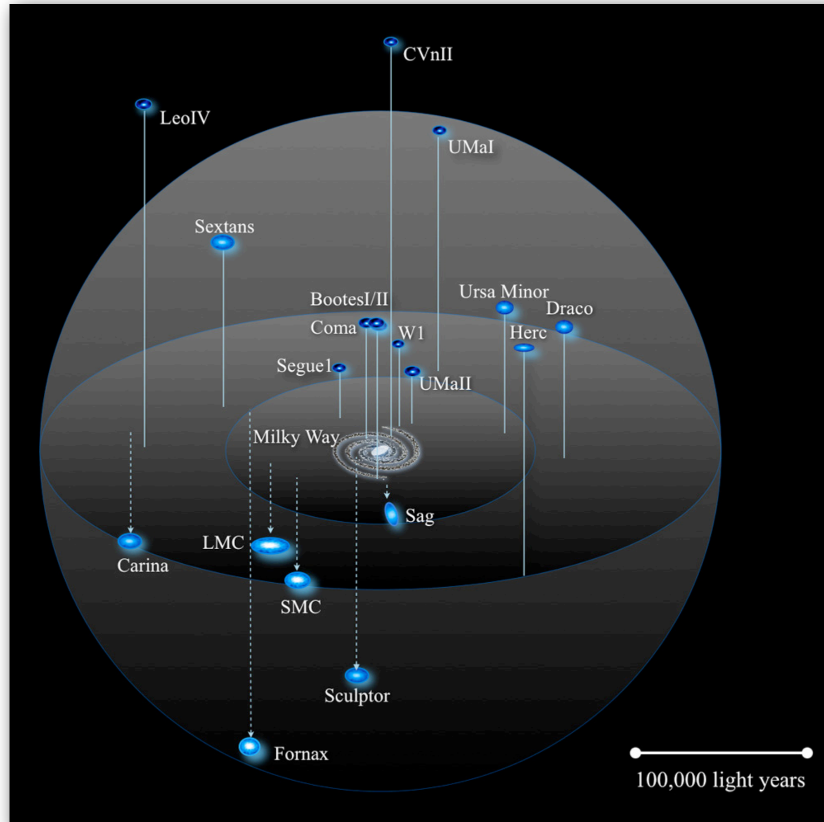
CDM and the Substructure Crisis

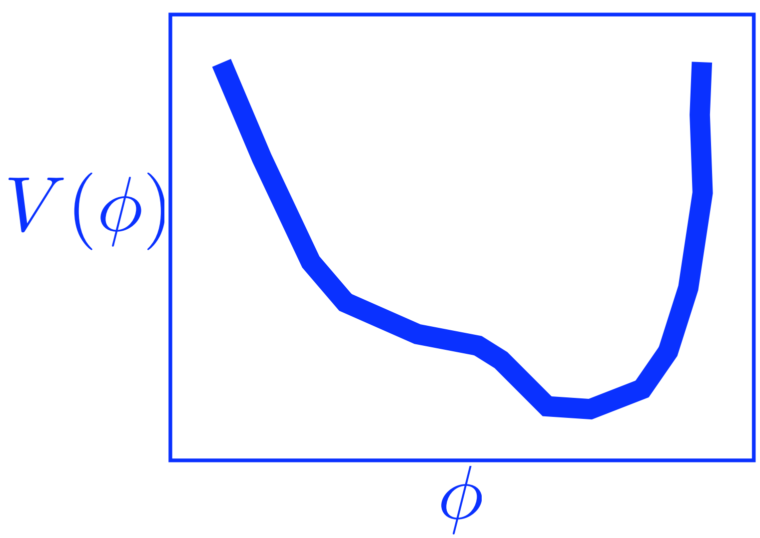
J. S. Bullock
XX Canary Islands Winter School, LG Cosmology

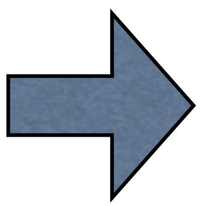


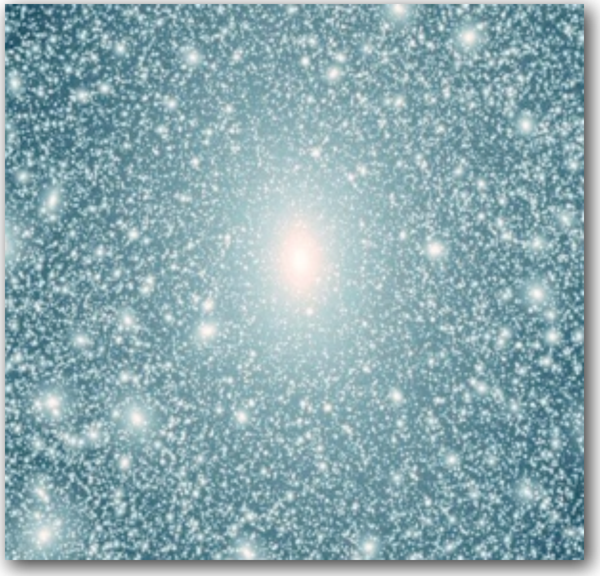
Theory: $N>10^{10}$

Observation: N~20

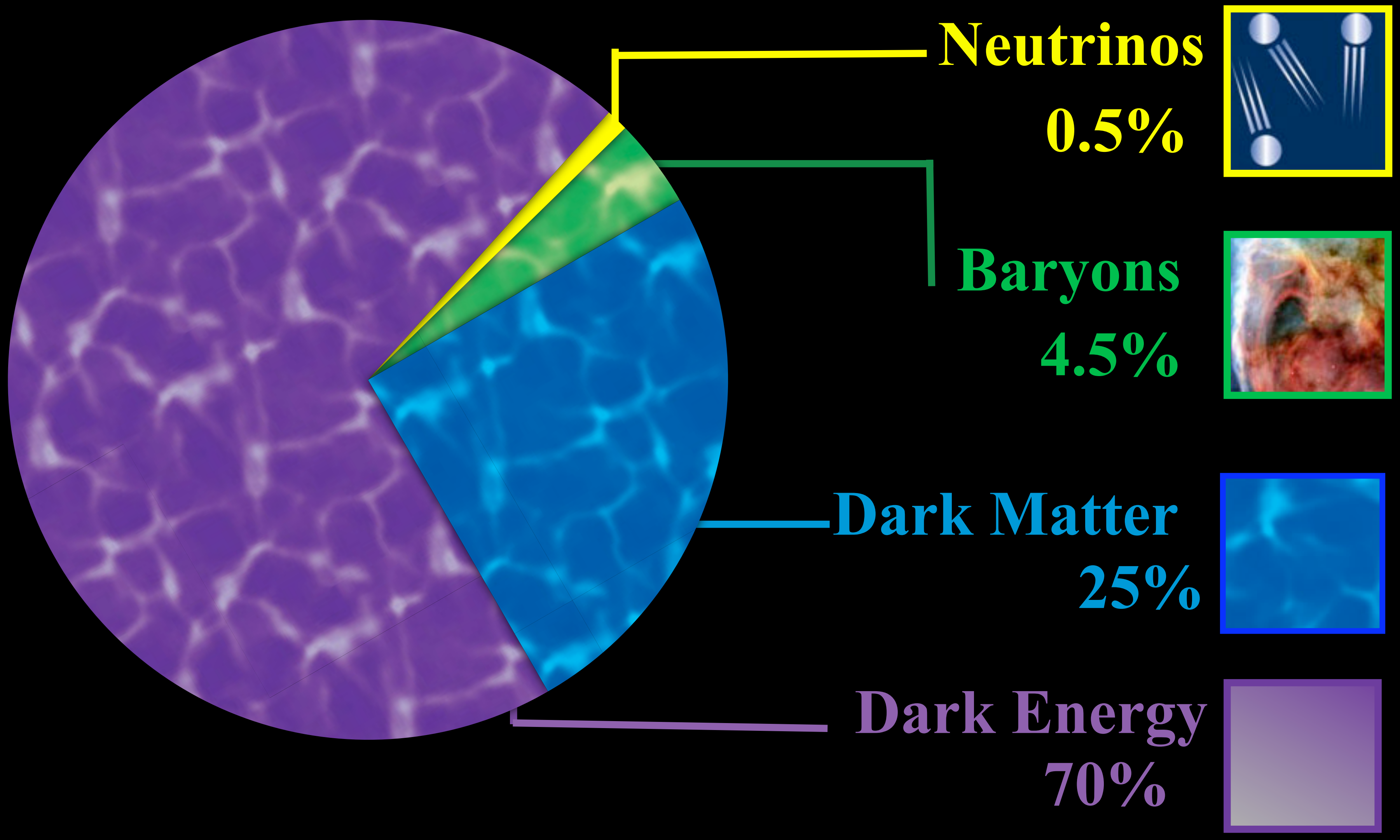
Lecture 1
From Inflation to DM substructure

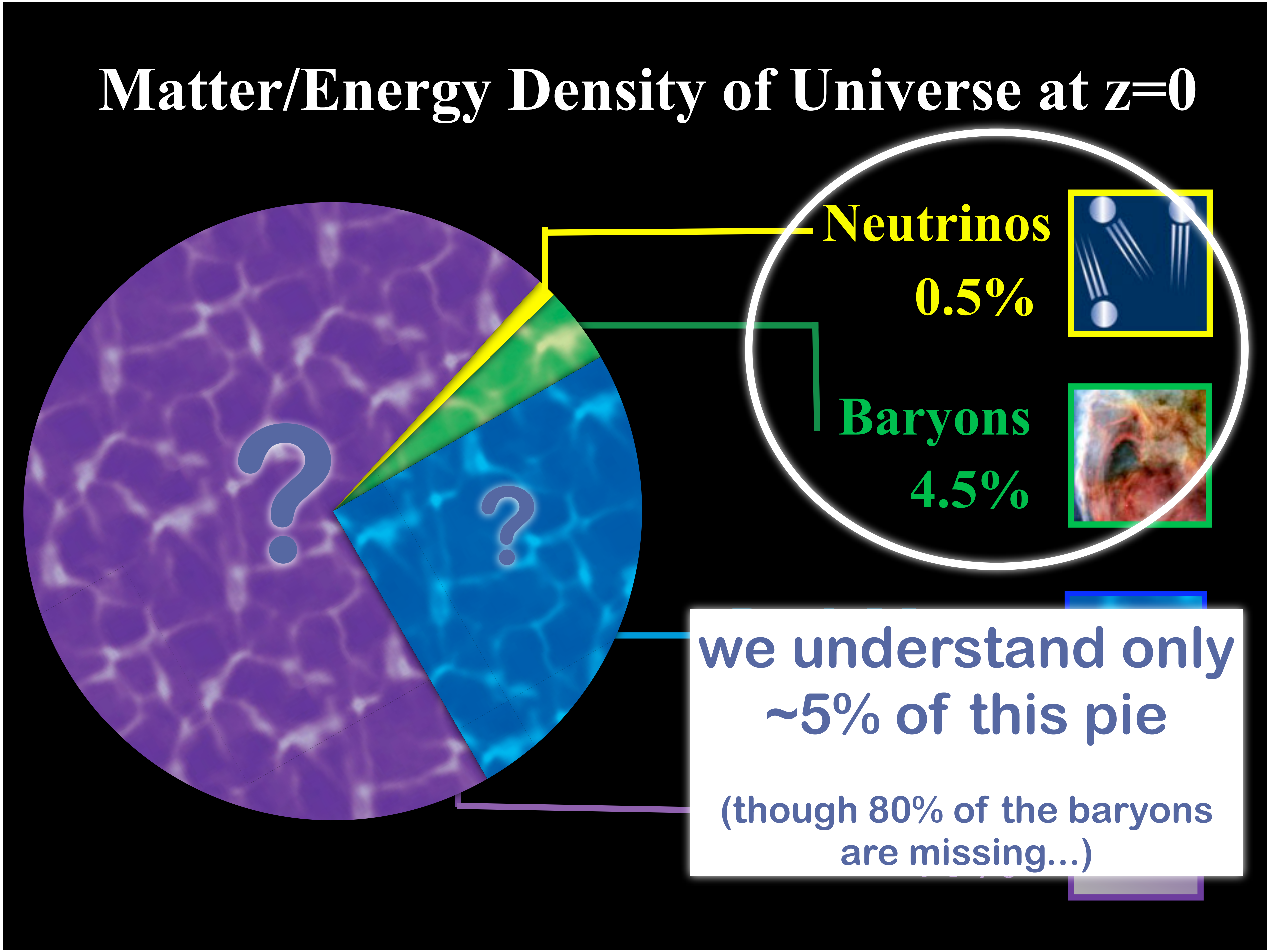


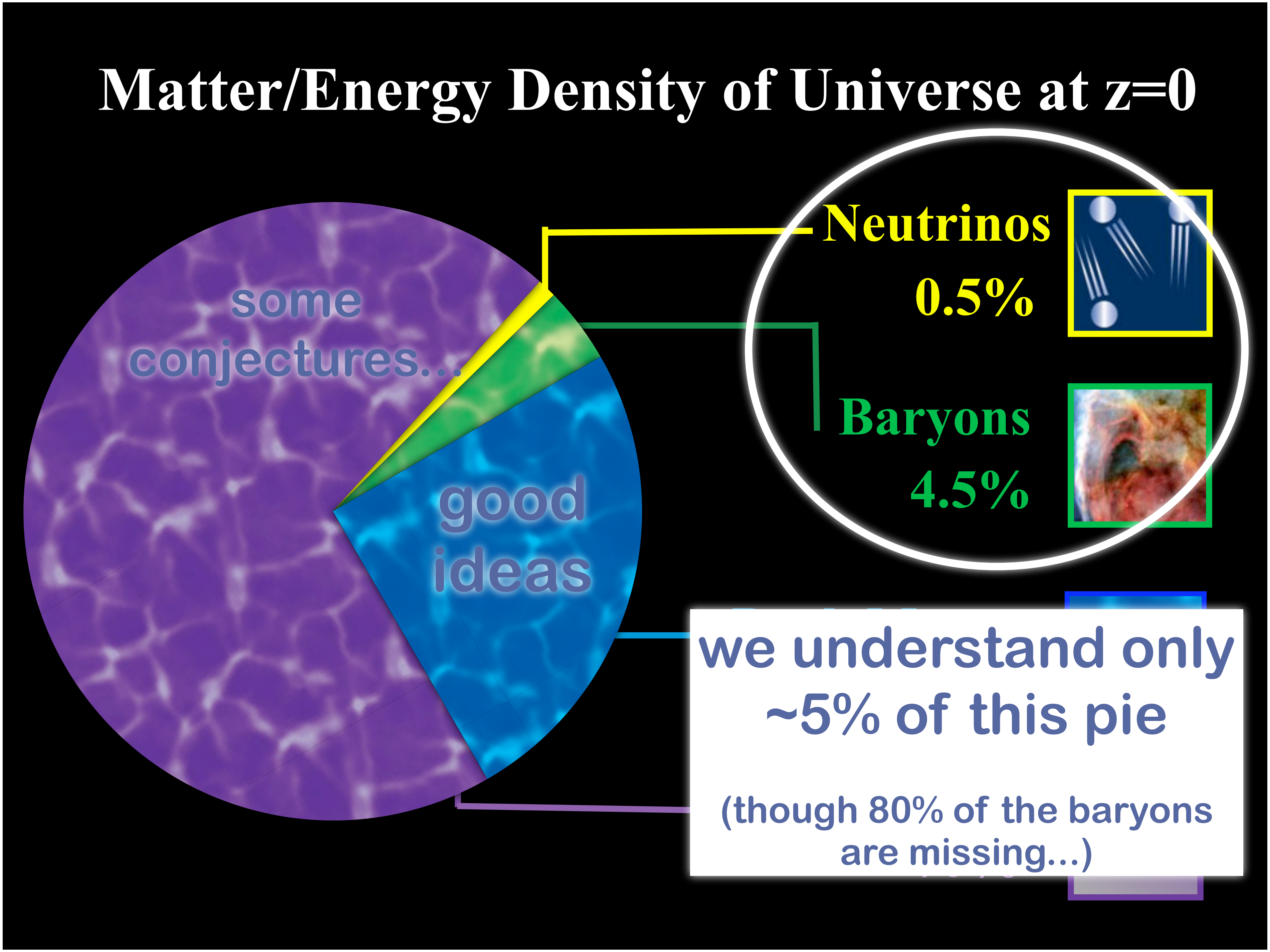




Matter/Energy Density of Universe at z=0

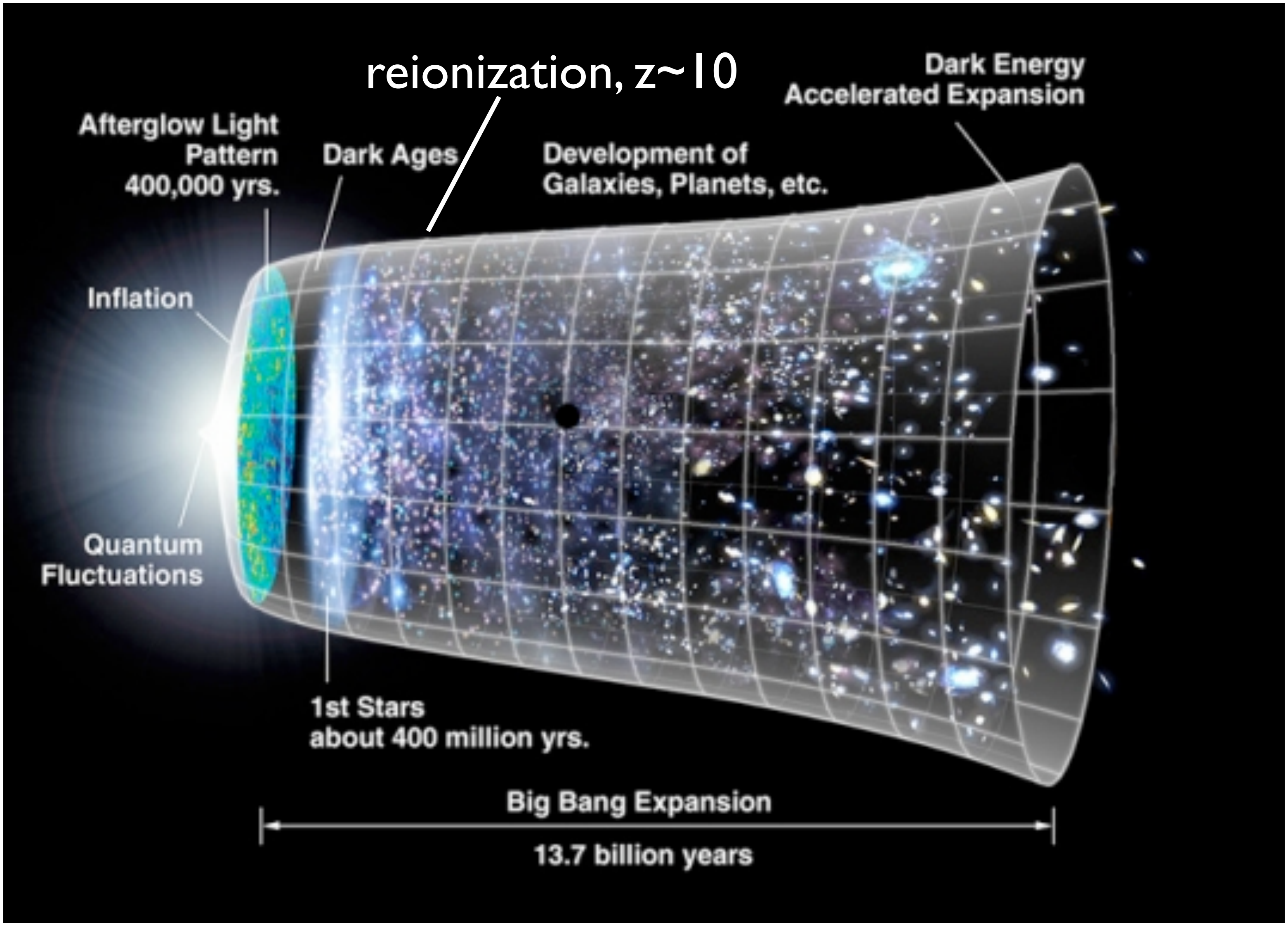



Matter/Energy Density of Universe at z=0
Neutrinos
0.5%
Baryons
4.5%
?
?
we understand only
~5% of this pie
(though 80% of the baryons
are missing...)

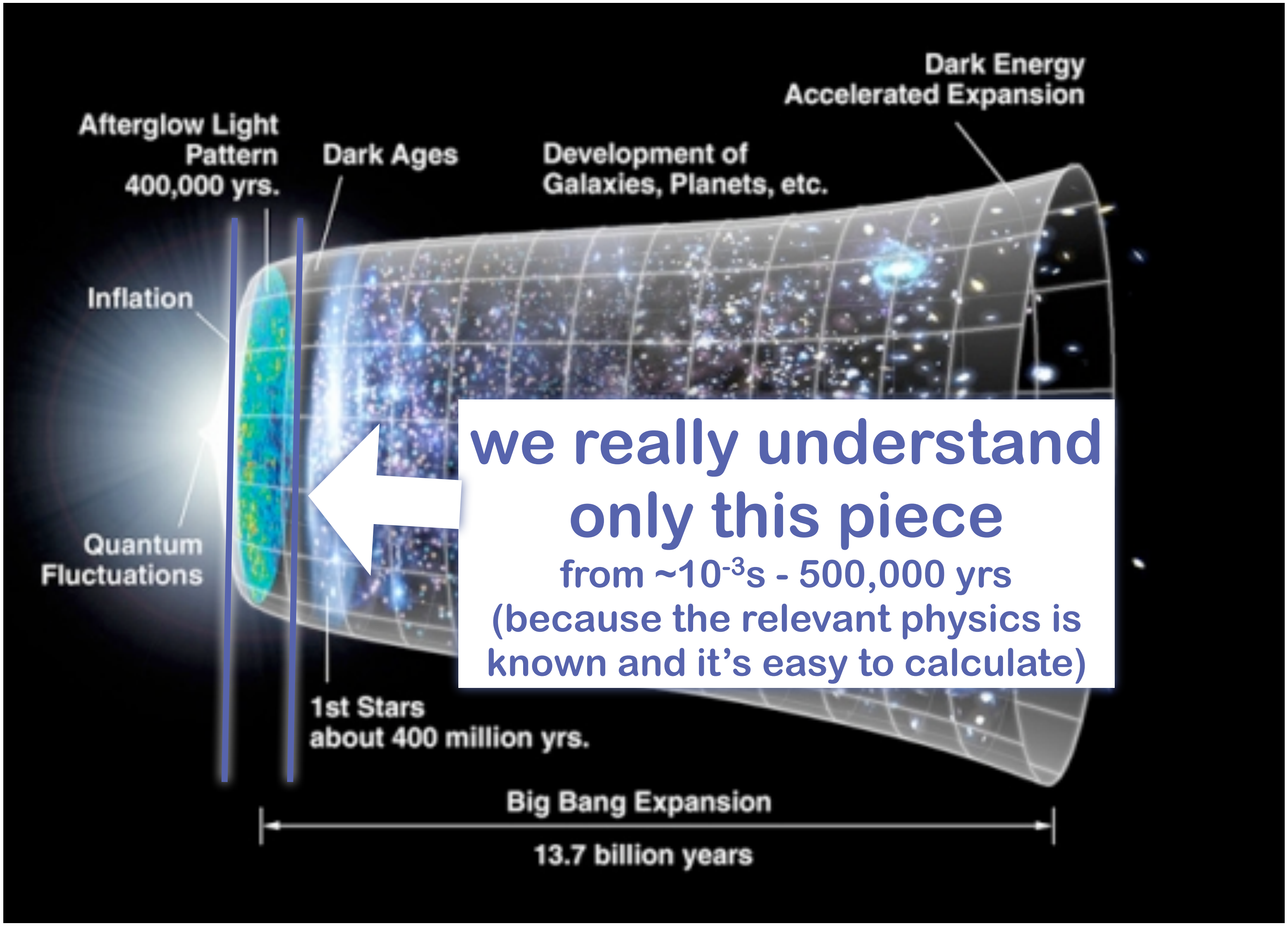


Matter/Energy Density of Universe at z=0
Neutrinos
0.5%
Baryons
4.5%
some conjectures...
good ideas
we understand only ~5% of this pie
(though 80% of the baryons are missing...)

