# The Evolution of Binary Systems

Philipp Podsiadlowski (Oxford)

- the majority of stars are in binary systems

$\rightarrow$ major cause of 'unusual' stellar systems

**Lecture 1: Fundamentals of Binary Evolution**

**Lecture 2: Current Topics in Binary Evolution**

- ▷ Short-Period Subdwarfs
- ▷ Symbiotic Binaries
- ▷ Ultracompact Binaries

**Lecture 3: Late Stellar Evolution and Supernovae in Binaries**

- ▷ The Progenitors of SNe Ia
- ▷ Gamma-Ray bursts

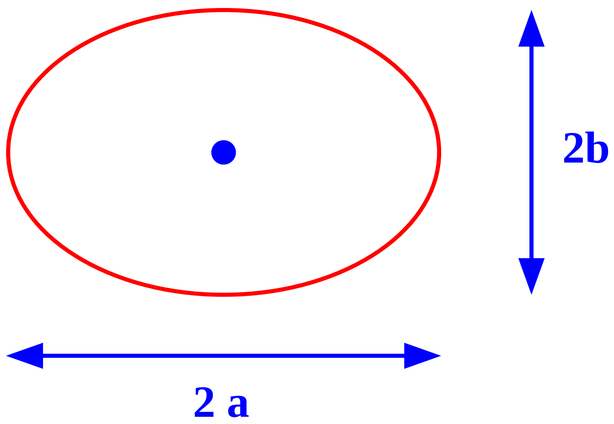
**Lecture 4: Low-mass X-Ray Binaries and Millisecond Pulsars**

**Lecture 5: High-Mass X-Ray Binaries**

- ▷ Double Neutron-Star Binaries
- ▷ Thorne-Żytkow Objects

# BINARY STARS

- most stars are members of binary systems or multiple systems (triples, quadruples, quintuplets, ...)
- orbital period distribution: $P_{orb} = 11\,\text{min to} \sim 10^7\,\text{yr}$
- the majority of binaries are wide and do not interact strongly
- close binaries (with $P_{orb} \lesssim 10\,\text{yr}$) can transfer mass $\rightarrow$ changes structure and subsequent evolution
- approximate period distribution: $f(\log P) \simeq \text{const.}$ (rule of thumb: 10 % of systems in each decade of log P from $10^{-3}$ to $10^7$ yr; 50 % less than 100 yr)
- generally large scatter in distribution of eccentricities $e^2 \equiv 1 - b^2/a^2$, a = semi-major, b = semi-minor axis




- systems with eccentricities $\lesssim 10\,\text{d}$ tend to be circular $\rightarrow$ evidence for tidal circularization

## Massive Binaries

- essentially all O stars are in close binaries
- masses are correlated (many stars of comparable masses)
- many are relatively close triples
  - ▷ third star can drive binary evolution
  - ▷ possibility of double interactions between inner binary and outer component (1 %?)

## Low-Mass Binaries

- lower binary frequency (e.g. M dwarfs)
- masses less correlated

## Metallicity Dependence?

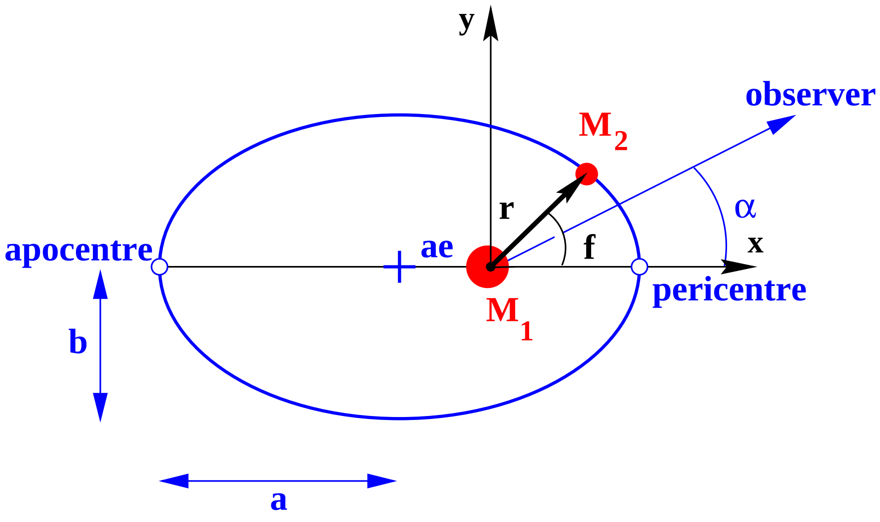
unknown question: are binary properties metallicity dependent?

