Lensing Basics:
IV. Power-law mass model and its applications

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Power-law mass profile

Spherical power-law lens profile with slope γ':

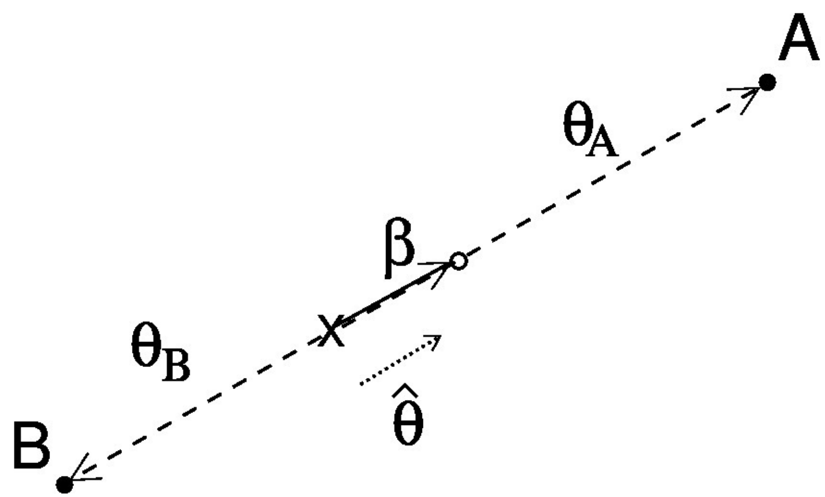
$$\kappa(\theta) = \frac{3-\gamma'}{2}\left[\frac{\theta_{\rm E}}{|\theta|}\right]^{\gamma'-1}$$

$$\alpha(\theta) = \theta_{\rm E}\left(\frac{\theta_{\rm E}}{|\theta|}\right)^{\gamma'-2}$$

Note:

- $\boldsymbol{\alpha} = \alpha\frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|}$
- γ'=2 is isothermal

Produces 2-image systems



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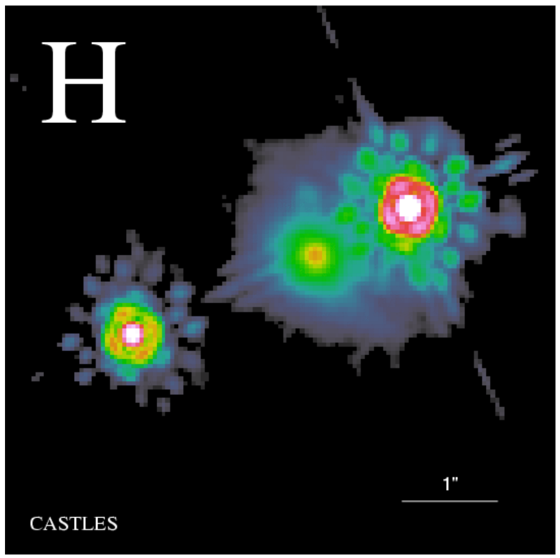


[CASTLES]

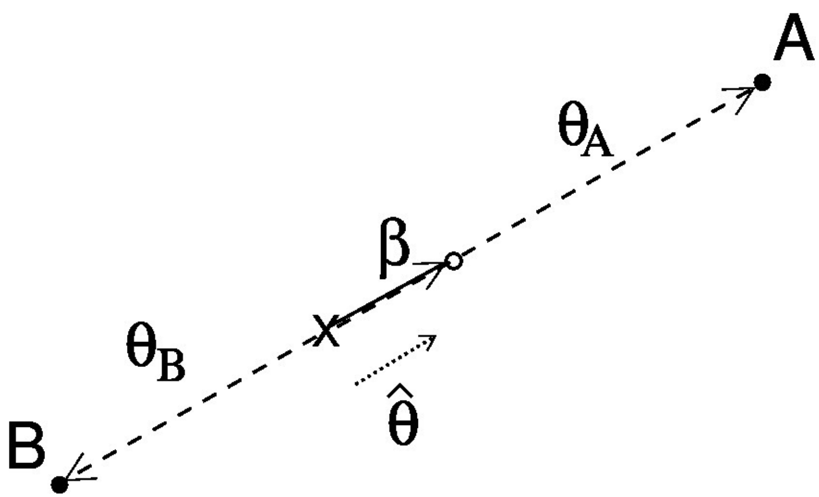
Single-component source I

Spherical power-law lens profile:

$$\kappa(\theta) = \frac{3-\gamma'}{2}\left[\frac{\theta_E}{|\theta|}\right]^{\gamma'-1}$$

$$\alpha(\theta) = \theta_E \left(\frac{\theta_E}{|\theta|}\right)^{\gamma'-2}$$

Produces 2-image systems



Given a lens system, how do we determine the lens mass distribution?
i.e., how to use lenses to measure mass?

➡ Use observables to constrain lens mass model

Observables:
- image positions
- image fluxes
- image shapes
- time delay

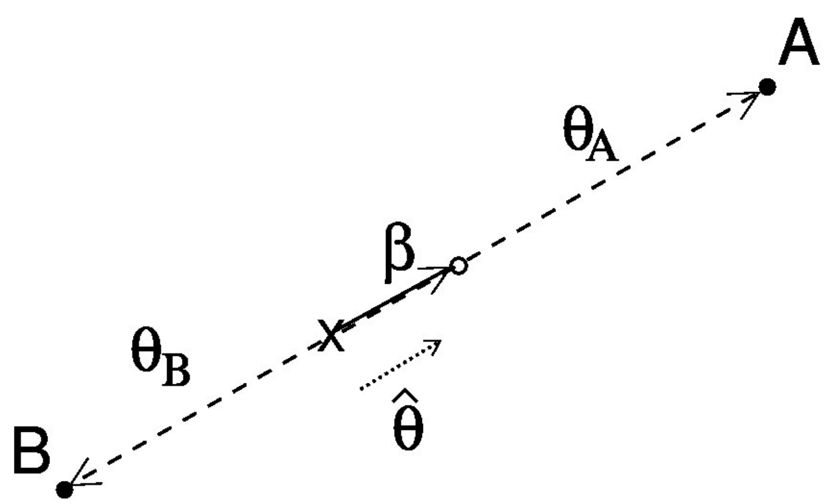
Single-component source II

Spherical power-law lens profile:

$$\kappa(\theta) = \frac{3-\gamma'}{2}\left[\frac{\theta_E}{|\theta|}\right]^{\gamma'-1}$$

$$\alpha(\theta) = \theta_E \left(\frac{\theta_E}{|\theta|}\right)^{\gamma'-2}$$

Produces 2-image systems



For simplicity, consider a 2-image system with observed image positions

To constrain lens mass parameters of power-law:

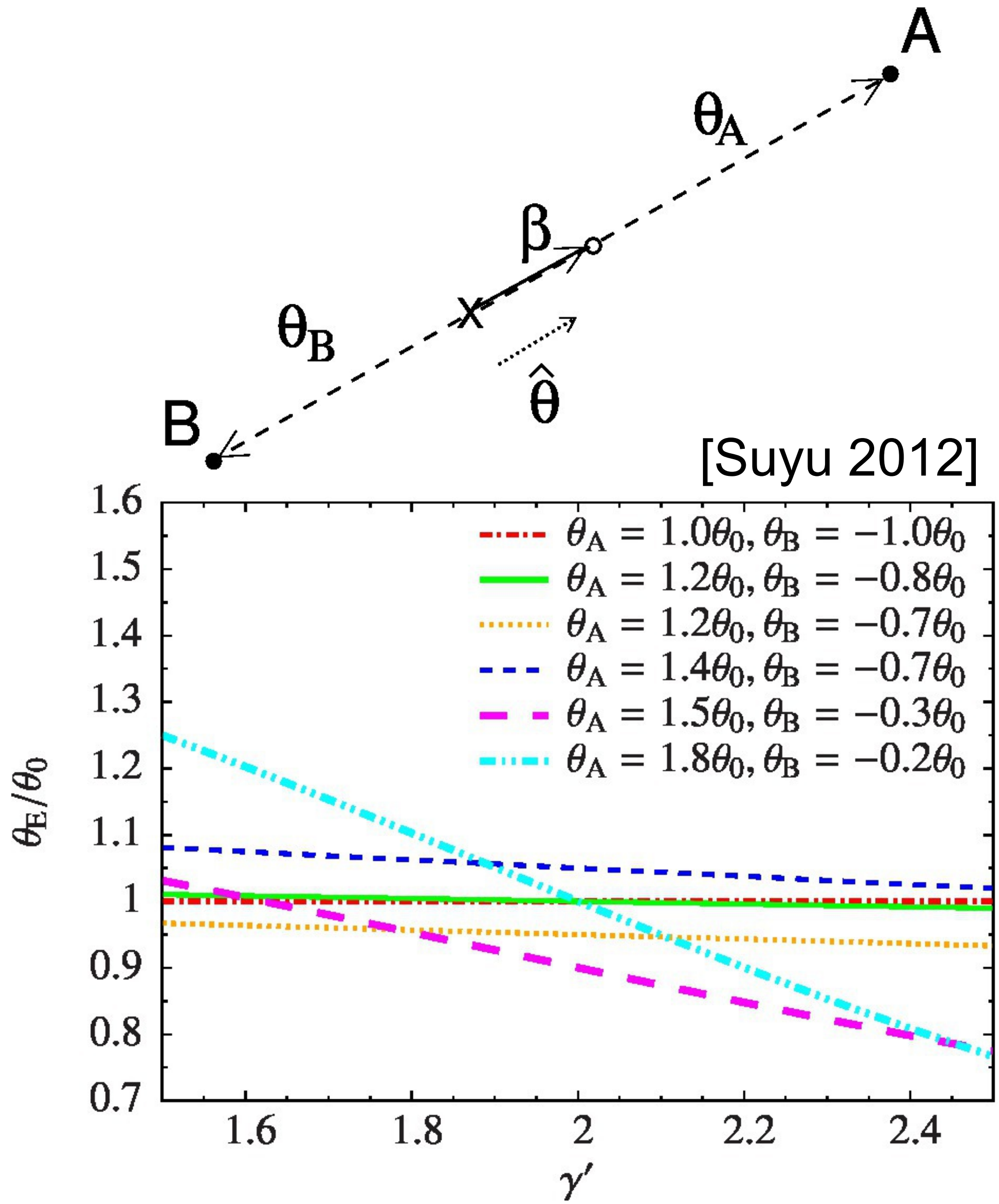
$$\begin{cases} \beta_s = \theta_A - \alpha(\theta_A)\dfrac{\hat{\theta}_A}{\hat{\theta}} \\ \beta_s = \theta_B - \alpha(\theta_B)\dfrac{\hat{\theta}_B}{\hat{\theta}} \end{cases}$$

2 equations

3 unknowns $(\beta_s, \theta_E, \gamma')$

cannot solve for all param.

Single-component source III



For simplicity, consider a 2-image system with observed image positions

To constrain lens mass parameters of power-law:

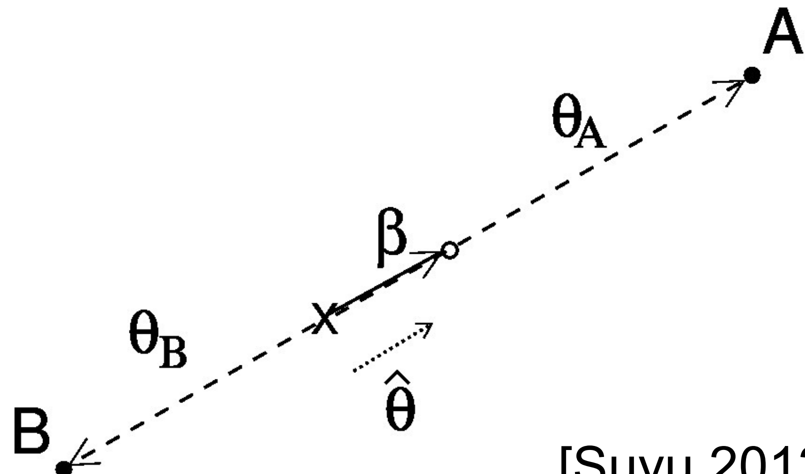
$$\begin{cases} \beta_s = \theta_A - \alpha(\theta_A)\dfrac{\hat{\theta}_A}{\hat{\theta}} \\ \beta_s = \theta_B - \alpha(\theta_B)\dfrac{\hat{\theta}_B}{\hat{\theta}} \end{cases}$$

