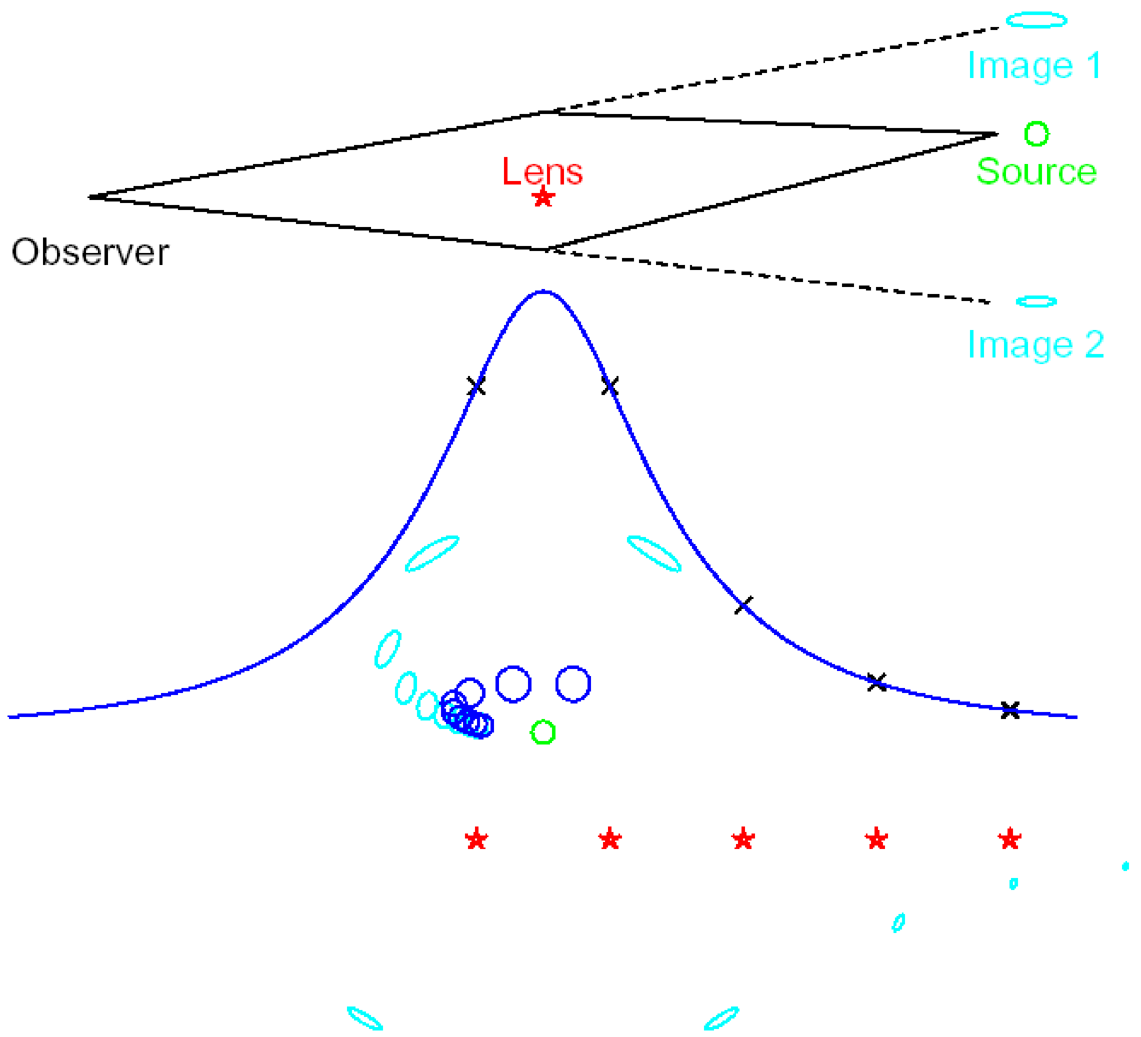
Exoplanet Microlensing I:

Simple Lens & Binary-Lens Basics

Andy Gould (Ohio State)



Generation 1

- Liebes 1964, Phys Rev, 133, B835
  - Many practical examples, including planets
- Refsdal 1964, MNRAS, 128, 259
  - Mass measurement of Isolated Star
- Refsdal 1966, MNRAS, 134, 315
  - Space-Based Parallaxes
- Paczynski 1986, ApJ, 304, 1
  - Proposed First Practical Experiment

Generation 0

- Eddington 1920, Space, Time, and Gravitation
- Chwolson 1924, Astron. Nachr. 221, 329
- Einstein 1936a, Science, 84, 506

"Some time ago R.W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request .... there is no great chance of observing this phenomenon."

- Einstein 1936b (private letter to Science editor)

"Let me also thank you for your cooperation with the little publication, which Mister Mandl squeezed out of me. It is of little value, but it makes the poor guy happy."

Generation -1: Einstein (1912)

[Renn, Sauer, Stachel 1997, Science 275, 184]

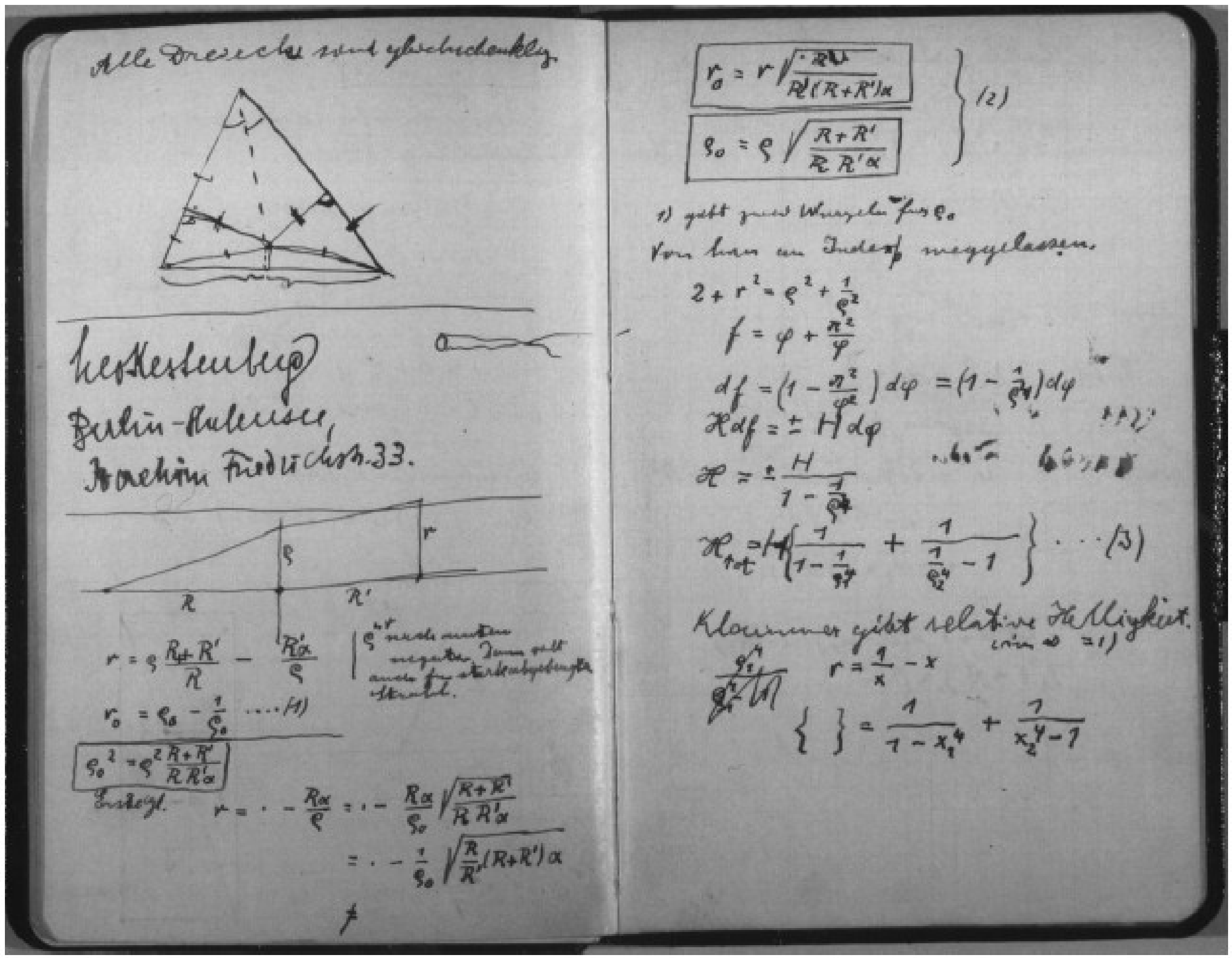


Fig. 1. Notes about gravitational lensing dated to 1912 on two pages of Einstein's scratch notebook (*12*). [Reproduced with permission of the Einstein Archives, Jewish National and University Library, Hebrew University of Jerusalem]

SCIENCE • VOL. 275 • 10 JANUARY 1997 185

Mao & Paczynski

Central Caustics

GRAVITATIONAL MICROLENSING BY DOUBLE STARS AND PLANETARY SYSTEMS

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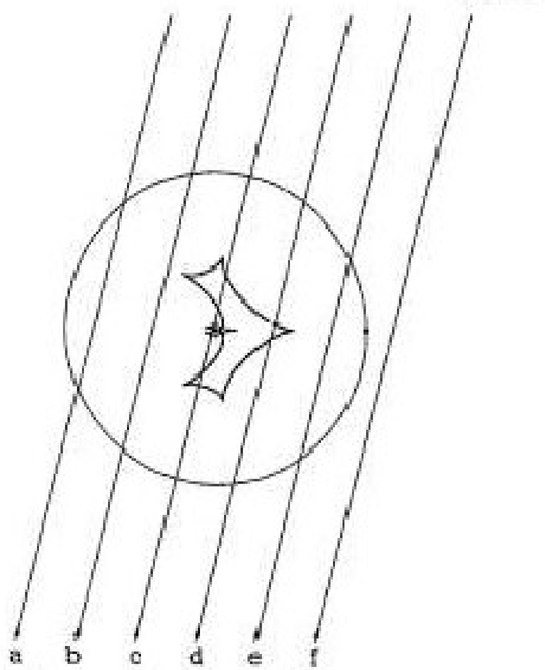
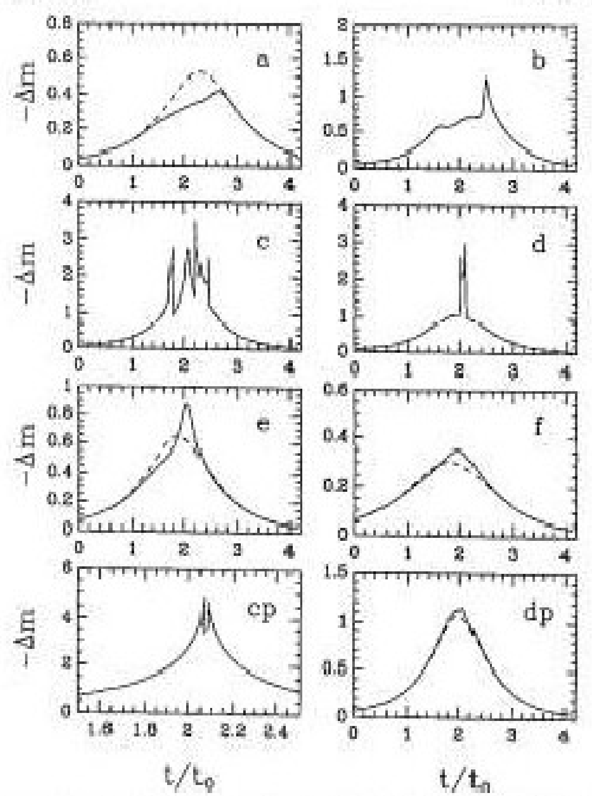
Received 1991 March 12; accepted 1991 April 2



. 1.—Geometry of microlensing by a binary, as seen in the sky. The y star of 1 $M_{\odot}$ is located at the center of the figure, and the secondary of ; or 0.001 $M_{\odot}$ is located on the right, on the Einstein ring of the y. The radius of the ring is 1.0 mas for a source located at a distance of 8 d the lens at 4 kpc. The two complicated shapes around the primary are



the lens. The effect is strong even if the companion is a planet. A massive search for microlensing of the Galactic bulge stars may lead to a discovery of the first extrasolar planetary systems.

Gould & Loeb

Planetary Caustics

DISCOVERING PLANETARY SYSTEMS THROUGH GRAVITATIONAL MICROLENSES

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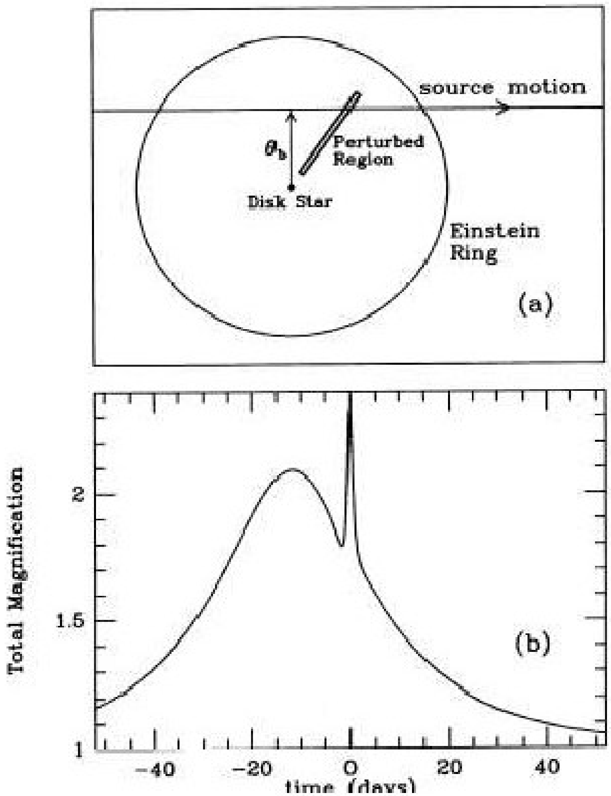
Received 1991 December 26; accepted 1992 March 9



5. OBSERVATIONAL REQUIREMENTS

Two distinct steps are required to observe a planetary system by microlensing. First, one must single out a disk star which happens to be microlensing a bulge star. Second, one must observe this star often enough to catch the deviation in the light curve due to the planet. The first step involves the observation of millions of bulge stars on the order of once per day. The second step involves the observation of a handful of stars many times per day. In the following we give a rough outline of what is required for each of these steps.

While observations from one site would be useful, there are advantages to be gained by observing from several sites. First,

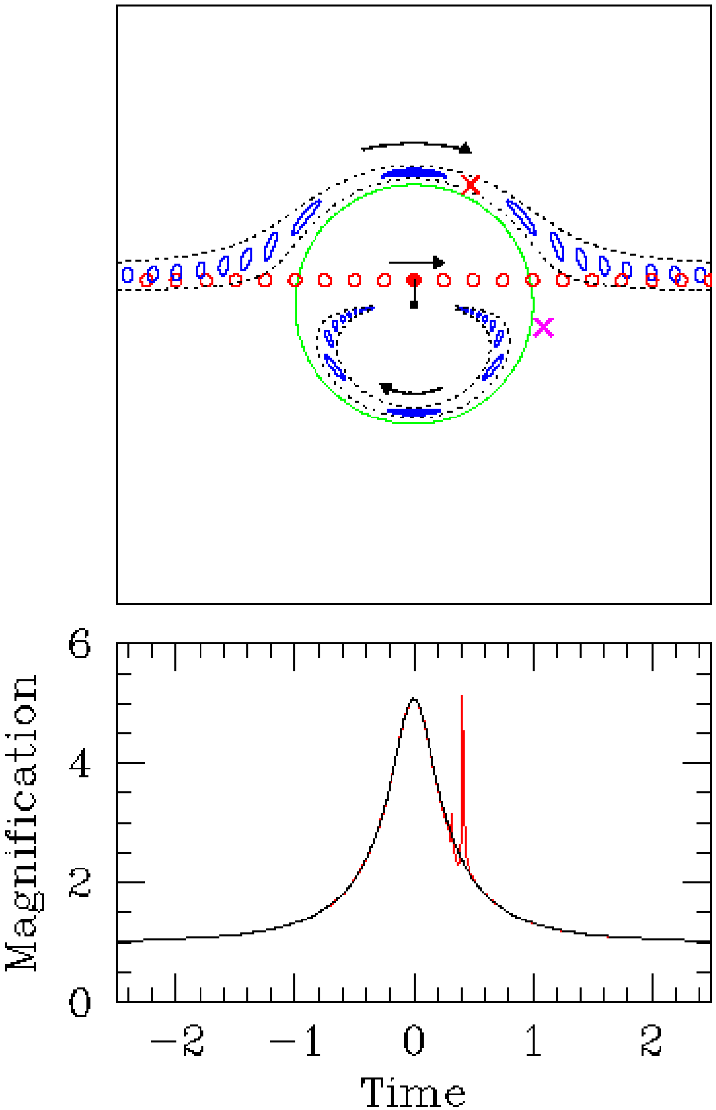
two telescopes that were totally committed. Third, in view of the fleeting nature of the events, it would seem prudent to build in some redundancy in case of bad weather at a particular site. Thus, the optimal scheme would employ, say, a dozen telescopes. Each of these would be committed to carry out two observations per night. During the near-December season,

6 Features & 6 Parameters

- Time of Peak
- Height of Peak
- Width of Peak
- Time of Perturbation
- Height of Perturbation
- Width of Perturbation

- t_0
- u_0
- t_E
- Trajectory angle: α
- Planet-star separation: s
- Planet/star mass ratio: q

How Microlensing Finds Planets



Clear Division Between

Known Knowns & Known Unknowns

- Planet/star mass ratio
- Separation in units of Einstein radius

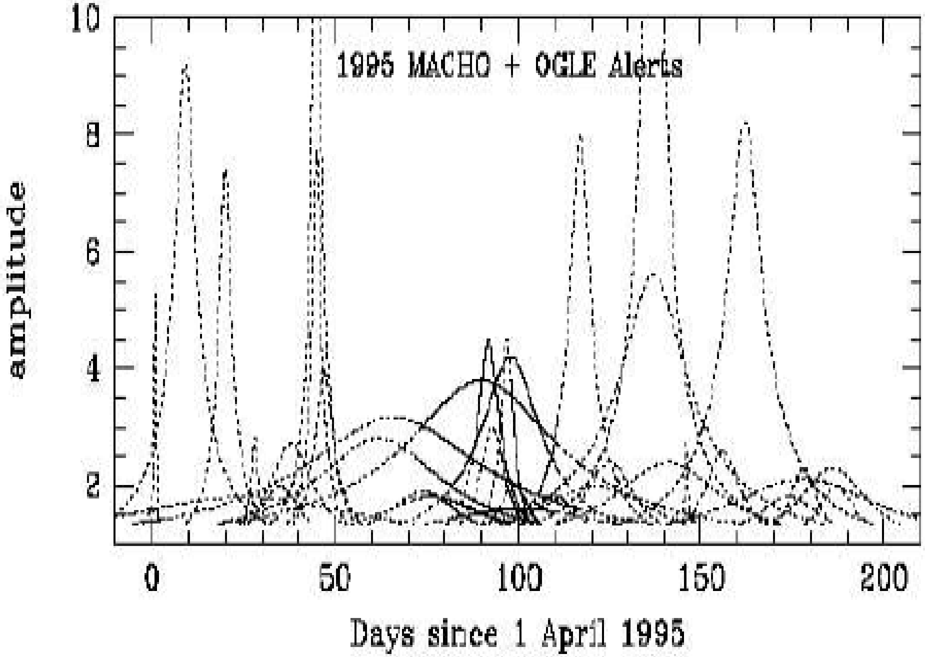
- Planet mass
- System distance
- Planet/star projected physical separation