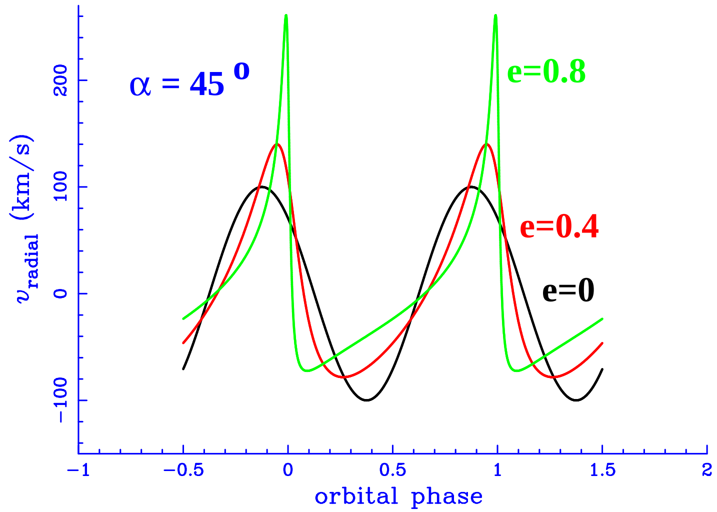
- depends on poorly understood binary formation process
- has implications, e.g., for X-ray binaries, SN and GRB progenitors, UV excess in elliptical galaxies

## Classification

- **visual binaries:** see the periodic wobbling of two stars in the sky (e.g. Sirius A and B); if the motion of only one star is seen: astrometric binary
- **spectroscopic binaries:** see the periodic Doppler shifts of spectral lines
  - ▷ **single-lined:** only the Doppler shifts of one star detected
  - ▷ **double-lined:** lines of both stars are detected
- **photometric binaries:** periodic variation of fluxes, colours, etc. are observed (caveat: such variations can also be caused by single variable stars: Cepheids, RR Lyrae variables)
- **eclipsing binaries:** one or both stars are eclipsed by the other one $\rightarrow$ inclination of orbital plane $i \simeq 90^\circ$ (most useful for determining basic stellar parameters)

## Radial Velocity (eccentric binaries)




for an eccentric binary

$$x(t) = a\,(\cos E - e) \qquad \tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\;\tan\frac{E}{2}$$
$$y(t) = b\,\sin E$$

where the eccentric anomaly E is defined by Kepler's equation

$$E - e\sin E = \frac{2\pi}{P}t = M \quad \text{(mean anomaly)}$$

## Eccentric Binaries

- consider a spectroscopic binary
- measure the radial velocity curve along the line of sight from $\frac{v_r}{c} \simeq \frac{\Delta\lambda}{\lambda}$ (Doppler shift)
- for an eccentric binary

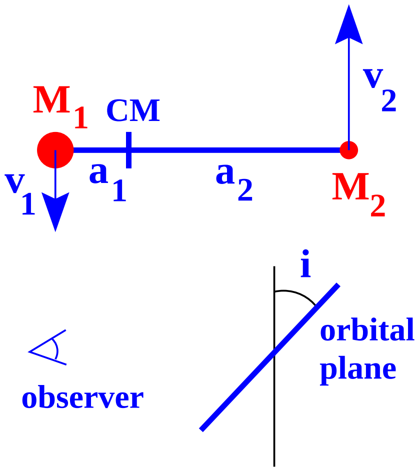
$$\begin{aligned} x(t) &= a\,(\cos E - e) \\ y(t) &= b\,\sin E \end{aligned}$$

▷ where the eccentric anomaly is defined by Kepler's equation

$$E - e\sin E = \frac{2\pi}{P}t \quad = M \quad \text{(mean anomaly)}$$

# THE BINARY MASS FUNCTION

- consider a spectroscopic binary
- measure the radial velocity curve along the line of sight from $\frac{v_r}{c} \simeq \frac{\Delta\lambda}{\lambda}$ (Doppler shift)