2 equations

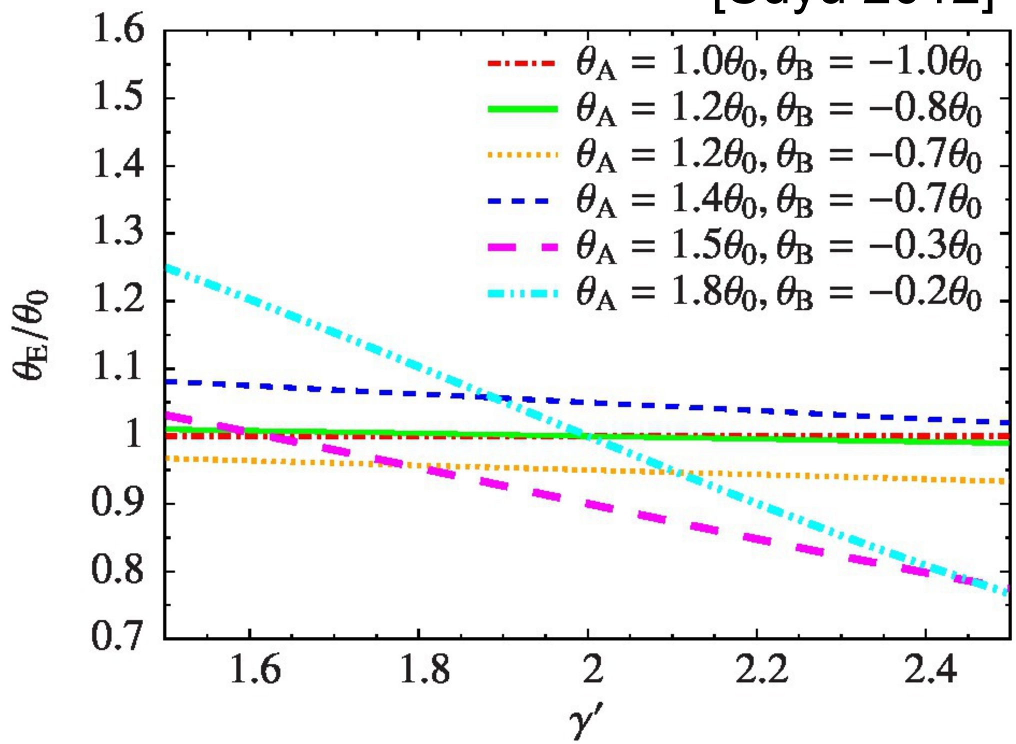
3 unknowns $(\beta_s, \theta_E, \gamma')$

➡ relations b/w parameters

Single-component source IV



[Suyu 2012]



For simplicity, consider a 2-image system with observed image positions

Recall:

$$\kappa(\theta) = \frac{3-\gamma'}{2}\left[\frac{\theta_E}{|\theta|}\right]^{\gamma'-1}$$

Mass enclosed within θ_E

$$M_E = \pi\theta_E^2\, D_d^2\, \Sigma_{cr}$$

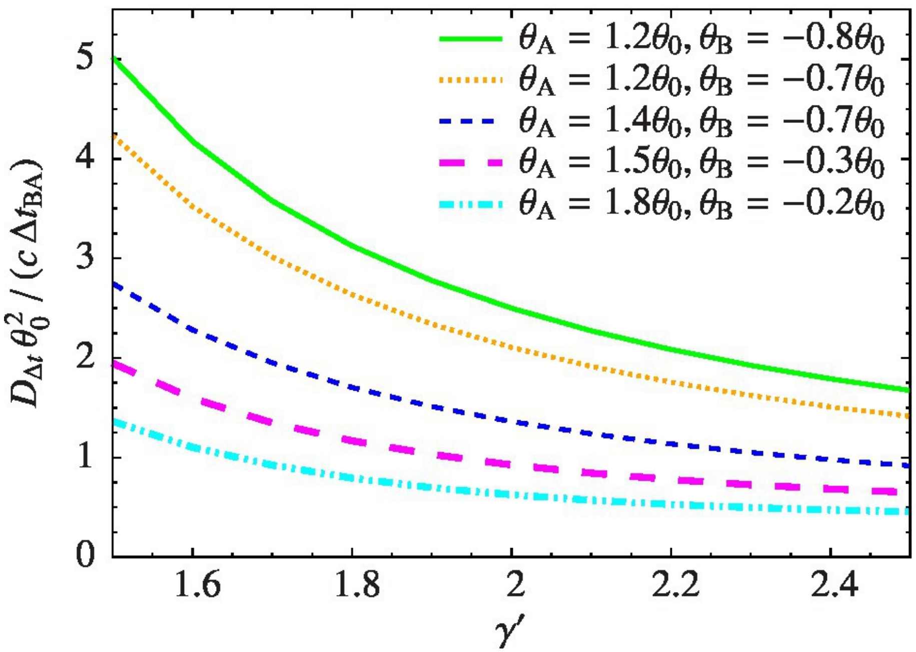
No direct dependence on γ'

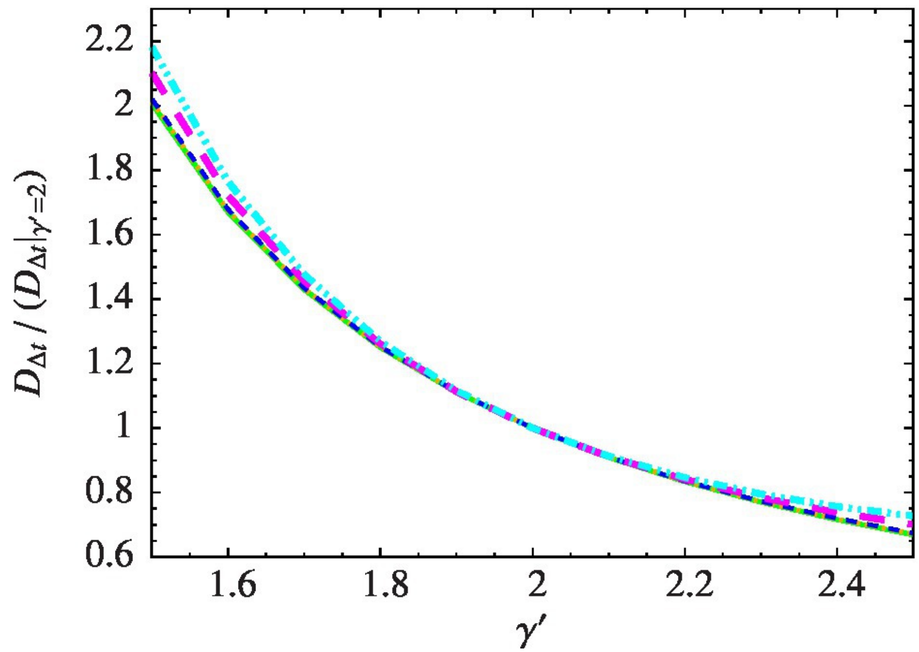
For systems with θ_A/θ_B<~2, lensing delivers accurate M_E within ~5%

Single-component source V

Degeneracy between $D_{\Delta t}$ and γ´

[Suyu 2012]



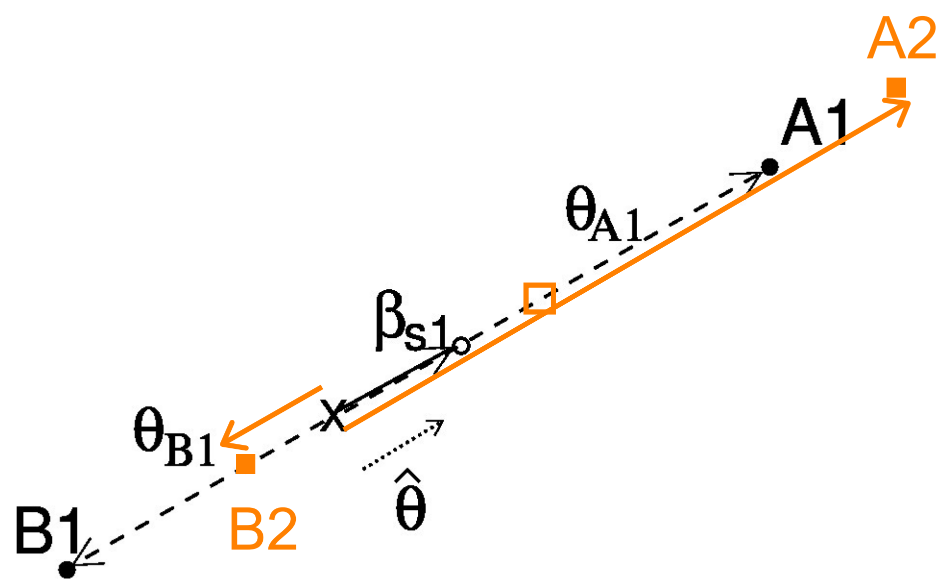


- different image configurations lead to similar relative $D_{\Delta t}$
- γ´ cannot be determined directly from lens system
- Need more information for precision cosmography

Two-component source I

Constraint equations:

$$\begin{cases} \beta_{s1} = \theta_{A1} - \alpha(\theta_{A1}) e_{A1} \\ \beta_{s1} = \theta_{B1} - \alpha(\theta_{B1}) e_{B1} \\ \beta_{s2} = \theta_{A2} - \alpha(\theta_{A2}) e_{A2} \\ \beta_{s2} = \theta_{B2} - \alpha(\theta_{B2}) e_{B2} \end{cases}$$



$$\alpha(\theta) = \theta_E \left(\frac{\theta_E}{|\theta|} \right)^{\gamma' - 2}$$

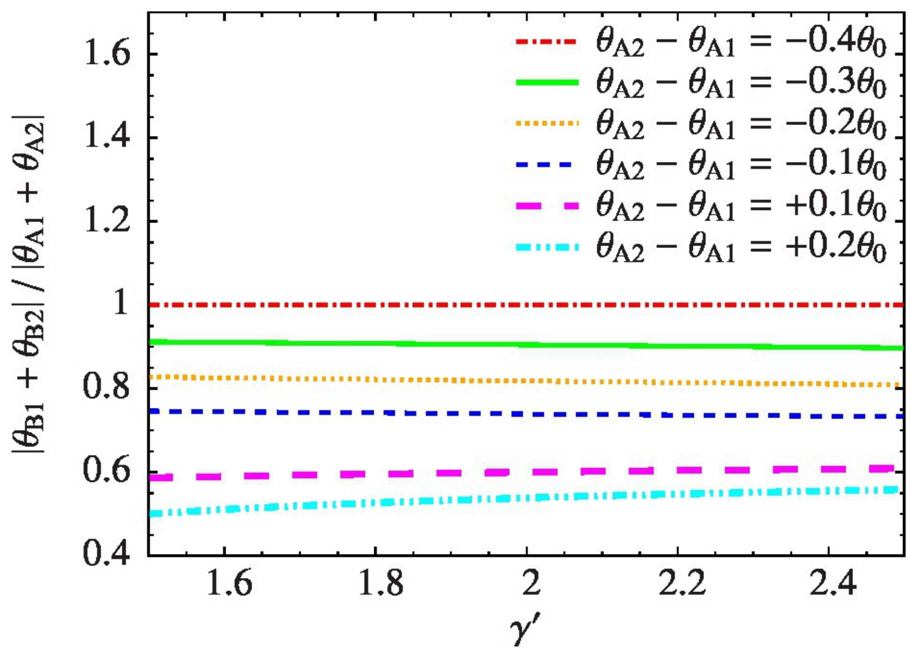
4 equations
4 unknowns $(\beta_{s1},\ \beta_{s2},\ \theta_E,\ \gamma')$

CAN solve for γ'! What is the required positional precision?

