Galaxy Dynamics for the Milky Way

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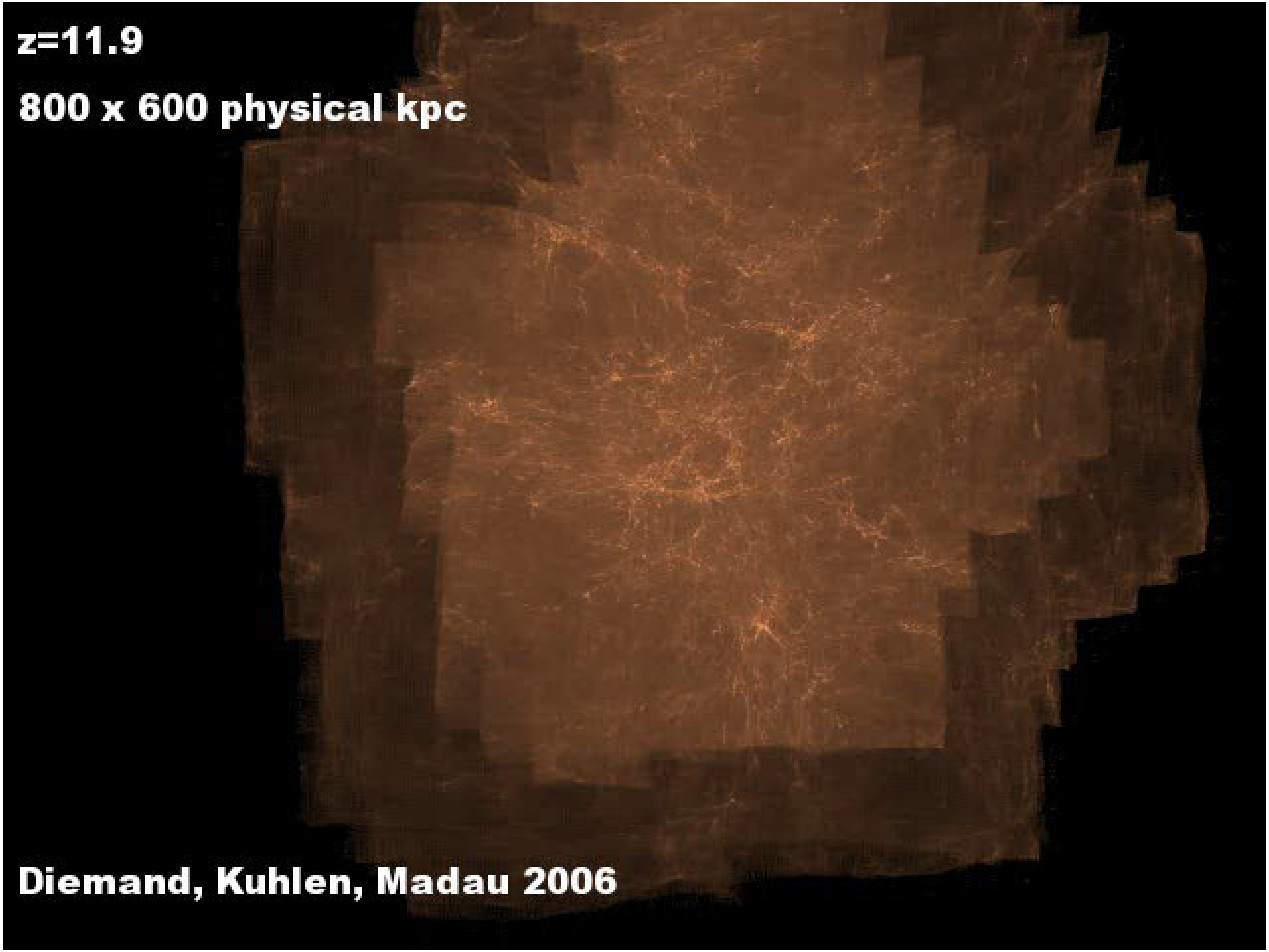
Lectures I + II

Describing them
Modelling them
Interpreting them

For more details on almost anything here, go to:
Binney & Tremaine 2nd Edition (still a right of passage)


z=11.9
800 x 600 physical kpc
Diemand, Kuhlen, Madau 2006

z: 49.5

Some conceptual basics:

- Constituents: Dark matter + Stars + Gas (+dust)
- Dynamical behaviour
  - Gas is dissipational:
    - can get heated: compression, shocks, radiation
    - Energy loss through radiation → disks
    - $T_{gas} \sim T_{virial} \sim 10^{5-6}$ K → hydrostatic equilibrium
    - $T_{gas} \ll T_{virial}$ → non-intersecting orbits → disks
  - Stars (+DM) are collisionless:
    - Stars know only about the potential (not other stars)

Some conceptual basics:

- What's 'dynamical model' for a galaxy?
  - A simultaneous description of the
    - gravitational potential $\Phi(r,t)$
    - (orbit) description of the 'tracers' that move within that potential.
  - 'Self-consistency' is merely a limiting case, where

    the stars/tracers that we observe generate the gravitational potential in which they orbit

- → Always consider separately:

  Kinematic tracers ←→ gravitational potential

How to describe stellar dynamical systems?

- On scales of galaxies, dynamics of stars are independent of their individual masses
  → consider set of N identical (mass) stars
- Then an instantaneous description of the stellar kinematics is given by the distribution function (DF), i.e. the probability of finding a star in a *phase-space* element (*x*,*v*),

$$f(\mathbf{x},\mathbf{v},t)\,\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{v}$$

Boltzmann Equation
phase-space density conservation

- In Lagrangian coordinates, where the coord. systems follows the stars: **df/dt =0.**
  - Merely probability conservation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \dot{\mathbf{w}} \cdot \frac{\partial f}{\partial \mathbf{w}}$$

with $\mathbf{w} = (\mathbf{x}, \mathbf{v})$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0.$$

(collisionless) Boltzmann Equation

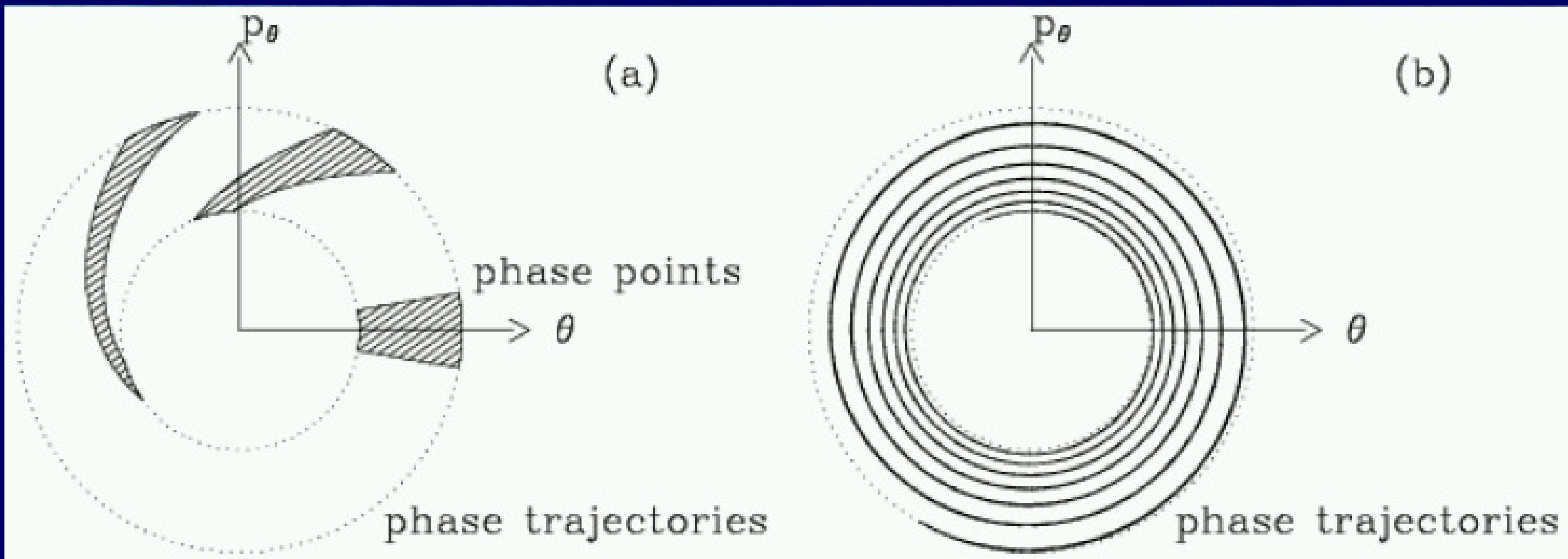
- The equation is valid when
  - Gravitational potential is independent of position of the individual particles (collisionless)
  - No symmetry or steady-state required

The promise of the collisionless Boltzmann Equation

- If phase space density is conserved, then the present-day phase-space distribution -which stars are on which 'orbits' - reflects the distribution at (much) earlier epoch
  → galactic archeology

- How can we make that work in practice?

Phase-mixing vs. fine-grain phase-space conservation



While the Boltzmann Equation always holds, any measurement of phase-space density (in dx,dc space) over a non-infinitesimal volume will always decrease.

What use is the Boltzmann Equation?

- Not much in its $\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0.$ form
  - holds only for infinitesimal phase-space volumes
  - Any finite volume: phase-mixing
    → *coarse-grained* DF is not conserved!
- Choose appropriate coordinates for DF
  - (isolating) integrals of motion
  - Action-angle variables
- Take moments of the Boltzmann Equation
  → Jeans Equation → virial theorem

I. Describing Stellar Dynamical Systems in Equilibrium

Modeling Collisionless Matter I: Jeans Equation

We have the **Boltzmann Equation** (= phase space continuity equation)
It says: if I follow a particle on its gravitational path (=Lagrangian derivative) through phase space, it will always be there.

$$\frac{Df(\vec{x},\vec{v},t)}{Dt} = \frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \vec{\nabla}\Phi_{grav}\frac{\partial f}{\partial \vec{v}} = 0$$

A rather ugly partial differential equation!

Note: we have substituted gravitational force for accelaration!

