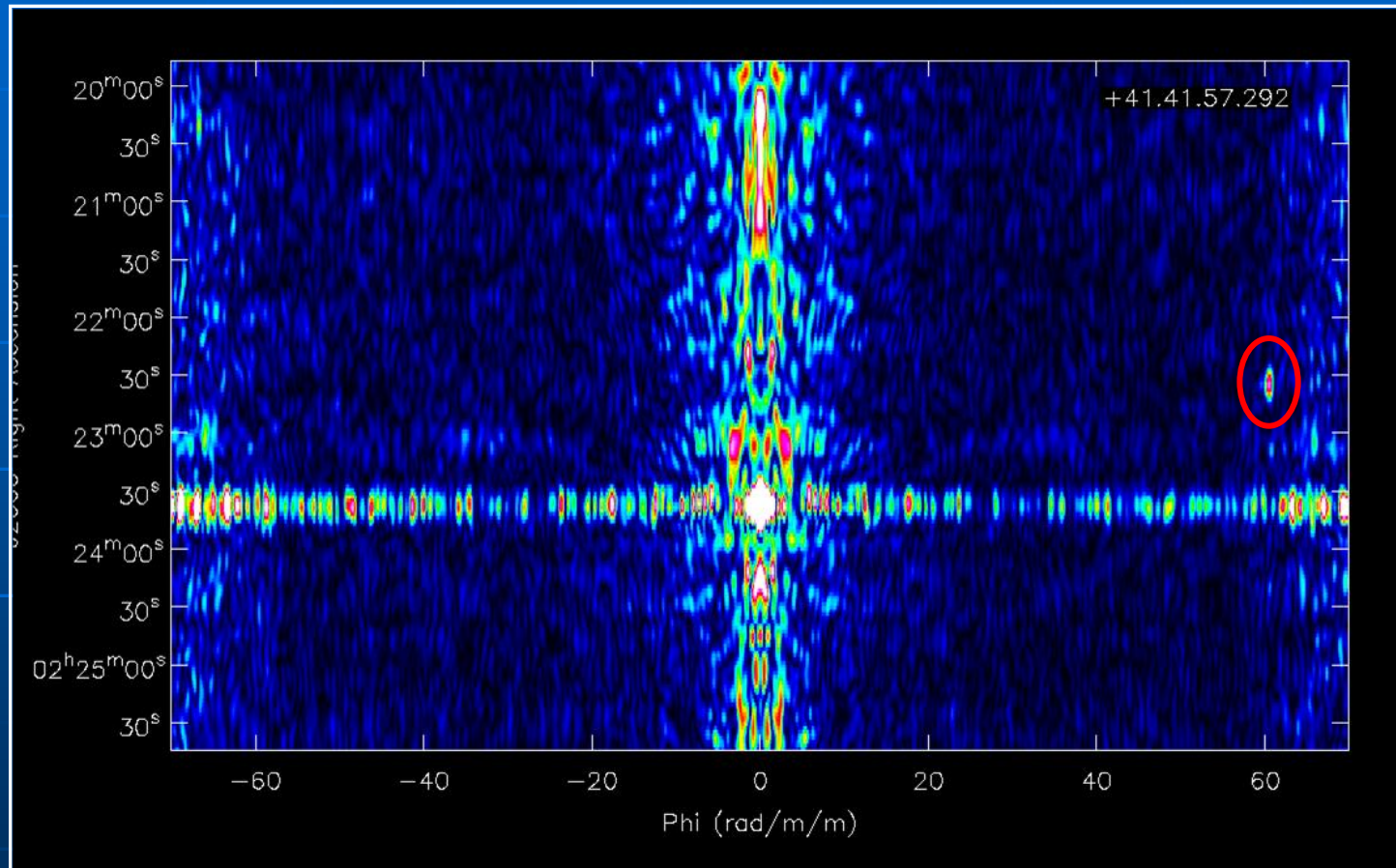
Many FD components:  
high FD resolution required (i.e. large coverage in  $\lambda^2$ )