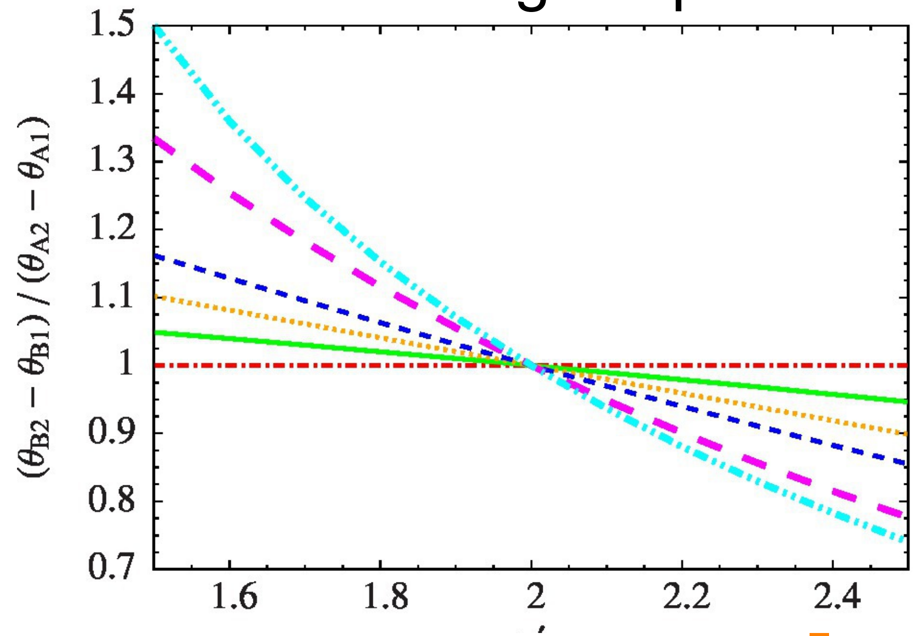
Two-component source II

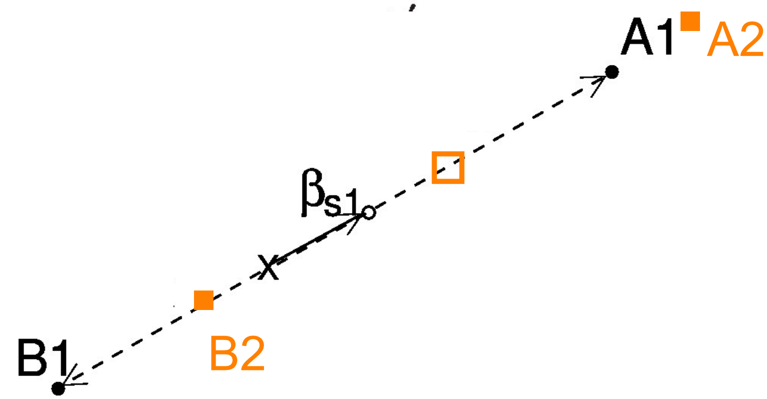
Consider $\theta_A = 1.2\theta_0$; $\theta_B = -0.8\theta_0$

Asymmetry



Relative image separation

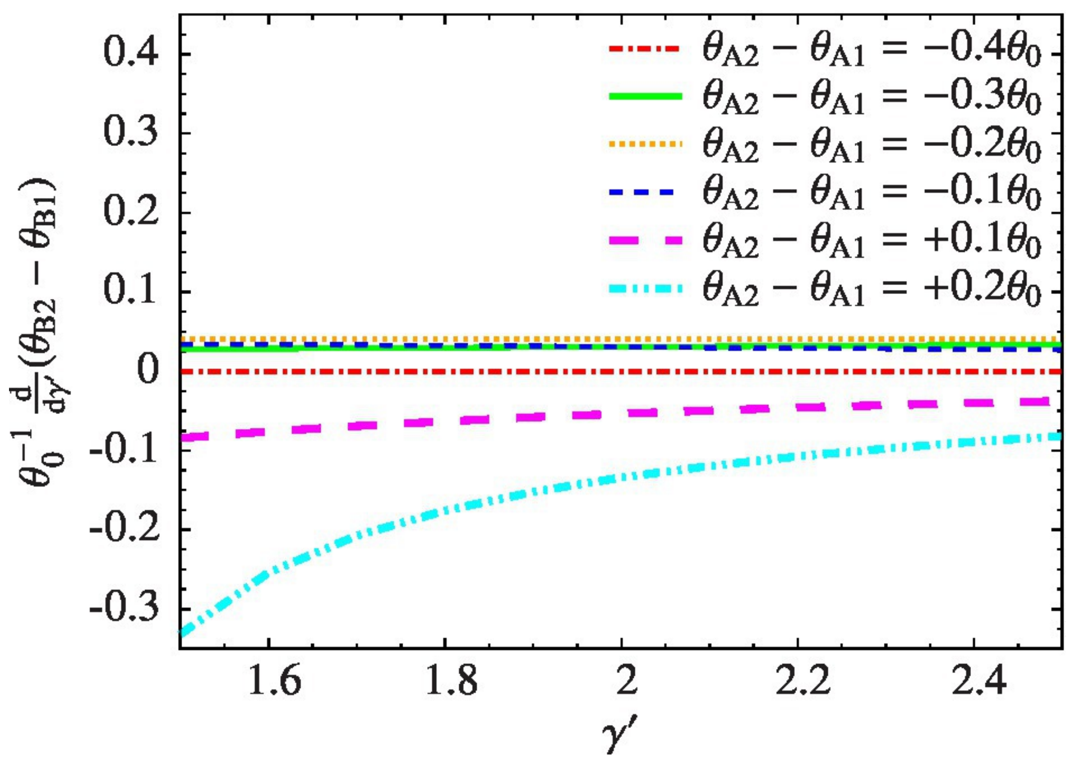


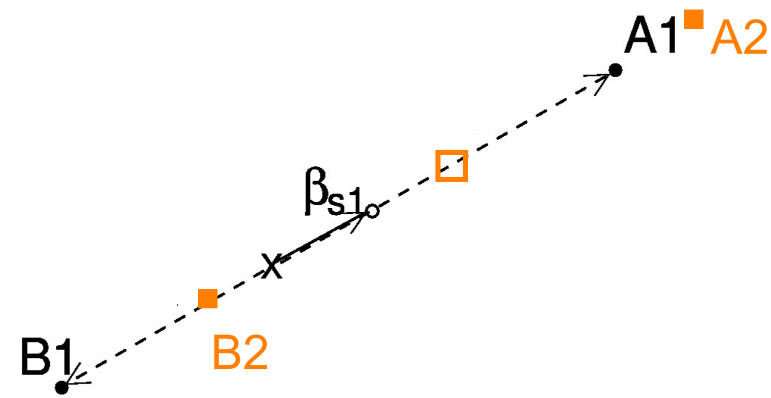


With asymmetric configuration, relative separation between the two components is a decreasing function of γ´

Two-component source III

Derivative of image separation with respect to γ´





[Suyu 2012]

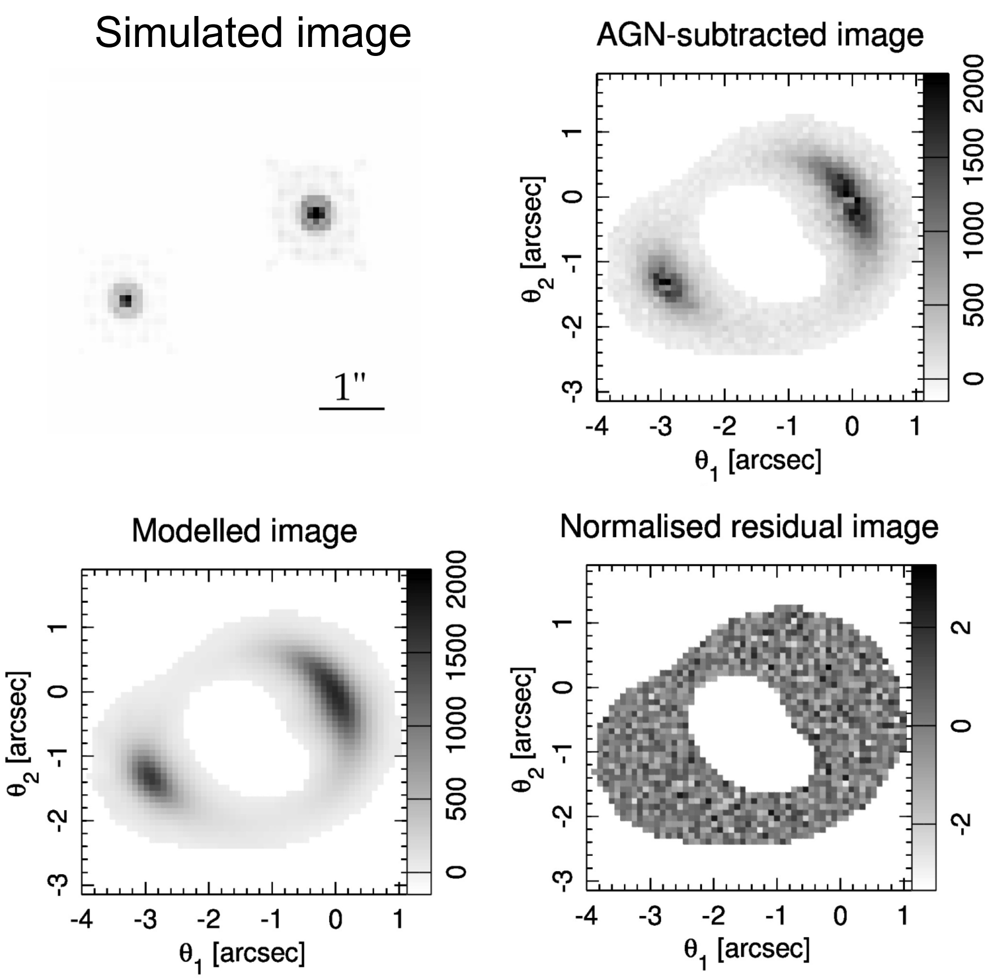
- derivative ~ 0.1θ_0
- If want Δγ´ to 0.03 (~3% in $D_{\Delta t}$), need $(\theta_{B2}-\theta_{B1})$ ~ 0.003θ_0

Need <~3 mas precision on separation! TOUGH!

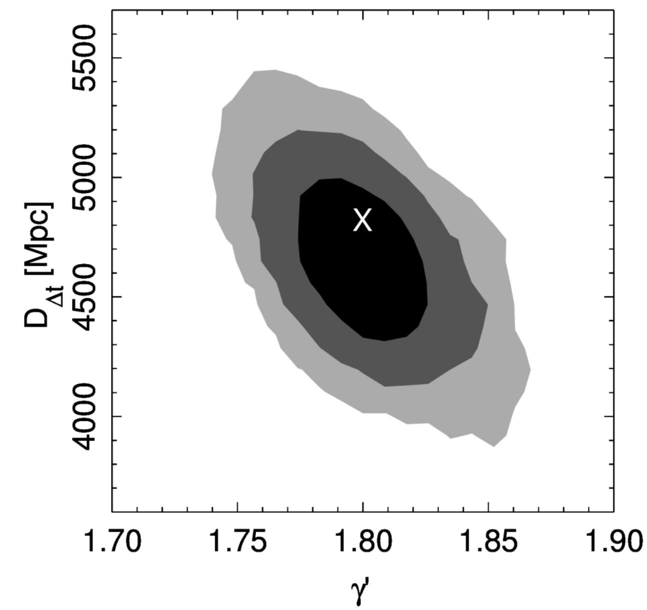
Spatially extended source?

Doubles with extended sources

elliptical mass distribution with external shear



[Suyu 2012]

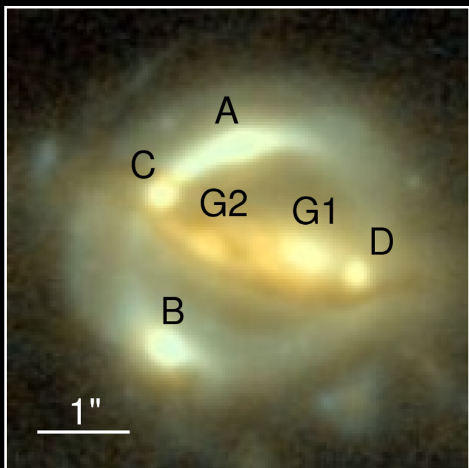


Extended images allow measurements of γ' to <0.05 precision or $D_{\Delta t}$ to ~5%

Application I: cosmography from time-delay lenses

Gravitational lens time delays

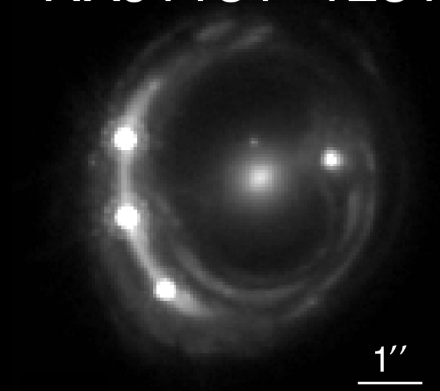
B1608+656



Discovery (CLASS):
Myers et al. 1995

Delays (radio obs):
Fassnacht et al. 2002

RXJ1131−1231



Discovery (CLASS):
Sluse et al. 2003

Delays (optical obs):
Tewes et al. 2012

Time delay:

$$t = \frac{1}{c} D_{\Delta t} \, \phi_{\rm lens}$$

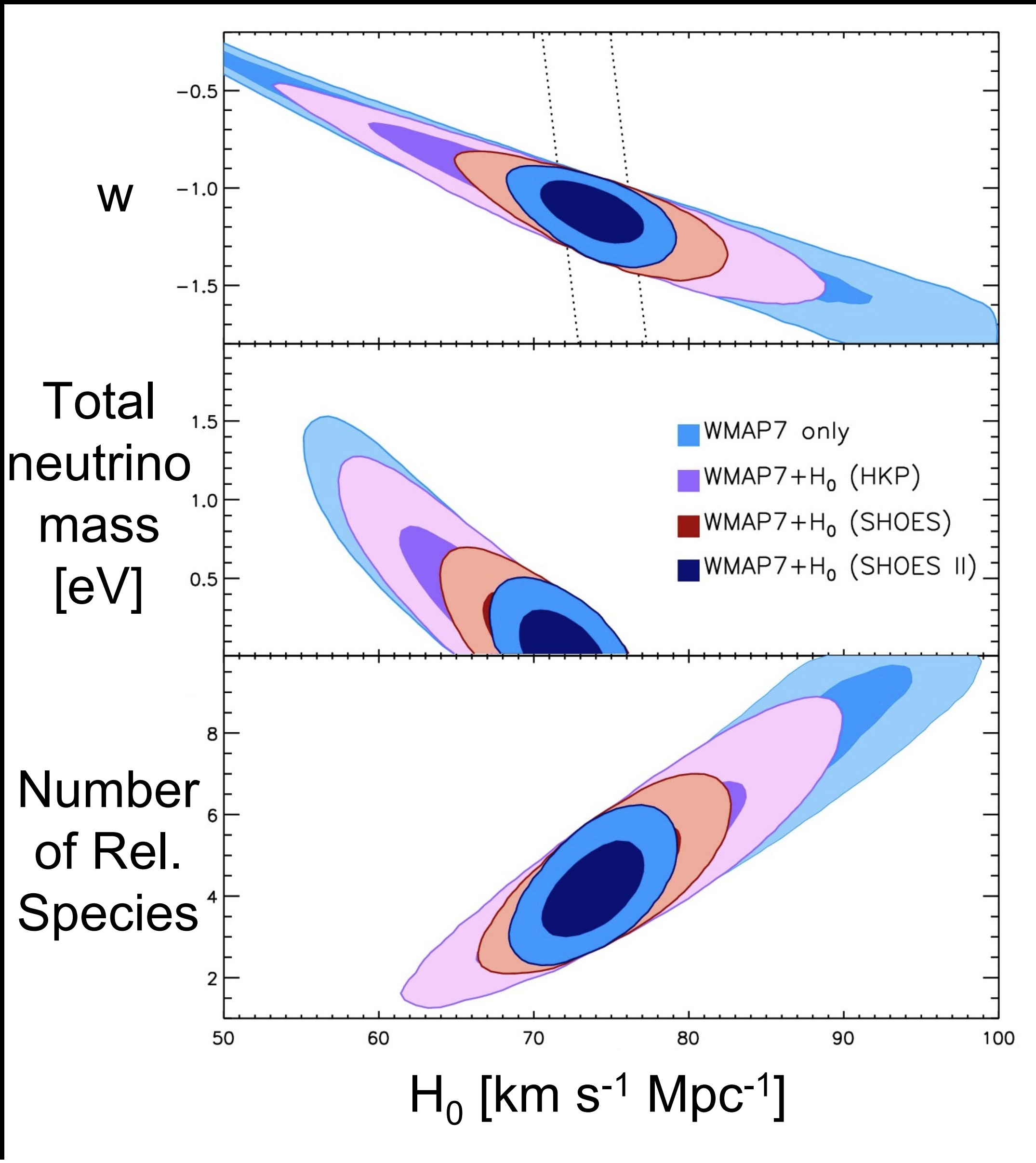
Time-delay distance:

$$D_{\Delta t} \propto \frac{1}{H_0}$$

Obtain from lens mass model

H_0, a key parameter

[Riess et al. 2011]



H_0 provides critical independent constraints on

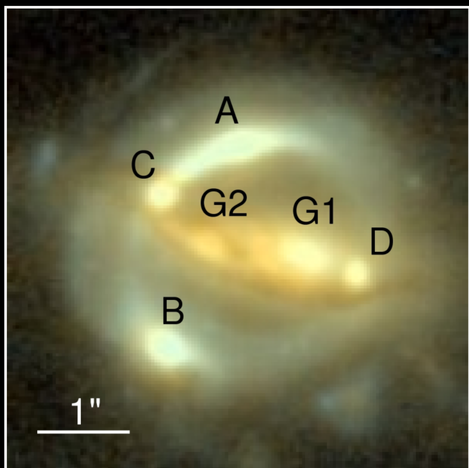
- nature of dark energy
- neutrino physics
- spatial curvature of the Universe

[e.g., Freedman et al. 2012, Suyu et al. 2012, Weinberg et al. 2012, Sekiguchi et al. 2010]

Independent methods are needed to overcome systematics, especially the unknown unknowns

Gravitational lens time delays

B1608+656



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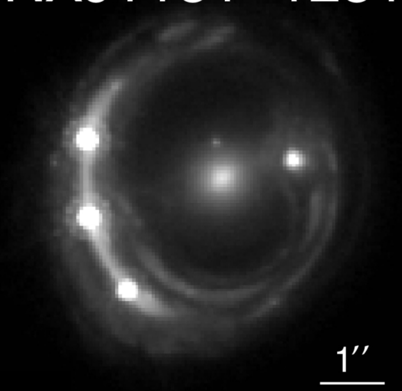

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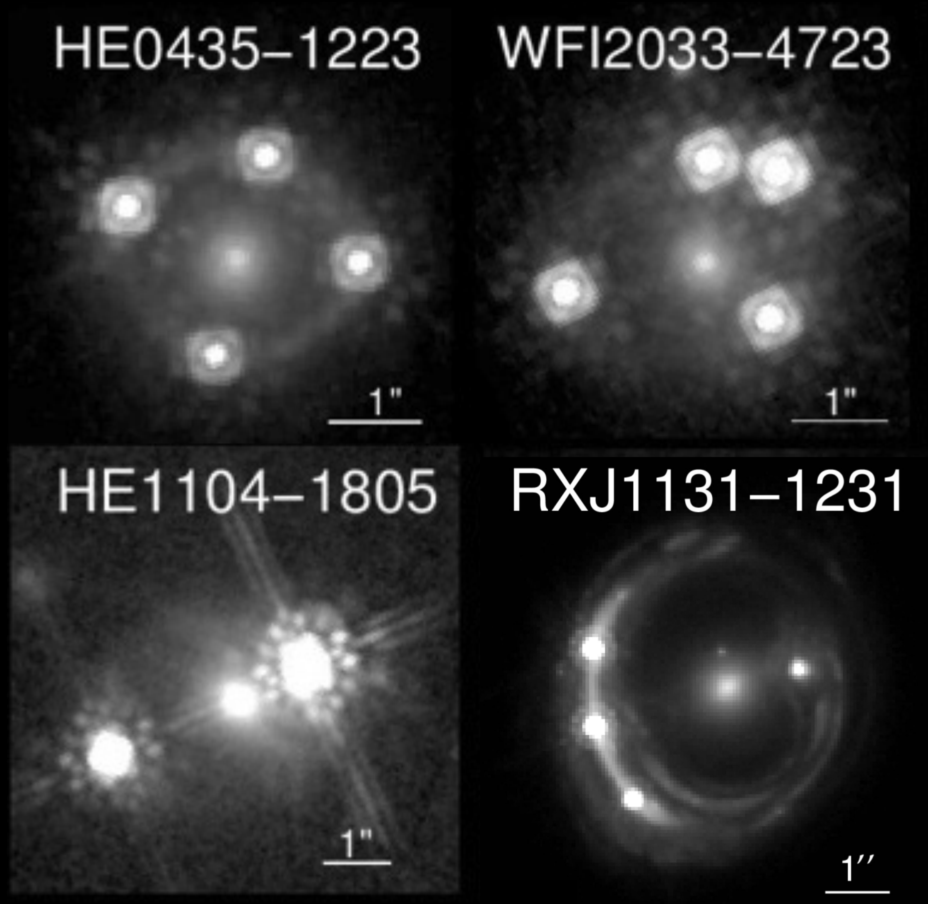
Obtain from lens mass model

For cosmography, need:
(1) time delays
(2) lens mass model
(3) mass along line of sight

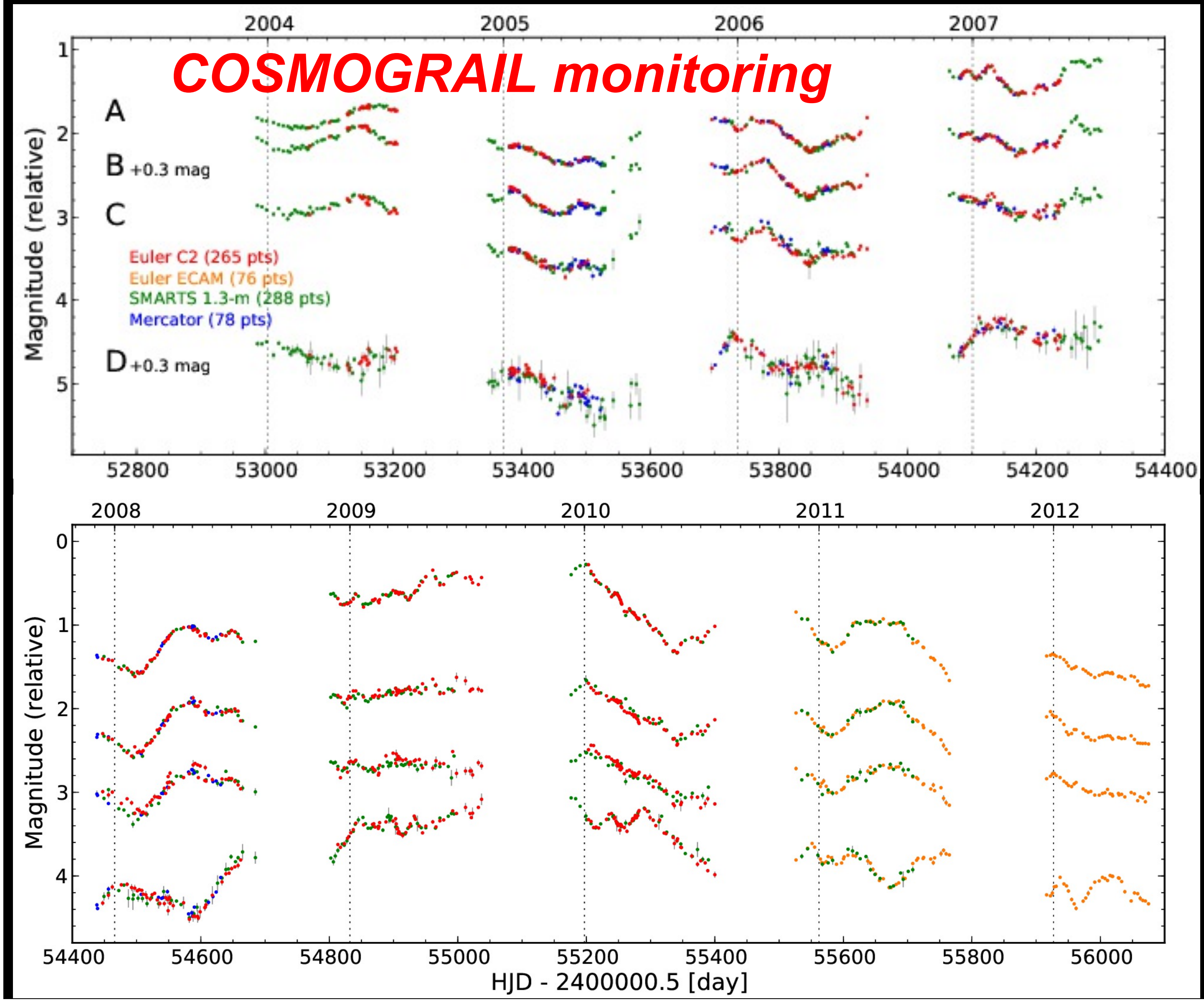
COSMOGRAIL:

the COSmological MOnitoring of GRAvItational Lenses

- time delays of lensed quasars from optical monitoring
- expect to have delays with a few percent error for ~20 lenses



Time delays of RXJ1131-1231



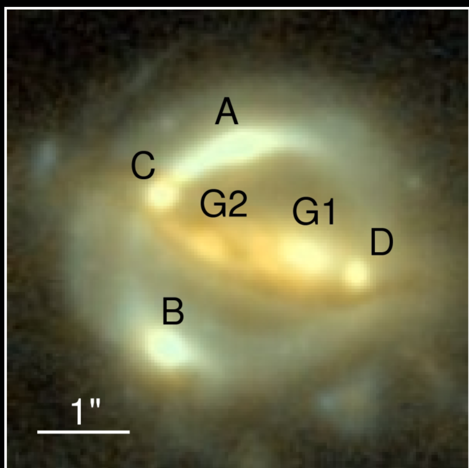
Time delay with 1.5% accuracy!