▷ $M_1 a_1 = M_2 a_2$

▷ $P = \frac{2\pi}{\omega} = 2\pi \frac{a_1 \sin i}{v_1 \sin i} = 2\pi \frac{a_2 \sin i}{v_2 \sin i}$

▷ gravitational force = centripetal force

$$\rightarrow \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_1 \sin i)^2}{a_1 \sin^2 i} M_1, \quad \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_2 \sin i)^2}{a_2 \sin^2 i} M_2$$

substituting

$(a_1 + a_2)^2 = a_1^2 (M_1 + M_2)^2 / M_2^2,$ etc.

$$\rightarrow \quad f_1(M_2) = \frac{M_2^3 \sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_1 \sin i)^3}{2\pi G}$$

$$f_2(M_1) = \frac{M_1^3 \sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_2 \sin i)^3}{2\pi G}$$

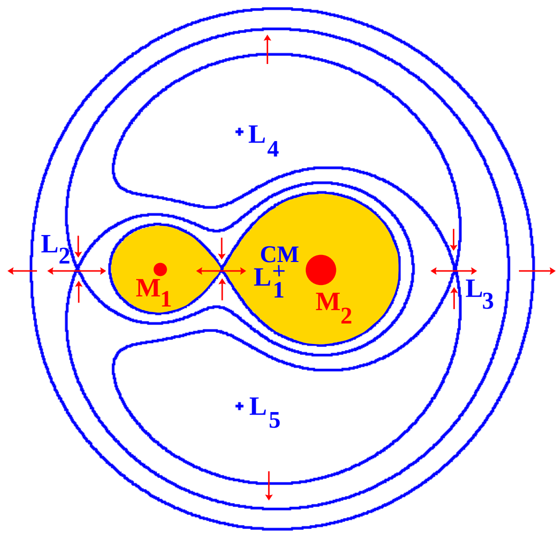
- $f_1$, $f_2$ mass functions: relate observables $v_1 \sin i$, $v_2 \sin i$, $P$ to quantities of interest $M_1$, $M_2$, $\sin i$
- measurement of $f_1$ and $f_2$ (for double-lined spectroscopic binaries only) determines $M_1 \sin^3 i$, $M_2 \sin^3 i$

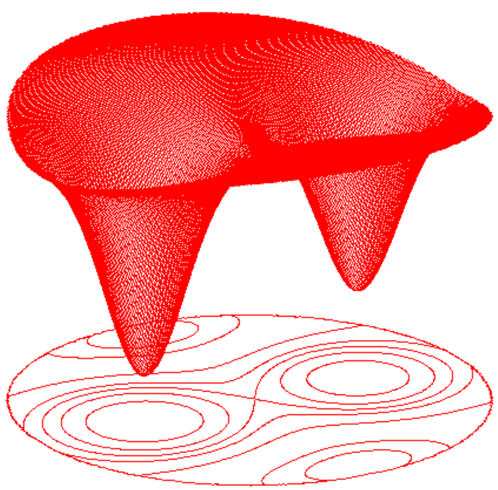
  ▷ if i is known (e.g. for visual binaries or eclipsing binaries) $\rightarrow$ $M_1$, $M_2$

  ▷ for $M_1 \ll M_2 \rightarrow f_1(M_2) \simeq M_2 \sin^3 i$ (measuring $v_1 \sin i$ for star 1 constrains $M_2$)

- for eclipsing binaries one can also determine the radii of both stars (main source of accurate masses and radii of stars [and luminosities if distances are known])

The Roche Potential



effective Roche-lobe radius (star 2):
$R_L = \frac{0.49}{0.6+q^{2/3}\ln(1+q^{-1/3})}\,A$, where $q = M_1/M_2$ is the mass ratio, A orbital separation.