*Applications of  
RM Synthesis*

# Polarized emission from PSR J0218+4232

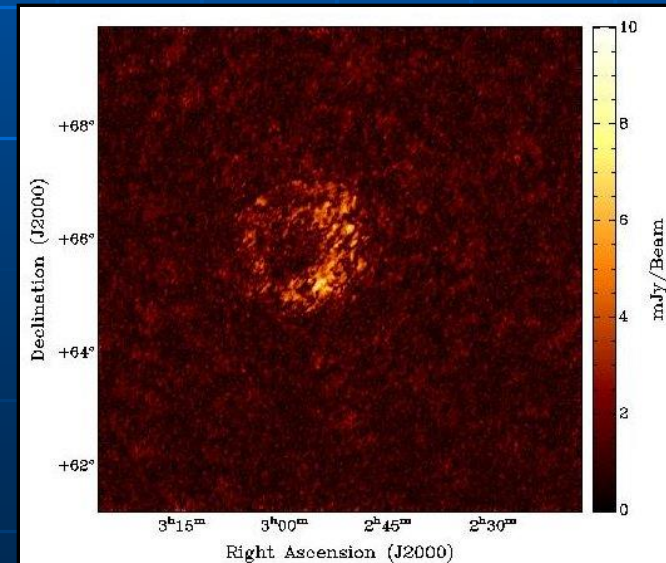
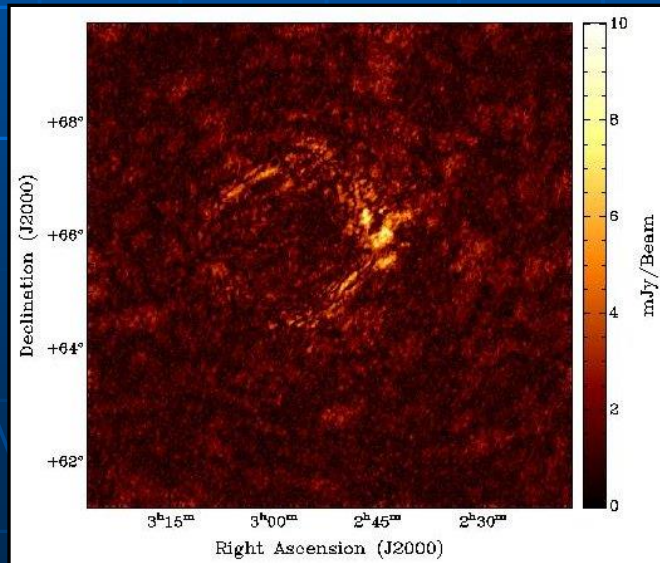
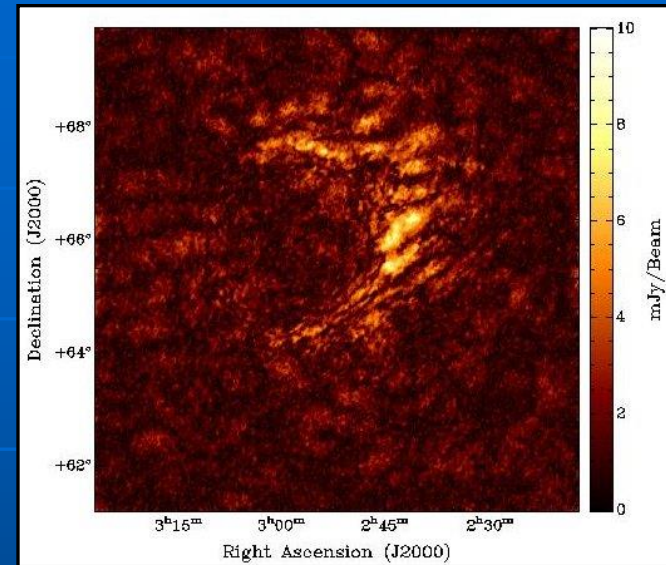
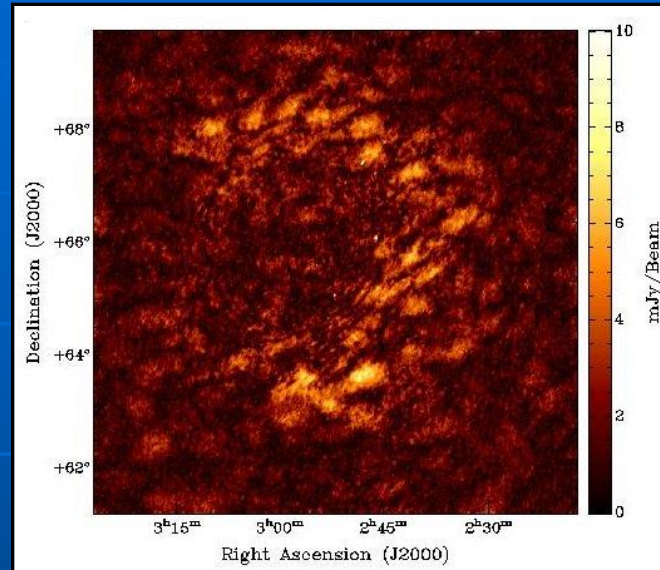
LOFAR - Cut along RA at constant DEC  
(A. Noutsos, C. Sobey, M. Bell, A. Horneffer)



- Strong instrumental polarization visible around 0 rad/m<sup>2</sup>
- Polarized pulsar detected at 61 rad/m<sup>2</sup>

# Polarization from the FAN region of the Milky Way

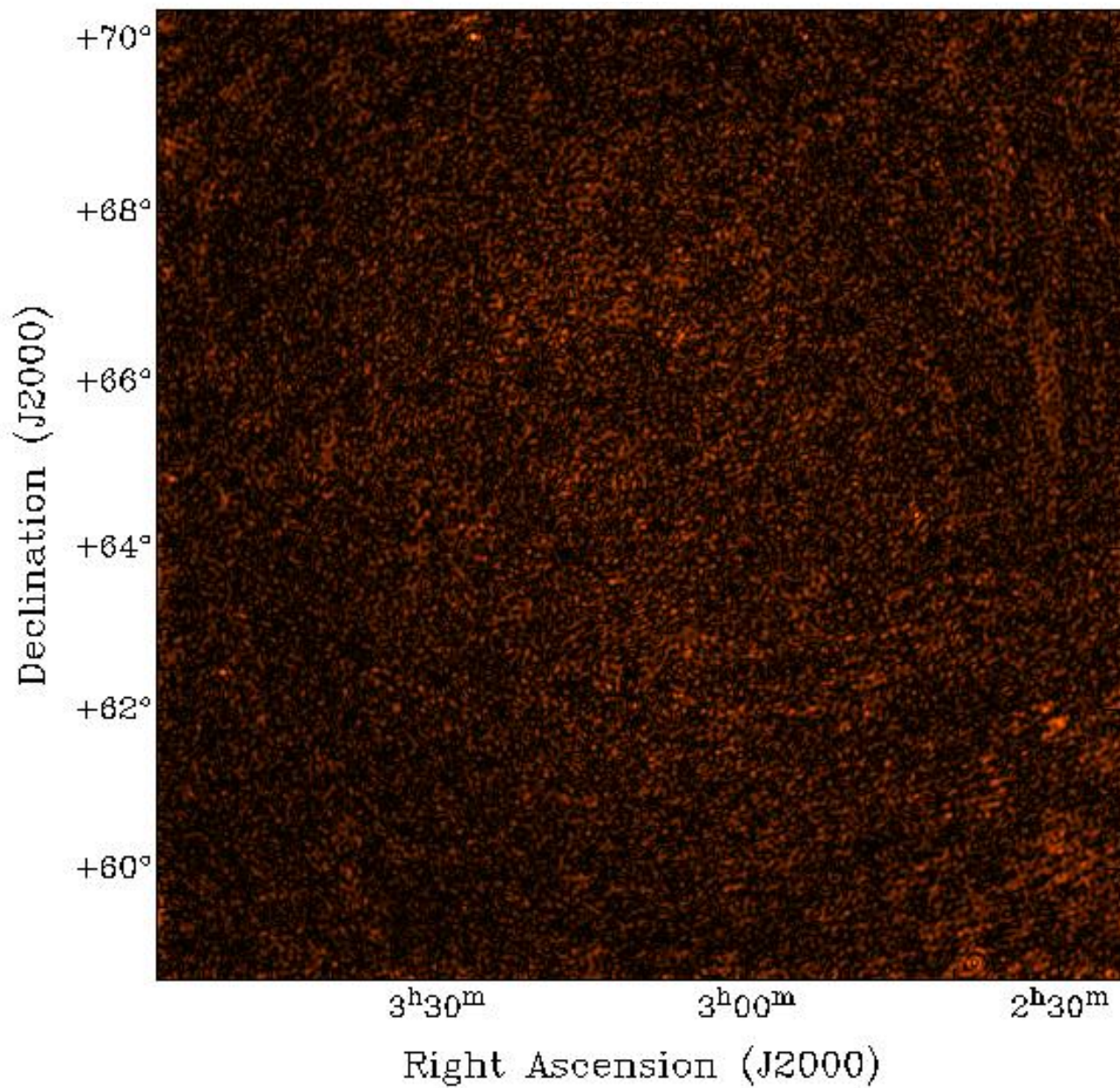
LOFAR HBA 120-180 MHz, Faraday depths  $\phi = -1 \dots -5 \text{ rad/m}^2$