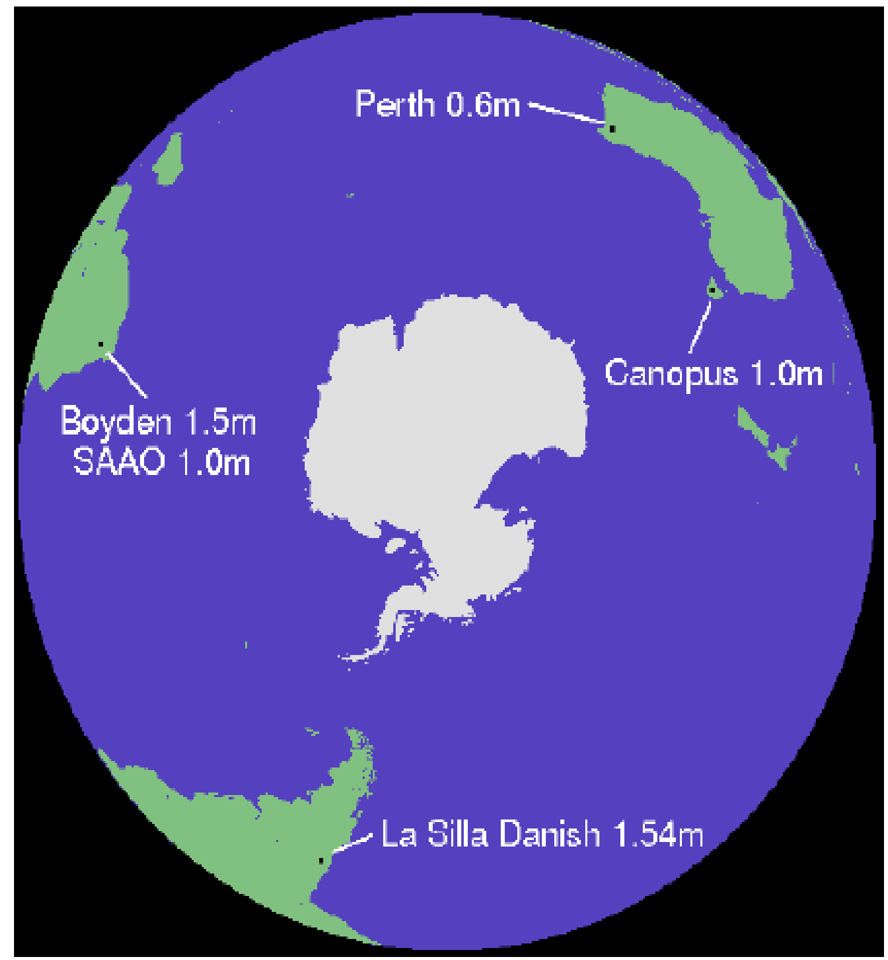
And inevitably: Unknown Unknowns

- Planet/star 3-D separation
- Planet Orbital Motion

1995 PLANET Pilot Season

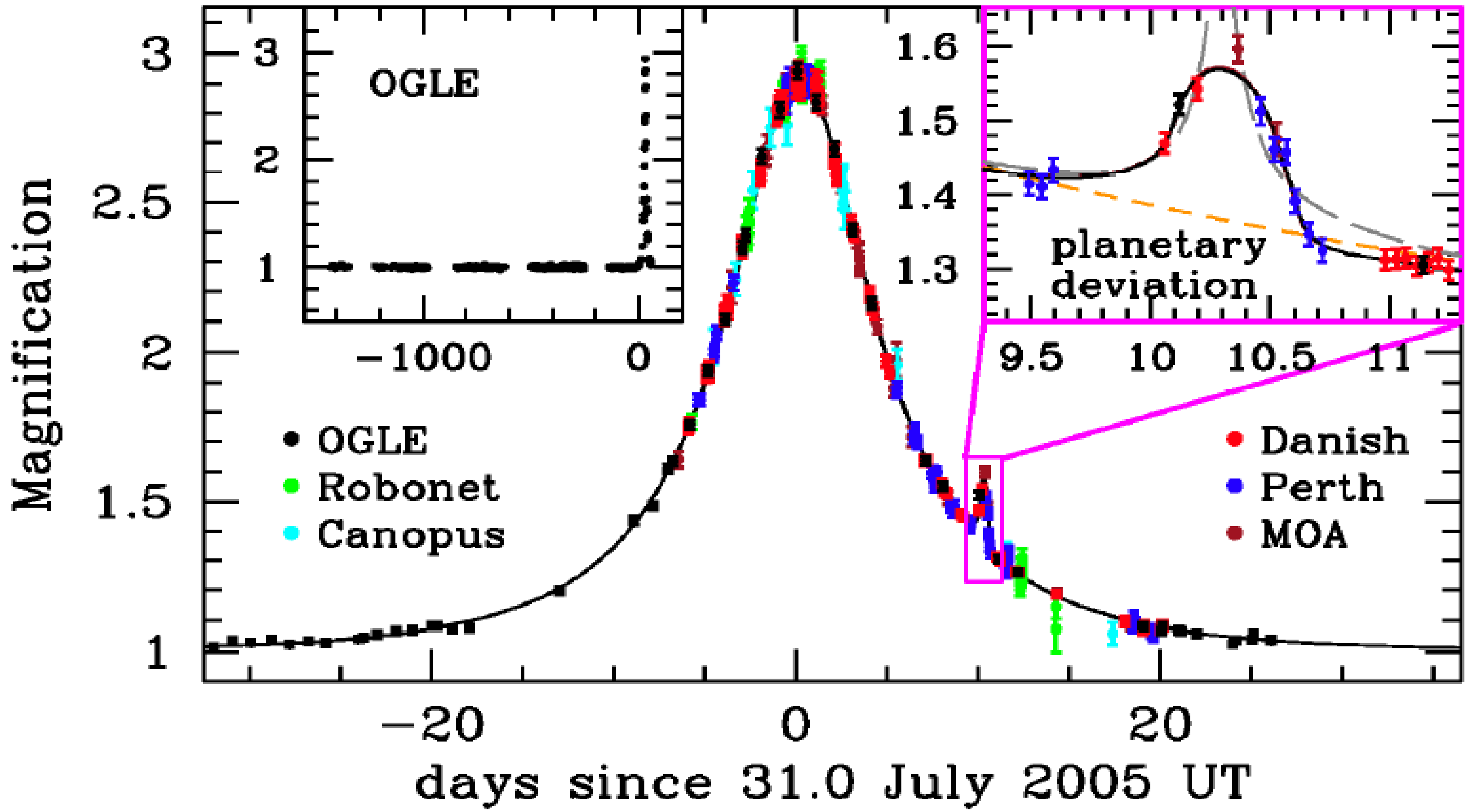
- Albrow et al. 1998
  ApJ, 509, 687



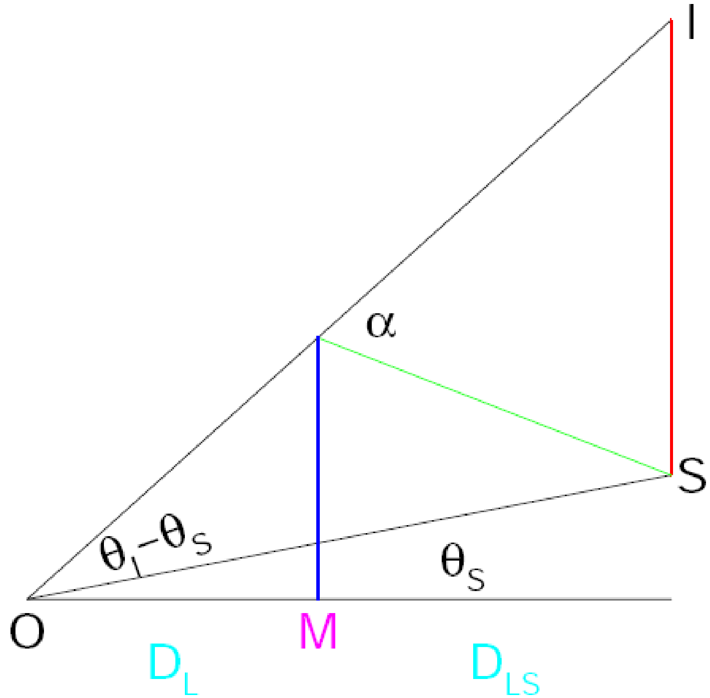


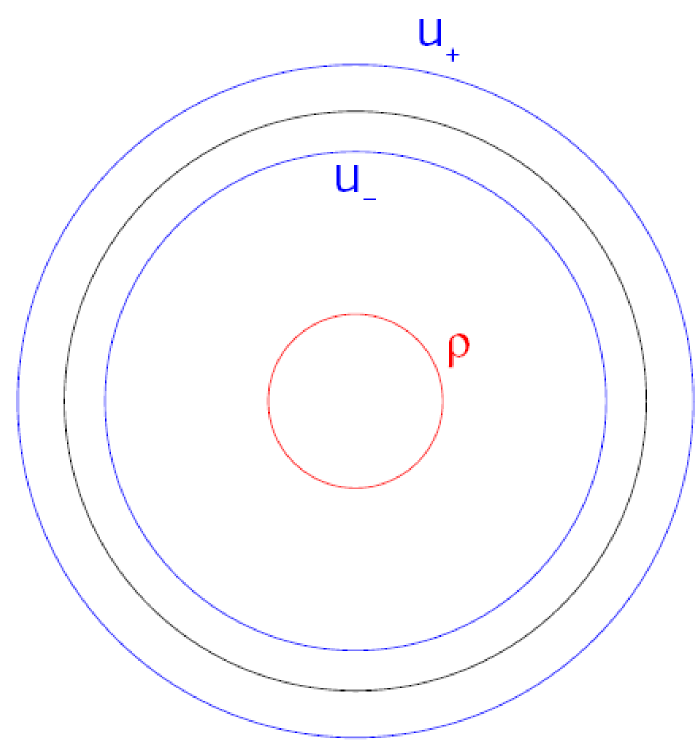
OGLE-2005-BLG-390

"Classical-Followup" Planetary Caustic



Beaulieu et al. 2006, Nature, 439, 437





$$(\theta_I - \theta_S) D_S = \alpha D_{LS}$$

$$\alpha = 4GM/(D_L \theta_I c^2)$$

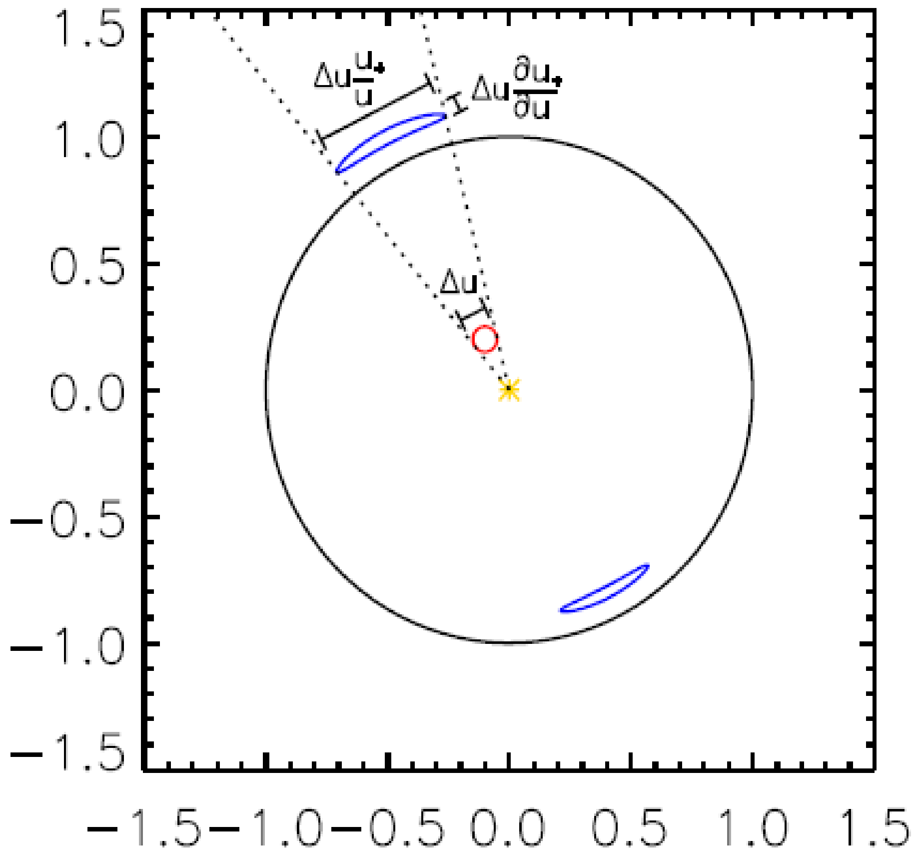
$$(\theta_I - \theta_S)\theta_I = \theta_E^{\ 2} = (4GM/c^2)(D_{LS}/D_L D_S)$$

$$\theta_I/\theta_E = [u +/- (u^2 + 4)^{1/2}]/2; \ \ u = \theta_S/\theta_E$$

Point-Lens Magnification

$$A = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$$\begin{aligned}
u_\pm &= \frac{\sqrt{u^2+4}}{2} \pm u \\
A_\pm &= \pm \frac{u_\pm}{u}\frac{\partial u_\pm}{\partial u} \\
&= \pm \frac{1}{2}\frac{\partial u_\pm^2}{\partial u^2} \\
A_+ - A_- &= \frac{1}{2}\left(\frac{\partial u_+^2}{\partial u^2} + \frac{\partial u_-^2}{\partial u^2}\right) = \frac{\partial(u^2+2)}{\partial u^2} = 1 \\
A_\pm &= \frac{A \pm 1}{2}
\end{aligned}$$



$$A = A_+ + A_- = \frac{1}{2}\left(\frac{\partial u_+^2}{\partial u^2} - \frac{\partial u_-^2}{\partial u^2}\right) = \frac{\partial(u\sqrt{u^2+4})}{2u\partial u} = \frac{u^2+2}{u\sqrt{u^2+4}}$$

Point-Lens Limiting Formulae

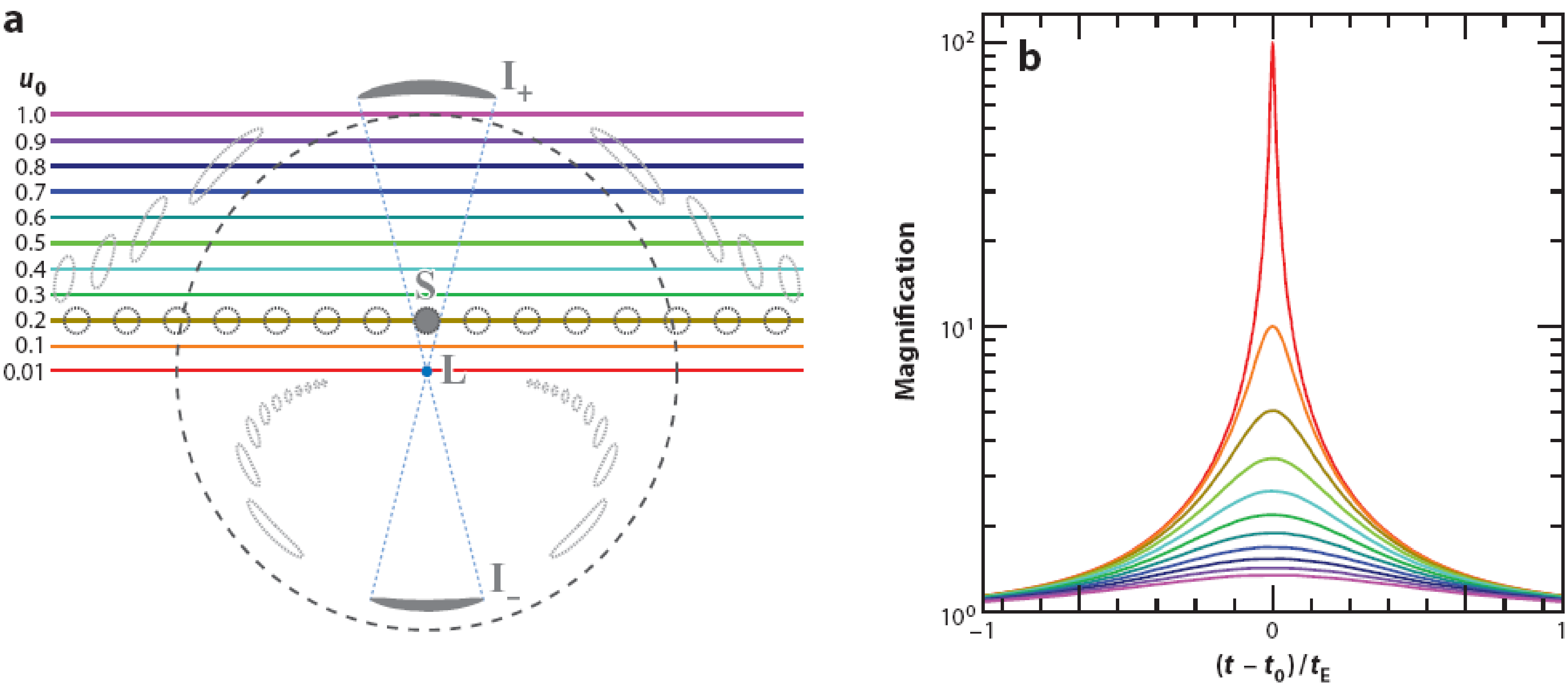
$$A(u) = \frac{1}{u}\frac{1+u^2/2}{\sqrt{1+u^2/4}} \rightarrow \frac{1}{u}\left(1+\frac{3}{8}u^2\right) \qquad (u \ll 1)$$

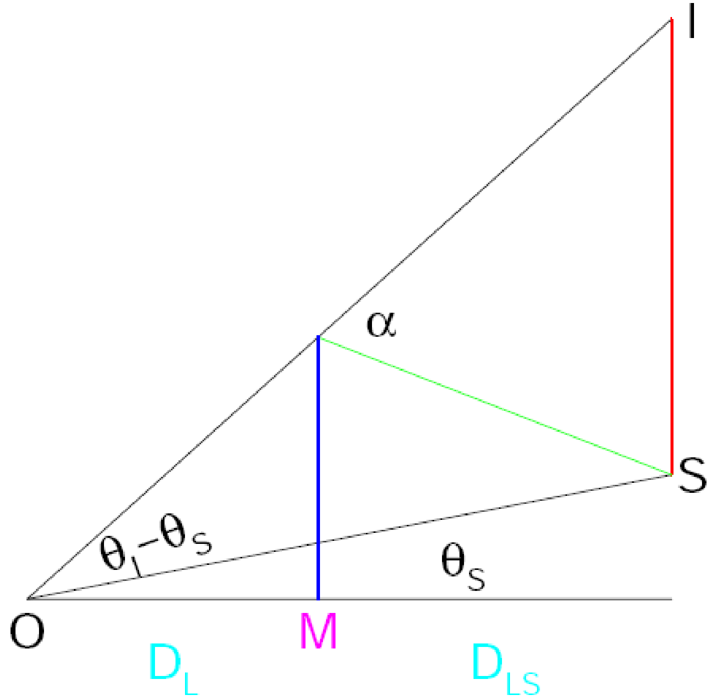
$$A(u) = \left(1-\frac{4}{(u^2+2)^2}\right)^{-1/2} \rightarrow 1+\frac{2}{(u^2+2)^2} \qquad (u \gg 1)$$

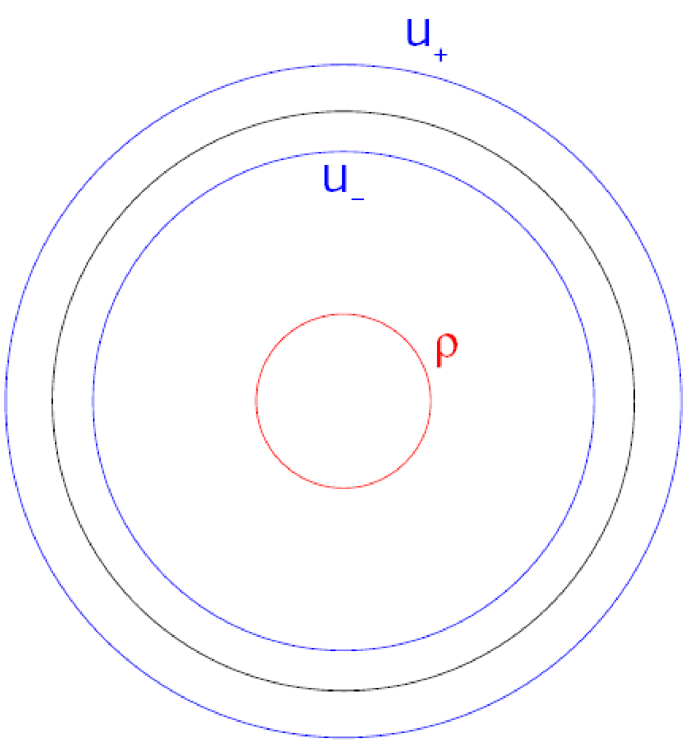
$$A(1) = \frac{3}{\sqrt{5}} \simeq 1.34$$

$$u(A) = \sqrt{2[(1-A^{-2})^{-1/2}-1]}$$

Point-Lens Light Curves







$$(\theta_I-\theta_S)D_S = \alpha D_{LS}$$

$$\alpha = 4GM/(D_L\theta_I c^2)$$

$$(\theta_I-\theta_S)\theta_I = \theta_E{}^2 = (4GM/c^2)(D_{LS}/D_L D_S)$$

$$\theta_I/\theta_E = [u +/- (u^2 + 4)^{1/2}]/2;\quad u=\theta_S/\theta_E$$

Source Centered on Point Lens

$$A = \frac{\pi(u_+^2 - u_-^2)}{\pi\rho^2}, \qquad u_\pm = \frac{\rho \pm \sqrt{\rho^2 + 4}}{2}$$

$$A = \sqrt{1 + \frac{4}{\rho^2}} \to 1 + \frac{2}{\rho^2}, \qquad \rho \equiv \frac{\theta_*}{\theta_{\rm E}}$$