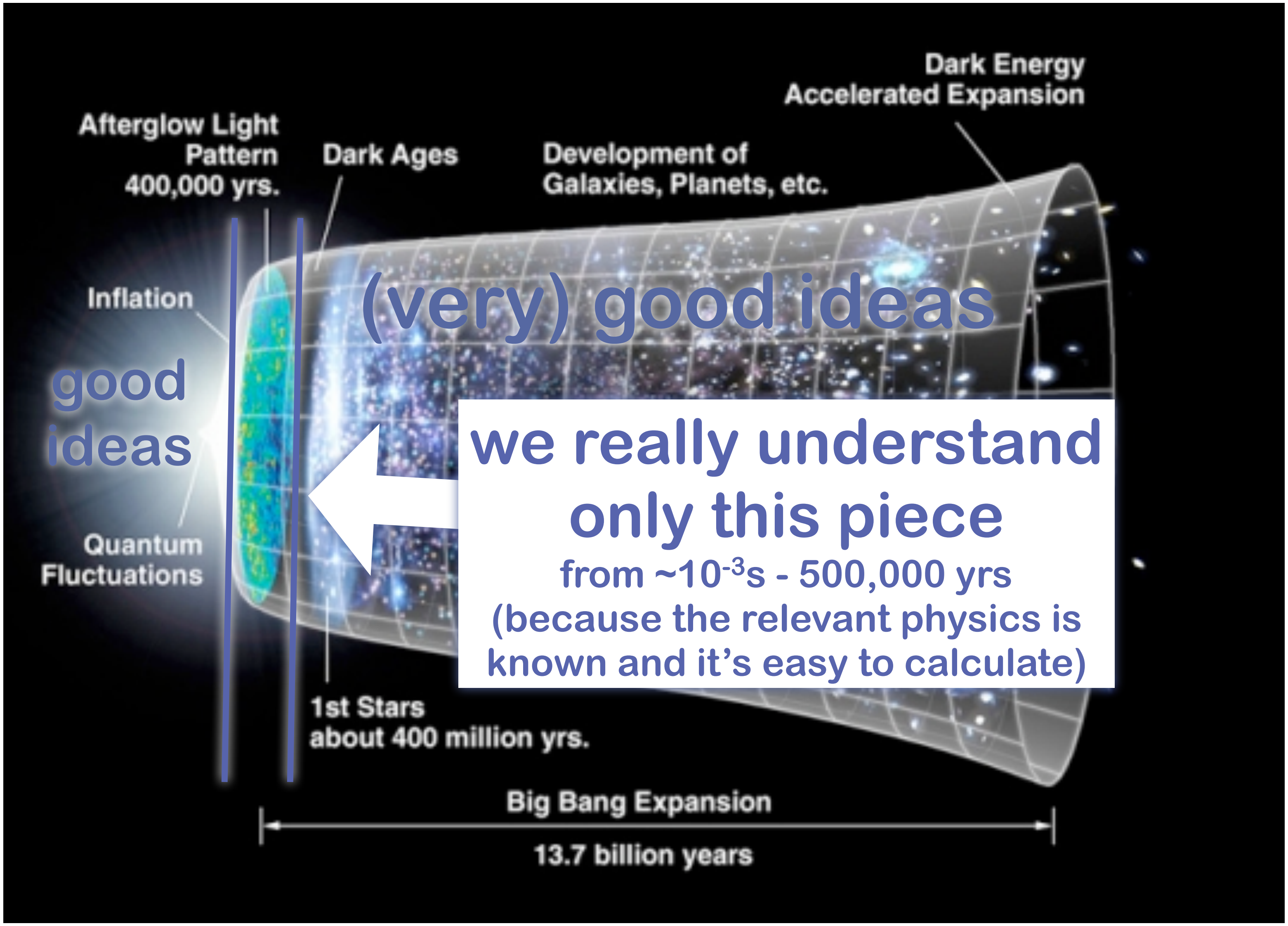
Timeline of the Universe

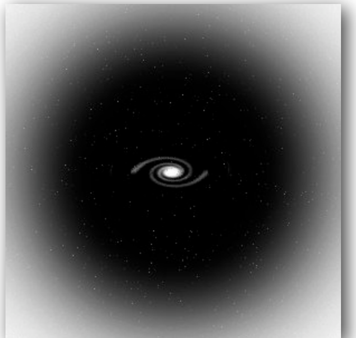


Timeline of the Universe

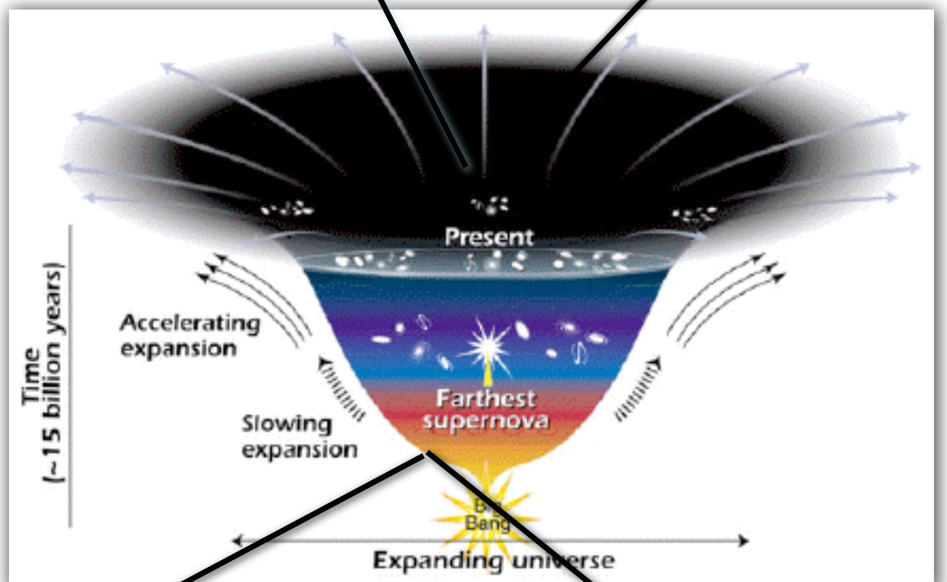


Timeline of the Universe

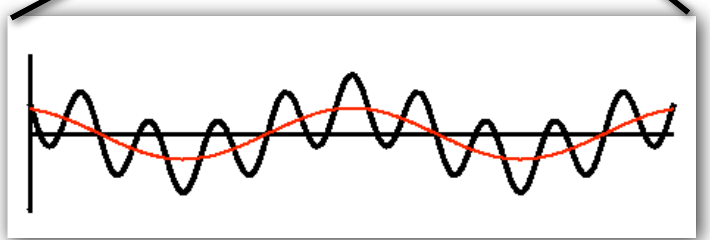




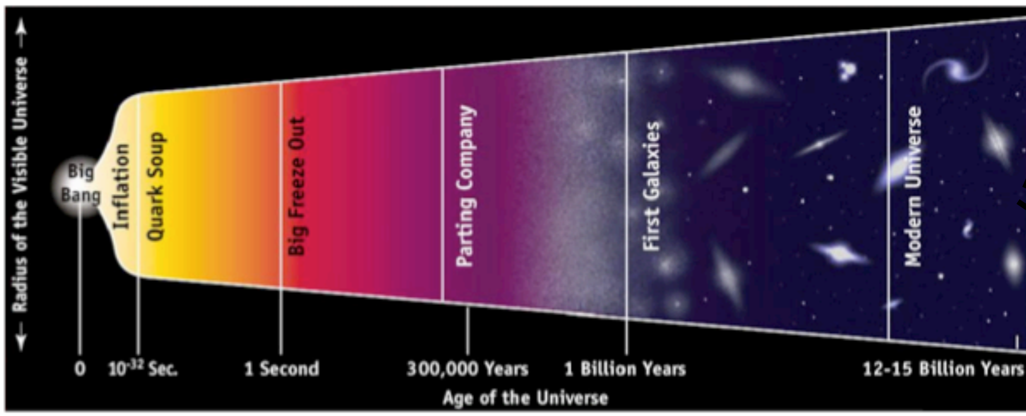
Form dark matter halos at z=0 and host galaxies, galaxy clusters, etc.



Amplified by gravity, density enhancements grow, overcome the expansion, collapse into bound objects



Initial density fluctuations (set during an early epoch of cosmic inflation)

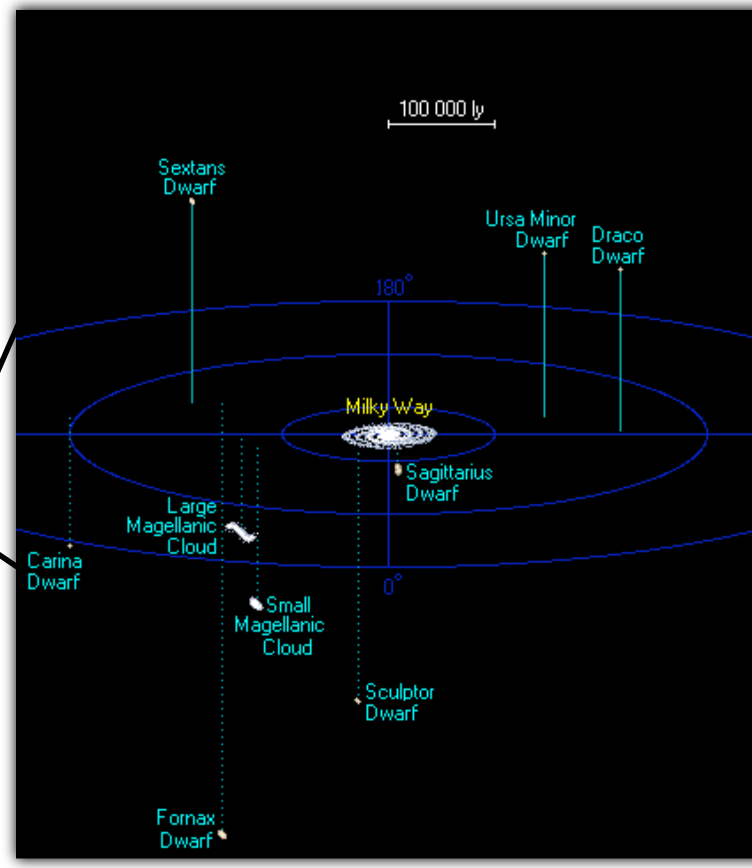


fairly well observed

observational frontier

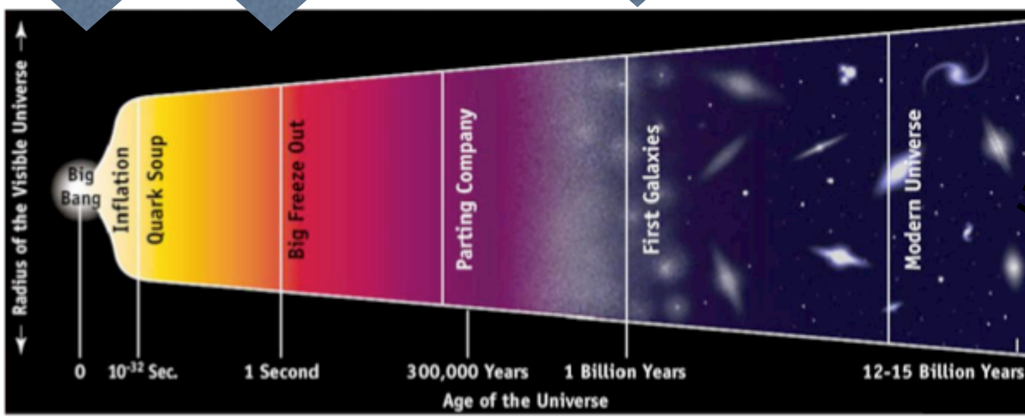
physics understood + indirect tests

beyond standard model / new physics



detailed observations (resolved stars)

Local Group Cosmology



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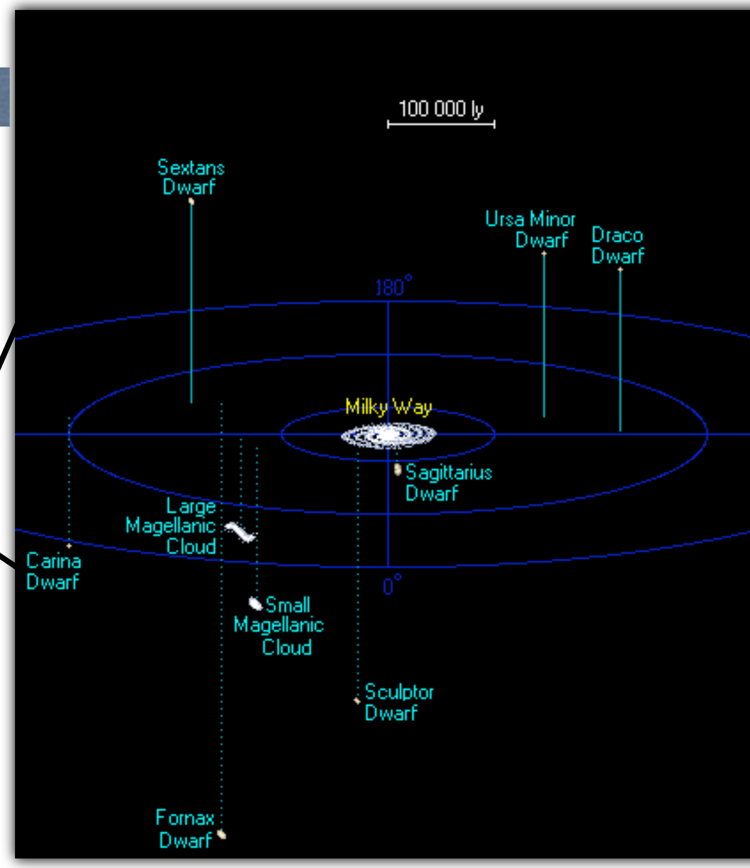
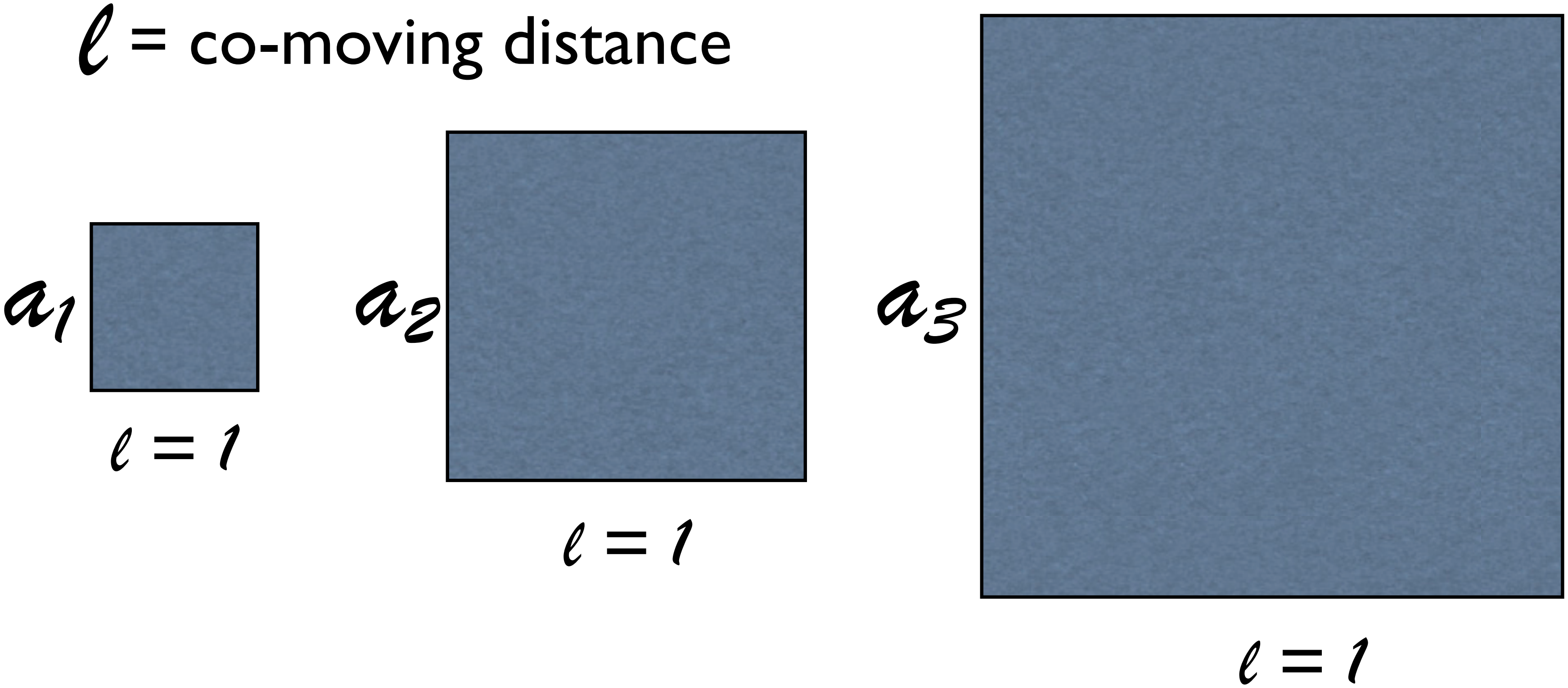


detailed observations (resolved stars)

Expanding Universe

a = scale factor = $(1+z)^{-1}$

ℓ = co-moving distance

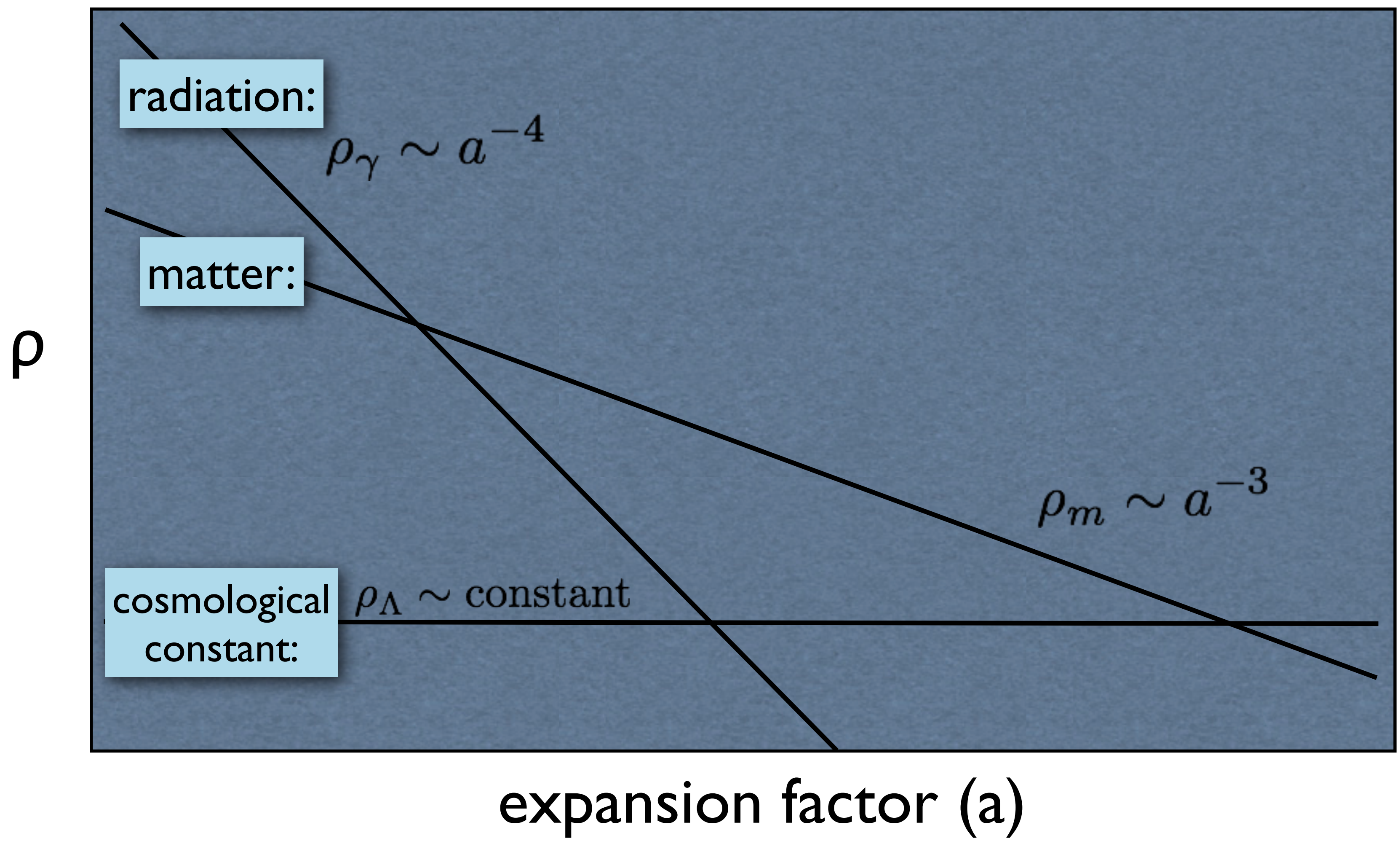


dense, hot universe

diffuse, cool universe

Energy Density of Universe Evolves with Expansion

matter: $\rho_m \sim a^{-3}$ radiation: $\rho_\gamma \sim a^{-4}$ cosmological constant: $\rho_\Lambda \sim \text{constant}$



