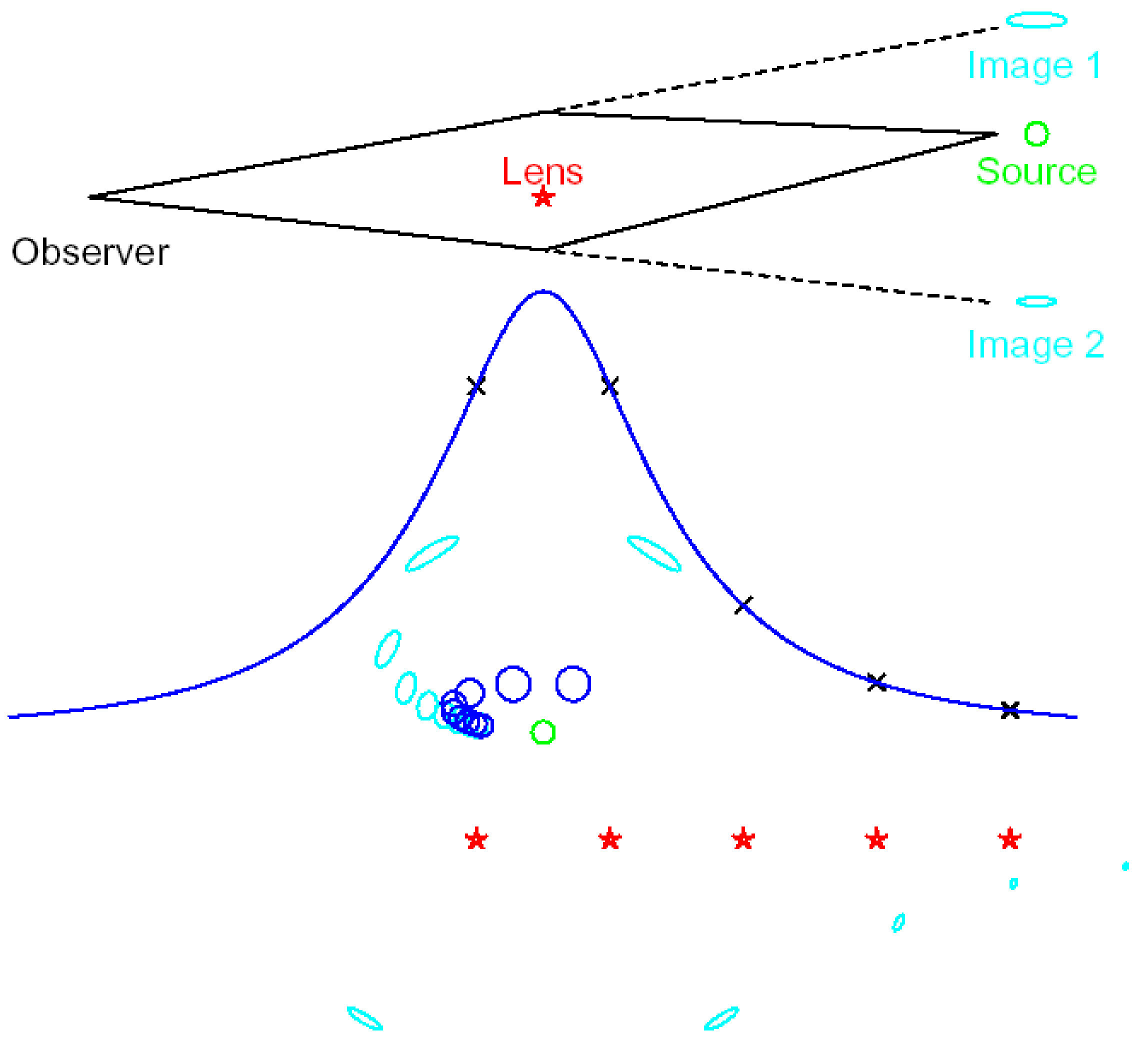
Exoplanet Microlensing III:

Inverting Lightcurves, μlens vs. world

Andy Gould (Ohio State)



Simple Point Lens

3 Features & 3 Parameters

- Time of Peak
- Height of Peak
- Width of Peak

- t_0
- u_0
- t_E

Simple Planetary (G&L) Lenses

6 Features & 6 Parameters

- Time of Peak
- Height of Peak
- Width of Peak
- Time of Perturbation
- Height of Perturbation
- Width of Perturbation

- t_0
- u_0
- t_E
- Trajectory angle: α
- Planet-star separation: s
- Planet/star mass ratio: q

Planetary Lenses usually have FS

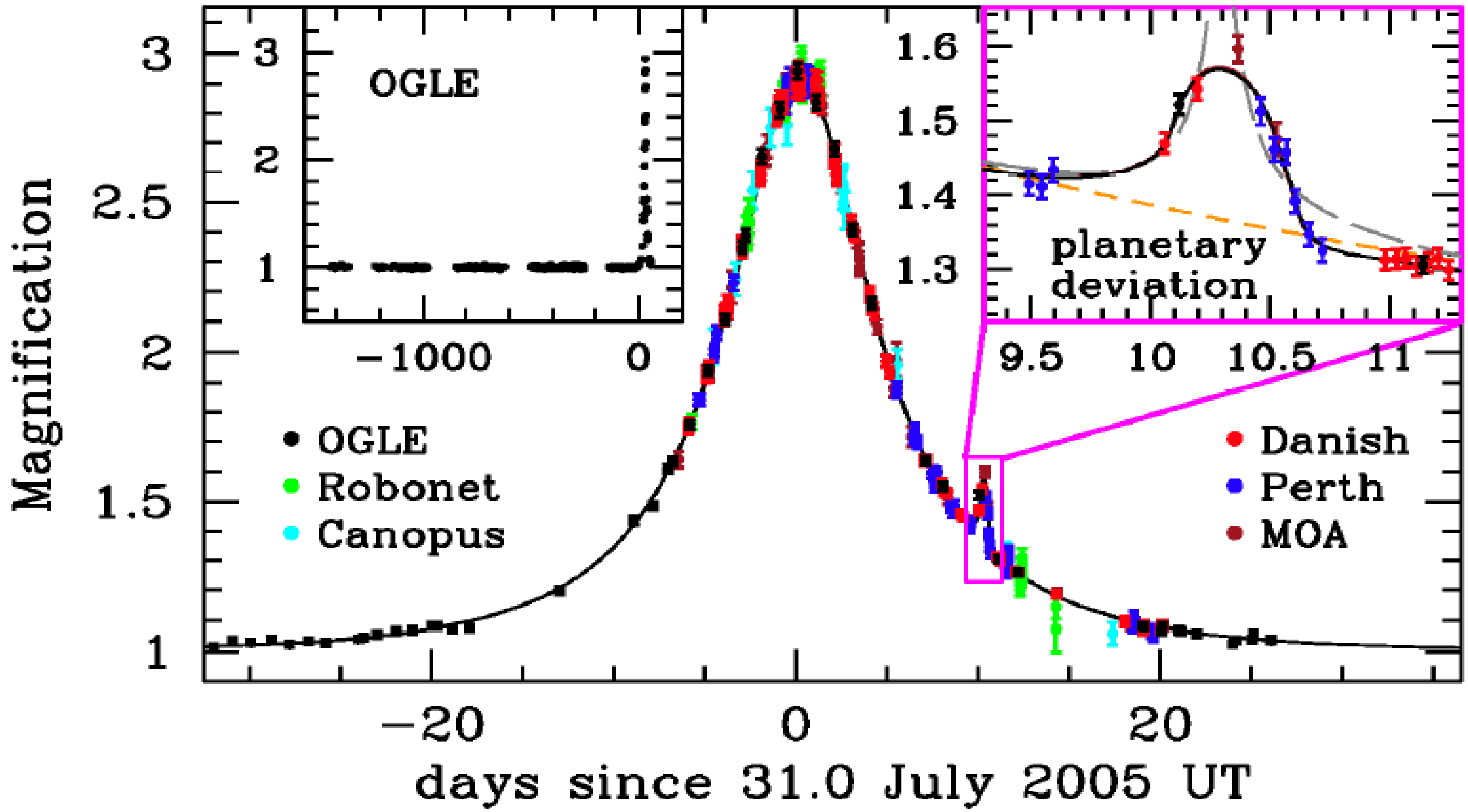
7 Features & 7 Parameters

- Time of Peak
- Height of Peak
- Width of Peak
- Time of Perturbation
- Height of Perturbation
- Width of Perturbation
- Width of Caustic Cr.

- t_0
- u_0
- t_E
- Trajectory angle: α
- Planet-star separation: s
- Planet/star mass ratio: q
- $t_* = \rho * t_E$

OGLE-2005-BLG-390

Solved by Inspection!



Beaulieu et al. 2006, Nature, 439, 437

Source Centered on Point Lens

$$A = \frac{\pi(u_+^2 - u_-^2)}{\pi\rho^2}, \qquad u_\pm = \frac{\rho \pm \sqrt{\rho^2 + 4}}{2}$$

$$A = \sqrt{1 + \frac{4}{\rho^2}} \to 1 + \frac{2}{\rho^2}, \qquad \rho \equiv \frac{\theta_*}{\theta_{\rm E}}$$

Conjecture for Big Source on Planet Caustic

$$A_p = 2\left(\frac{\theta_{{\rm E},p}}{\theta_*}\right)^2$$

Plus Simple Timing Argument

$$\frac{t_p}{t_{\rm E}} = \frac{\theta_*}{\theta_{\rm E}}$$

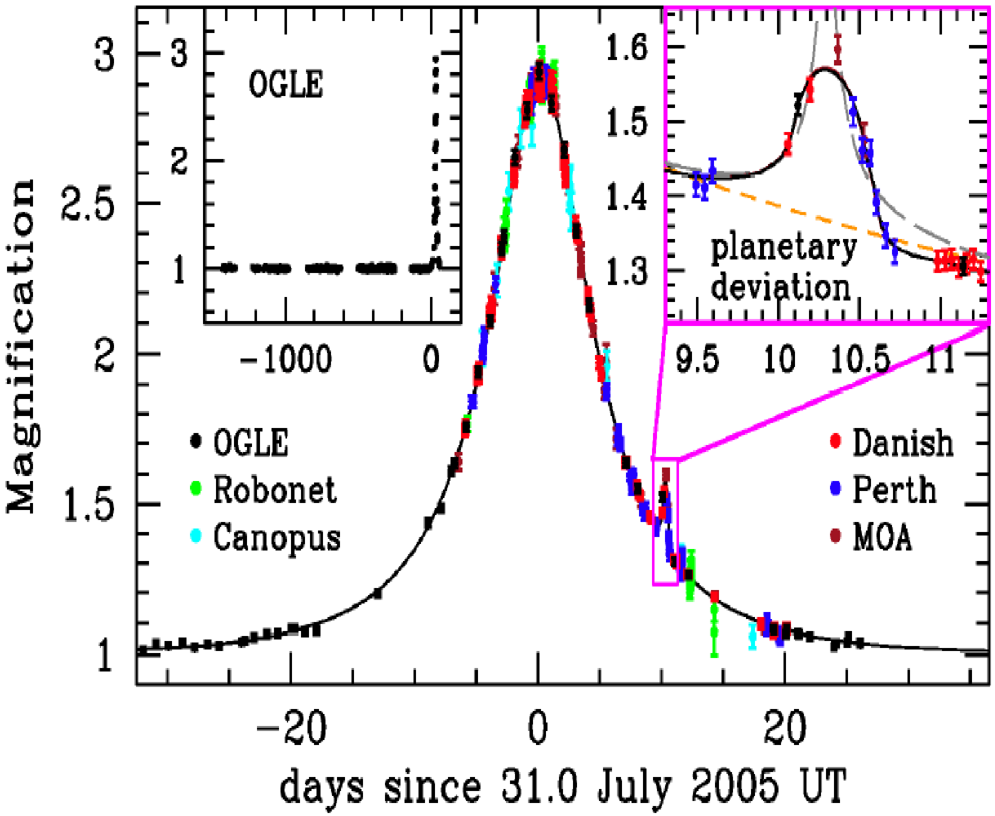
Yields Mass-Ratio Estimate

$$q = \frac{M_p}{M} = \frac{\theta_{{\rm E},p}^2}{\theta_{\rm E}^2} = \frac{\theta_{{\rm E},p}^2}{\theta_*^2}\,\frac{\theta_*^2}{\theta_{\rm E}^2} = \frac{A_p}{2}\,\frac{t_p^2}{t_{\rm E}^2}$$

Mass-Ratio Estimate

a la Gould & Loeb

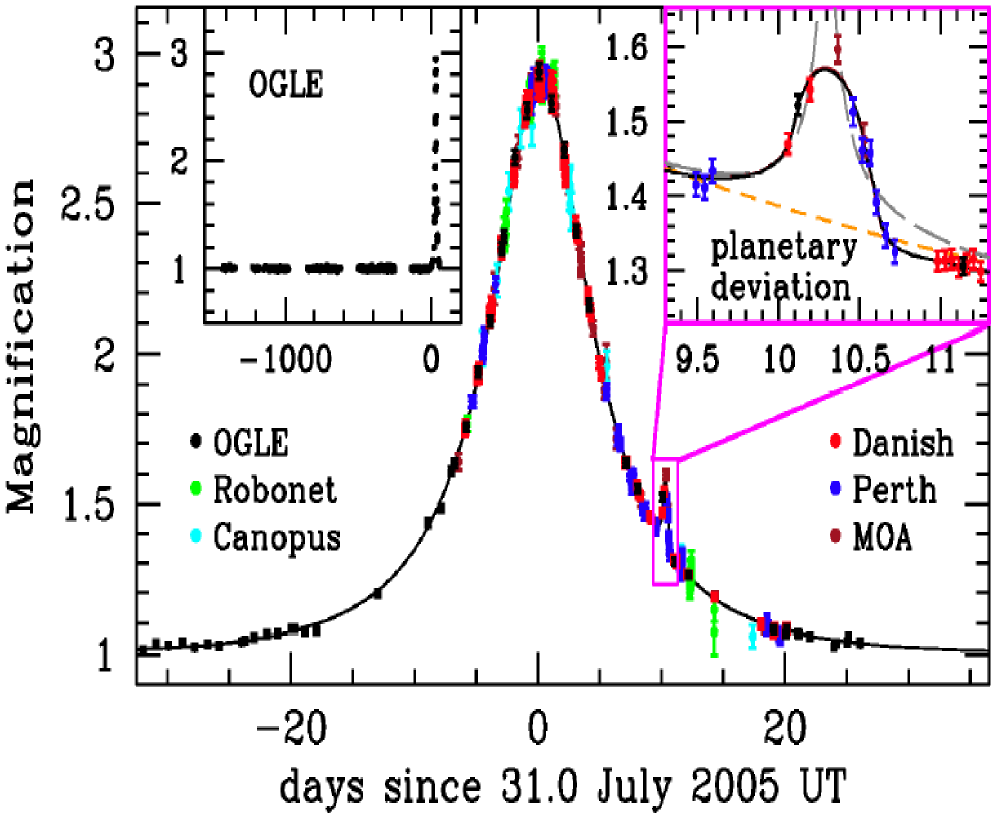
- q=(A_p/2)(t_p/t_E)^2
- A_p = 0.2
- t_p =0.3 day
- t_E=10 day
- q=9e-5
- q_actual = 8e-5



Separation Estimate

a la Gould & Loeb

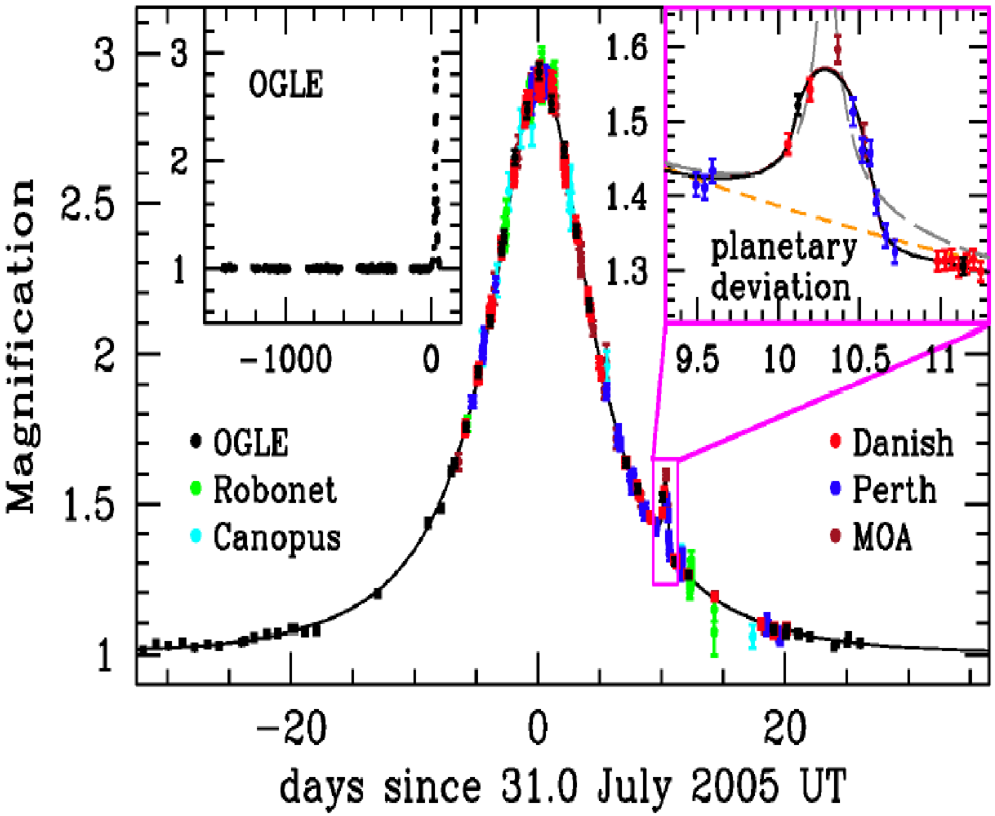
- A_perturb = 1.37
- u(A_perturb) = 0.96
- s = $u_+(u)$ = 1.59
- s_actual = 1.61



Angle Estimate

a la Gould & Loeb

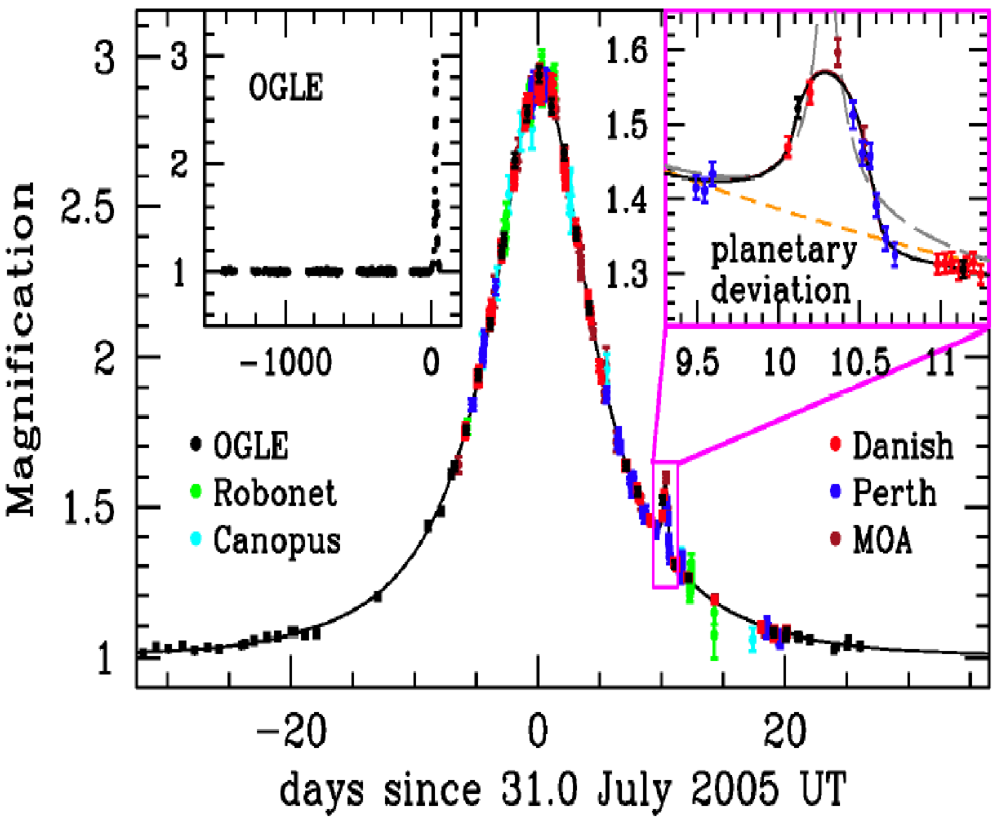
- A_perturb = 1.37
- u(A_perturb) = 0.96
- A_peak = 2.9
- u_0(A_peak) = 0.36
- α = arcsin(u0/u(A_p))
  = 22 deg
- α_actual = 22 deg



