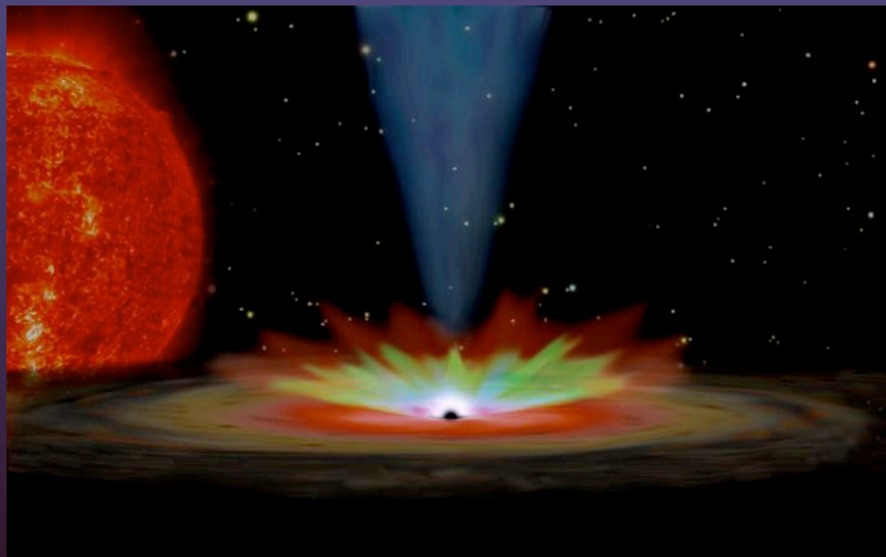
Accretion

introduction:

http://www.mpa-garching.mpg.de/~henk/disksn.pdf

disks09.pdf



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Accretion:

Mass accumulating on a (compact) object by the action of its gravity

Examples:

- Formation of stars, planetary systems
- 'cataclysmic variables' (accreting white dwarfs)
- X-ray binaries (accreting black holes, neutron stars)
- Active galactic nuclei (massive accreting black holes)

Close connection with jets:

- protostellar jets (Herbig-Haro objects) $v \sim 100 - 300\ \mathrm{km/s}$
- microquasars, SS433
- radio galaxies
- gamma-ray bursts

relativistic: $v \approx c$ $(\Gamma = 10 - 1000)$

Gravitational potential of spherically symmetric mass M of radius R

$$\Phi = -\frac{GM}{r} \qquad (r > R)$$

Acceleration of gravity

$$\mathbf{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{r}$$

Particles freely falling from $r \to \infty$ to r :

$$E_{\rm K} = \frac{1}{2}v^2 \qquad \text{(kinetic energy per unit mass)}$$

Energy conservation: $E_{\rm K} + \Phi = E = \text{cst.}$

At r : $v^2 = \frac{2GM}{r}$ (free-fall or escape speed)

(def:) Compact star: M/R *large*

Example: neutron star, $M = 1.4M_\odot,\ R = 10\ \text{km}$: $\frac{v_{\rm ff}}{c} \approx 0.6$
(Newtonian approximation!)

Accretion of gas

Equation of state of ideal gas: $P = \rho \mathcal{R} T / \mu$
(pressure P, density ρ, gas constant $\mathcal{R}$, 'molecular weight' μ)

Internal energy of gas at temperature T: $u = \dfrac{P}{\rho(\gamma - 1)}$ $\quad (\gamma = \dfrac{c_p}{c_v} = \text{cst.})$

Freely falling gas, dissipating its kinetic energy at accreting surface:

$$T = \frac{1}{2}(\gamma - 1) T_{\rm v}$$

$T_{\rm v}$: *virial temperature,* $T_{\rm v} \equiv \dfrac{GM}{\mathcal{R} r}$

(assumption: *adiabatic* flow) (=?)

Neutron star, $v_{\rm ff}/c = 0.6$: $T_{\rm vir} \sim 10^{12}$ K $\approx 300\,\text{MeV}$

$$v_{\rm ff}/c = 0.6 \quad \longrightarrow \quad T_{\rm vir} \sim 10^{12}\ \mathrm{K} \ \approx 300\,\mathrm{MeV}$$

$$kT \sim m_{\rm p}c^2 \gg m_{\rm e}c^2$$

Actual temperatures limited by:
- radiative energy loss during accretion (disks)
- energy density in radiation
- $e^{\pm}$ pair creation
- energy loss by neutrinos

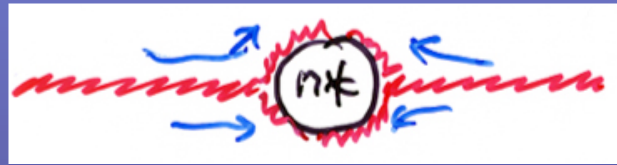
} core collapse SN, GRB

But: if $T_{\rm electron} \neq T_{\rm ion}$:

ion *temperatures* 10^{12} *possible in 'ion supported' accretion*

Radiative loss

BB: $F_{\rm rad} = \sigma T^4$



Accretion rate $\dot M$ (g/s) on star of radius R :

$$\dot E_{\rm rad} = -\Phi \dot M = \frac{GM\dot M}{R} = 4\pi R^2 \sigma T^4$$

$$\dot M \approx 10^{-8} M_\odot \ {\rm yr}^{-1} \rightarrow \ T \sim 1{\rm keV} \sim 10^7{\rm K}$$

Blackbody approx: never very good, but can be fair for high *optical depth* τ

$$\tau = n \sigma_{\rm c} R$$

particle density ↗ ↖ *cross section*

Escape time of photon from object of size R : $t_{\rm esc} \sim \frac{R}{c}\tau^2 \quad (\tau > 1)$

Accretion time : $t_{\rm acc} = R/v_r$

Radiation cooling important when $t_{\rm esc} < t_{\rm acc}$

Radiation processes

Photon production:

- *Atomic transitions*
- *Fully ionized: bremsstrahlung*
- *in   : synchrotron/cyclotron emission*
- $e^{\pm}$ *pair production* $(T > 100\ \mathrm{keV})$

Opacity :

- same processes, plus
- electron scattering

Thomson: elastic $e-\gamma$ scattering

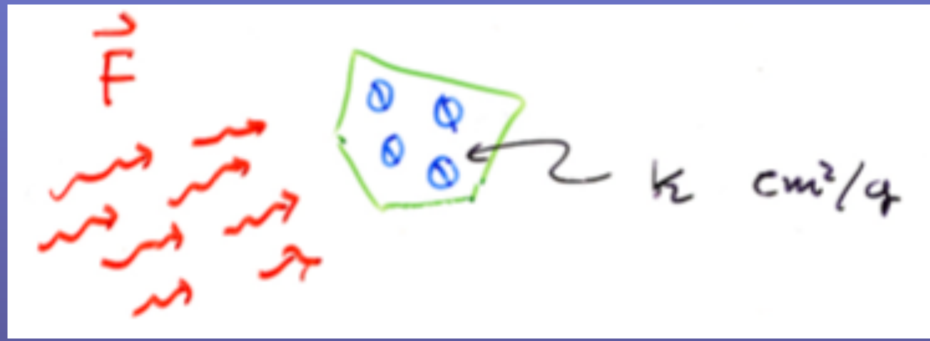
$$\sigma_\mathrm{T} = \frac{8\pi}{3} r_0^2 \qquad (<\sim 100\mathrm{keV})$$

$$\kappa_\mathrm{T} = \sigma_\mathrm{T}/m_\mathrm{p} \approx 0.3\ \mathrm{cm}^2/\mathrm{g}$$

Comptonization (inelastic, $>\sim 100\mathrm{keV}$):

- $e-\gamma$ scattering with change of photon energy

Eddington limit



Radiative force: $\dfrac{F_{\rm rad}}{c}\kappa$

gravity: $\dfrac{GM}{r^2}$

Equate: $F_{\rm E} \equiv \dfrac{c}{\kappa}\dfrac{GM}{r^2}$ (Eddington flux)

Eddington Luminosity (spherical, $\times 4\pi r^2$*) :* $L_{\rm E} = 4\pi GM\dfrac{c}{\kappa}$

$$M = 1M_\odot,\ \ \kappa = 0.3 \qquad \rightarrow L_{\rm E} \approx 10^6 L_\odot$$

If L from accretion: $\eta\dfrac{GM\dot{M}}{R} = L_{\rm E} \rightarrow \dot{M} = \dot{M}_{\rm E} \equiv \dfrac{1}{\eta}4\pi R\dfrac{c}{\kappa}$

$$\eta = 1: \qquad \dot{M}_{\rm E} \sim 10^{-8} M_\odot\ {\rm yr}^{-1} \quad (10\ {\rm km},\ \ \kappa = 0.3)$$

Eddington critical accretion rate.

Eddington limit, optically thick

static: $\nabla(P_{\rm gas} + P_{\rm rad}) = \rho \mathbf{g}$

$$\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}\tau} = F_{\rm rad}$$

$$\mathrm{d}\tau = \kappa\rho\mathrm{d}x$$

radiative flux balances gravity if $F_{\rm rad} = F_{\rm E}$

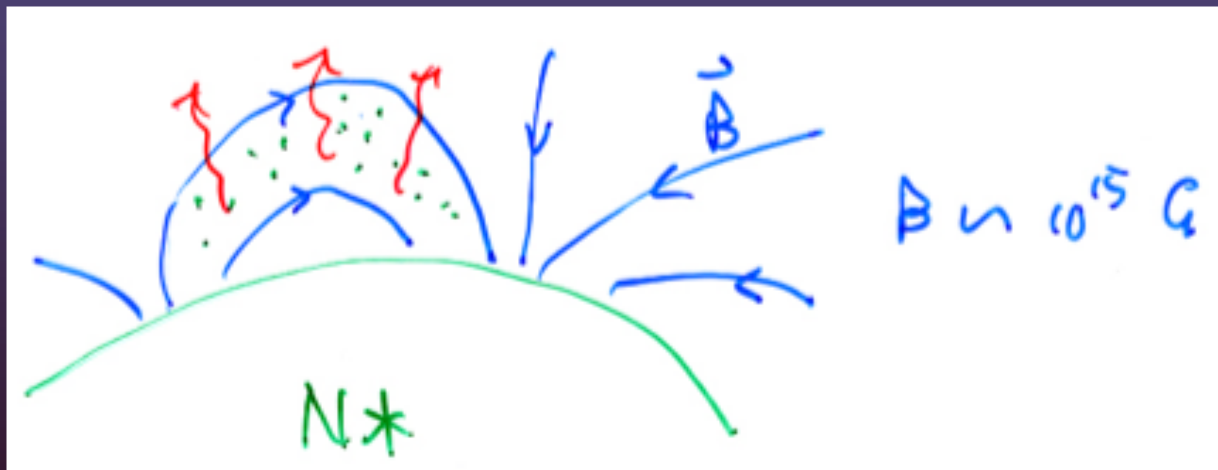
→ maximum luminosity is Eddington if energy carried by radiation

Applicability of the Eddington luminosity limit

1 - static radiating object assumed
2 - only gravity, no other restraining forces
3 - energy transported by something else (convection, B-fields)