## THE ROCHE POTENTIAL

- restricted three-body problem: determine the motion of a test particle in the field of two masses $M_1$ and $M_2$ in a circular orbit about each other
- equation of motion of the particle in a frame rotating with the binary $\Omega = 2\pi/P$:

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}\,U_{\rm eff} - \underbrace{2\vec{\Omega}\times\vec{v}}_{\text{Coriolis force}},$$

where the effective potential $U_{\rm eff}$ is

$$U_{\rm eff} = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \underbrace{\frac{1}{2}\Omega^2\left(x^2+y^2\right)}_{\text{centrifugal term}}$$

- Lagrangian points: five stationary points of the Roche potential $U_{\rm eff}$ (i.e. where effective gravity $\vec{\nabla}U_{\rm eff} = 0$)
  - ▷ 3 saddle points: $L_1$, $L_2$, $L_3$
- Roche lobe: equipotential surface passing through the inner Lagrangian point $L_1$ ('connects' the gravitational fields of the two stars)

## Classification of close binaries

- **Detached binaries:**
  - ▷ both stars **underfill their Roche lobes,** i.e. the **photospheres** of both stars lie **beneath** their respective Roche lobes
  - ▷ **gravitational interactions** only (e.g. tidal interaction, see Earth-Moon system)
- **Semidetached binaries:**
  - ▷ one **star fills its Roche lobe**
  - ▷ the Roche-lobe filling component **transfers matter** to the detached component
  - ▷ **mass-transferring binaries**
- **Contact binaries:**
  - ▷ **both stars fill or overfill their Roche lobes**
  - ▷ formation of a common photosphere surrounding both components
  - ▷ **W Ursae Majoris stars**

## The Algol Paradox

- Algol is an eclipsing binary with orbital period 69 hr, consisting of a B8 dwarf ($M = 3.7\,M_\odot$) and a K0 subgiant ($M = 0.8\,M_\odot$)
- the eclipse of the B8 star is very deep $\rightarrow$ B8 star more luminous than the more evolved K0 subgiant
- the less massive star is more evolved
- inconsistent with stellar evolution $\rightarrow$ Algol paradox
- explanation:
  - ▷ the K star was initially the more massive star and evolved more rapidly
  - ▷ mass transfer changed the mass ratio
  - ▷ because of the added mass the B stars becomes the more luminous component

# Binary Interactions

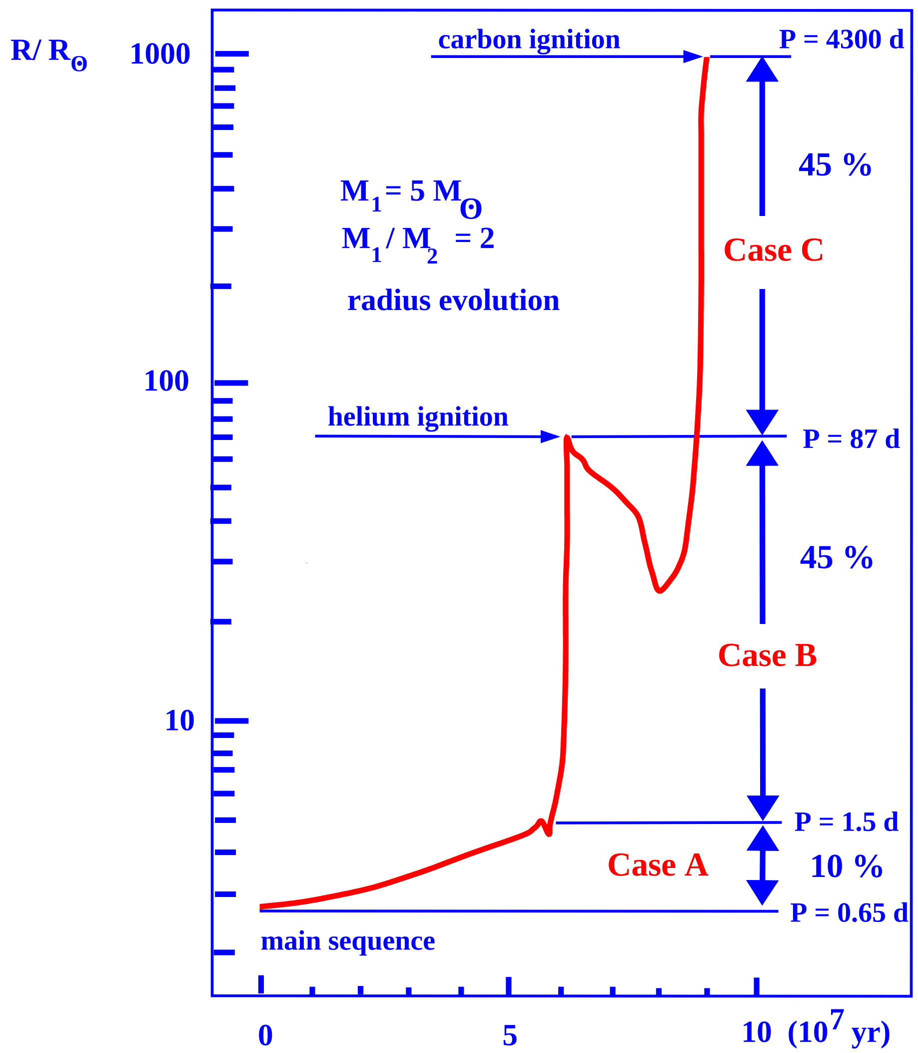
- a large fraction are members of interacting binaries $(30 - 50\,\%)$