To simplify it, one takes velocity moments:
i.e. $\int_{\mathbb{R}^3} \ldots v^n d^3v$ n = 0,1, ... on both sides

Moments of the Boltzmann Equation

0th Moment $$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$ **mass conservation**

ρ: tracer (!) density; v/u: indiv/mean particle velocity

1st Moment $\int \ldots v_j d^3v$

$$\frac{\partial}{\partial t}(\rho \vec{u}) + \vec{\nabla} \cdot \left(\rho \left(\underline{\underline{T}} + \vec{u} \cdot \vec{u}\right)\right) + \rho \vec{\nabla} \Phi = 0$$

$$\text{with } \rho \underline{\underline{T}} = \int f \cdot (v_i - u_i)(v_j - u_j)\, d^3v$$

this is the "Jeans Equation"

Virial Theorem

Consider for simplicity the one-dimensional analog of the Jeans Equation in steady state:

$$\frac{\partial}{\partial x}\left[\rho v^2\right] + \rho \frac{\partial \Phi}{\partial x} = 0$$

After integrating over velocities, let's now

integrate over : $\vec{x}$ $\int \ldots x d\vec{x}$

[one needs to use Gauss' theorem etc..]

$$-2E_{kin} = E_{pot}$$

Application of the Jeans Equation

- Goal:
  - Avoid "picking" right virial radius.
  - Account for spatial variations
  - Get more information than "total mass"

- Simplest case
- spherical: $\rho(\vec{r}) = \rho(r)$

  static: $\vec{v} \equiv 0, \frac{\partial}{\partial t} \equiv 0$

$$\vec{\nabla}\left(\rho \underline{\underline{T}}\right) = -\rho \vec{\nabla} \Phi$$

Choose spherical coordinates: $$\frac{d}{dr}\left(\rho\sigma_r^2\right) + \frac{2\rho}{r}\left(\sigma_r^2 - \sigma_t^2\right) = -\rho\frac{d\Phi}{dr}$$

σ_r is the radial and σ_t the tangential velocity dispersion

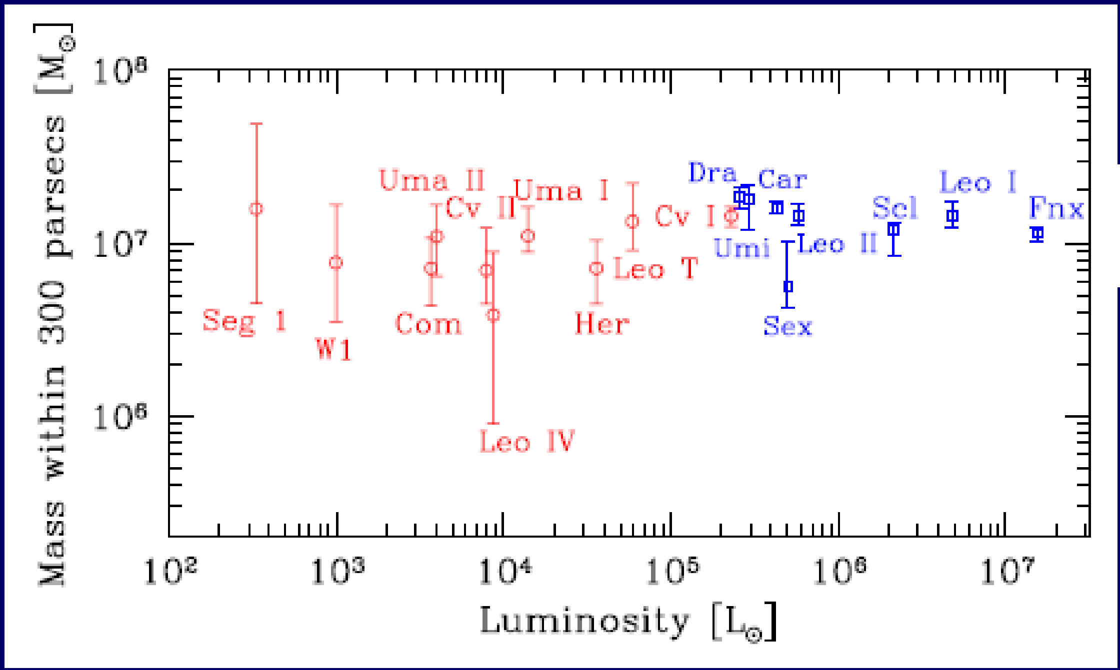
$$\frac{d}{dr}\left(\rho\sigma_r^2\right) = -\rho\frac{d\Phi}{dr}$$

for the „isotropic" case! („isotropy" is assumption, not Physics here)

An Example: When Jeans Equation Modeling is Good Enough:

The Masses within 300pc of Faint Milky Way Satellites

Strigari et al 2008



$$r\frac{d(\rho_\star \sigma_r^2)}{dr} = -\rho_\star(r)\frac{GM(r)}{r} - 2\beta(r)\rho_\star\sigma_r^2.$$

Anisotropy $\beta = 1 - \sigma_\theta^2(r)/\sigma_r^2(r)$

Density of the Tracers $\rho_{\rm pl}(r) = \frac{\rho_0}{[1+(r/r_{\rm pl})^2]^{5/2}}$

+p($V_{l.o.s.}$) of 10-50 stars

DM density creating the potential $\rho(r) = \frac{\rho_0}{(r/r_0)^a[1+(r/r_0)^b]^{(c-a)/b}}$

Which of these can lead to a match to the data?

Orbits and Integrals of Motion

- (Isolating) Integrals of Motion (IOM):
  - Are quantities *I(x,v)* that are conserved along the orbit

$$\frac{\mathrm{d}}{\mathrm{d}t} I[\mathbf{x}(t), \mathbf{v}(t)] = 0$$

  - Isolating: integrals that differentiate among orbits

- Examples of integrals (that can be 'written down')
  - Orbital energy, E, in any time-independent Φ
  - Angular momentum, L, in spherical symmetry
  - One component of L, e.g. L_z, in axisymmetry

What use are Integrals of Motion?

- Jeans Theorem: any function of the IOM is a solution of the Boltzmann Equation

$$\frac{\mathrm{d}}{\mathrm{d}t} I[\mathbf{x}(t), \mathbf{v}(t)] = 0 \quad \Rightarrow \quad \frac{\mathrm{d}I}{\mathrm{d}t} = \mathbf{v} \cdot \frac{\partial I}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial I}{\partial \mathbf{v}} = 0.$$

  – Compare to

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0.$$

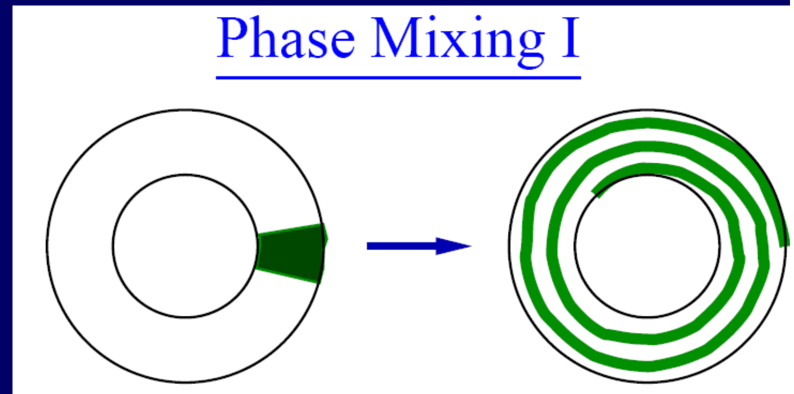
Boltzmann Equation

- **Or, any steady state solution to the Boltzmann Eq. can only depend on IOMs**
- Solutions can be specified as (time-independent) functions of (generally 3) integrals of motion

Integrals of Motion and Conservation of phase-space structure

In stationary, or slowly-varying potentials:

- Sub-structure is phase-mixed in 'real space'
- Phase-space density (e.g. E,L_z space) is conserved
  → dynamics basis of 'galactic archeology'



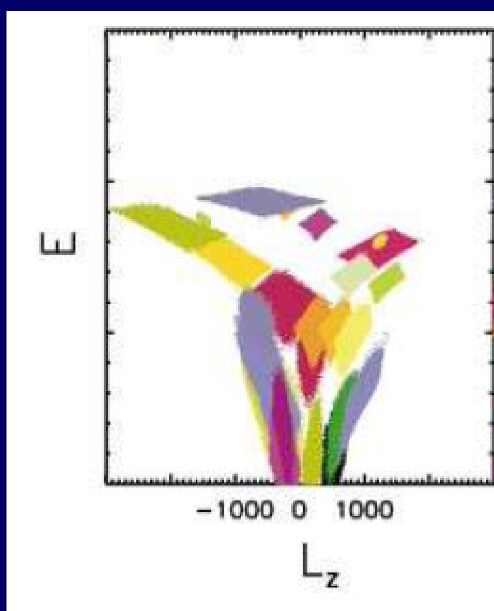
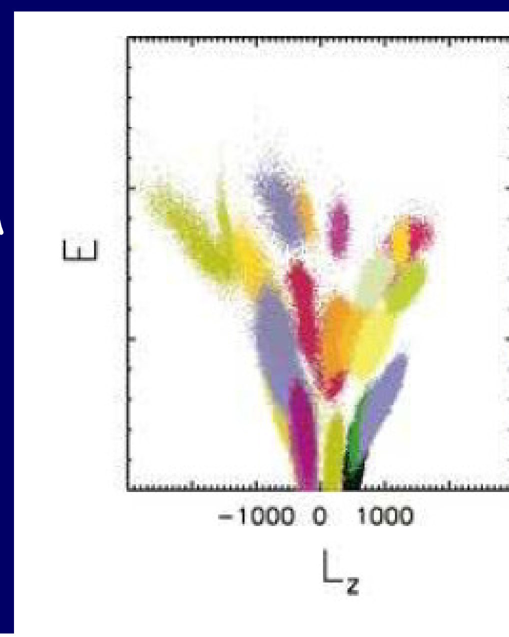


Figure 3. Initial distribution of partic

Initial clumps in phase-space →. observed 10Gyrs later with tha GAIA satellite



Notes on Integrals of Motion

- The problems of phase-mixing (that bedeviled the dxdv description of the DF) disappear when considering a DF-f(I.o.M.)
- I.o.M.s are not directly observable, need *X, V* (observable) and $\Phi(r)$ [which is desirable, but not observable]
- Some I.o.M. are preserved in slowly varying potentials (*adiabadic invariance* , e.g. angular momentum), others shift coherently…
- **Rapid and strong** potential changes mess with the I.o.M

Describing Collisionless Systems: Approach II

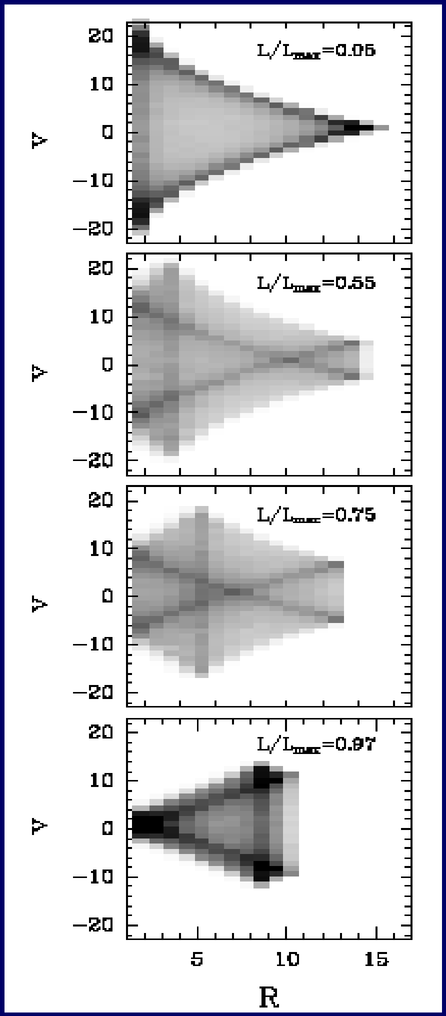
"Orbit-based" Models

Schwarzschild Models (1978)