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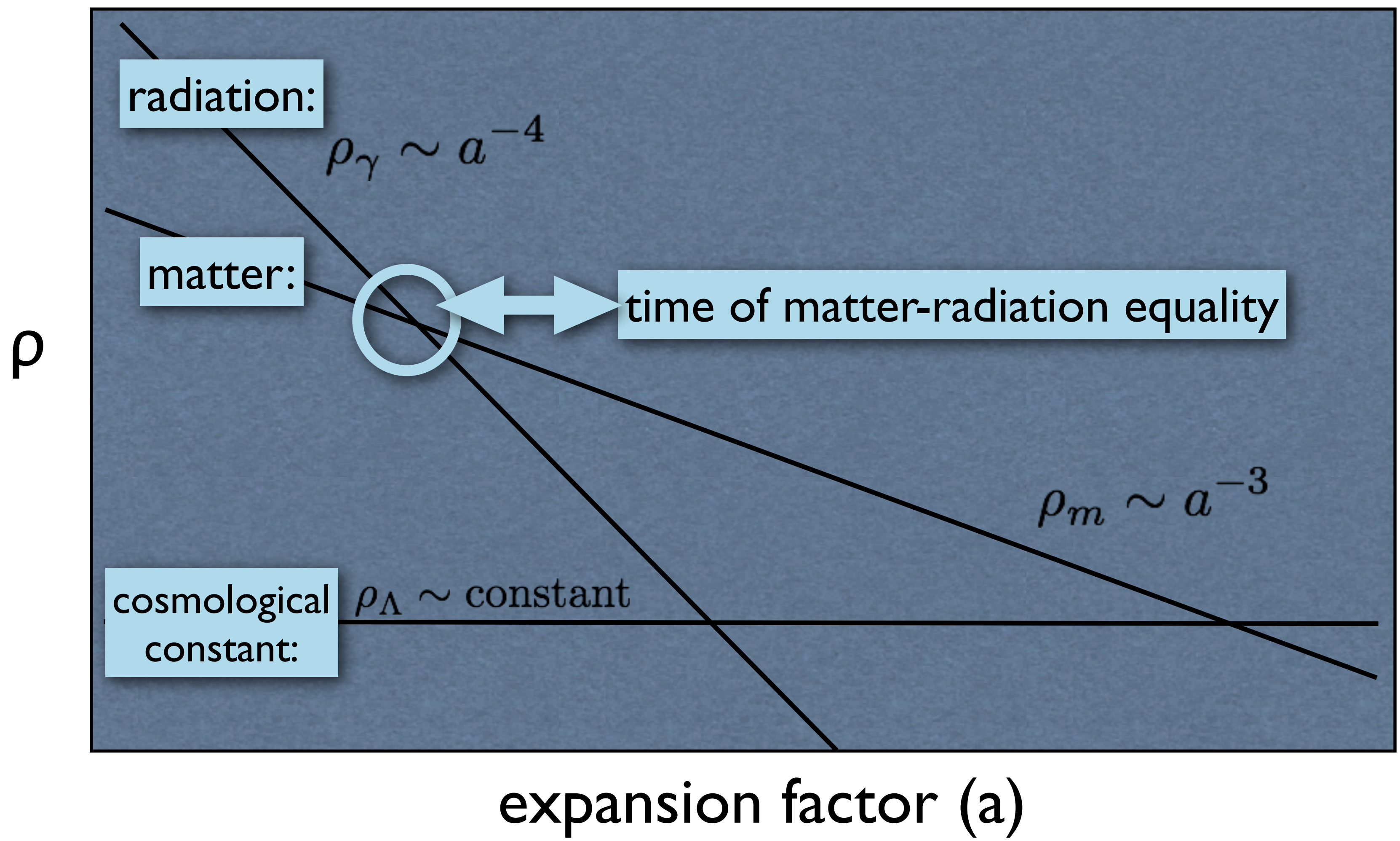
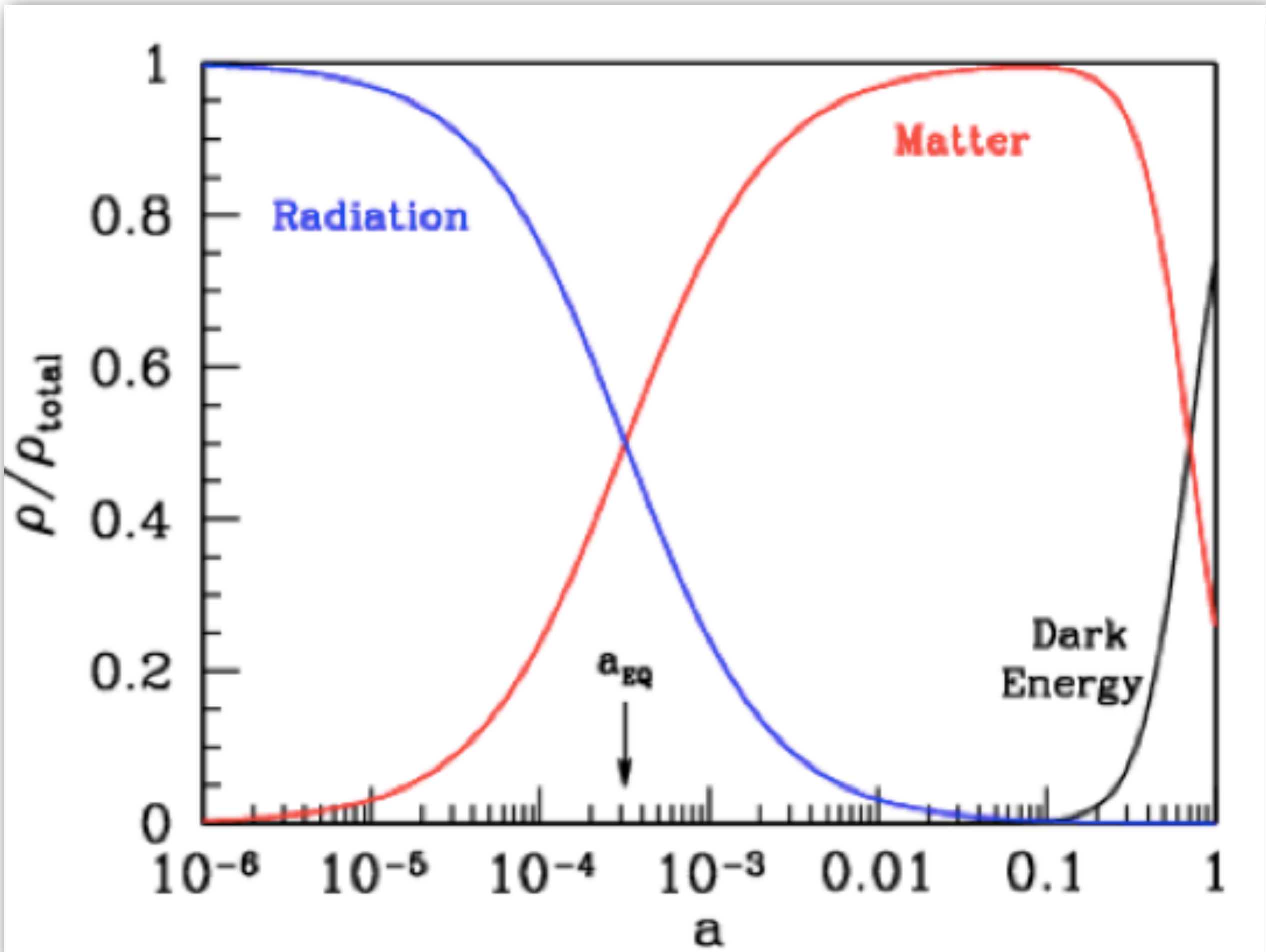


Energy Density Evolution with Expansion



Energy Density Controls Expansion History

Friedmann Equation (flat universe):

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8}{3}\pi G\rho(t)$$

Before Matter Dominates ($a<a_{eq}$): $H^2 \propto a^{-4}$ ⇒ $a \propto t^{1/2}$

When Matter Dominates: $H^2 \propto a^{-3}$ ⇒ $a \propto t^{2/3}$

When Cosmological Constant Dominates (future): $H^2 \propto \mathrm{const}$ ⇒ $a \propto e^{H\,t}$

Expansion History of the Universe

Friedmann Equation (flat universe): $$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8}{3}\pi G\rho(t)$$

very early on (T~10[13] GeV) Inflation potential dominates: $H^2 \propto V(\phi) \sim \text{const}$ ⇒ $a \propto e^{H\,t}$

Before Matter Dominates ($a<a_{eq}$): $H^2 \propto a^{-4}$ ⇒ $a \propto t^{1/2}$

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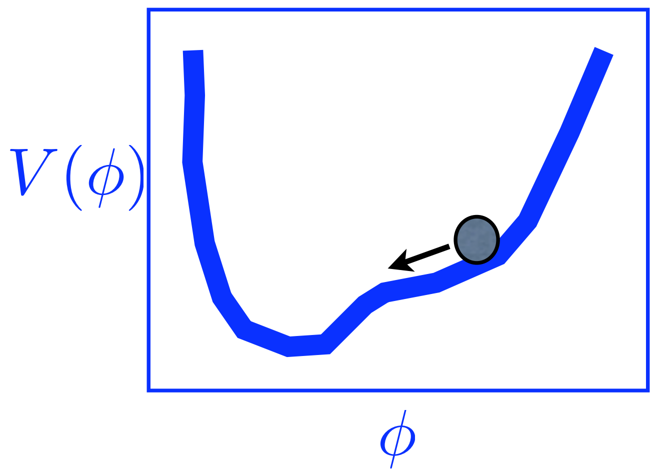
t~10^{-34} seconds: Inflation



$$H^2 \propto V(\phi) \sim \text{const} \Rightarrow a \propto e^{H\,t}$$

inflaton field has quantum fluctuations

$$\delta\phi \sim H$$

different regions of the universe end inflation at slightly different times

$$\delta t \simeq \frac{\delta\phi}{\dot{\phi}} \simeq \frac{H}{\dot{\phi}}$$

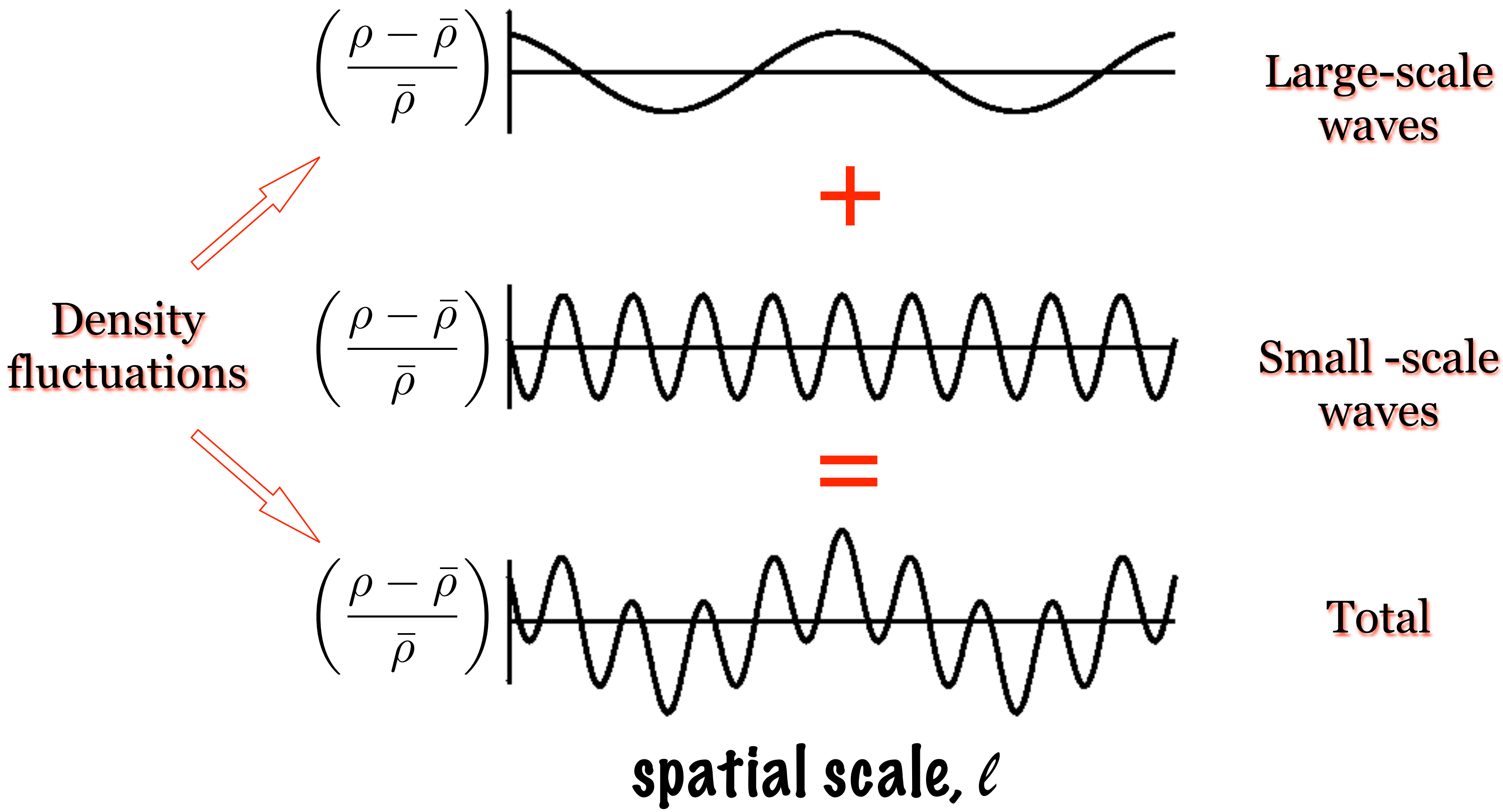
⇒ shape of inflation potential gives fluctuation amplitudes and distribution with length scale

spatial curvature is left with ripples, which correspond to density perturbations:

$$\frac{\delta\rho}{\rho} \simeq H\delta t \simeq \frac{H^2}{\dot{\phi}}$$

⇐ fluctuations are **Gaussian** (in simple models). with this RMS value.

Initial fluctuations in the density on different scales, with amplitudes set by inflation



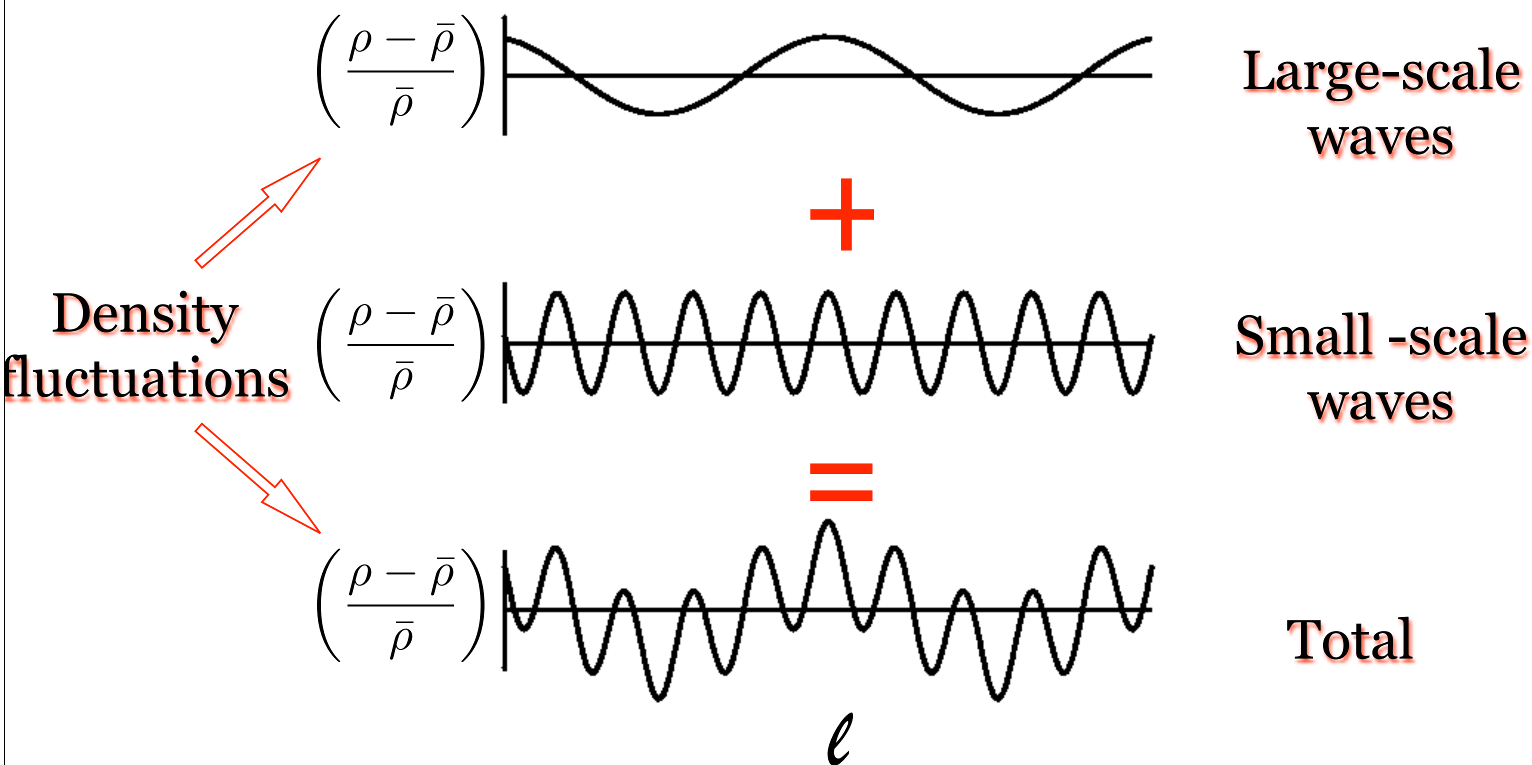
Characterizing Fluctuations

P(k)=power spectrum

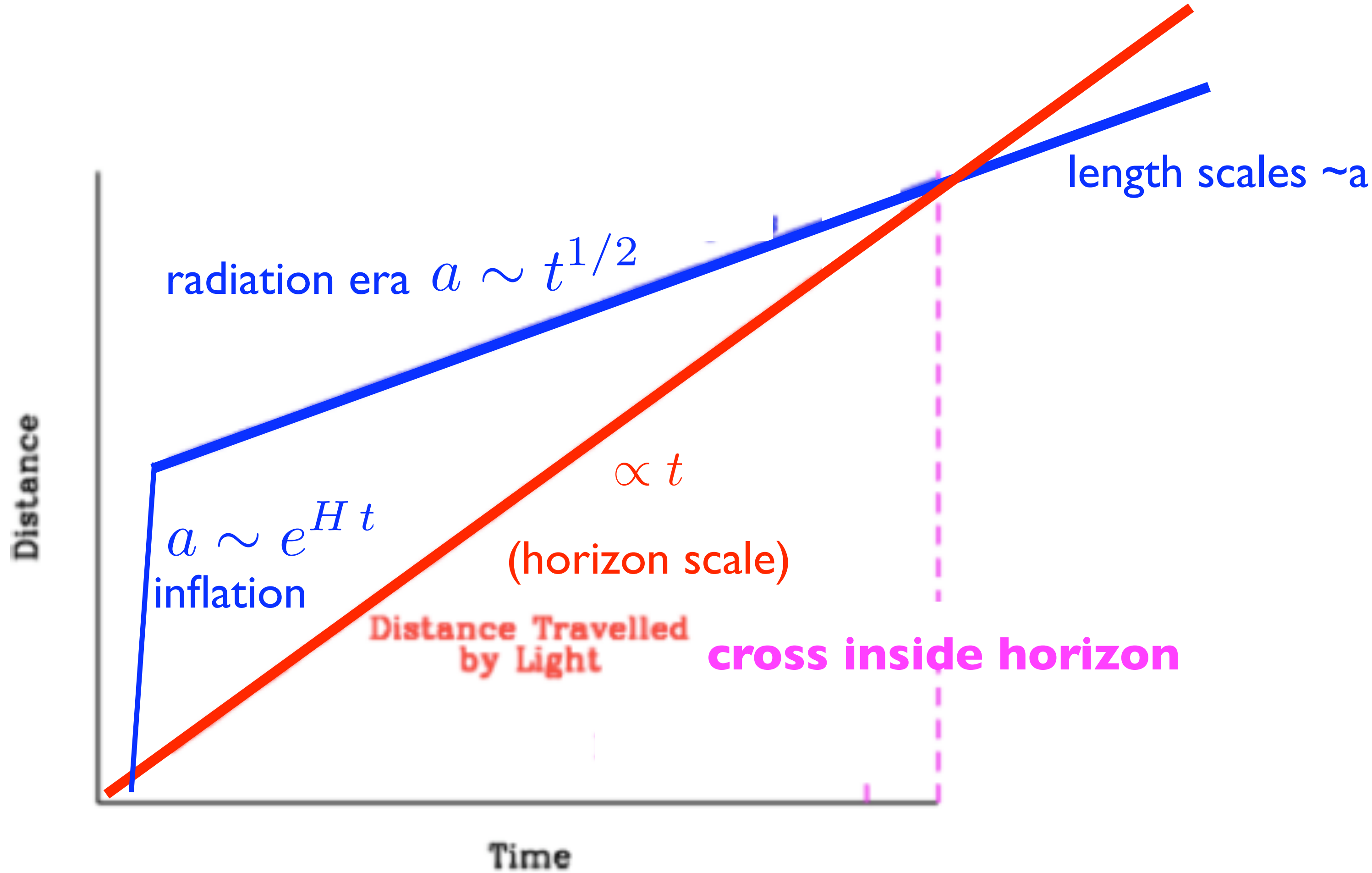
$$\delta^2(R) \simeq \int_0^{1/R} \Delta^2(k)\, d\ln k \qquad \Delta^2(k) \equiv k^3 P(k)/2\pi^2$$

Mass variance on a scale R

Contribution due to modes with λ~1/k



a fluctuation of size λ will cross inside horizon when $(a\,\lambda) = (c\,t)$



Inside horizon, perturbations only grow during matter phase

Friedmann Equation (flat universe):

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8}{3}\pi G\rho(t)$$

$$t_{exp} \sim H^{-1} \sim \frac{1}{\sqrt{G\rho_u}} \qquad t_{collapse} \sim \frac{1}{\sqrt{G\rho_{matter}}}$$

if $\rho_{matter} < \rho_u$ $\Rightarrow$ $t_{collapse} > t_{exp}$

universe expands too fast for fluctuations to grow.

$\Rightarrow$ $\dfrac{\delta\rho}{\rho} \sim \text{constant}$

early times.
during radiation phase

Perturbations always grow during matter phase

Friedmann Equation (flat universe):

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8}{3}\pi G\rho(t)$$

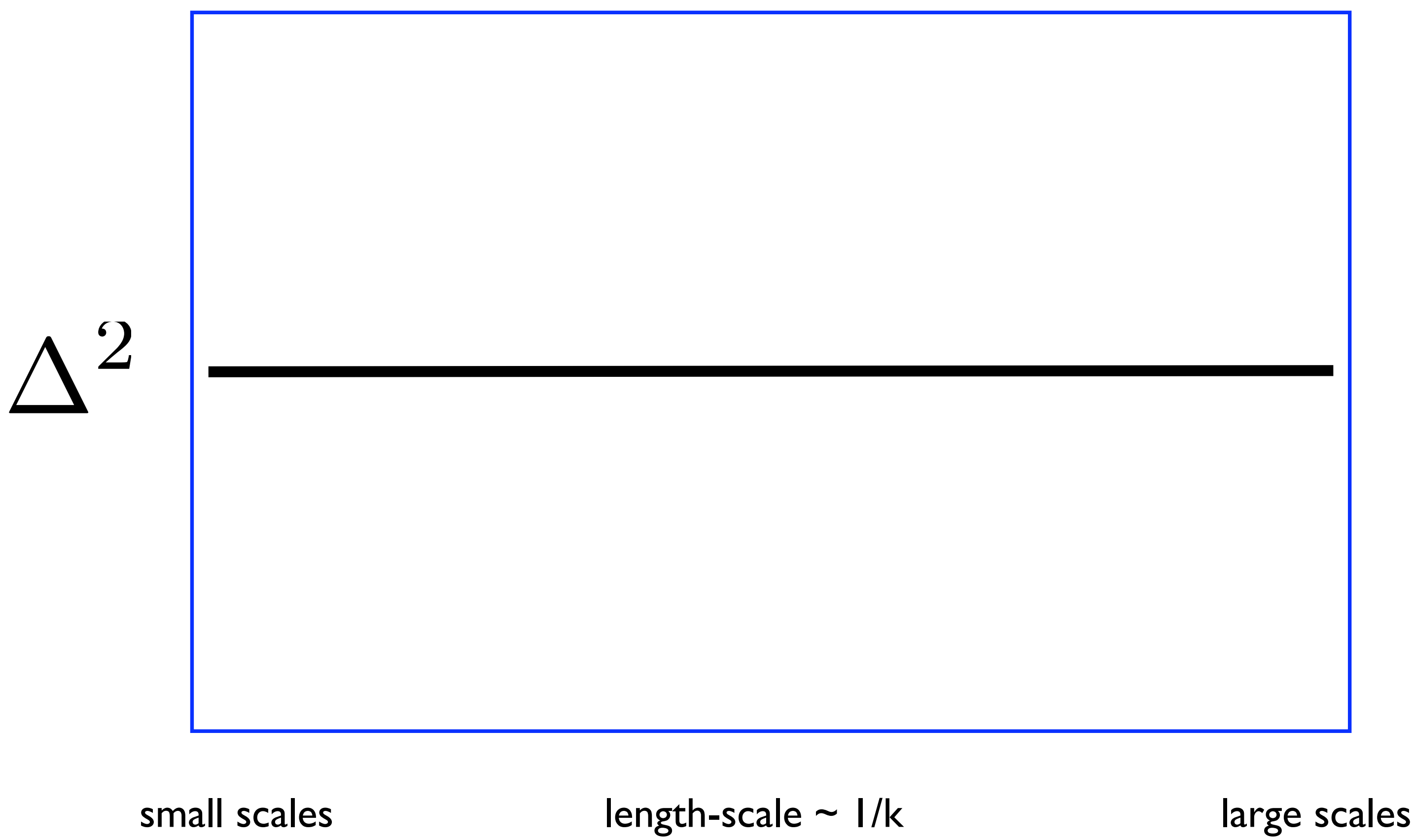
if $\rho_{matter} = \rho_u$ ⇒ overdense regions expand slightly slower than the average