  $(50\,\%$ of all stars are in binaries with $P_{\rm orb} < 100\,{\rm yr})$

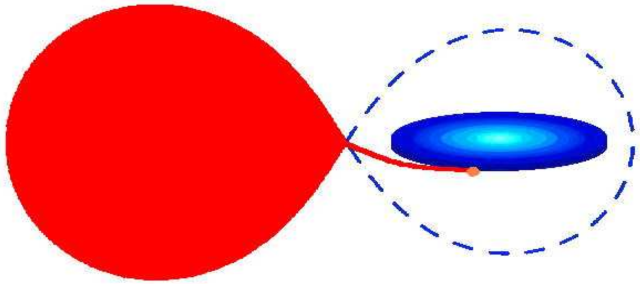
- **note:** mass transfer is more likely for post-MS systems
- binary interactions
  - ▷ common-envelope (CE) evolution
  - ▷ stable Roche-lobe overflow
  - ▷ binary mergers
  - ▷ wind Roche-lobe overflow

Classification of Roche-lobe overflow phases

(Paczynski)

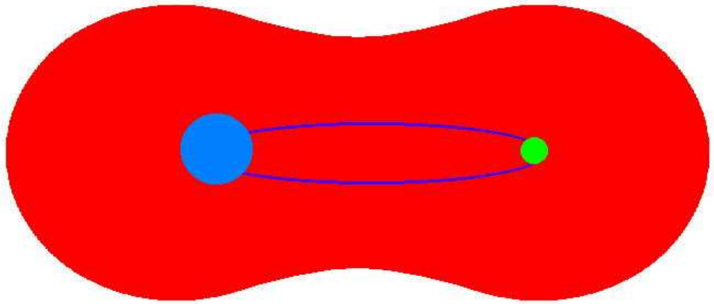


## Stable Mass Transfer



- mass transfer is 'largely' conservative, except at very mass-transfer rates
- mass loss + mass accretion
- the mass loser tends to lose most of its envelope → formation of helium stars
- the accretor tends to be rejuvenated (i.e. behaves like a more massive star with the evolutionary clock reset)
- orbit generally widens

## Unstable Mass Transfer



- dynamical mass transfer → common-envelope and spiral-in phase (mass loser is usually a red giant)
  - ▷ mass donor (primary) engulfs secondary
  - ▷ spiral-in of the core of the primary and the secondary immersed in a common envelope
- if envelope ejected → very close binary (compact core + secondary)
- otherwise: complete merger of the binary components → formation of a single, rapidly rotating star

# Binary Mergers

- one of the most important, but not well studied binary interactions
- BPS: $\sim 10\,\%$ of all stars are expected to merge with a companion star $\rightarrow$ 1 binary merger in the Galaxy every 10 yr!
- efficient conversion of orbital-angular momentum to spin orbital-angular momentum
- if mergers occur early in the evolution $\rightarrow$ subsequent spin-down just as for single stars
- need late mergers to affect the nearby CSM, get rapidly rotating progenitors (GRB progenitors?) (e.g. case C mass transfer)

Merger candidates: SN 1987A, FK Comae, V Hyd, B[e] supergiants [R4], Sher 25, HD168625, Car, V838 Mon.