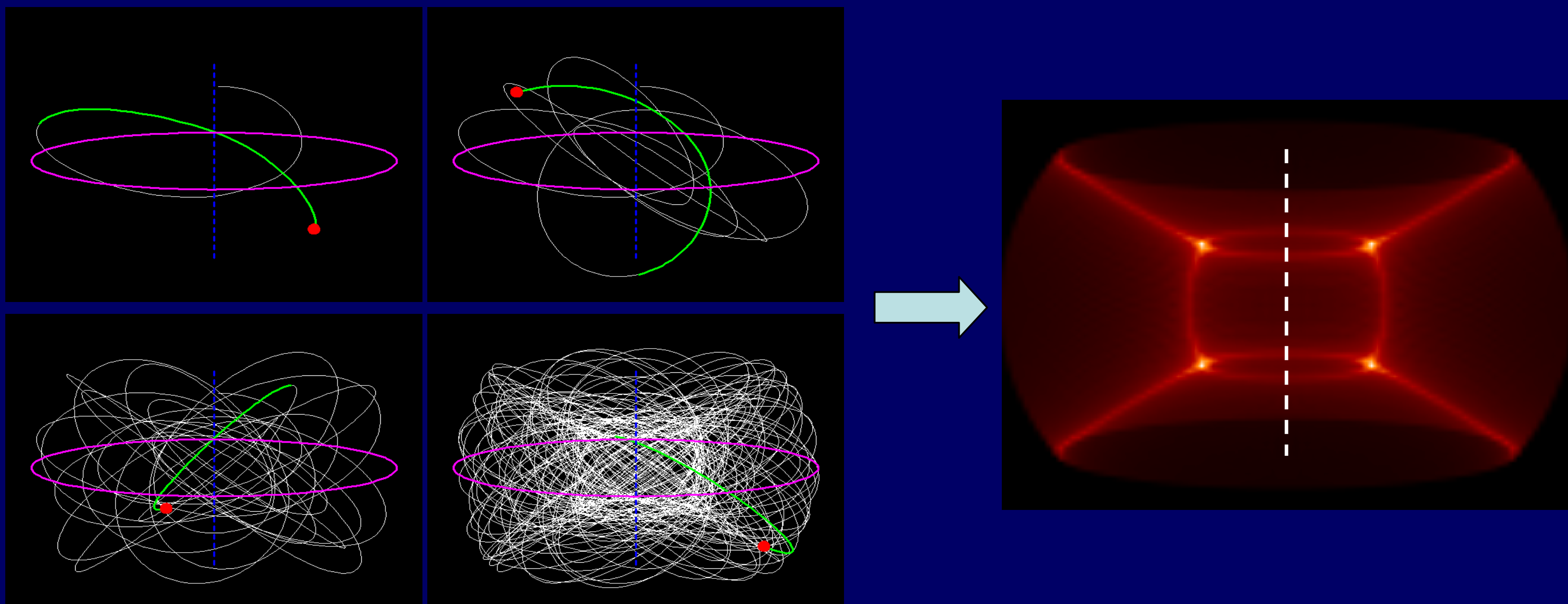
- What would the galaxy look like, if all stars were on the same orbit?
  - pick a potential Φ
  - Specify an orbit by its "isolating integrals of motion", e.g. E, J or J_z
  - Integrate orbit to calculate the
    - time-averaged
    - projected

    properties of this orbit

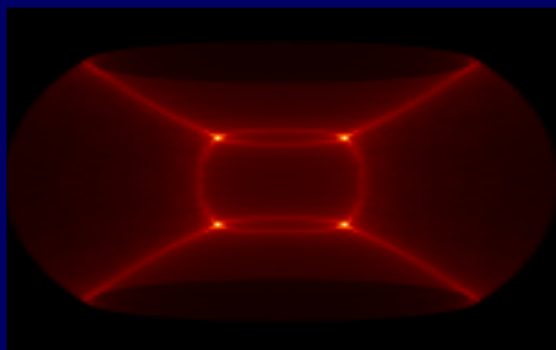
    (NB: time average in the calculation is identified with ensemble average in the galaxy at on instant)
  - Sample "orbit space" and repeat



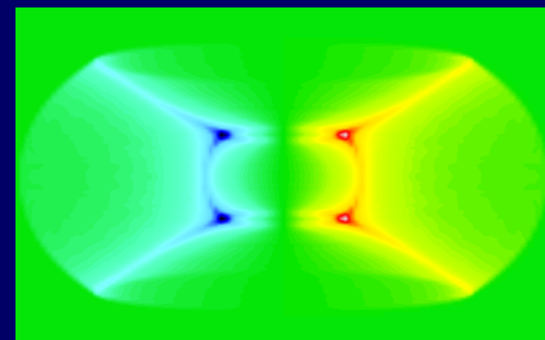
from Rix et al 1997



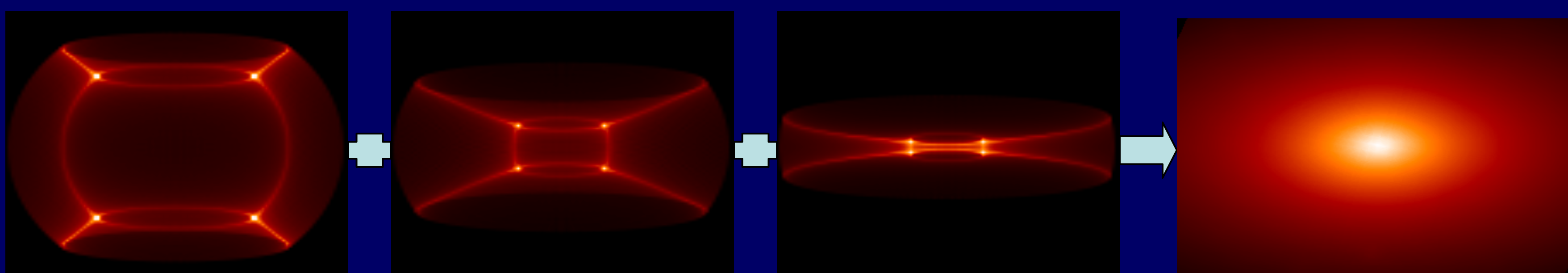
Figures courtesy Michele Capellari 2003



Projected density

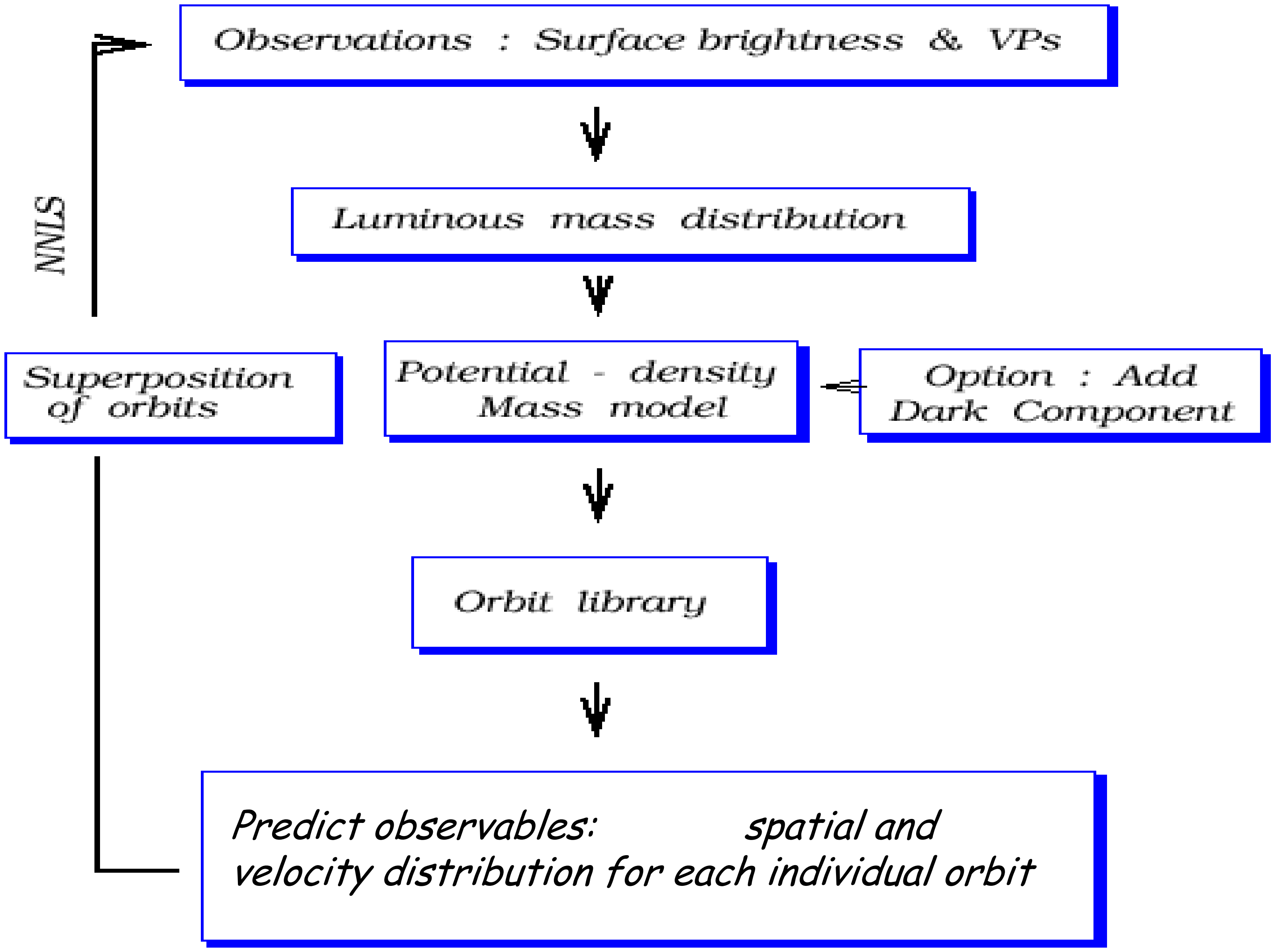


$V_{line\text{-}of\text{-}sight}$



images of model orbits

Observed
galaxy image

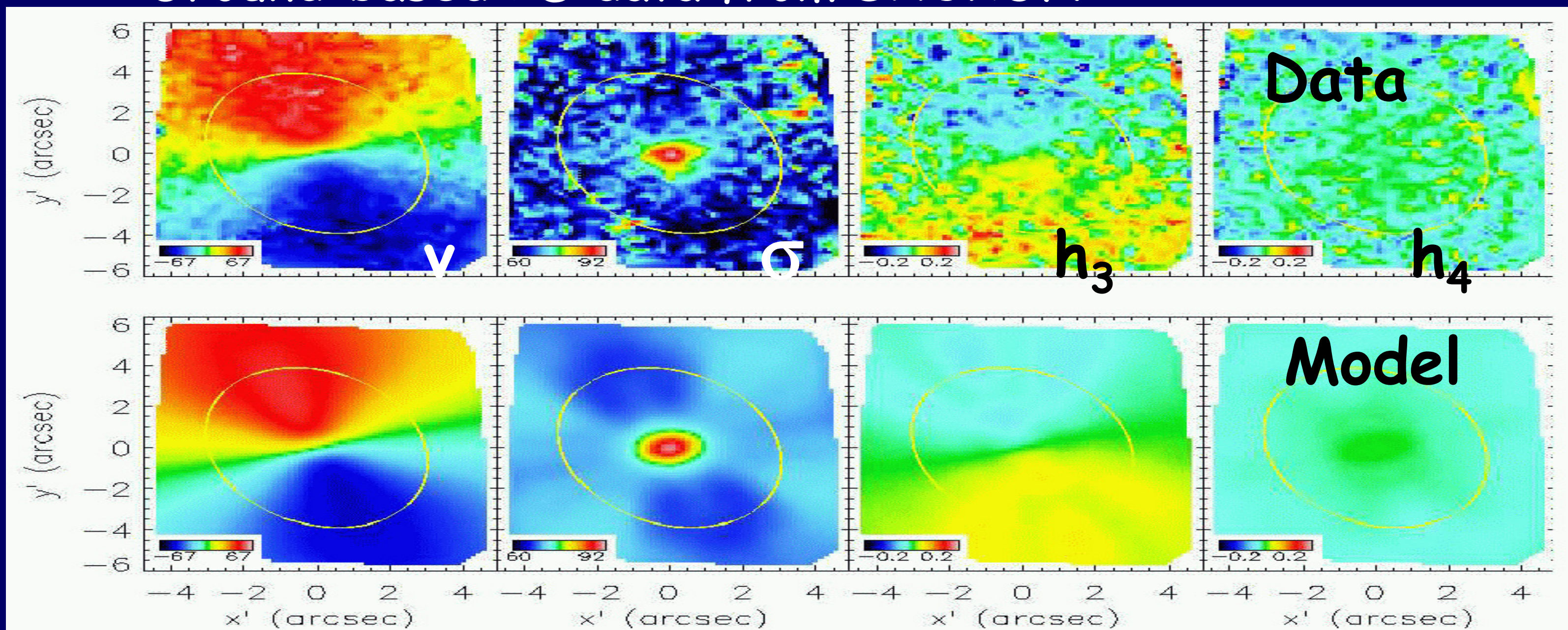

Observations : Surface brightness & VPs
Luminous mass distribution
Potential - density
Mass model
Option : Add
Dark Component
Superposition
of orbits
NNLS
Orbit library
Predict observables: spatial and
velocity distribution for each individual orbit