Exceptions to applicability

ad 1: nova, supernova explosions
ad 2: other forces: magnetic fields (e..g. *magnetars*)



$$U_{\rm rad} = aT^4 \quad F_{\rm rad} = \frac{1}{4} acT^4$$

Magnetic confinement: $U_{\rm rad} < E_{\rm mag} = \dfrac{B^2}{8\pi}$

$\longrightarrow$ $F_{\rm rad} < \dfrac{B^2}{8\pi} c$

Can be $\gg F_{\rm Edd}$

Example: magnetars

$$L_{\rm max} = 4\pi R^2 \frac{B^2}{8\pi} c$$
$$\sim 10^7 L_\odot B_{14}^2 R_6^2$$

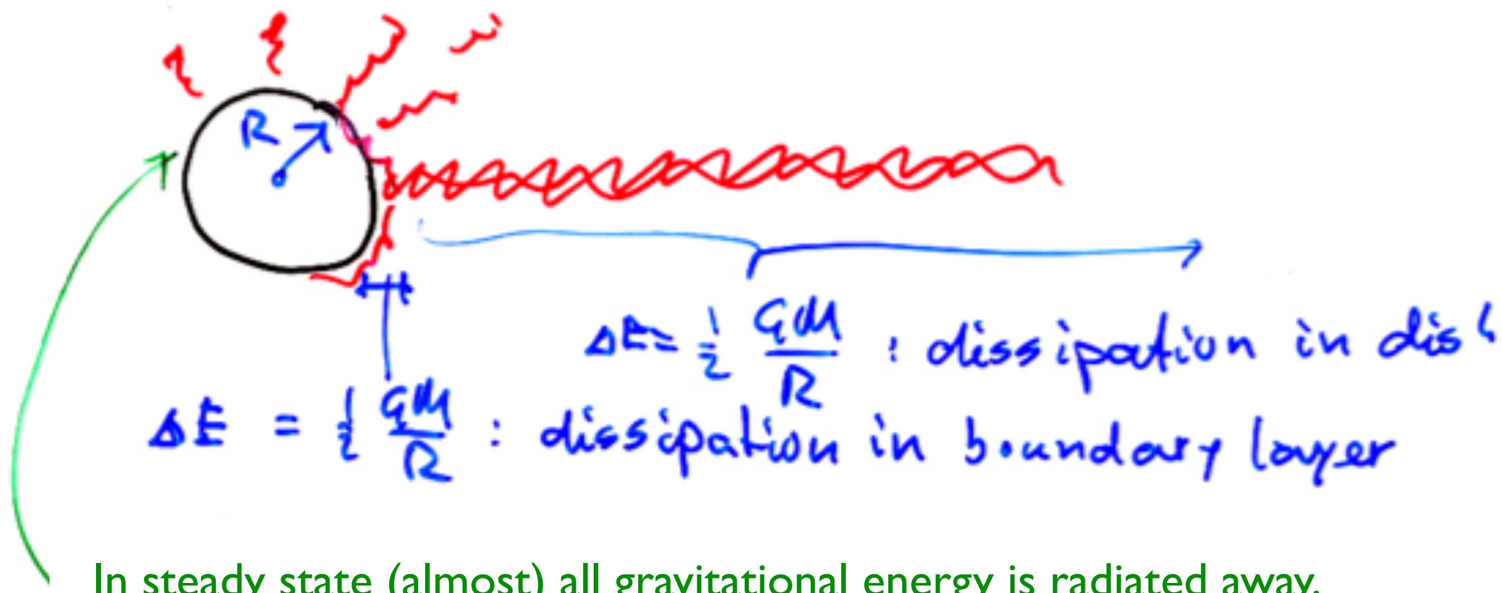
(decrease of B w. distance!)

Eddington accretion rate:

the difference between black hole and neutron star accreters

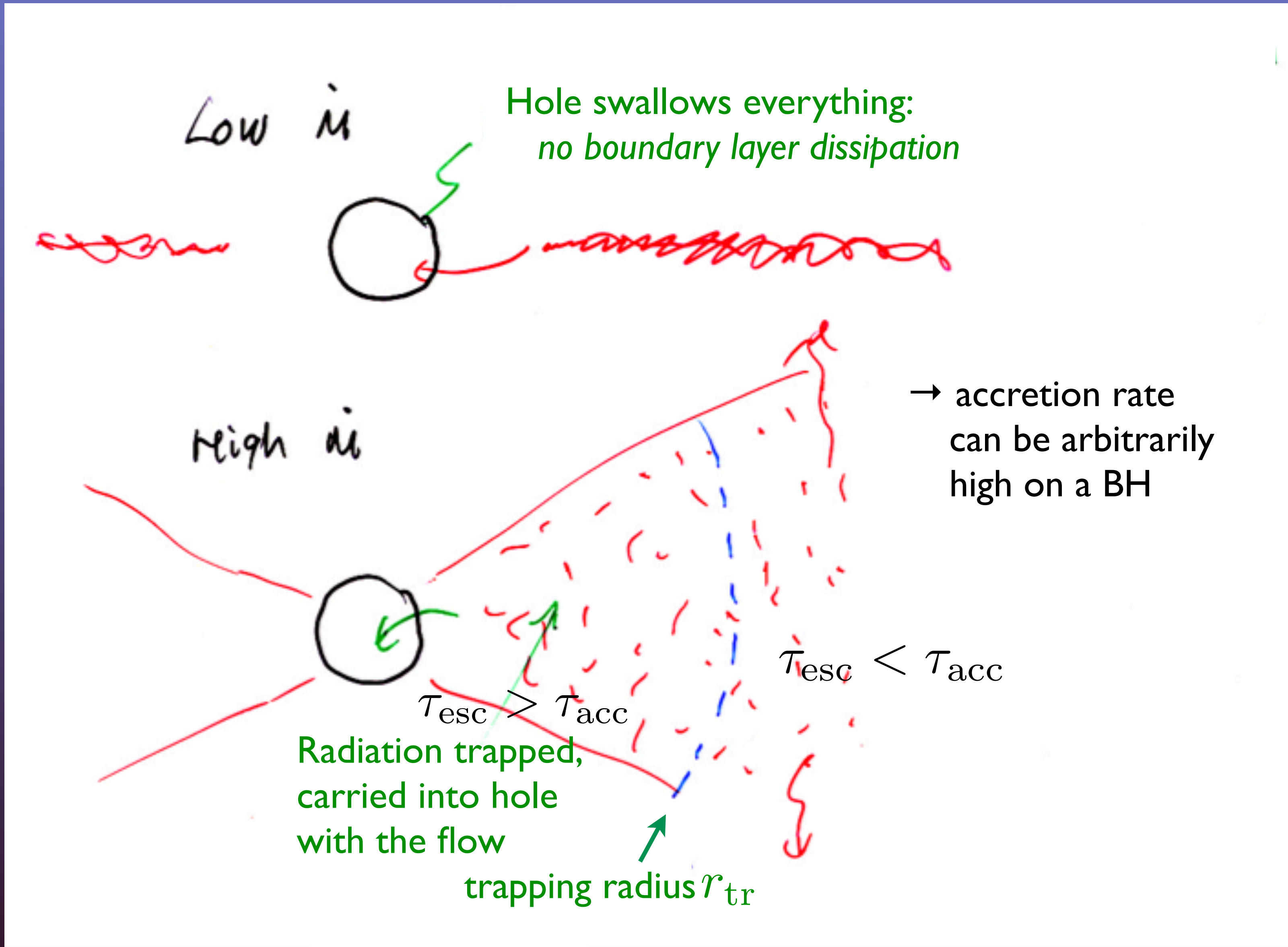
Accretion on *neutron stars*:

Orbital kinetic energy: $E_{\rm orb} = \frac{1}{2}\frac{GM}{r}$ *(Kepler orbits)*



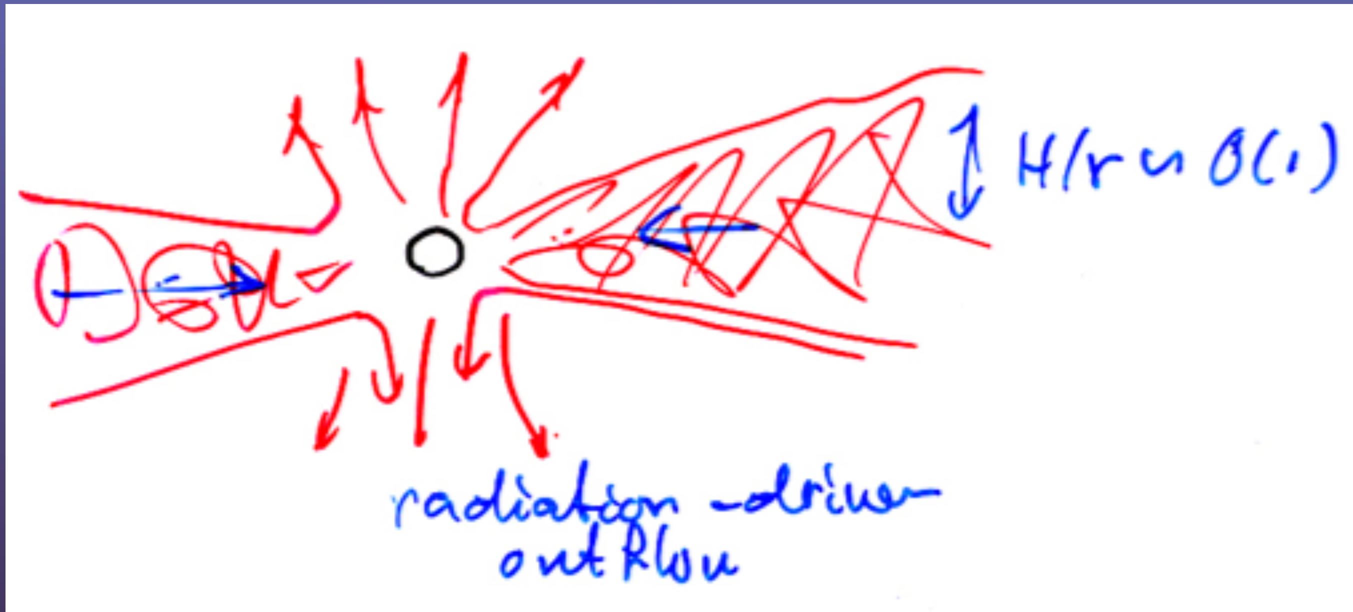
In steady state (almost) all gravitational energy is radiated away.
Half from disk, half from boundary layer on star.
(Some stays as thermal and rotation energy on the star)

Accretion on *black holes:*



non-spherical super-Eddington accretion:

n-star with $\dot{M} > \dot{M}_{\rm E}$:



Example: SS433 ?