FD = 9.000000e+00

# FAN region of the Milky Way

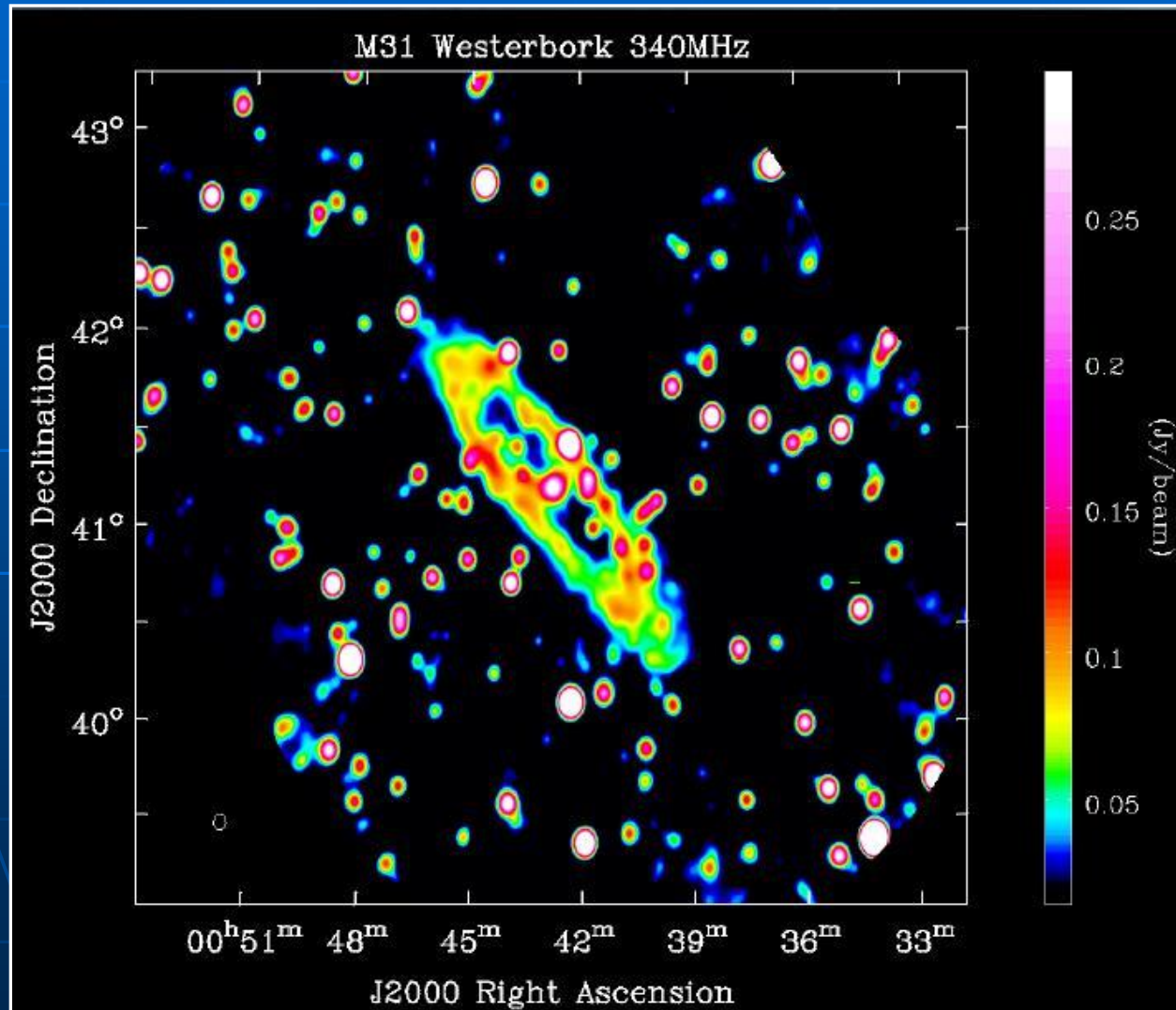
Westerbork 310-376 MHz



Bernardi et al.  
(2009)