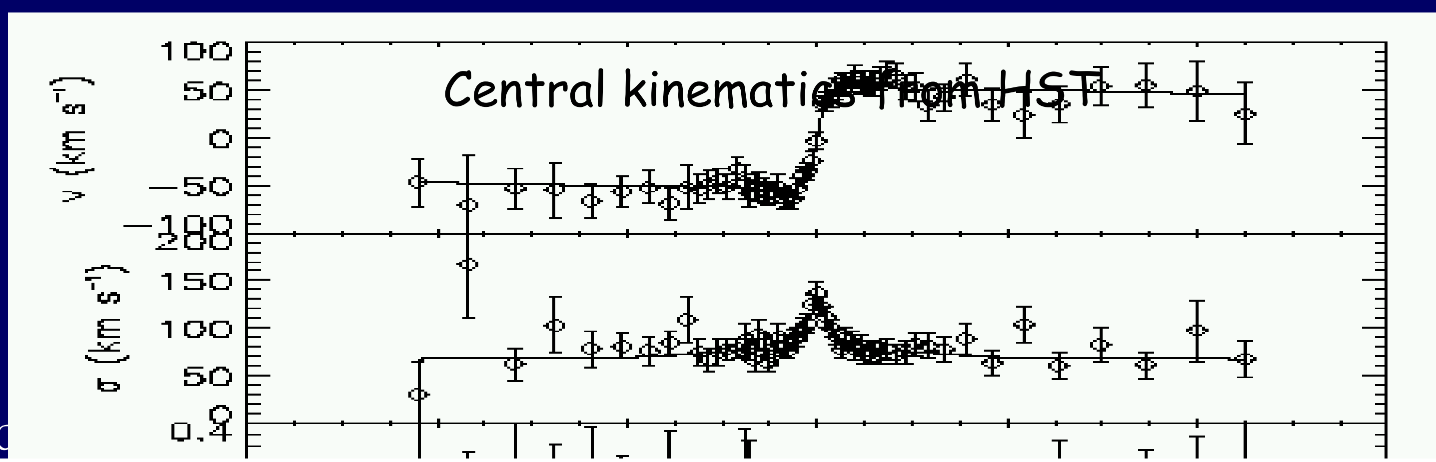
Example of Schwarzschild Modeling

M/L and M_{BH} in M32

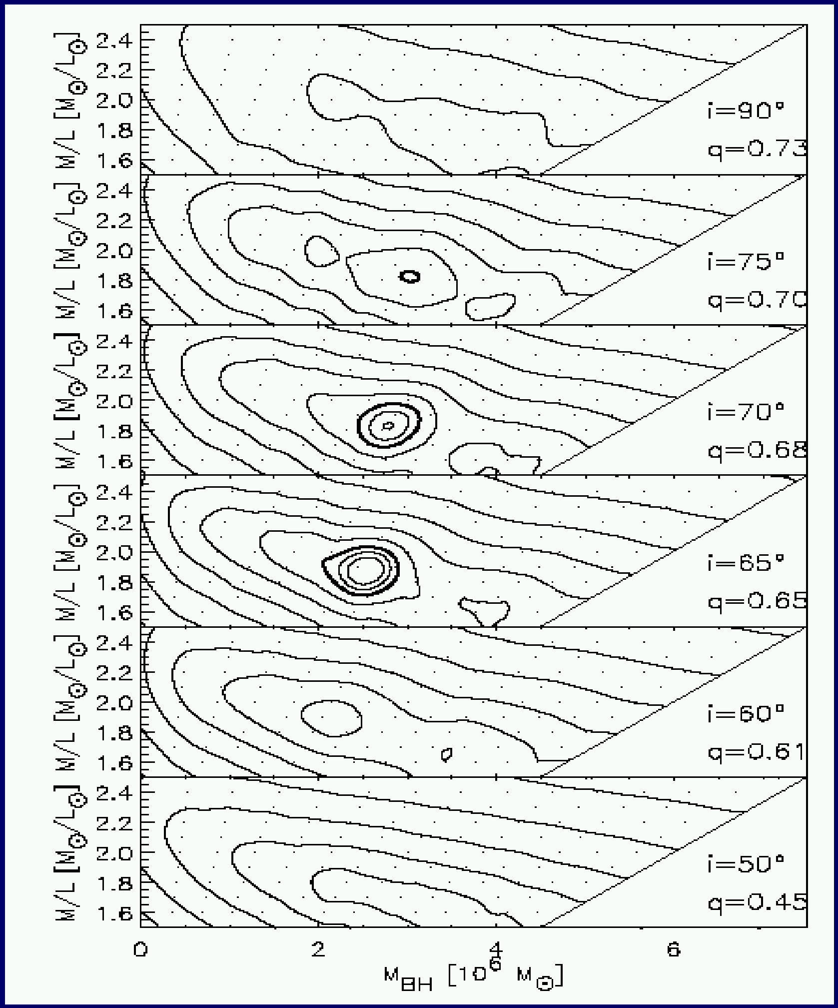
Verolme et al 2001

Ground-based 2D data from SAURON





- Then ask:
  for what potential and what orientation, is there a combination of orbits that matches the data well ?



Determine: inclination, M_{BH} and M/L simultaneously

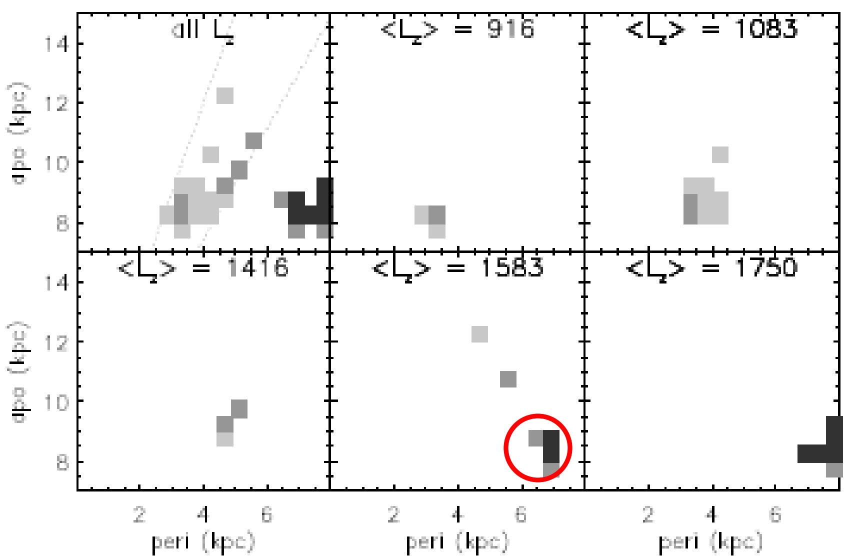
NB: assumes axisymmetry

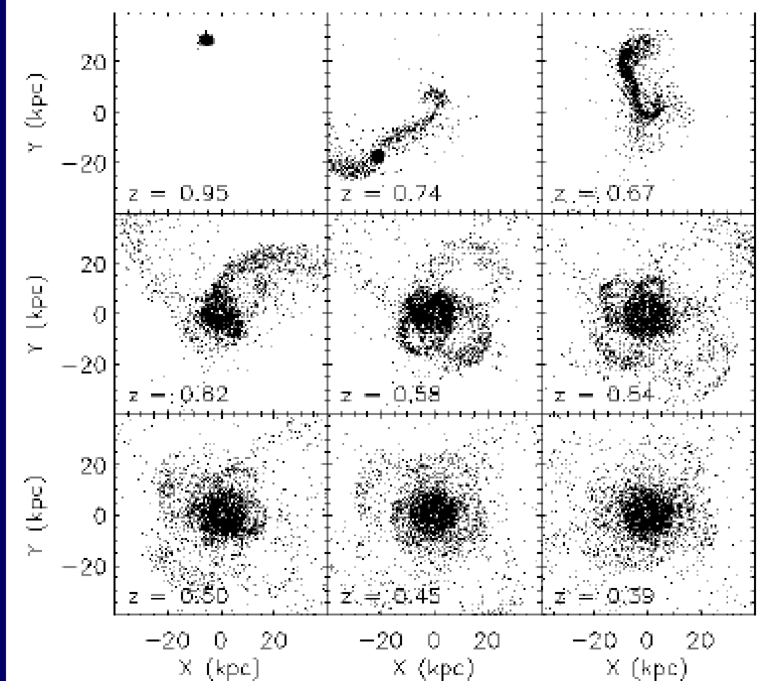
Thinking about galaxies as ensembles of orbits..

Application of 'Finding stellar streams'

- Are there remnants of disrupted satellites in the neighborhood of the Sun?
  - e.g. Helmi et al 2001, 2006
  - Klement et al 2008

Overdensities in I.o.M. space (APL space)





From Helmi et al 2006:

Phase-mixed → look for groups of stars with similar integrals of motion

HW

Violent relaxation

Mon. Not. R. astr. Soc. (1967) **136**, 101–121.

STATISTICAL MECHANICS OF VIOLENT RELAXATION IN STELLAR SYSTEMS

D. Lynden-Bell

(Communicated by the Astronomer Royal)

(Received 1966 December 19)

Summary

An explanation of the observed light distributions of elliptical galaxies is sought and found.

The violently changing gravitational field of a newly formed galaxy is effective in changing the statistics of stellar orbits.