Where disks form

Star formation
Galactic nuclei
Mass transfer in close binary stars

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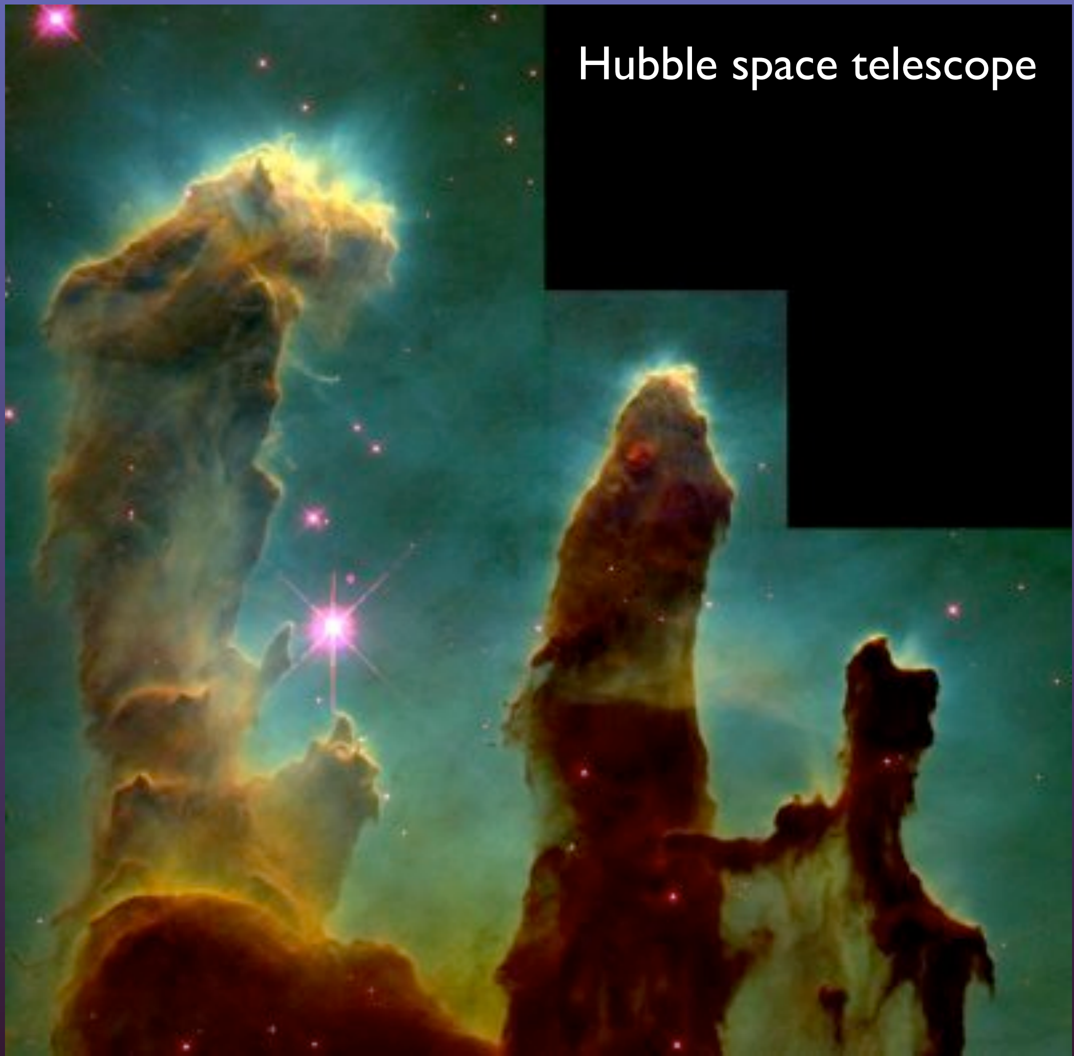
Star forming region



'protodisks'

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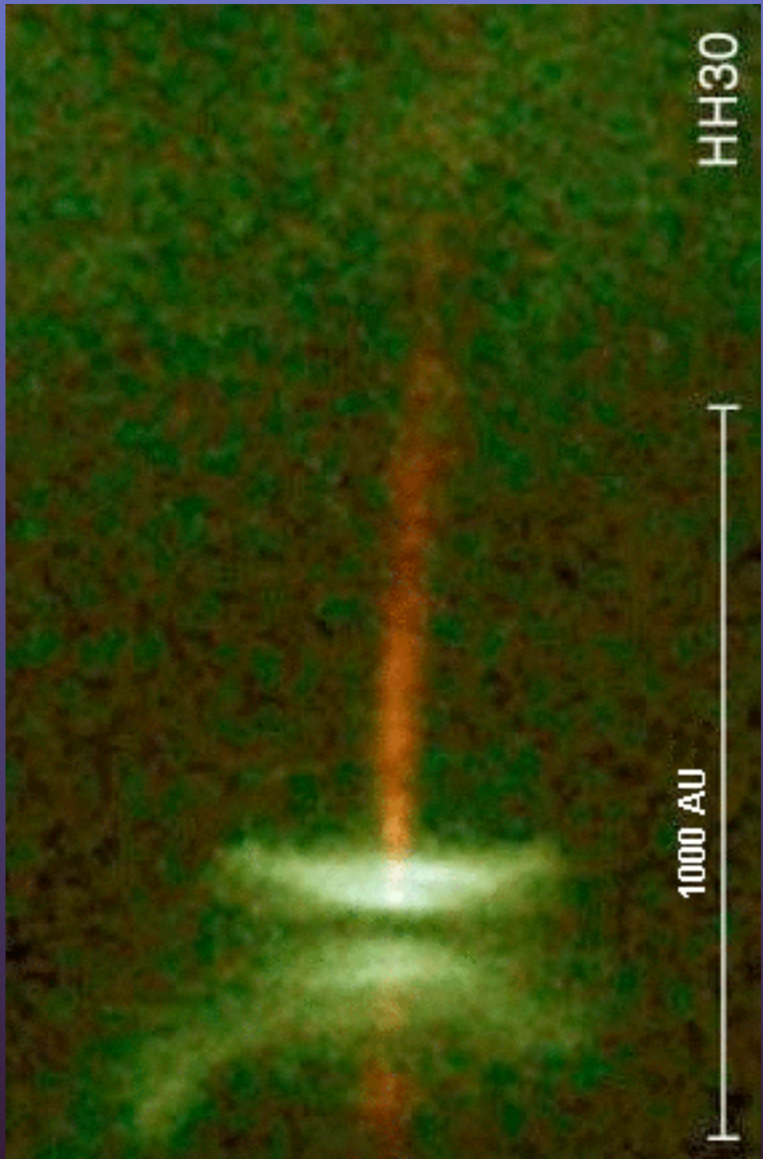

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Disks around active galactic nuclei

Disks around active galactic nuclei

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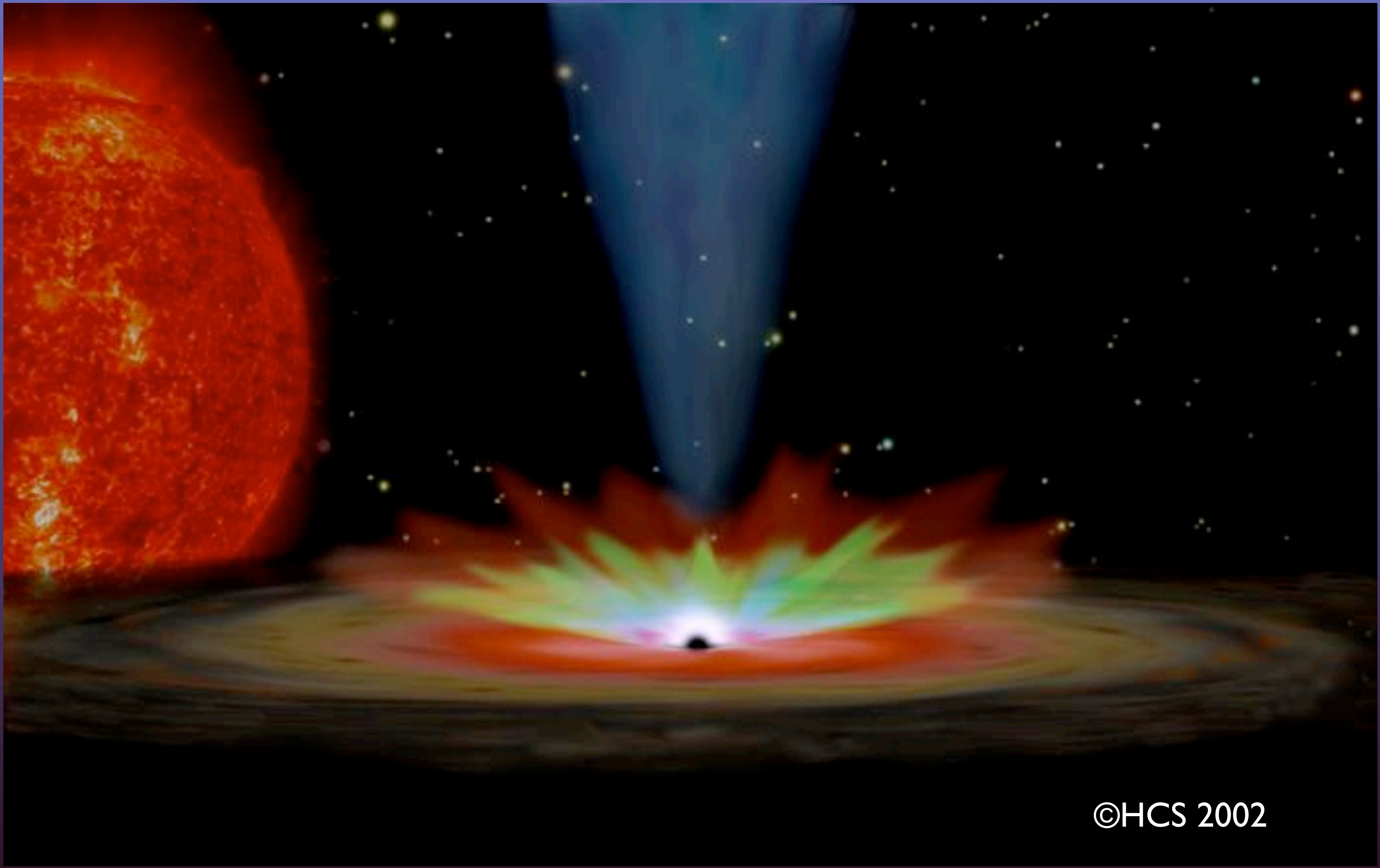
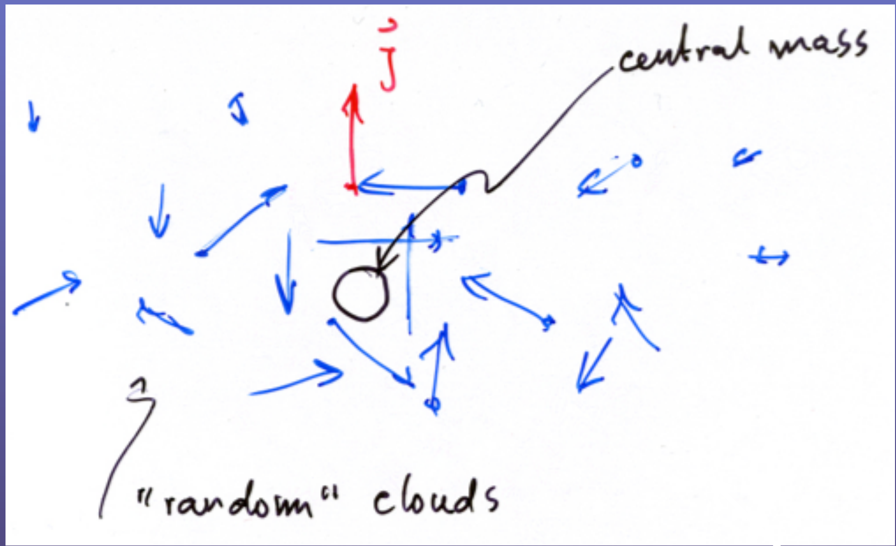
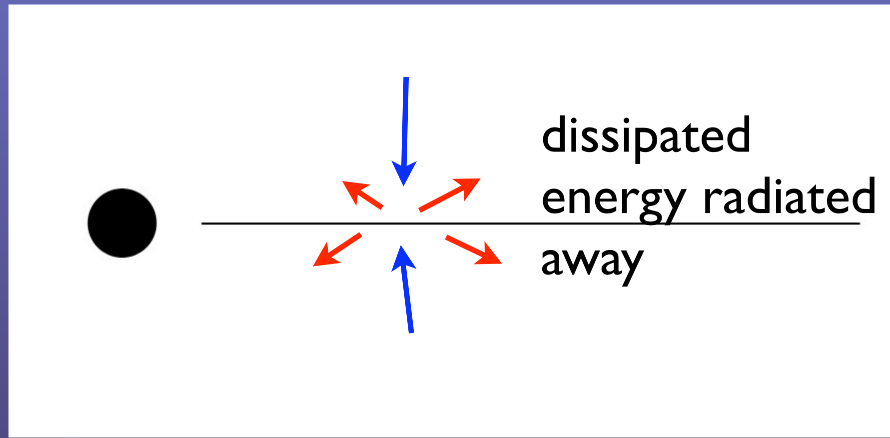
Disks formed by mass transfer in a binary