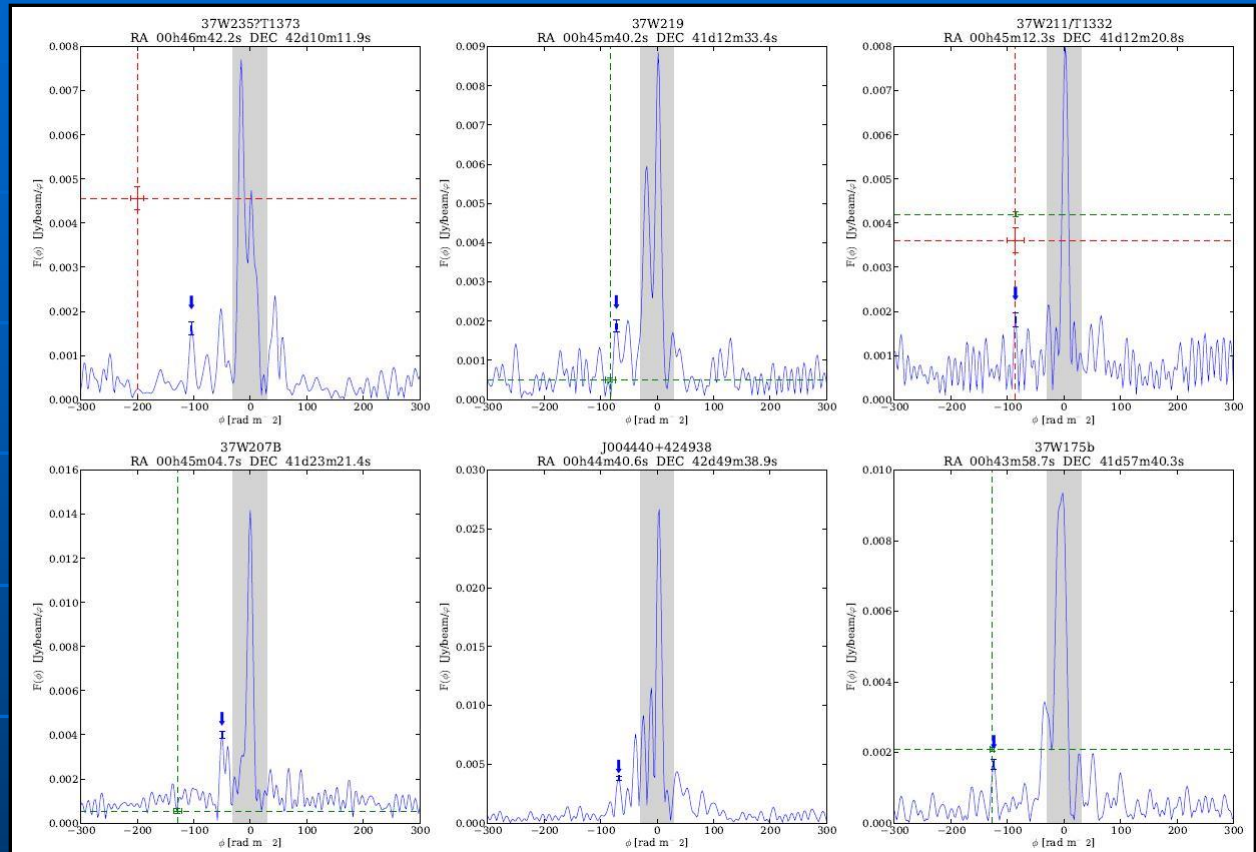
# M31 WSRT 310-376 MHz: Total intensity

Gießübel et al. 2013



# M31 WSRT 310-376 MHz Faraday spectra

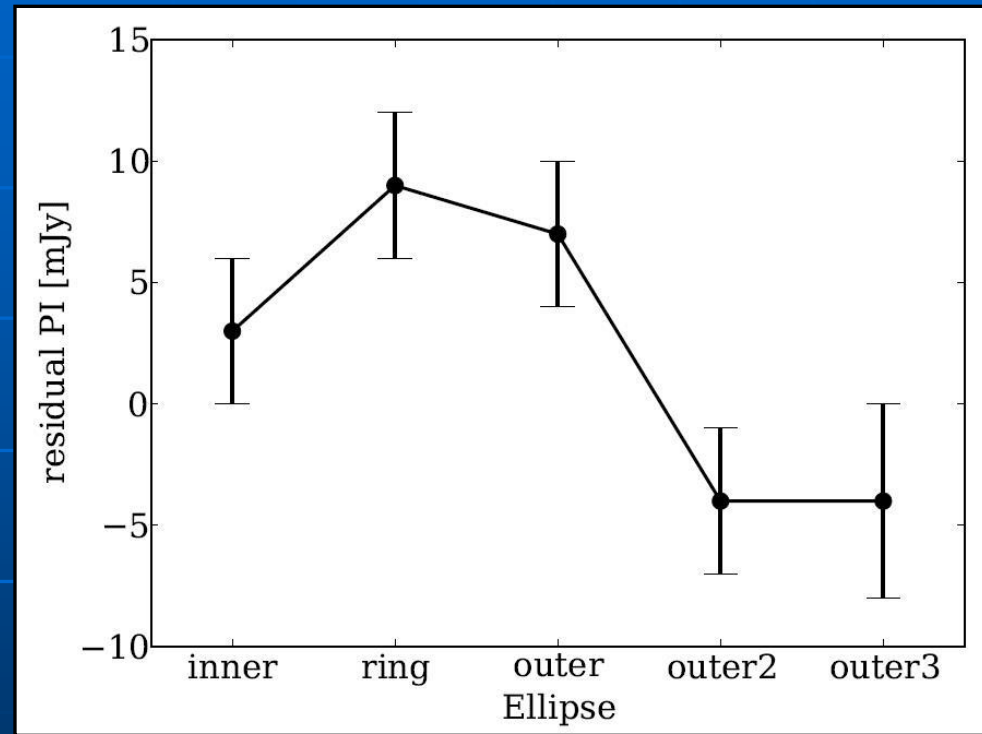
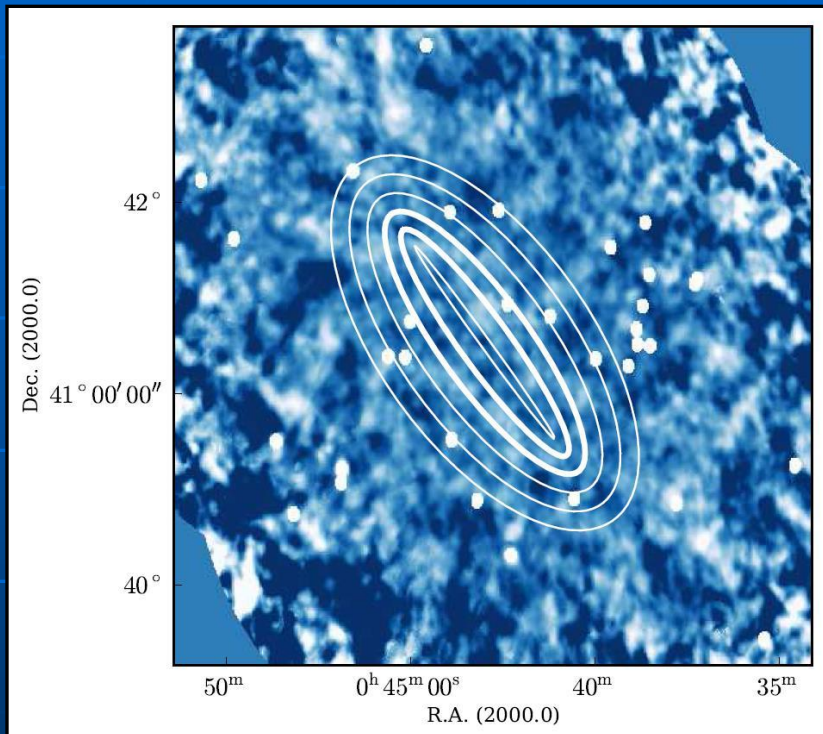
Gießübel et al. 2013



- 18 of 33 sources have complex spectra
- All 8 sources with deviating FDs have complex spectra
- Both sources with less DP have simple spectra

# M31 WSRT 310-376 MHz: Polarized intensity

Gießübel et al. 2013



- Detection of very low polarization ( **$0.21 \pm 0.05\%$**  !) by integration of Faraday spectra
- Strong depolarization:  $DP(90\text{cm}/6\text{cm}) = 0.005 \pm 0.002$

# Strengths of RM Synthesis

- Multi-component sources can be resolved in Faraday space even if not angularly resolved
- Multi-component medium can be resolved in Faraday space even if not angularly resolved
- The 3-D Faraday cube yields a 3-D picture of the magneto-ionic ISM or IGM (*Faraday tomography*) – but ...