Basic idea:

- (rapidly) time-varying potential changes energies of particles
- Different change for different particles

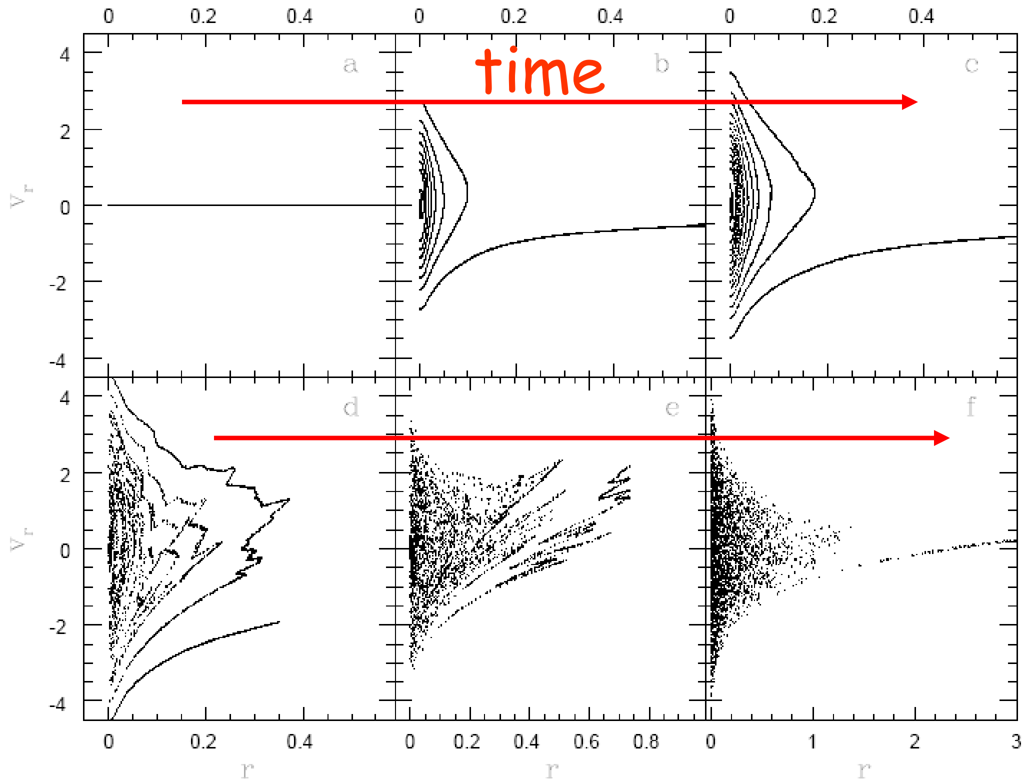
$$E = \frac{1}{2}v^2 + \Phi \text{ and } \Phi = \Phi(\vec{x}, t)$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial E}{\partial \vec{v}} \cdot \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \frac{\partial E}{\partial \Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\partial \Phi}{\partial t}$$

The **time-scale** for violent relaxation is

$$t_{\rm vr} = \left\langle \frac{({\rm d}E/{\rm d}t)^2}{E^2} \right\rangle^{-1/2} = \left\langle \frac{(\partial\Phi/\partial t)^2}{E^2} \right\rangle^{-1/2} = \frac{3}{4}\langle \dot{\Phi}^2/\Phi^2 \rangle^{-1/2}$$

How violent relaxation works in practice (i.e. on a computer)



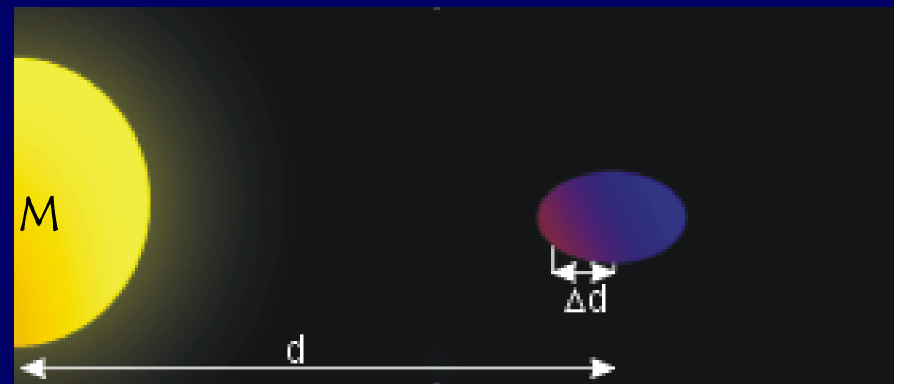
(from: Henriksen & Widrow 1997)

Collapse of a spherical system with $\rho_{\rm init} \propto r^{-3/2}$

Tidal forces and tidal disruption

"Roche limit": for existence of a satellite, its self-gravity has to exceed the tidal force from the 'parent'

Tidal force $\mathbf{F}_\mathrm{T} = -\frac{3GM}{2d^3}\Delta d^2$

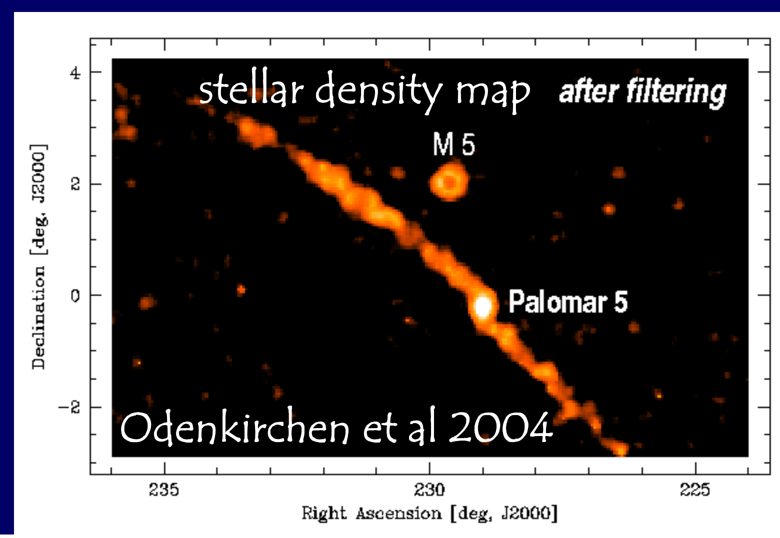


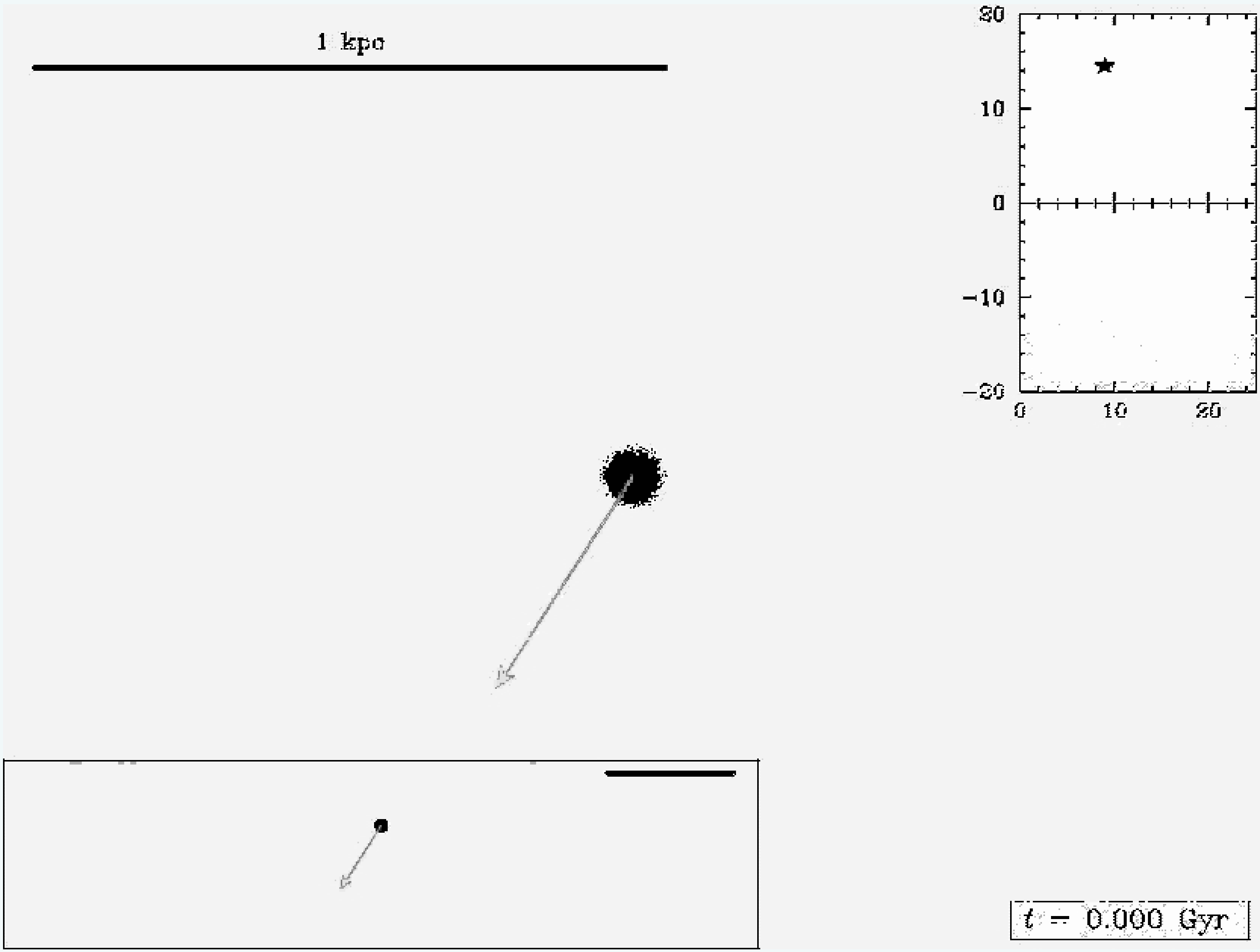
Tidal radius: $R_{tidal}(satellite) = f\left[\frac{M_{satellite}}{M_{host}(<R_{peri})}\right]^{1/3} \times R_{peri}$ with $f \approx 2/3[1-\ln(1-e)]^{-1/3}$

or… $\langle \rho_\mathrm{main\ galaxy}(<R_\mathrm{peri})\rangle \approx \langle \rho_\mathrm{satellite}(<r_\mathrm{tidal})\rangle$

In cosmological simulations, many DM sub-halos get tidally disrupted.

- How important is it, e.g. in the Milky Way?
- The GC Pal 5 and the Sagittarius dwarf galaxy show that it happens





From Dehnen, Rix, Odenkirchen & Grebel 2004

Dynamical friction

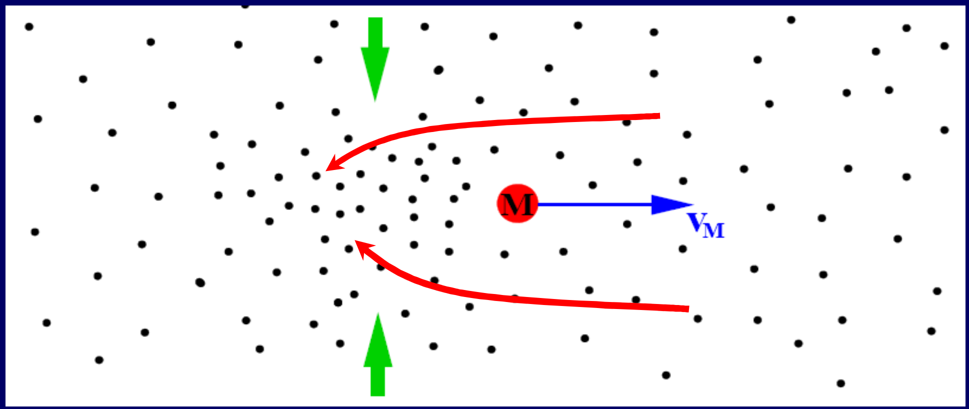
A "heavy" mass, a satellite galaxy or a bound sub-halo, will experience a slowing-down drag force (dynamical friction) when moving through a sea of lighter particles

Three ways to look at the phenomenon

a) There is a formula due to Chandrasekhar: average over all particle encounters

b) A system of many particles is driven towards "equipartition", i.e.
$E_{kin}(M) \sim E_{kin}(m)$
$\Rightarrow V^2_{\text{of particle } M} < V^2_{\text{of particle } m}$

c) Heavy particles create a 'wake' behind them



$$F_{dyn.fric} = -\frac{4\pi G M^2}{V_M^2} \rho_m \cdot \ln\Lambda$$

Where m « M and ρ_m is the (uniform) density of light particles m, and $\Lambda = b_{max}/b_{min}$ with $b_{min} \sim \rho_M/V_2$ and $b_{max} \sim$ size of system typically $\ln \Lambda \sim 10$