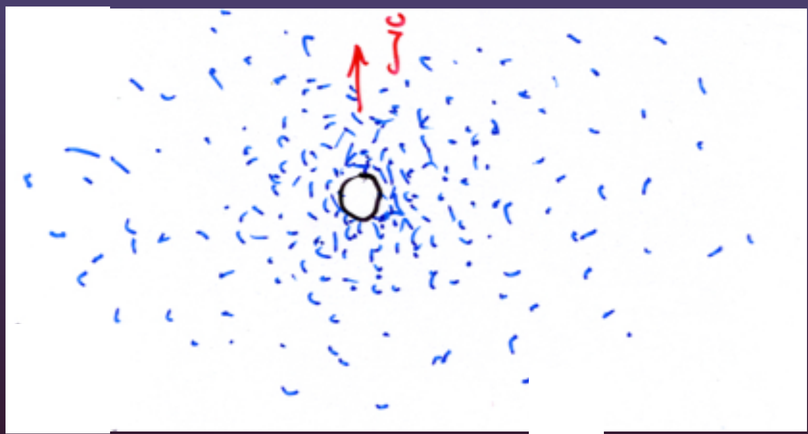


Why disks form: the role of angular momentum



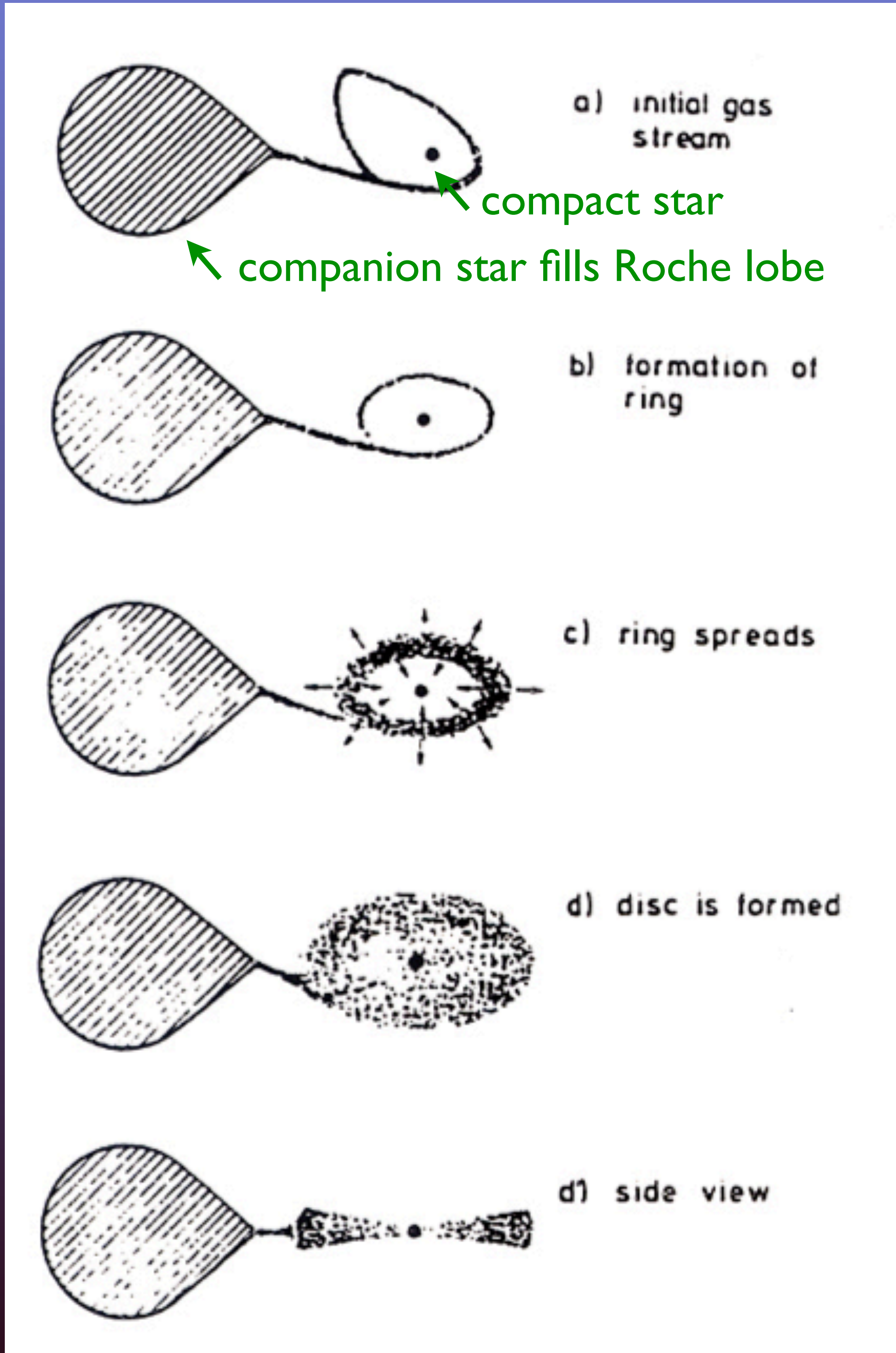


Virialized:



→

cooled:
(example: gas disks in galaxies)



Formation of a disk in a mass-transfering binary

Mass transfering binaries

Frank, King & Raine, "Accretion Power in Astrophysics", CUP, Ch.4

X-ray binary: n-star or BH + main sequence star
Cataclysmic variable: white dwarf + main sequence star (or WD)

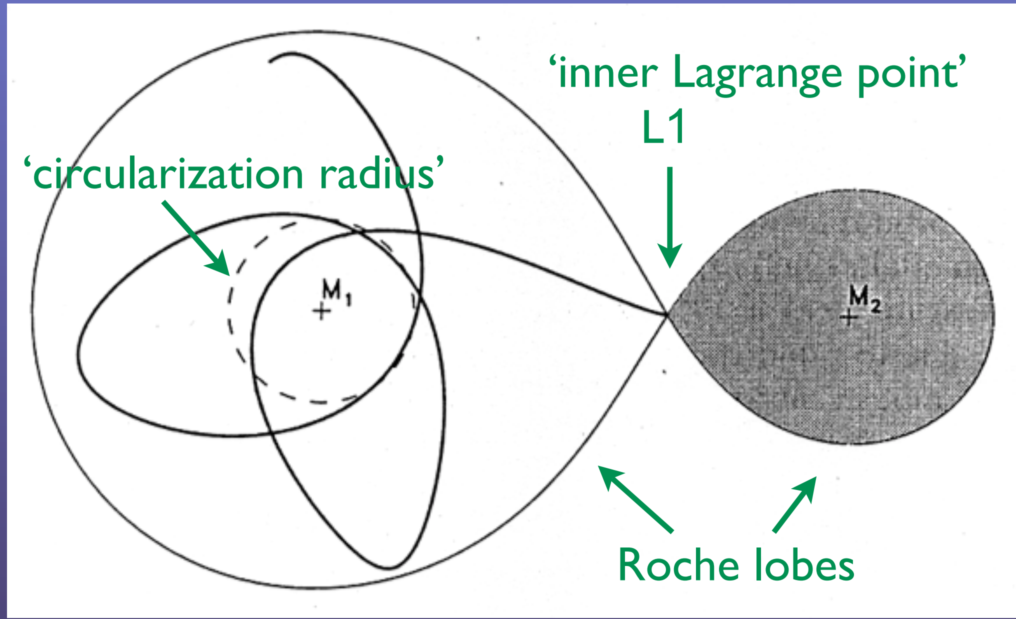
Binary: $M_1, \; M_2, \; \; q \equiv M_2/M1$ (mass ratio)
Circular orbit, separation a, orbital frequency $\Omega = 2\pi/P$

Kepler III: $\Omega^2 = \dfrac{G(M_1 + M_2)}{a^3}$ ('period-mean density relation')

$$\rightarrow \quad a = 3.5\,10^{10}\ \mathrm{cm}\ (\frac{M_1}{M_\odot})^{1/3}(1+q)^{1/3}P_{\mathrm{hr}}^{2/3}$$

(c.f. $R_\odot = 7\,10^{10}\mathrm{cm}$)

Roche lobe overflow



Roche surface:

first equipotential surface in a rotating frame (with centrifugal force) connecting the two stars

Nomenclature:

'Roche lobe' ↔ 'Hill sphere'

M1: primary, accreter

M2: secondary, donor

Useful approximations for
- location of Lagrange points,
- shape, volume of Roche lobes:

B. Warner, 1995, "Cataclysmic Variable Stars", CUP, pp 30-40

Equation of motion of free test-particles in a binary

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\Phi_1 - \nabla\Phi_2 \qquad\qquad \Phi_{1,2} = -\frac{GM_{1,2}}{|\mathbf{r}-\mathbf{r}_1(t)|}$$

In corotating frame (rate Ω):

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\Phi_1 - \nabla\Phi_2 + 2\mathbf{v}\times\boldsymbol{\Omega} + (\Omega\times\mathbf{r})\times\Omega$$

Coriolis

Centrifugal

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\Phi_{\mathrm{R}} + 2\mathbf{v}\times\boldsymbol{\Omega} \qquad\qquad \Phi_{\mathrm{R}} = -\frac{GM_1}{|\mathbf{r}-\mathbf{r_1}|} - \frac{GM_2}{|\mathbf{r}-\mathbf{r_2}|} - \frac{1}{2}(\Omega\times\mathbf{r})^2$$

‘Centrifugal potential’