One can show: ⇒ $\frac{\delta\rho}{\rho} \propto a$

late(r) times.
during matter phase

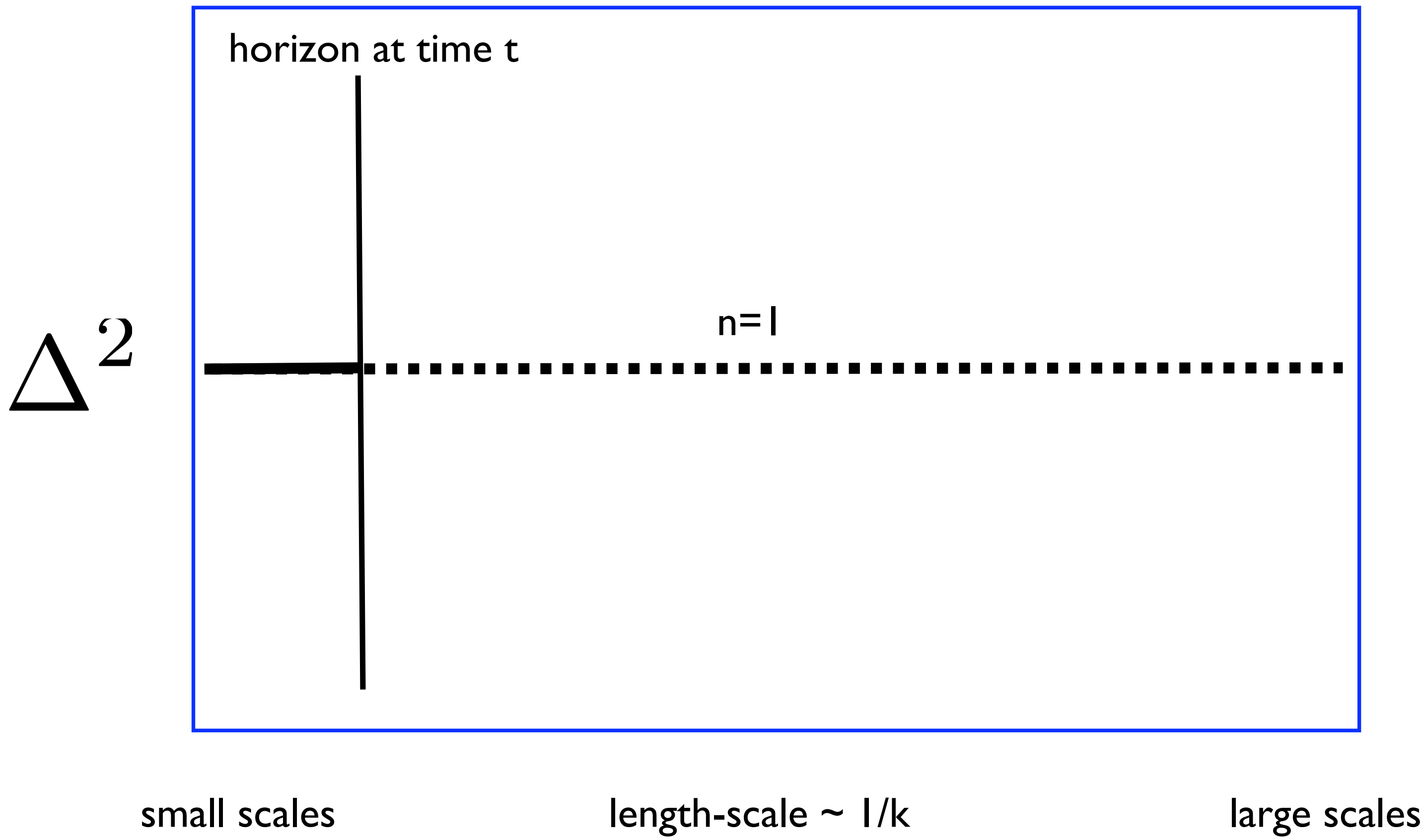
consider a model where fluctuation amplitudes are the same on all scales as they first cross inside horizon

this is true to first-order in inflation -- model is said to have a “tilt” of n=1



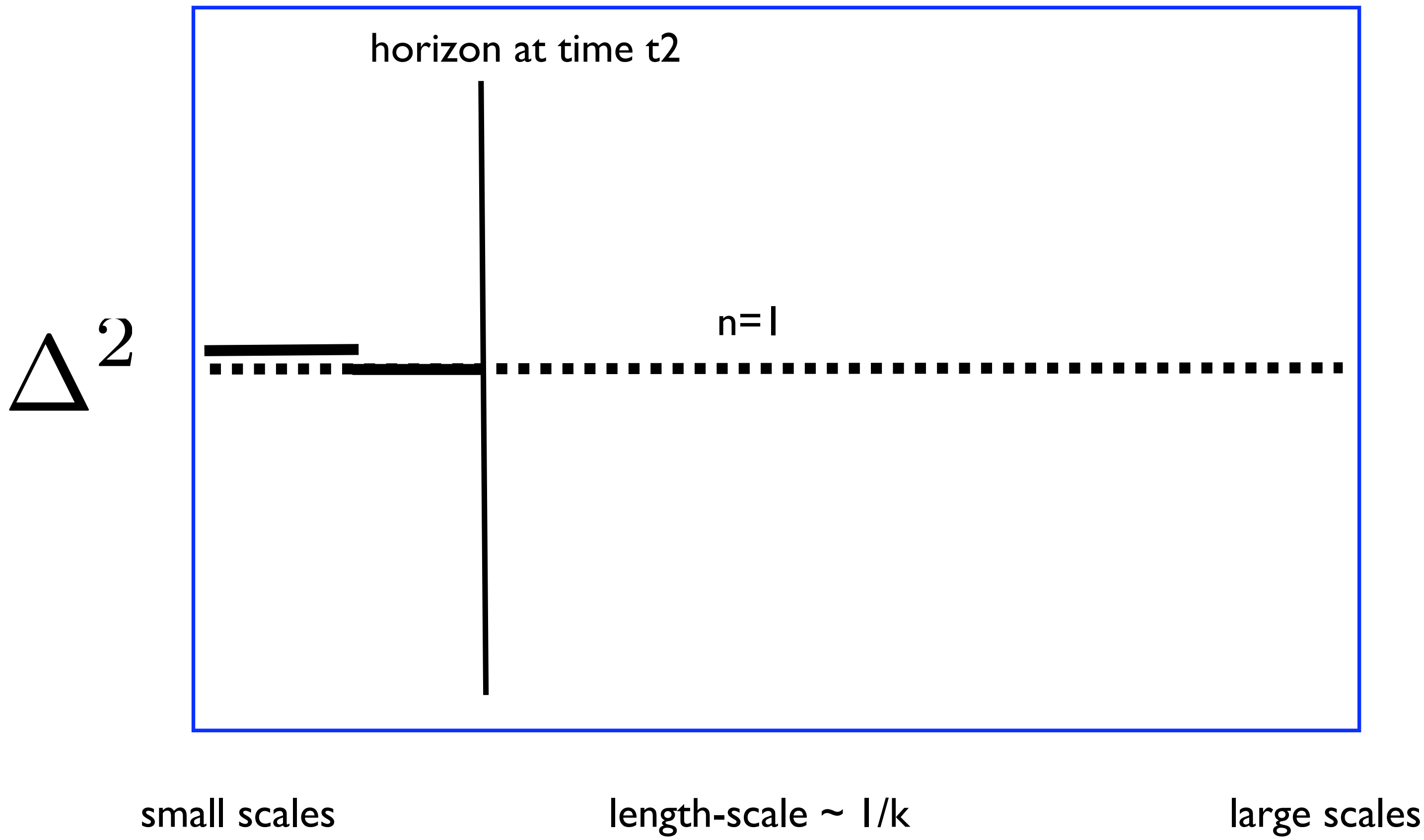
time $t_1 << t_{mat=rad}$

small scale fluctuations cross inside horizon during radiation dominated era: grow very slowly



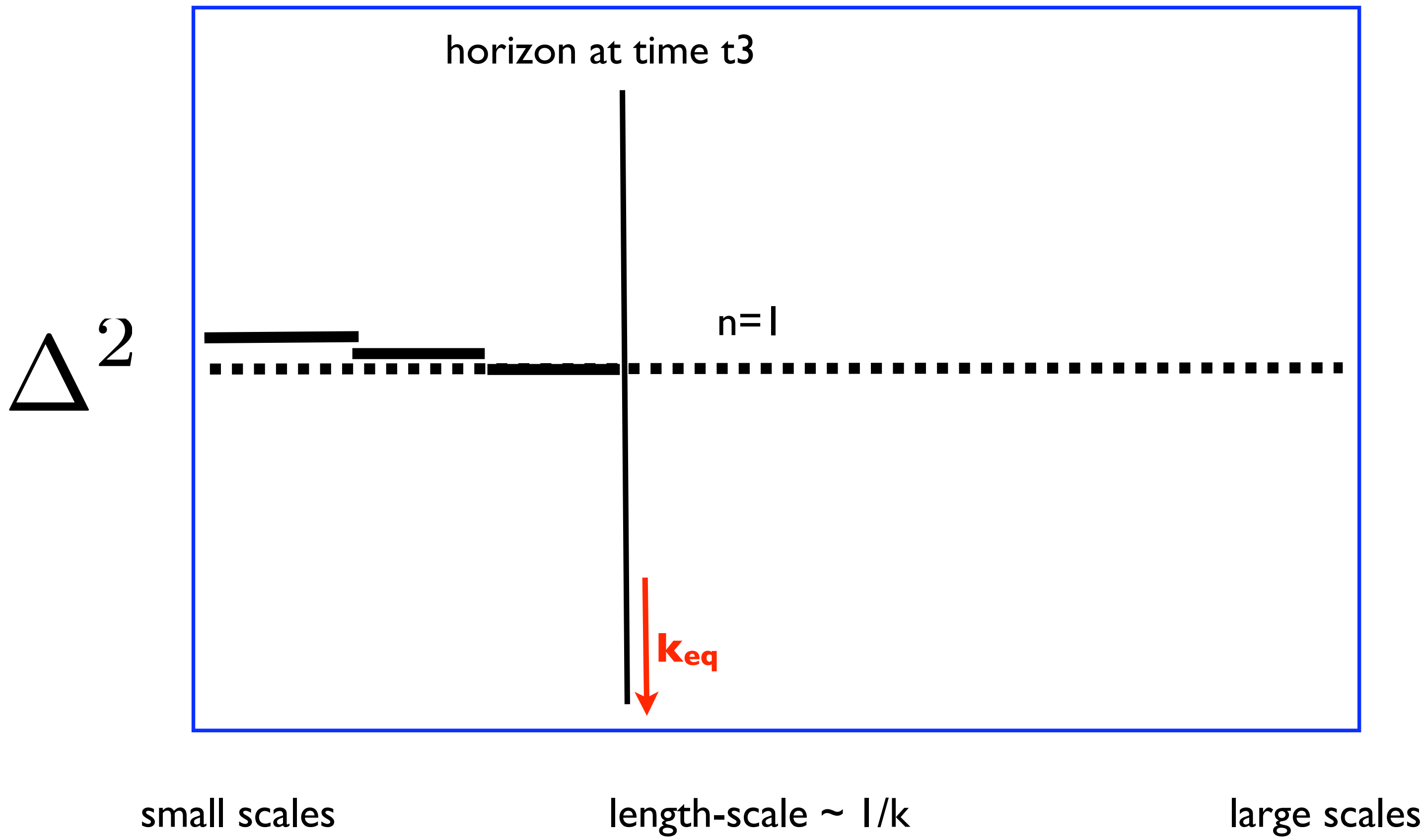
time $t_2 << t_{mat=rad}$

small scale fluctuations cross inside horizon during radiation dominated era: grow very slowly



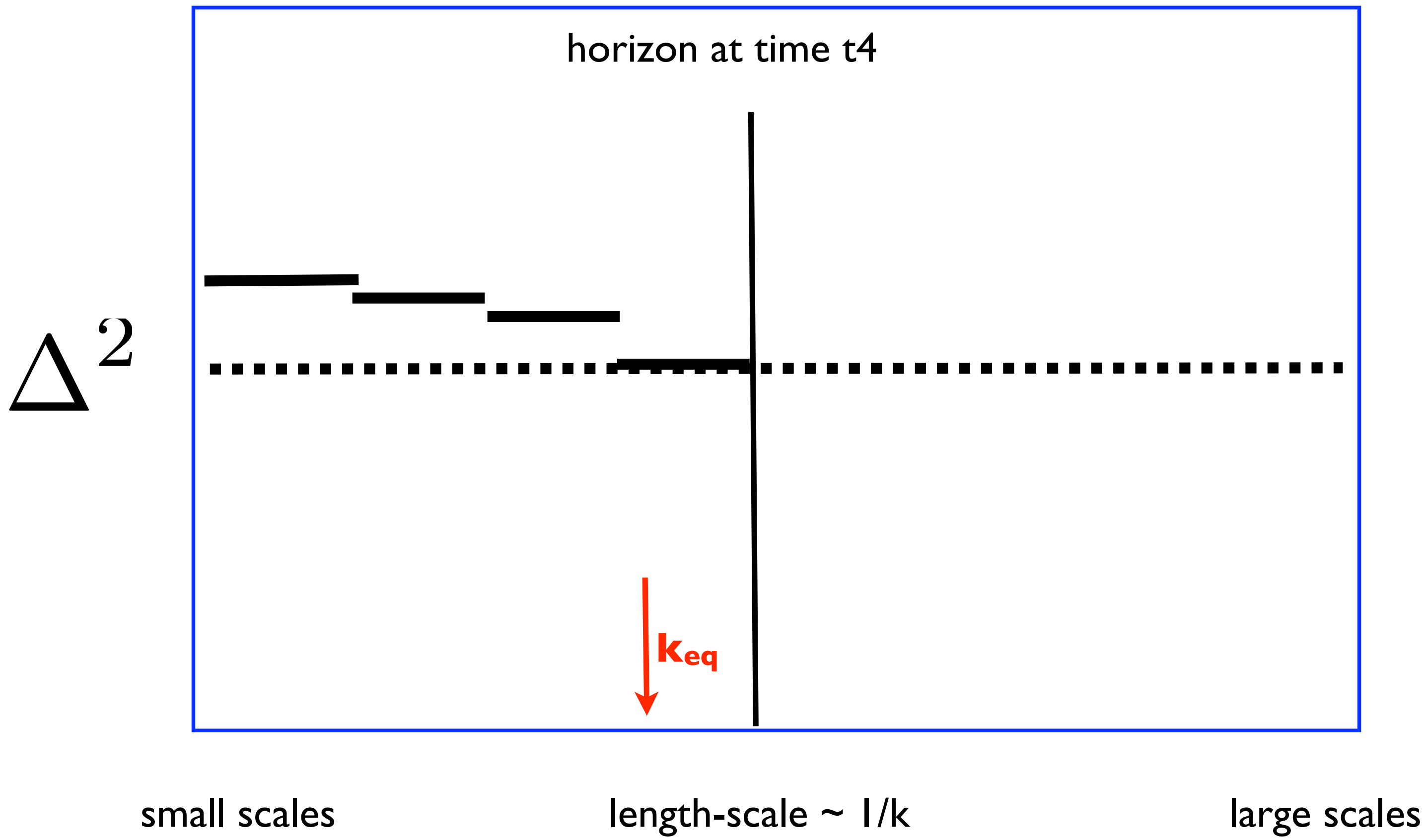
time t_3 ~< $t_{mat=rad}$

small scale fluctuations cross inside horizon during radiation dominated era: grow very slowly