Conjecture for Big Source on Planet Caustic

$$A_p = 2\left(\frac{\theta_{{\rm E},p}}{\theta_*}\right)^2$$

Plus Simple Timing Argument

$$\frac{t_p}{t_{\rm E}} = \frac{\theta_*}{\theta_{\rm E}}$$

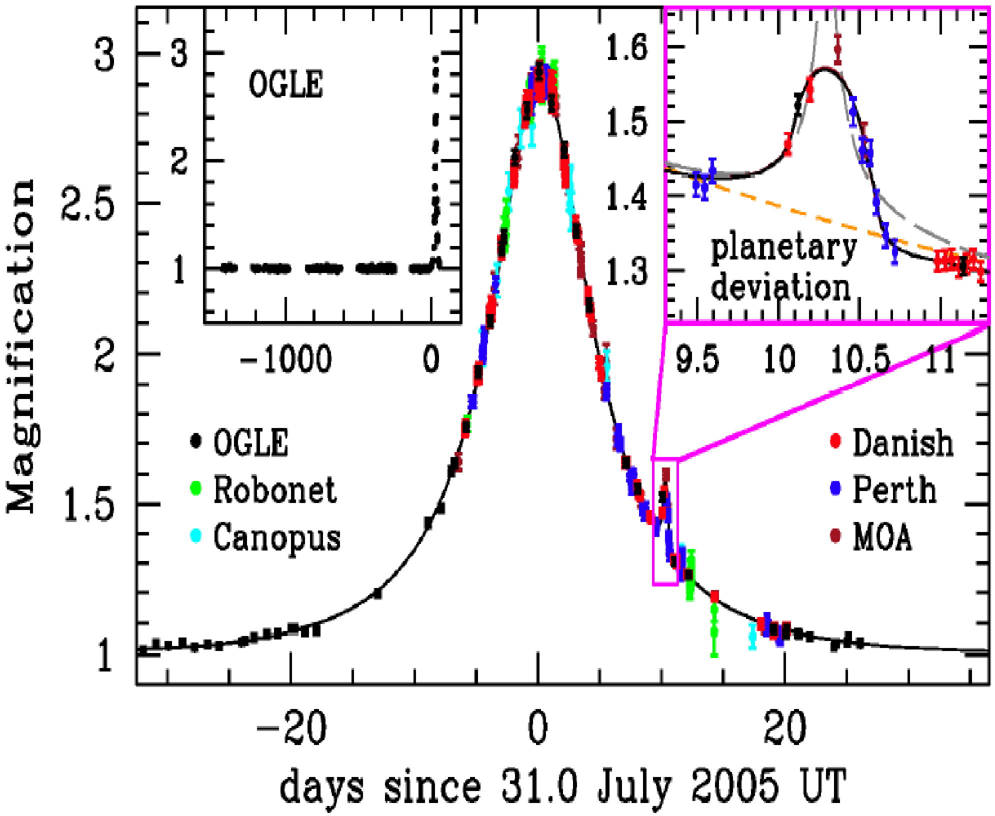
Yields Mass-Ratio Estimate

$$q = \frac{M_p}{M} = \frac{\theta_{{\rm E},p}^2}{\theta_{\rm E}^2} = \frac{\theta_{{\rm E},p}^2}{\theta_*^2}\,\frac{\theta_*^2}{\theta_{\rm E}^2} = \frac{A_p}{2}\,\frac{t_p^2}{t_{\rm E}^2}$$

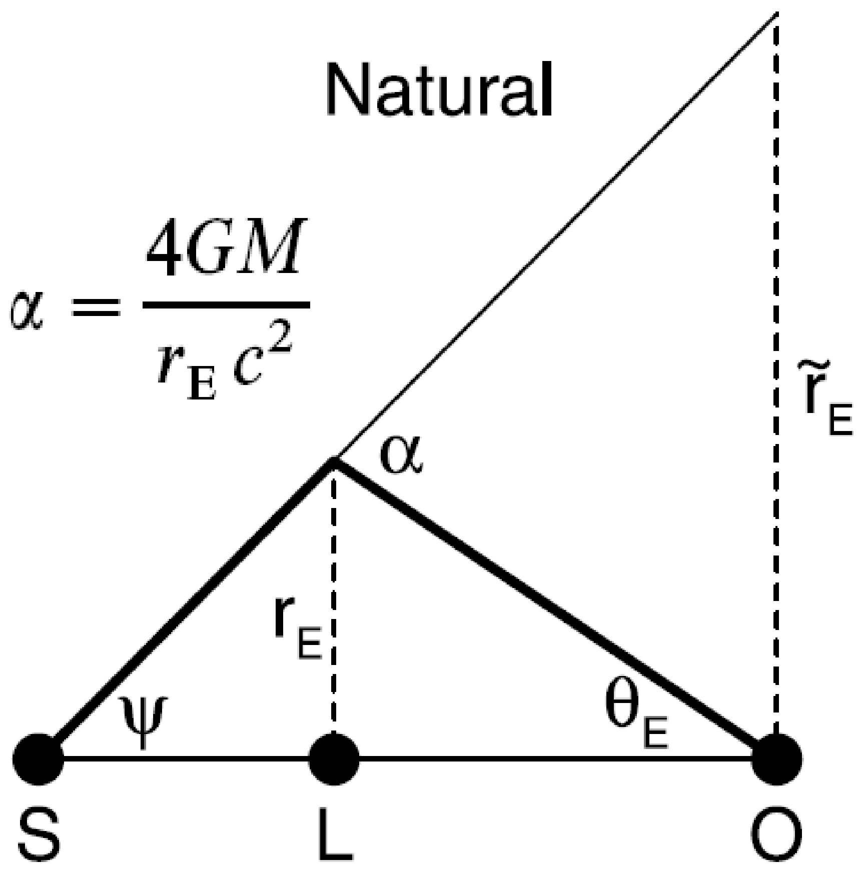
Mass-Ratio Estimate

a la Gould & Loeb

- q=(A_p/2)(t_p/t_E)^2
- A_p = 0.2
- t_p =0.3 day
- t_E=10 day
- q=9e-5
- q_actual = 8e-5



Relation of Mass and Distance to Lensing Observables



$$\alpha/\tilde{r}_E = \theta_E/r_E$$

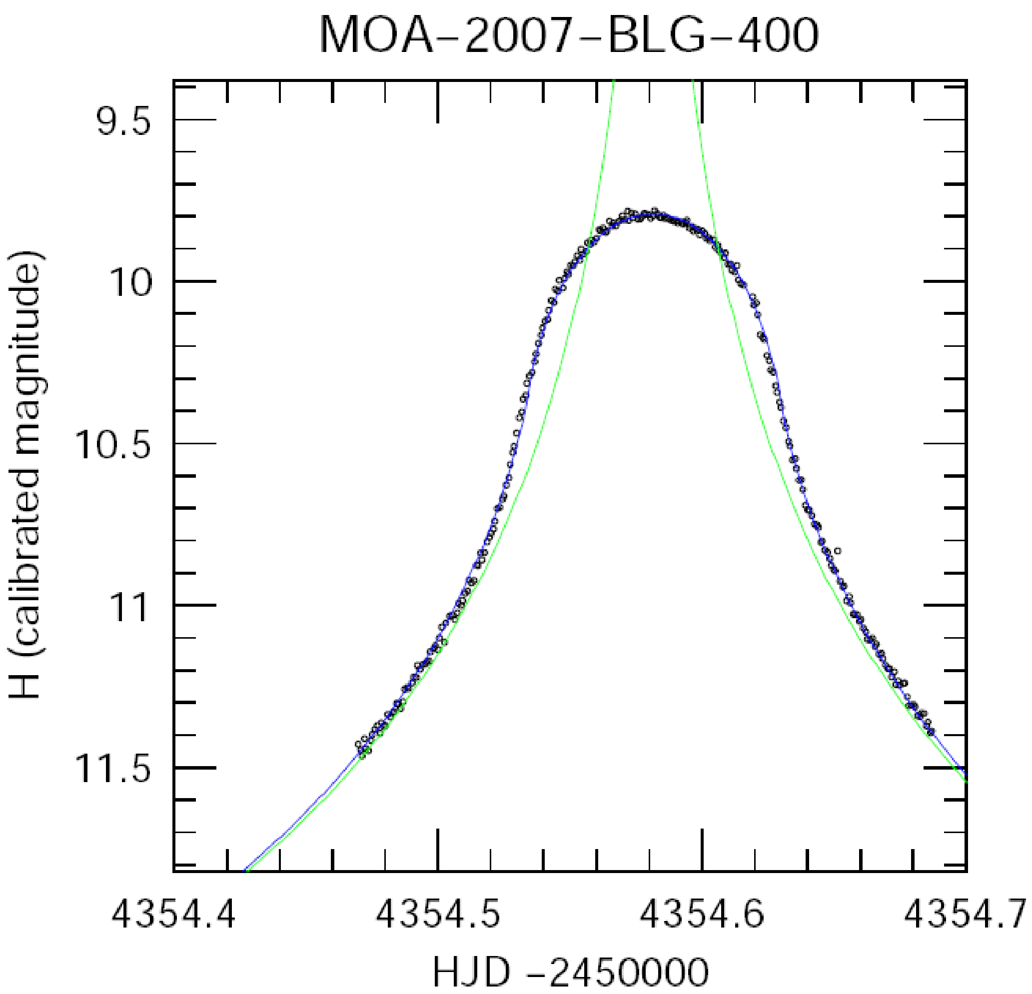
$$\theta_E \tilde{r}_E = \alpha r_E = \frac{4GM}{c^2}$$

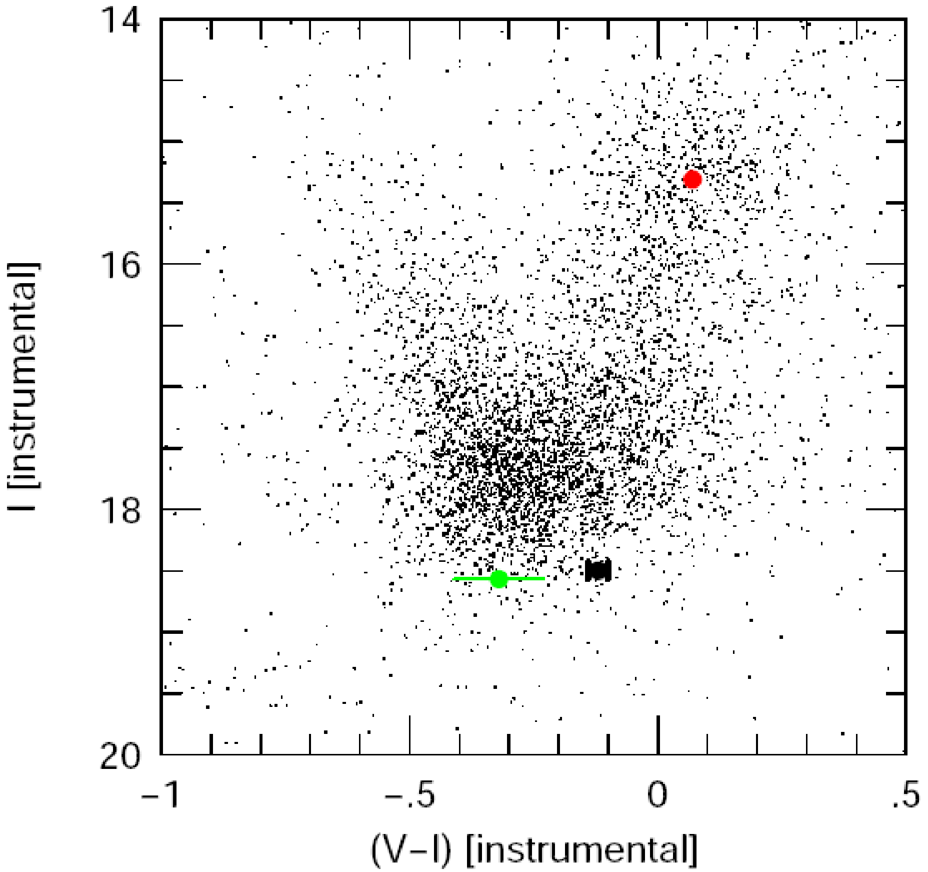
$$\theta_E = \alpha - \psi = \frac{\tilde{r}_E}{D_l} - \frac{\tilde{r}_E}{D_s} = \frac{\tilde{r}_E}{D_{rel}}$$

$$\tilde{r}_E = \sqrt{\frac{4GMD_{rel}}{c^2}} \qquad \theta_E = \sqrt{\frac{4GM}{D_{rel} c^2}}$$

To measure angular Einstein radius:

Standard Sky-Plane Rulers





Finite Source "Attenuation"

$$A(u) \to \frac{1}{u}$$

$$A_{\rm fin}(u|\rho) = \frac{1}{\pi\rho^2}\int_0^{2\pi} d\theta \int_0^{\rho} d\bar\rho\,\rho A[(u+\bar\rho\cos\theta)^2 + (\bar\rho\sin\theta)^2]$$

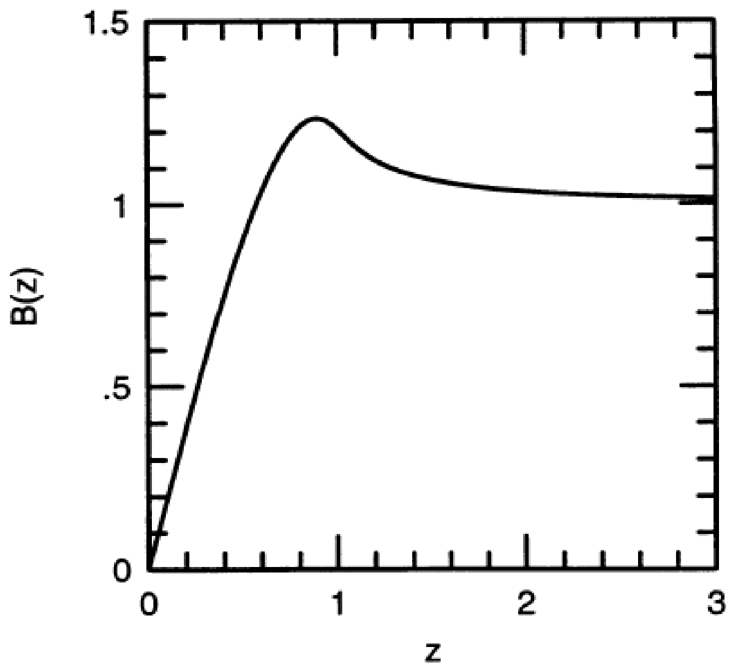
$$A_{\rm fin}(u|\rho) = \frac{B(z)}{u}; \qquad z \equiv \frac{u}{\rho}$$

$$B(z) = \frac{z^2}{\pi}\int_0^{2\pi} d\theta \int_0^{1/z} dq\,q(1+q^2+2q\cos\theta)^{-1/2}$$

$$A_{\rm fin}(u|\rho) = \frac{B(z)}{u} \to A(u)B(z)$$

Finite Source "Attenuation"

$$A_{\text{fin}}(u|\rho) = \frac{B(z)}{u} \rightarrow A(u)B(z)$$

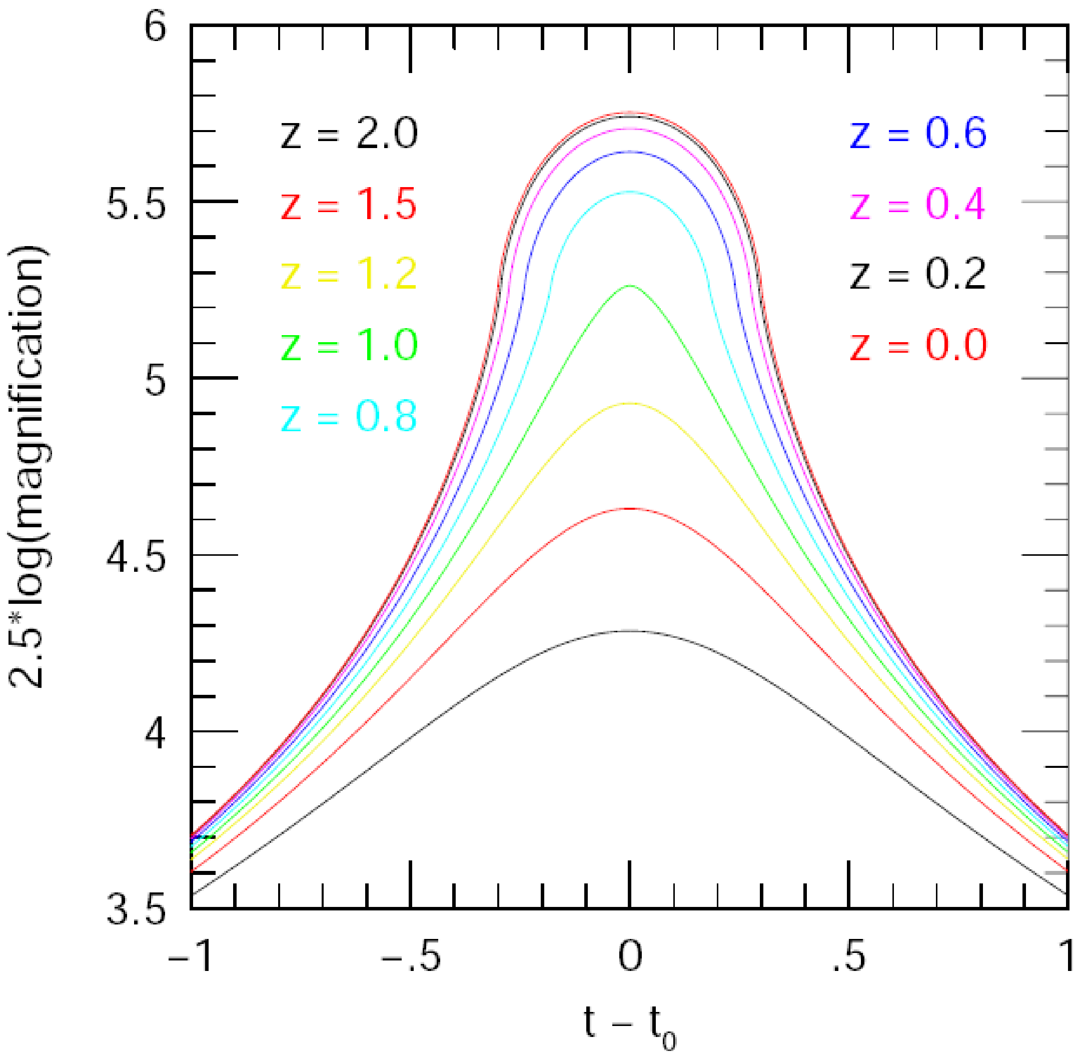


PROPER MOTIONS OF MACHOs

ANDREW GOULD

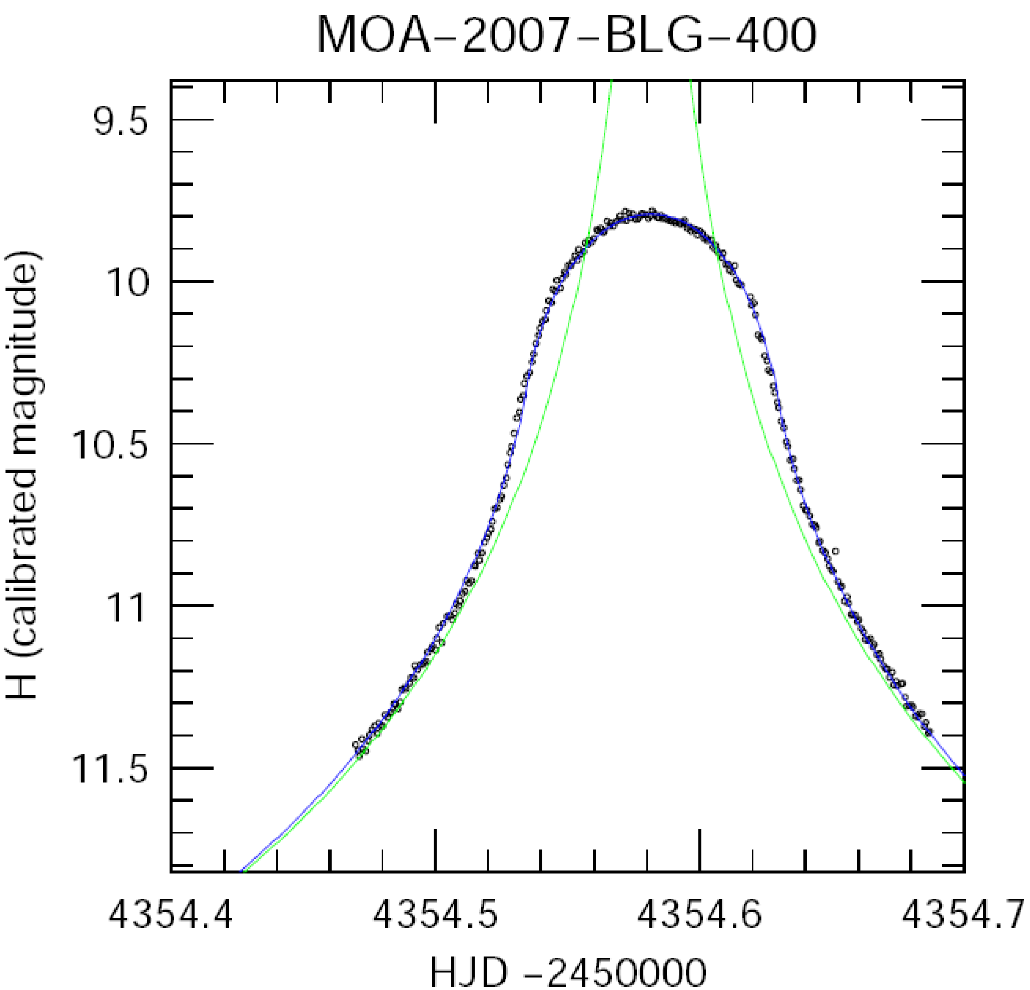
THE ASTROPHYSICAL JOURNAL, 421:L71–L74, 1994 February 1