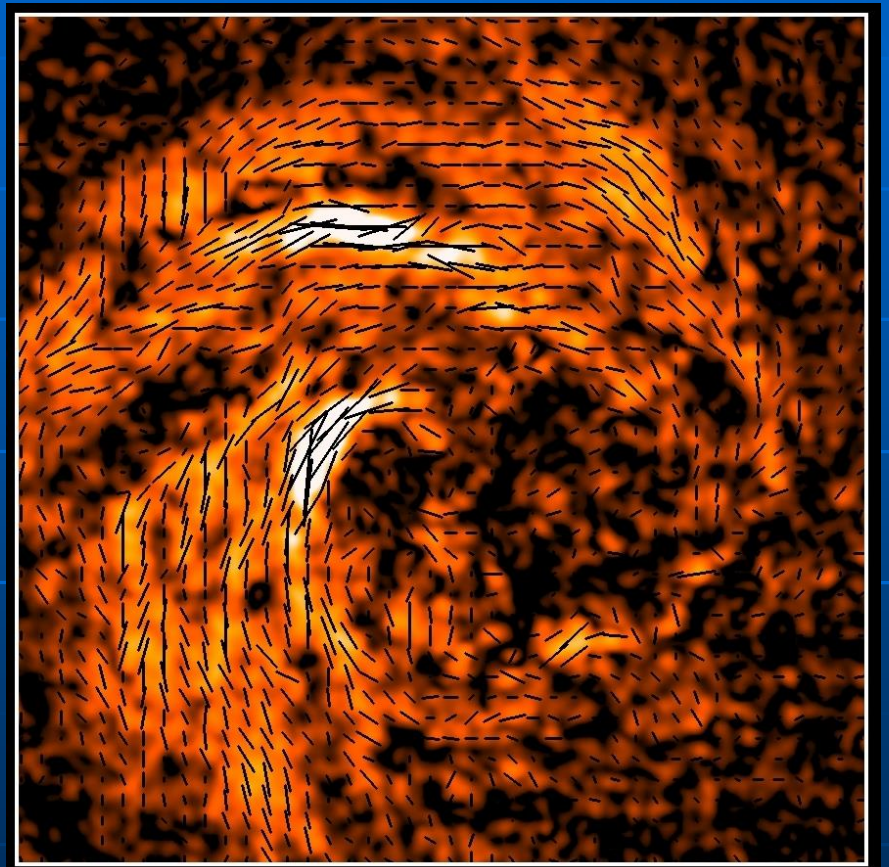
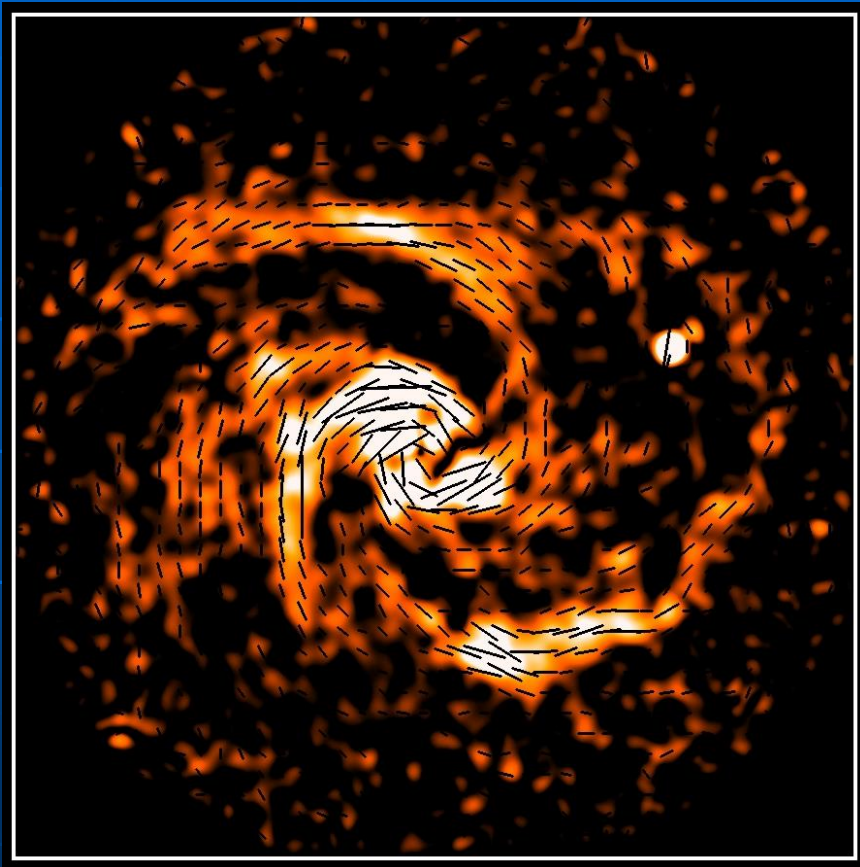


# Practical problems with RM Synthesis

- Negative  $\lambda^2$  cannot be observed: Fourier transform is limited
- Present-day radio telescopes cover only a small range in  $\lambda^2$
- Faraday depth (FD) is **not** geometrical depth
- Only emitting & rotating regions generate separate components
- *The interpretation of Faraday spectra is not straight forward: modelling needed*