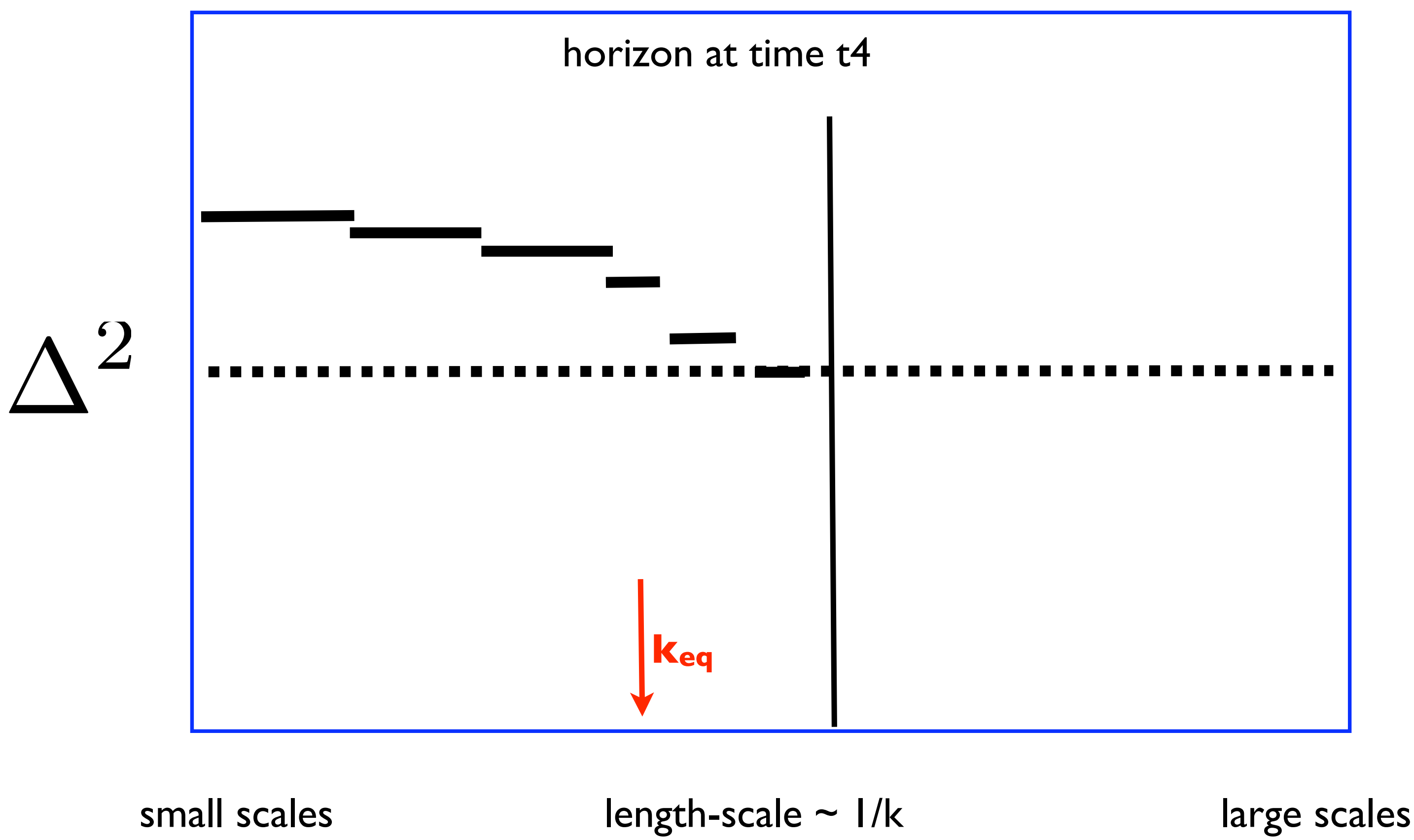
time t_4 >~ $t_{mat=rad}$

after matter dominated era: growth proceeds quickly



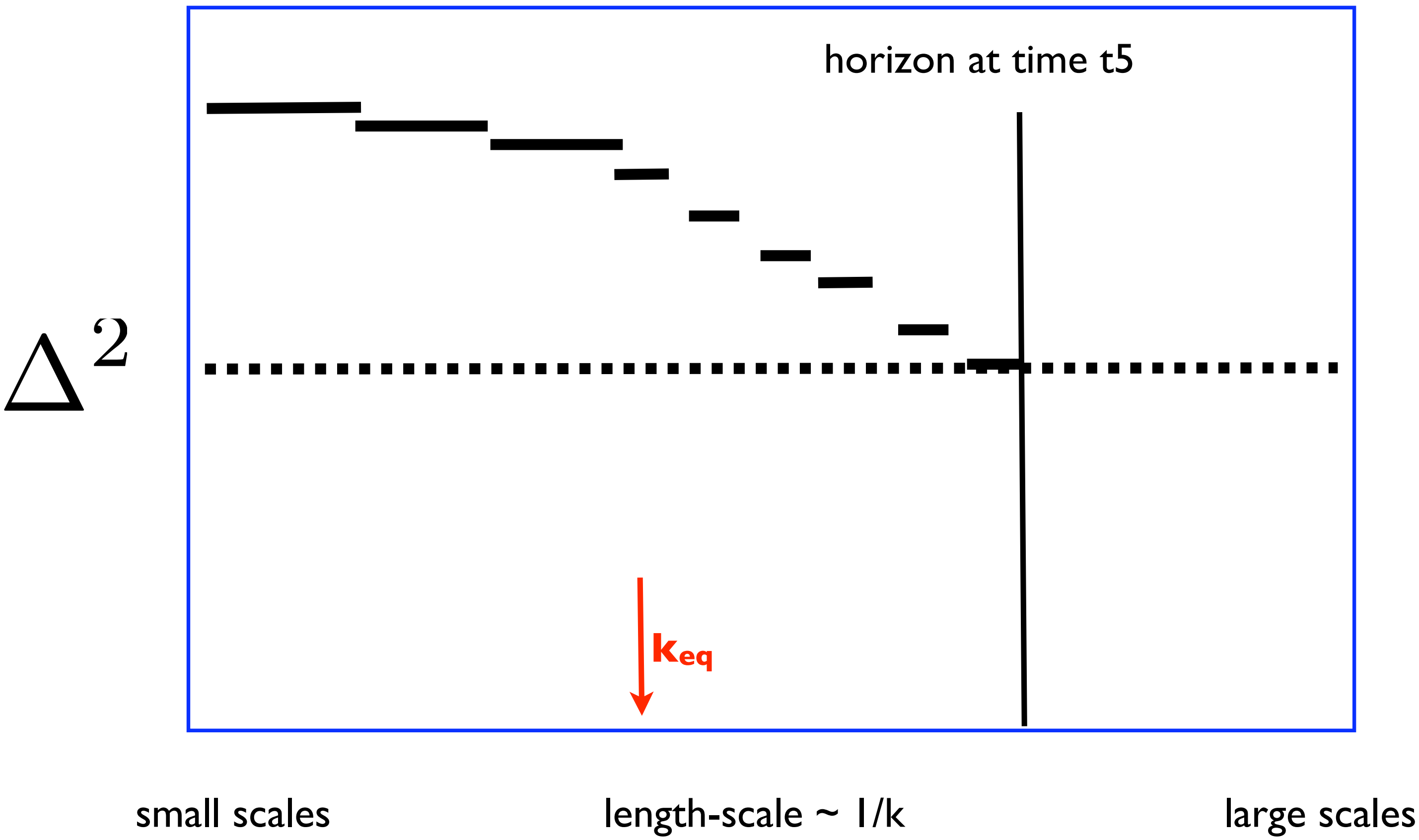
time $t_4 > t_{mat=rad}$

after matter dominated era: growth proceeds quickly

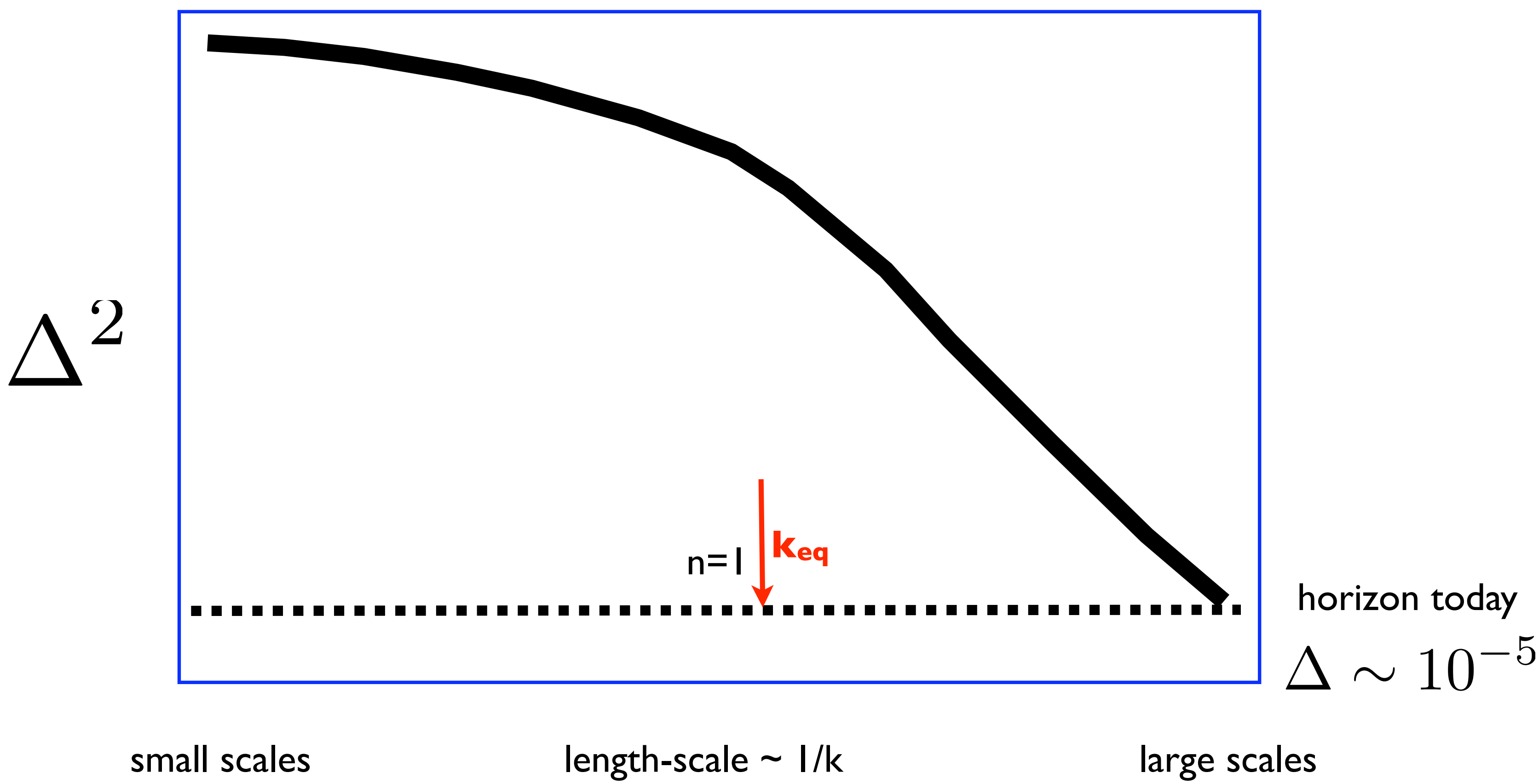


time t_5 >> $t_{mat=rad}$

after matter dominated era: growth proceeds quickly



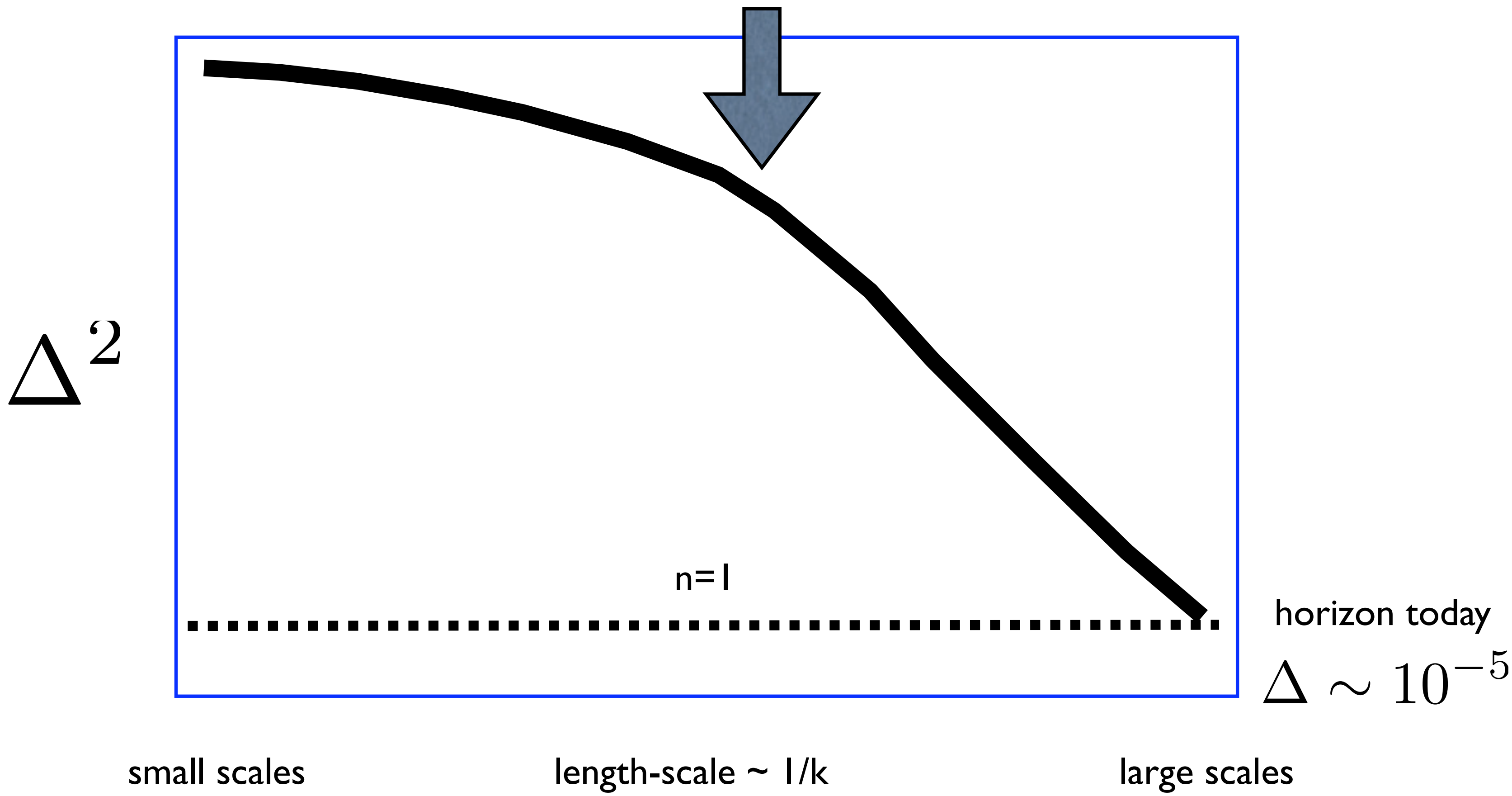
eventually



Δ^2
n=1
k_{eq}
horizon today
$\Delta \sim 10^{-5}$
small scales
length-scale ~ 1/k
large scales

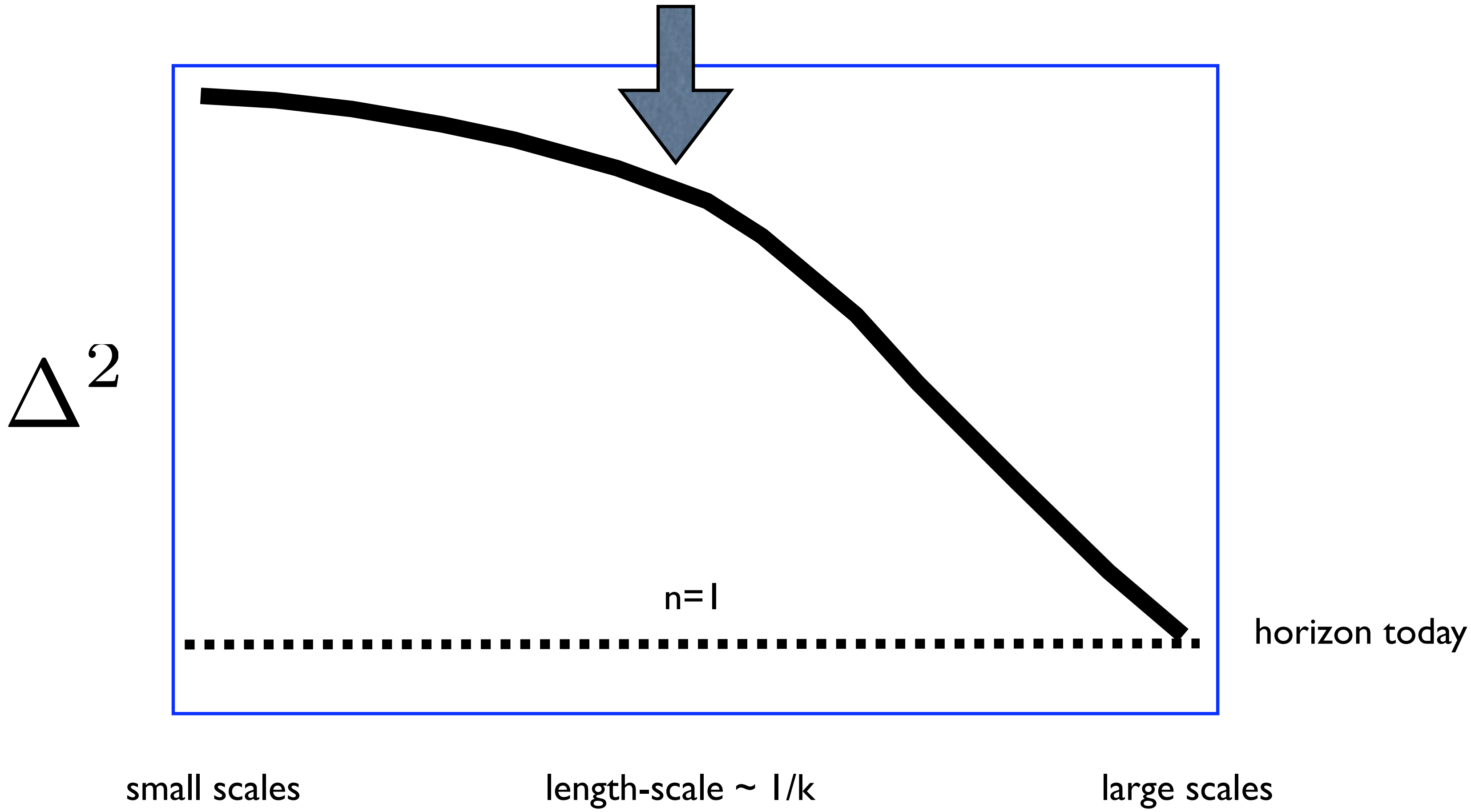
eventually

this is horizon scale at matter-radiation equality



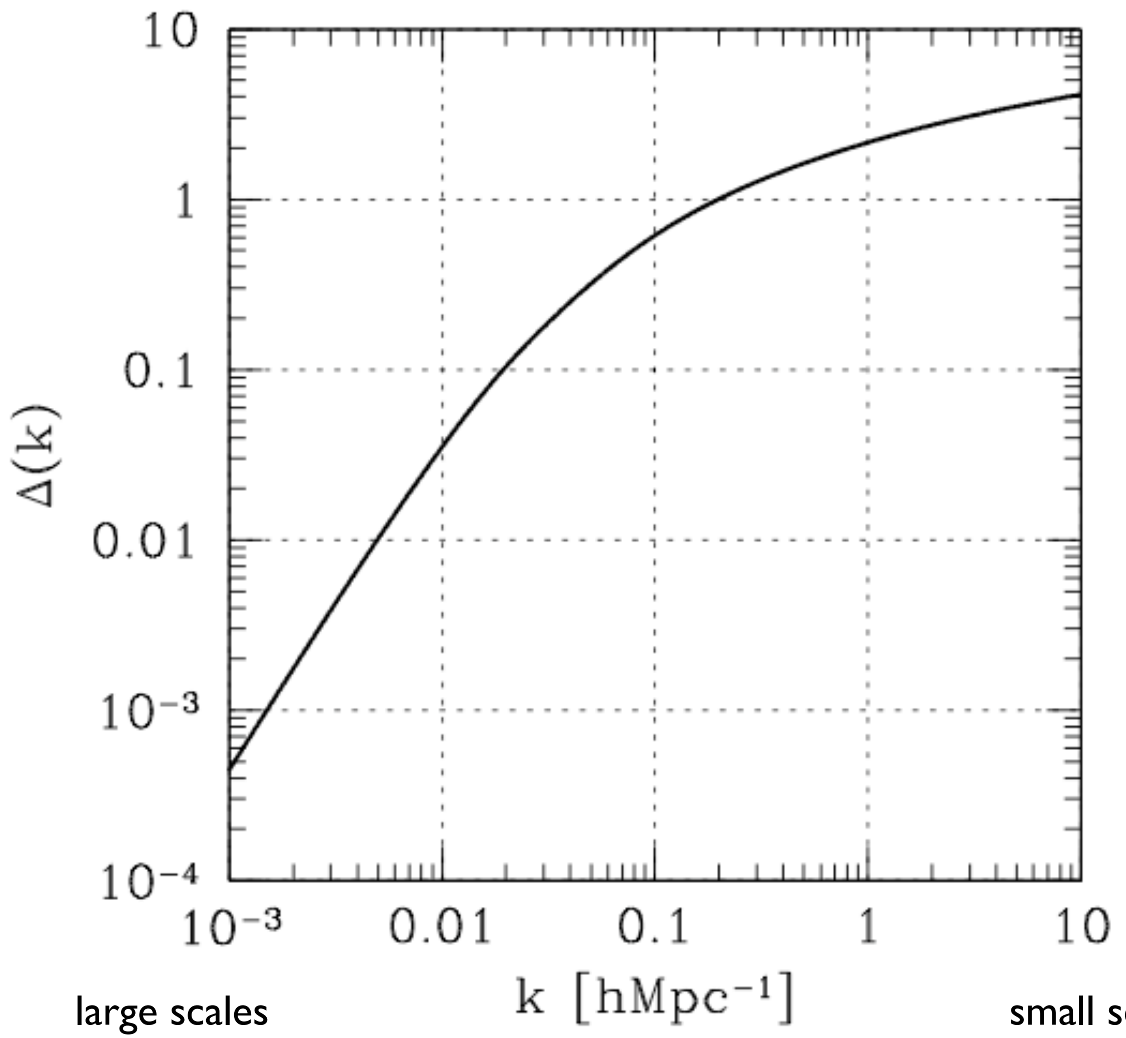
scale is set by matter-radiation equality

$$\lambda_{eq} a_{eq} \equiv c t_{eq} \quad \Rightarrow \quad k_{eq} = \frac{2\pi}{\lambda_{eq}} \simeq 0.1\, Mpc^{-1} \left(\frac{0.15}{\Omega_m h^2} \right)^{-1}$$



more precisely...

$$\Omega_\Lambda - 1 = \Omega_M = 0.3 \quad h = 0.7 \; \sigma_8 = 0.93 \; \Omega_b h^2 = 0.02$$



large scales

small scales

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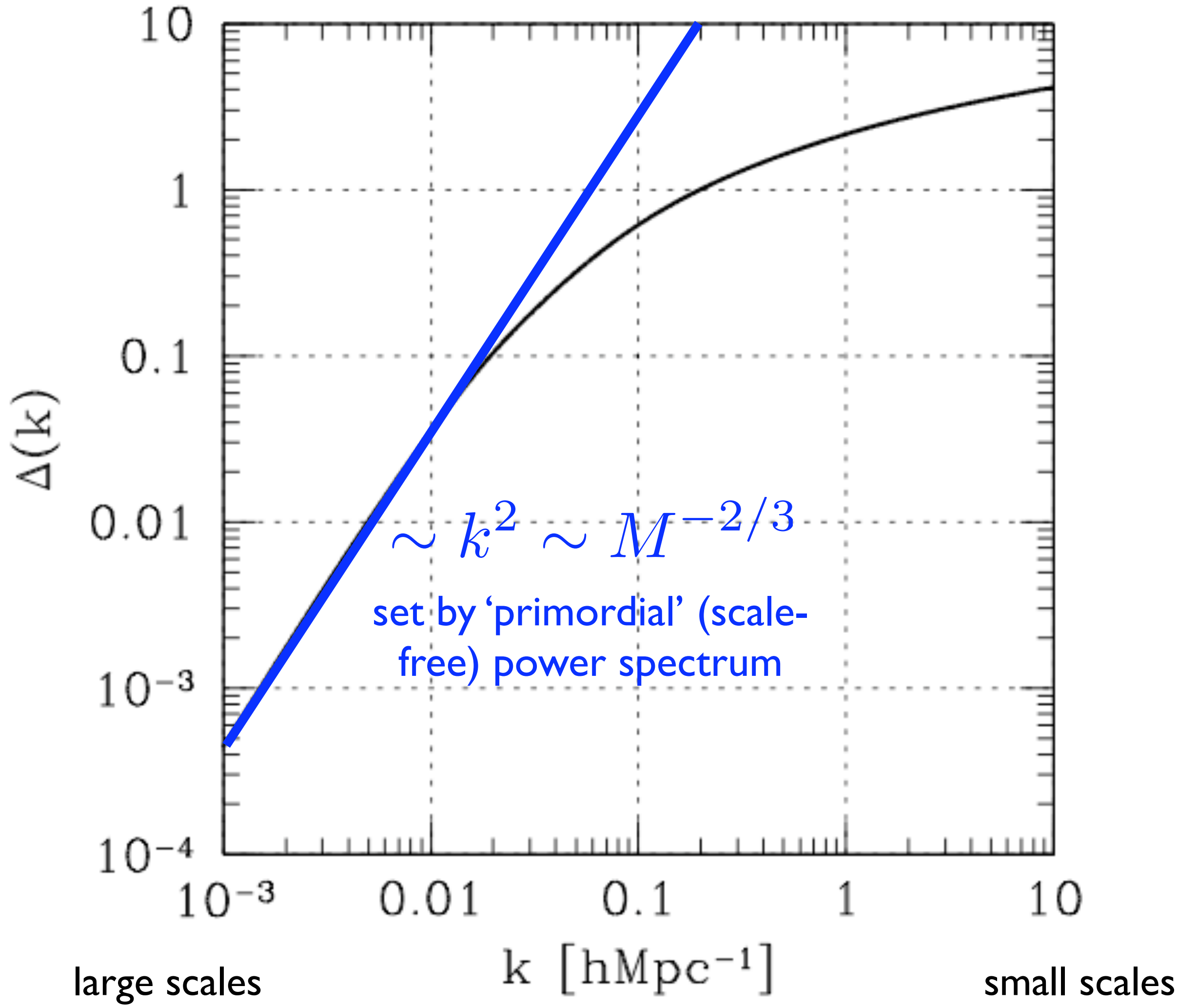

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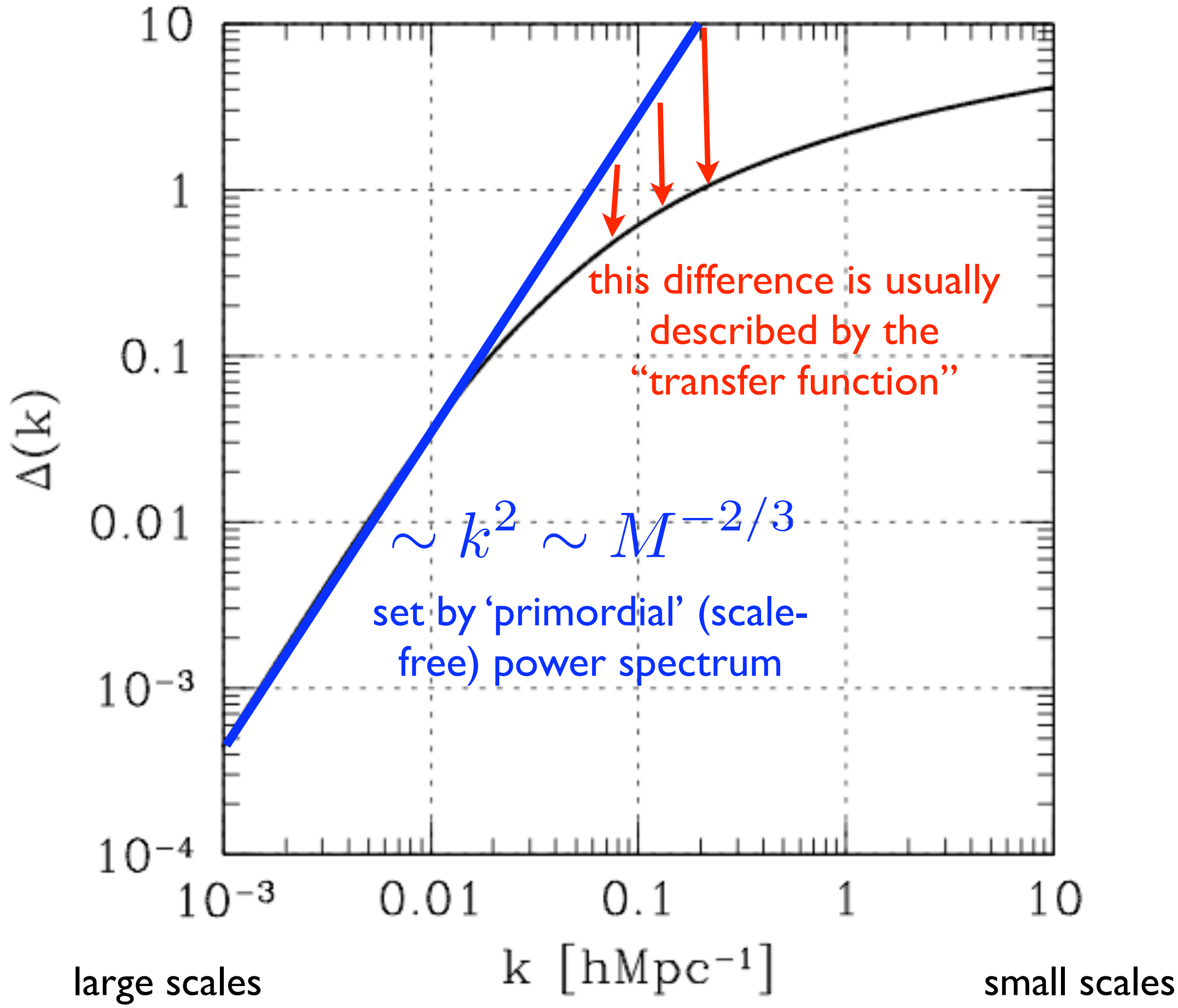