[Tewes et al. 2012b]

Based on state-of-the-art curve modeling techniques [Tewes et al. 2012a]

Gravitational lens time delays

B1608+656



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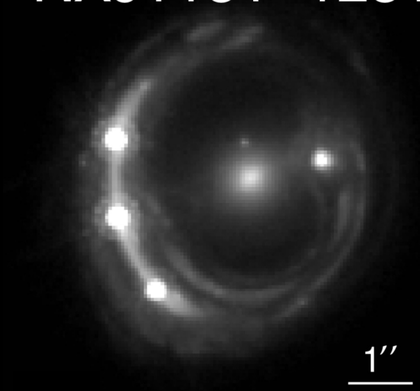
RXJ1131−1231



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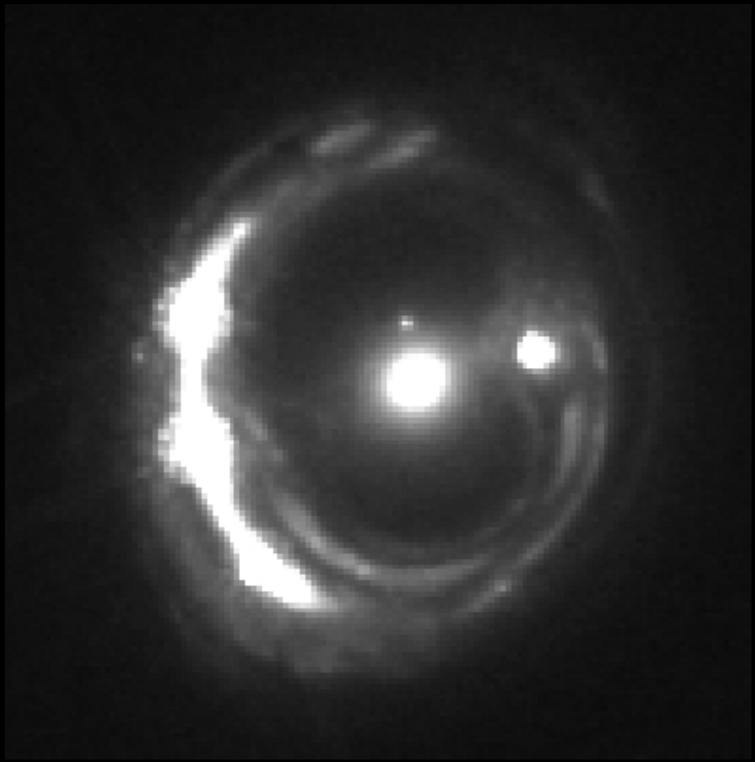
Obtain from lens mass model

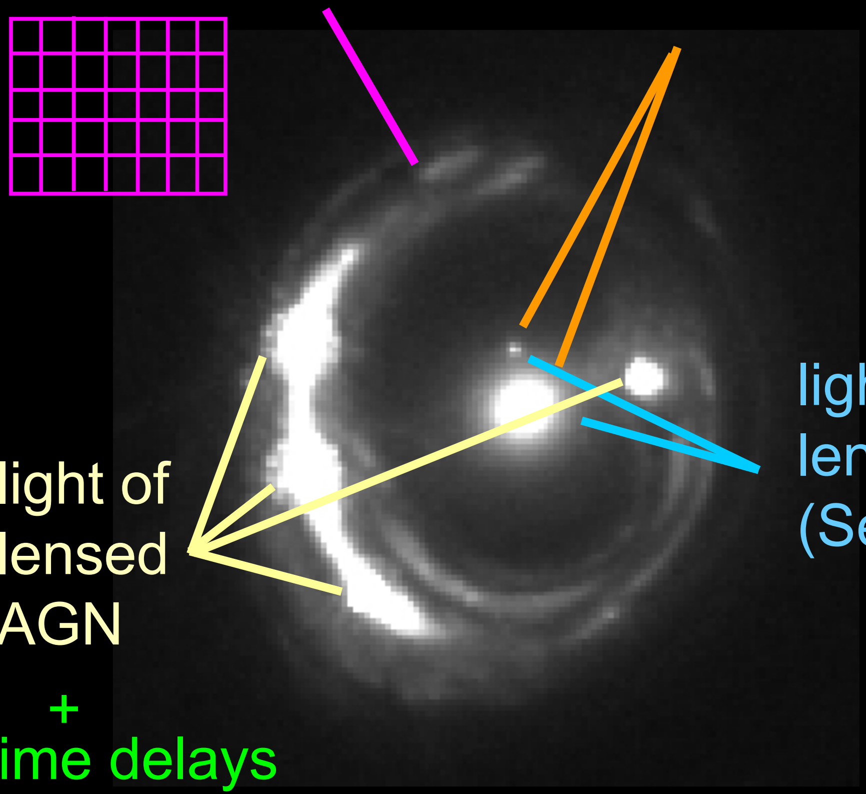
For cosmography, need:
(1) time delays ✓
(2) lens mass model
(3) mass along line of sight

Lens Model

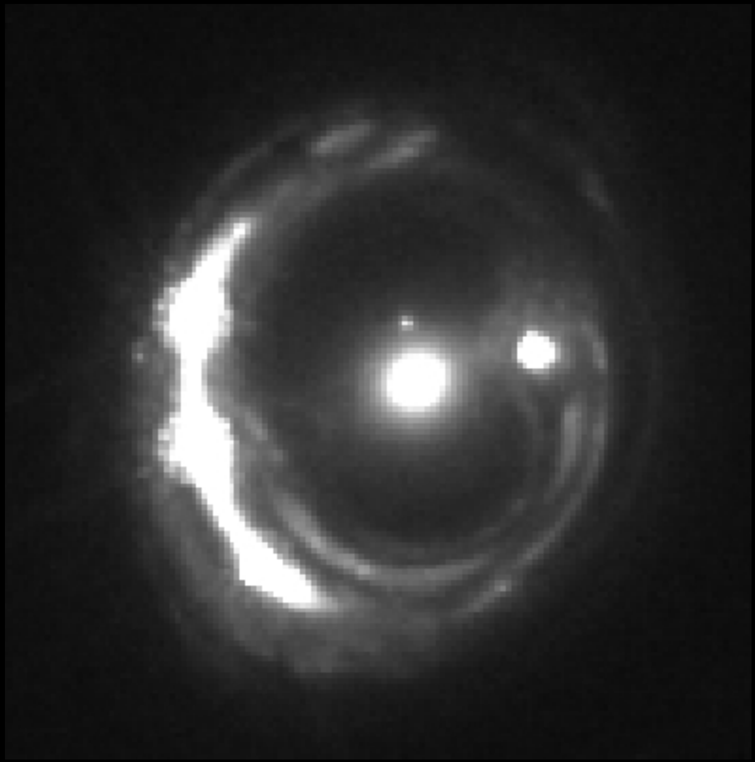
Observed Image

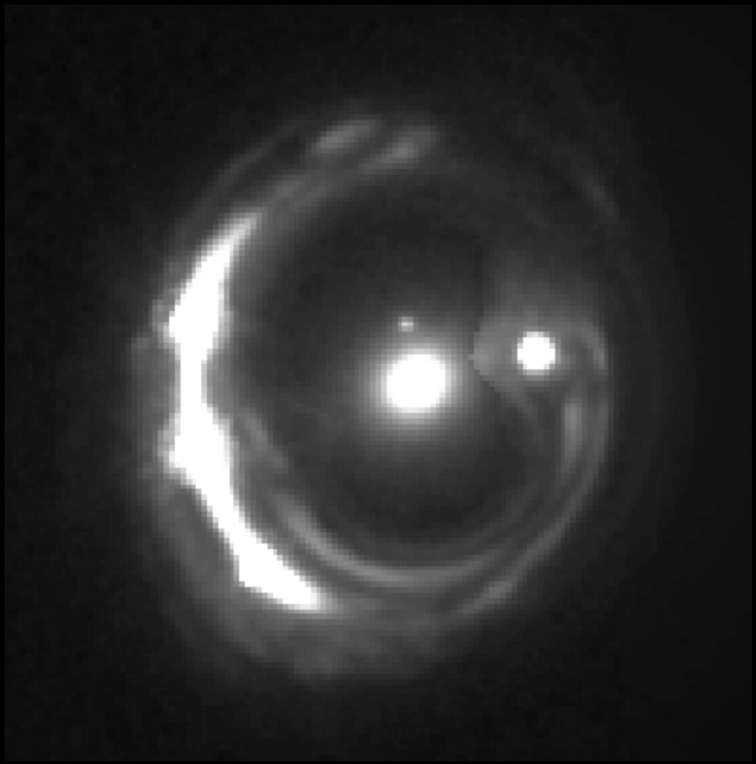


Lens Model


light distribution
of extended source
mass distribution
of lens
light of
lens
(Sersic)
light of
lensed
AGN
+
time delays

Lens Model

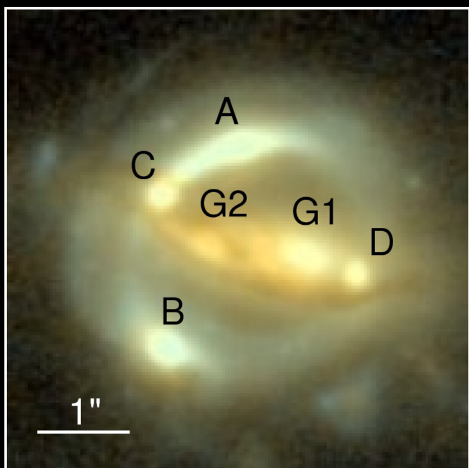




[Suyu et al. 2012]

Gravitational lens time delays

B1608+656



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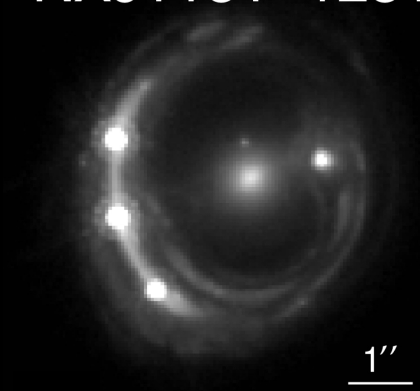

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Obtain from lens mass model

For cosmography, need:
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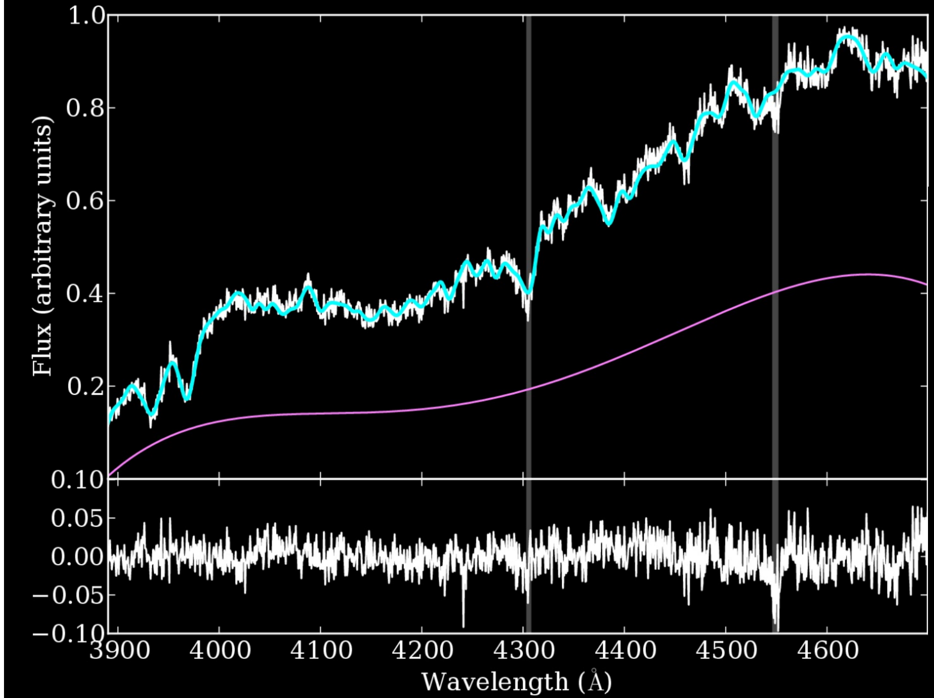
(2) lens mass model ✓
(3) mass along line of sight