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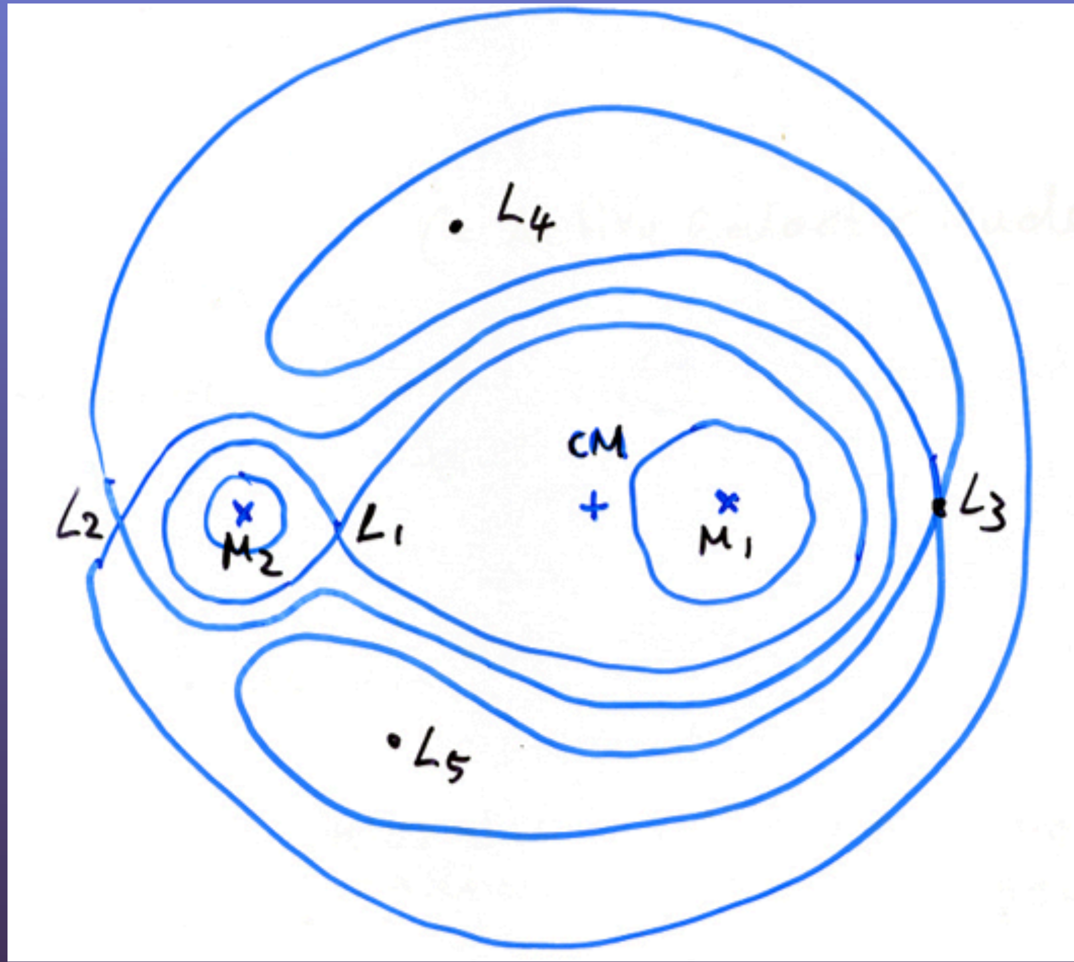
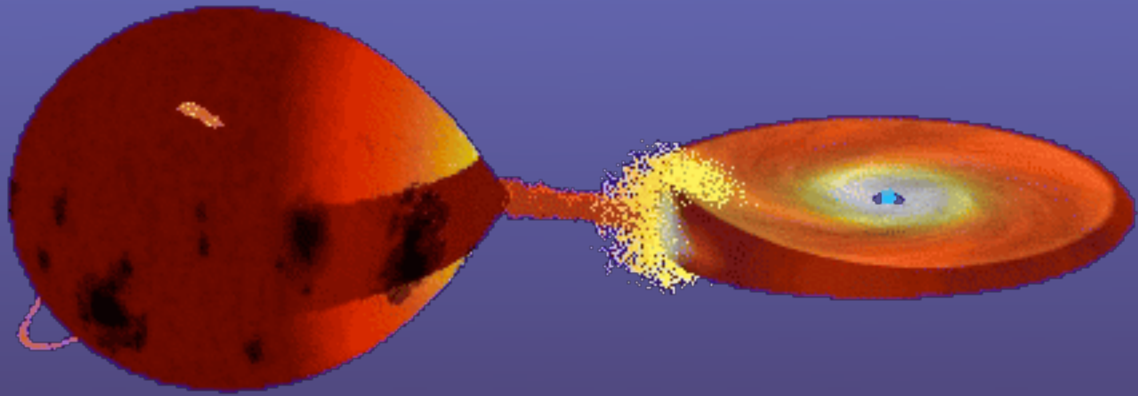
Equipotential surfaces



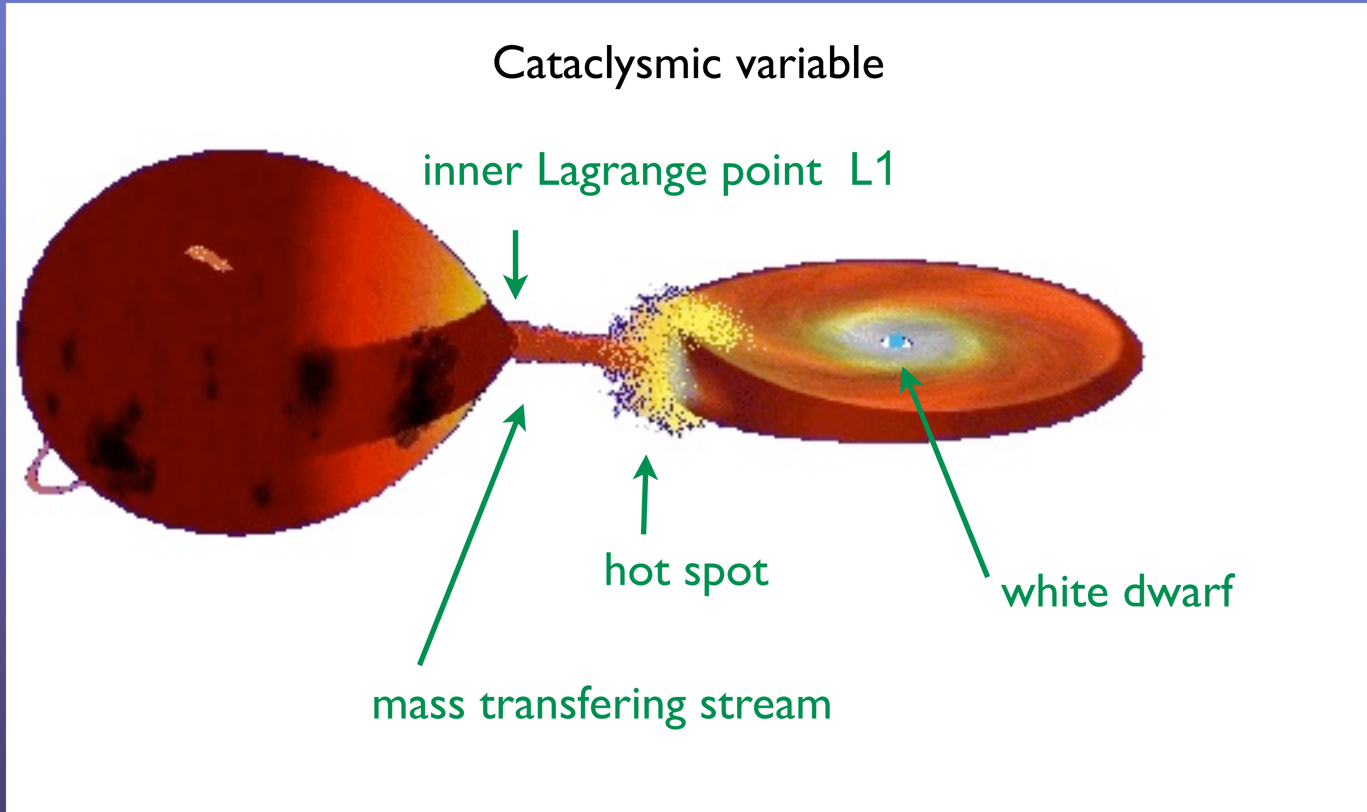
5 Equilibrium points:

Langrange points
unstable: L_1, L_2, L_3
stable: L_4, L_5
('Greeks, & Trojans')

Validity of $\Phi_{\rm R}$:
- strictly corotating objects
(the stars, not for orbits between them!)



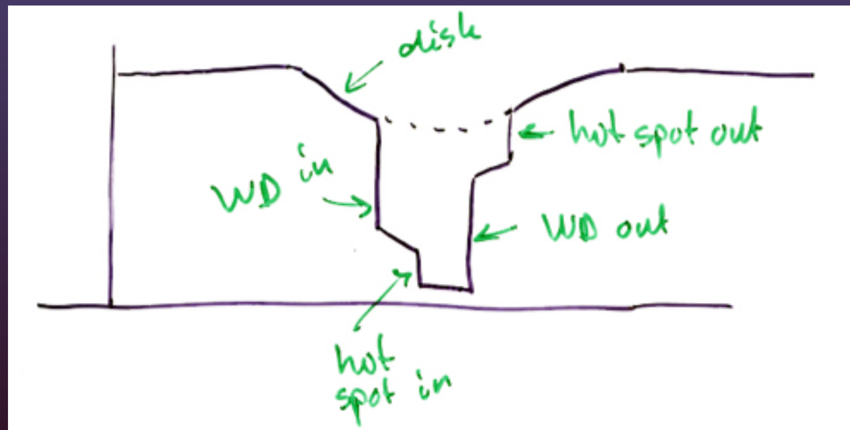
http://physics.technion.ac.il/~astrogr/research.html