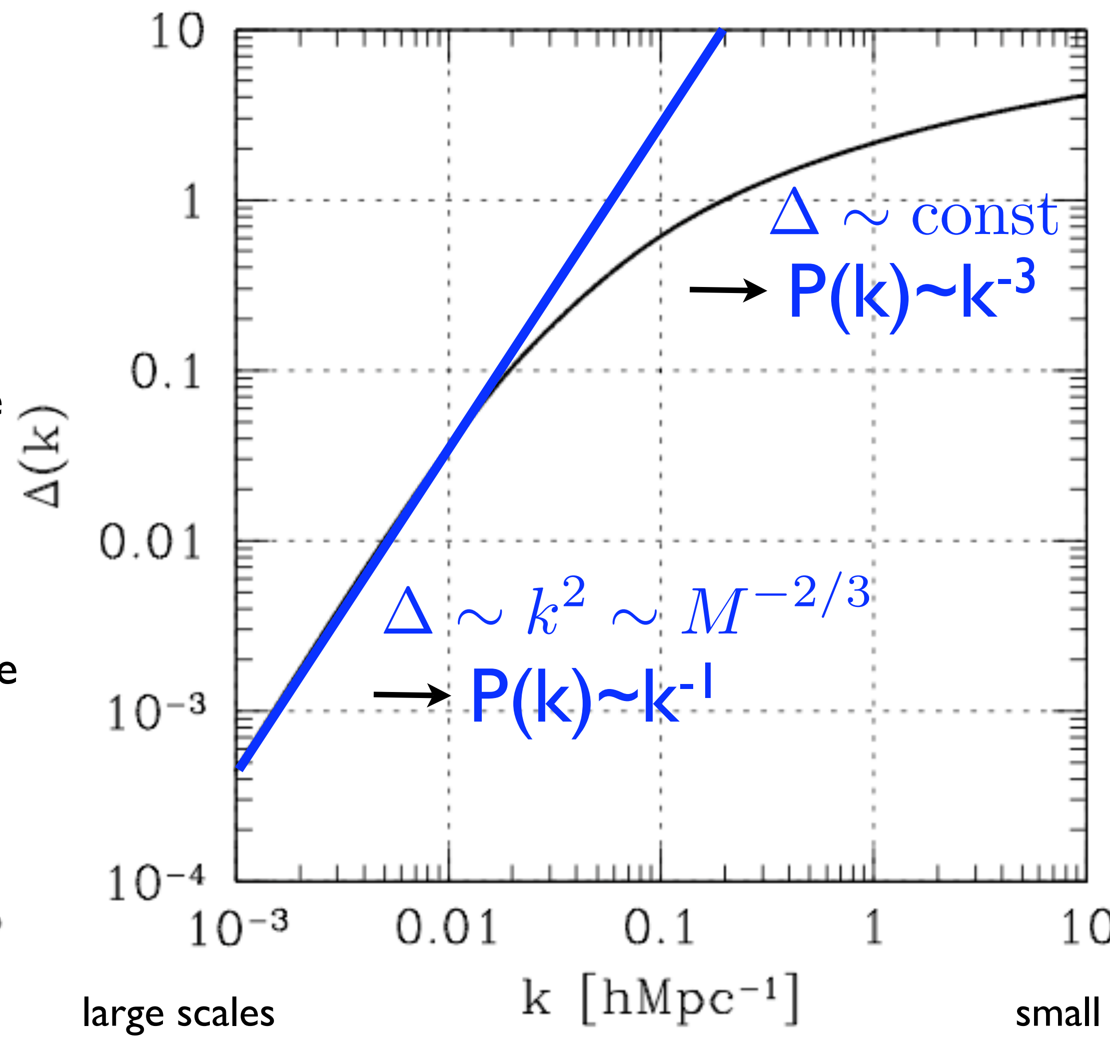
Relating to primordial power spectrum

$$\Delta^2 \propto k^3 P(k) \propto k^3 k^n$$

P(k) is the power spectrum

$P(k) \sim k^n$
n=1 gives scale invariance at horizon crossing

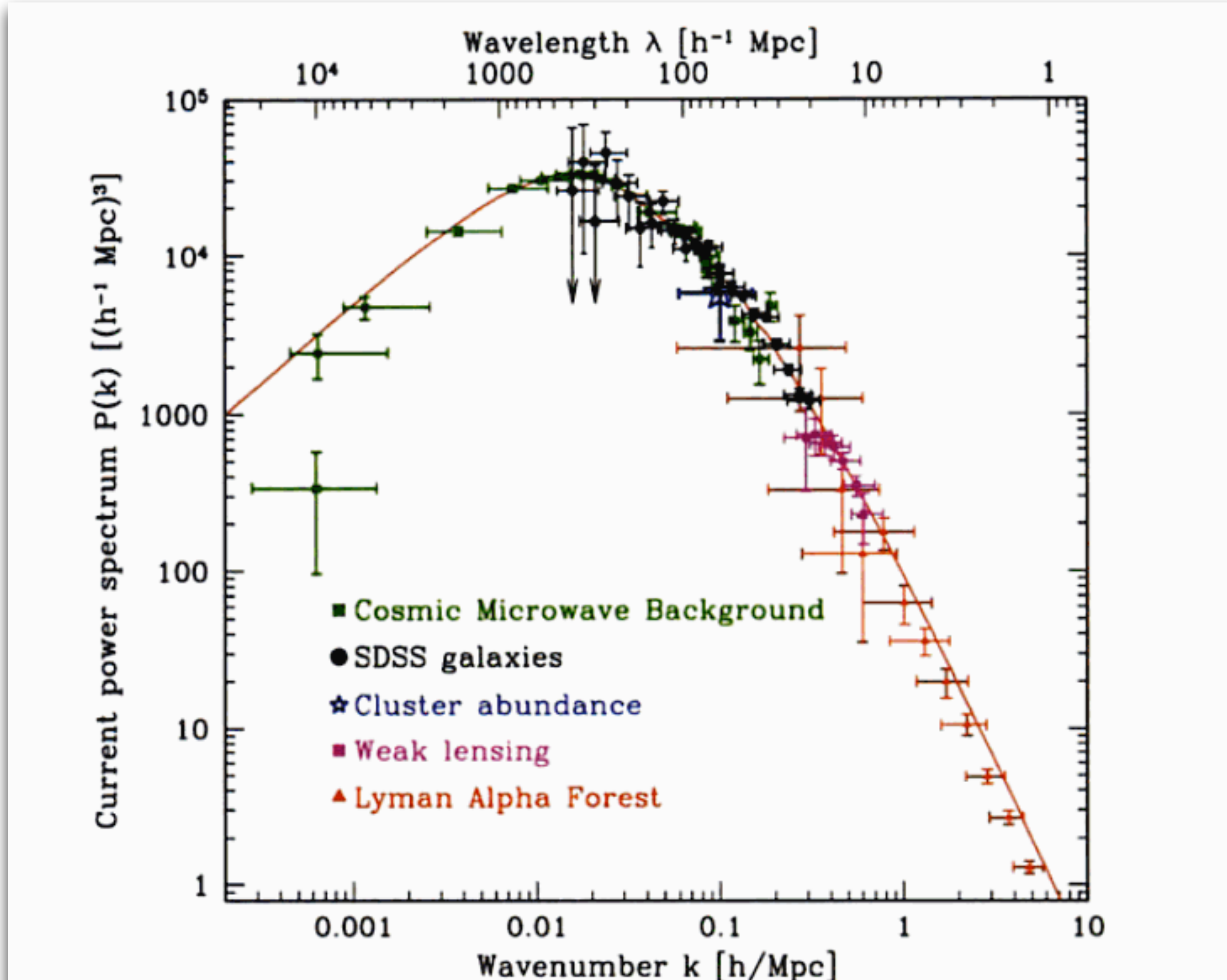
the 'tilt' n is usually used as the variable to describe the 'primordial' power spectrum



large scales

small scales

Clustering of non-linear universe looks a lot like LCDM...



$\Delta^2 \propto k^3 P(k) \propto k^3 k^n$

Tegmark compilation

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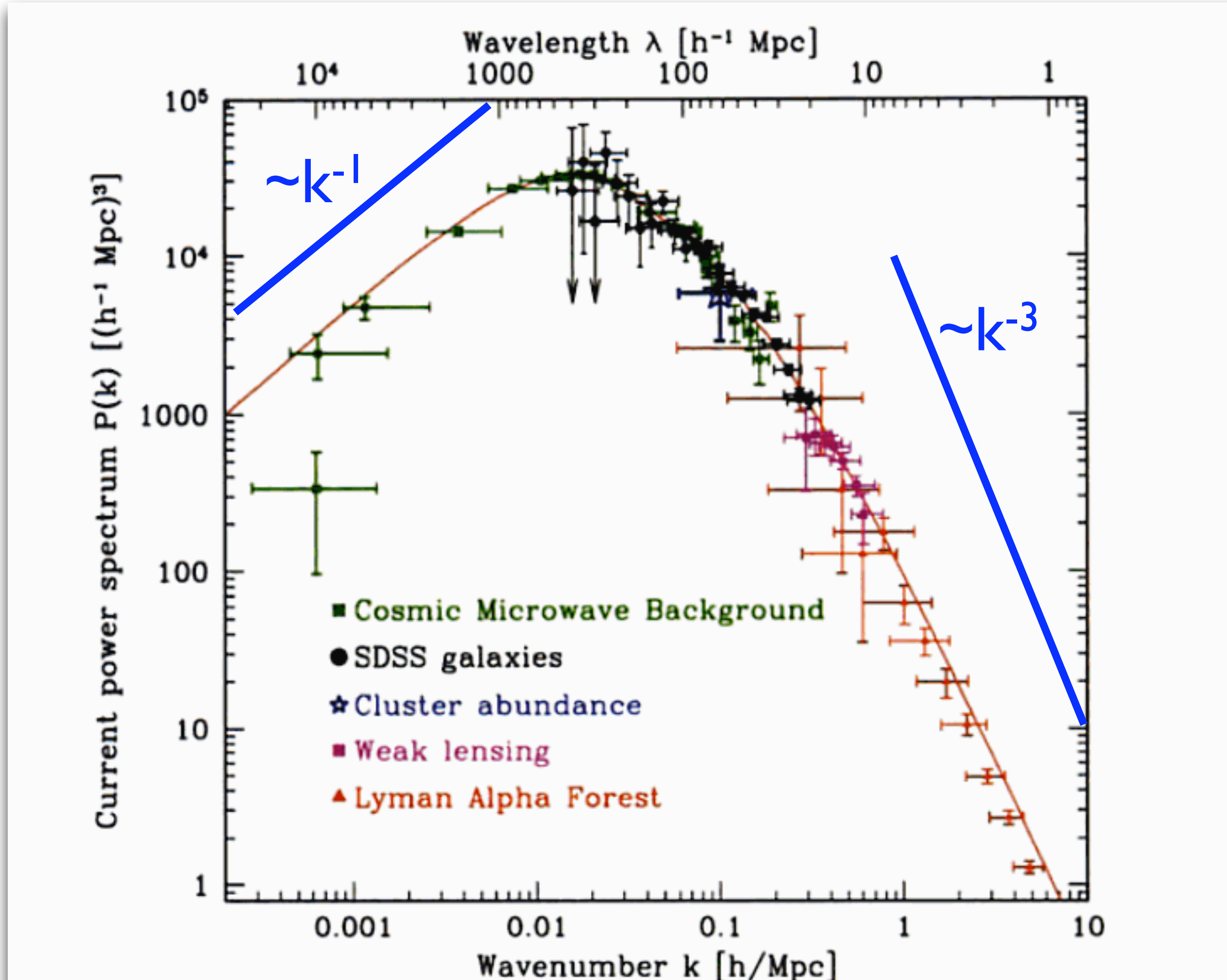


$\Delta^2 \propto k^3 P(k) \propto k^3 k^n$

Tegmark compilation

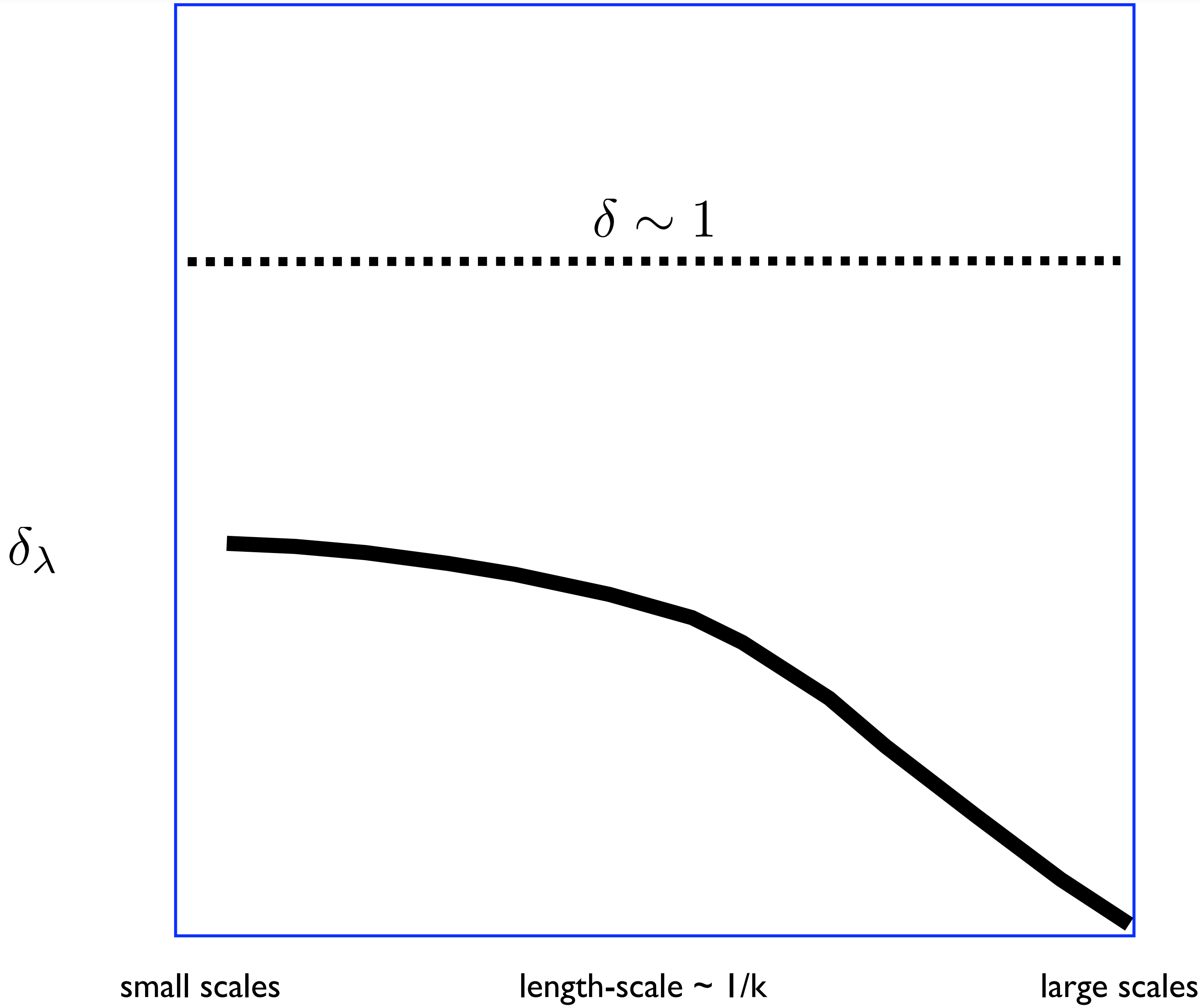
Growth of linear perturbations

$$\delta_\lambda(a) \propto D(a)$$ if $\lambda < \lambda_{eq}$

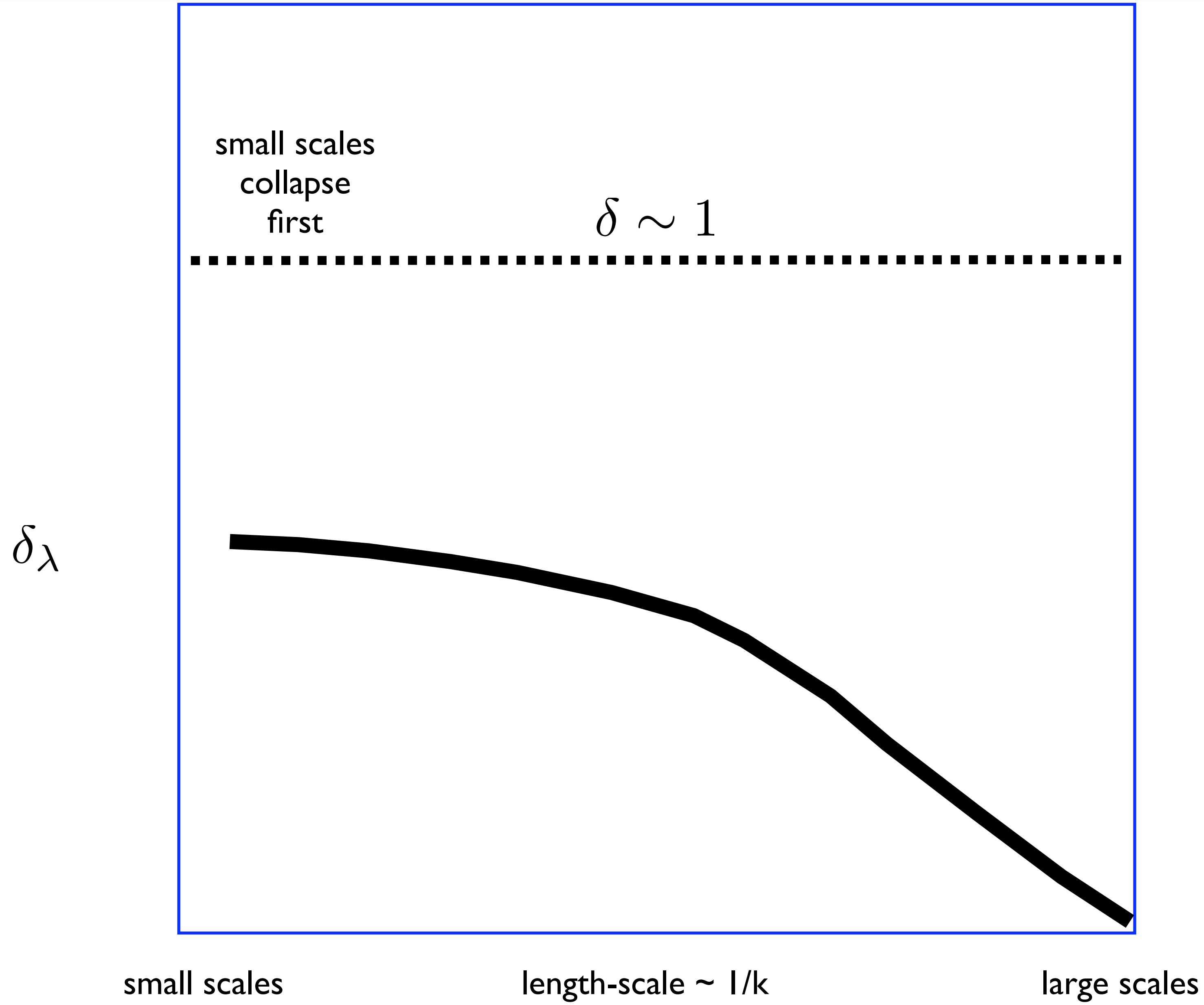
$$\delta_\lambda(a) \propto D(a) \left(\frac{\lambda}{\lambda_{eq}}\right)^{-2}$$ if $\lambda > \lambda_{eq}$

D(a) is the "growth function", which is D(a)~a for flat, matter universe.
For a LCDM universe, D(a)~a at early times, but D(a)~constant at late times

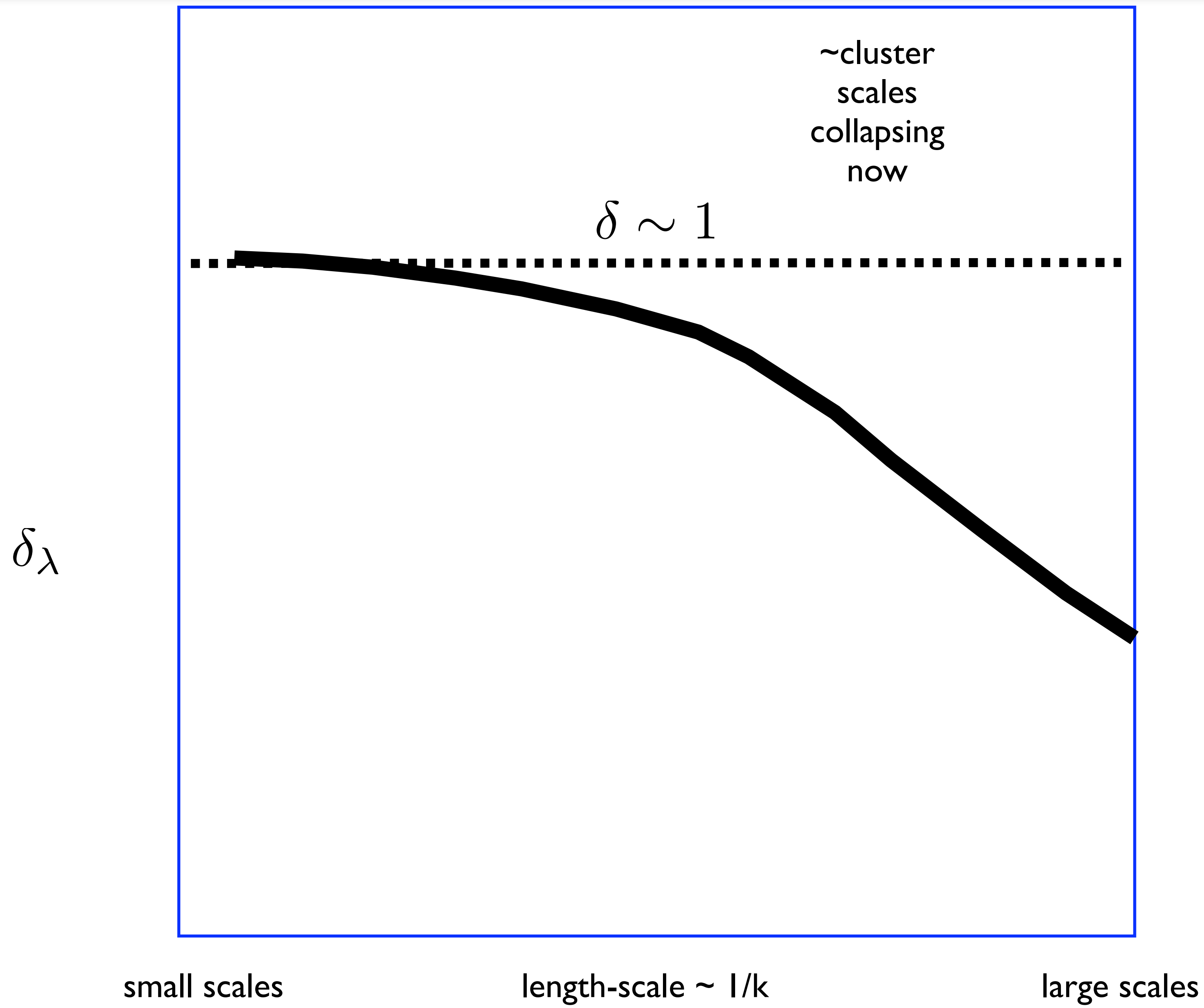
Growth of perturbations



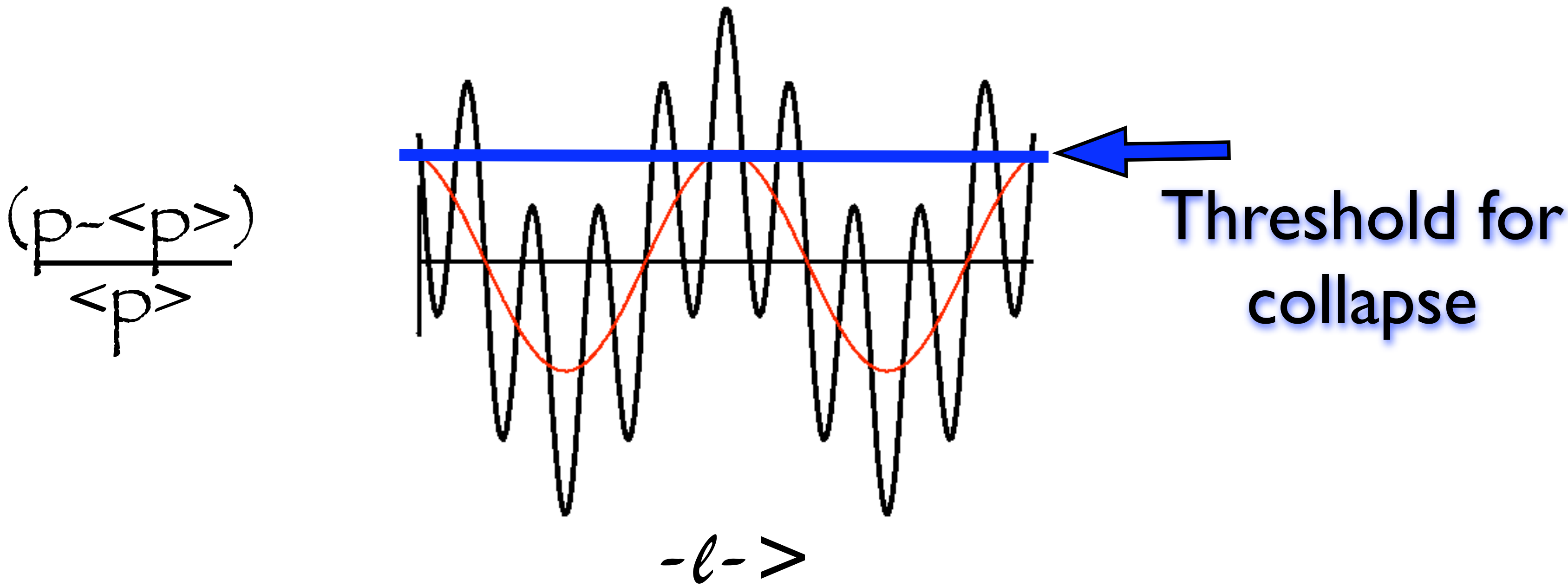
Early times


small scales
collapse
first
$\delta \sim 1$
δ_λ
small scales
length-scale ~ 1/k
large scales