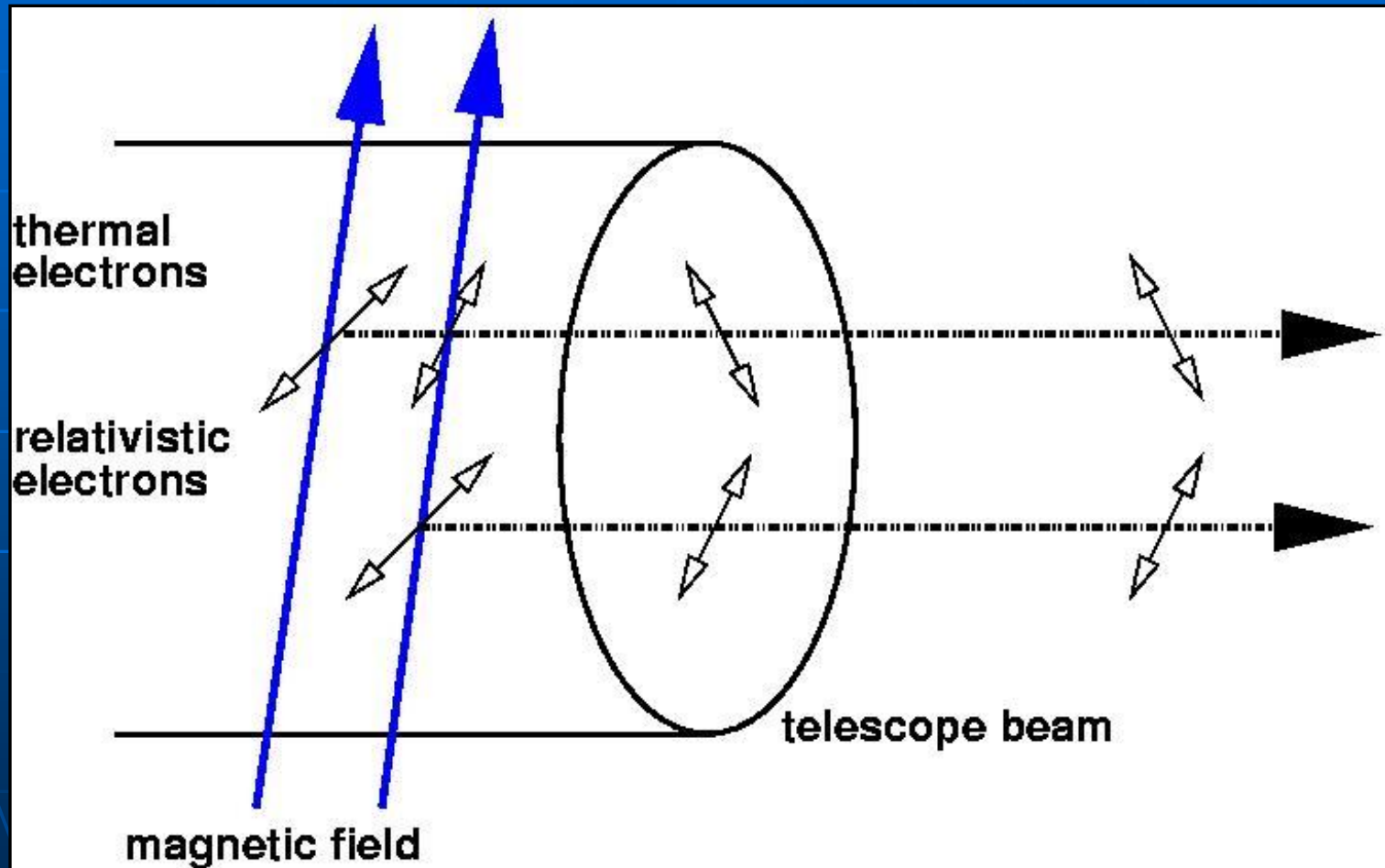
# Faraday depolarization

Beck 2007

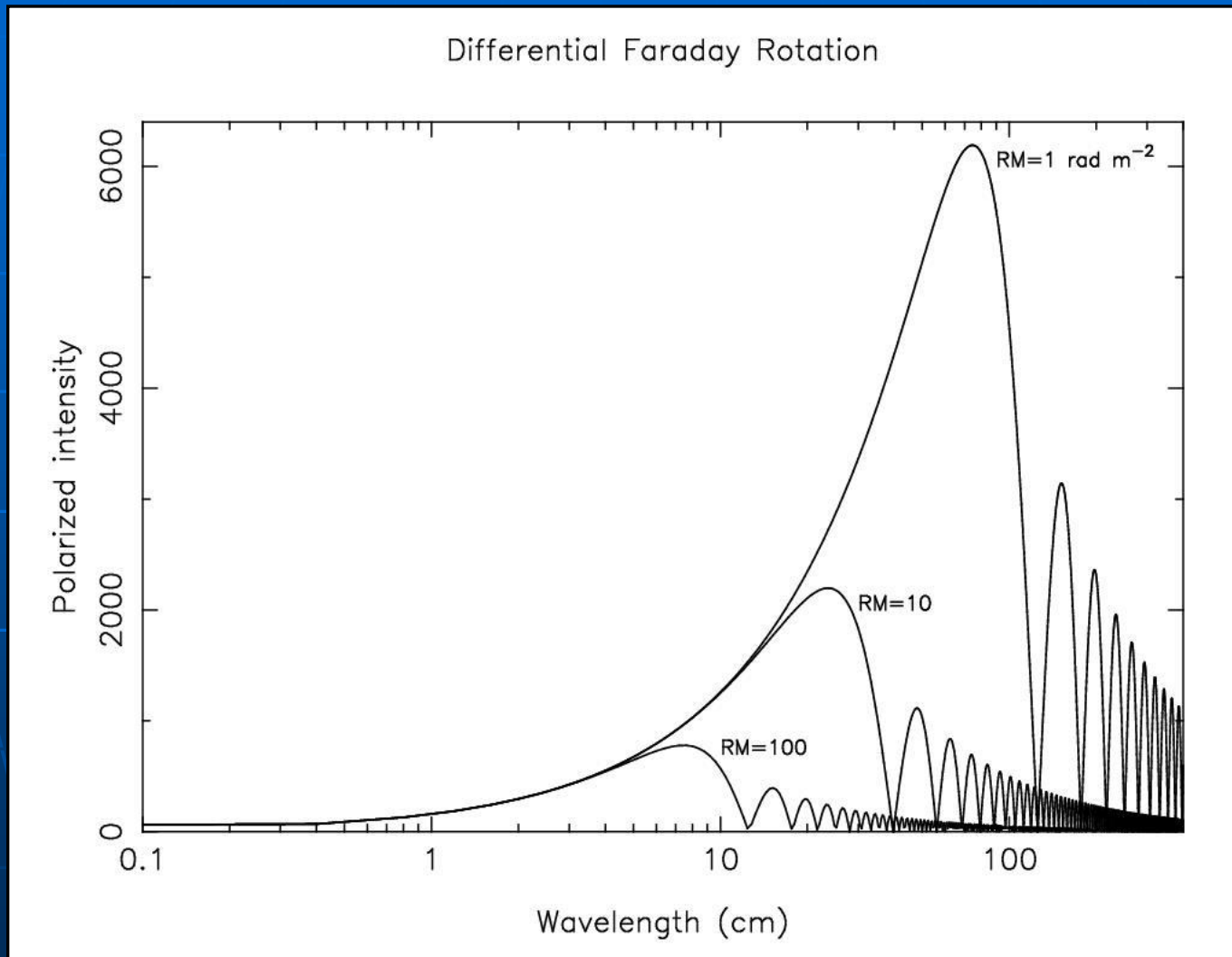


NGC 6946: Polarized intensity at 6 cm and 20.5 cm