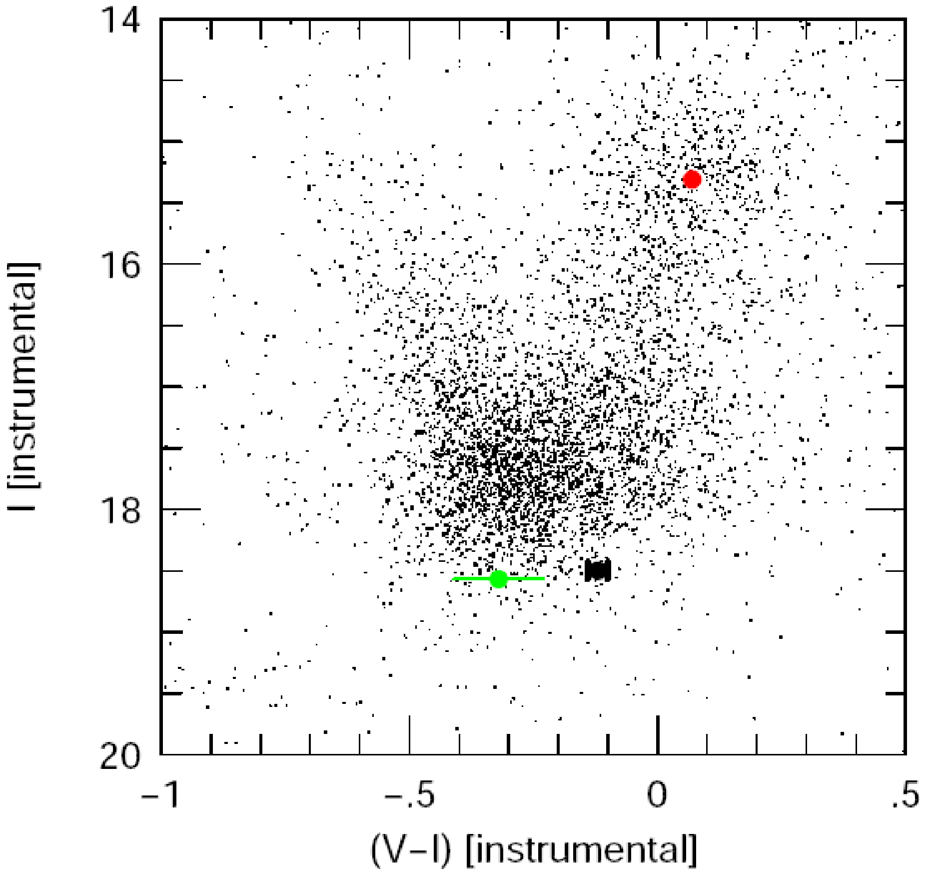
Finite Source "Attenuation"



To measure angular Einstein radius:

Standard Sky-Plane Rulers





To measure angular Einstein radius:

Standard Sky-Plane Rulers

$$\rho \equiv \frac{\theta_*}{\theta_{\rm E}}$$

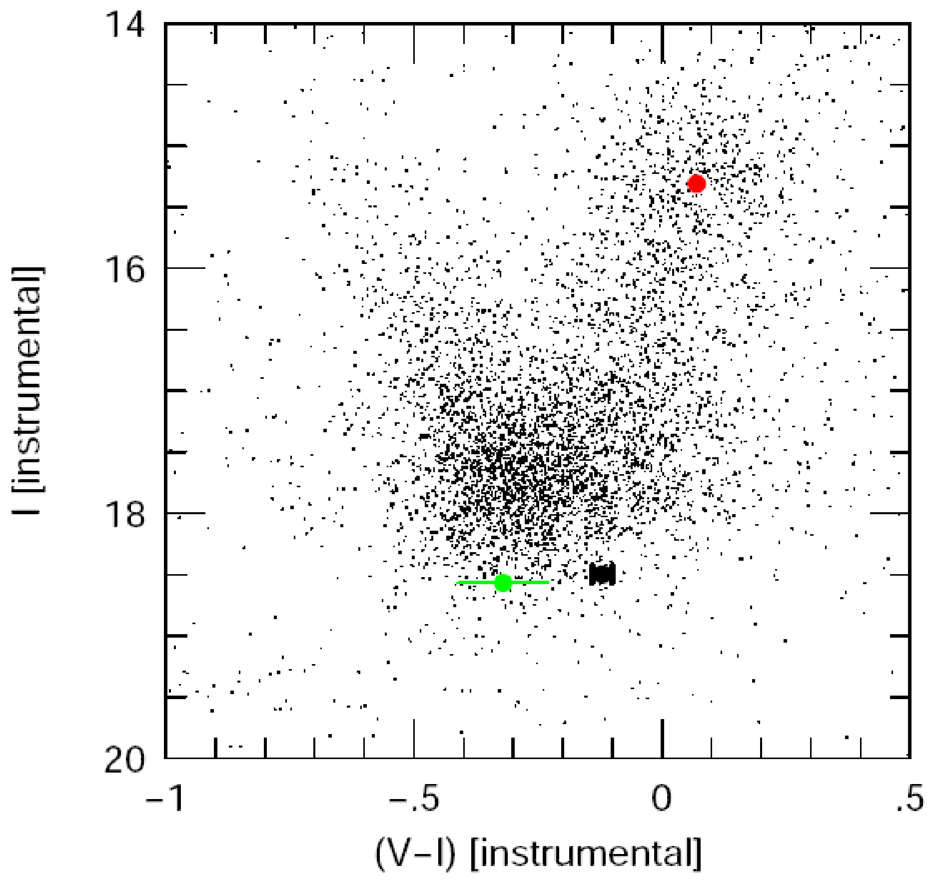
$$\theta_{\rm E} = \frac{\theta_*}{\rho}$$

$$f_{s,0} = \pi \theta_*^2 S_0$$

$$\theta_* = \sqrt{\frac{f_s(I_0)}{\pi S[(V-I)_{s,0}]}}$$

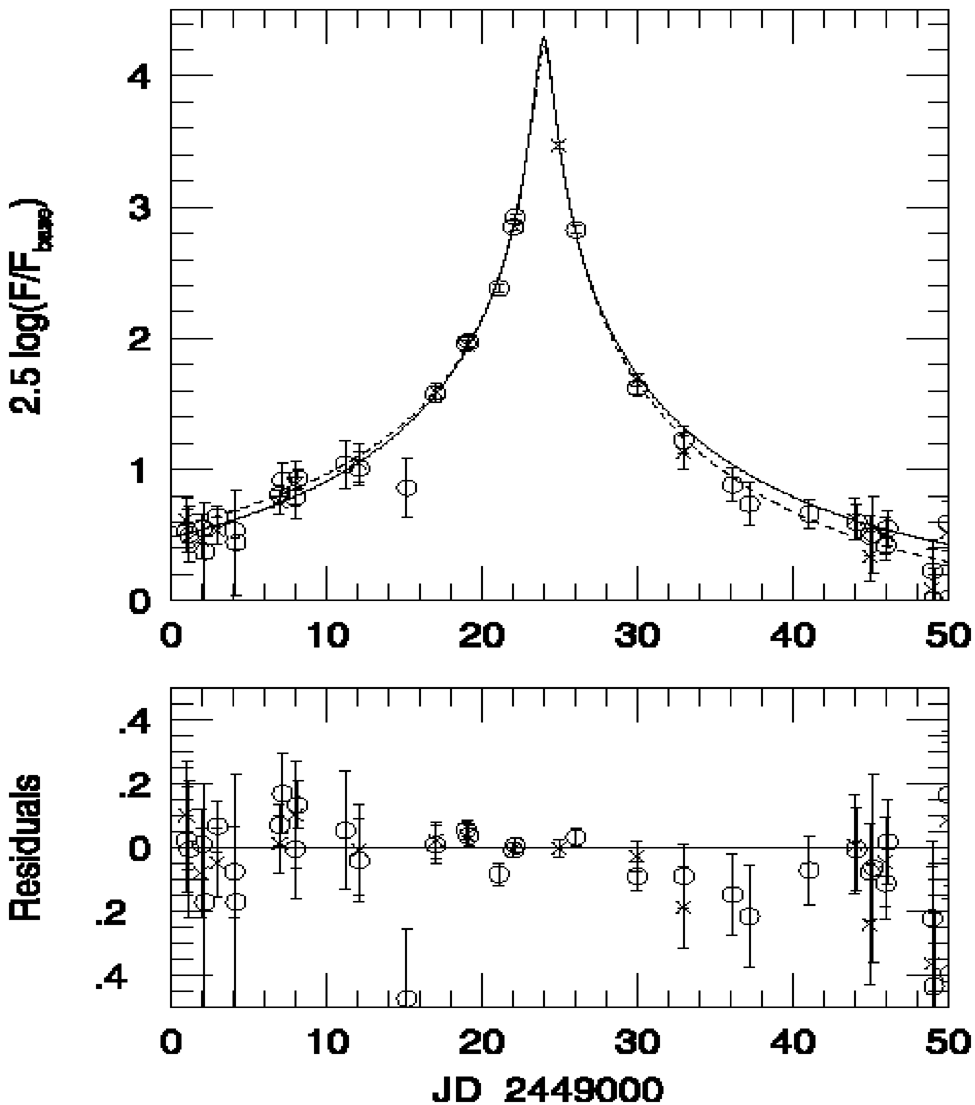
$$(V-I)_{s,0} = (V-I)_{clump,0} + \Delta(V-I)$$

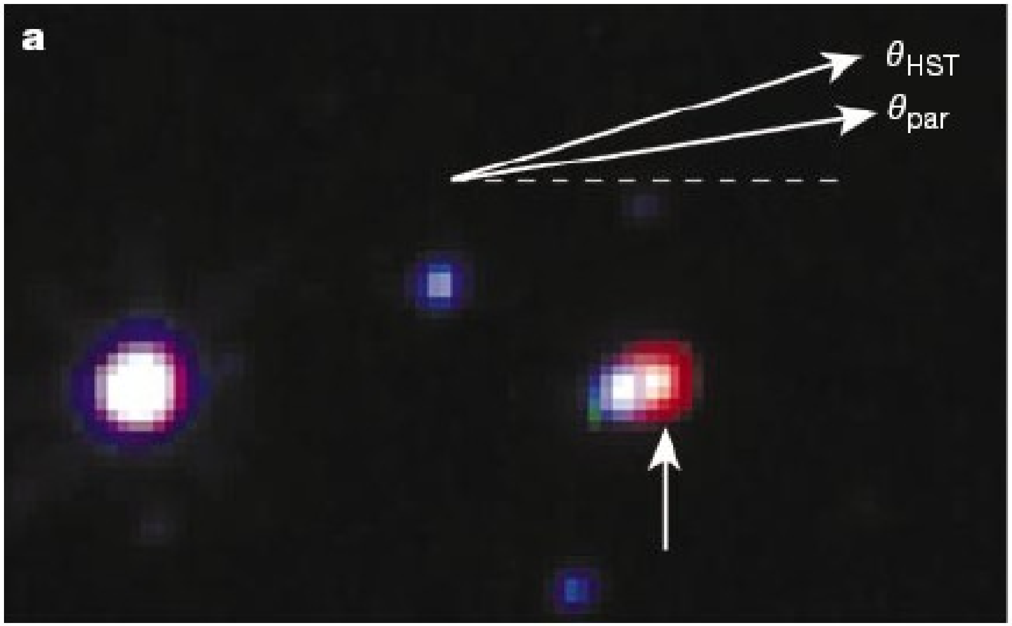
$$I_{s,0} = I_{clump,0} + \Delta I$$



MACHO-LMC-5

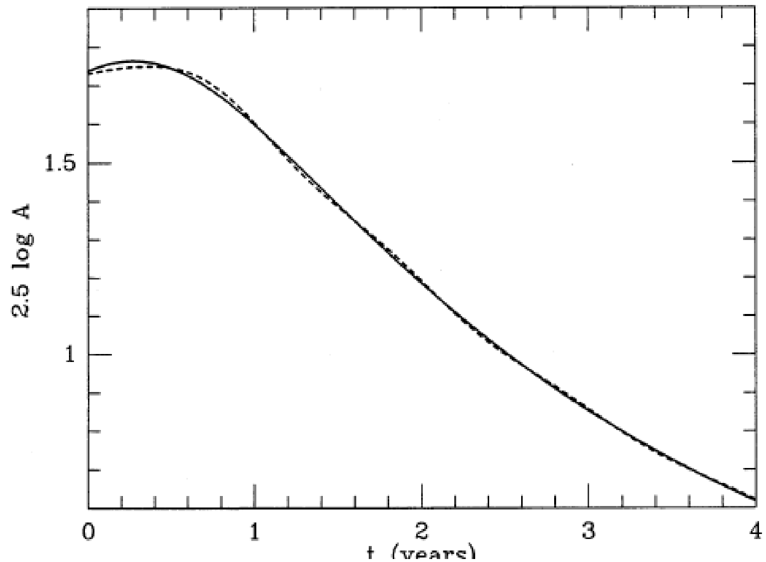
Angular Einstein radius from proper motion

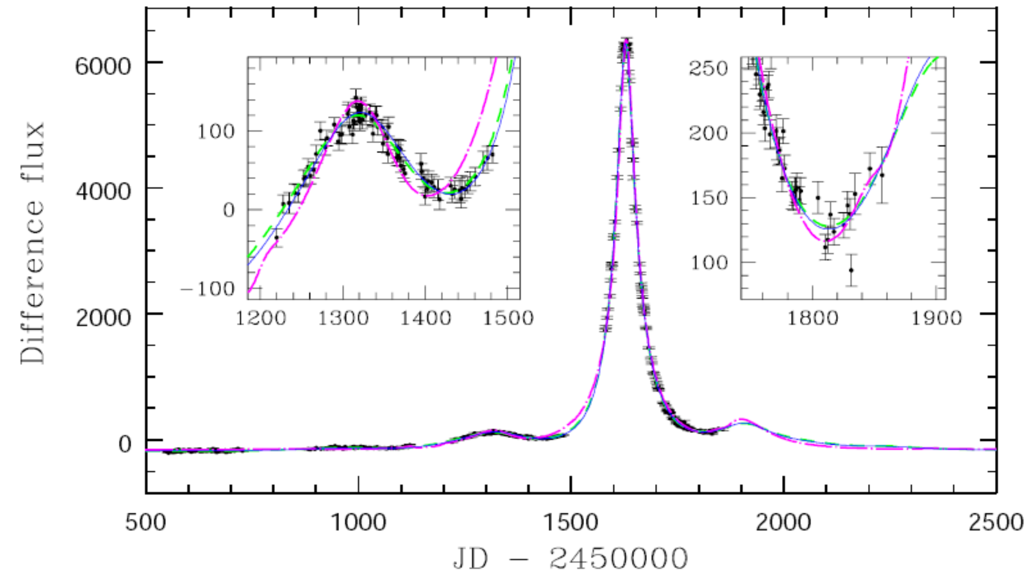


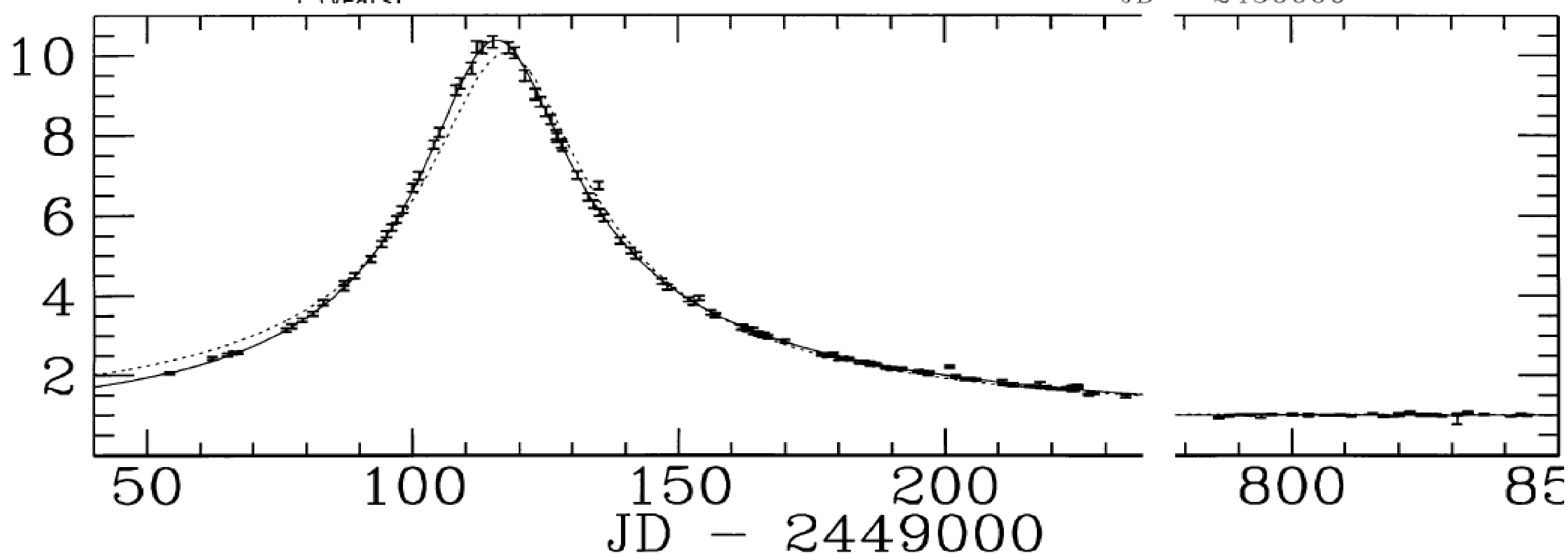


To measure parallax:

Standard Observer-Plane Rulers

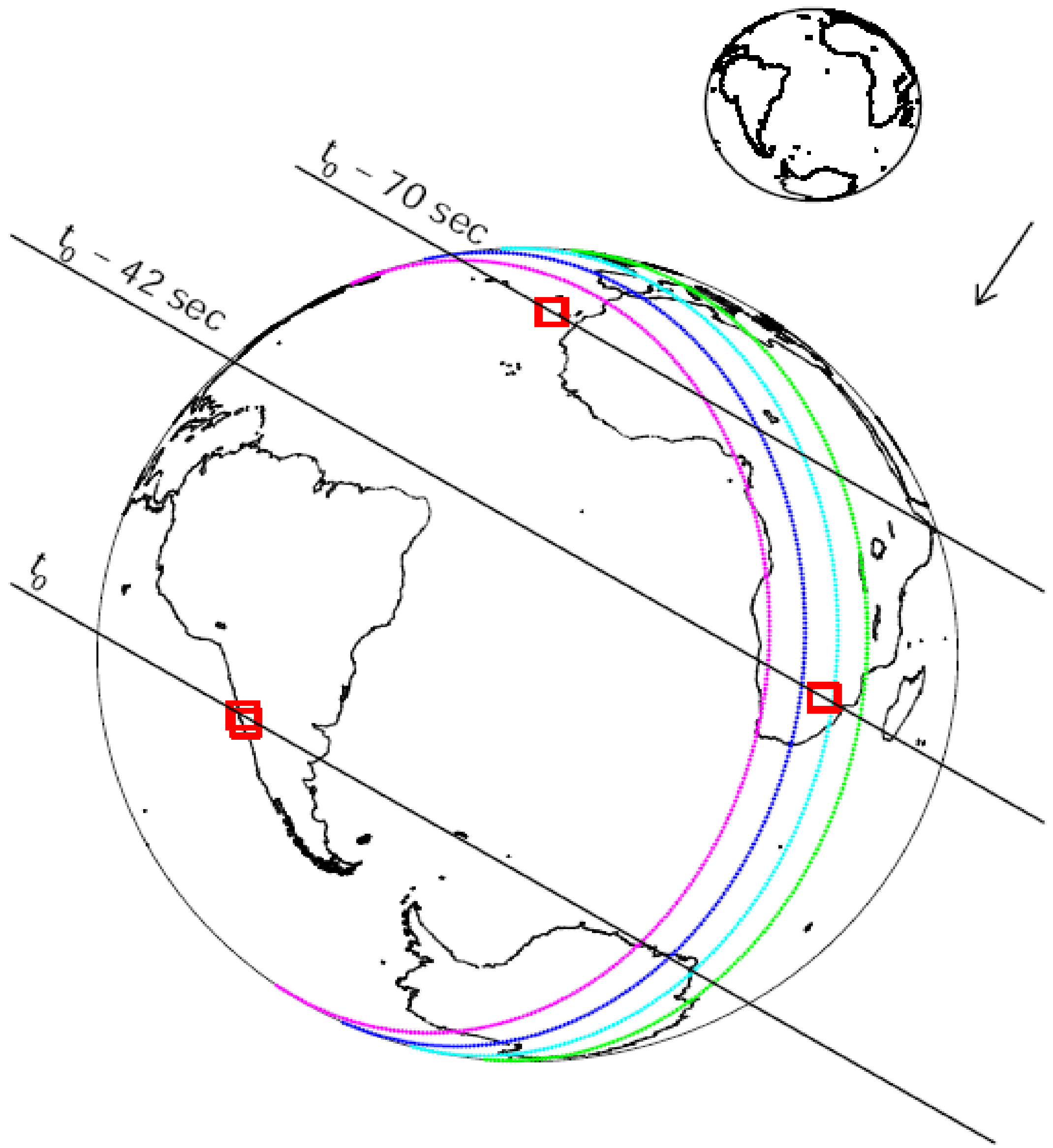






Terrestrial Parallax:

Simultaneous Observations on Earth



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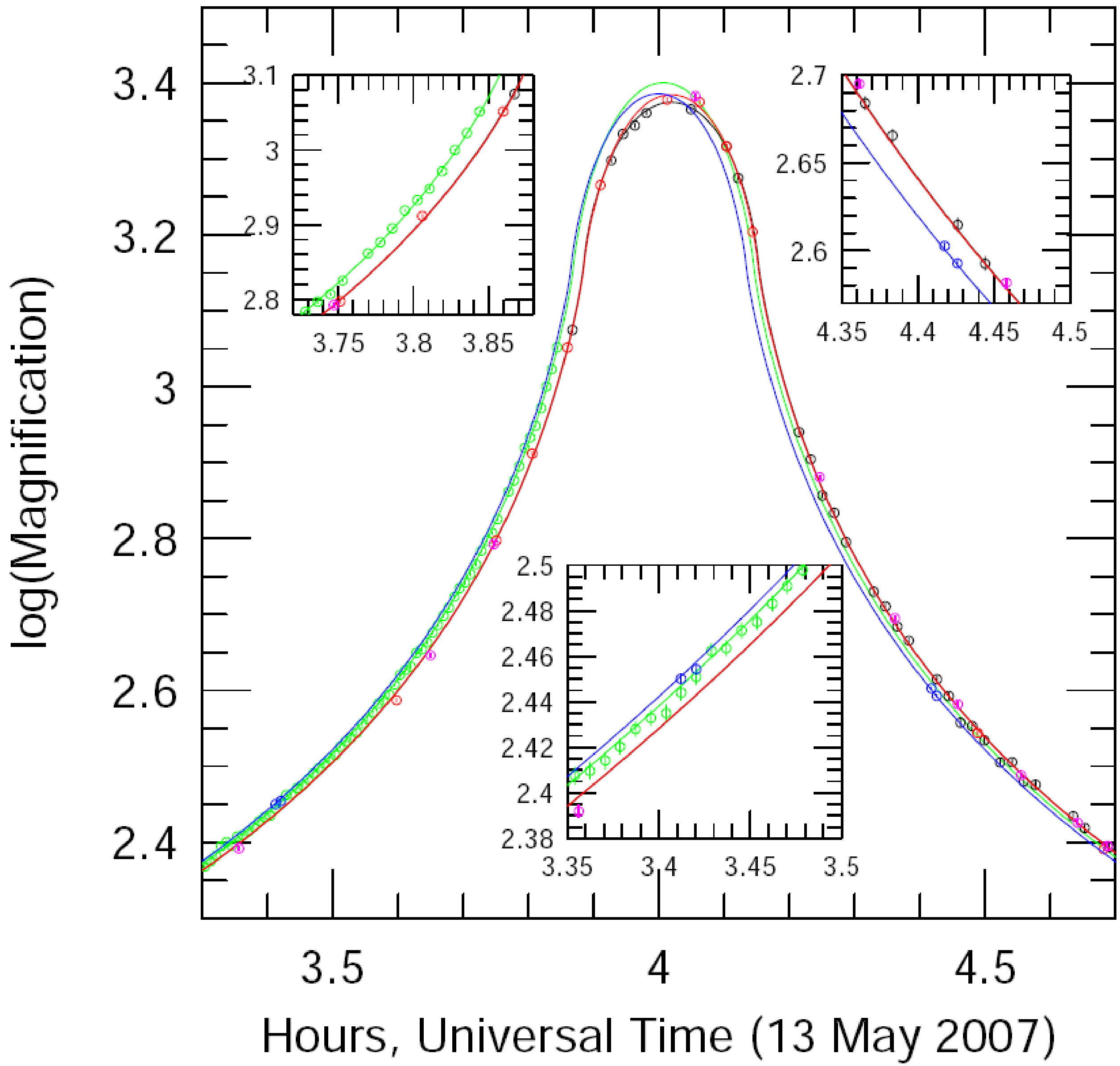
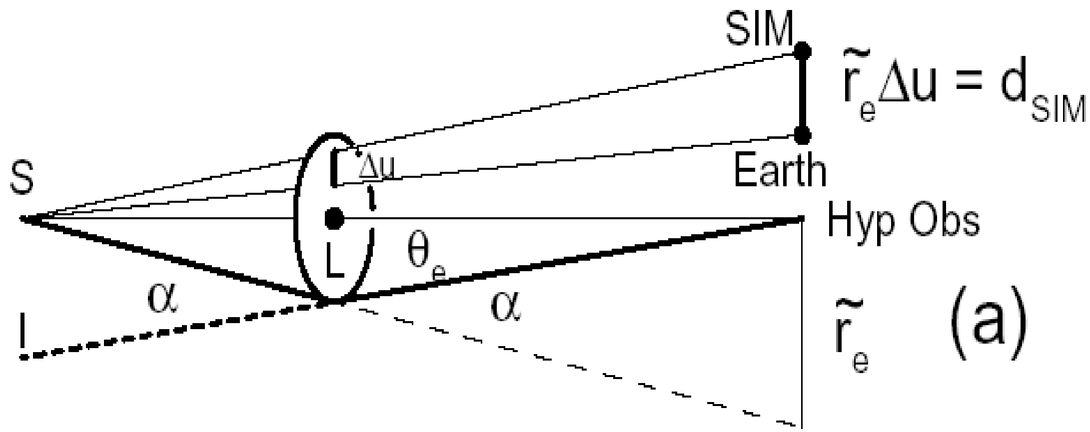
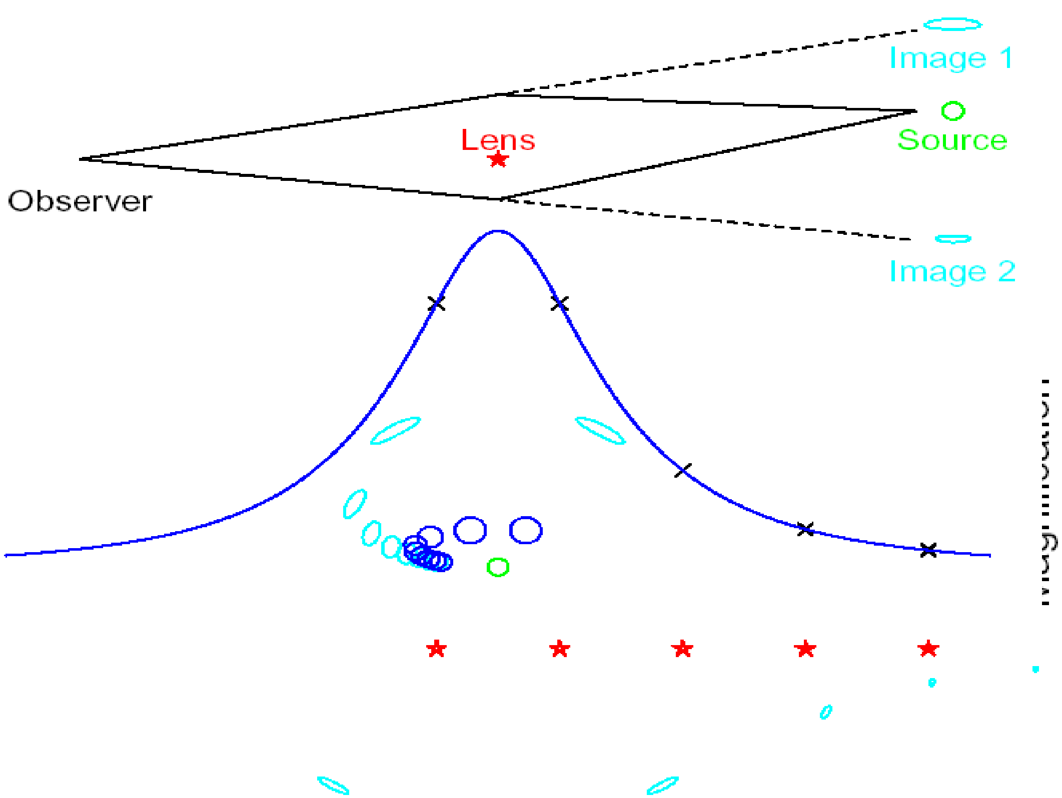
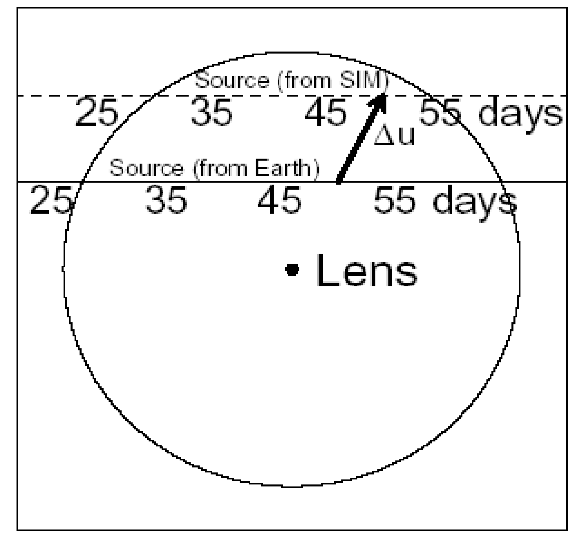
Canaries South Africa Chile



Space-Based Parallaxes & Einstein Radii : SIM

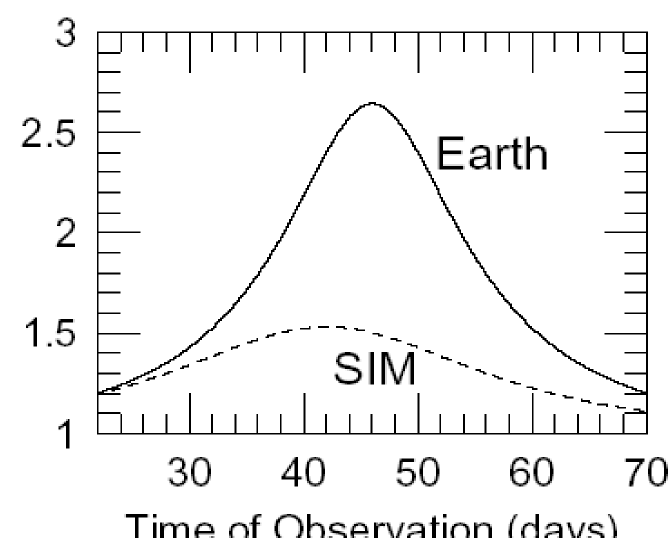






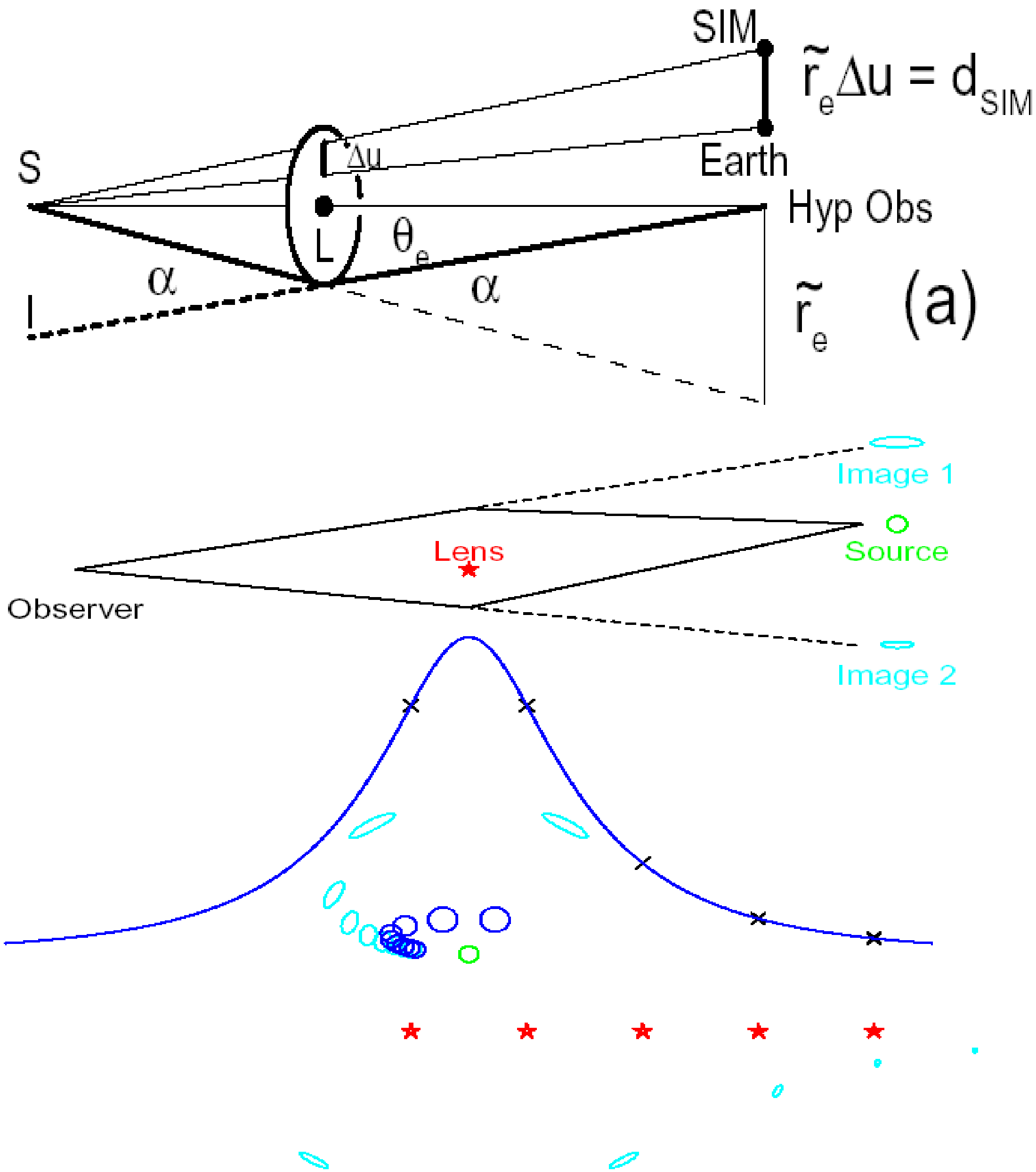
$$\tilde{r}_e = \frac{d_{SIM}}{\Delta u}$$

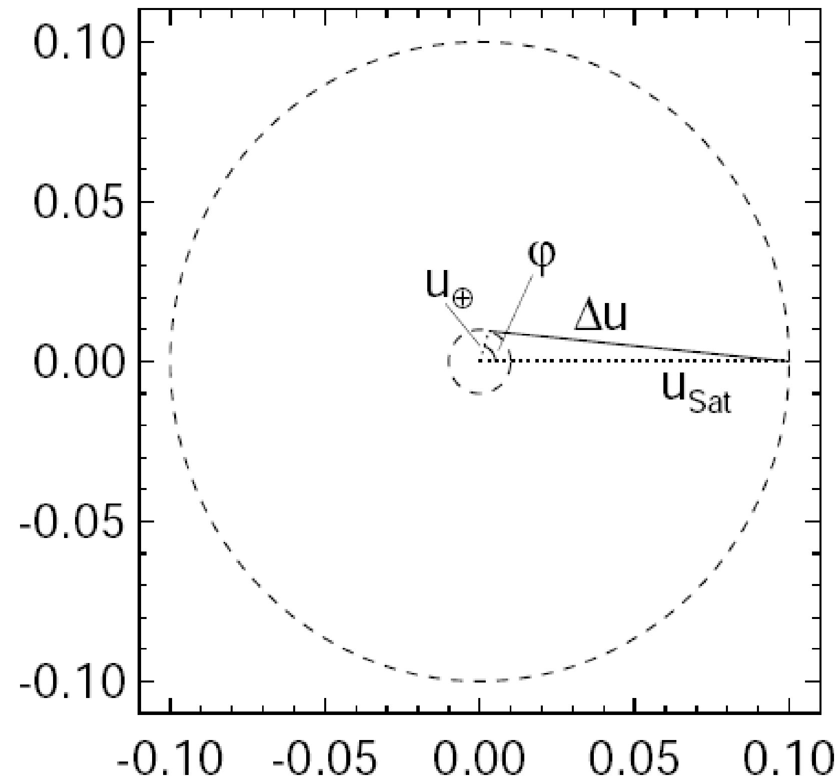
(b)



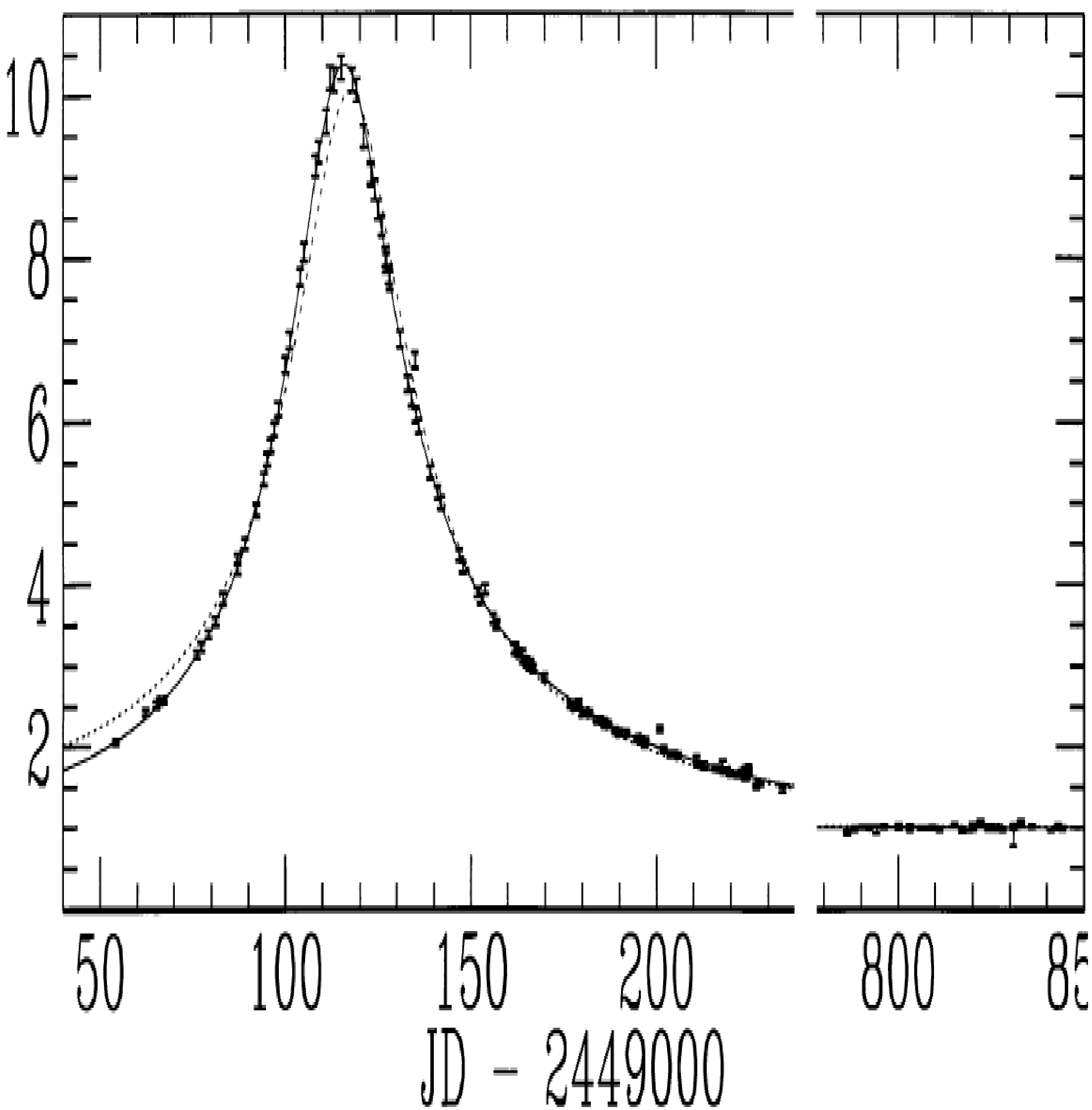
(c)