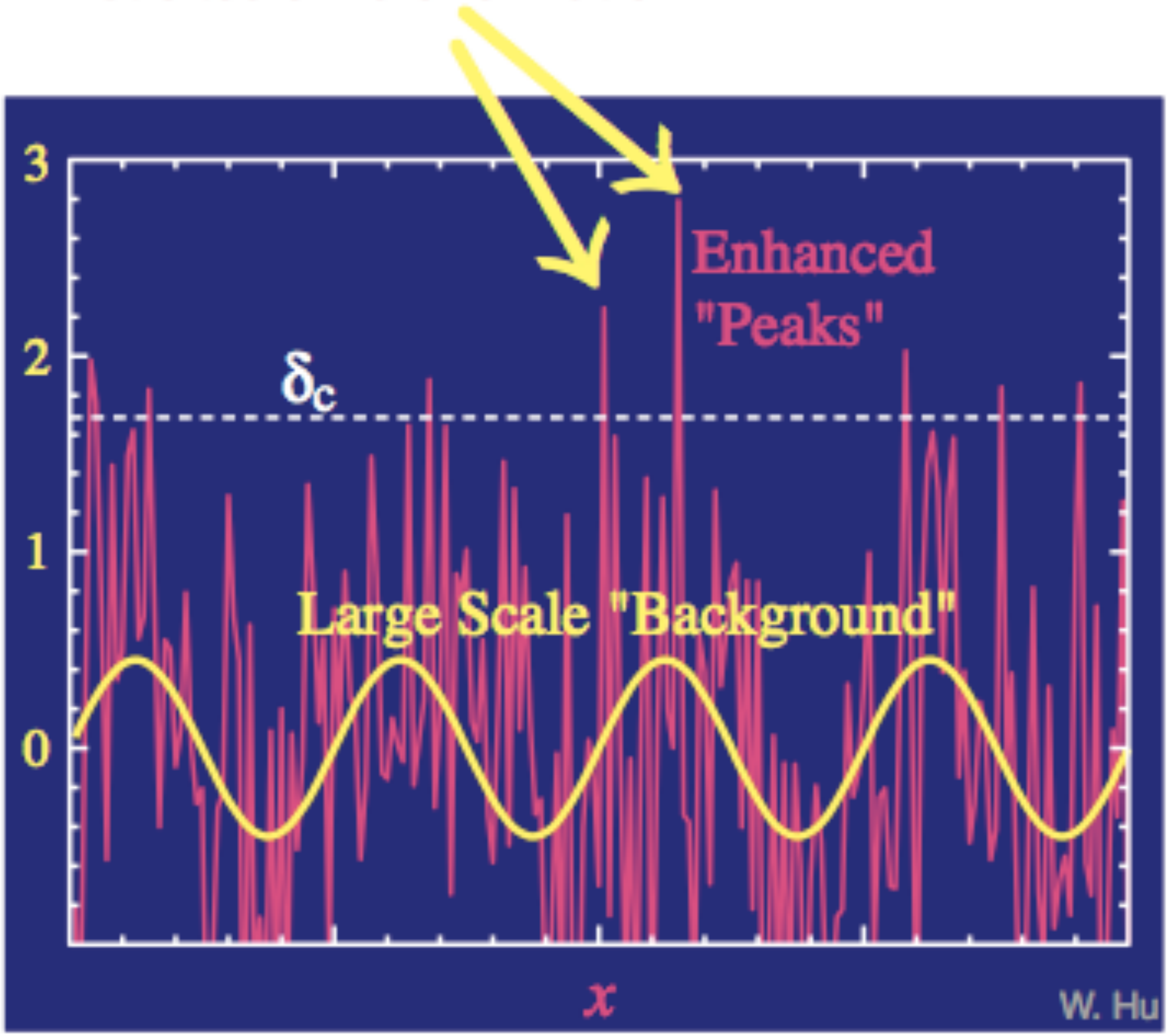
Late times


~cluster
scales
collapsing
now
$\delta \sim 1$
δ_λ
small scales
length-scale ~ 1/k
large scales

Fluctuation growth: small systems collapse first

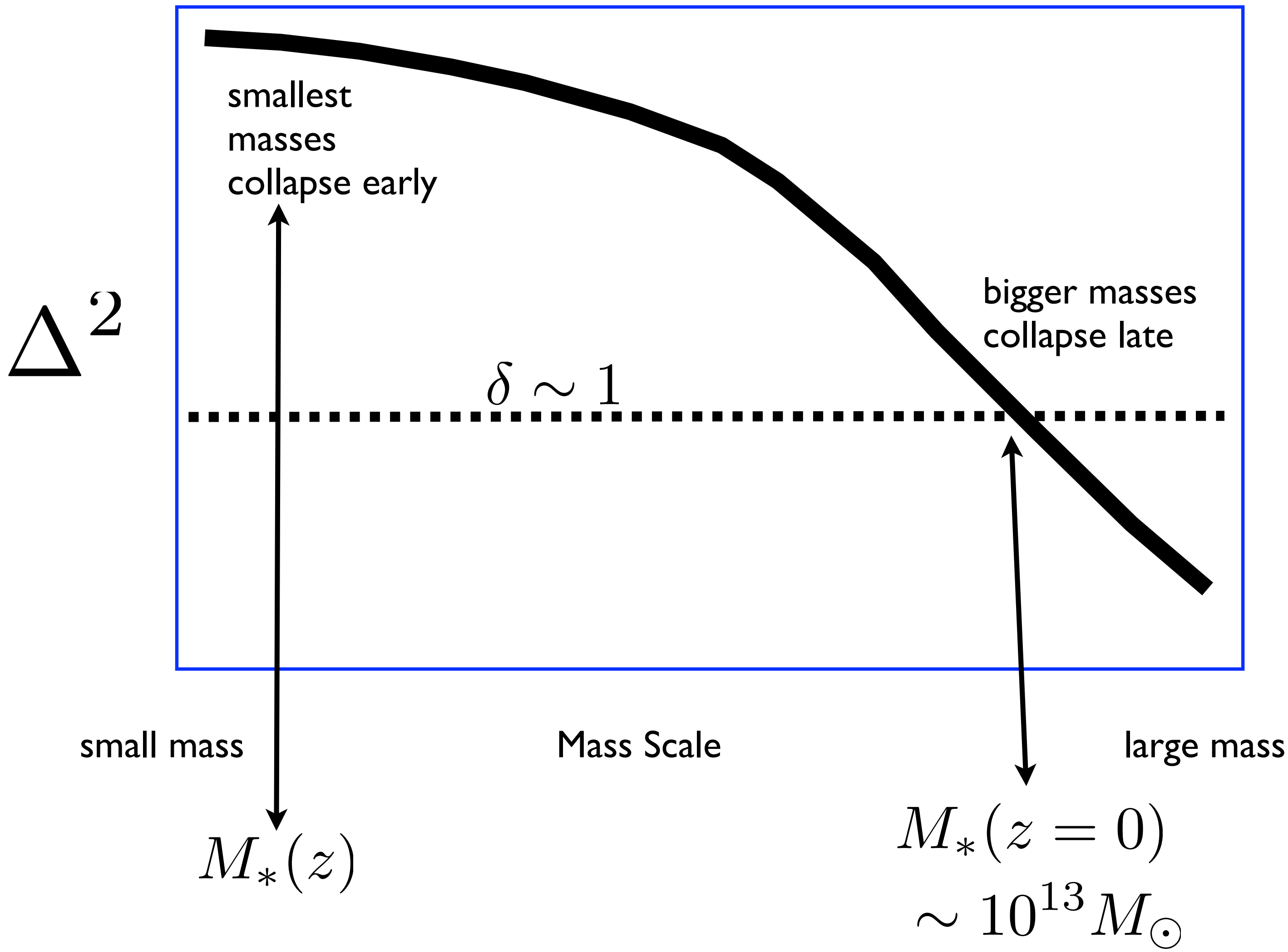


first sites of halo formation



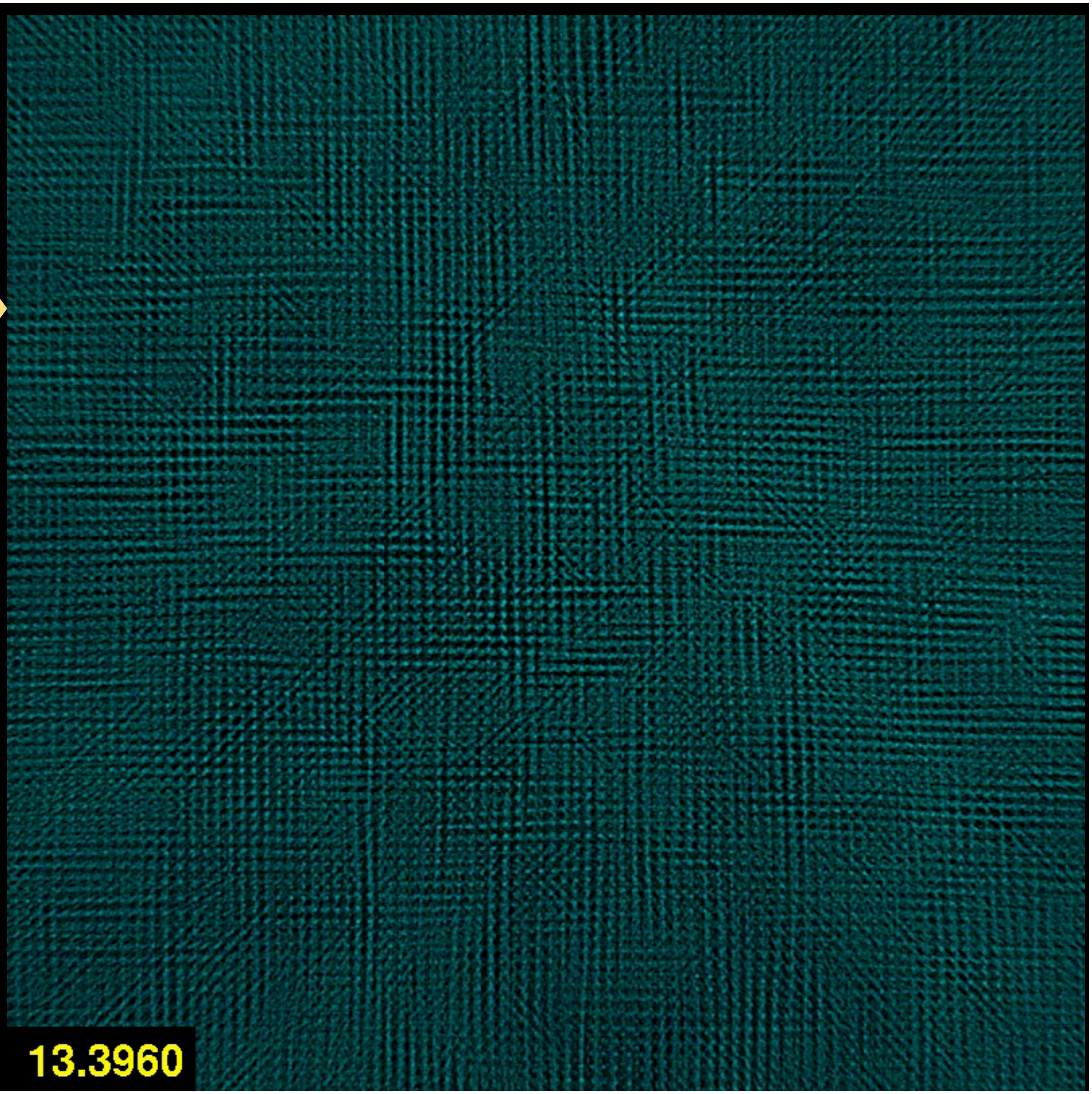
Mass of Typical Collapsing Objects: M*(z)

$$M \simeq \lambda^3 \rho \simeq k^{-3} \rho$$



Allgood et al. 06

LCDM simulations: Hierarchical growth

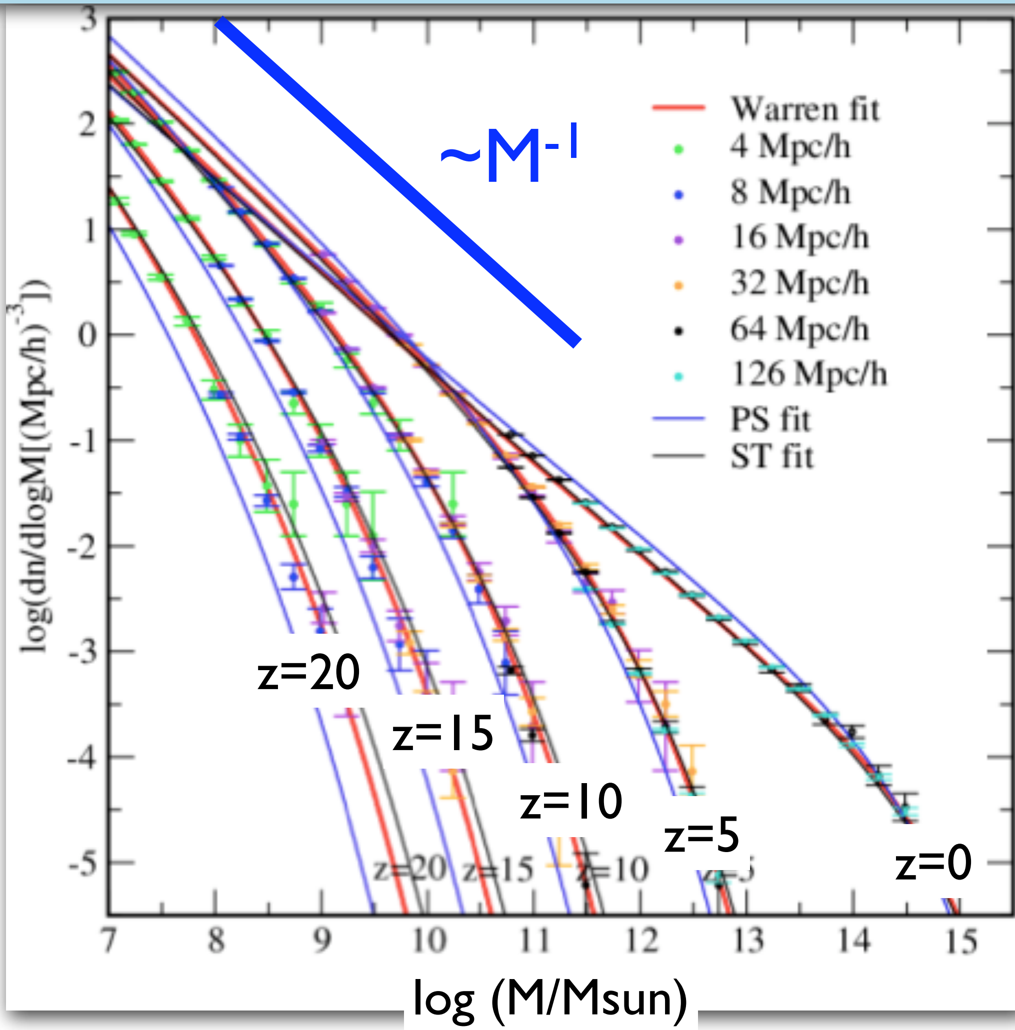


$20h^{-1}$Mpc Sphere within 120 h^{-1}Mpc box.

$m_p = 10^8$ Msun

$T_{lookback}$(Gyr)= 13.3960

Collapsed Structures:
Dark Matter Halo Mass Function from N-body simulations



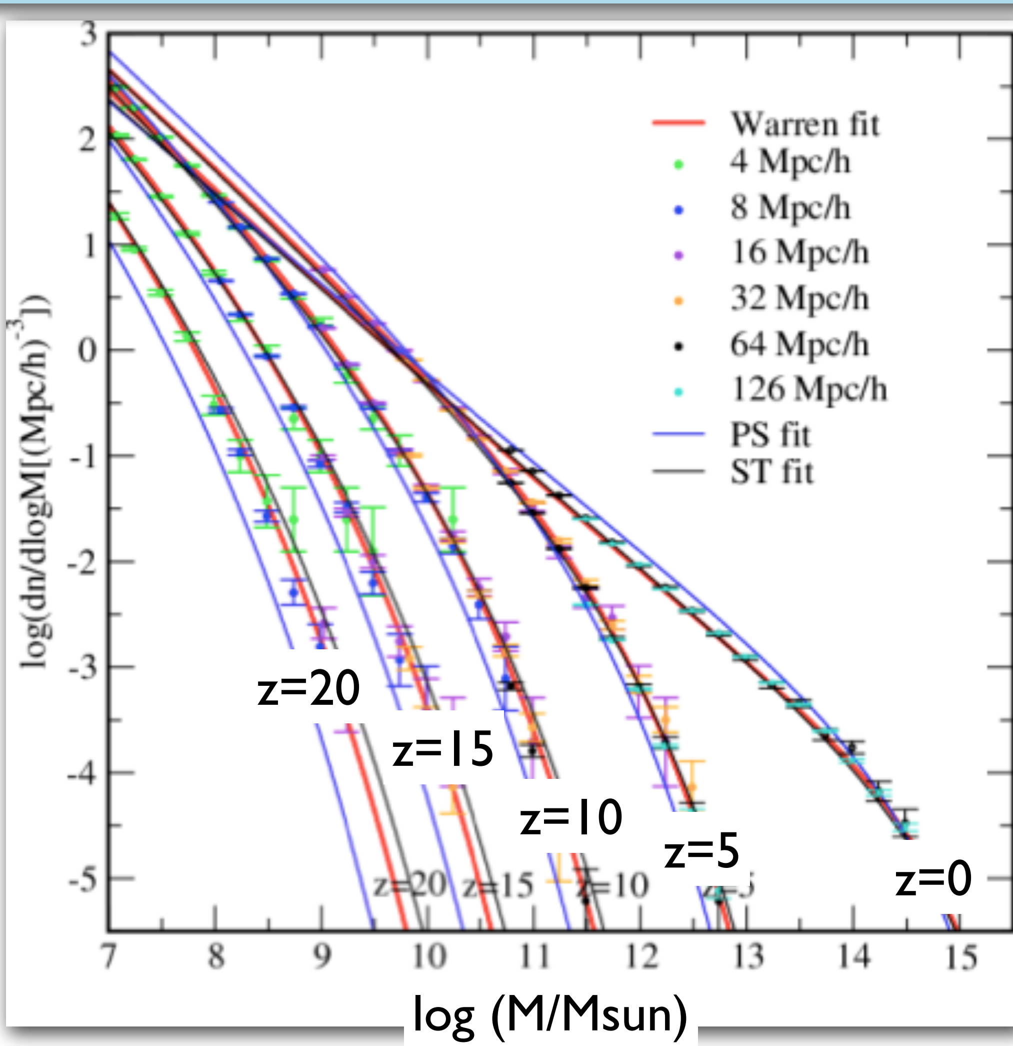
Lukic, Heitmann et al. 2007

$$\frac{dN}{d\ln M} \sim M^{-1} \quad M \ll M_*$$

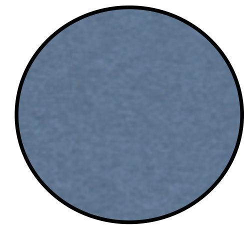
$$\frac{dN}{d\ln M} \sim e^{-(M/M*)^{\alpha}} \quad M \gg M_*$$

M* decreases rapidly with z

Collapsed Structures:
Dark Matter Halo Mass Function from N-body simulations

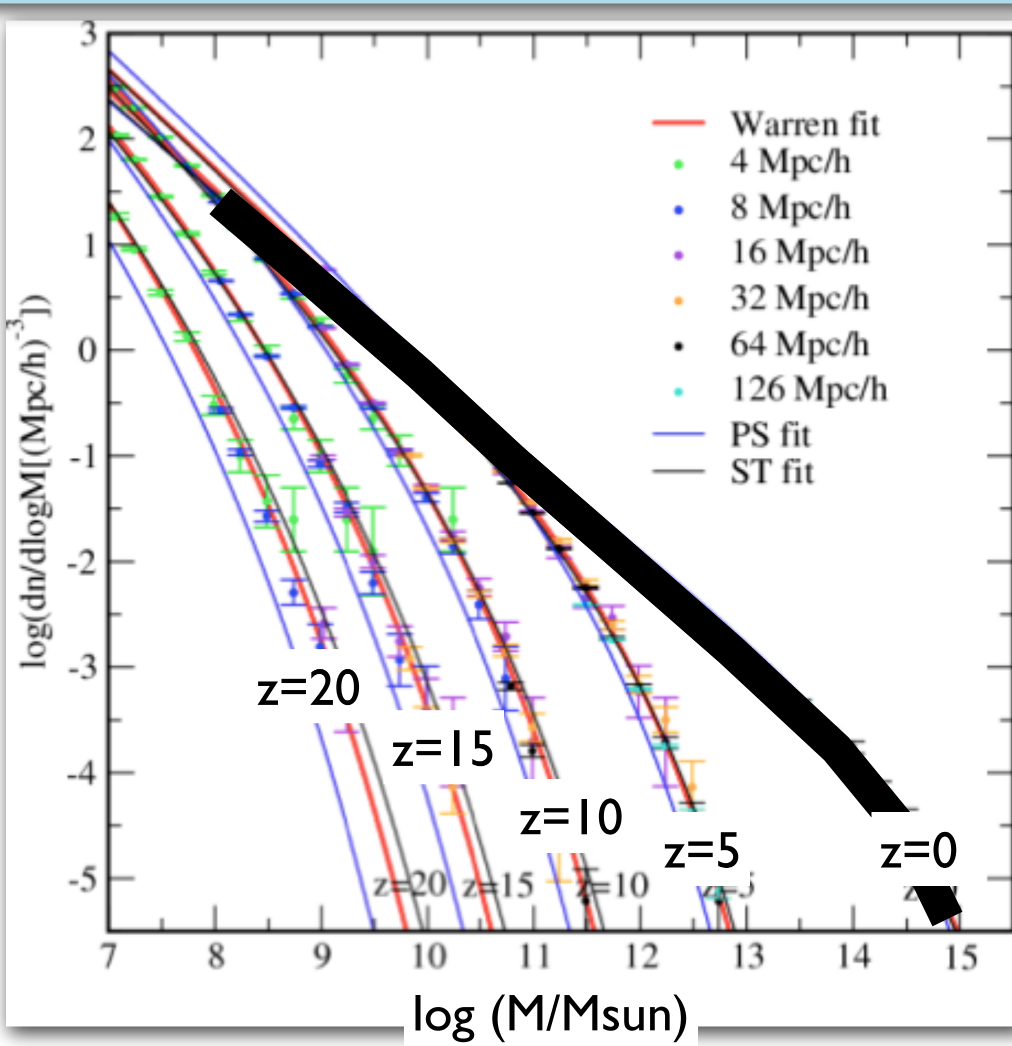


Lukic, Heitmann et al. 2007



Dark matter halos in simulations need to be **defined** in some way (e.g by size and by mass). Typically one adopts a 'virial' mass, defined by the radius within which the halo has a density of ~200 times the background density / or critical density.