## Mass-Transfer Driving Mechanisms

- mass transfer is driven either by the expansion of the mass donor or because the binary orbit shrinks due to angular momentum loss from the system
- expansion of the donor:
  - ▷ due to nuclear evolution ("evolutionary driven mass transfer"; then $\dot{M} \sim M/t_{\rm nuclear}$) or
  - ▷ non-thermal-equilibrium evolution ("thermal timescale mass transfer"; then $\dot{M} \sim M/t_{\rm KH}$)

  conservative mass transfer:
  - ▷ total angular momentum of binary:

    $$J = \frac{M_1 M_2}{M_1 + M_2} \underbrace{\sqrt{G(M_1 + M_2)\,A}}_{\text{specific angular momentum}}$$

    (A: orbital separation)
  - ▷ if J, $M_1 + M_2$ conserved $\rightarrow (M_1 M_2)^2\,A = \text{constant}$ (implies minimum separation if $M_1 = M_2$)

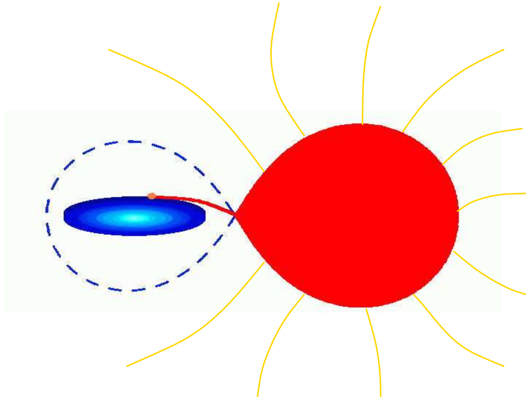
## angular momentum loss from the system:

### gravitational radiation:

- ▷ effective for $P_{orb} \lesssim 12\,hr$

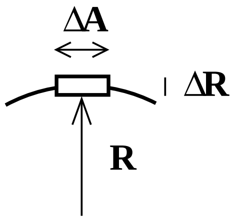
### magnetic braking

- ▷ red dwarf loses angular momentum in magnetic wind
- ▷ tidal locking of secondary
- ▷ extracts angular momentum from orbit



## The Eddington Limit

- **Definition:** the maximum luminosity for which the gravitational force on a fluid element exceeds the radiation pressure force (i.e. the maximum luminosity at which matter can be accreted)




▷ fluid element with cross section $\Delta A$ and height $\Delta R$ at a distance $R$ from the centre of gravity of mass $M$,

- the (inward) gravitational force on the element is

$$F_{grav} = \underbrace{-\frac{GM}{R^2}}_{\text{gravity}} \quad \underbrace{\Delta A \Delta R}_{\text{mass}}$$

- the (outward) radiative force on the element (due to the deposition of momentum by photons absorbed or scattered): $F_{rad} = \underbrace{\frac{L}{4\pi R^2\, c}\Delta A}_{\text{momentum flow}} \quad \underbrace{\kappa\rho\, \Delta R}_{\text{momentum "deposited"}}$

- maximum luminosity: $F_{grav} + F_{rad} = 0$ and solving for $L$ then yields $\boxed{L_{edd} = \frac{4\pi GMc}{\kappa}}$

- for Thomson scattering in a solar-type plasma ($\kappa = 0.034\,\mathrm{m^2\,kg^{-1}}$), $L_{edd} \simeq 3.8 \times 10^4\,L_\odot\ (M/M_\odot)$.

### Eddington accretion rate (maximum accretion rate)

- if the luminosity is due to accretion luminosity (i.e. gravitational energy release) $L_{grav} = GM\dot{M}/R$, where $R$ is the inner edge of the accretion flow, equating $L_{edd} = L_{grav}$: $\boxed{\dot{M}_{edd} = \dfrac{4\pi c R}{\kappa}}$
- for the Sun, $\dot{M} \simeq 10^{-3}\,M_\odot\,\mathrm{yr^{-1}}$
- For a neutron star, $\dot{M} \simeq 1.8 \times 10^{-8}\,M_\odot\,\mathrm{yr^{-1}}$

### The Response of the Accreting Star

- if the accretion timescale ($t_{acc} \equiv M/\dot{M}$) is shorter than the envelope thermal timescale
  - ▷ star swells up (may fill its Roche lobe)
- on main sequence, rejuvenation: star behaves like a more massive star
- post-main sequence
  - ▷ core mass fixed → different structure → favours more compact (blue) subsequent evolution (blue supernova progenitors)