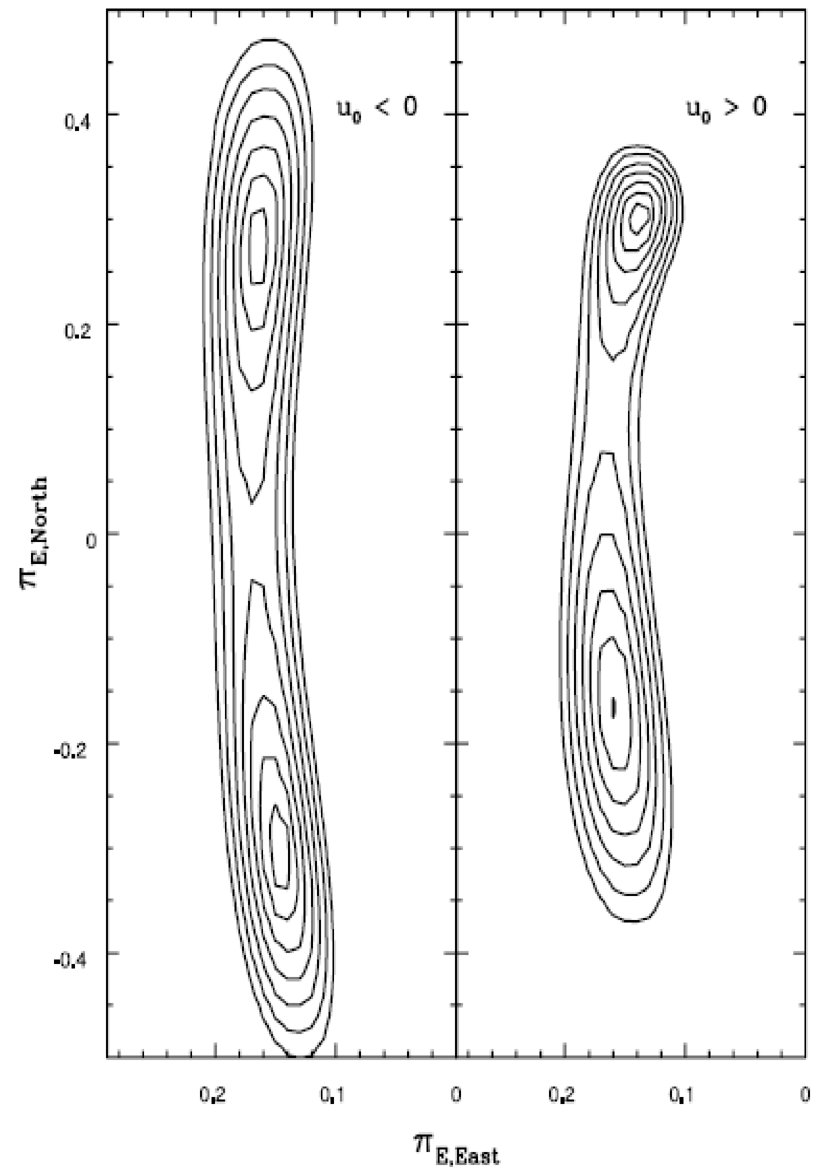
Space-Based Parallaxes & Einstein Radii : Deep Impact





Parallax Degeneracies: Macho 104-C





Introducing Jerk-Parallax:

$$\pi_j \equiv \frac{4}{3}\frac{\boldsymbol{j}}{\alpha^2 t_{\rm E}}$$

$$\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{\omega} t + \pi_{\rm E}\left(\frac{1}{2}\boldsymbol{\alpha} t^2 + \frac{1}{6}\boldsymbol{j} t^3 + \ldots\right)$$

$$u^2 = \sum_{i=0}^{\infty} C_i t^i \qquad C_0 = u_0^2, \quad C_1 = 0$$

$$C_2 = -\alpha\, u_0 \pi_{{\rm E},\perp} + t_{\rm E}^{-2}$$

$$C_3 = \alpha\frac{\pi_{{\rm E},\parallel}}{t_{\rm E}} + \frac{1}{4}\alpha^2 t_{\rm E} u_0 \boldsymbol{\pi}_{\rm E} \times \boldsymbol{\pi}_j$$

$$C_4 = \frac{\alpha^2}{4}(\pi_{\rm E}^2 + \boldsymbol{\pi}_j \cdot \boldsymbol{\pi}_{\rm E}) + \frac{1}{12}\frac{\Omega_\oplus^2}{\alpha} u_0 \pi_{{\rm E},\perp}$$

Theory vs. Observation

$$u_0 = 0 \Rightarrow C_0 = C_1 = 0; \qquad C_2 = t_{\rm E}^{-2}$$

$$C_3 = \frac{\alpha}{t_{\rm E}} \pi_{{\rm E},\parallel}; \qquad C_4 = \frac{\alpha^2}{4} \vec{\pi}_{\rm E} \cdot (\vec{\pi}_{\rm E} + \vec{\pi}_j)$$

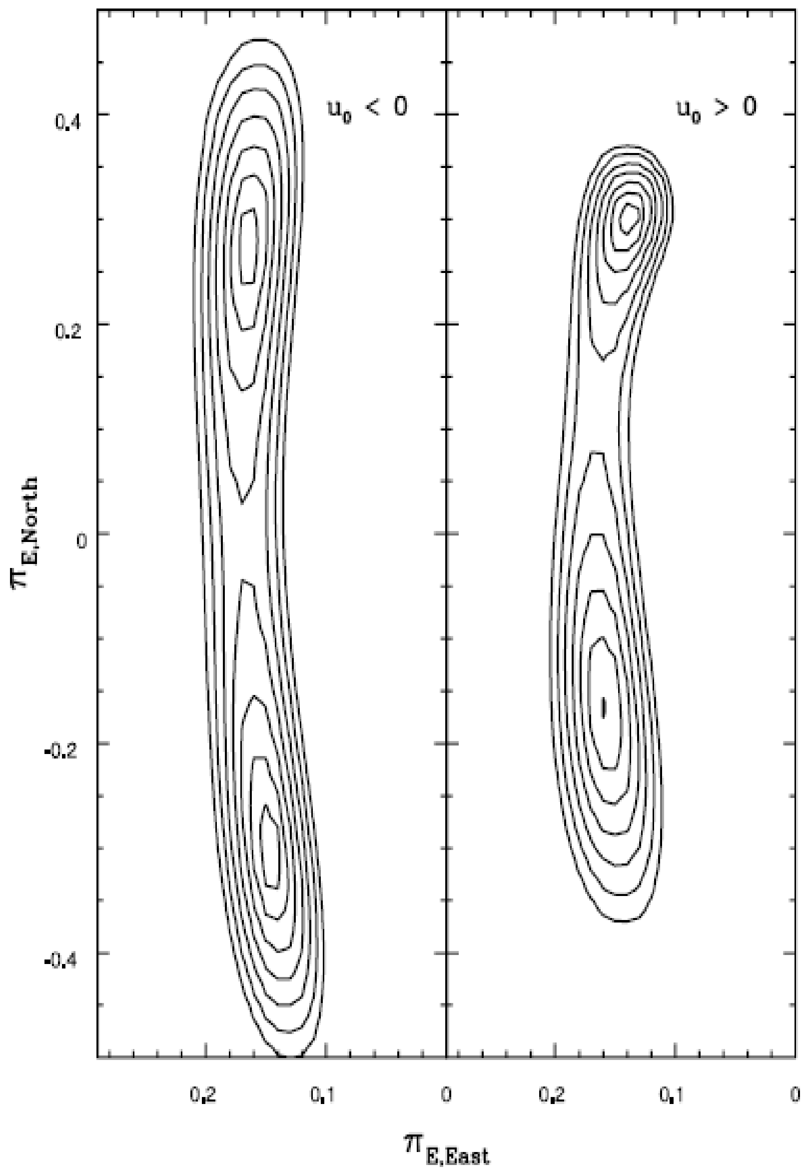
$$t'_{\rm E} = t_{\rm E}; \qquad \pi_{{\rm E},\parallel} = \pi'_{{\rm E},\parallel}$$

$$\vec{\pi}'_{\rm E} \cdot (\vec{\pi}'_{\rm E} + \vec{\pi}_j) = \vec{\pi}_{\rm E} \cdot (\vec{\pi}_{\rm E} + \vec{\pi}_j)$$

$$\pi'_{{\rm E},\perp}(\pi'_{{\rm E},\perp} + \pi_{j,\perp}) = \pi_{{\rm E},\perp}(\pi_{{\rm E},\perp} + \pi_{j,\perp})$$

$$\pi'_{{\rm E},\perp} = -(\pi_{{\rm E},\perp} + \pi_{j,\perp})$$

$$\pi_{j,\perp} \to -\frac{4}{3} \frac{\rm yr}{2\pi t_{\rm E}} \frac{\sin\beta}{(\cos^2\psi \sin^2\beta + \sin^2\psi)^{3/2}}$$



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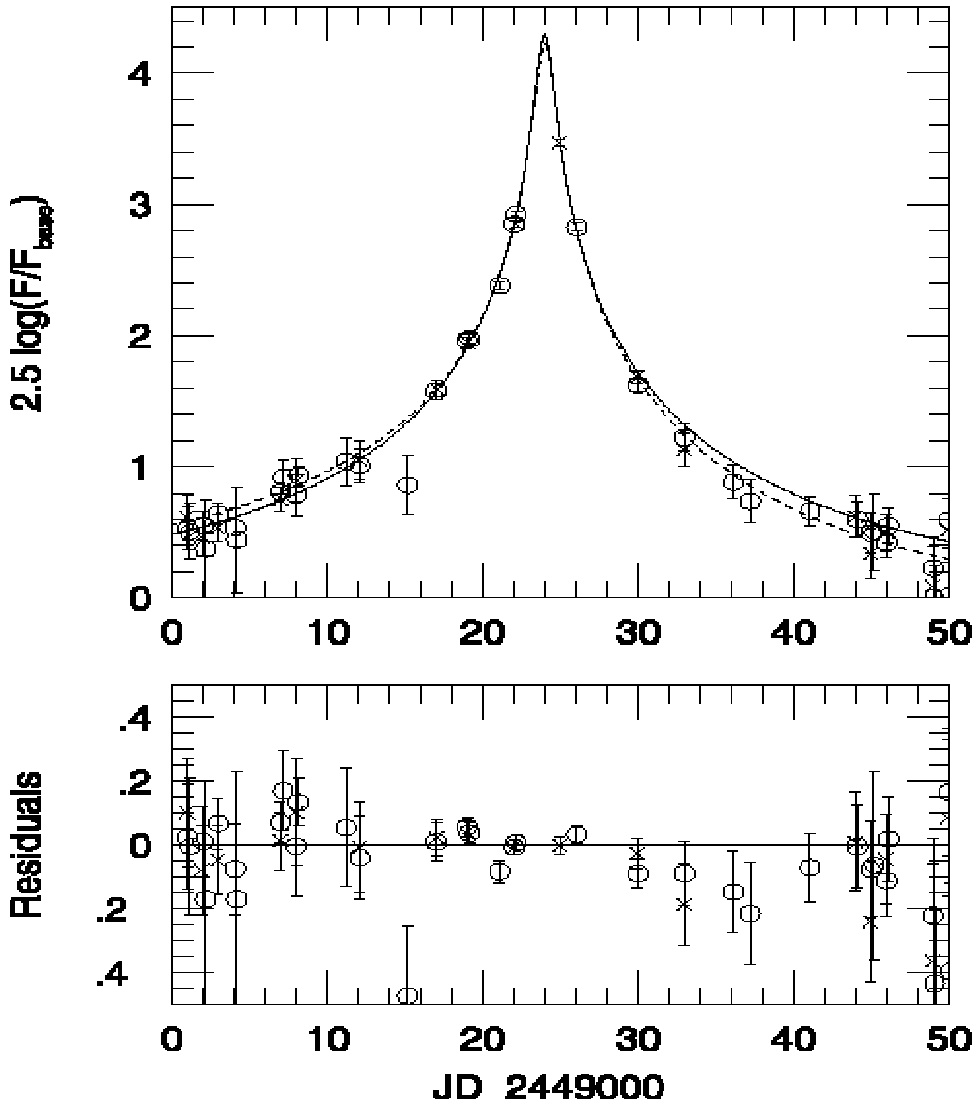
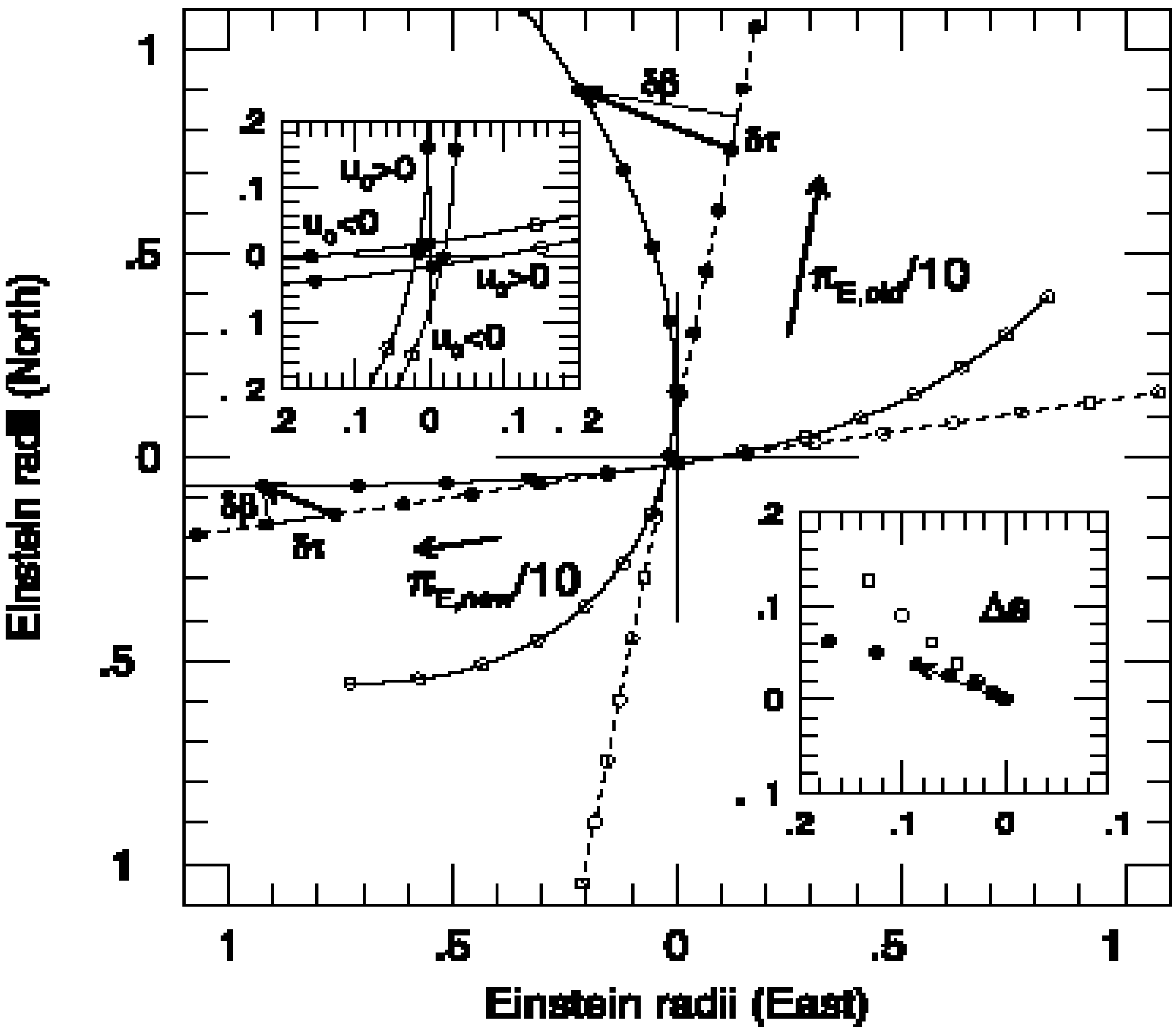
First Jerk-Parallax Degeneracy



Implies that very different trajectories can generate the same lightcurve.



Macho LMC-5:

Analytic formula works

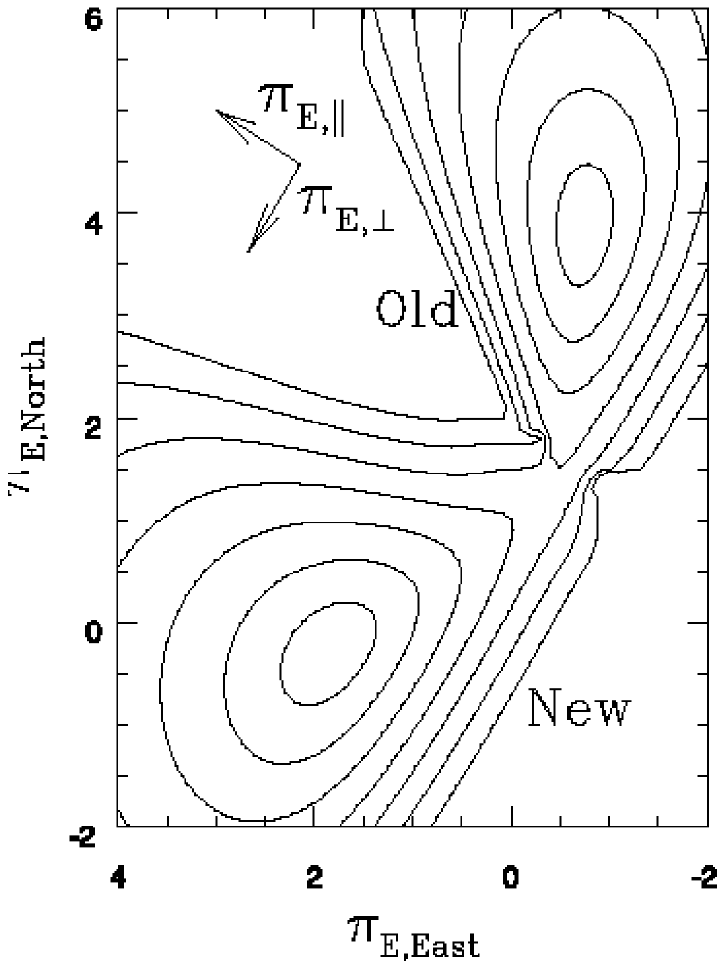
$$\pi_{j,\perp} \to -\frac{4}{3}\frac{\mathrm{yr}}{2\pi t_\mathrm{E}}\frac{\sin\beta}{(\cos^2\psi\sin^2\beta+\sin^2\psi)^{3/2}}$$

$$\mathrm{LMC} \Rightarrow \beta \sim -90^\circ \Rightarrow \pi_{j,\perp} \to \frac{78\,\mathrm{day}}{t_\mathrm{E}} \to 2.4$$

$$\pi_{\mathrm{E},\perp,\mathrm{old}} = -3.7$$

$$\Rightarrow \pi_{\mathrm{E},\perp,\mathrm{new}} = -(\pi_{\mathrm{E},\perp,\mathrm{old}} + \pi_{j,\perp})$$

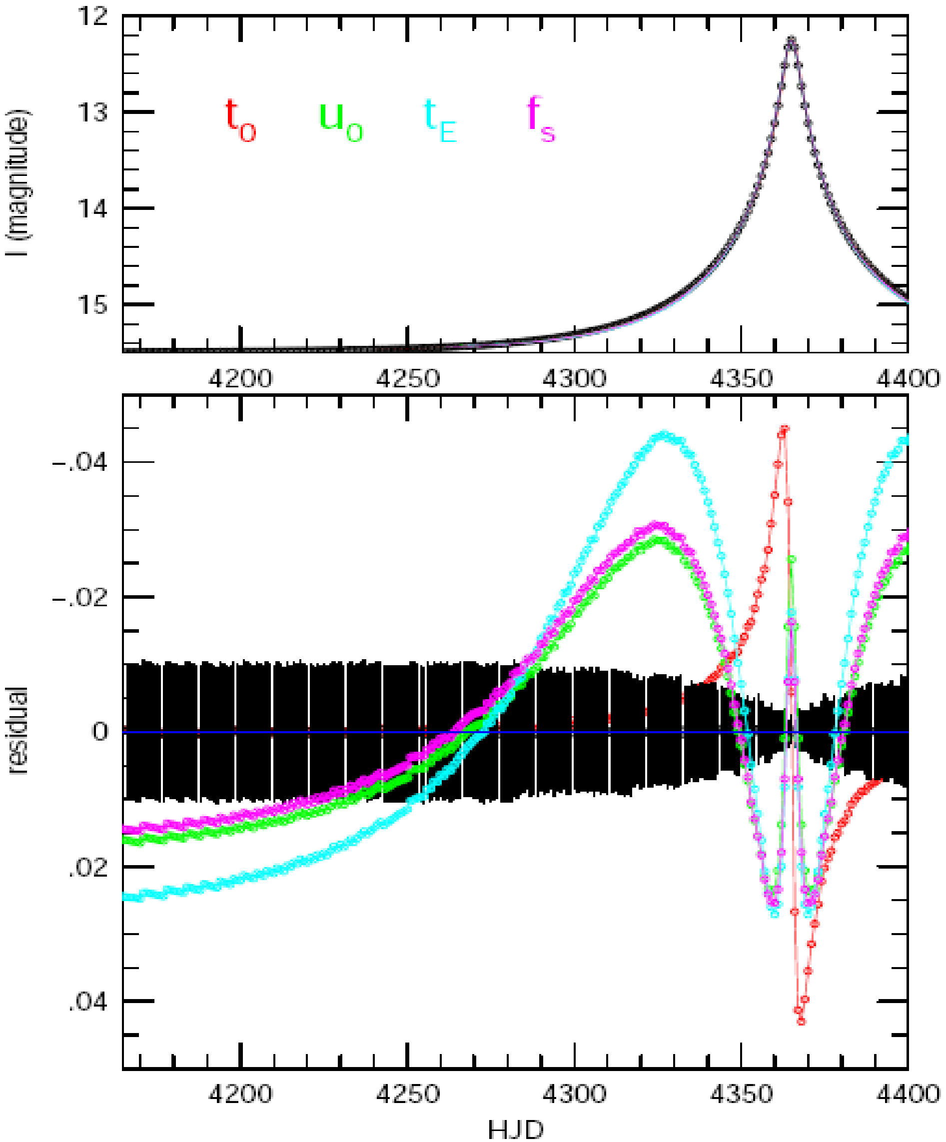
$$= -(-3.7+2.4) = +1.3$$



Continuous Degeneracies:

Which curve is most different?