Disk size in steady state:
- viscous spreading → r↑
- tidal torques → r↓

equilibrium → *tidal radius*

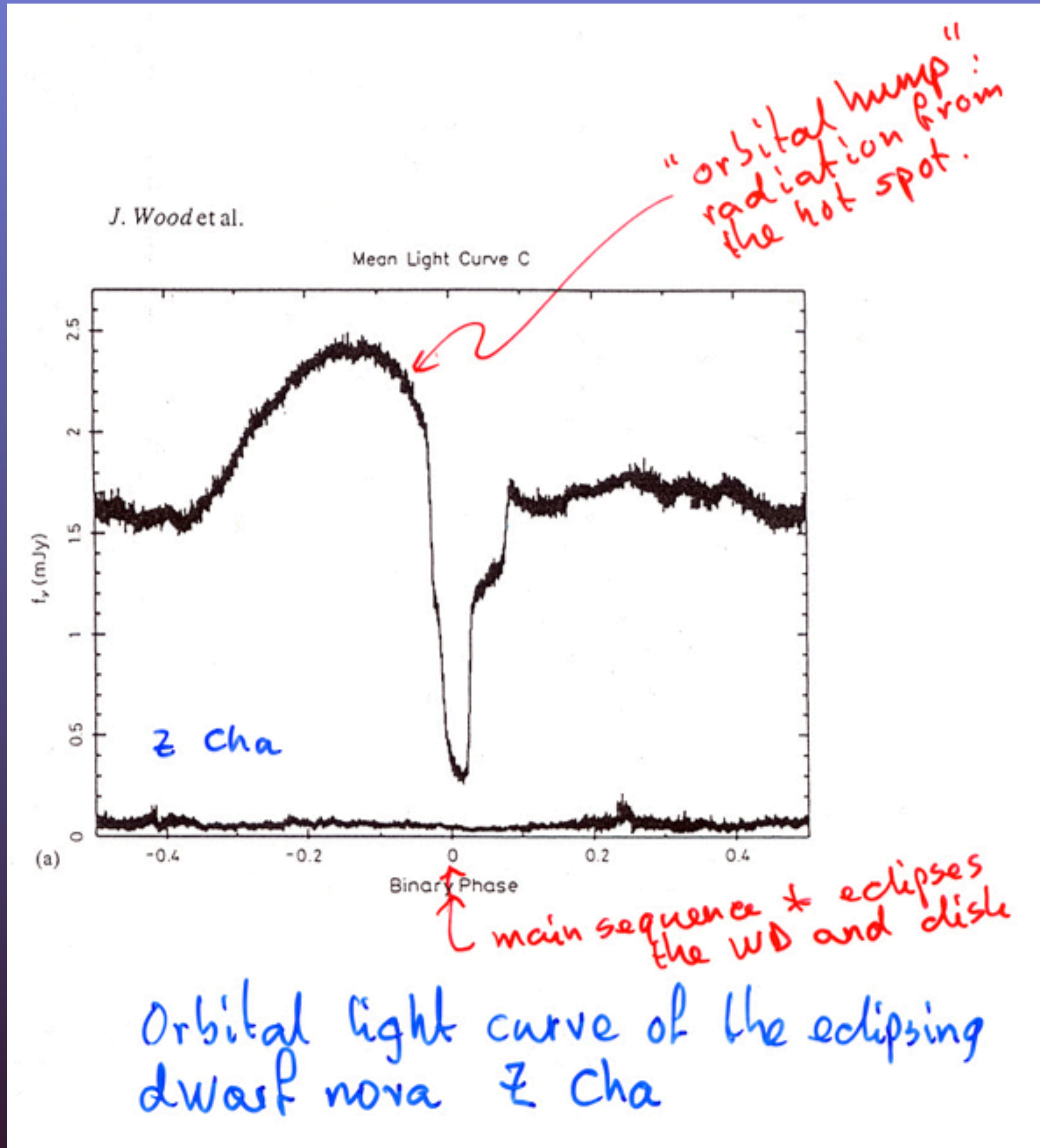
Predicted eclipse light curve:



Actual light curve

used to determine
- orbital parameters
- disk size
- stream impact region
- 'disk turbulence'

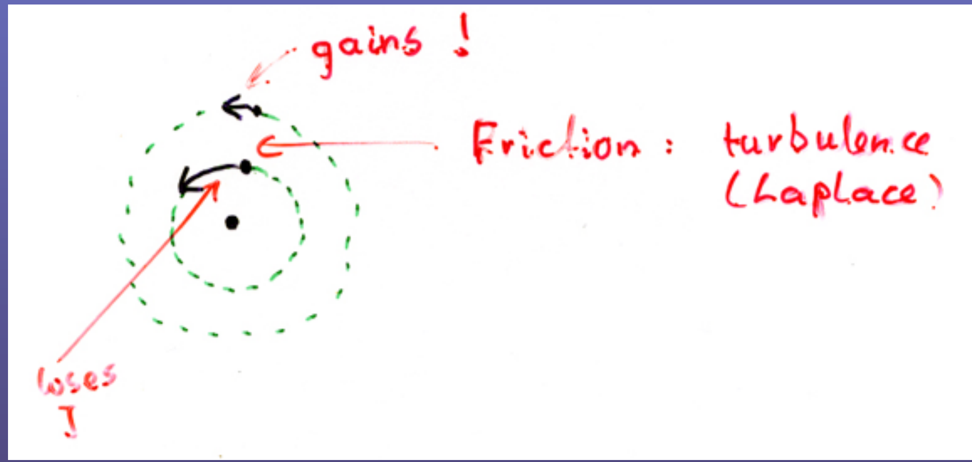
CV's are the best studied disk systems



Angular momentum transport and viscosity in disks

('central problem in accretion disks theory')

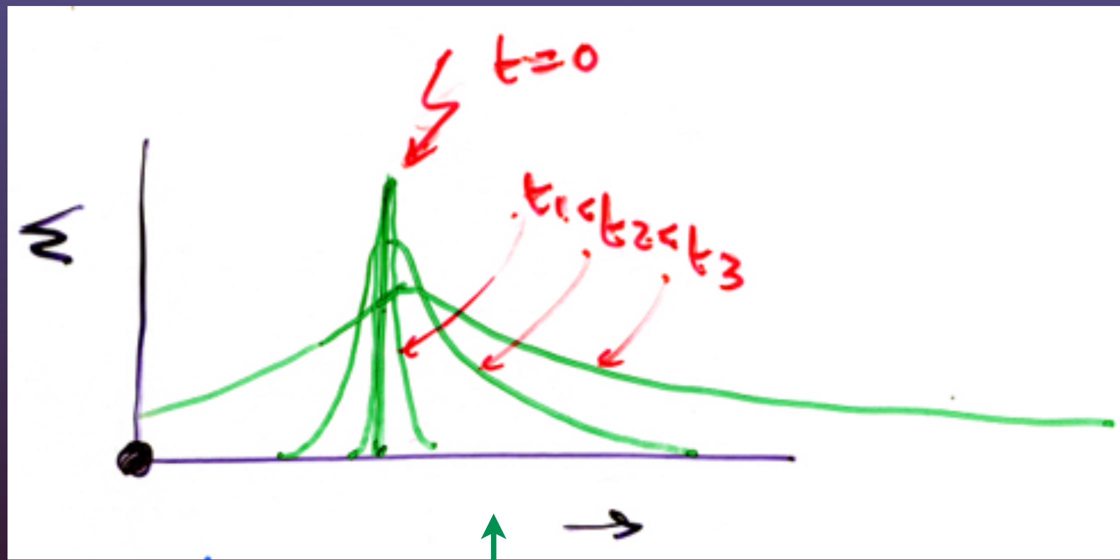
For accretion, angular momentum loss necessary



Model: viscous friction inner orbits faster than outer: shear flow

→ *viscous spreading*

Predicted spreading of a ring, $t \to \infty$:



- (almost) all mass accreted
- (almost) all J to $r \to \infty$

(mass and angular momentum conserved !)

Find the error in this sketch!

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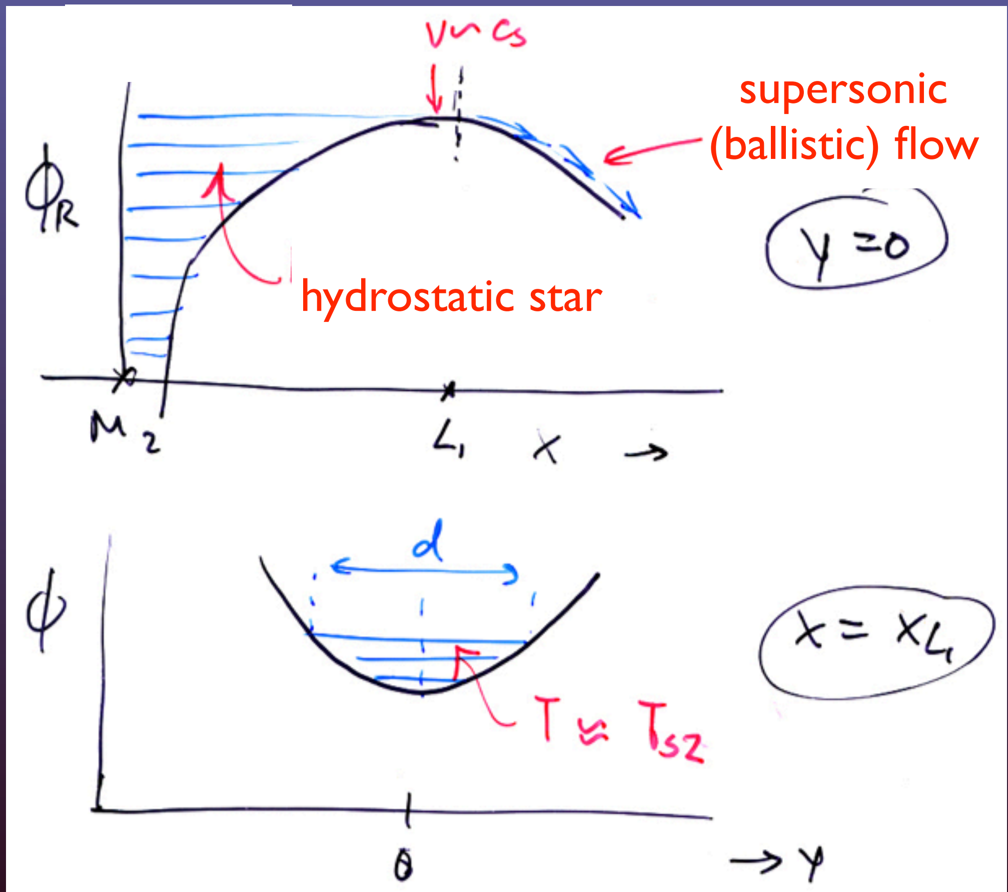
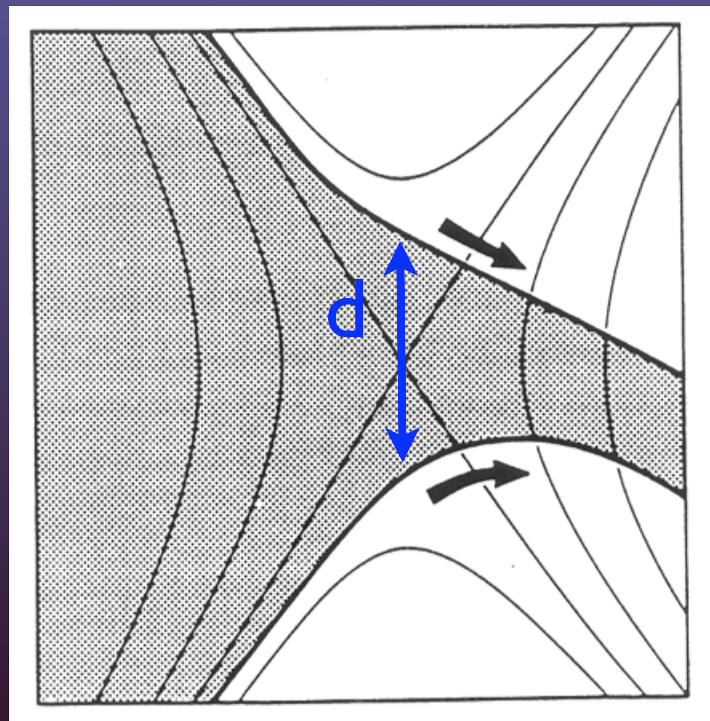
Flow through L1

How wide is the stream? Determined by temperature of gas.