Source Size Estimate

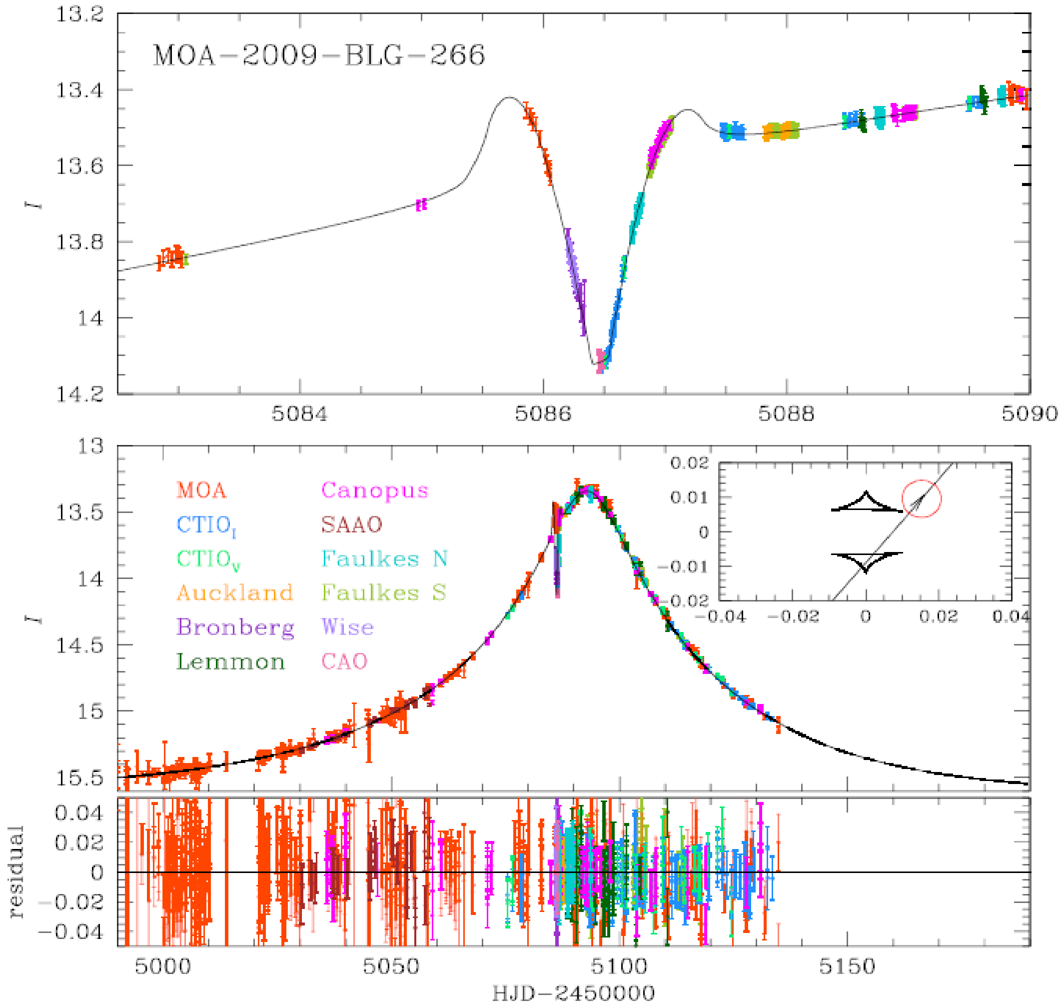
a la Gould & Loeb

- $u_0 = 0.36$; $t_* = 0.3$ day
- $\tau(u=1) = (1-0.36^2)^{1/2} = 0.93$
- $t(A=1.34) = 10$ day
- $t_E = t(1.34)/\tau(1.34) = 10.7$ days
- $\rho = t_*/t_E = 0.28$
- ρ_actual = 0.26



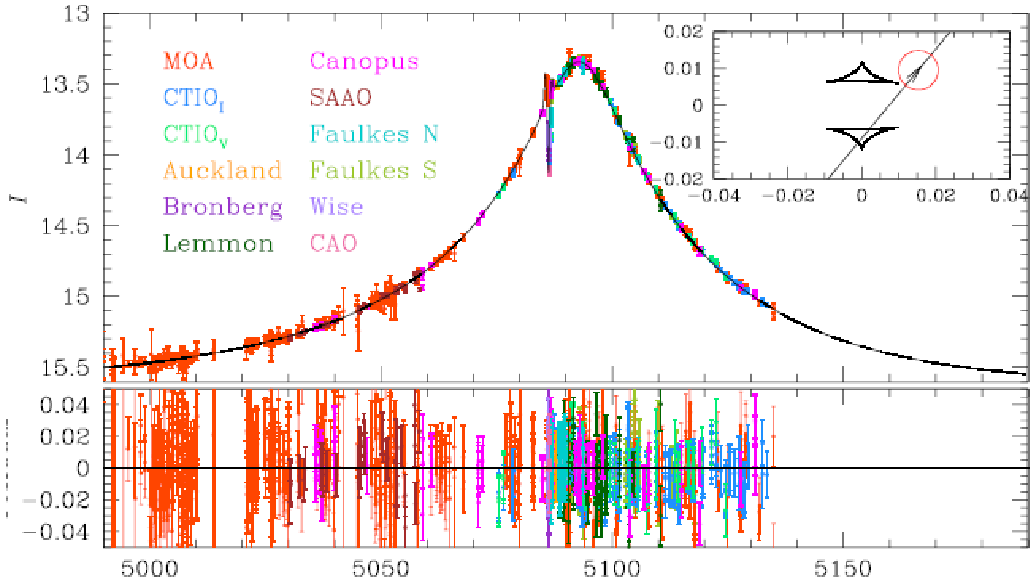
MOA-2009-BLG-266

Minor Image Planetary Caustic



Preliminary Model (Cheongho Han)

MOA-2009-BLG-266



Main Parameters

$$t_0 = 5093.1 \qquad \text{[inspection]}$$

$$u_0 \simeq A_{\rm max}^{-1} = 10^{0.4(I_{\rm peak} - I_{\rm base})} = 0.132$$

$$[I_{\rm peak} = 13.35, \qquad I_{\rm base} = 15.55]$$

$$t_{\rm E,1} = \frac{t_{\rm eff}}{u_0} = \frac{t_0 - t_{1/2,-}}{u_0} = \frac{5093.1 - 5084.7}{0.132} = 64\,\text{day}$$

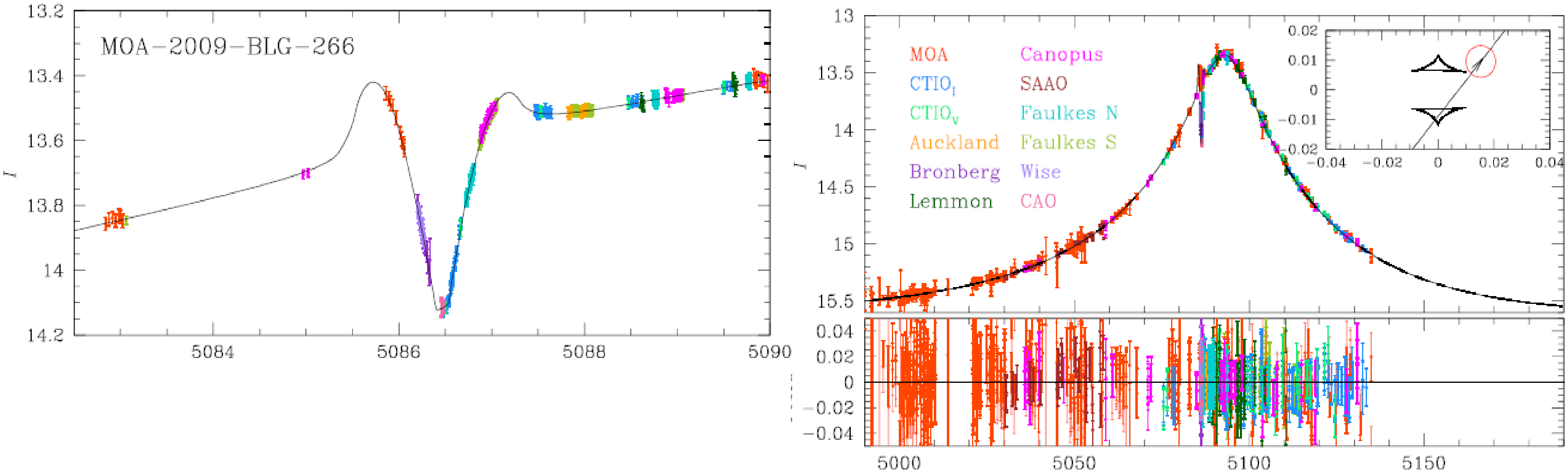
$$t_{1/2,-} = t[I = I_{\rm peak} + 2.5\log 2^{1/2}] = t[I = 13.73] = 5084.7$$

$$t_{\rm E,2} = t_0 - t_{\rm ring,-} = 5093 - 5036 = 57\,\text{day}$$

$$t_{\rm ring,-} = t[I = I_{\rm base} - 2.5\log(9/5)^{1/2}] = t[I = 15.23] = 5036$$

$$t_{\rm E} \to \frac{t_{\rm E,1} + t_{\rm E,2}}{2} = 60\,\text{day}$$

MOA-2009-BLG-266



$$t_{0,\mathrm{planet}} = 5086.5 \qquad \tau_{\mathrm{planet}} = \frac{t_0 - t_{0,\mathrm{planet}}}{t_{\mathrm{E}}} = \frac{6.6}{60} = 0.11$$

$$u_{\mathrm{planet},1} = A^{-1}_{\mathrm{planet}} = 10^{0.4(I_{\mathrm{plan}} - I_{\mathrm{base}})} = 0.154 \qquad [I_{\mathrm{planet}} = 13.58]$$

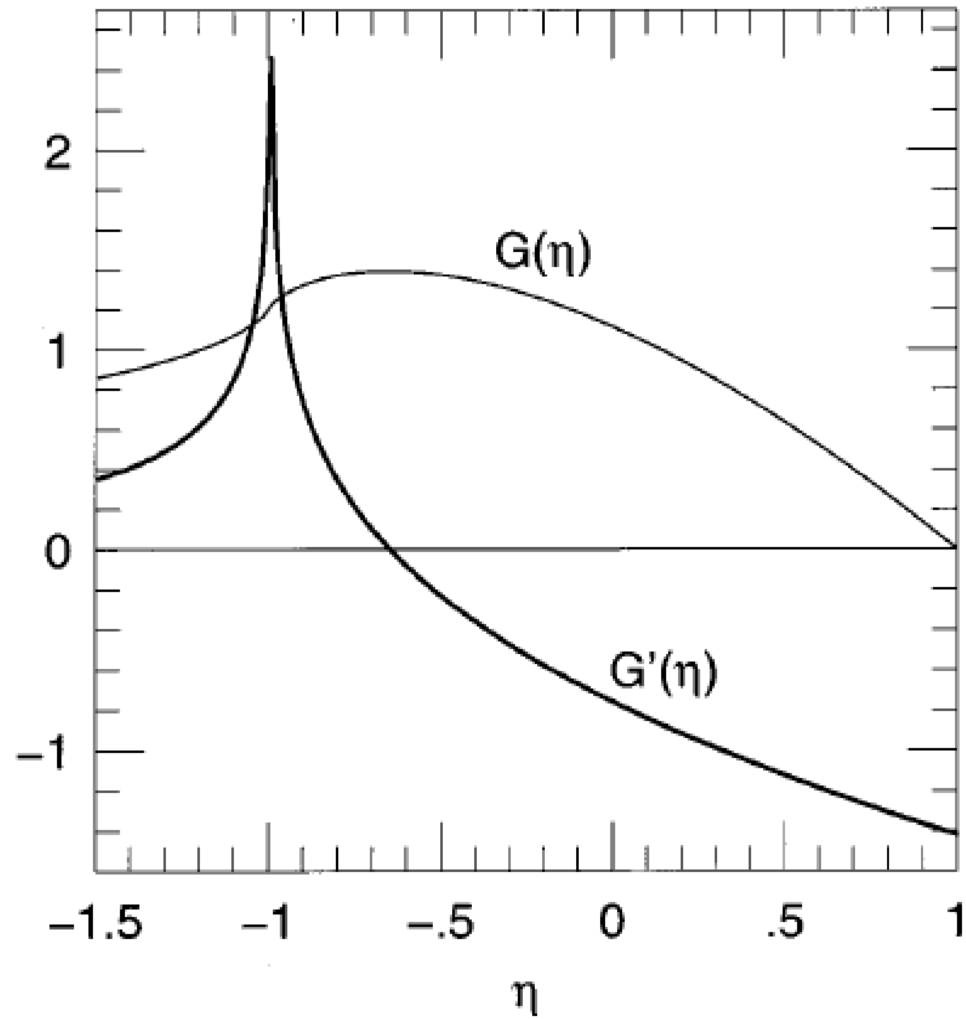
$$u_{\mathrm{planet},2} = \sqrt{u_0^2 + \tau^2_{\mathrm{planet}}} = 0.172$$

$$u_{\mathrm{planet}} = \frac{u_{\mathrm{planet},1} + u_{\mathrm{planet},2}}{2} = 0.163$$

$$s = \frac{-u_{\mathrm{planet}} + \sqrt{u^2_{\mathrm{planet}} + 4}}{2} = 0.922$$

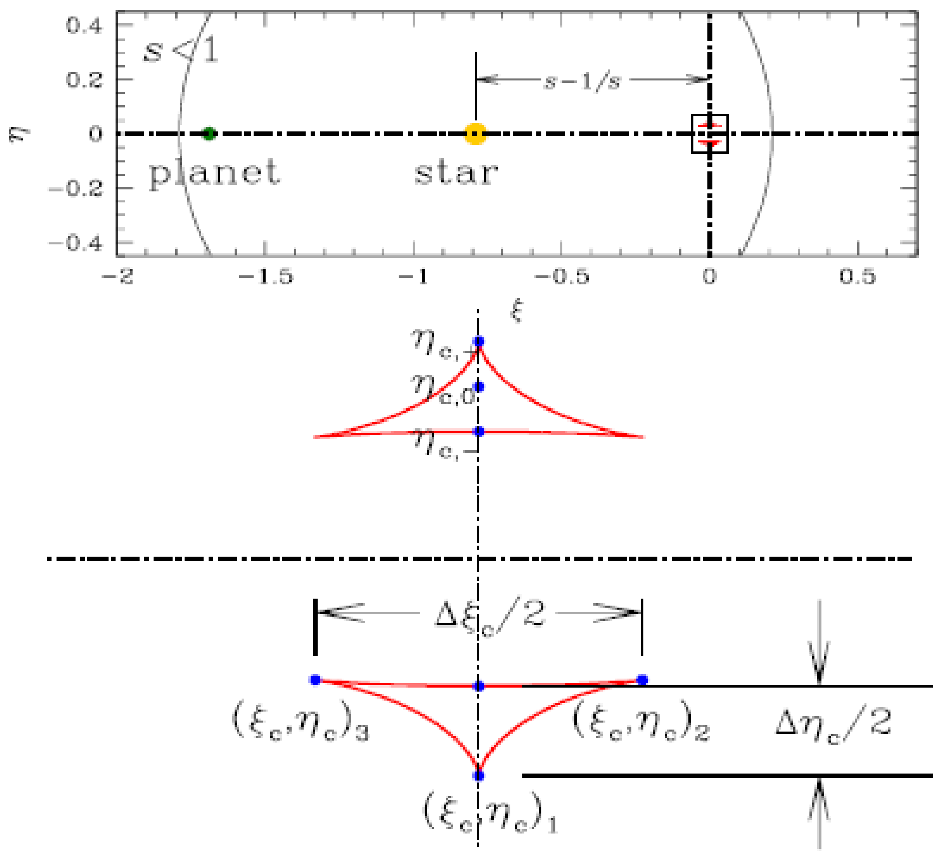
$$\alpha = \sin^{-1}\frac{u_0}{u_{\mathrm{planet}}} = 54^\circ$$

Generic Caustic Exit



Gould & Andronov 1999, ApJ, 516, 236

Minor Image Analytic Formulae



$$\eta_{c,-} = 2q^{1/2}(1-s^2)^{1/2}/s$$

Han 2006, ApJ, 638, 1080

MOA-2009-BLG-266

Planet Parameters II: harder

$$t_{\rm cross,1} = \frac{t_{\rm planet-peak,1} - t_{\rm planet-trough,1}}{1.7} = 0.41\,{\rm day}$$

$$t_{\rm cc,1} = t_{\rm planet-peak,1} + 0.7 * t_{\rm cross,1} = 5085.98$$

$$t_{\rm planet-peak,1} = 5085.7, \qquad t_{\rm planet-trough,1} = 5086.4$$

$$t_{\rm cross,2} = \frac{t_{\rm planet-peak,2} - t_{\rm planet-trough,2}}{-1.7} = 0.38\,{\rm day}$$

$$t_{\rm cc,2} = t_{\rm planet-peak,2} - 0.7 * t_{\rm cross,2} = 5086.93$$

$$t_{\rm planet-peak,2} = 5087.2, \qquad t_{\rm planet-trough,1} = 5086.55$$

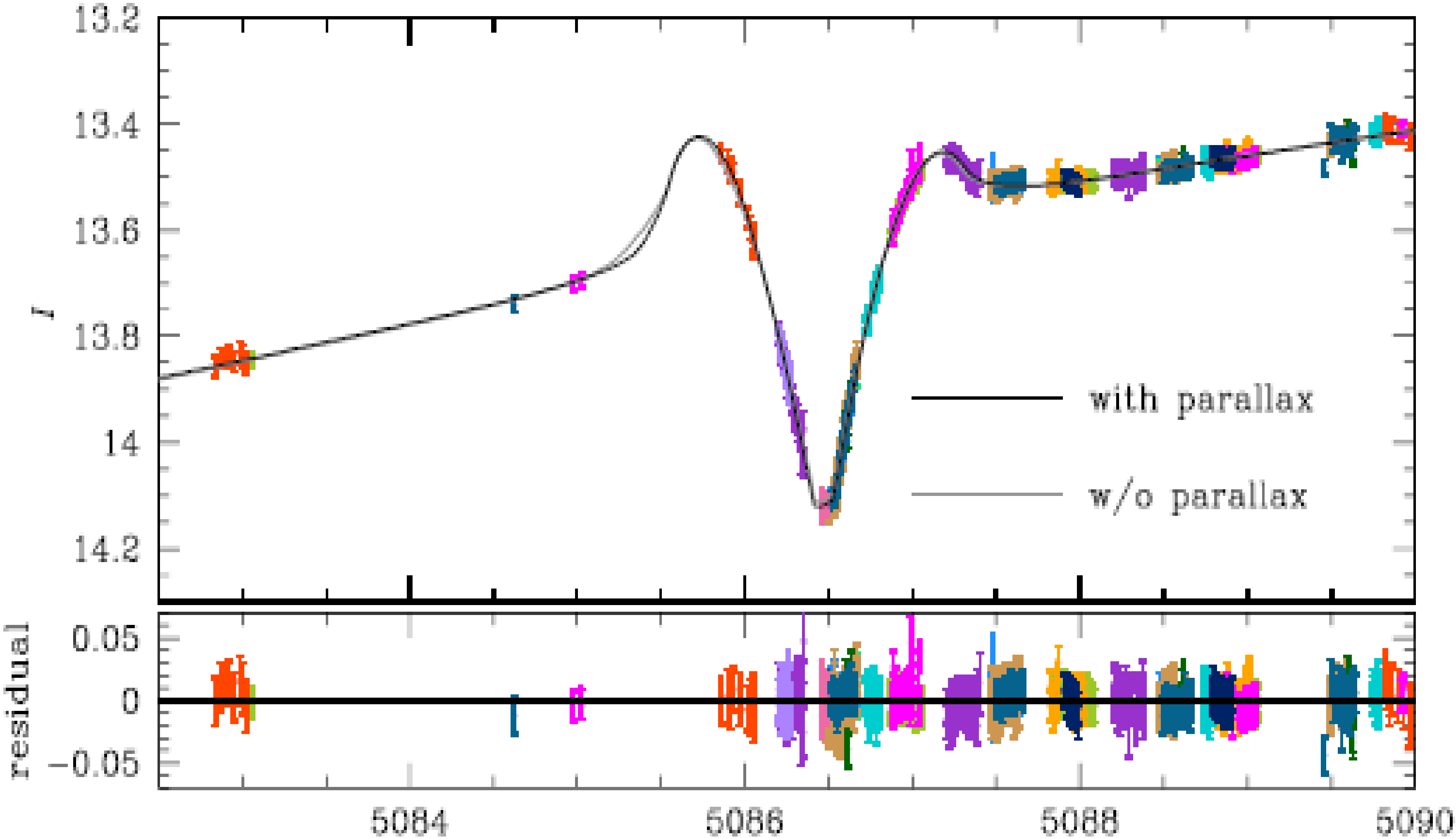
$$t_{\rm cross} = \frac{t_{\rm cross,1} + t_{\rm cross,2}}{2} = 0.397\,{\rm day}$$

$$\Delta u = \frac{t_{\rm cc,2} - t_{\rm cc,1}}{t_{\rm E}} \sin\alpha = 0.0128$$

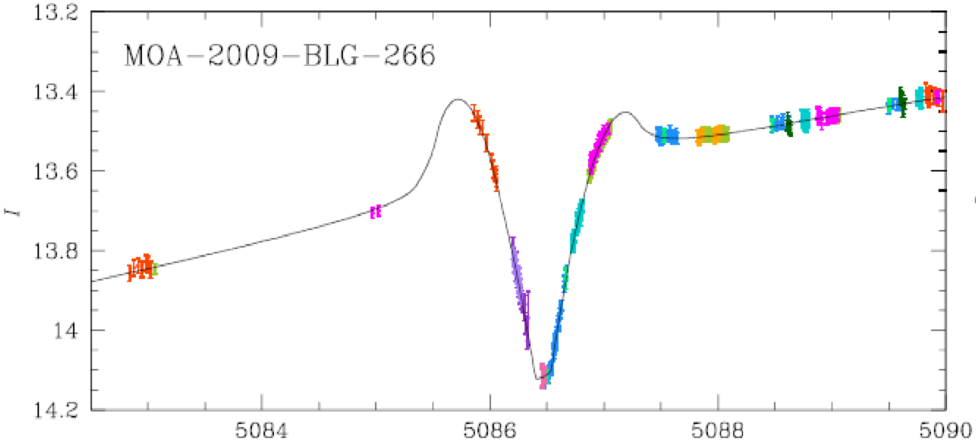
$$\Delta u = 4\sqrt{\frac{q u_{\rm planet}}{s}} \Rightarrow q = \frac{s}{u_{\rm planet}}\left(\frac{\Delta u}{4}\right)^2 = 5.8 \times 10^{-5}$$

$$t_* = t_{\rm cross}\sin\alpha = 0.32\,{\rm day}, \qquad \rho = \frac{t_*}{t_{\rm E}} = 5.3 \times 10^{-3}$$