- F = f_s*A + f_b
- A = A(u)
- u^2=u0^2+[(t-t0)/t_E]^2



Binary-Lens Equation

$$\mathbf{u} - \mathbf{y} = -\frac{\mathbf{y} - \mathbf{y}_L}{|\mathbf{y} - \mathbf{y}_L|^2}$$

$$\mathbf{y}_L = 0 \rightarrow \mathbf{u} - \mathbf{y} = -\frac{\mathbf{y}}{y^2} \Longrightarrow u - y = -\frac{1}{y}$$

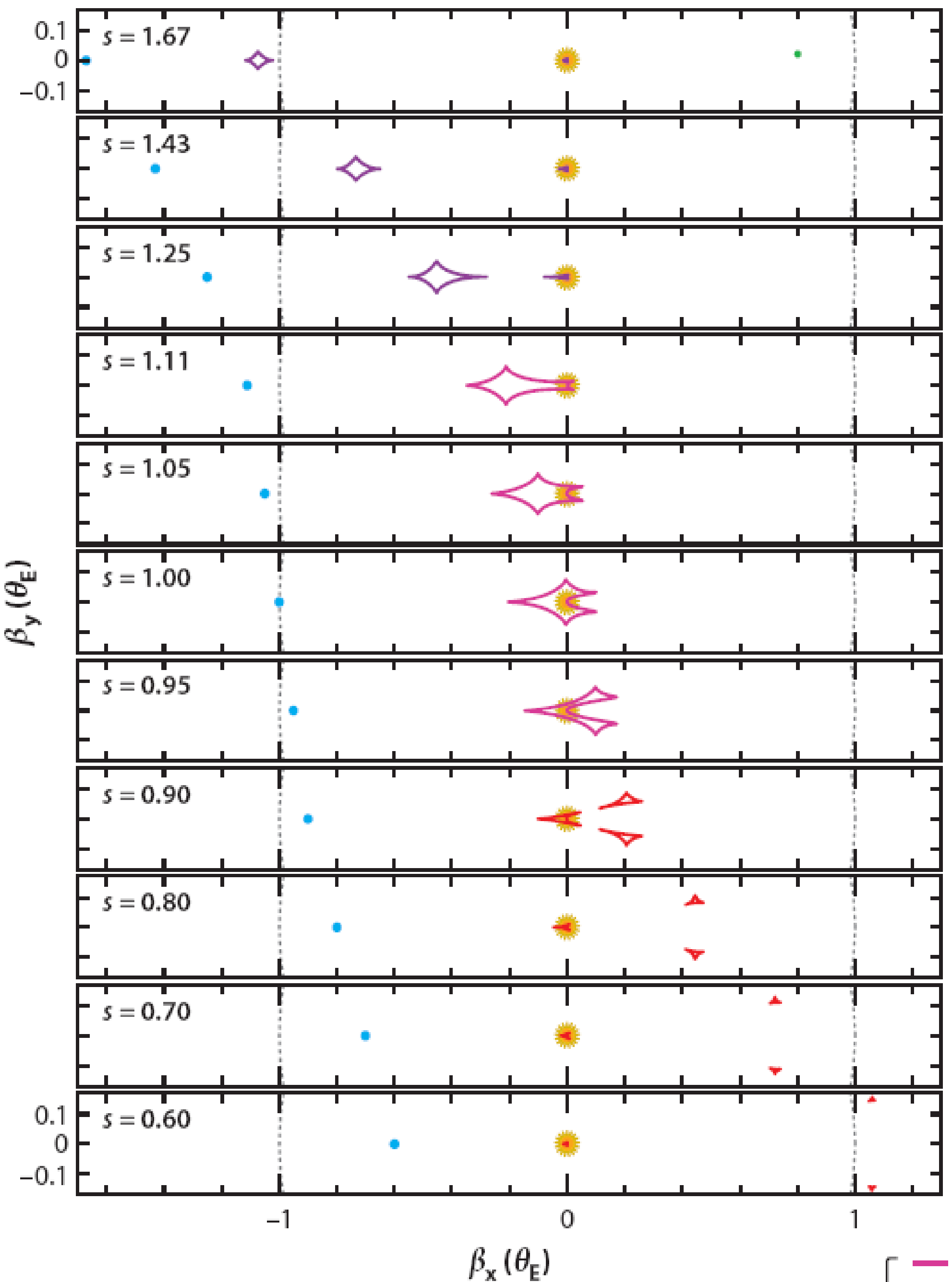
$$\Longrightarrow (y - u)y = 1 \Longrightarrow (\theta_I - \theta_S)\theta_I = \theta_{\mathrm{E}}^2$$

$$\mathbf{u} = \mathbf{y} - \sum_i \epsilon_i \frac{\mathbf{y} - \mathbf{y}_{m,i}}{|\mathbf{y} - \mathbf{y}_{m,i}|^2} \qquad \epsilon_i \equiv \frac{m_i}{M_{\mathrm{tot}}}$$

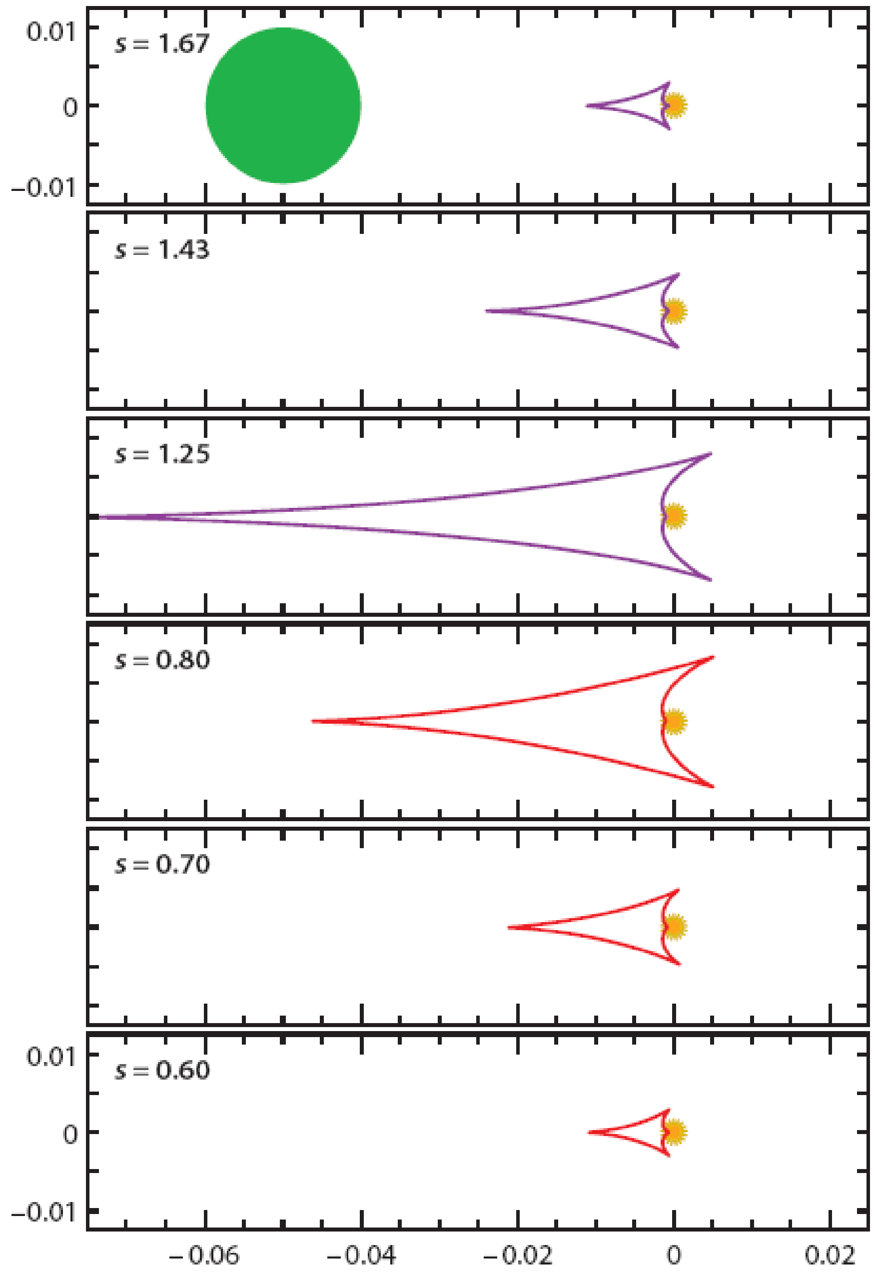
$$\zeta = z - \sum_i \frac{\epsilon_i}{\bar{z} - \bar{z}_{m,i}}$$