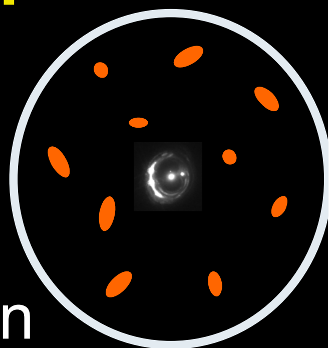
Line-of-sight Contribution

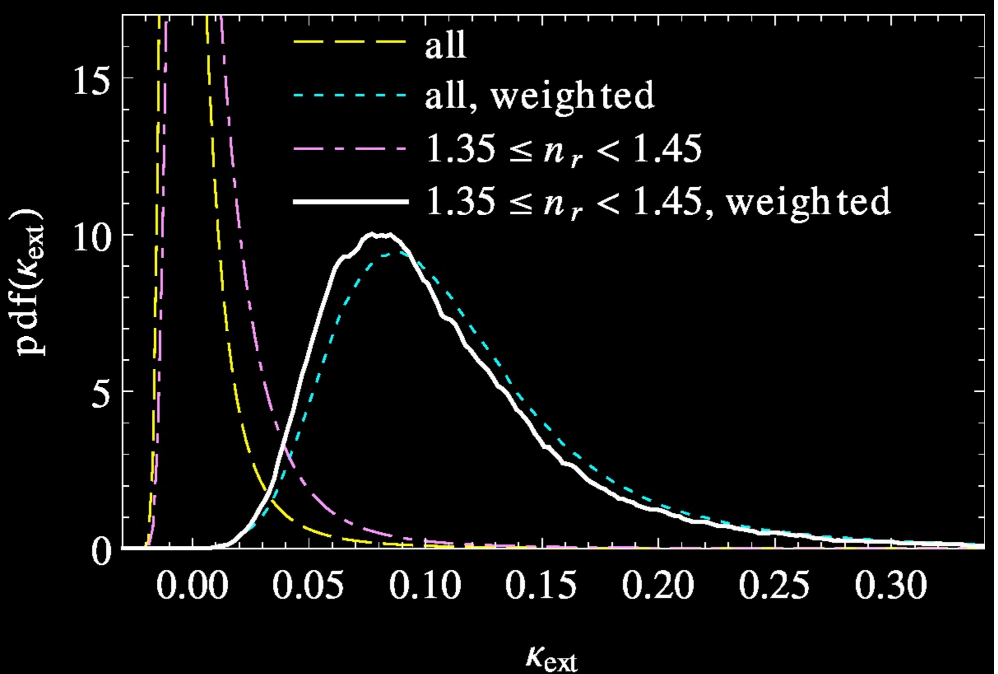
Keck LRIS



Velocity dispersion:
323 ± 20 km/s

Lens environment +
Millennium Simulation



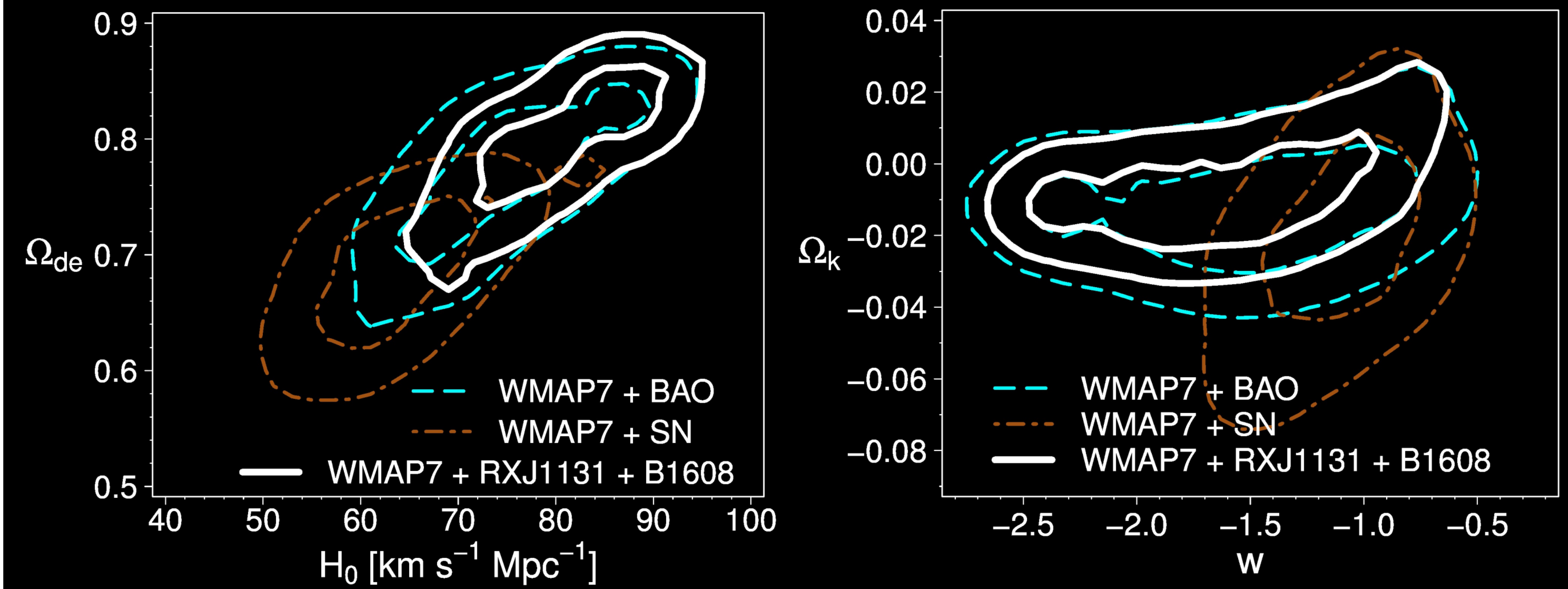


[Suyu et al. 2012]

Cosmological Probe Comparison I

WMAP7owCDM prior

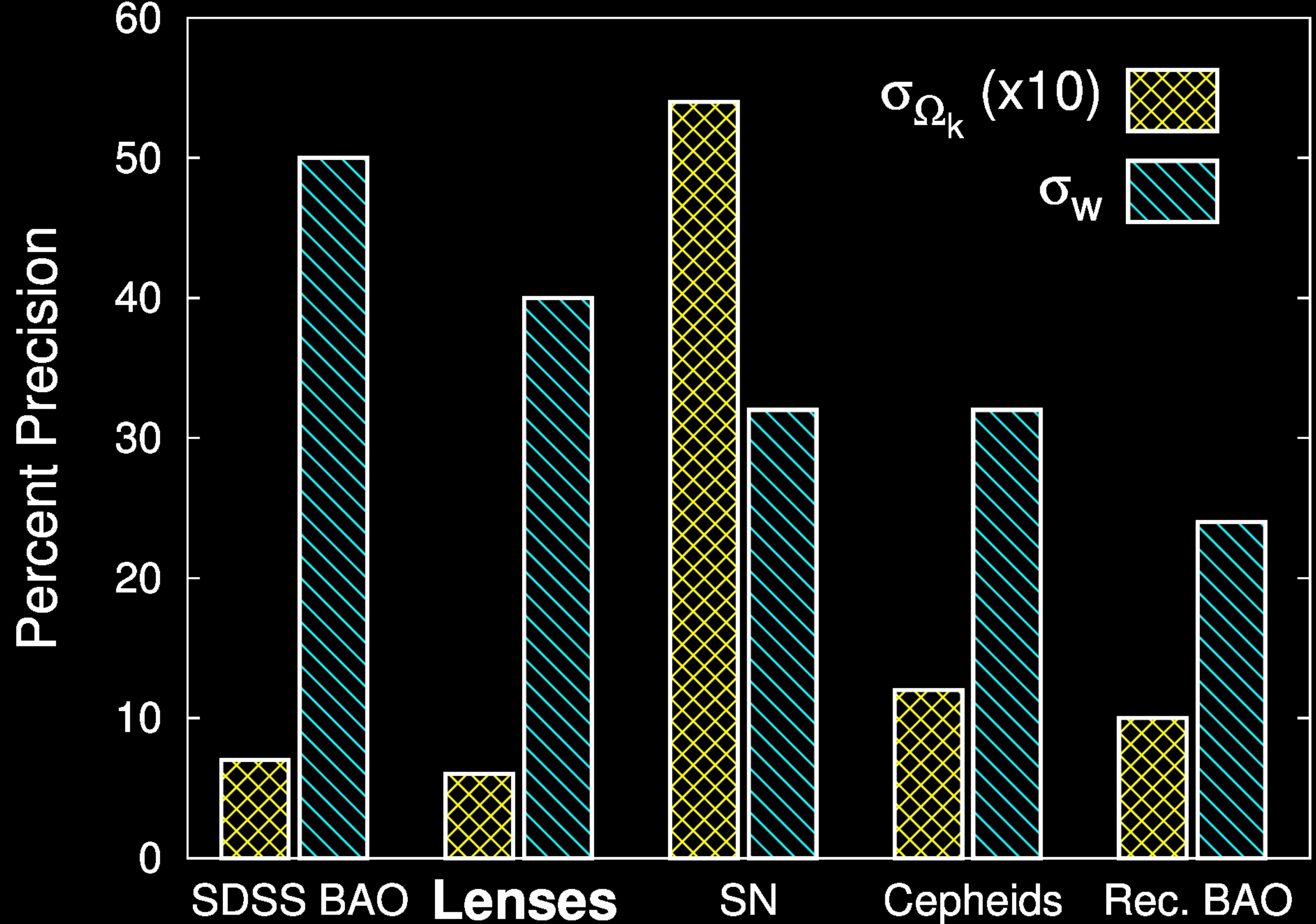
[Suyu et al. 2012]



- contour orientations are different: complementarity b/w probes
- contour sizes are similar: lensing is a competitive probe