$d \sim (H_2 R_2)^{1/2} \; (\ll R_2)$

pressure scale height at L1: $H_2 = \dfrac{\mathcal{R}T_{s2}}{\mu g_2} \qquad (g_2 = \dfrac{GM}{R_2^2})$





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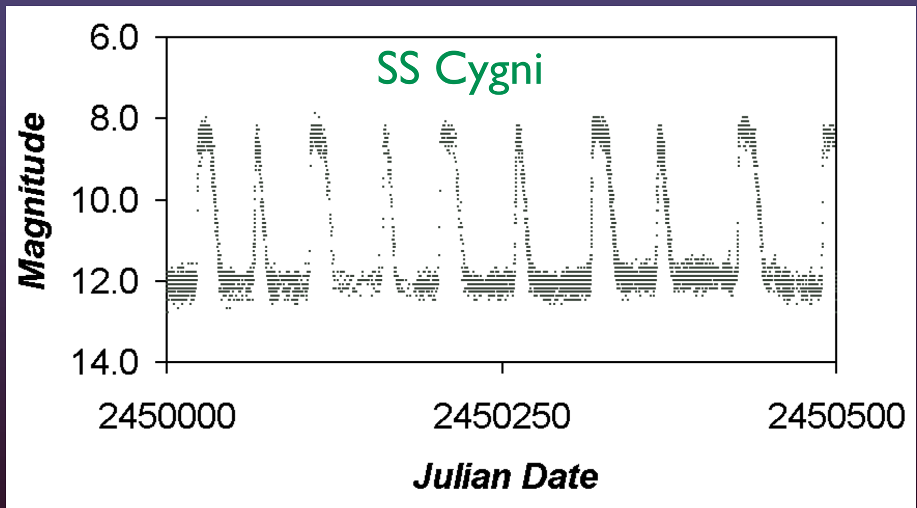
Viscosity: how large?

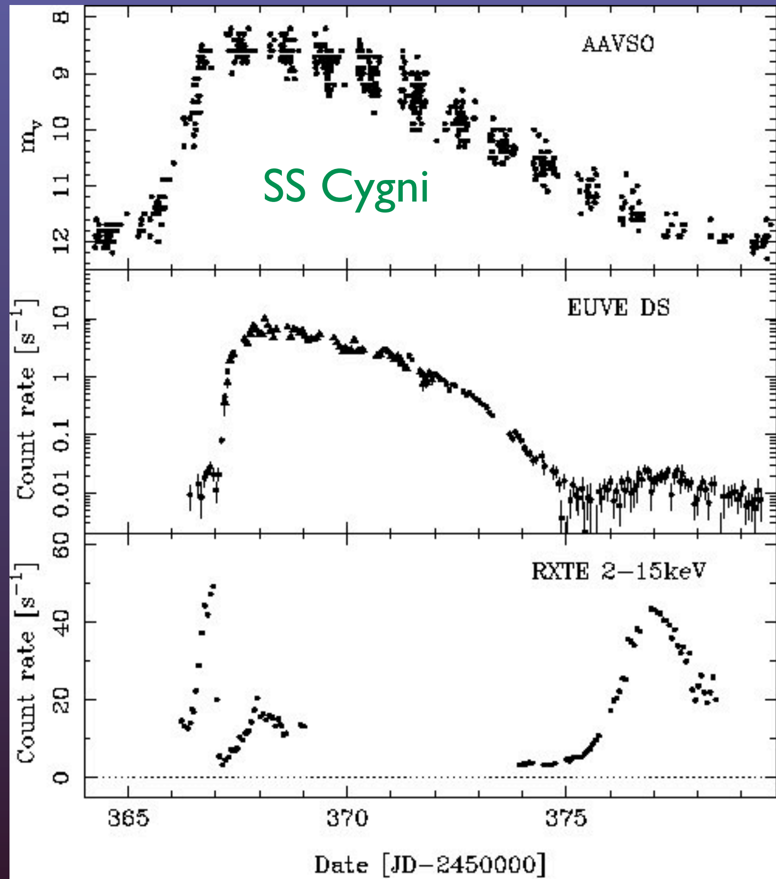
Observations of CV outbursts: decay time of outburst
Theory: disk instability, viscous time scale

$$\sim 5\text{d} \rightarrow t_{\text{decay}} = \frac{r_{\text{disk}}^2}{\nu} \leftarrow \sim 2\,10^{10}\ \text{cm}$$

$$\rightarrow \quad \nu \sim \frac{4\,10^{20}}{4\,10^{5}} \sim 10^{15}\ \text{cm}^2\text{s}^{-1}$$

Ionized gas: $\nu \sim 1 - 10\ \text{cm}^2\text{s}^{-1}$





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Cool disks

Gas pressure is unimportant in cool disks (cool: $T \ll T_{\rm vir}$)

Equation of motion in potential of a point mass

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{\rho}\nabla P - \frac{GM}{r^2}\hat{\mathbf{r}}$$

fluid velocity   gas density   gas pressure   gravity

'typical' scales of quantities:

length: arbitrary distance r_0

time: Kepler time scale $t_0 = \Omega_0^{-1} = (\frac{r_0^3}{GM})^{1/2}$

velocity: Kepler velocity $v_0 = \Omega_0 r_0$

Isothermal gas (assume): $\nabla P = \mathcal{R}T\nabla\rho$

write $\mathbf{v} = \tilde{\mathbf{v}}\, v_0,\ \mathbf{r} = \tilde{\mathbf{r}}\, r_0,\ t = \tilde{t}\, t_0$

$$\underbrace{\frac{\partial \tilde{\mathbf{v}}}{\partial \tilde{t}}}_{\mathcal{O}(1)} + \underbrace{\tilde{\mathbf{v}}\cdot\tilde{\nabla}\tilde{\mathbf{v}}}_{\mathcal{O}(1)} = -\frac{T}{T_{\rm vir}}\underbrace{\tilde{\nabla}\ln\rho}_{\mathcal{O}(1)} - \underbrace{\frac{\hat{\mathbf{r}}}{\tilde{\mathbf{r}}^2}}_{\mathcal{O}(1)} \qquad T_{\rm v} \equiv \frac{GM}{\mathcal{R}r}$$

gas pressure comes in with factor $T/T_{\rm v}$

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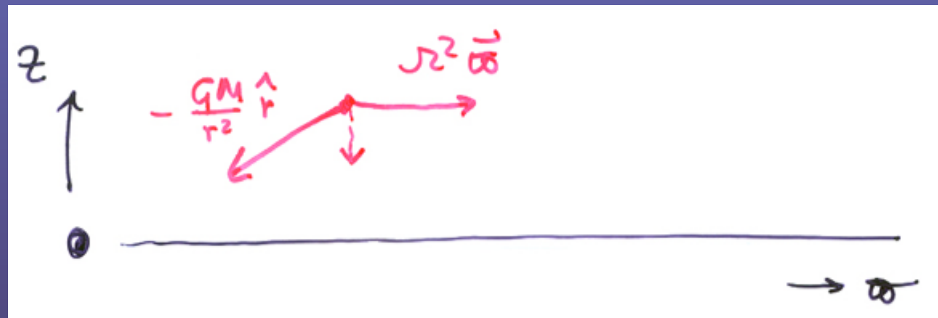
Thin disks

'disk aspect ratio'

Cool disks ($\dfrac{T}{T_{\rm vir}} \ll 1$) *are thin* ($\dfrac{H}{r} \ll 1$)

Calculate disk thickness for an 'isothermal' disk: $\partial_z T = 0$

coordinates:
cylindrical (ϖ, φ, z)



Forces on particle rotating with Kepler rate $\Omega_{\rm K}$ at ϖ :

$$\Omega_{\rm K}^2 \varpi = \frac{GM}{\varpi^2} \quad (z=0)$$

$$g_z \approx \Omega^2\varpi - \frac{GM}{r^2}\hat{\mathbf{r}} \approx -\Omega_{\rm K}^2 z + \mathcal{O}(z^3) \qquad z/\varpi \ll 1$$

$$\frac{{\rm d}P}{{\rm d}z} = g_z\rho \;\rightarrow\; \frac{\mathcal{R}T}{\mu}\frac{{\rm d}\rho}{{\rm d}z} = -z\Omega_{\rm K}\rho \;\rightarrow\; \frac{{\rm d}\ln\rho}{{\rm d}z} = -z\frac{\mu\Omega_{\rm K}^2}{\mathcal{R}T} \;\rightarrow\; \rho = \rho_0\exp[-\frac{z^2}{2H^2}]$$

where H nominal disk thickness $H = (\dfrac{\mathcal{R}T}{\mu\Omega_{\rm K}})^{1/2} = \dfrac{c_{\rm si}}{\Omega_{\rm K}}$

(isothermal sound speed: $c_{\rm si} = (\mathcal{R}T/\mu)^{1/2}$)

Consequences of thin disk approximation:

$\partial_r P$ negligible

1 radial equation of motion: $\Omega^2 r = \dfrac{GM}{r^2}$ (circular Kepler orbits)

Internal energy $u = P/\rho$:

2 → advection of internal energy negligible

→ viscous dissipation is radiated away locally:

$$Q_{\rm visc} = \rho\nu(r\partial_r\Omega)^2 = \quad (\frac{3}{2}\Omega)^2\nu\rho = {\rm div}\,\mathbf{F}_{\rm rad}$$

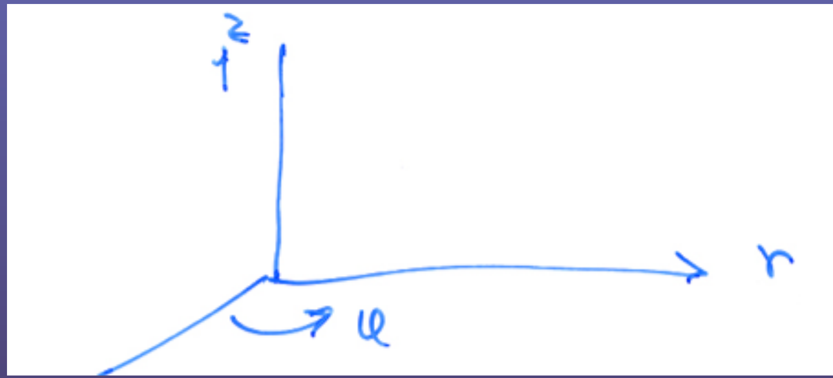
$$\int {\rm d}z\ {\rm div}F = \text{surface flux} \quad \rightarrow \quad \frac{9}{4}\Omega^2\int_{-\infty}^{\infty}\nu\rho{\rm d}z = 2\sigma T_{\rm s}^4$$

[note: this is not equal to the local gravitational energy release. Radial energy flux associated with viscous torques t.b. accounted for]

3 All unknown physics enters through viscosity ν

Thin axisymmetric viscous disks

Frank, King & Raine Accretion power Astrophsyics
Pringle, J.E., 1981, Ann rev. Astron. Astrophys.



midplane of disk at $z = 0$

surface mass density: $\Sigma = \int_{-\infty}^{\infty} \rho \, \mathrm{d}z$

radial (accretion-)velocity v_r
continuity (mass conservation): $\partial_t(r\Sigma) + \partial_r(r\Sigma v_r) = 0$
radial equation of motion: $v_\varphi^2 = \Omega^2 r^2 = GM/r$ (Kepler orbits)
azimuthal equation of motion:

Angular momentum

$$\partial_t(r\Sigma\overline{\Omega r^2}) + \partial_r(r\Sigma v_r\overline{\Omega r^2}) = \partial_r(r^2\Sigma\nu\partial_r\Omega)$$

local rate of change of angular momentum

'advection' of angular momentum

viscous torque = viscous angular momentum flux

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Viscous disk theory

$$\partial_t(r\Sigma) + \partial_r(r\Sigma v_r) = 0 \quad (1)$$

$$\partial_t(r\Sigma\Omega r^2) + \partial_r(r\Sigma v_r\Omega r^2) = \partial_r(r^2\Sigma\nu\partial_r\Omega) \quad (2)$$

(2) $-\Omega r^2$ (1) $\rightarrow$ $r\Sigma v_r\partial_r(\Omega r^2) = \partial_r(r^2\nu\Sigma r\partial_r\Omega)$ (3)

Accretion rate: $\dot{M} = -2\pi r\Sigma v_r$

$\Omega \sim r^{-3/2}$ (Kepler) $\rightarrow$ $\dot{M} = 6\pi r^{1/2}\partial_r(r^{1/2}\nu\Sigma)$ (4)

(2) + (3) $\rightarrow$ $r\partial_t\Sigma = 3\partial_r[r^{1/2}\partial_r(\nu\Sigma r^{1/2})]$

(thin disk evolution equation)

- all relevant physics condensed in the viscosity ν
- diffusion equation: *viscous spreading*

Steady thin disks

$$\partial_t = 0 \;\rightarrow\; \dot{M} = \mathrm{cst.} \qquad \dot{M} = 6\pi r^{1/2}\partial_r(r^{1/2}\nu\Sigma)$$

(For accretion: $\dot{M} > 0$*)*

(4) $\rightarrow \; r^{1/2}\nu\Sigma = \dfrac{\dot{M}}{3\pi}r^{1/2} + c_2$ (c_2 : integration cst)

equivalent: $\nu\Sigma = \dfrac{\dot{M}}{3\pi}[1 - \beta(\dfrac{r_\mathrm{i}}{r})^{1/2}]$ (3) (r_i: inner edge of the disk)

Interpretation: Angular momentum flux (= 'torque') F_J

$$F_J = \dot{M}\Omega r^2 + 2\pi r^2\nu\Sigma r\partial_r\Omega$$

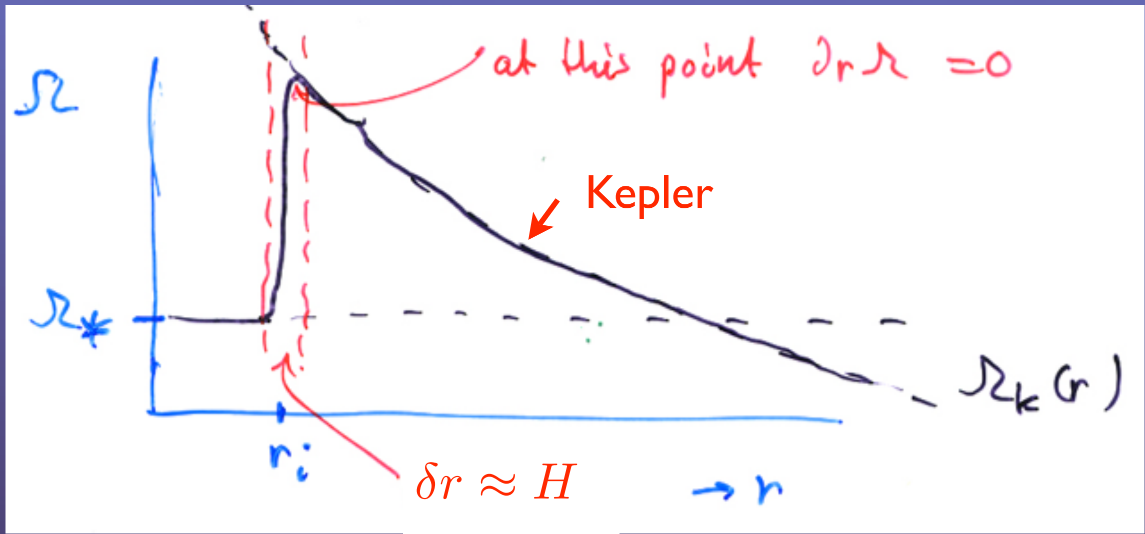
$r\partial_r\Omega = -\dfrac{3}{2}\Omega$ (Kepler) + (3) :

$$F_J = \beta\dot{M}\Omega(r_\mathrm{i})r_\mathrm{i}^2 \;\; (= (GMr_\mathrm{i})^{1/2})$$

→ β *is the angular momentum flux through the disk, in units of the angular momentum advected with* $\dot{M}$*, at* r_i

Steady thin disks

Thin disk solution for accretion onto a "slowly rotating" object



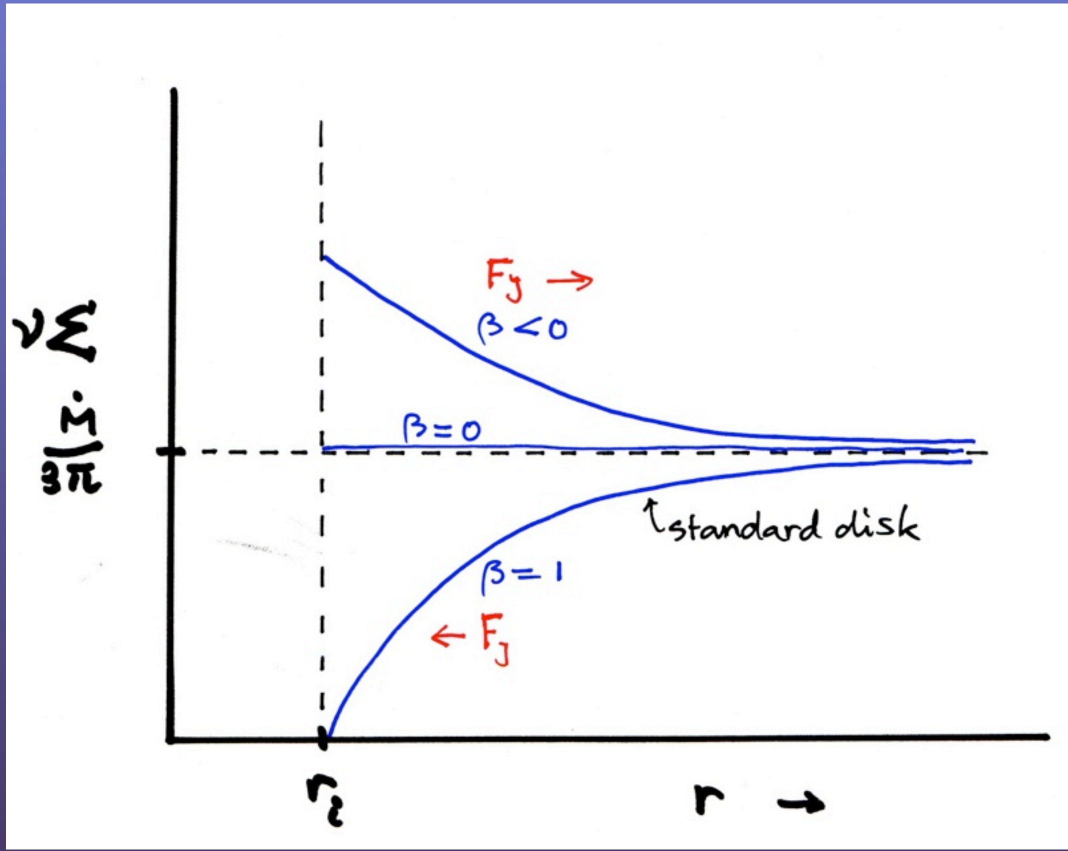
at r_i $\partial_r \Omega = 0$: no shear, no viscous stress

$$\rightarrow F_J = \dot{M}\Omega(r_\mathrm{i}) r_\mathrm{i}^2 \rightarrow \beta = 1 \quad \rightarrow \nu\Sigma = \frac{\dot{M}}{3\pi}[1 - (\frac{r_\mathrm{i}}{r})^{1/2}]$$

Standard steady thin disk result.
Holds (to lowest order in H/r) for all stars rotating with $\Omega_* < \Omega_\mathrm{K}(r_\mathrm{i})$ (i.e. all stars)

Exceptions : 1 stars rotating near maximum
2 stars with a magnetosphere

Surface density profile, relation with angular momentum flux



$\beta = 1$

'standard disk': accretion on slow rotator

$0 < \beta \leq 1$

angular momentum flux inward ('spinup')

$\beta = 0$

zero net angular momentum flux

$$\nu\Sigma = \frac{\dot{M}}{3\pi}[1 - \beta(\frac{r_i}{r})^{1/2}]$$

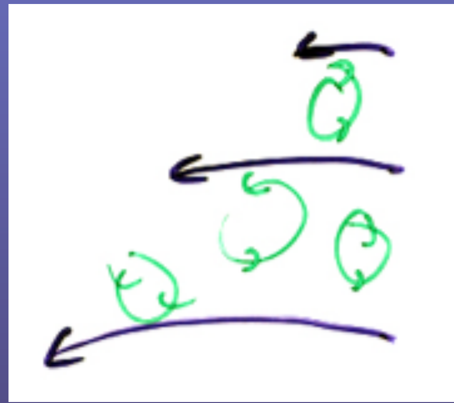
$\beta < 0$

outward angular momentum flux ('spindown'): *in accretion on a star with a strong magnetic field*

Source of viscosity: a limit

'Molecular' (microscopic) viscosity too low by many orders of magnitude

"Turbulence" (Laplace)



assume: hydrodynamic instability generates turbulence

"reason": Reynolds number:

$$\mathrm{Re} = \frac{LV}{\nu_{\mathrm{mic}}} = \frac{r_{\mathrm{disk}}^2 \Omega_{\mathrm{K}}}{\nu_{\mathrm{mic}}} \gg 1$$

(CV: $\mathrm{Re} \sim 10^{14}$)

Upper limit on turbulent viscosity: causality

speed of sound limits physical connection in radial direction

shear rate $\eta \equiv |\partial_r v| \sim \Omega_{\mathrm{K}}$

Eddy velocity $< c_{\mathrm{s}}$:

$$V = L\eta < c_{\mathrm{s}} \;\rightarrow\; L < c_{\mathrm{s}}/\Omega = H$$

1: $\nu \approx LV = L^2\eta < c_{\mathrm{s}}^2/\Omega$

2: 'vortices' in disks have radial length $L < H$

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hydrodynamic turbulence?

Is the shear flow in a thin accretion disk turbulent?

Earth-based intuition: $\mathrm{Re} = 10^{14}$ will be unstable & turbulent
Ap intuition: Kepler orbits are stable

Evidence.

- Laboratory experiments (rotating Couette flow):
  conflicting claims.
  Recent result: Princeton experiment at $\mathrm{Re} = 10^{6}$
  No angular momentum transport seen *when minimizing boundary effects*
  (see http://www.cmso.info/cmsopdf/general_aug06/Talks/Talks-pdf/Schartman.pdf)
- Numerical simulations
  Expected shear turbulence so far not seen

Major ideological consequences if confirmed: high Re may not be sufficient for turbulence or even instability.

De facto Ap attitude: *we have something better* ('MRI', magnetic turbulence)