Main effect of dynamical Friction

Orbital decay (e.g. of satellite galaxy) : $t_{df} \sim r / (dr/dt)$

$V_{circ}\, dr/dt = -0.4 \ln \Lambda\, \rho M/r$

or $$t_{df} \approx \frac{1.2}{lu\Lambda} \frac{r_i^2 V_c}{\rho M}$$

Dynamical Modelling in the Milky Way: A few worked examples

- Modelling gas motions near the center of the Galaxy
  - Binney et al 1991
- The Mass of the MW's Dark Matter Halo
  - Battaglia et al 2006, Smith et al 2007, Xue, Rix et al 2008
- Dynamics from 'tracing out an orbit'
  - Koposov, Rix et al 2009

Gas in the Inner Galaxy

- Remember: Gas is not collisionless → two regimes:
  - $KT \approx V^2_{characteristic}$ hot gas
  - $KT \ll V^2_{characteristic}$ warm, cold gas
- Dynamics of 'hot' gas

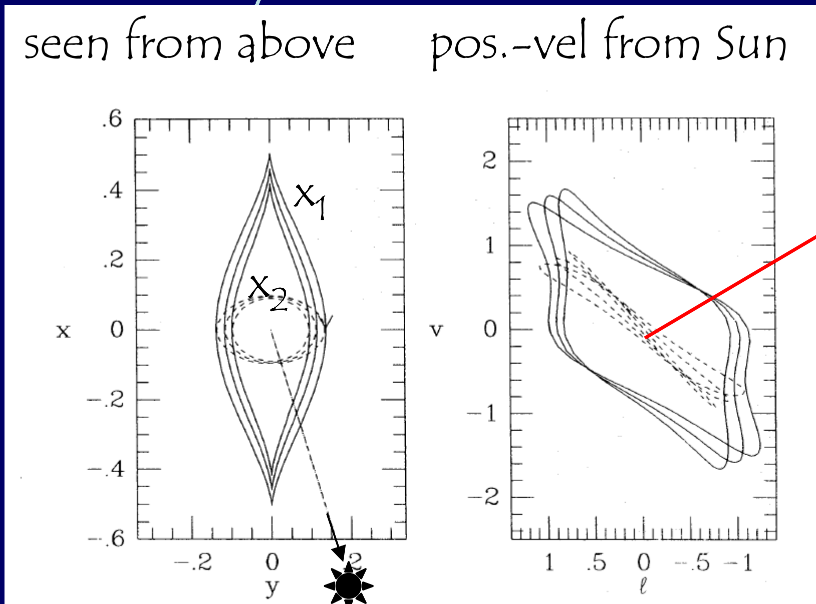
  'approximate hydrostatic equilibrium' → X-ray gas, 10^{5-6} K

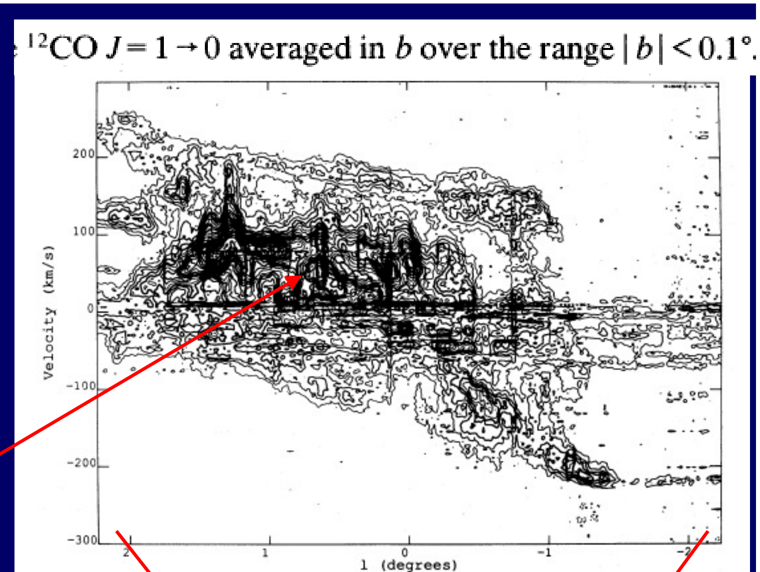
- Dynamics of COLD gas
  - To avoid shocks gas will settle on non-inersecting loop orbits
    - concentric circles (in axisymm. case)
    - ellipses in (slightly distorted) potentials, e.g. weak spiral arms
- In barred potentials:

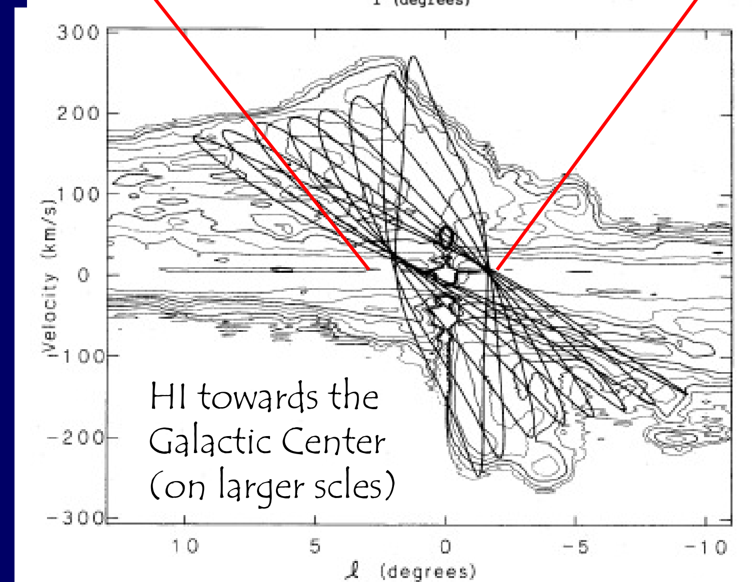
  closed-orbit ellipticity changes at resonances → shocks, inflow

Gas in the Inner Galaxy:
Diagnostics of the Central Bar

- Binney et al 1991







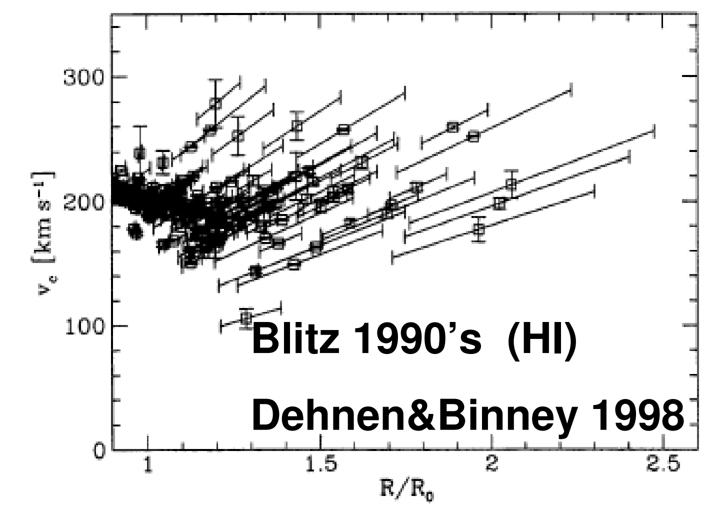
Gas flow near GC (100pc → 1kpc) dominated by bar

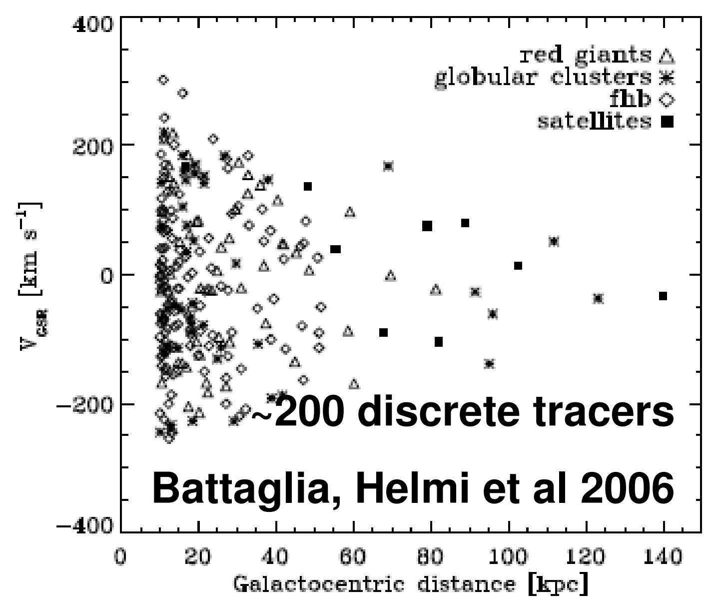
- $r_{co\text{-}rotation}$=2.4+-0.3 kpc
- viewing angle 16° +-2°

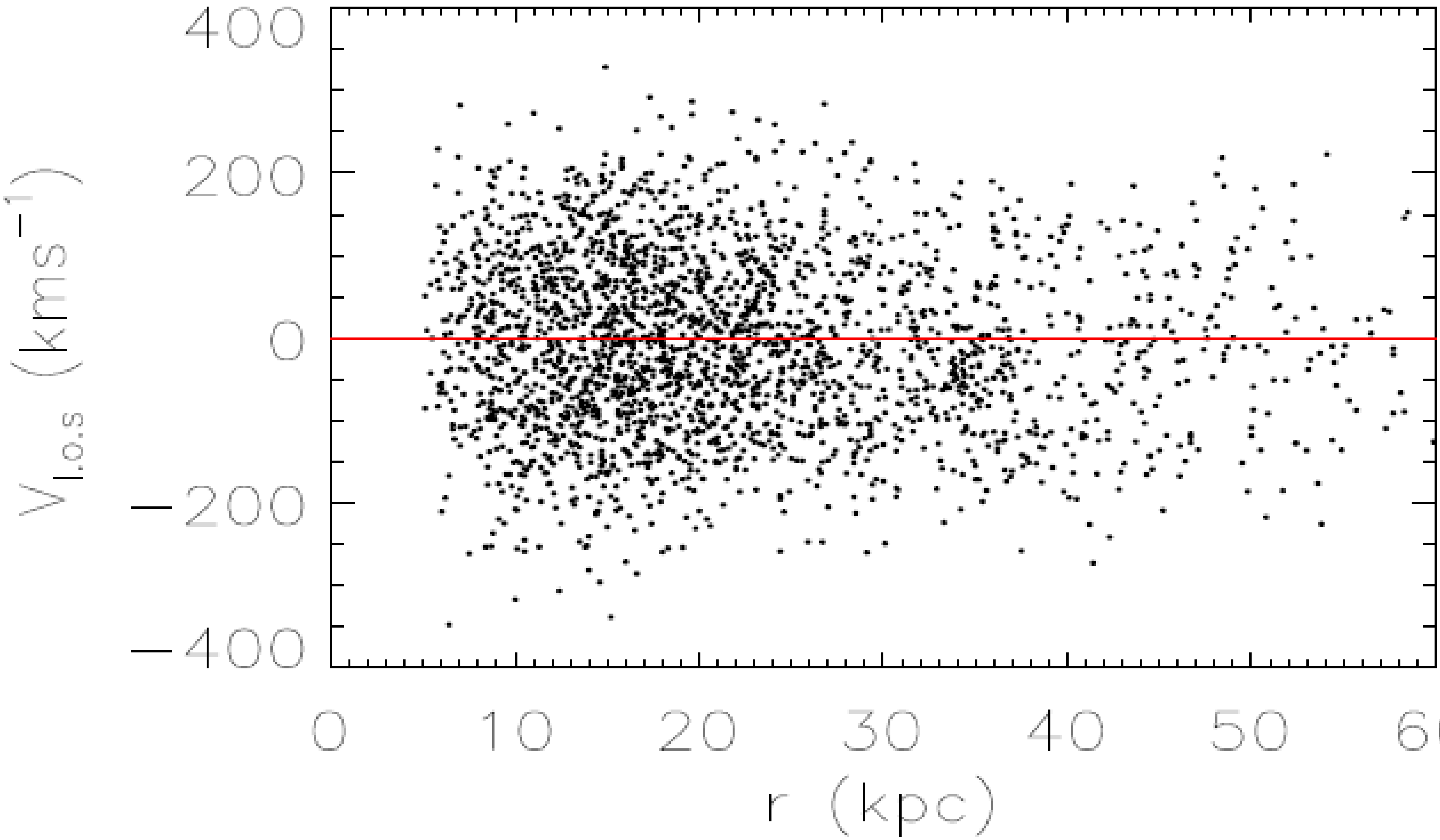
Mapping the MW Halo Mass Distribution

Xue, Rix, et al 2008

- Halo Mass
  - Normalizes all other quantities
    - Sub-halo abundance
    - Fraction of baryon → stars ?
  - Recent lit. values 0.8–2.5 x 10^{12} $M_{\odot}$
  - Are all satellites bound?
- Experiment
  - Use blue Horizontal Branch Stars
    - 5% distances to D~60 kpc
    - δv ~ 10 km/s + Fe/H estimates
  - Derive $p(v_{l.o.s.})$ at different R_{GC}