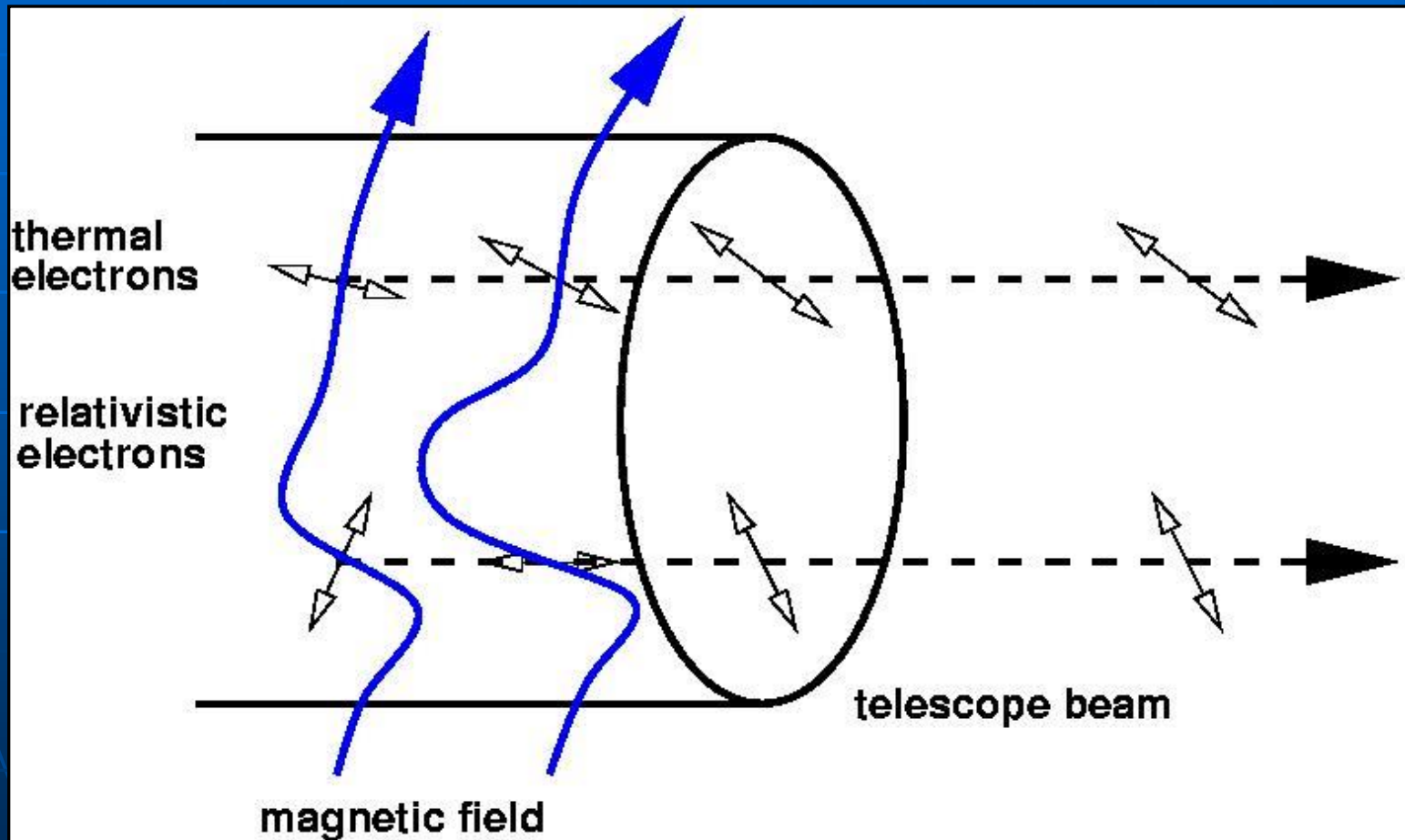
# Depolarization by differential Faraday rotation (regular field)



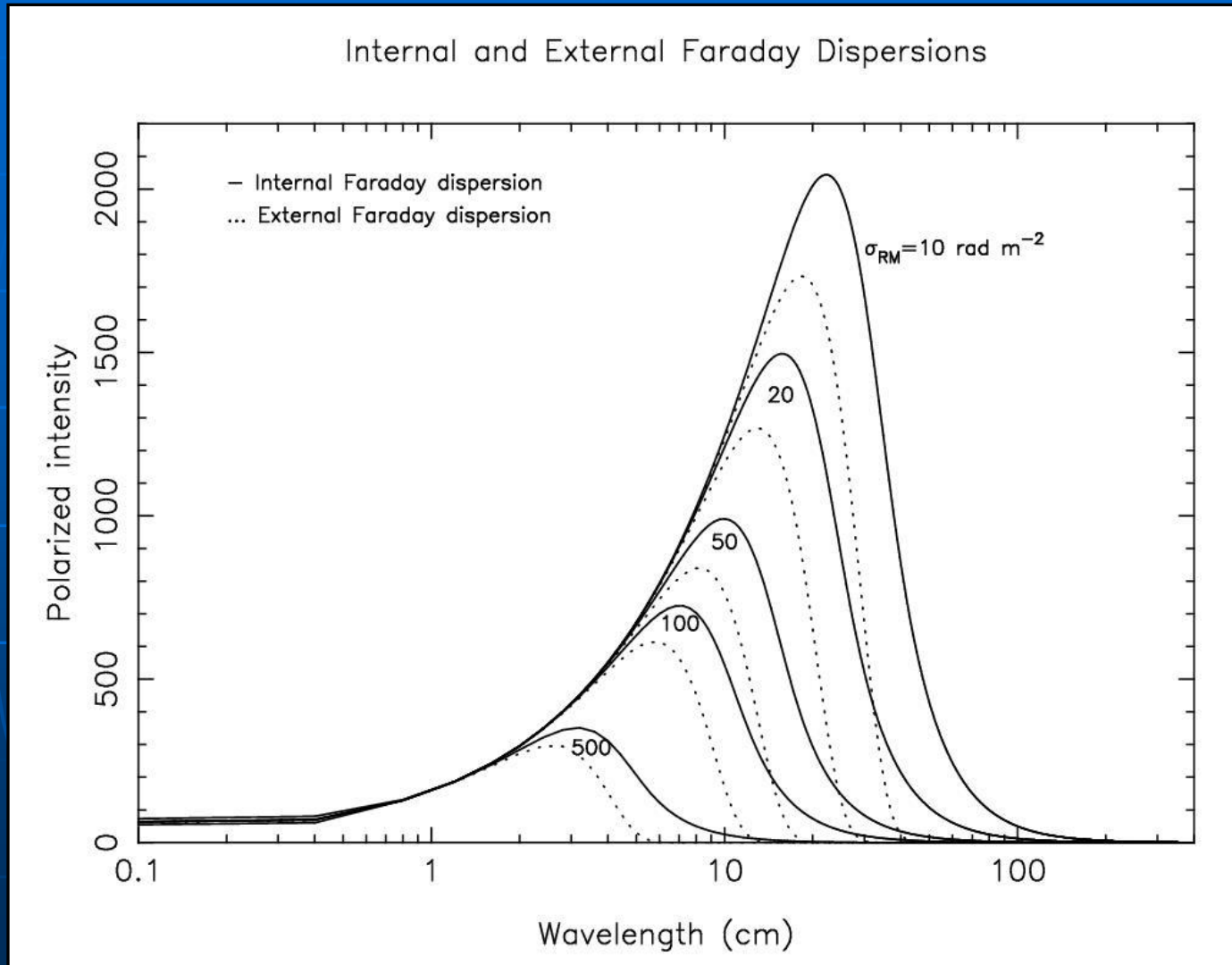
# Maximum polarized intensity (regular field)