MOA-2009-BLG-266



MOA-2009-BLG-266



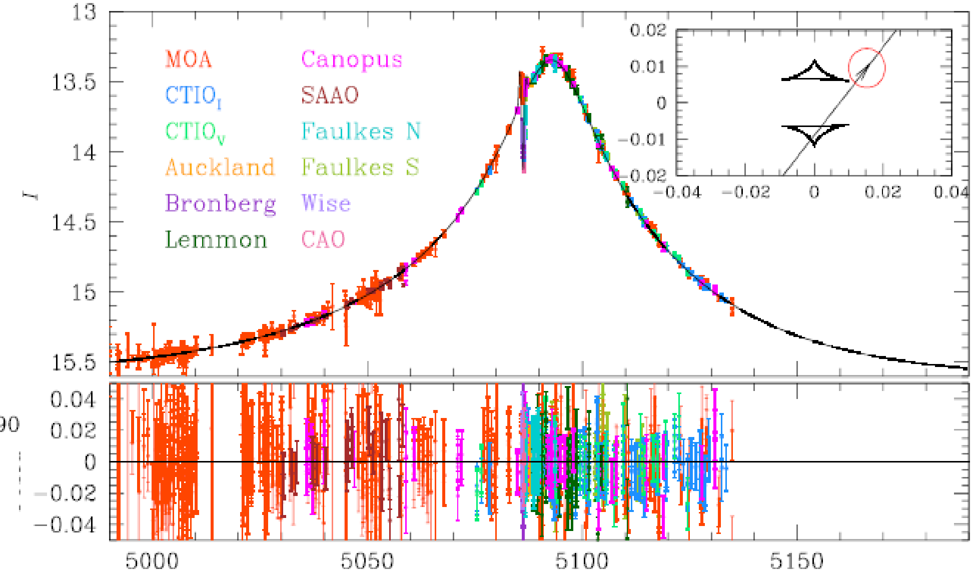


TABLE 1

MB09266: Eye vs. Computer

Parameter	Eye	Computer
t_0	5093.1	5093.07
u_0	0.13	0.13
t_E	60 d	60.2 d
q	5.8×10^{-5}	5.4×10^{-5}
s	0.922	0.914
α	54°	51°
ρ	5.3×10^{-3}	5.3×10^{-3}

Minor Image Test

$$\frac{A_{\rm trough}}{A_{\rm planet}} = 10^{0.4(I_{\rm planet} - I_{\rm trough})}$$

$$= 10^{0.4(13.58-14.02)} = 0.667$$

$$\frac{A_{\rm planet}+1}{2A_{\rm planet}} = 0.657$$

Why Planetary Caustics?

$$\gamma = \frac{q}{s^2} \qquad (\gamma \ll 1)$$

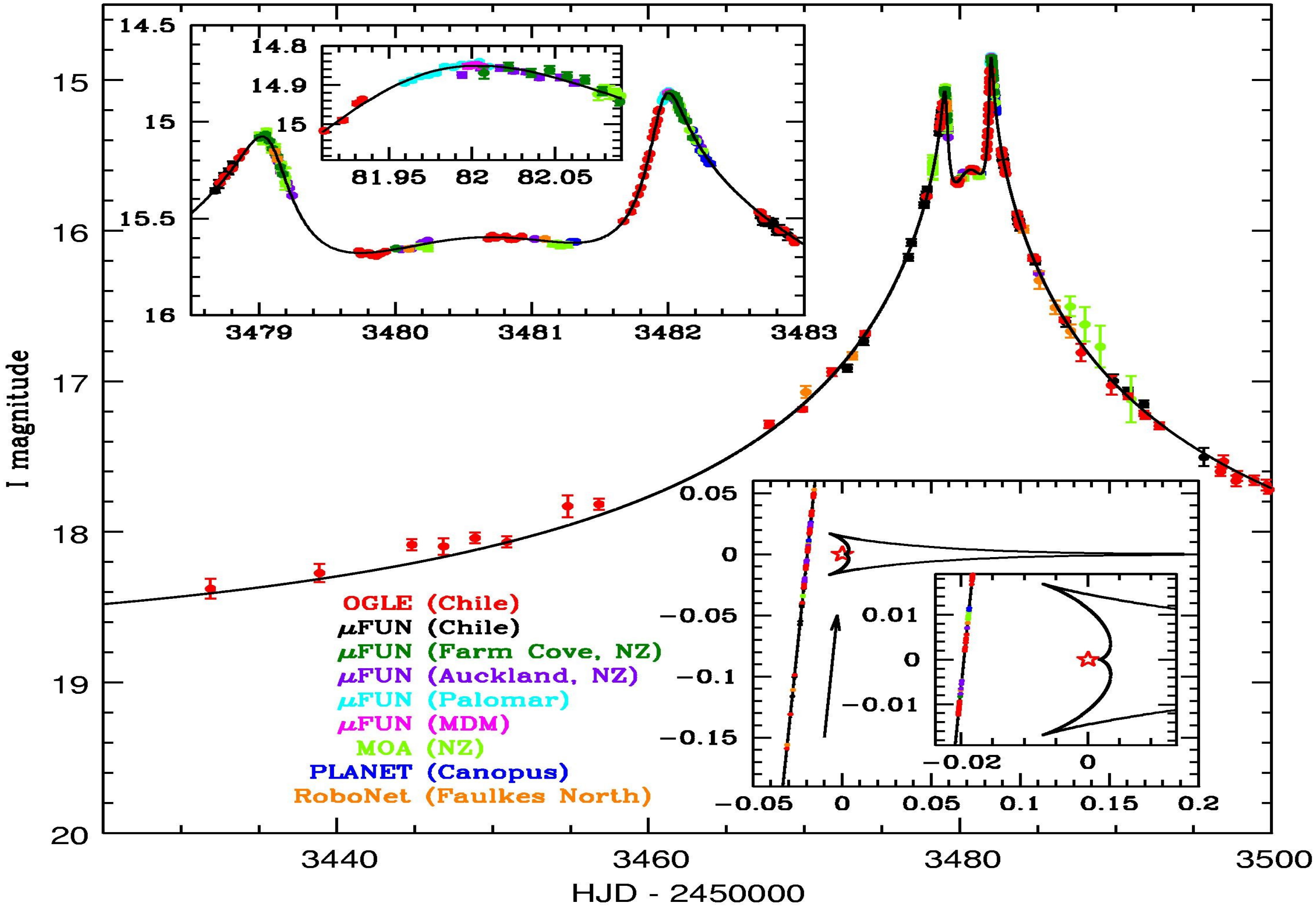
$$\theta_\gamma = 4\gamma\theta_{\rm E} = 4\frac{q}{s^2}\theta_{\rm E}$$

$$\theta_\gamma = 4\frac{m/M}{(\Delta\theta/\theta_{\rm E})^2}\theta_{\rm E} = 4\frac{m\theta_{\rm E}^3}{M(\Delta\theta)^2}$$

$$\theta_\gamma = 4\frac{m(\kappa\pi_{\rm rel}M)^{3/2}}{M(\Delta\theta)^2} = 4\sqrt{mM}\frac{(\kappa\pi_{\rm rel})^{3/2}}{(\Delta\theta)^2}m^{1/2}$$

Why Central Caustics?

First "High-Mag" Event



Udalski et al. 2005, ApJ, 628, L109

Amateurs + Professionals

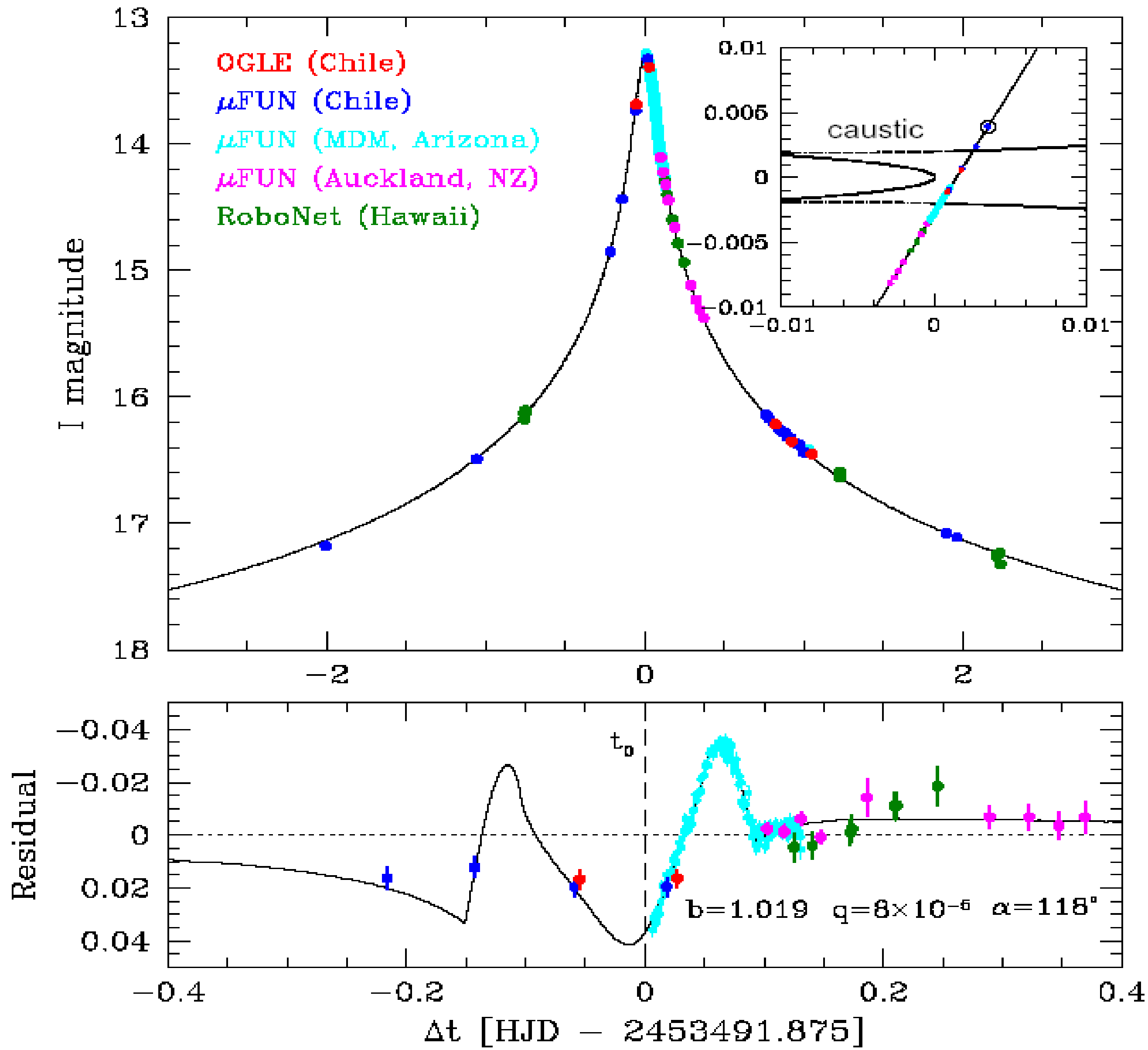
Grant, Ian, Jennie, Phil

Amateurs + Professionals

"It just shows that you can be a mother, you can work full-time, and you can still go out there and find planets."

Jennie McCormick
(Amateur Astronomer, Auckland, New Zealand)

OGLE-2005-BLG-169:
Second High-Mag Event

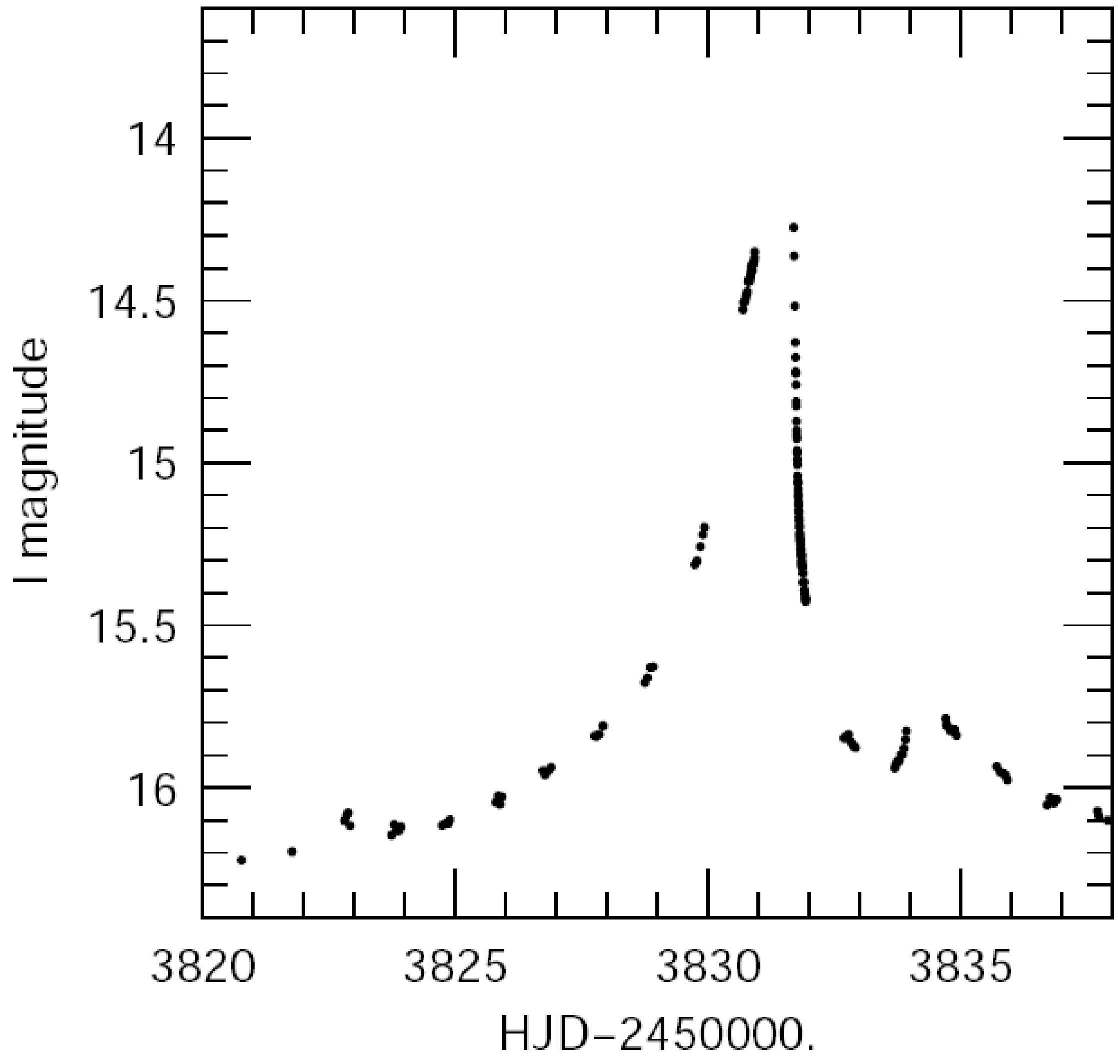


Deokkeun An


ASTRONOMY TODAY
MODERN PHYSICS
NUMERICAL RECIPES in C
OHI STATE

OGLE-2006-BLG-109:

Third High-Mag Event (OGLE only)



Solution Search Methods I:

(s,q) Grid

- Construct a grid of (s,q) maps ("map making")
- Make plausible guess for (t_0, u_0, t_E, ρ)
- Work on sequence of α, ($0<\alpha<2\pi$)
- MCMC
  - Start (s,q,α,t_0, u_0, t_E, ρ), vary (α,t_0, u_0, t_E, ρ)
- Fit: $f_i(t_k) = f_{s,i} * A(t_k) + f_{b,i}$ (i=1 ... n_obs)

Solution Search Methods II:

Intelligent Seeds

- Make plausible guess for (t_0, u_0, t_E, ρ)
- Choose set of (s,q) seeds that sample topologies
- Work on sequence of α, $(0<\alpha<2\pi)$
- MCMC vary $(s,q,\alpha,t_0, u_0, t_E, \rho)$ [All]
- Fit: $f_i(t_k) = f_{s,i} * A(t_k) + f_{b,i}$ (i=1 ... n_obs)
- Requires Contour or Adaptive Ray Shooting

Microlensing vs. Other Methods

- No Light from Planet/Host System
  - Distant planets
  - Low-mass planets
  - Free floating planets
- Einstein-Ring/Snow-Line Coincidence
- Host Rarely Seen
- Usually Just a Snap Shot

Microlensing vs. Other Methods I: Free-Floating Planets

- Crucial to Planet-Formation Theories
- Microlensing: Only FFP Method
- Key characteristic: Mass/Not Light

Free-Floating Planets

Point-Lens Events w/o FFPs (short)

$$\Gamma \propto \int dM F(M) \int dD_L D_L^2 n(D_L) \int d^2\mu \mu f_\mu(\mu) \theta_E(M, D_L)$$

$$t_E = \frac{\theta_E}{\mu}, \qquad \theta_E = \sqrt{\kappa M \pi_{rel}}$$

$$t_E \text{ small} \Rightarrow D_{LS} \ll D_S$$

$$dD_L D_L^2 n(D_L) \to dD_{LS} D_S^2 n(D_S) = K dD_{LS}; \quad \theta_E \to \sqrt{\frac{\kappa \mathrm{AU}\, M}{D_S^2} D_{LS}}$$

$$\Gamma \propto \int dM F(M) M^{1/2} \int d^2\mu\, \mu f_\mu(\mu) \int d \ln D_{LS} D_{LS}^{3/2}$$

$$\frac{d\Gamma}{d \ln t_E} \propto t_E^3 \int dM F(M) M^{-1} \int d^2\mu \mu f_\mu(\mu)$$

Free-Floating Planets

Point-Lens Events w/o FFPs (long)

$$\Gamma \propto \int dM F(M) \int dD_L D_L^2 n(D_L) \int d^2\mu \mu f_\mu(\mu) \theta_\mathrm{E}(M, D_L)$$

$$t_\mathrm{E} = \frac{\theta_\mathrm{E}}{\mu}, \qquad \theta_\mathrm{E} = \sqrt{\kappa M \pi_\mathrm{rel}}$$