$$\zeta \equiv u_1 + iu_2 \qquad z \equiv y_1 + iy_2$$

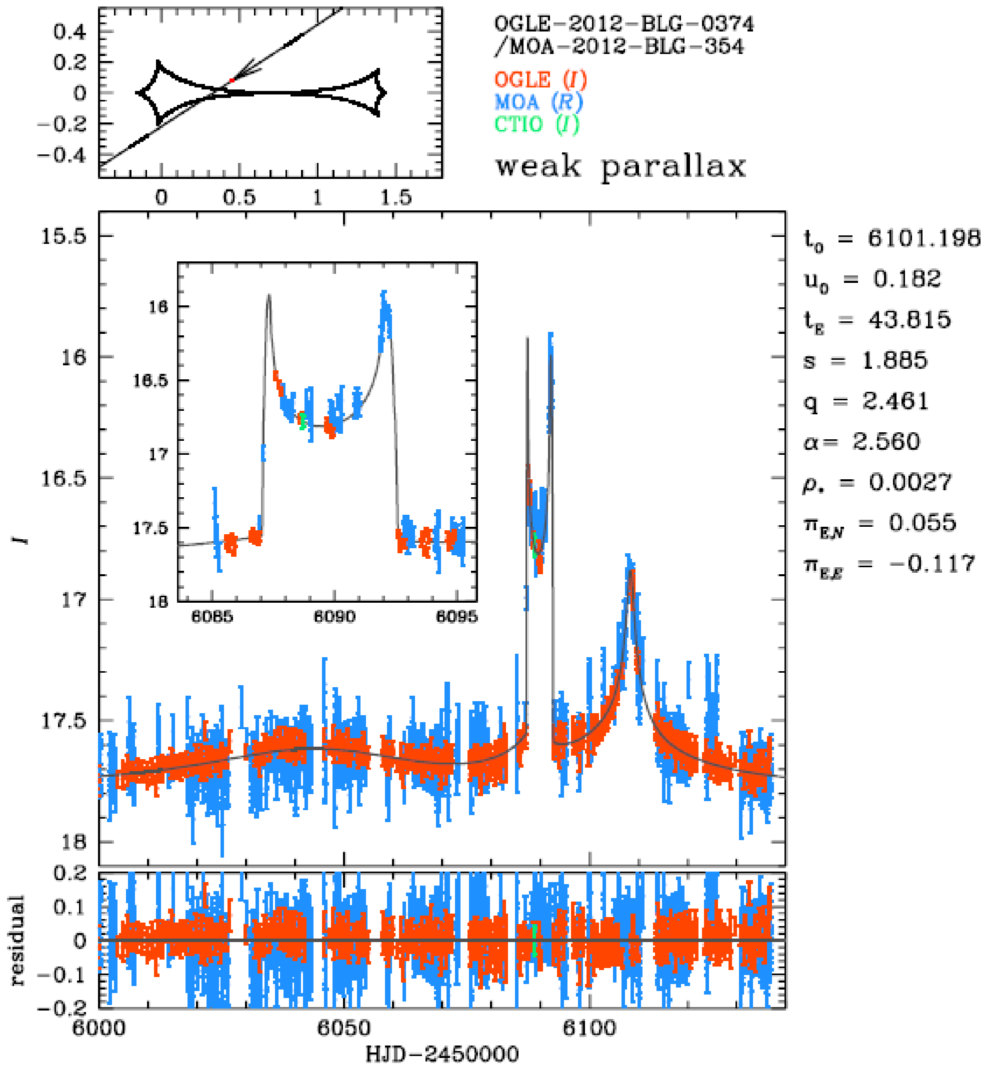
Binary-Lens Topologies



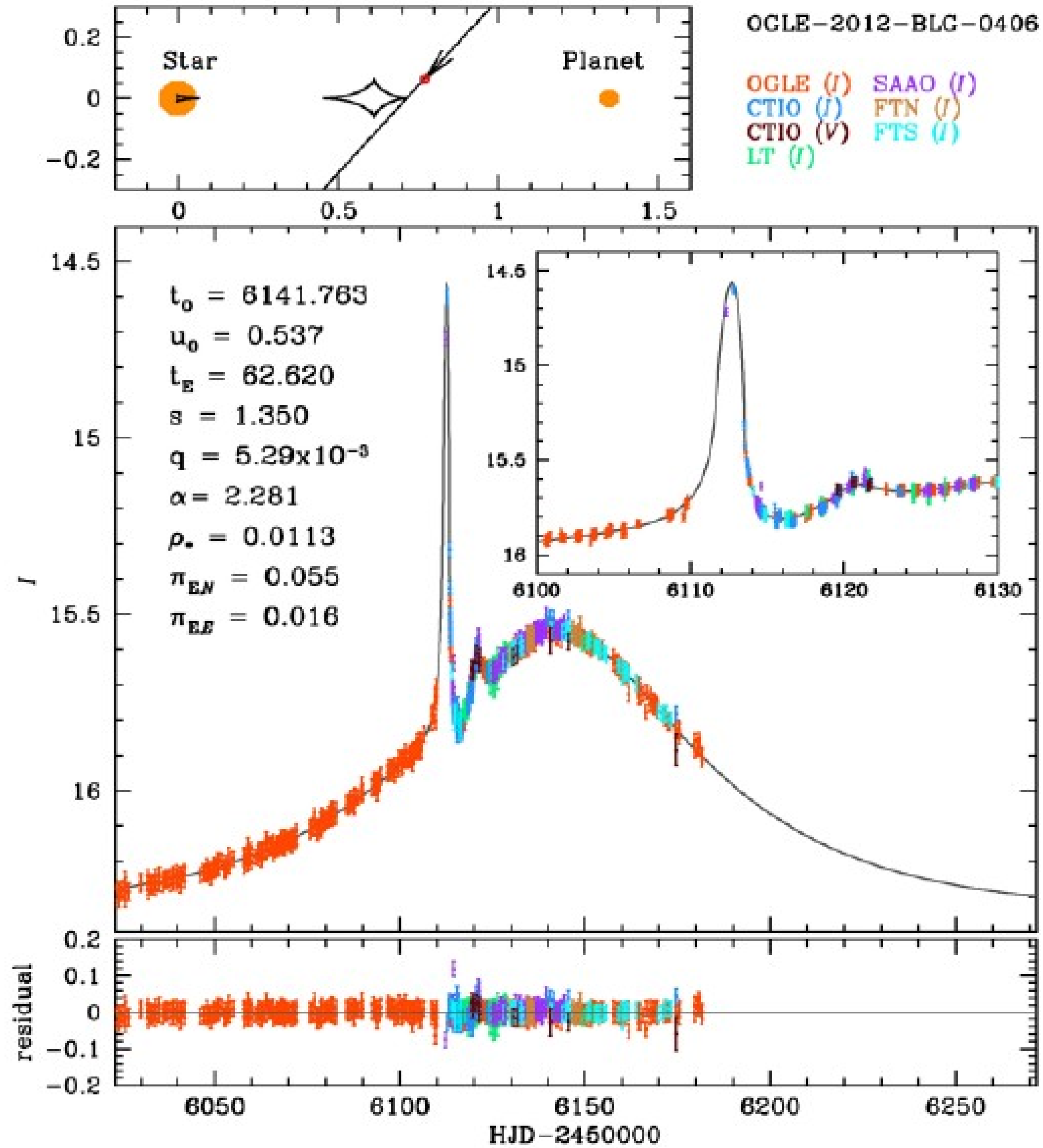
Planetary Central Caustics



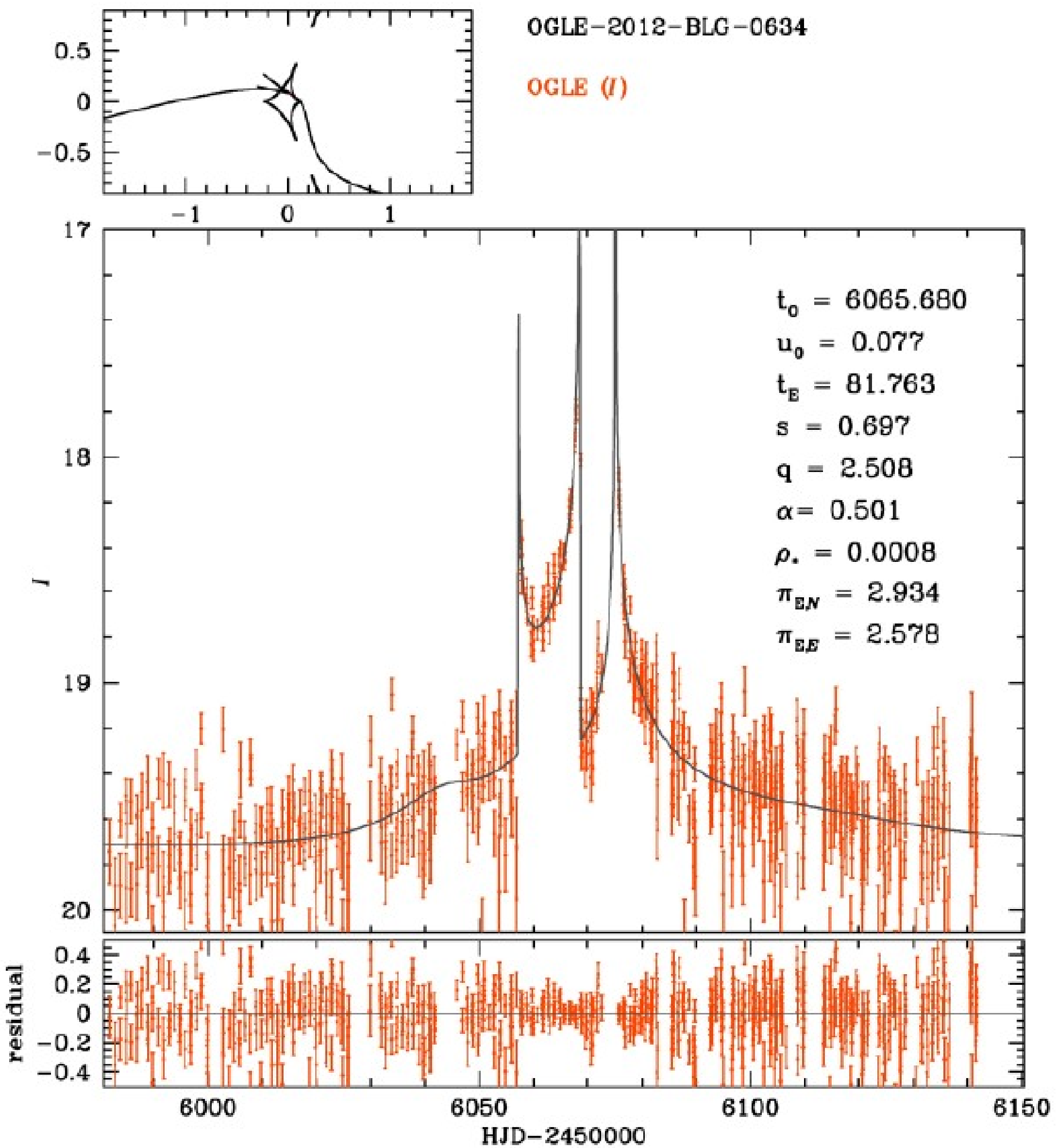
Binary + Parallax = curved trajectory



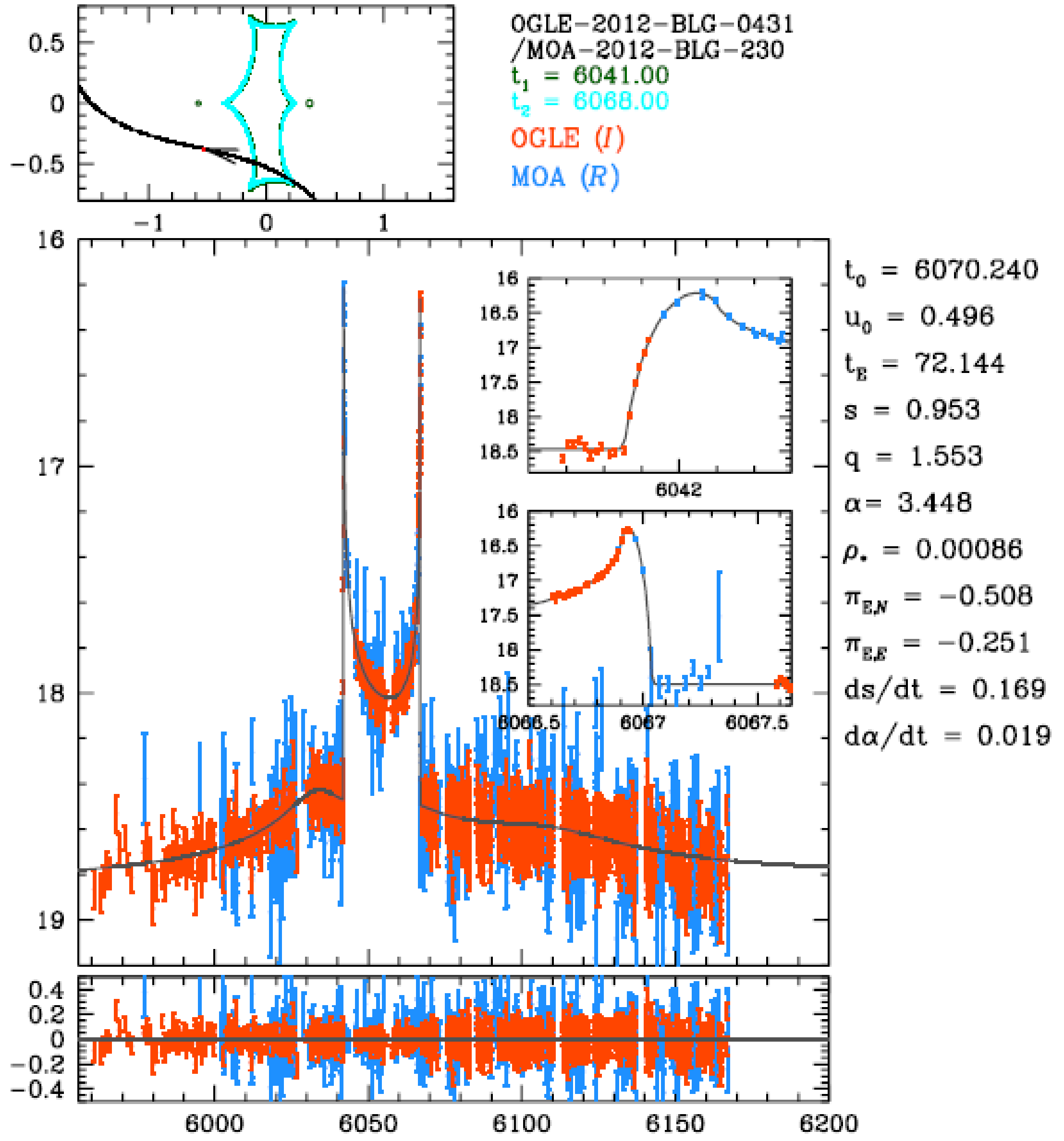
Binary + Parallax = curved trajectory



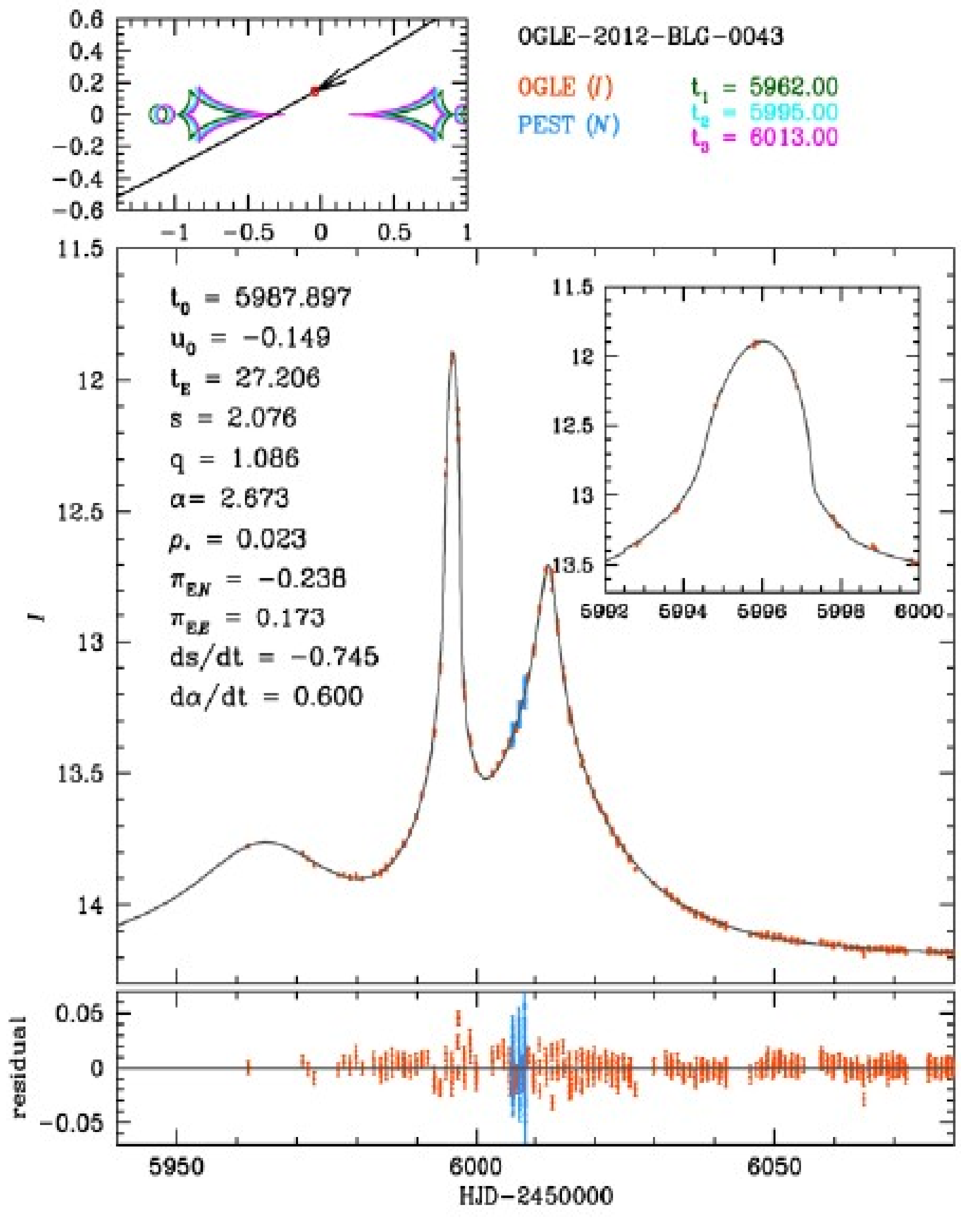
Binary + Parallax = curved trajectory



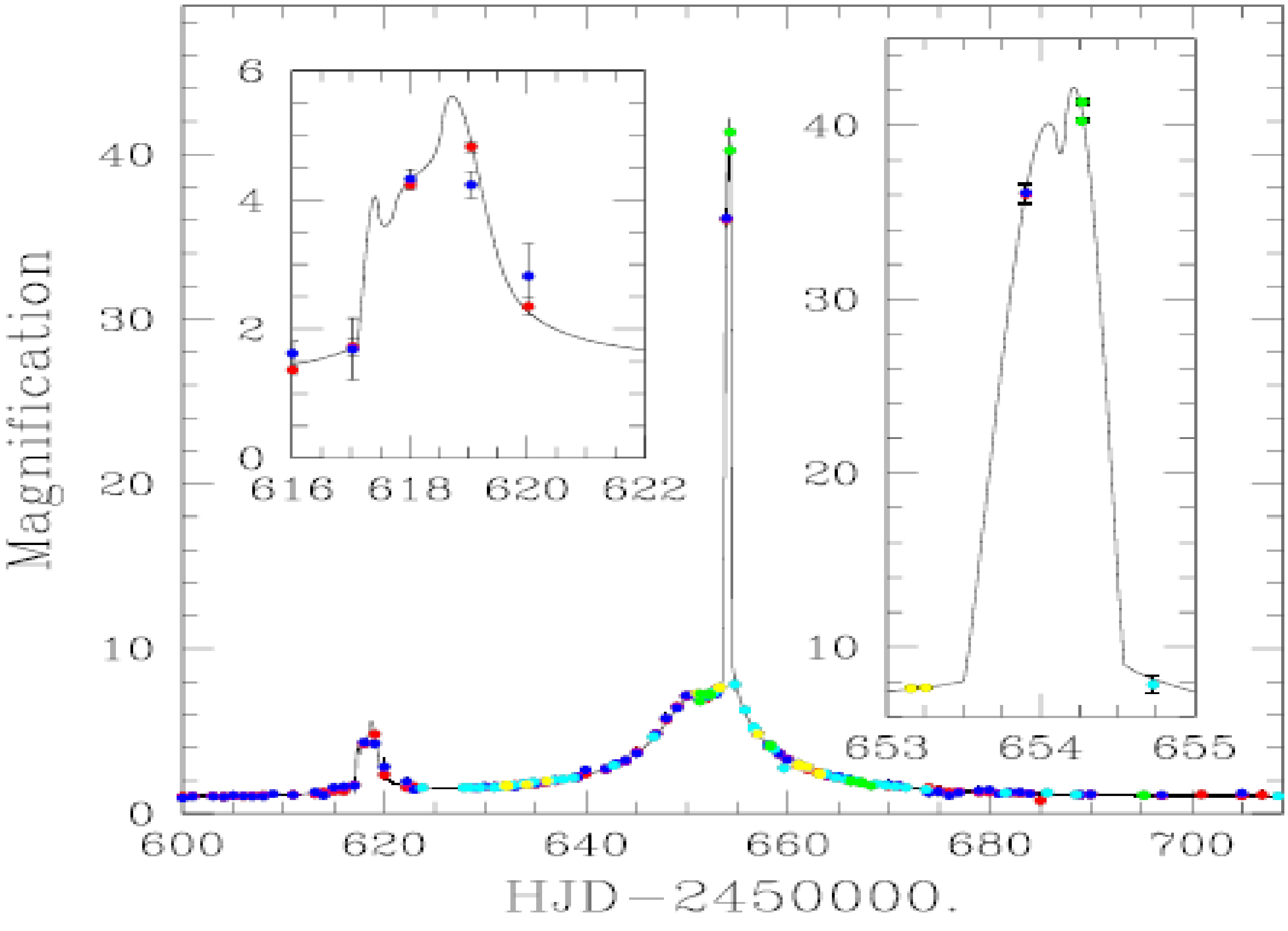
Binary Orbit = changing caustic



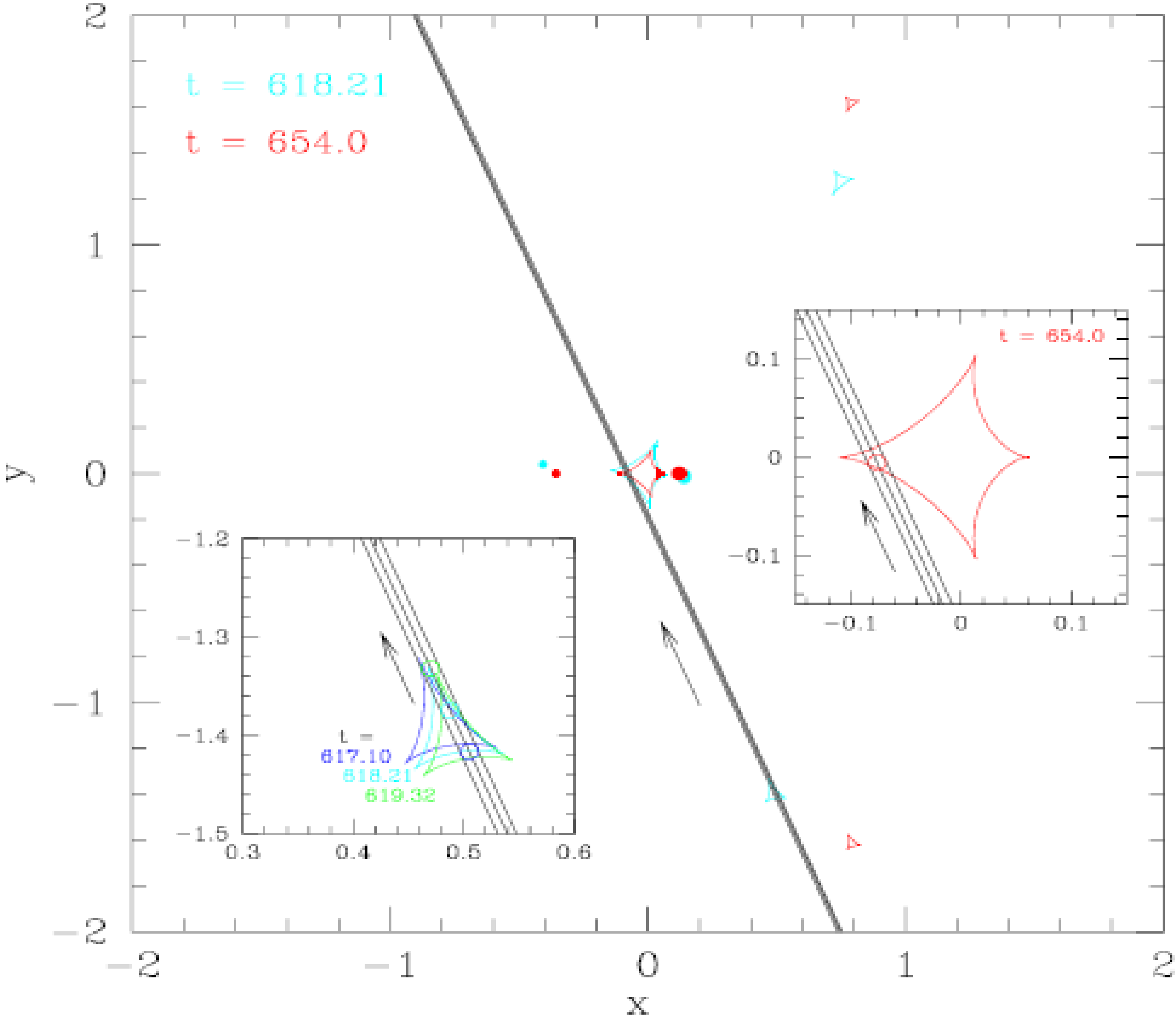
Binary Orbit = changing caustic



Macho 97-41: First Orbital Motion

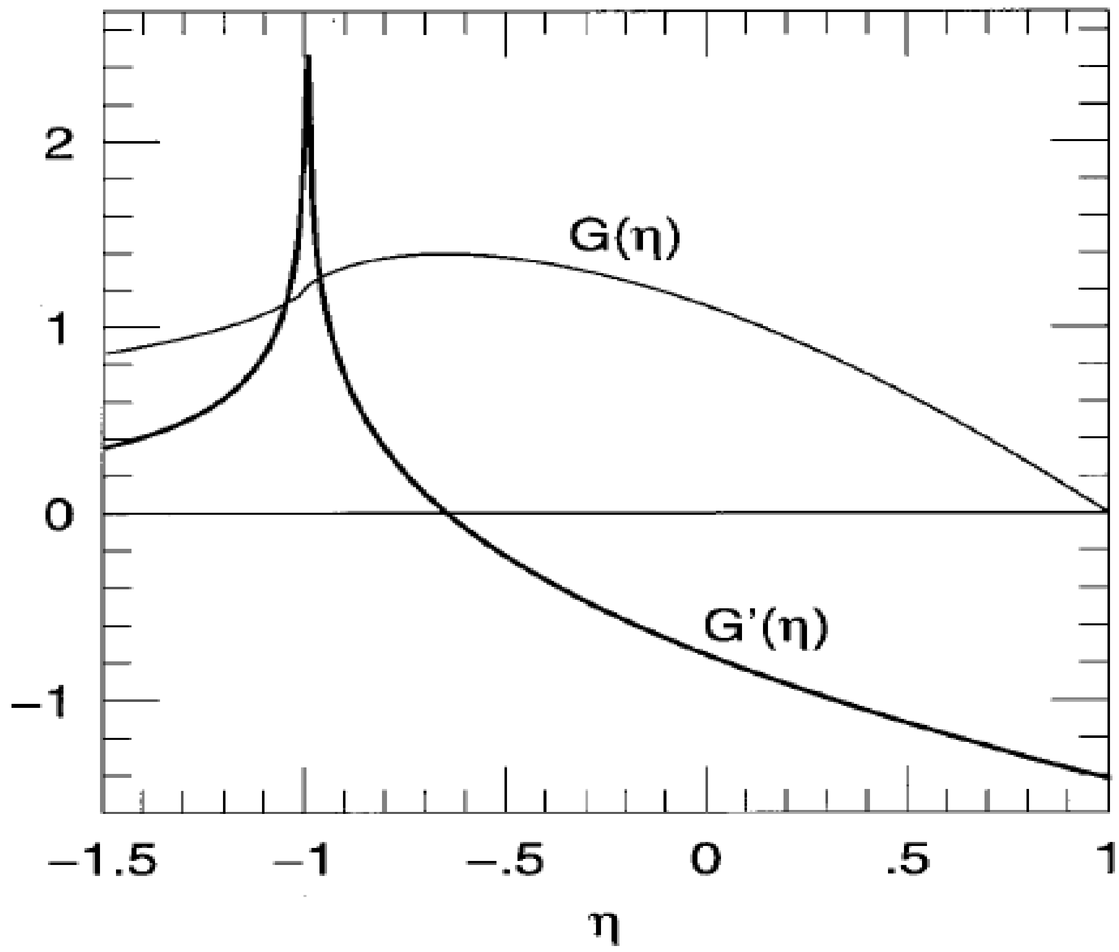


Macho 97-41: First Orbital Motion



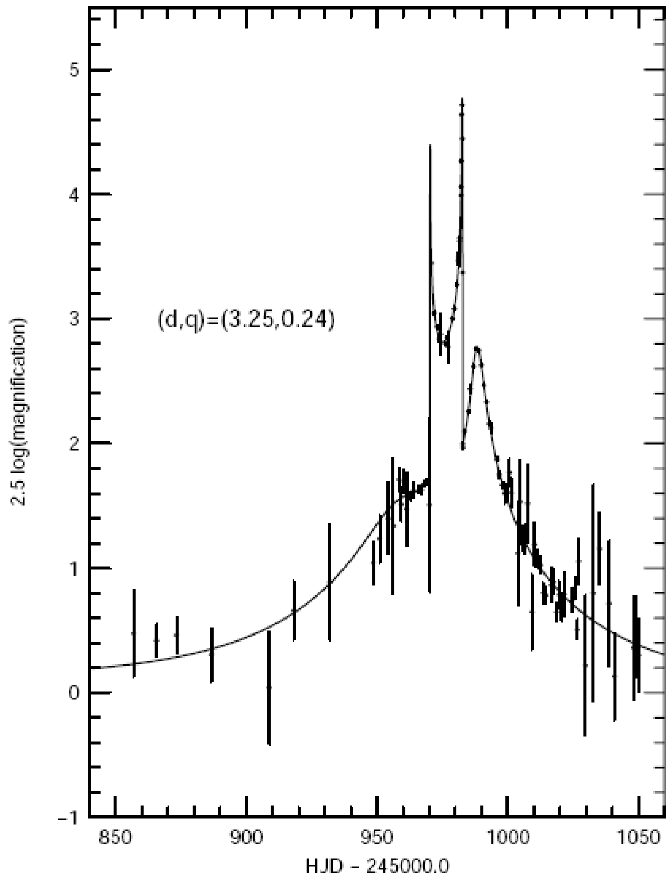
Finite-Source Effects

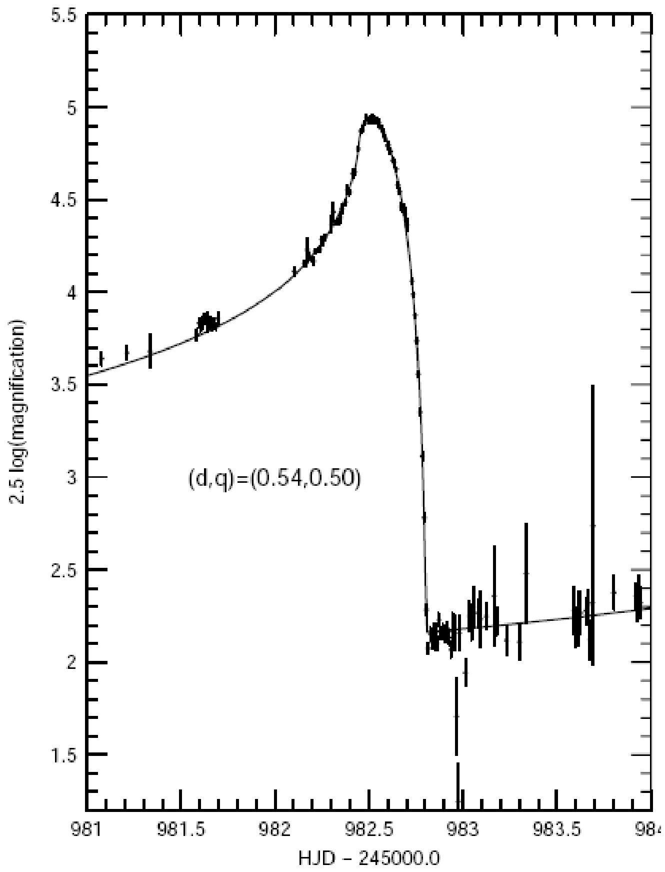
In Generic Caustic Crossing



Finite-Source Effects

Macho 98-SMC-1





Finite-Source Effects

Macho 98-SMC-1