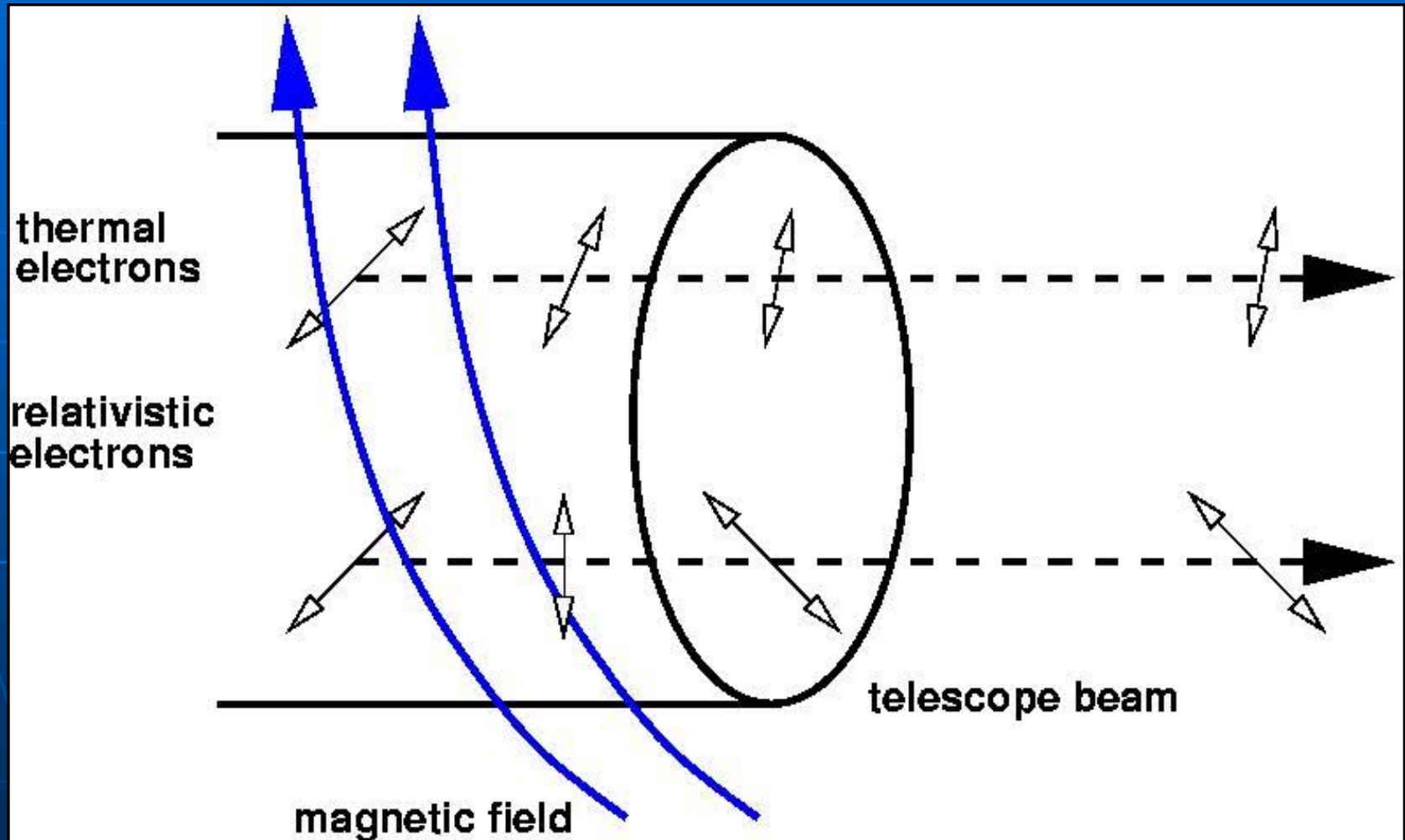
# Depolarization by internal Faraday dispersion (turbulent field)



# Maximum polarized intensity (turbulent field)



# Depolarization by rotation measure gradient



# Summary: Depolarization effects

- Beam DP by turbulent fields (wavelength-independent):

$$p = p_0 / N^{1/2}$$

(where  $N$  is the number of turbulent field cells)

- Differential Faraday rotation (wavelength-dependent):

$$p = p_0 |\sin(2 RM \lambda^2)| / (2 RM \lambda^2)$$

- Internal Faraday dispersion (wavelength-dep.):

$$p = p_0 (1 - \exp(-2 \sigma_{RM}^2 \lambda^4)) / (2 \sigma_{RM}^2 \lambda^4)$$

- External Faraday dispersion (wavelength-dependent):

$$p = p_0 \exp(-2 \sigma_{RM}^2 \lambda^4)$$

- RM dispersion:

$$\sigma_{RM}^2 = (0.81 n_e B_r)^2 L d f_v$$

( $B_r$  is the turbulent field strength,  $d$  the size of the turbulent cells,  $f_v$  the volume filling factor)



Martin Harwit, Cosmic Discovery  
(1981) :

*Polarization is an  
independent dimension  
in the phase space  
of observations*

# Summary

- Polarization is essential to observe cosmic magnetic fields
- The Universe looks different through “polarization goggles”
- However, some new telescopes are designed without taking care of polarization purity
- Join program committees and telescope design commissions !