Cosmological Probe Comparison II

WMAP7owCDM prior



SDSS BAO:
Percival et al. 2010

Lenses:
Suyu et al. 2012

Supernovae:
Suzuki et al. 2012

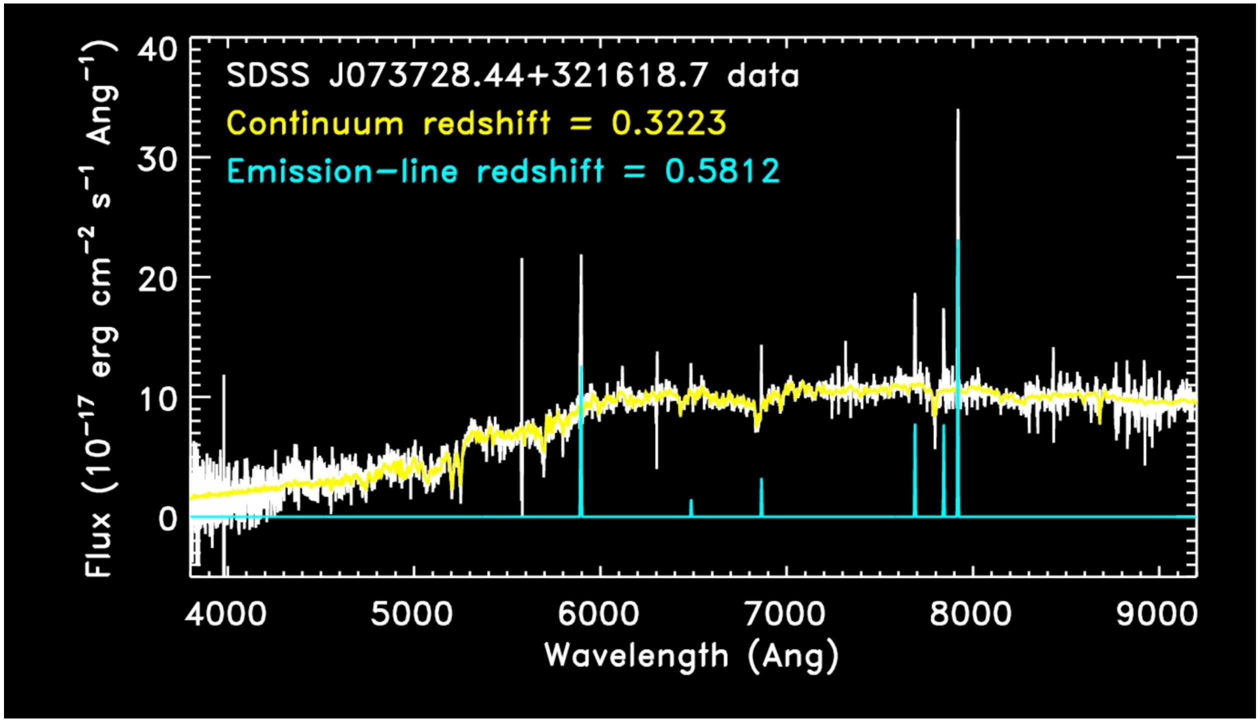
Cepheids:
Riess et al. 2011

Reconstructed BAO:
Mehta et al. 2012

Application II: internal structures of early-type galaxies

SLACS: Sloan Lens ACS Survey

SDSS spectra

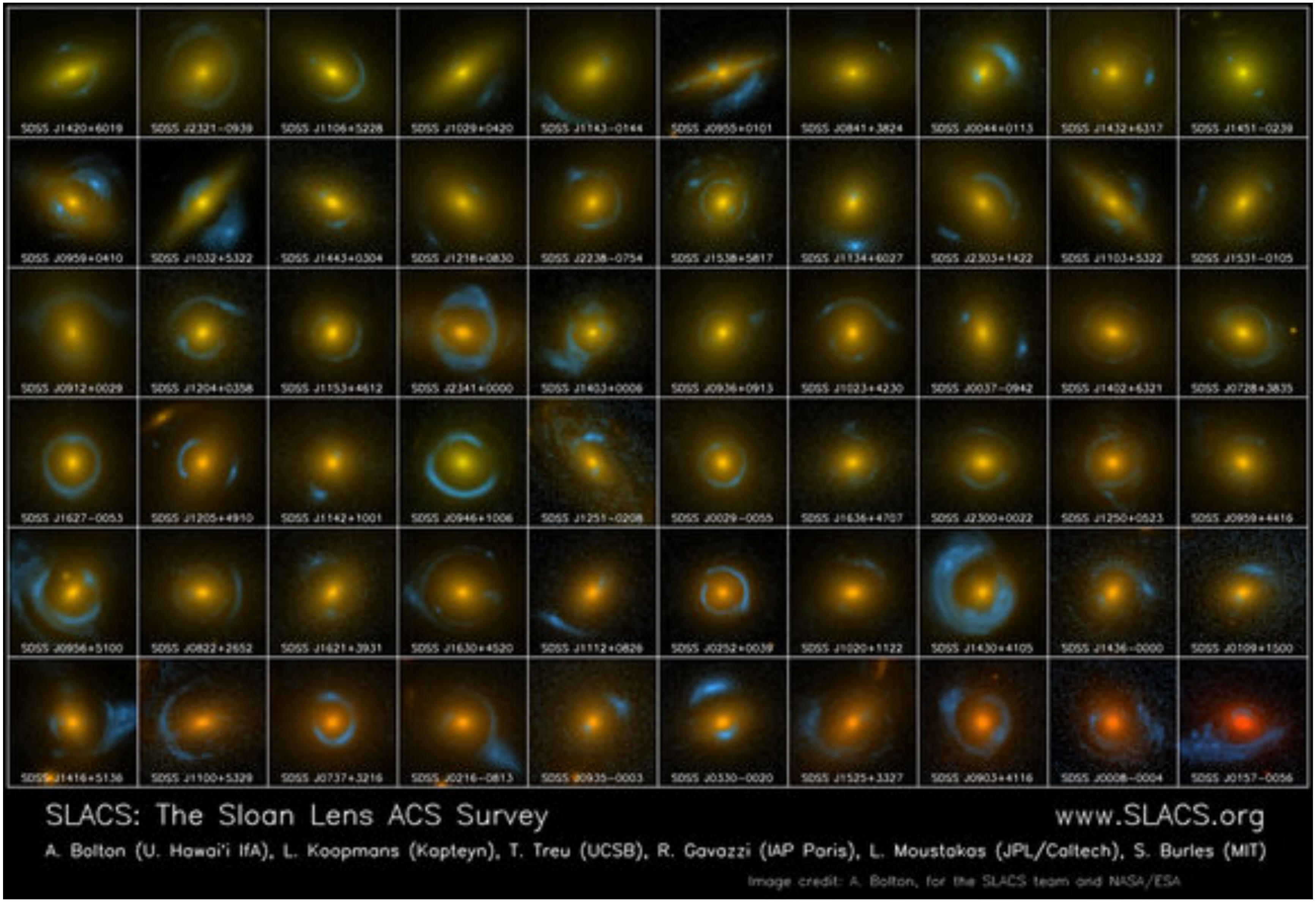


SDSS image

HST ACS image

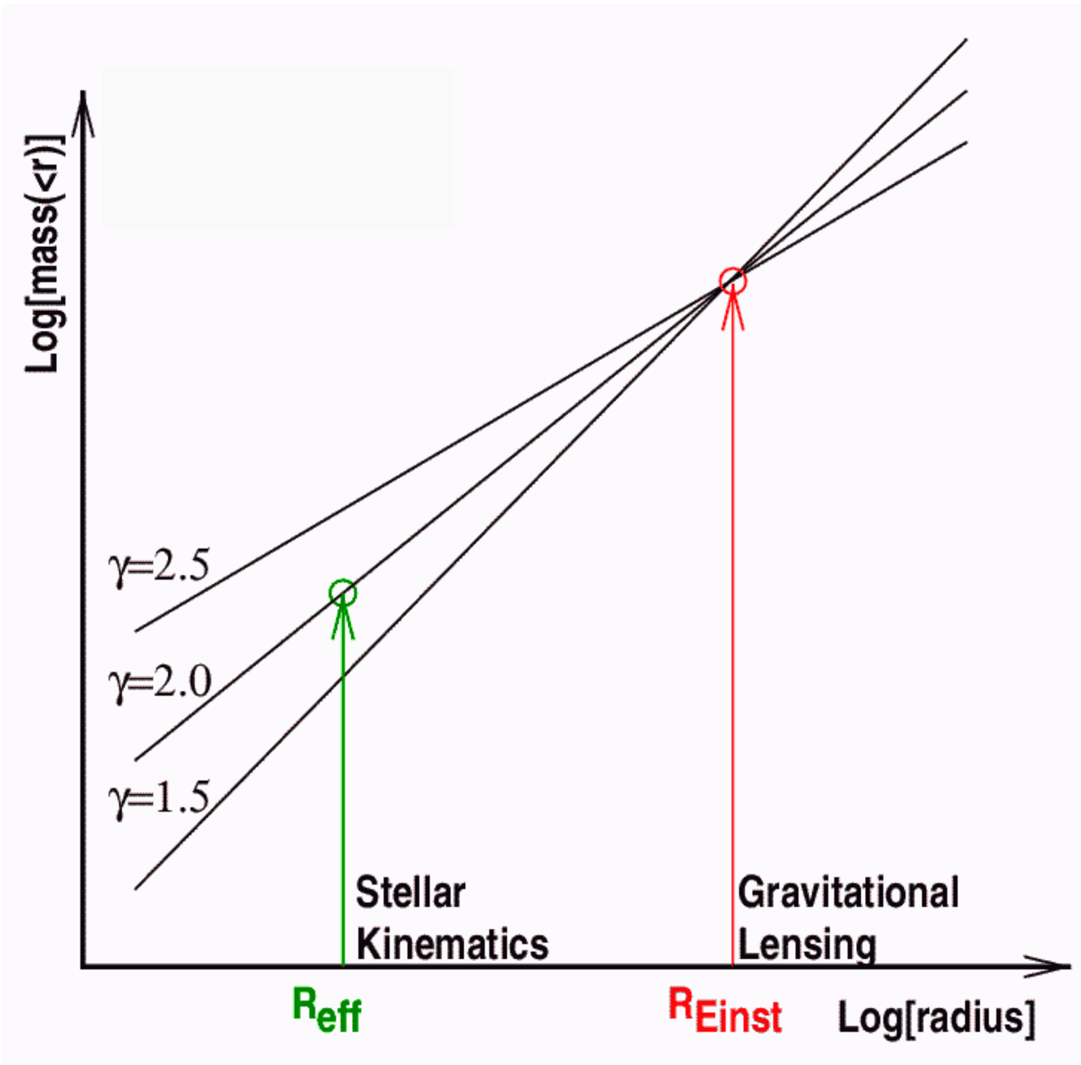
[images credit: A. Bolton]

SLACS: Sloan Lens ACS Survey



Lensing and stellar kinematics

[Treu & Koopmans 2004]



[credit: M. Barnabè]

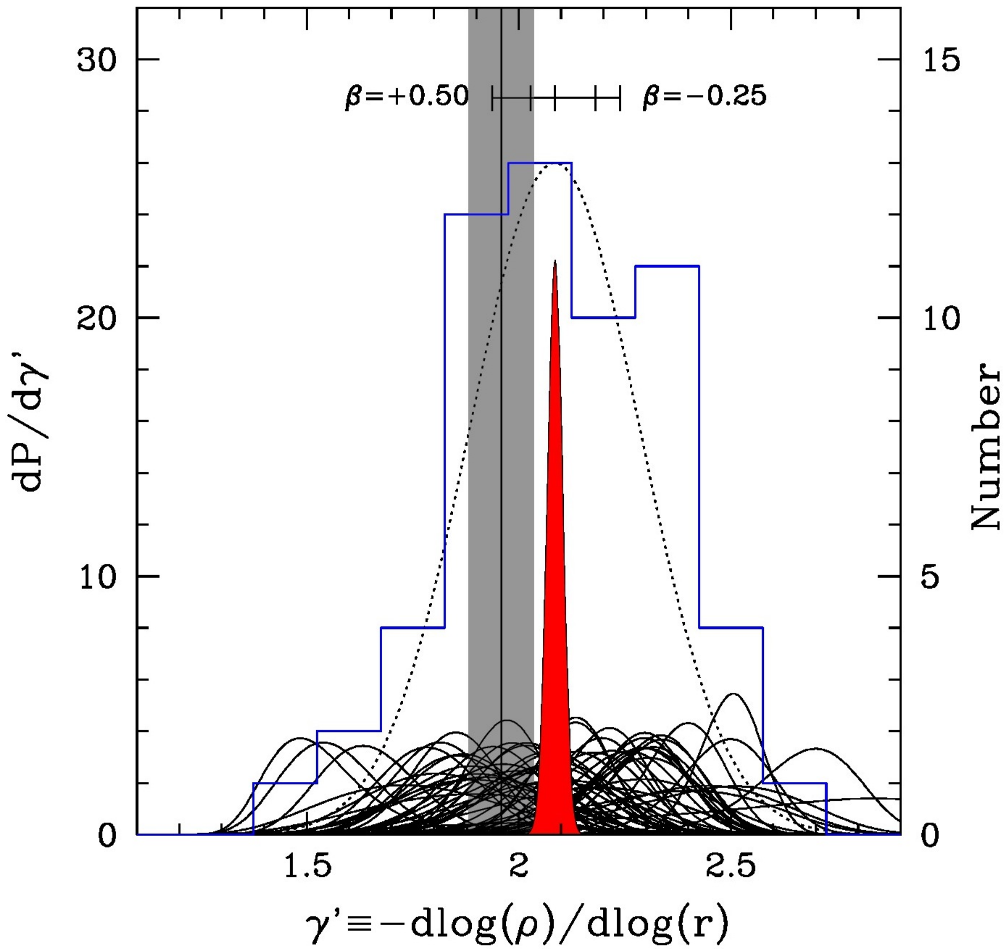
Lensing

- use SIE model (elliptical)
- determine $M_{Ein} = M(R<R_{Ein})$ and impose it as a constraint for the dynamical models

Kinematics

- adopt total density profile $\rho(r) \propto r^{-\gamma'}$
- assume constant anisotropy
- solve spherical Jean's eqns

Near isothermality



[Koopmans et al. 2009]

Mean slope from 73 early-type galaxies within the effective radius:

$$\langle \gamma' \rangle = 2.078 \pm 0.027$$

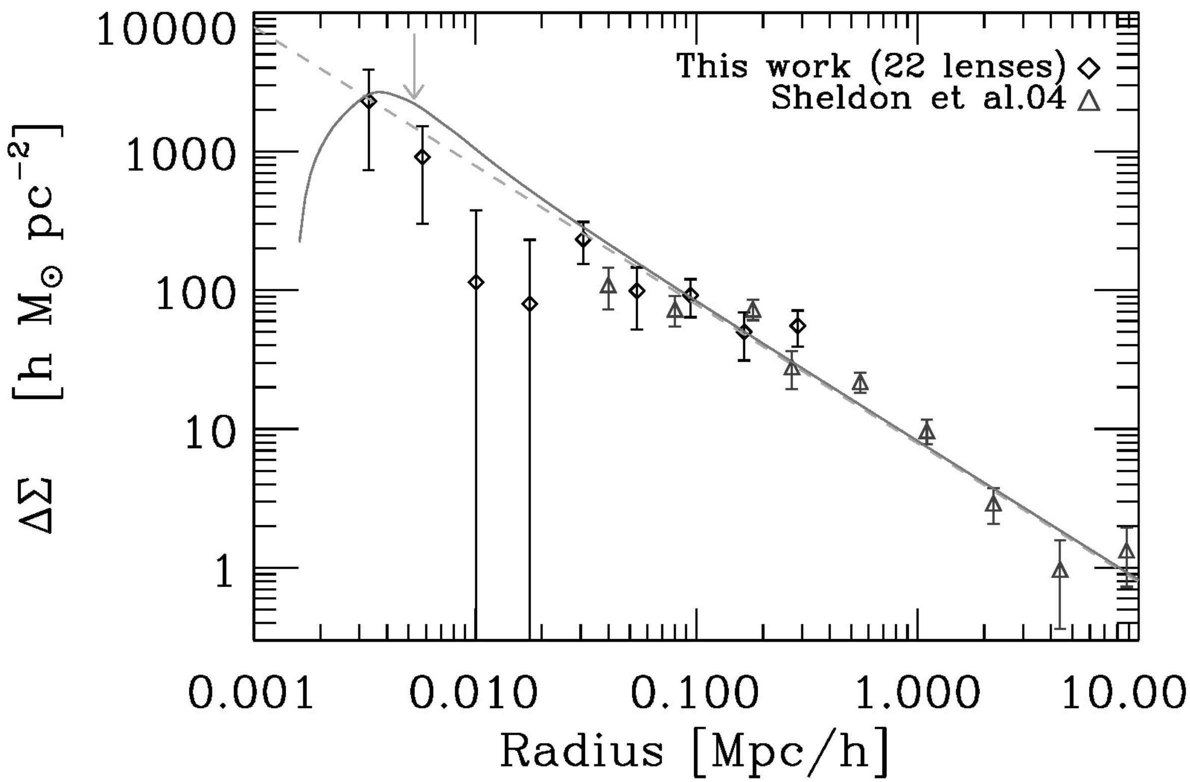
with intrinsic scatter:

$$0.16 \pm 0.02$$

[Auger et al. 2010; similar results from Barnabè et al. 2011 based on a smaller sample with 2D kinematics]