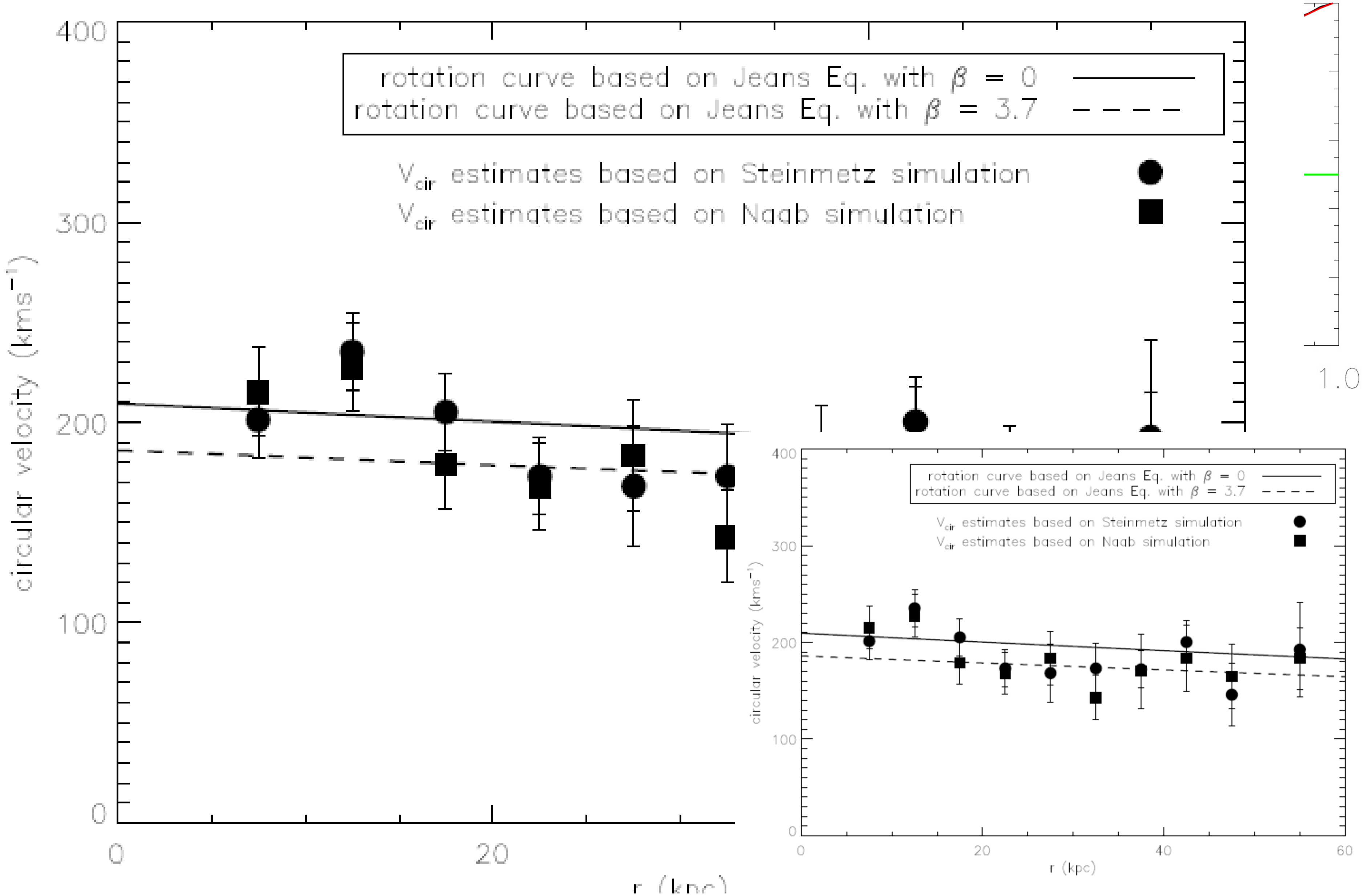






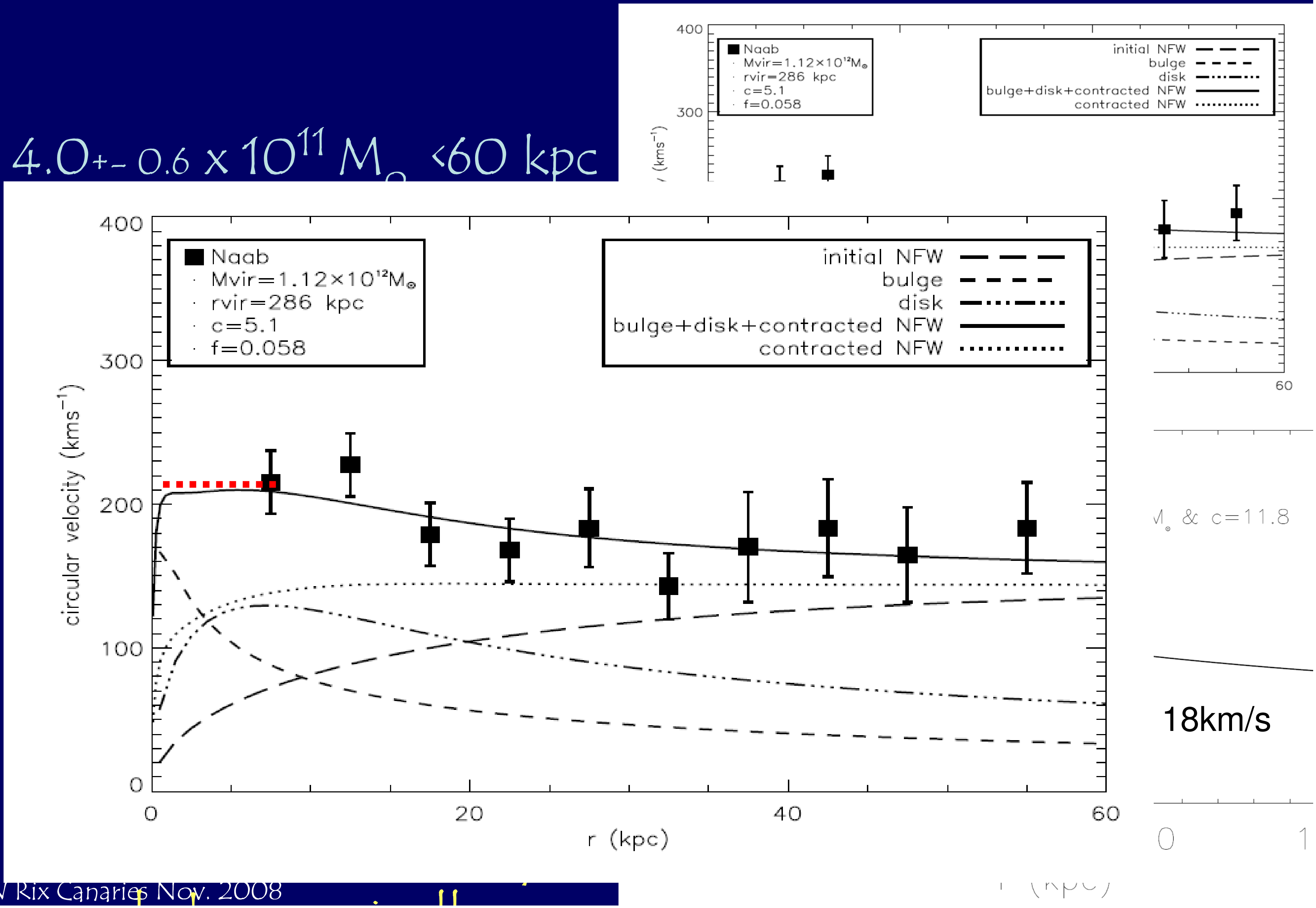
400
200
0
−200
−400
$V_{l.o.s}$ (kms^{-1})
0
10
20
30
40
50
60
r (kpc)

How to interpret/model these data?



Halo Mass: Results and Implications

- $4.0 \pm 0.6 \times 10^{11} M_\odot$ <60 kpc



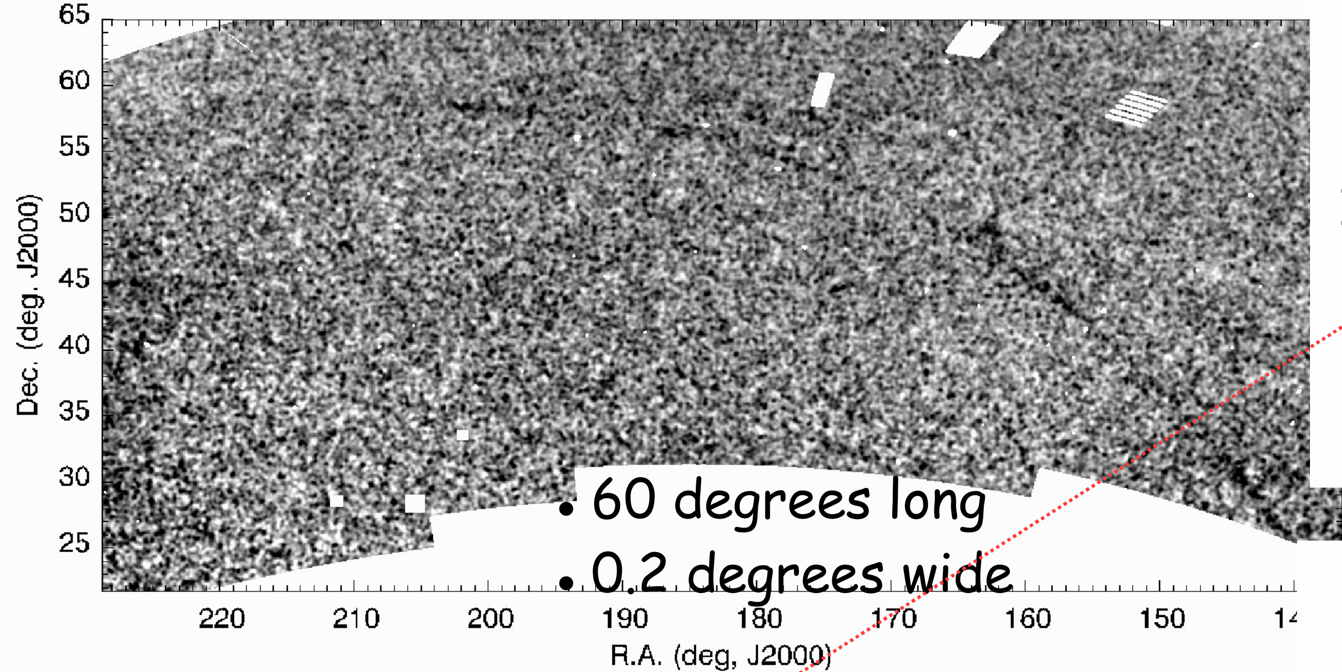
Tracing the Milky Way Potential by 'Following an Orbit'

- We could measure forces/accelerations directly if we could follow an orbit
- In a very cold stream, particles should only differ by orbital phase, but little in E,J…
- Perhaps best example:
  - Grillmair&Dionatos stream (2005)
  - Disrupted globular cluster (??)