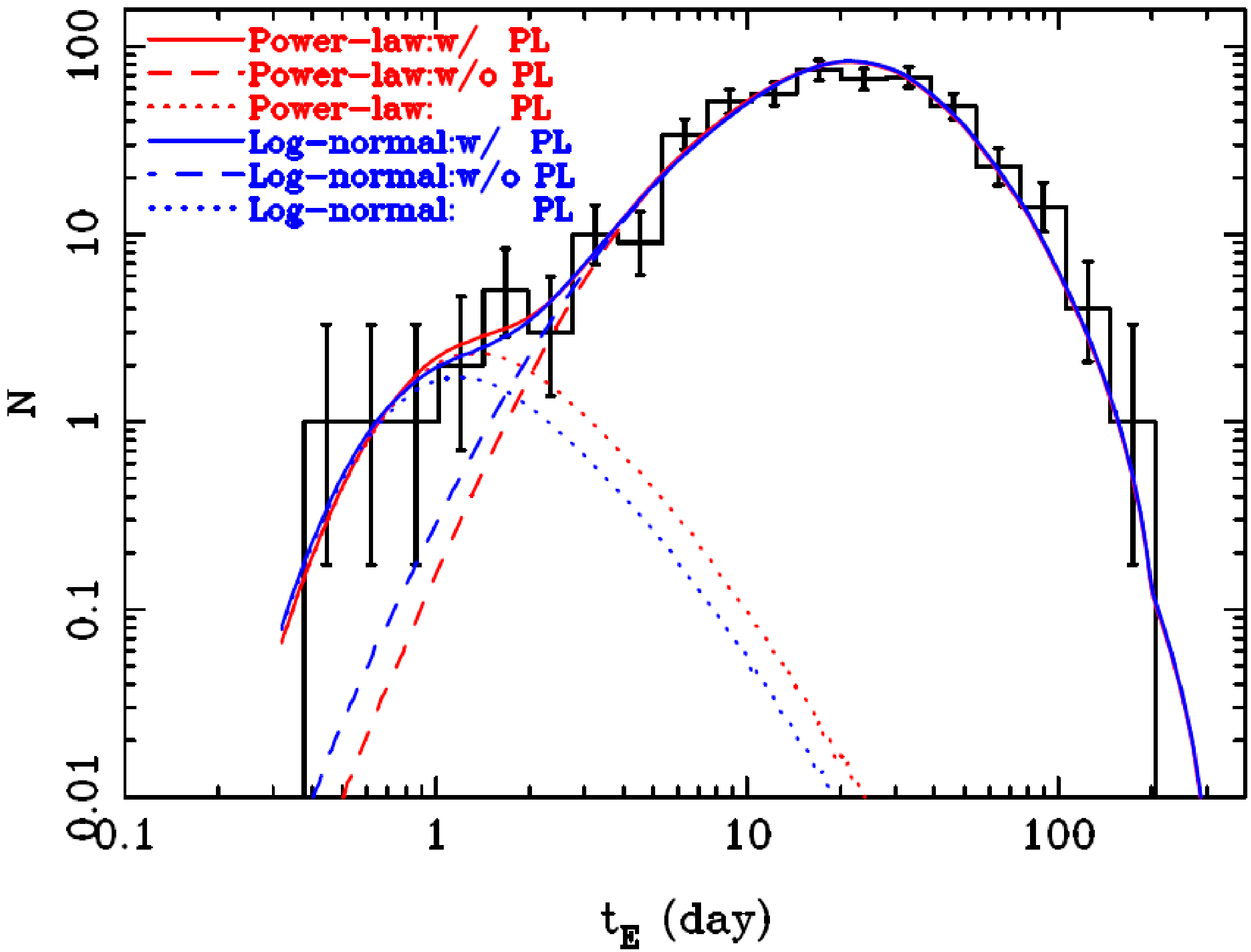
$$t_\mathrm{E} \text{ big} \Rightarrow \mu \rightarrow 0$$

$$d^2\mu \mu f_\mu(\mu) \rightarrow d\mu\, \mu^2 f_\mu(0) = K\, d\ln\mu\, \mu^3$$

$$\frac{d\Gamma}{d\ln t_\mathrm{E}} \propto t_\mathrm{E}^{-3} \int dM F(M) \int dD_L\, D_L^2 n(D_L) \theta_\mathrm{E}^4$$

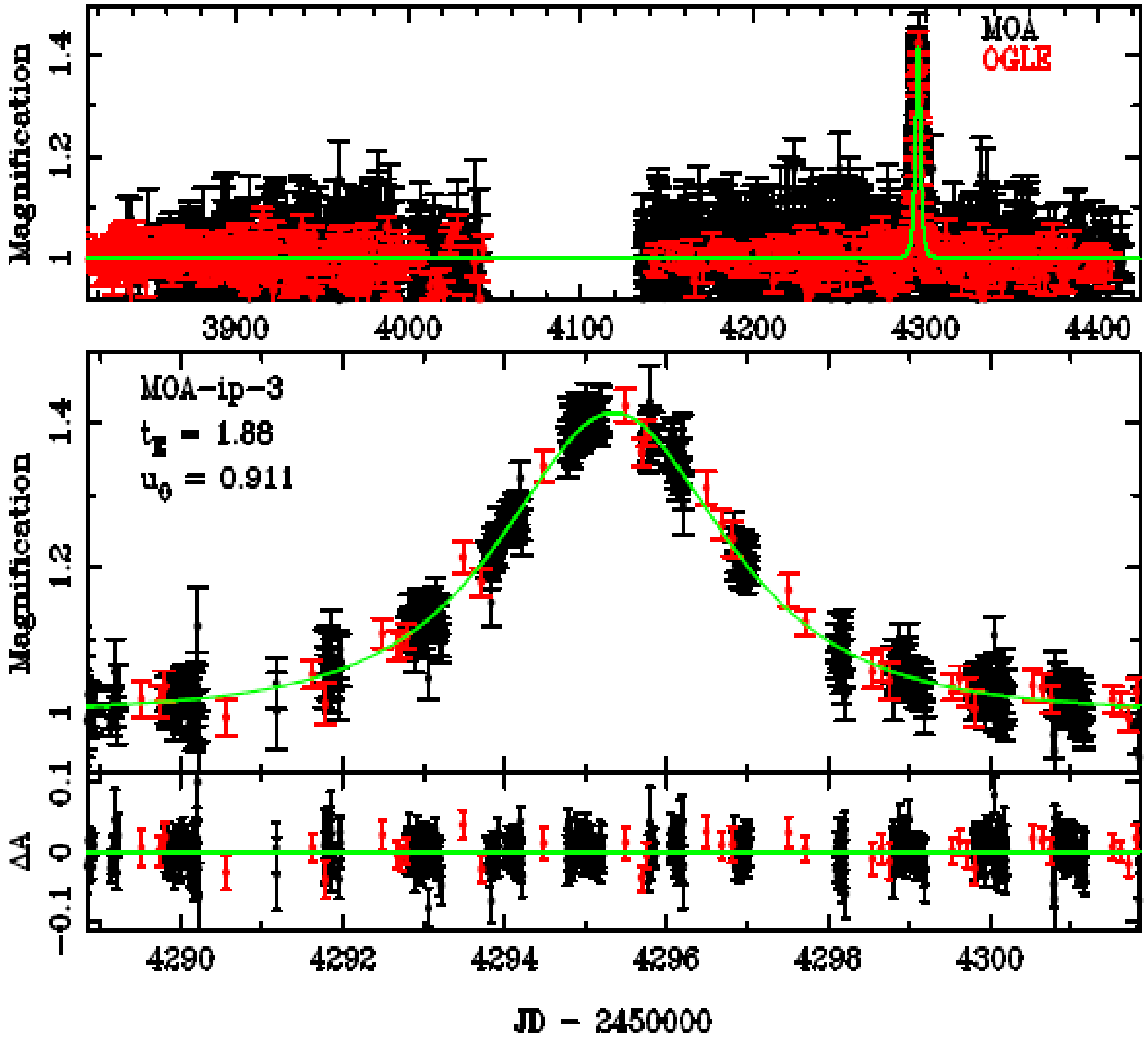
$$\frac{d\Gamma}{d\ln t_\mathrm{E}} \propto t_\mathrm{E}^{-3} \int dM F(M) M^2 \int dD_{LS}\, D_{LS}^2 n(D_{LS})$$

MOA Point-Lens Events

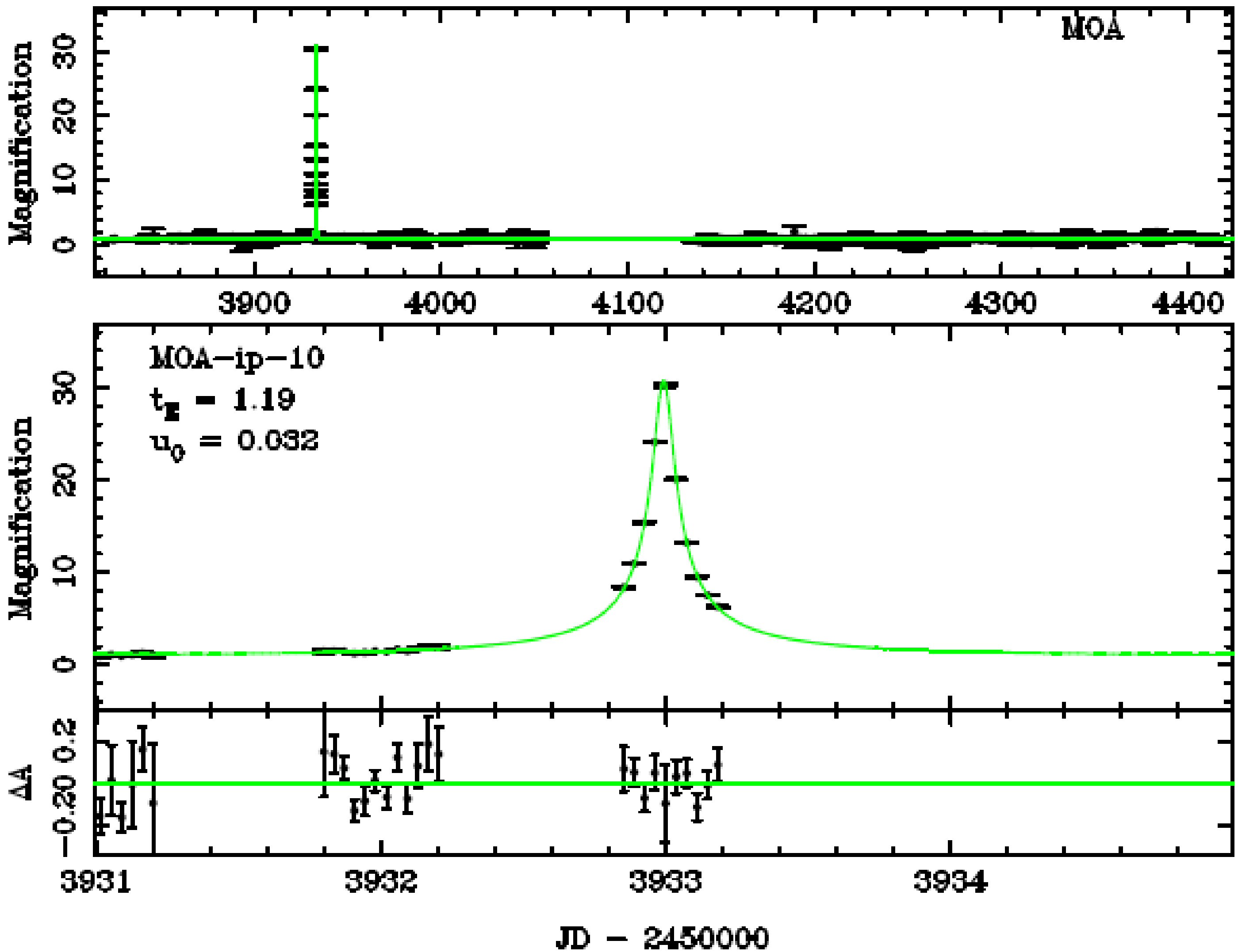


Sumi et al. 2011, Nature, 473, 349

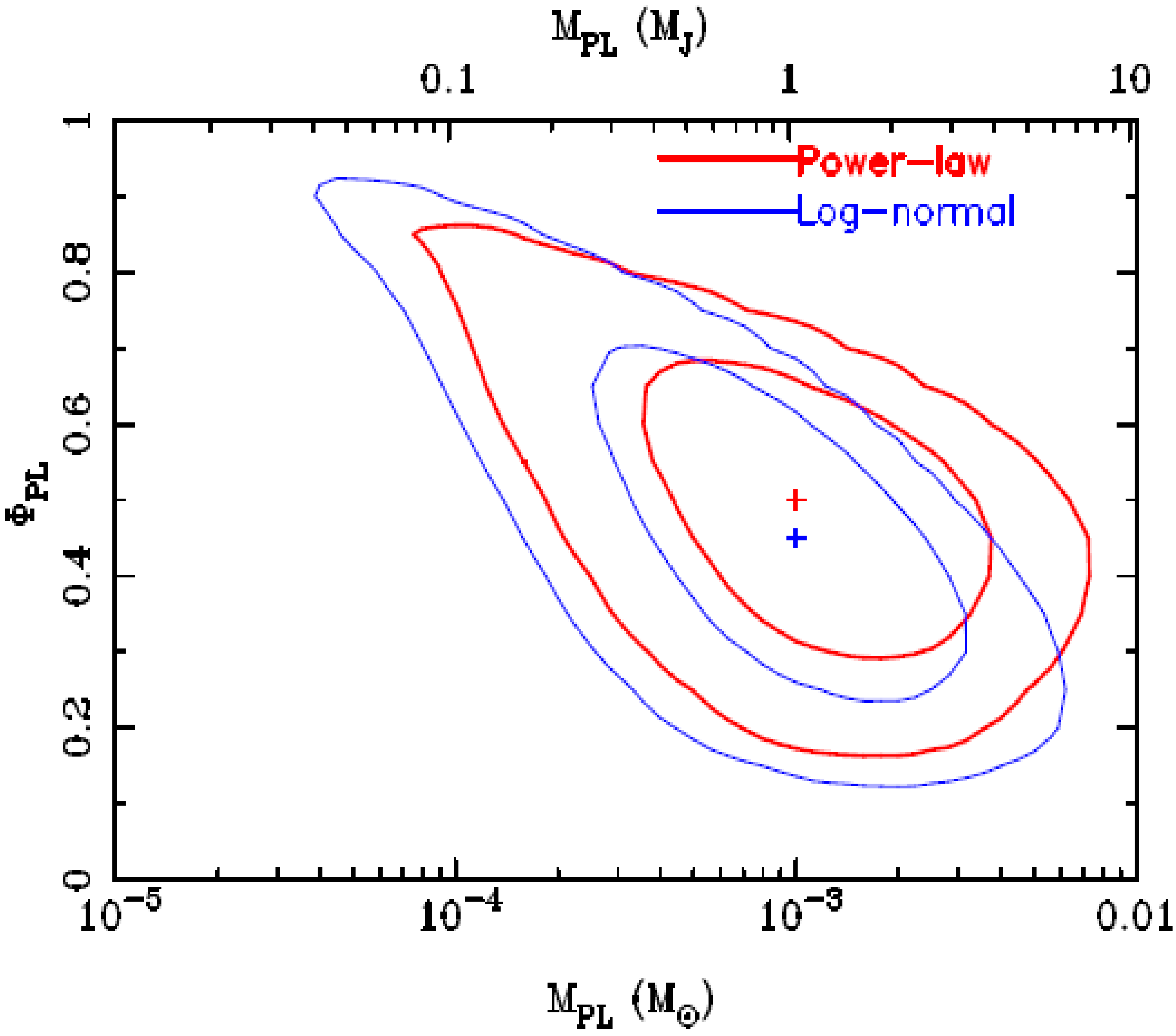
Sample Event 1



Sample Event 2



FFP Best-Fit Characteristics



Microlensing vs. Other Methods II:

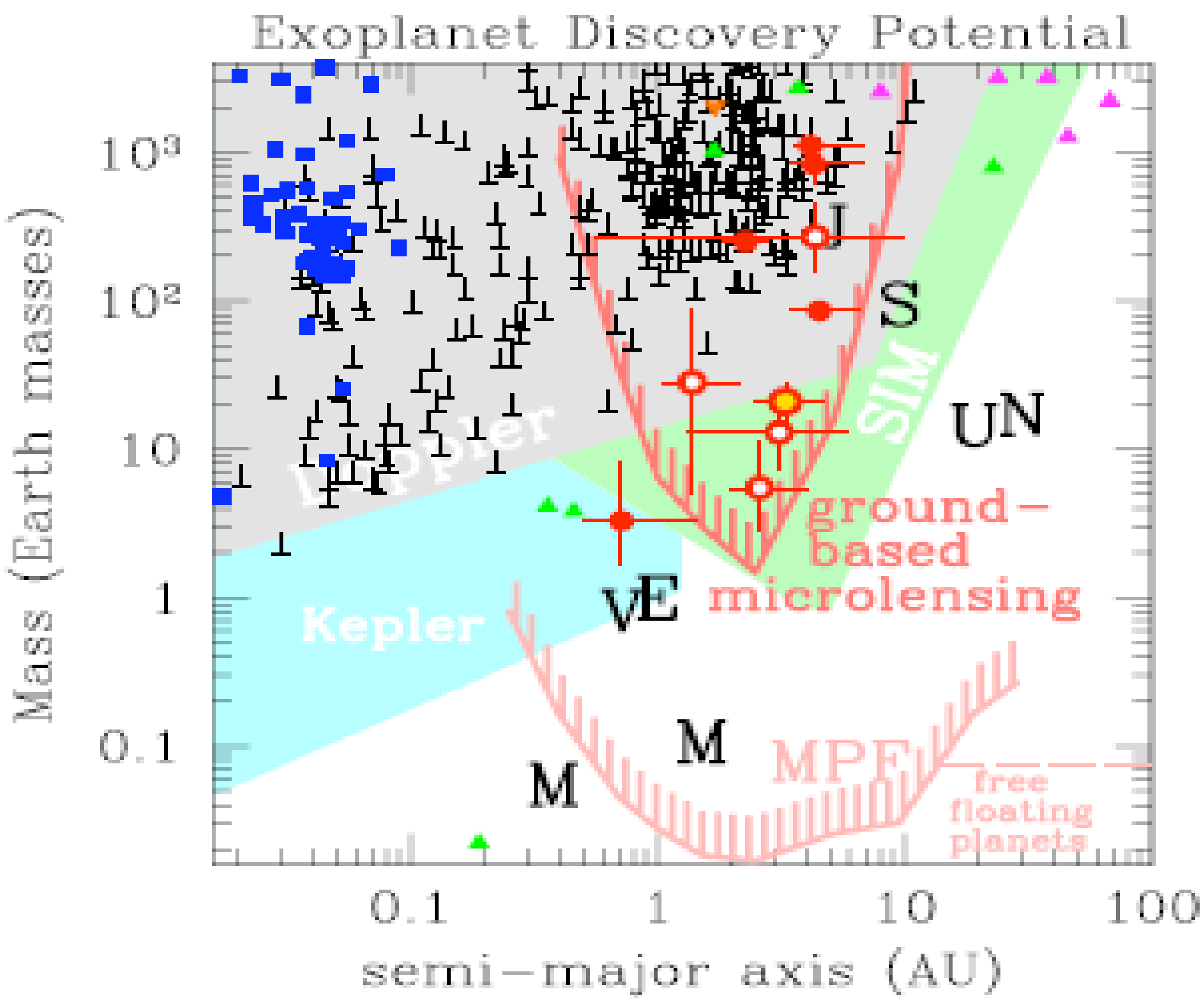
Bound-Planet Parameter Space

- Sensitivity Peaks Just Beyond Snow Line
  - Where giant planets are thought to form
- Key characteristic: Einstein radius