Isothermality out to large radii

Weak lensing analysis of 22 SLACS lenses:



$$\Delta\Sigma(R) = \Sigma_{\rm cr}\,\gamma(R) = \bar{\Sigma}(< R) - \Sigma(R)$$

↓ = mean eff. radius

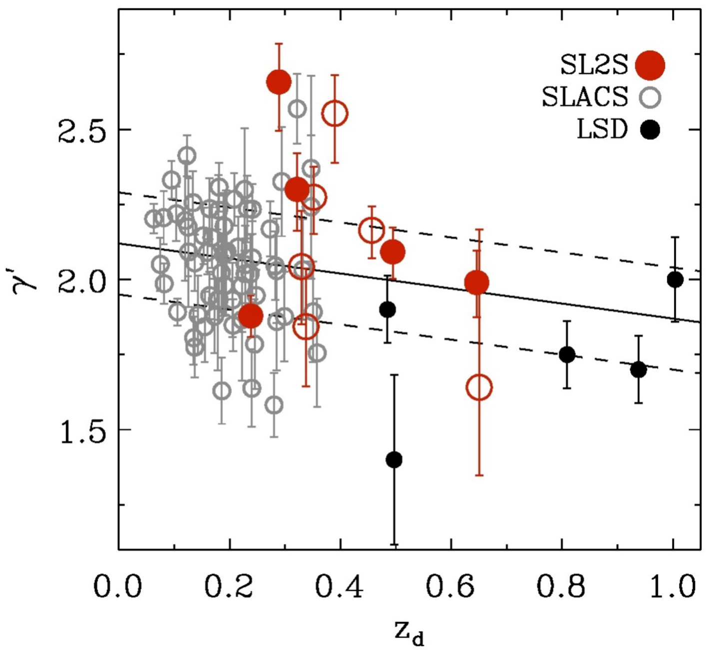
SIS shear profile shown is *NOT A FIT*

Total mass profile is consistent with being isothermal between 1-100 R_{eff}

[Gavazzi et al. 2007]

Evolution

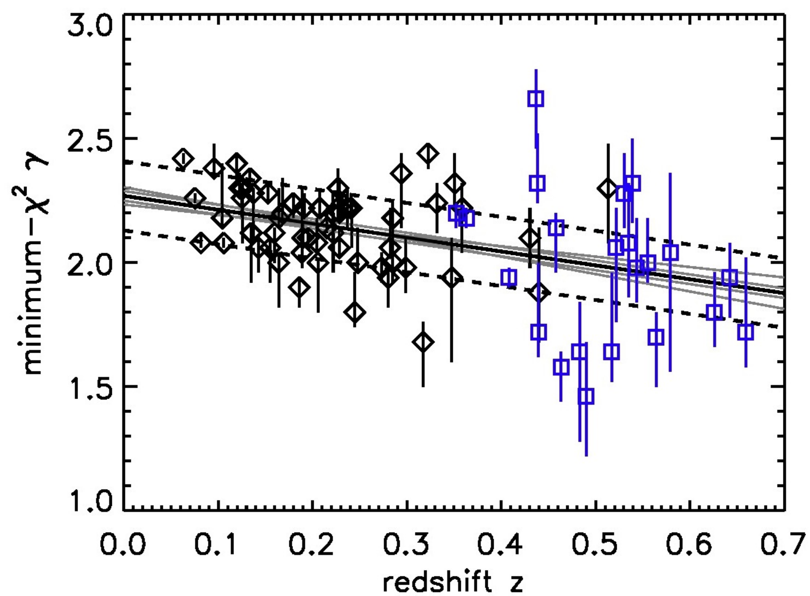
SL2S + LSD



$$\frac{d\langle\gamma'\rangle}{dz} = -0.25^{+0.10}_{-0.12}$$

[Ruff et al. 2011]

BELLS

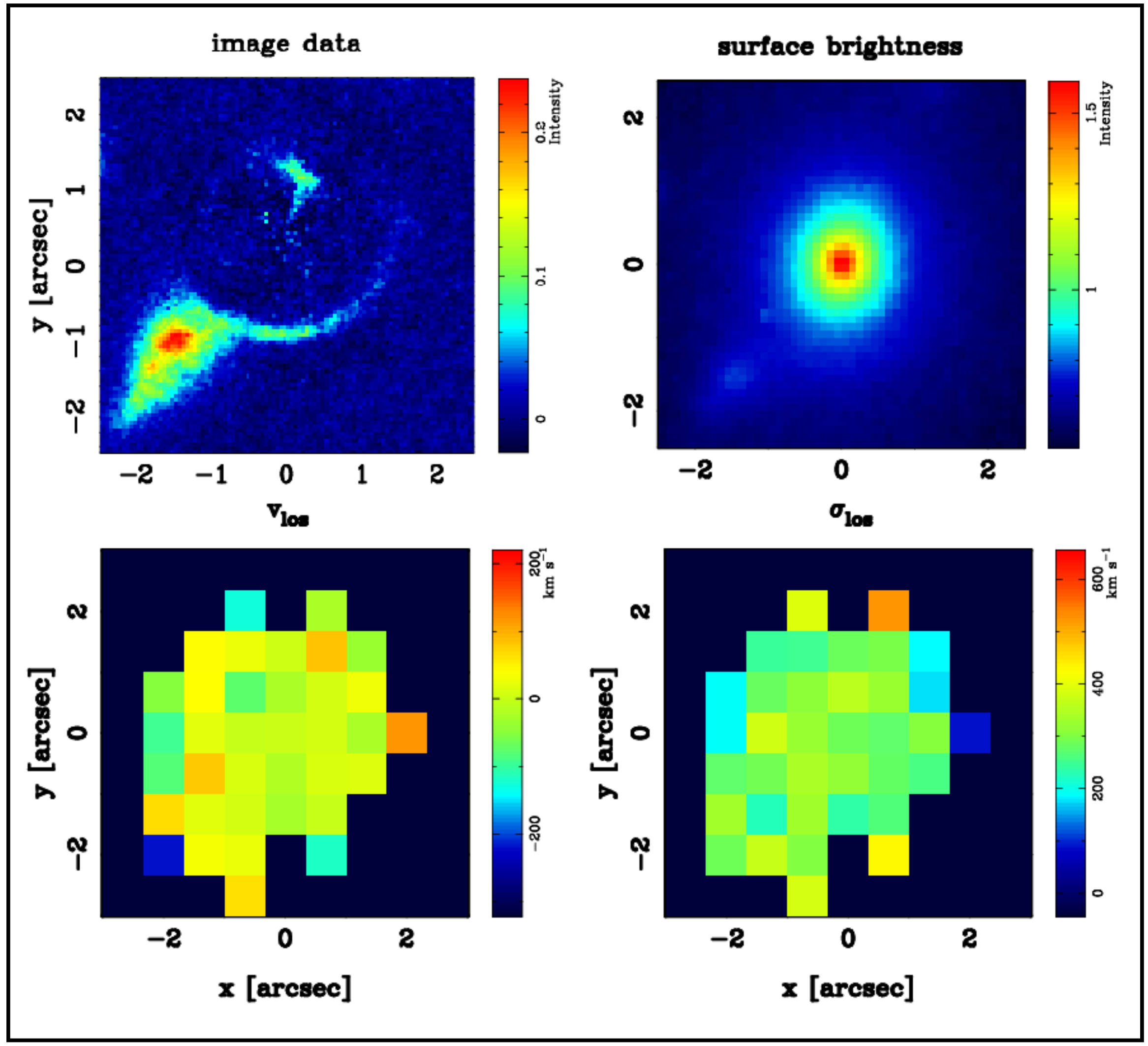


$$\frac{d\langle\gamma'\rangle}{dz} = -0.56 \pm 0.14$$

[Bolton et al. 2012]

Need larger sample of high-redshift lenses!

Lensing and 2D kinematics



Data:

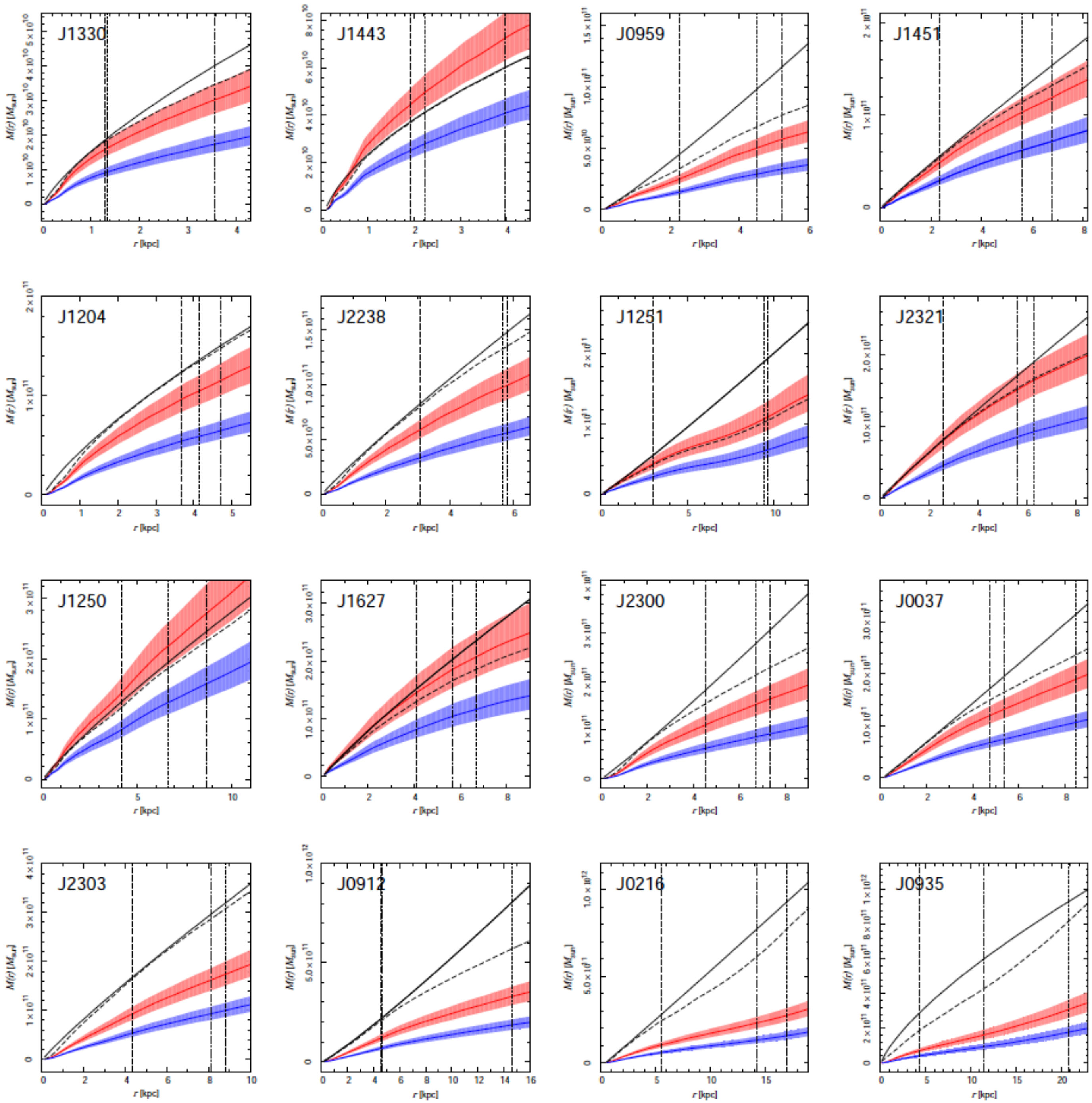
- lensed arcs
- lens surface brightness
- 2D map of line-of-sight velocity and velocity dispersion

Model:

self-consistent axi-symmetric mass model for both lensing and kinematics data

[Czoske et al. 2008; Barnabè et al. 2009; Barnabè et al. 2011; Czoske et al. 2012]

Dark and Luminous Mass Profiles



Ordered by increasing M

DM fraction lower limit: 0% to 40%

Chabrier or Salpeter IMFs: ***more massive systems contain more DM, and are DM-dominated already in the inner regions***

total mass
max. bulge
Salpeter IMF
Chabrier IMF

[Barnabè et al. 2011]