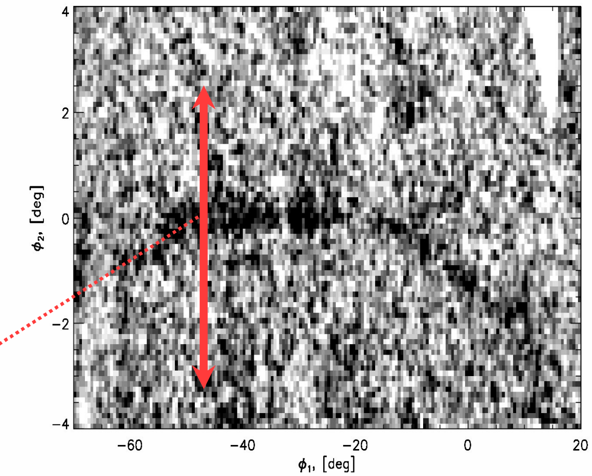
The cold/narrow stellar stream

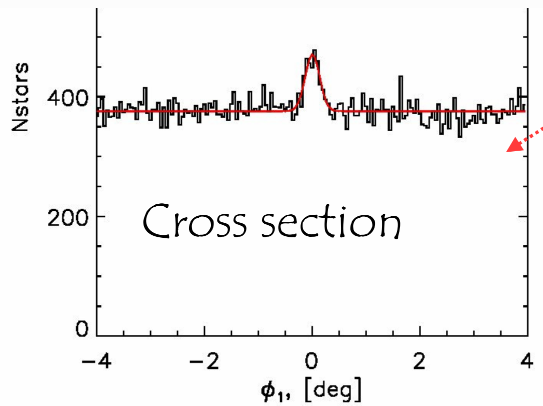
Grillmair, C.J., & Dionatos, O. 2006, ApJL, 643, L17

Stream in best-fit great circle coordinates



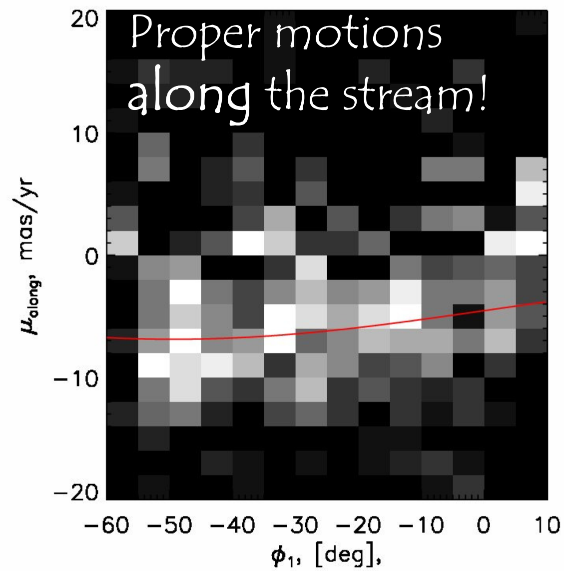
- 60 degrees long
- 0.2 degrees wide





Distances from color-magnitude diagram

Koposov, Hogg, Rix 2009

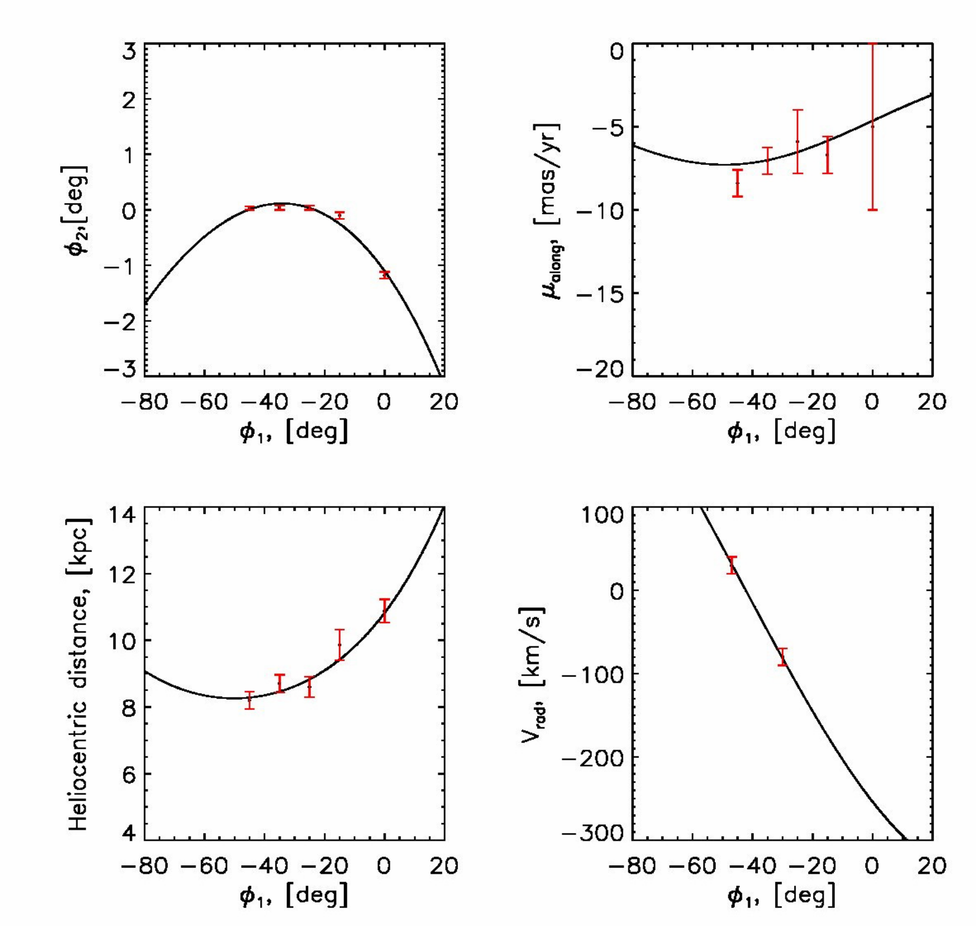


Matching the 6D (=r,v) constraints for the GD1 stream

Koposov et al 2009

- For what potentials can a single orbit be fit?
- Parameterize potential:
  - Myamoto-Nagai disk,
  - Hernquist bulge
  - DM halo
    - V_{circ}~const
    - Constant flattening

$$\Phi = v_0^2 \log(x^2+y^2+(z/q)^2+d^2)$$



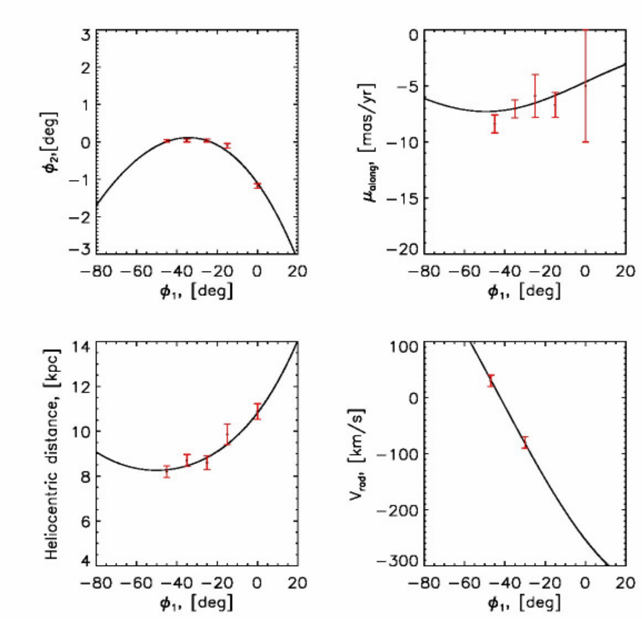
..tracing an orbit in the Milky Way's halo..

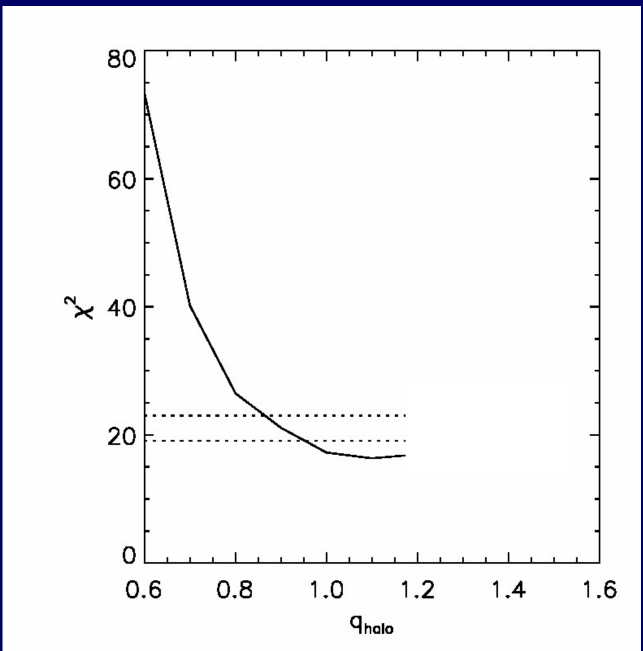
Koposov, Rix et al 2009

- Results:
  - DM halo cannot be significantly oblate

Note: $\varepsilon_{potential} < 0.05 \leftrightarrow \varepsilon_{mass} < 0.15$

  - Consistent with Sagittarius strem (little precession)





$q_{halo} > 0.95$ with 90% confidence

$q_{halo} > 0.85$ with 99% confidence

Some Guidance on

Galaxy Dynamics in the Context of the Milky Way

- The Milky Way has been in a state of approximately symmetrical near-equilibrium that is incessantly perturbed by a variety of effects.
- Phase-space distribution (Integrals of Motion) retain some memory.
- It is the ONLY galaxy in which the correlations between the properties of stars and the orbit they're on can be studied in detail → THINK ORBITS
- There is a dirth of modeling tools to put 10^4-10^9 measurements of star kinematics into context
  i.e. none of you could do justice to the information content of GAIA data!!!

Galaxy Dynamics Problems and the Milky Way

How to pose them? How to tackle them?

- Can Milky Way dynamics rule out MOND?
- What is $\rho_{DM}(r)$, what is the (3D) shape?
  - Dark matter at small radii? (<8kpc)
  - Dark matter > 50kpc?
- Is there direct evidence for small (dark) sub-halos?
  - Those who may not have any stars in them…
- What is the stellar/total density at the solar radius?
  - Oort limit…

Galaxy Dynamics Problems and the Milky Way

How to pose them? How to tackle them?

- What sets the distribution of stellar orbits around the black hole?

- Are the thick/thin disk components distinct?

- How much fine-grain structure does the stellar DF have?