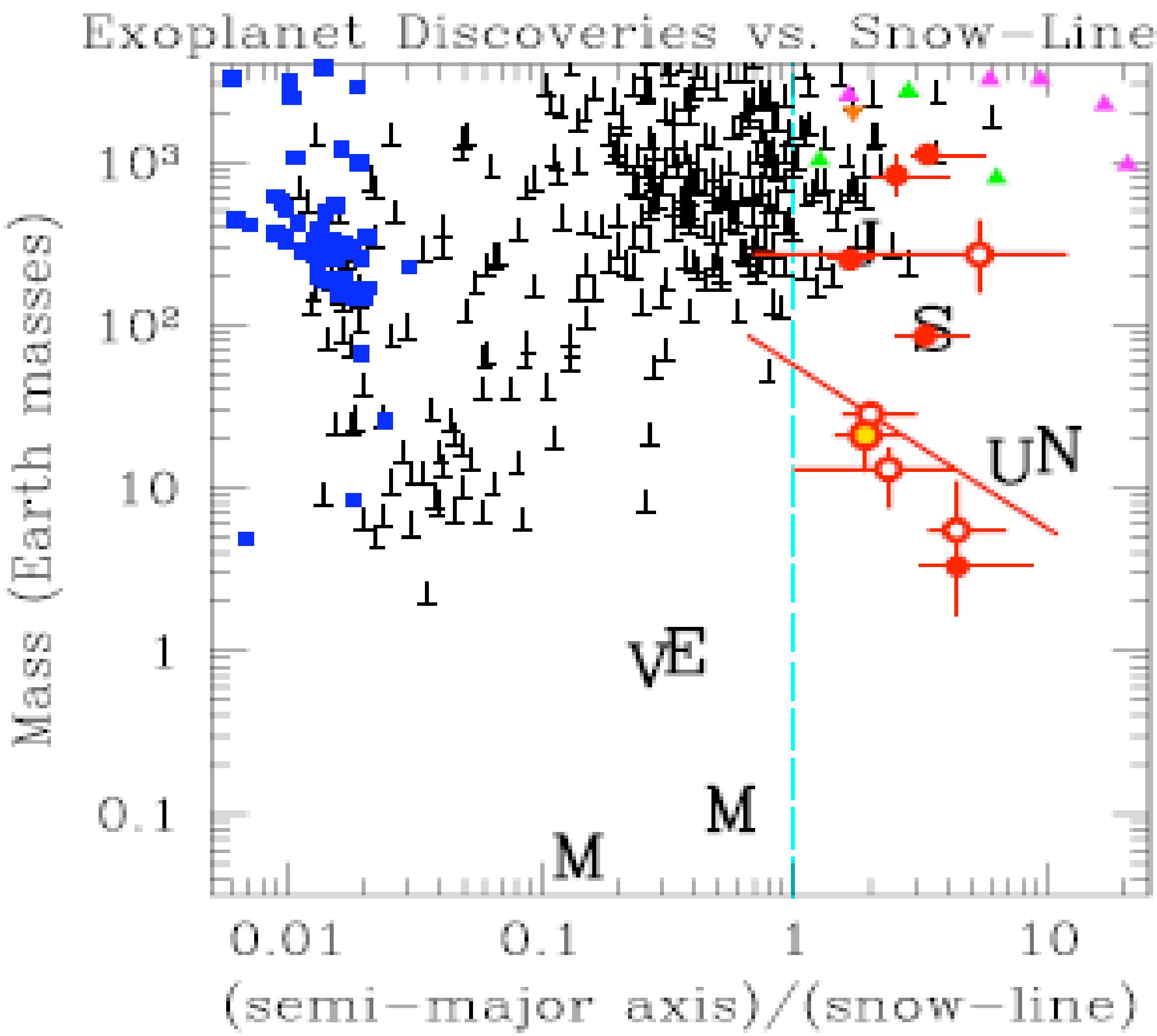
$$r_{\rm E} = 4\,{\rm AU}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D_L D_{LS}}{16\,{\rm kpc}^2}\right)^{1/2}$$

$$r_{\rm snow} = 2.7,{\rm AU}\frac{M}{M_\odot}$$

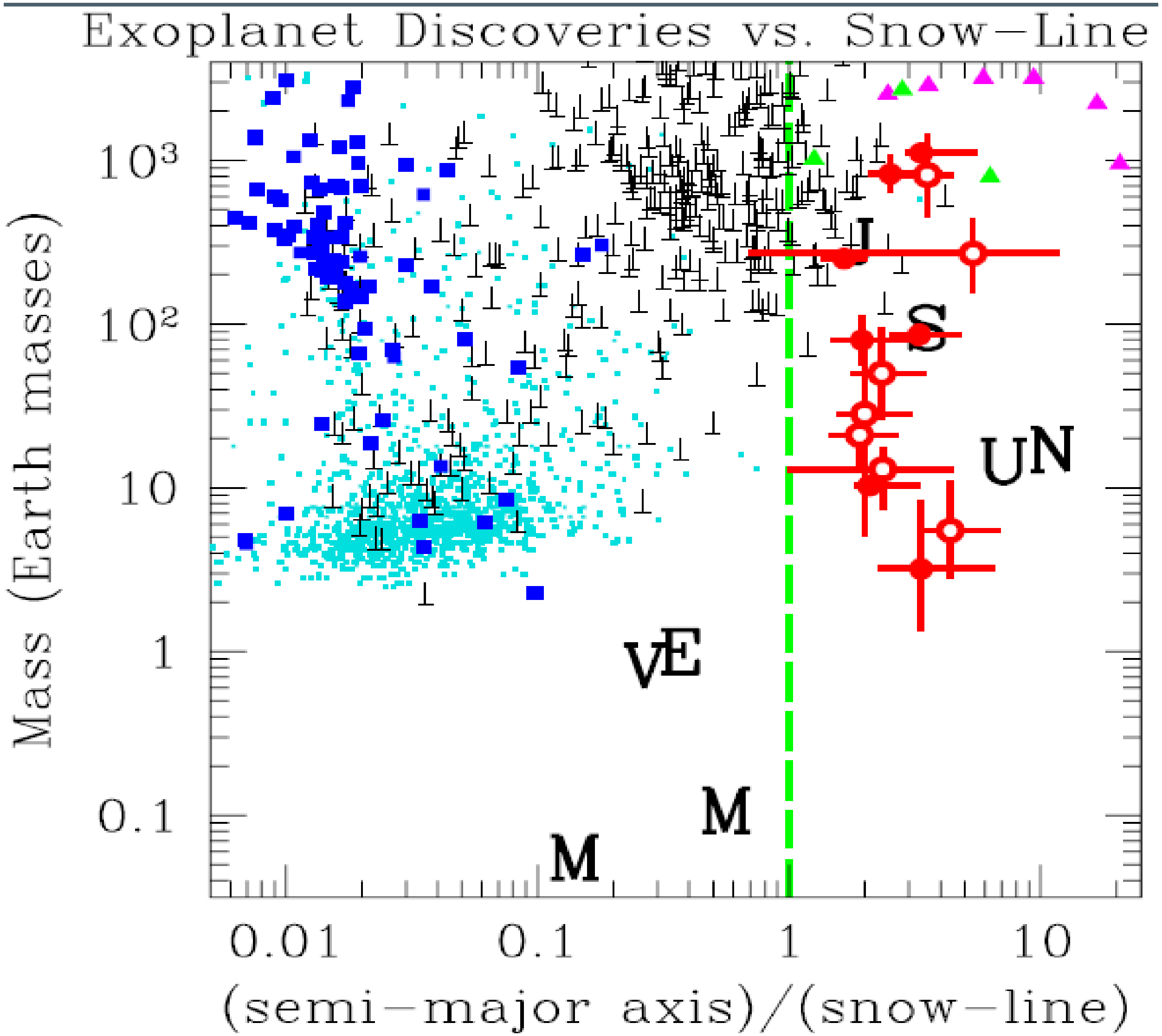
Planets 2010



Planets 2010



Planets 2011

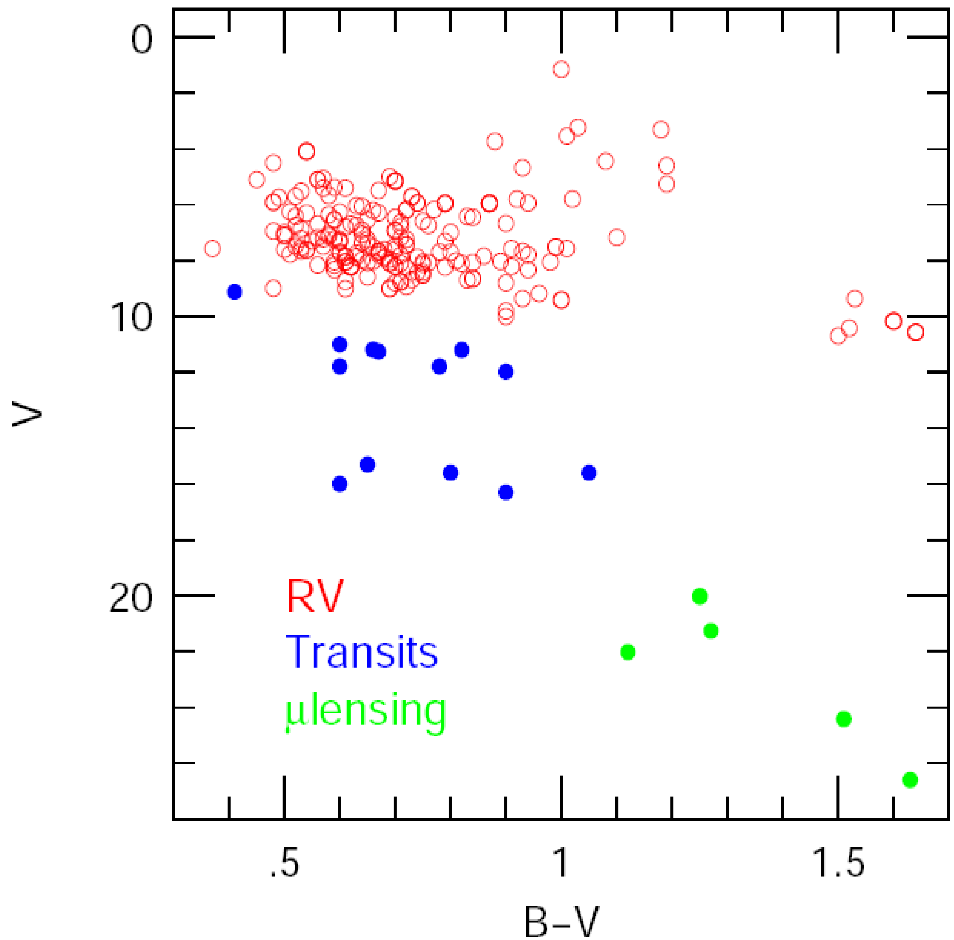


Microlensing vs. Other Methods III: Faint/Dim Hosts

- Other methods: struggle
- Microlensing: comes naturally
- Key Characteristic: Mass/Not Light

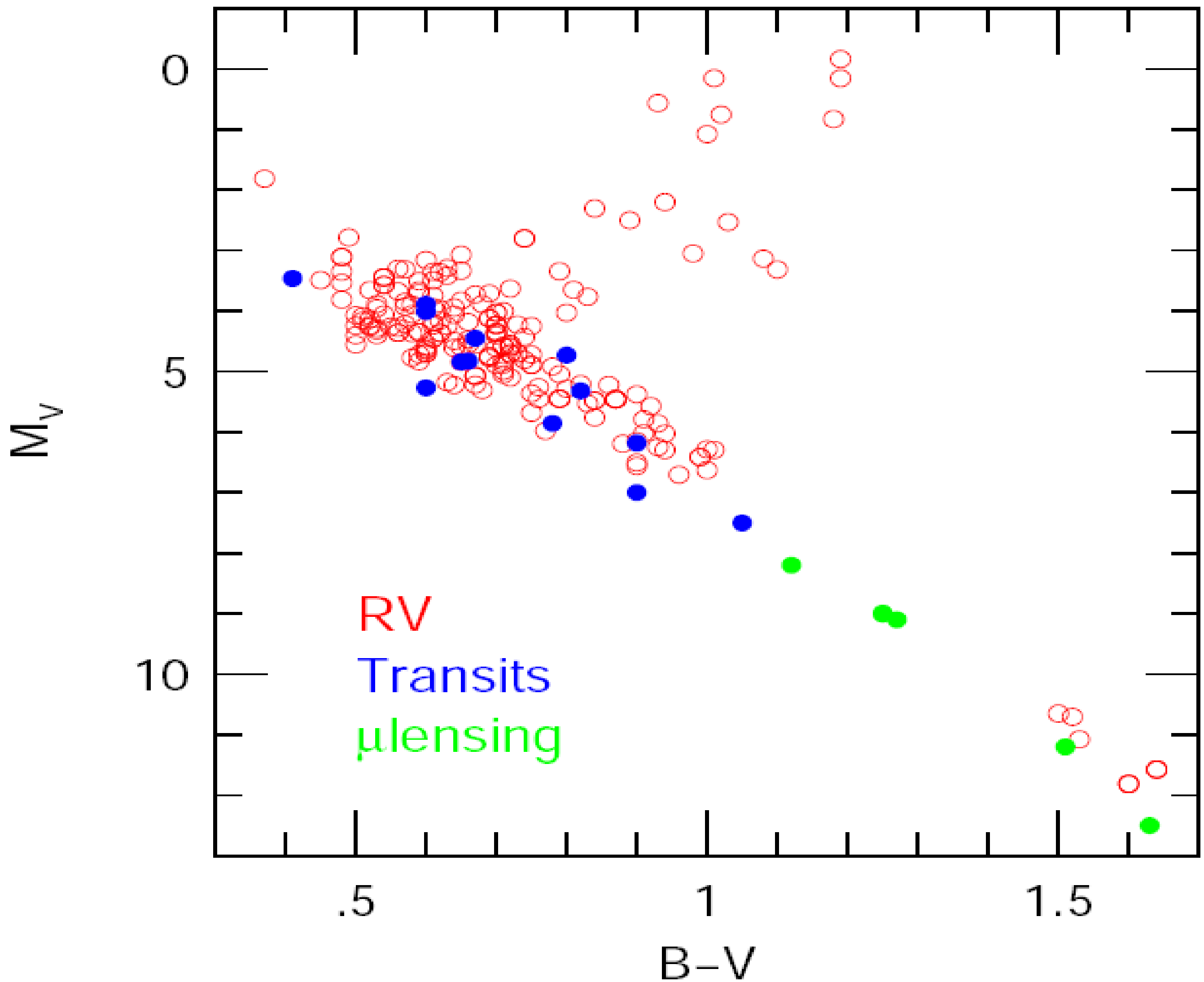
Faint Hosts:

CMD (Apparent Mags)



Dim Hosts:

CMD (Absolute mags)



Microlensing vs. Other Methods IV:

Biggest Challenge: Host ID

- Host superposed on source (glare)
- Typically moves at 4 mas/yr
  - 10+ years to separately resolve
- μlens masses require θ_E, π_E
  - Usually: θ_E si; π_E non
  - Hence: "Bayesian Estimates"

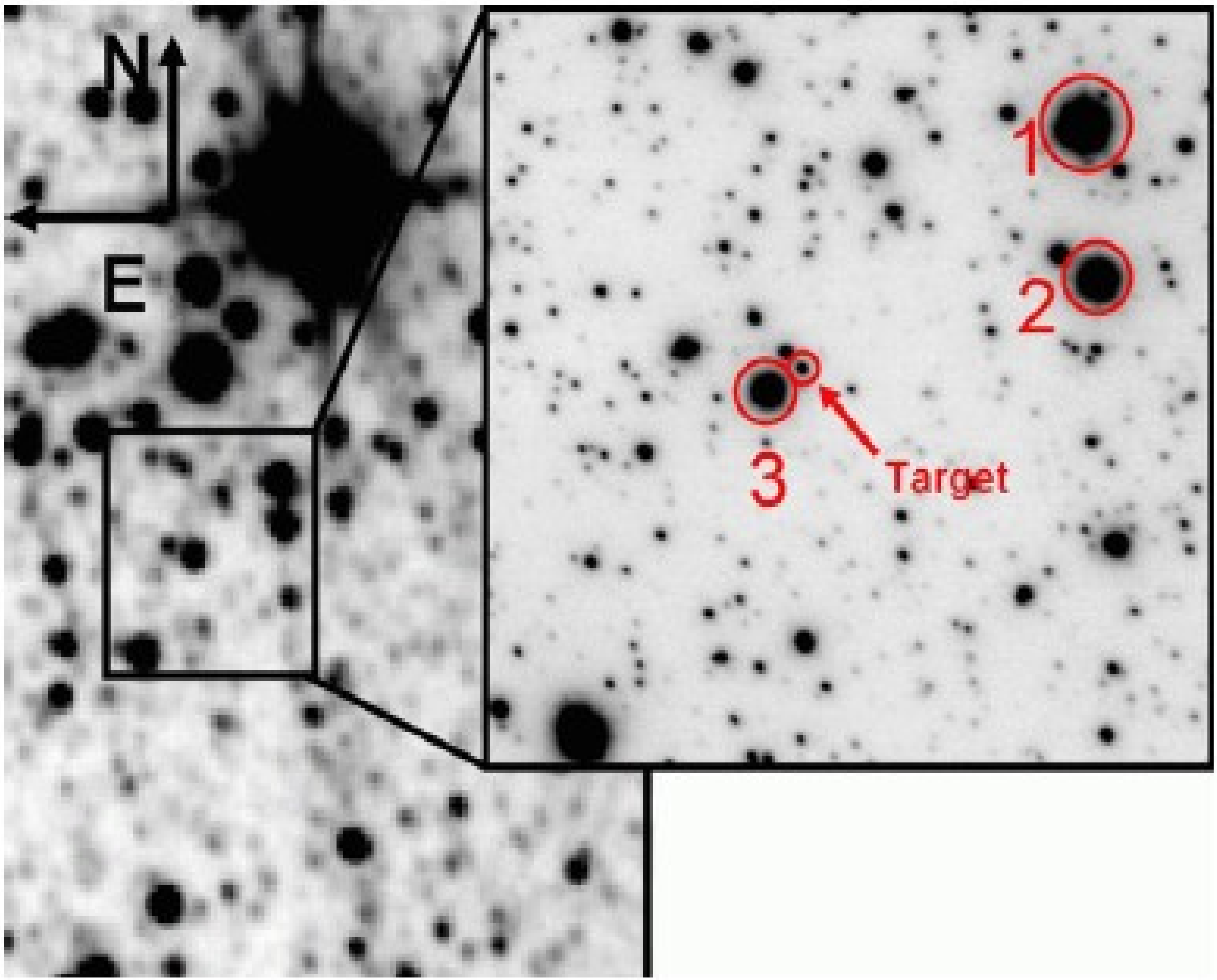
Bayesian Information Flow

(typical: finite source/no parallax)

- Line of Sight toward Bulge Source known well
- Proper motion $\mu = \theta_E/t_E$ known well
- θ_E ==> ($M\ \pi_{rel}$) known well
- Galactic model favors bulge lenses, unless μ big
- Galactic model favors low mass, unless θ_E big

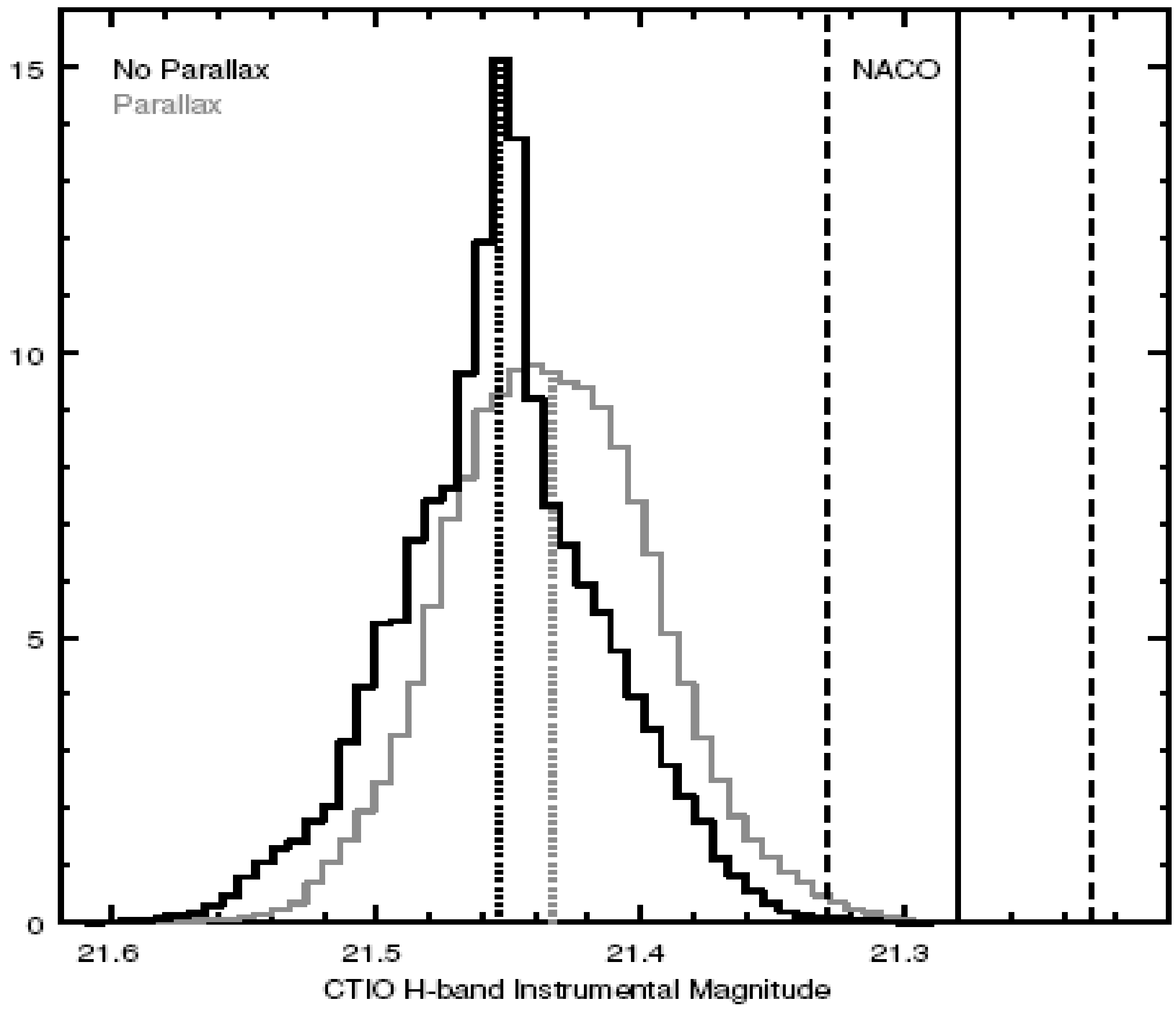
Extra Info: High Resolution Imaging

MOA-2008-BLG-310 (Janczak et al)



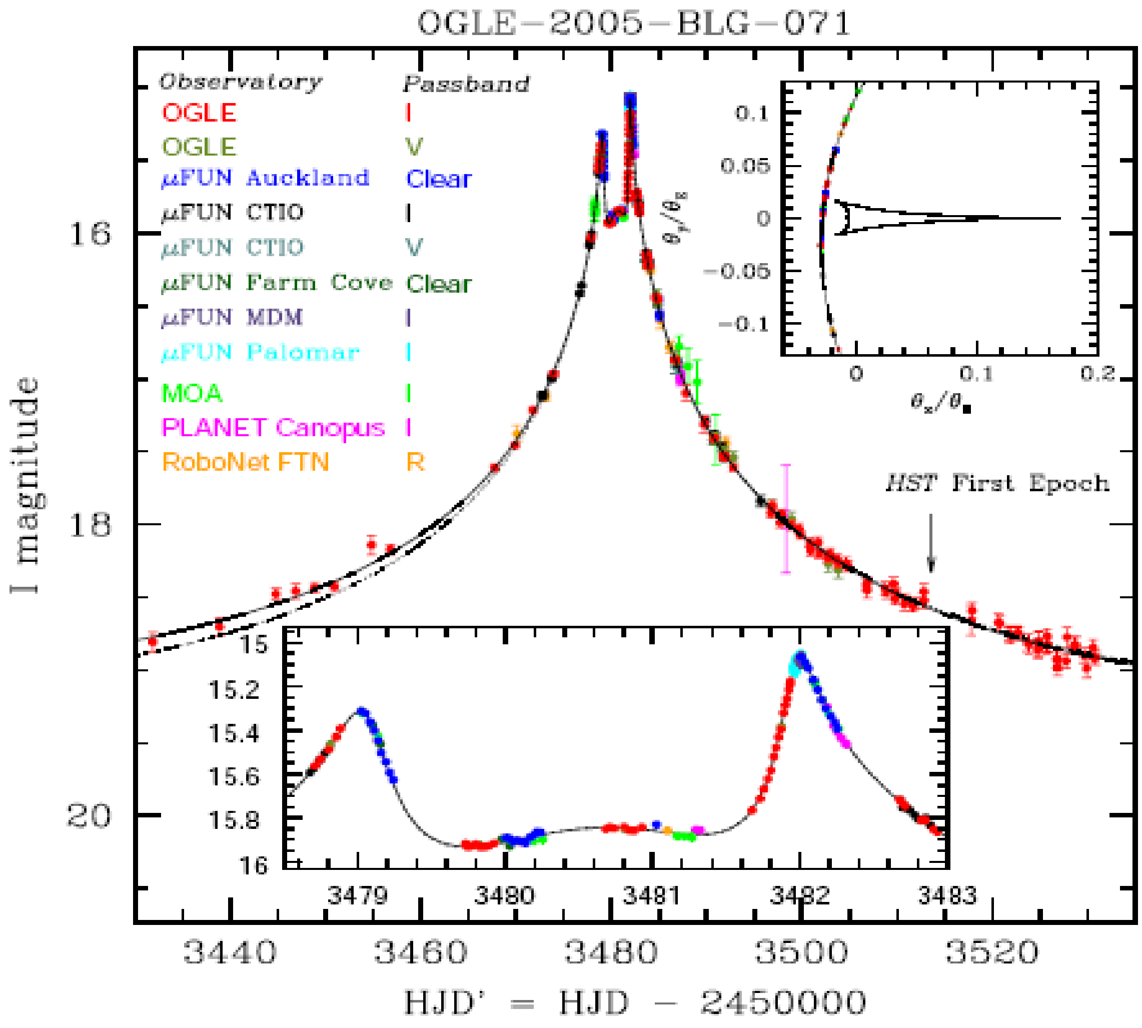
Extra Light Definitely Detected, but ..

Host, host-comp, source-comp, random?

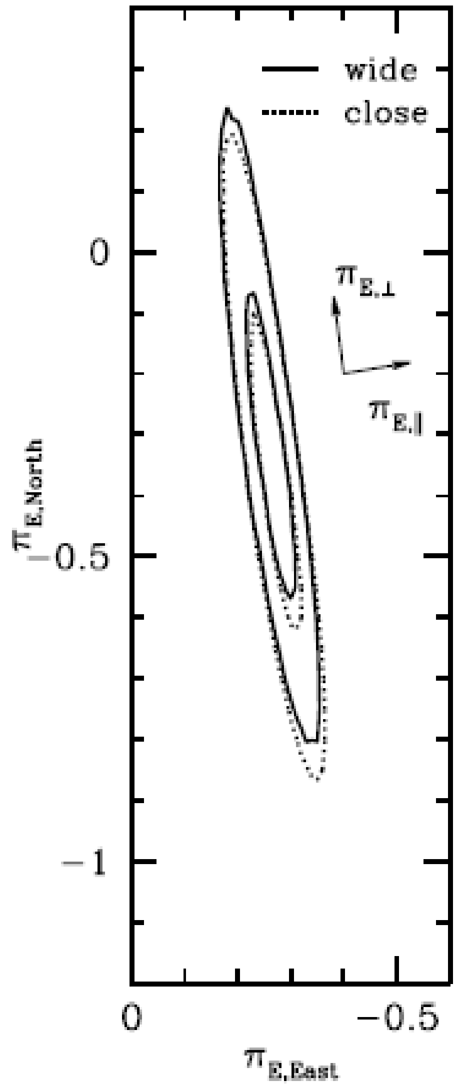


Multiple Partial Information

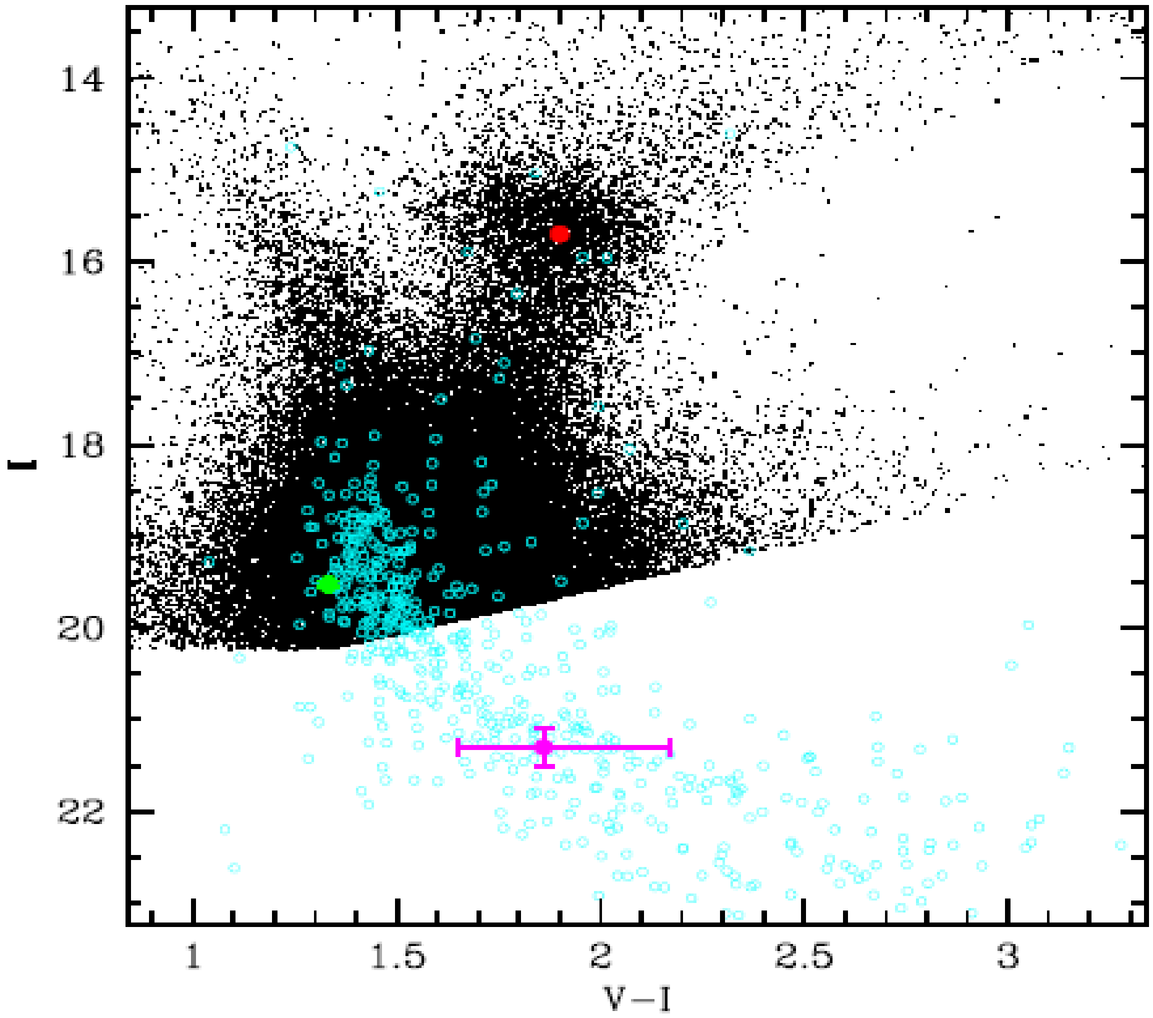
OGLE-2005-BLG-071 (Dong et al)



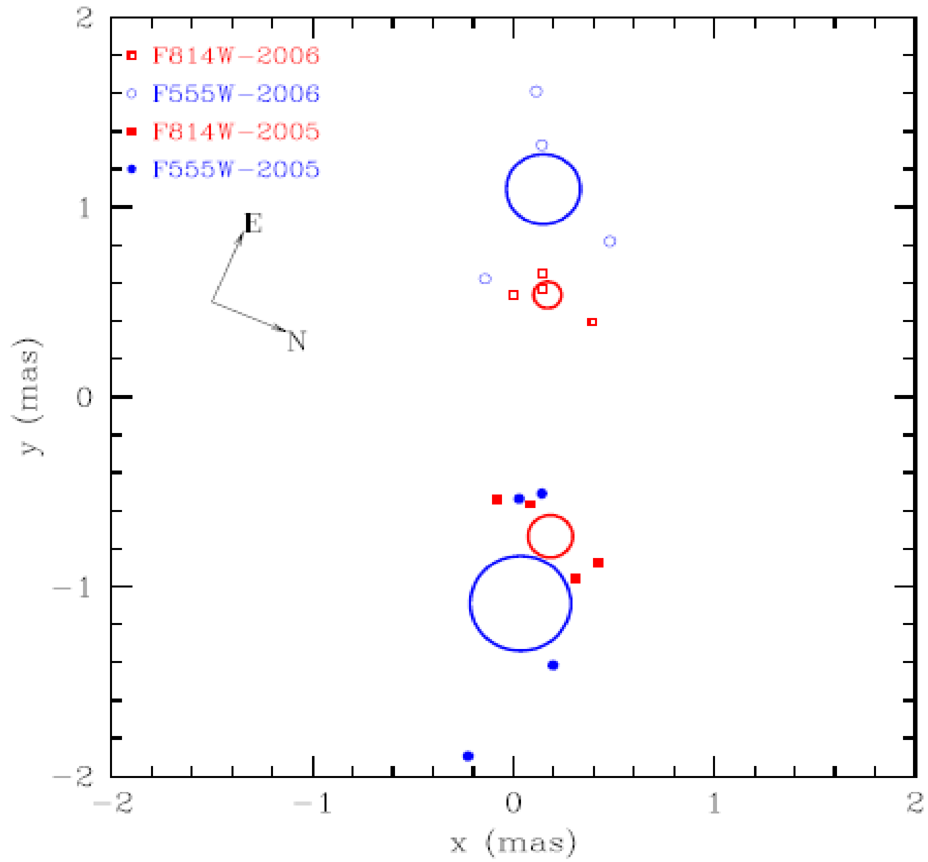
1-D Parallax Measurement



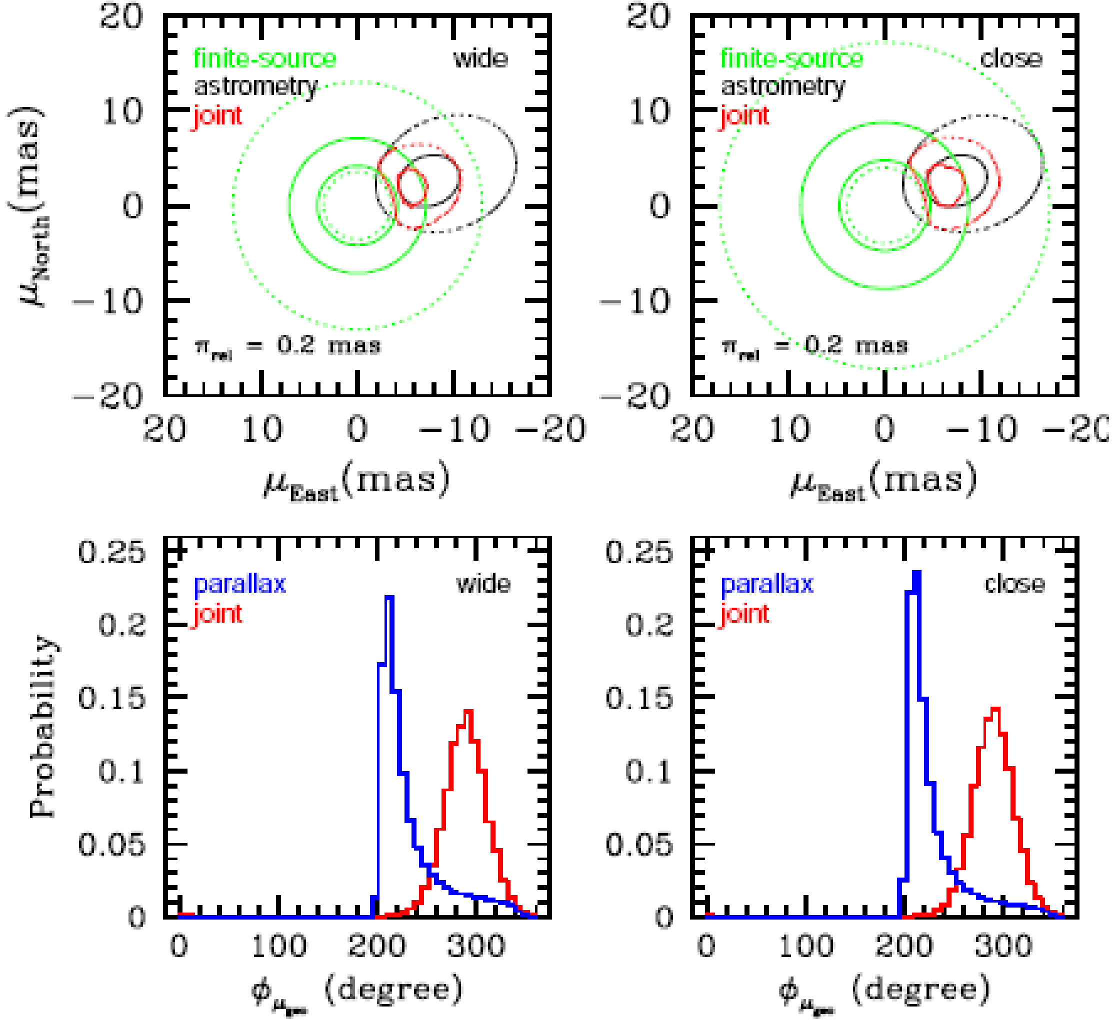
Blended Light Detected with HST



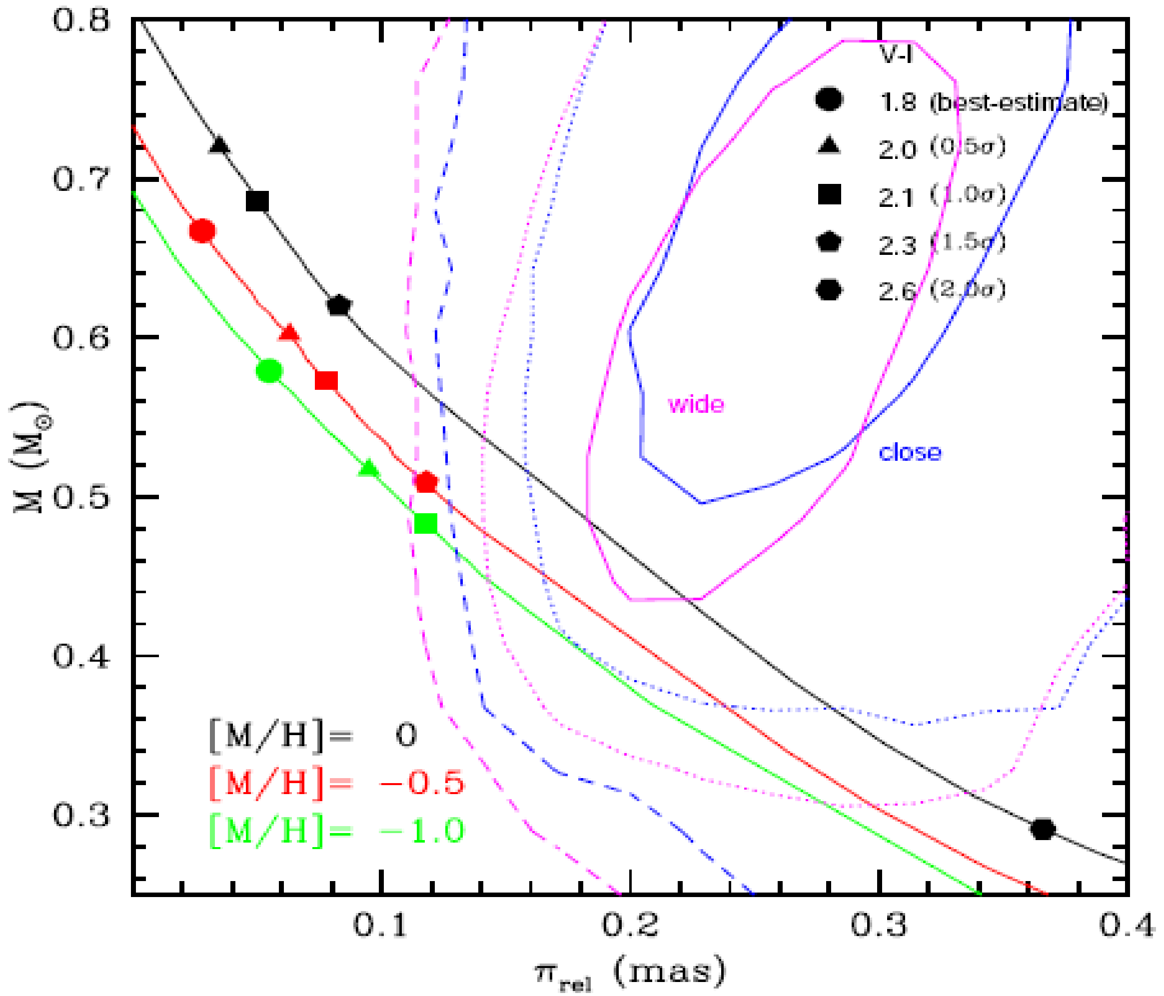
Differential V/I source+blend proper motion from 2 HST epochs



Some Tension Among Measurements



Likely to be an M dwarf

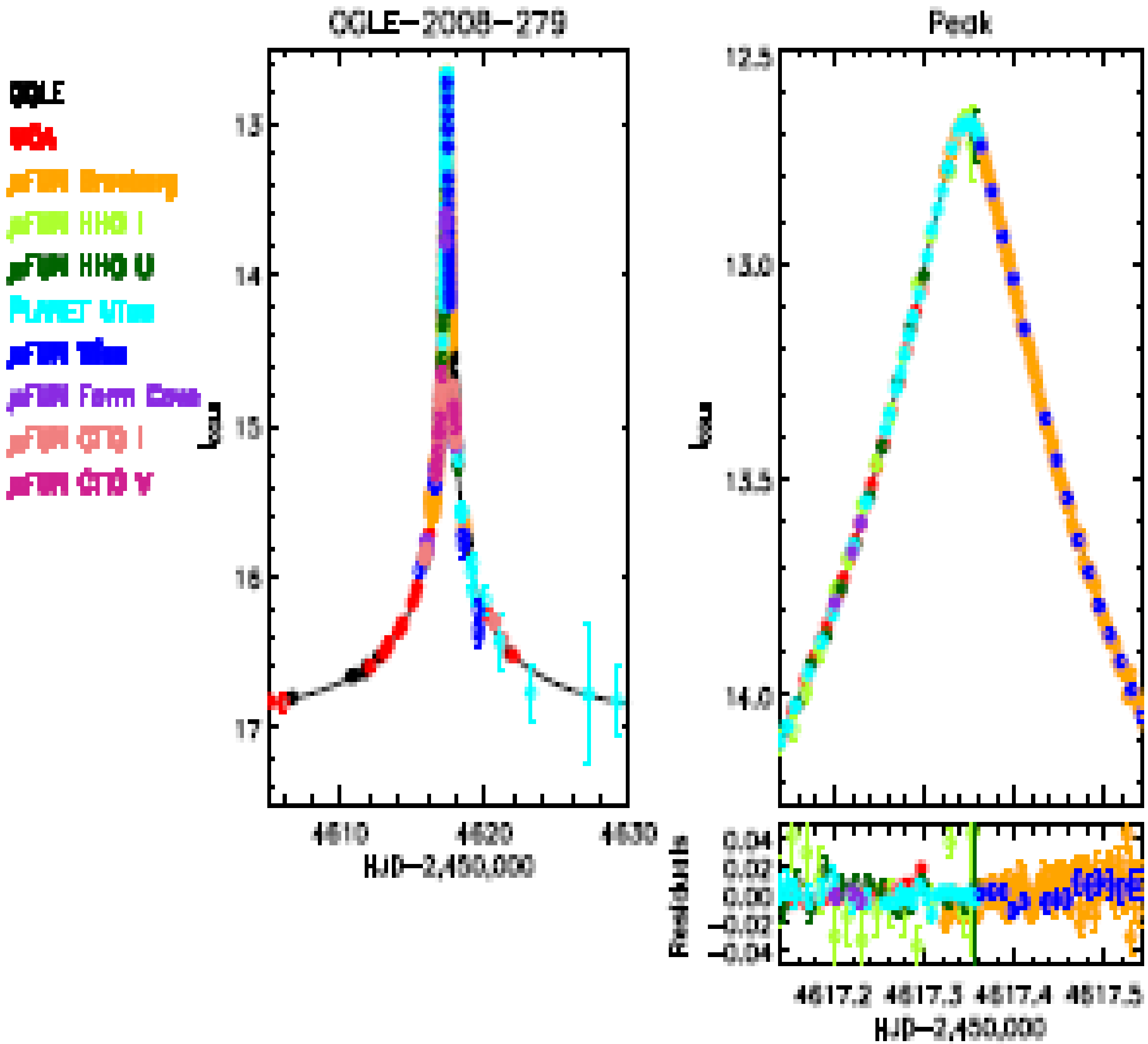


μlens Planet Frequency Estimates

- Gould et al. 2010, ApJ, 720, 1073
- Cassan et al. 2012, Nature, 481, 167
  - Gould et al. (2010)
  - Sumi et al. 2010, ApJ, 710, 1641

OGLE-2008-BLG-279:

A = 1600

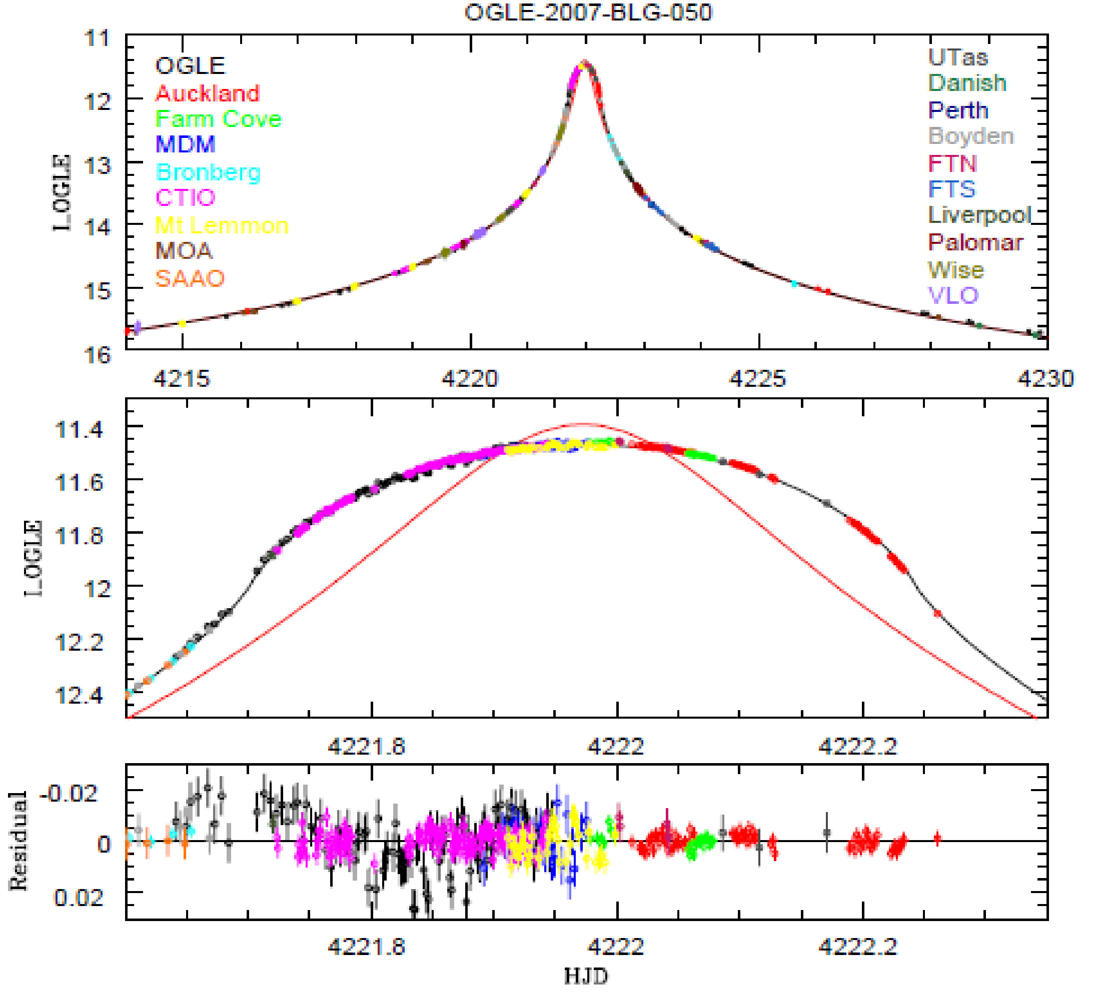


Yee et al. 2009, ApJ, 730, 2082

Jennifer Yee

OGLE-2007-BLG-050:

A = 432

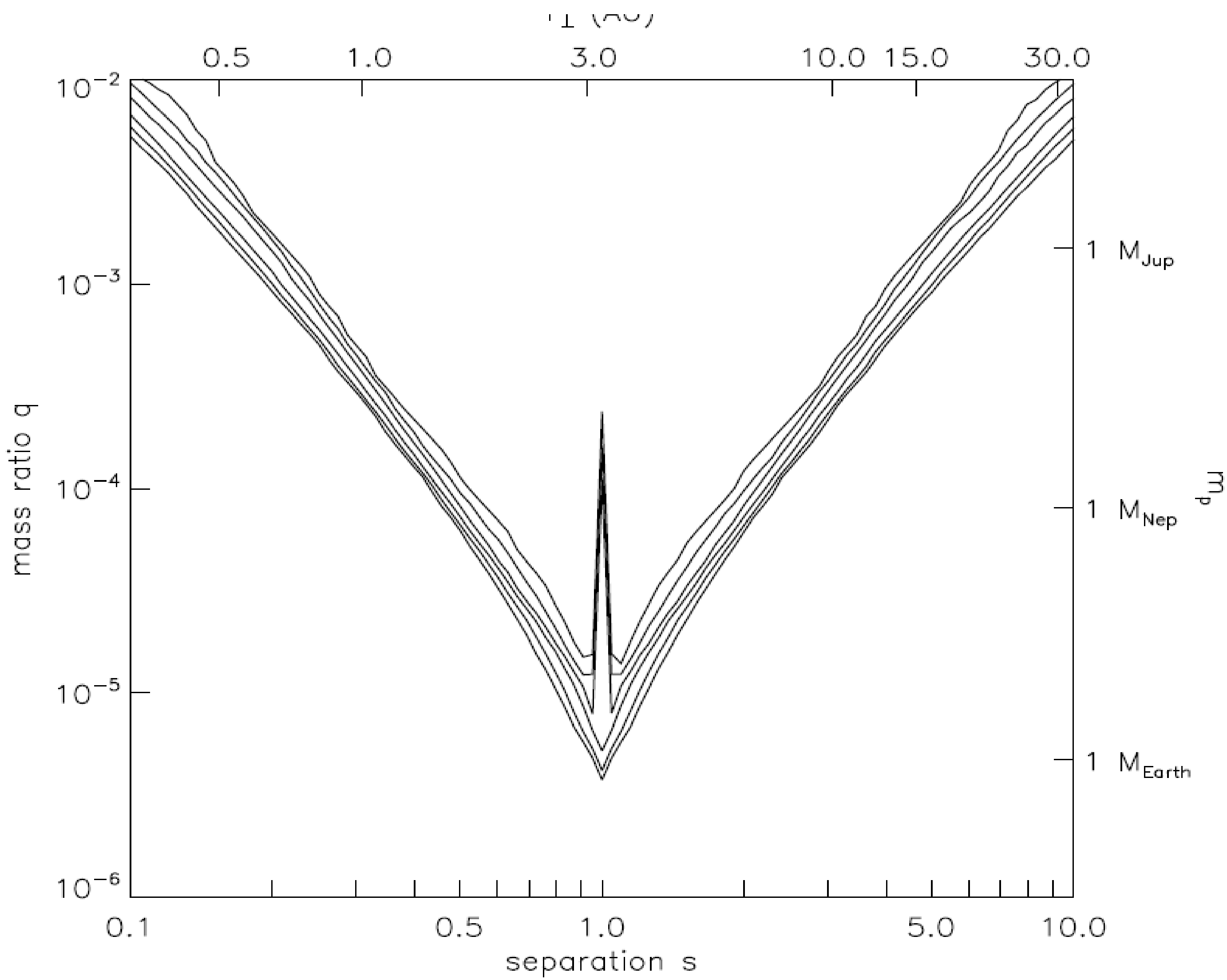


Batista et al. 2009, A&A, 508, 467

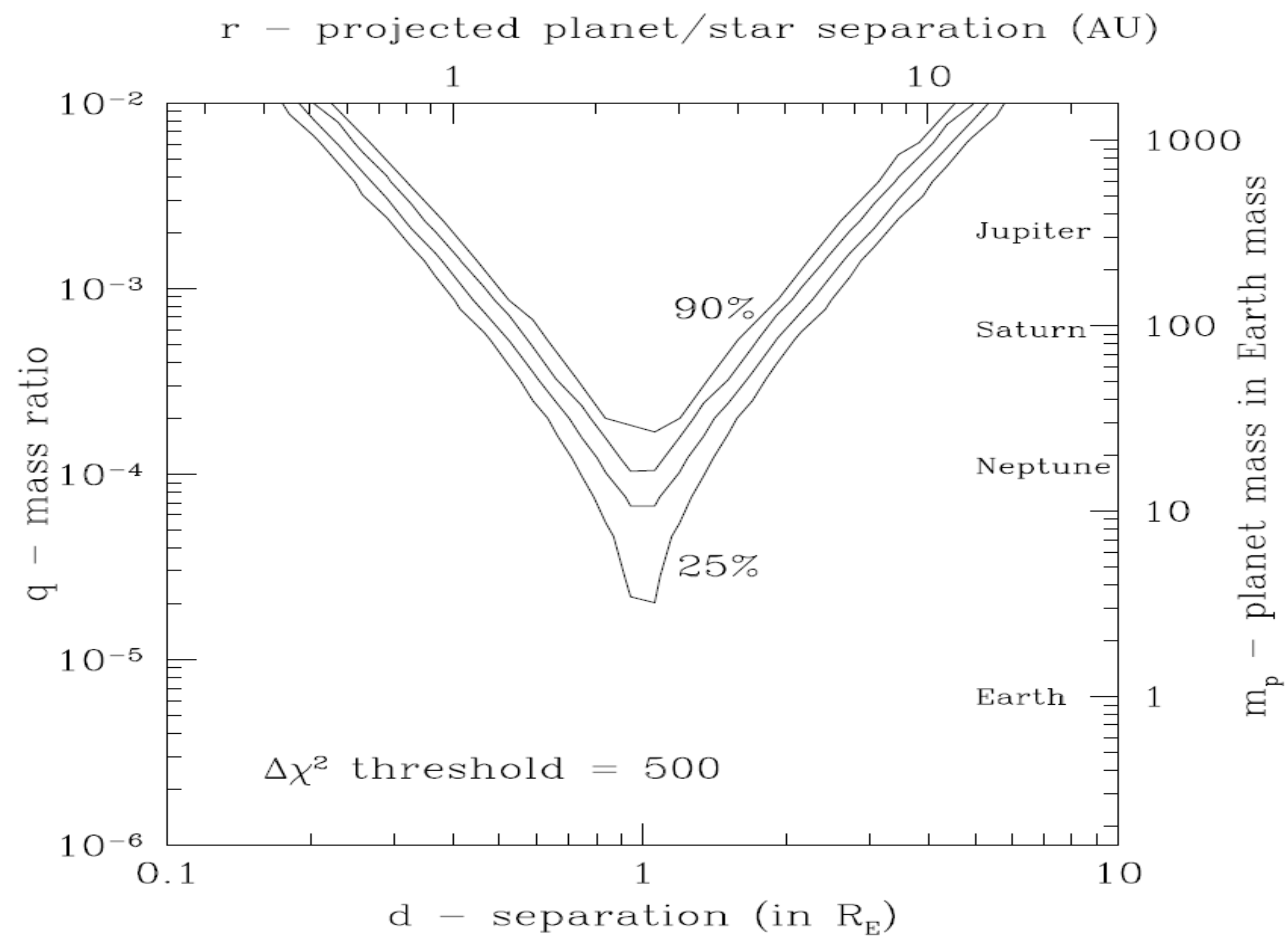
Virginie Batista

Planet Sensitivity

OGLE-2008-BLG-279



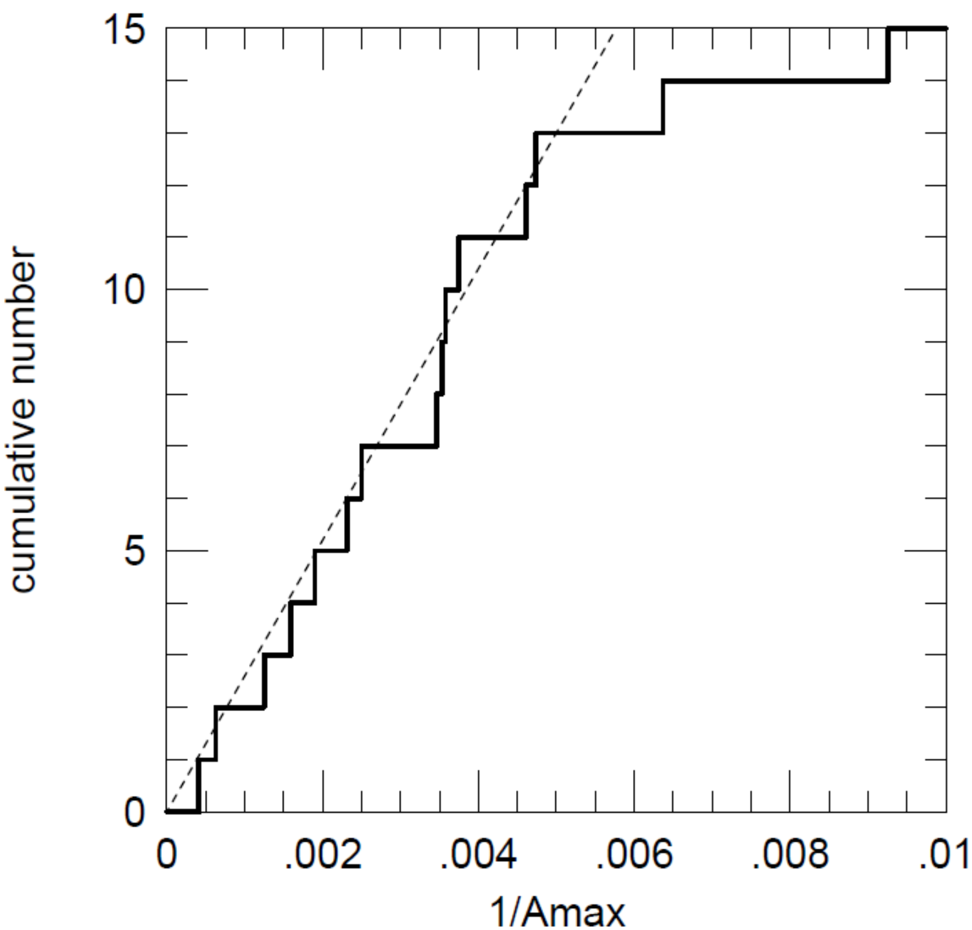
Planet Sensitivity
OGLE-2007-BLG-050



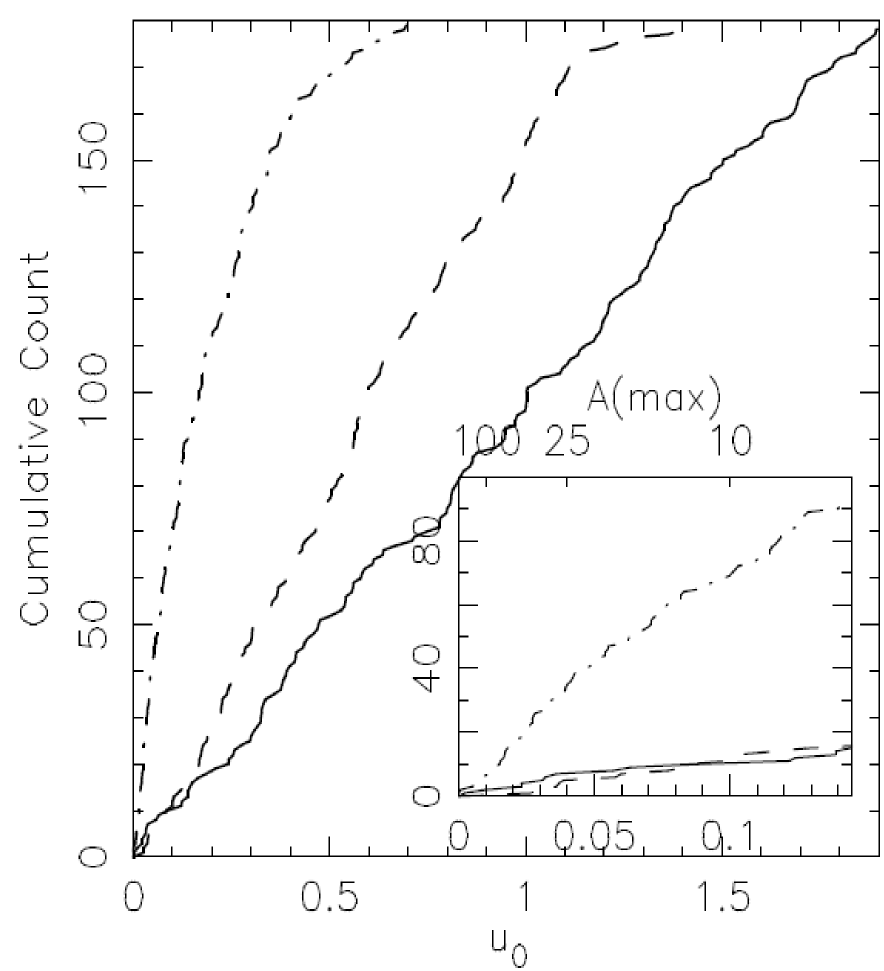
Well-covered events: fair sample

(Amax>200)

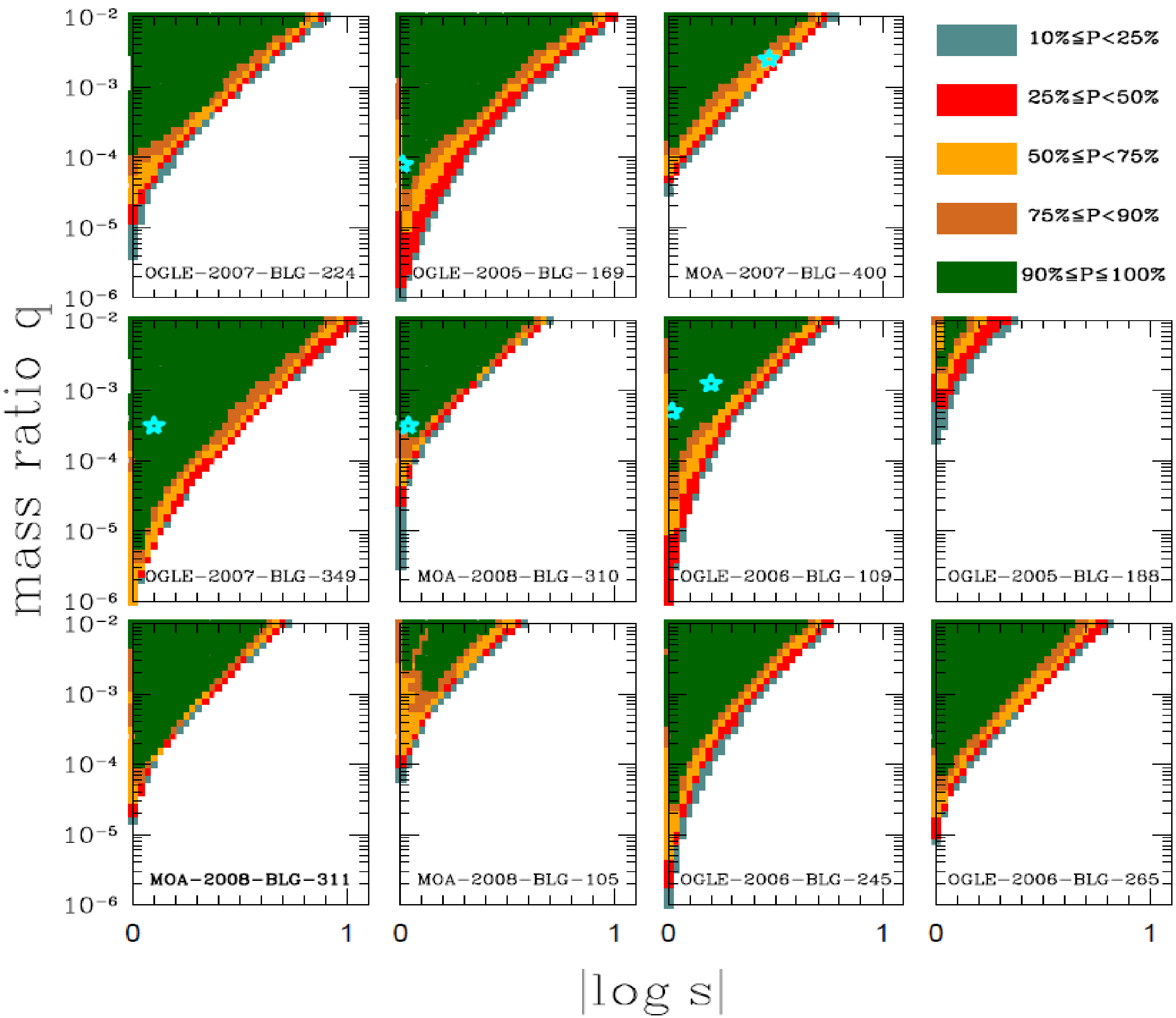
Well-Covered



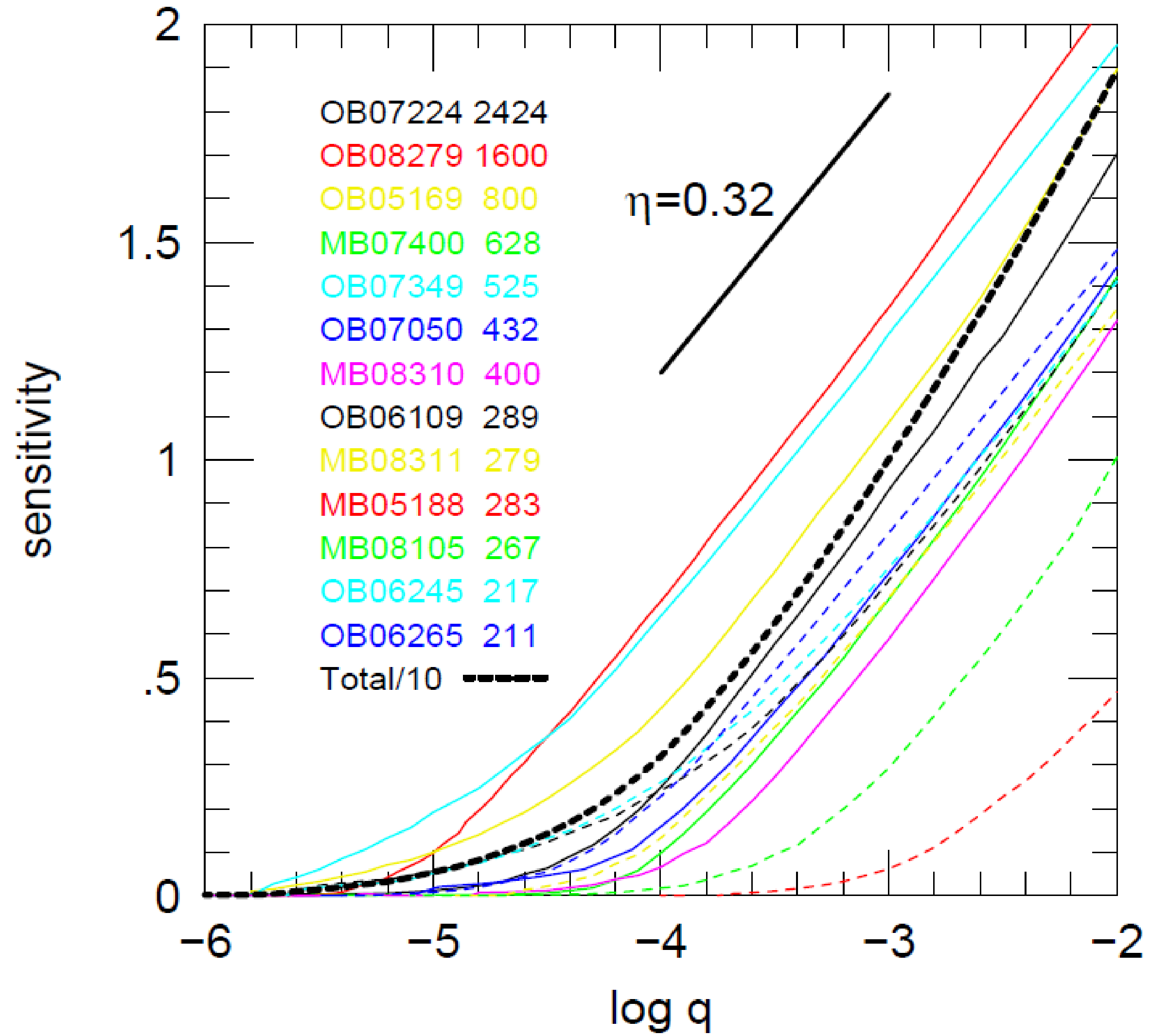
All Events



Planet Sensitivity Vs. Detections

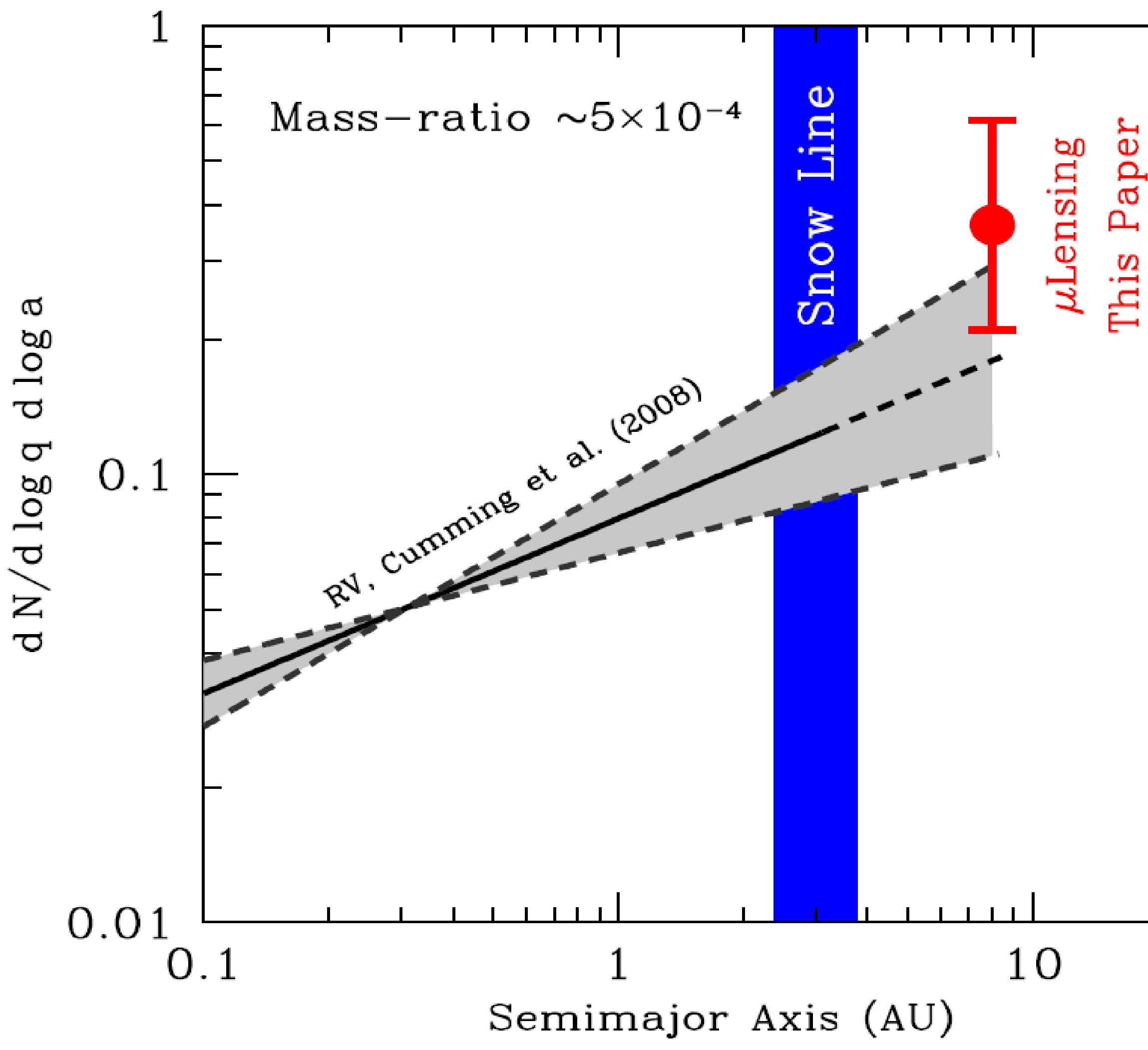


Planet Sensitivity Vs. Mass Ratio



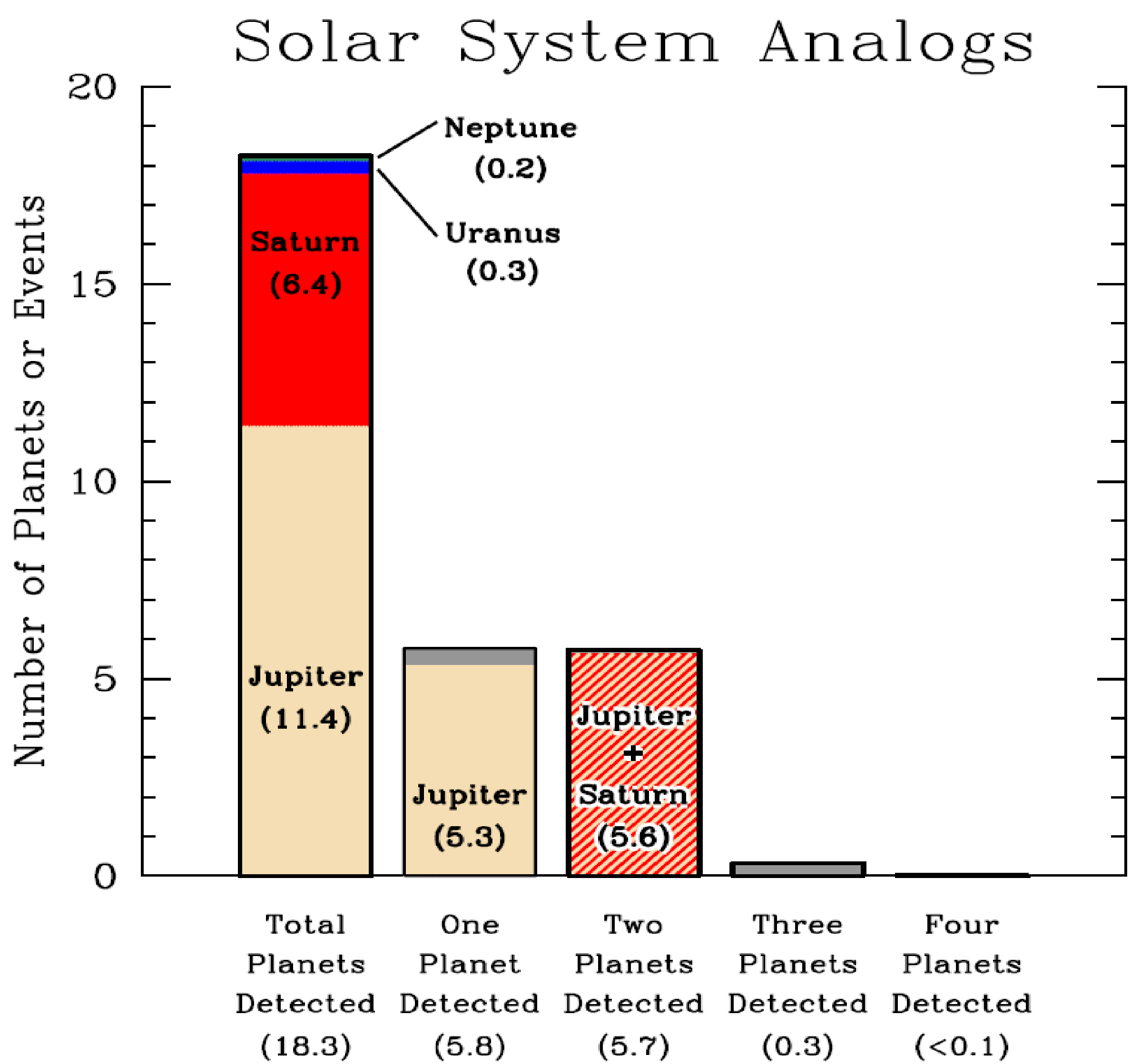
RV & Microlensing

Inside vs Outside Snow Line

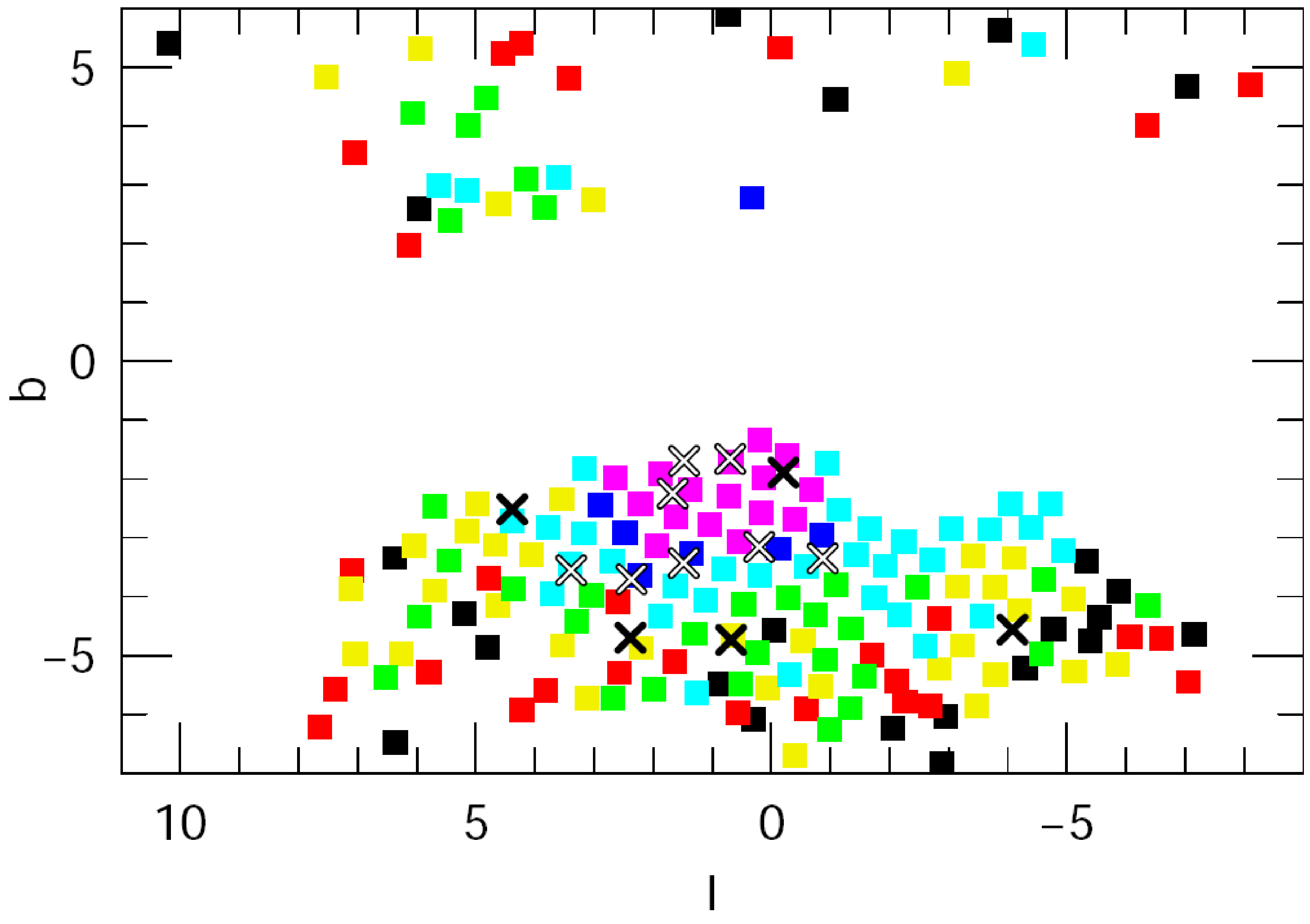


Solar System is Richer than Average

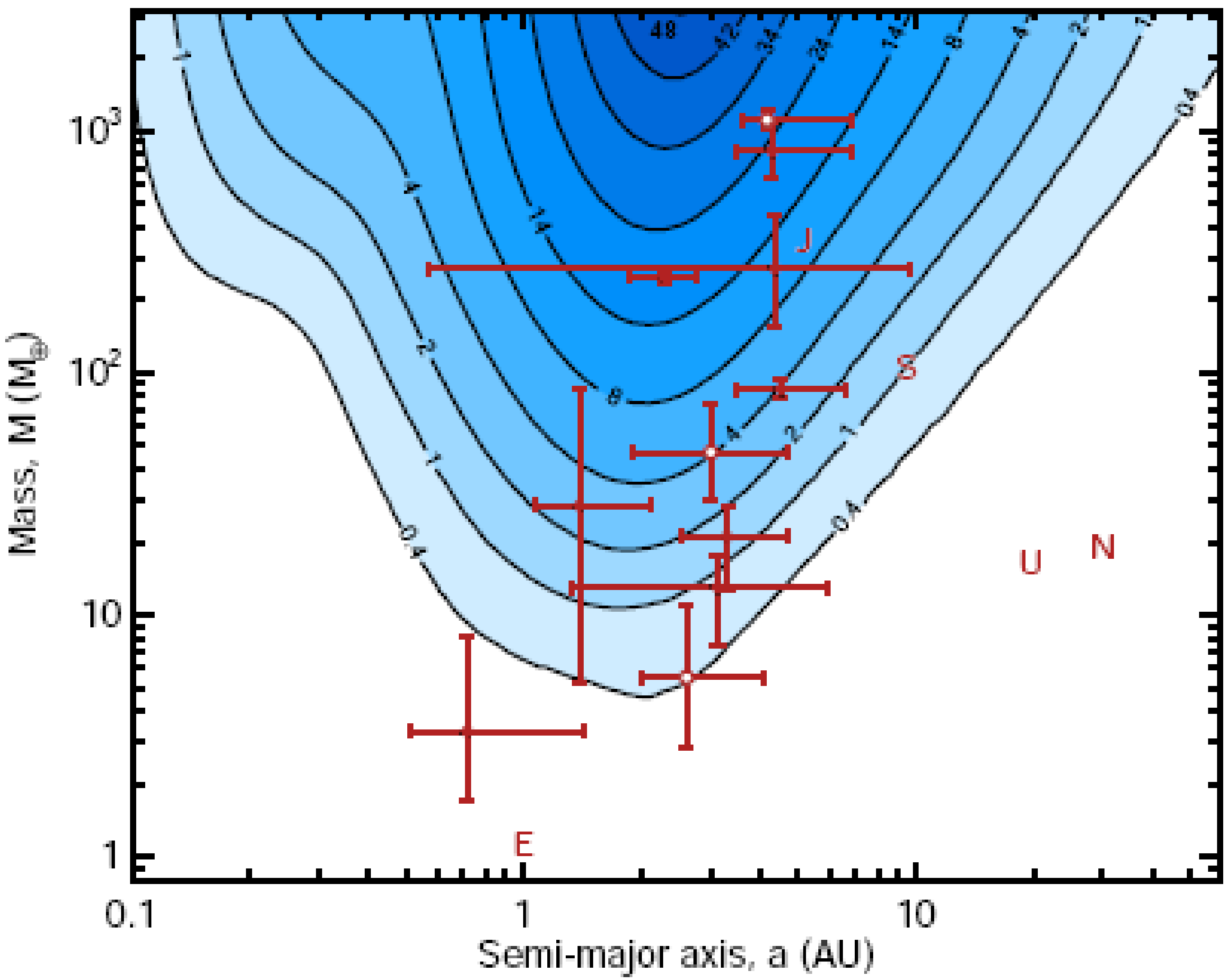
... But Not Dramatically So



Distribution of Planets on Sky



Combined Planet Frequency
Cassan et al (2012)



Combined Planet Frequency Cassan et al (2012)