Collapsed Structures:
Dark Matter Halo Mass Function from N-body simulations



Lukic, Heitmann et al. 2007

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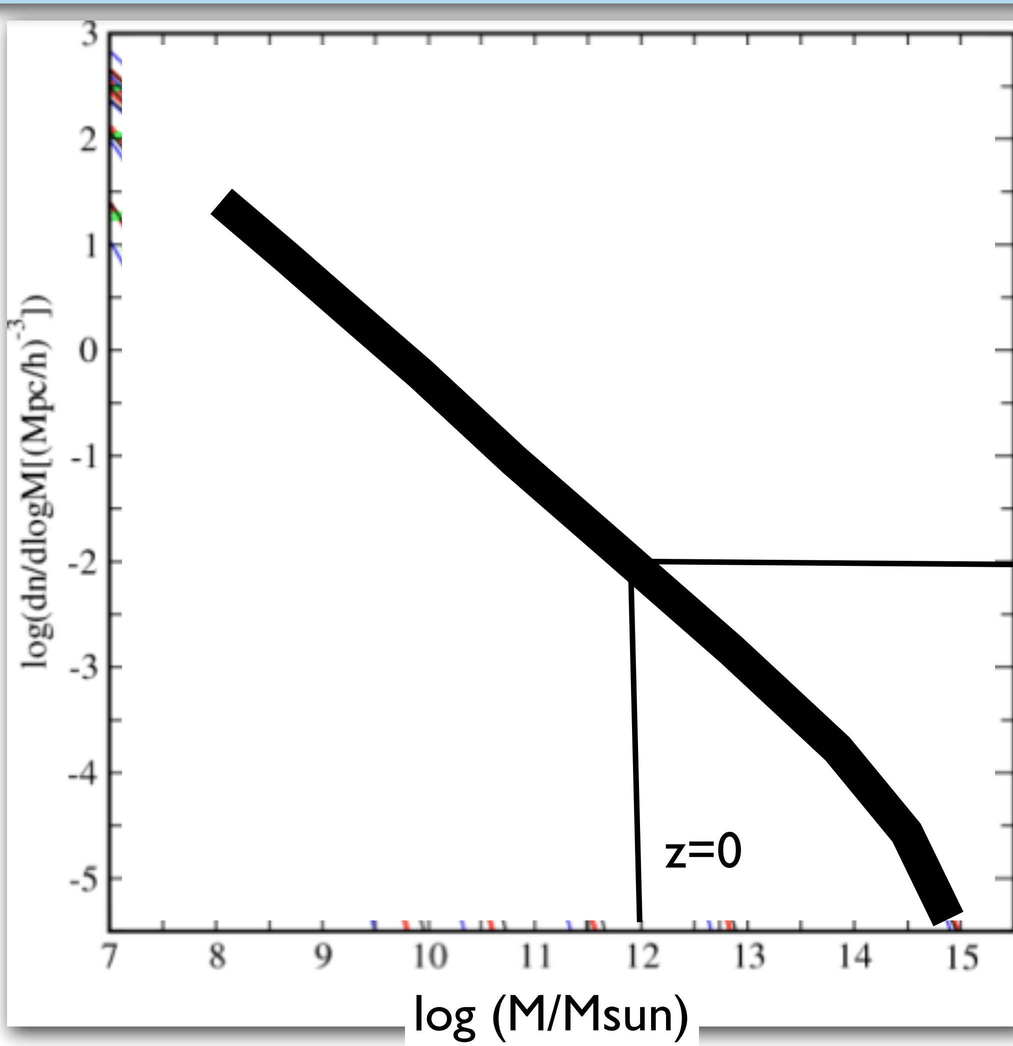

Note: number density of $M \sim 10^{12}$ M_{sun} halos is **~0.01 Mpc^{-3}** is similar to # density of bright galaxies like the Milky Way.

Lukic, Heitmann et al. 2007

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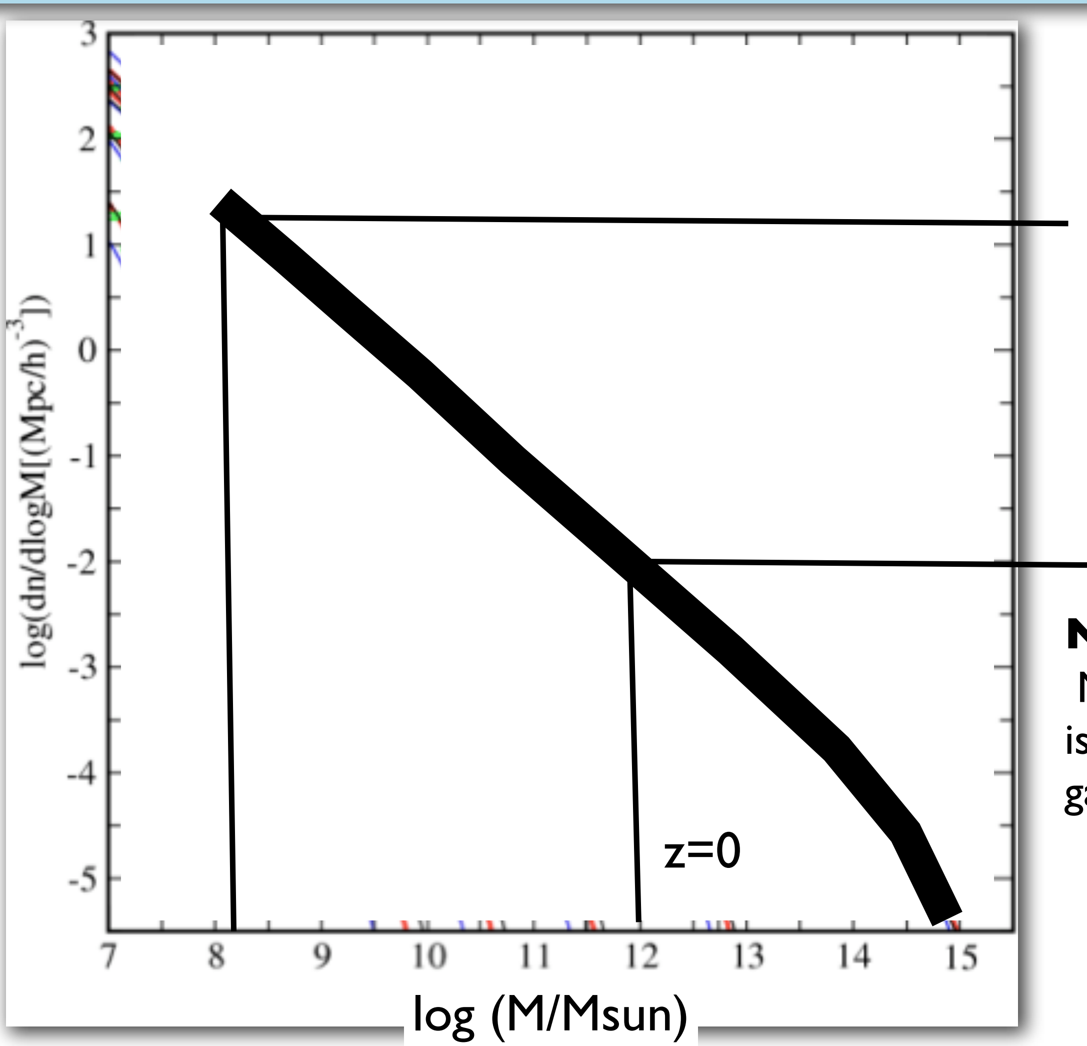

Lukic, Heitmann et al. 2007

However: number density of $M \sim 10^8\ M_{sun}$ halos is **~50 Mpc^{-3}**
-- this is very large, even compared to faint galaxy counts.
-- suggests that little halos must host very dim galaxies (or no galaxies...)

➡ feedback (e.g. White & Rees 78)

Note: number density of $M \sim 10^{12}\ M_{sun}$ halos is **~0.01 Mpc^{-3}** is similar to # density of bright galaxies like the Milky Way.

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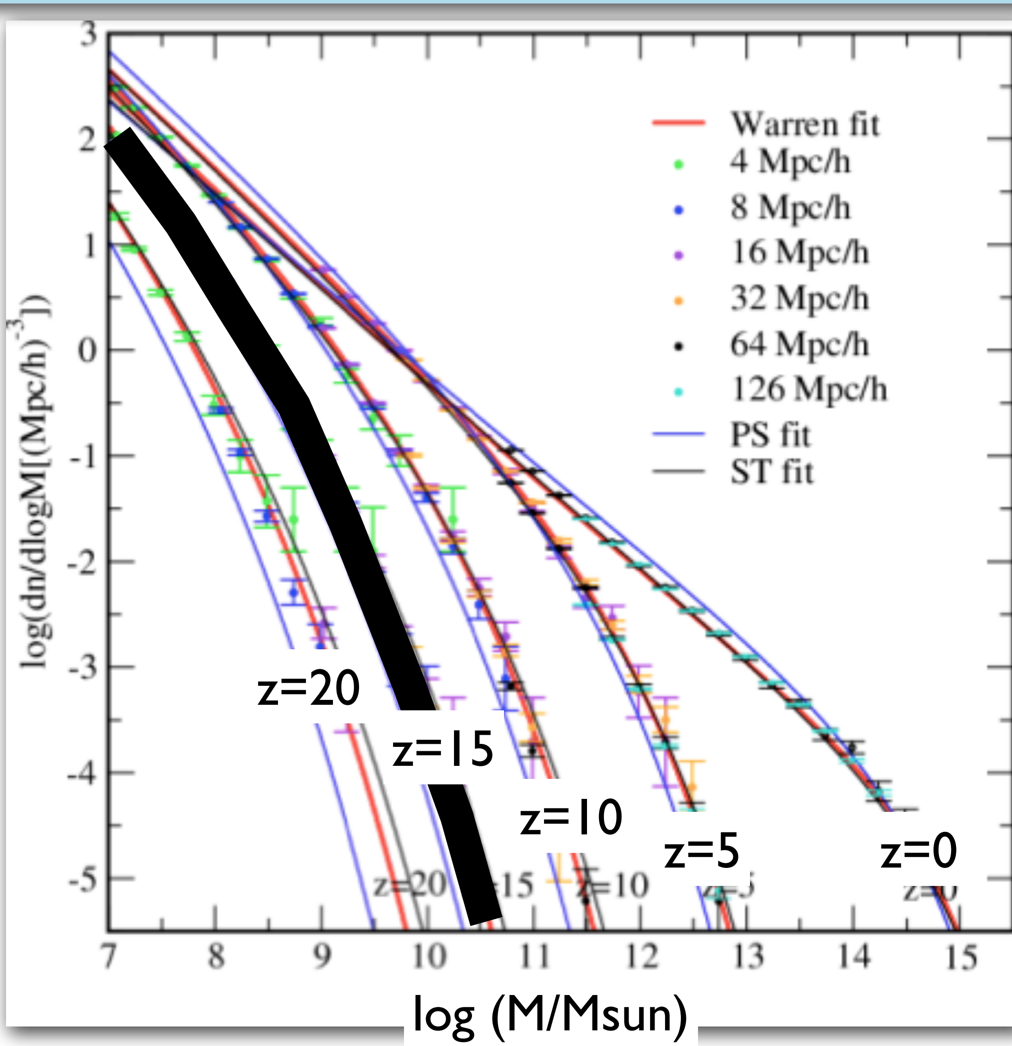

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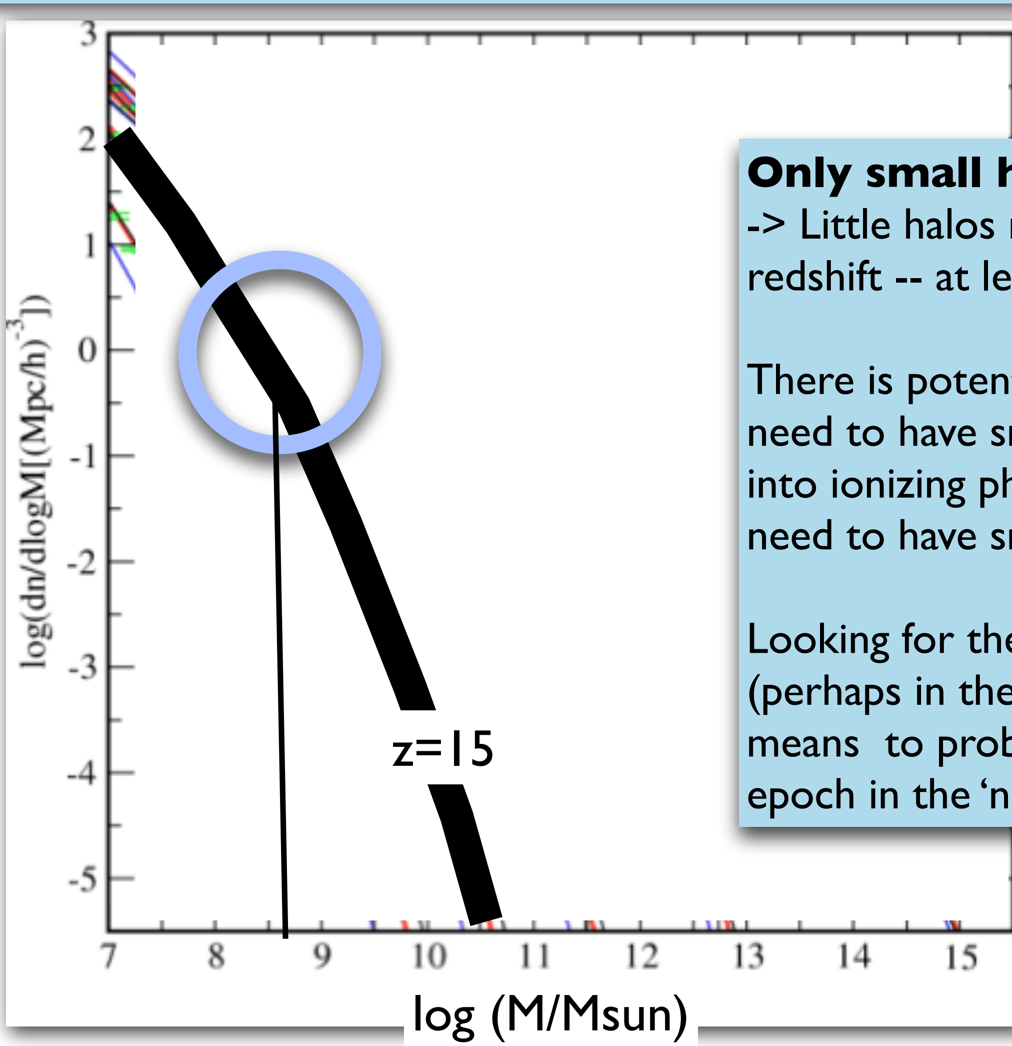


Only small halos exist at high z.
-> Little halos must produce stars efficiently at high redshift -- at least enough to ionize the universe.

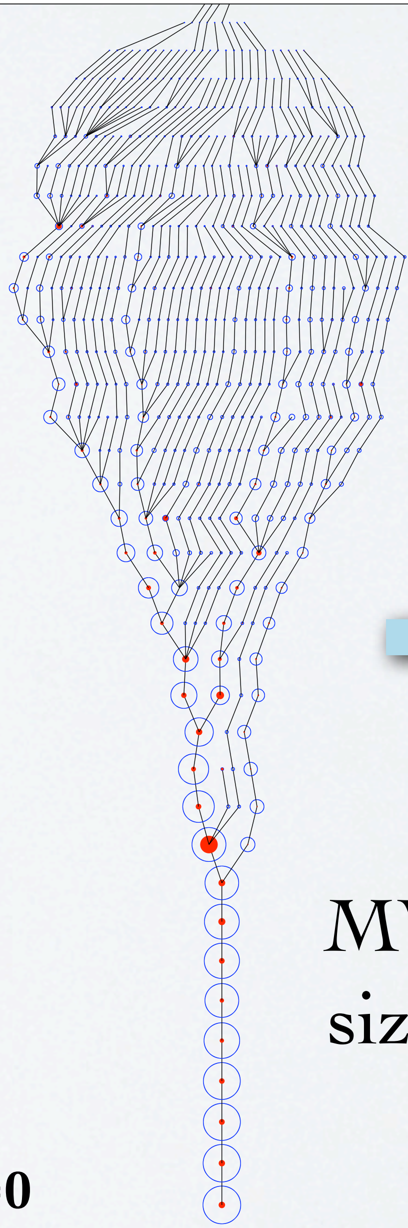
There is potentially some tension between the need to have small halos converting their baryons into ionizing photons efficiently at high z, and the need to have small halos very dark at low z.

Looking for the progenitors of these halos at low-z (perhaps in the halo of our galaxy?) provides a means to probe reionization / early star formation epoch in the 'near field'...

Lukic, Heitmann et al. 2007

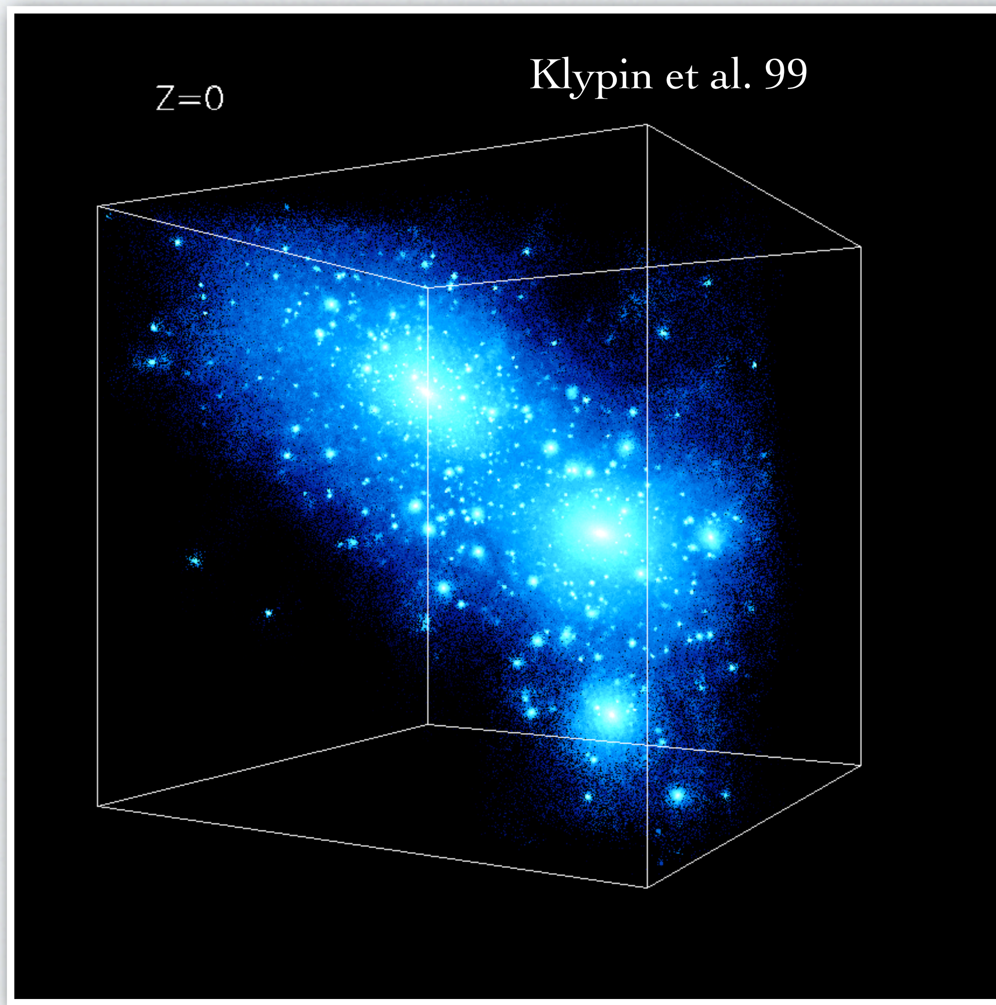
Surviving Substructure is abundant

z=8

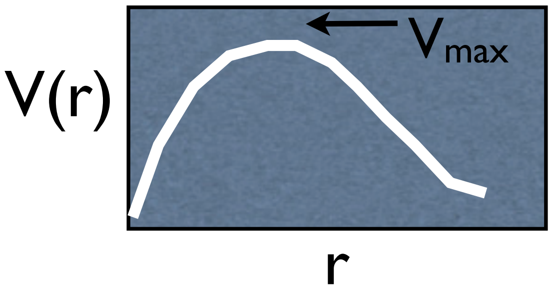


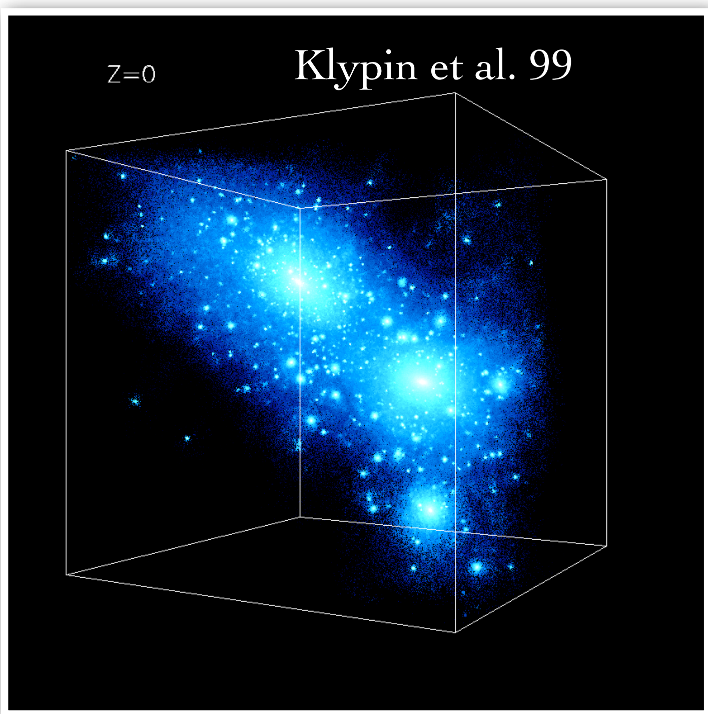
z=0

MW / M31
size system

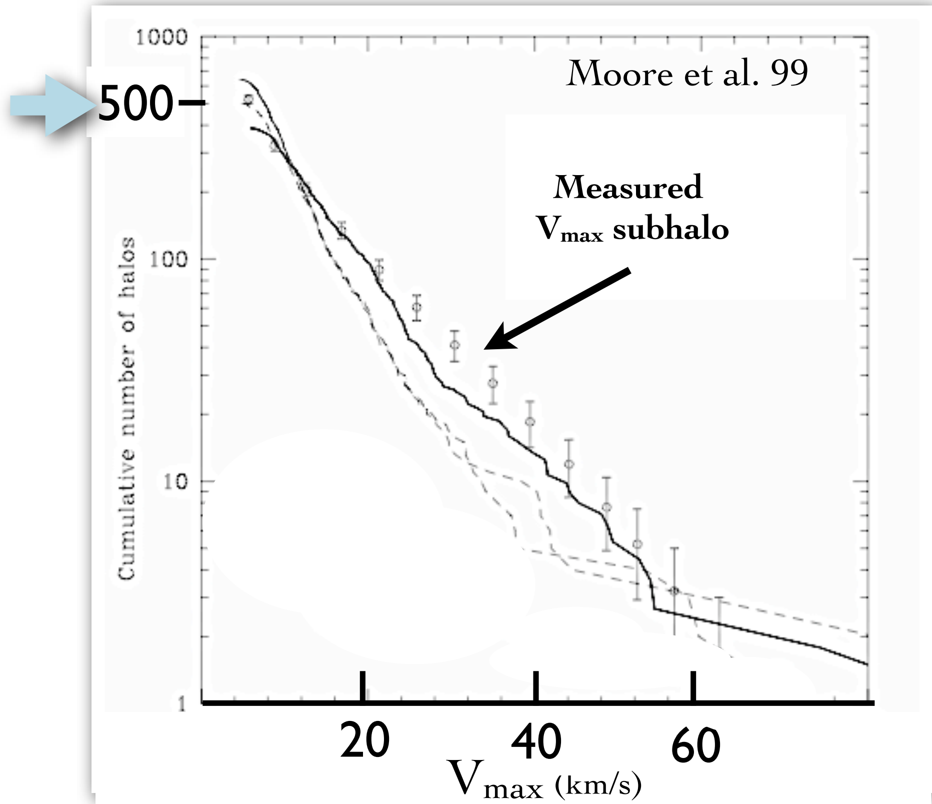


Mass definitions are (even more) ambiguous for subhalos in simulations. It is common instead to use maximum circular velocity, V_{max}





count subhalos within virial radius
of a simulated $M{\sim}10^{12}\ M_{sun}$ halo



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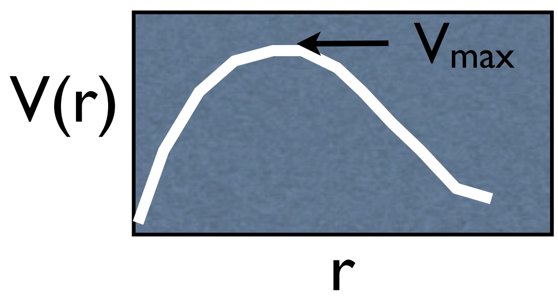

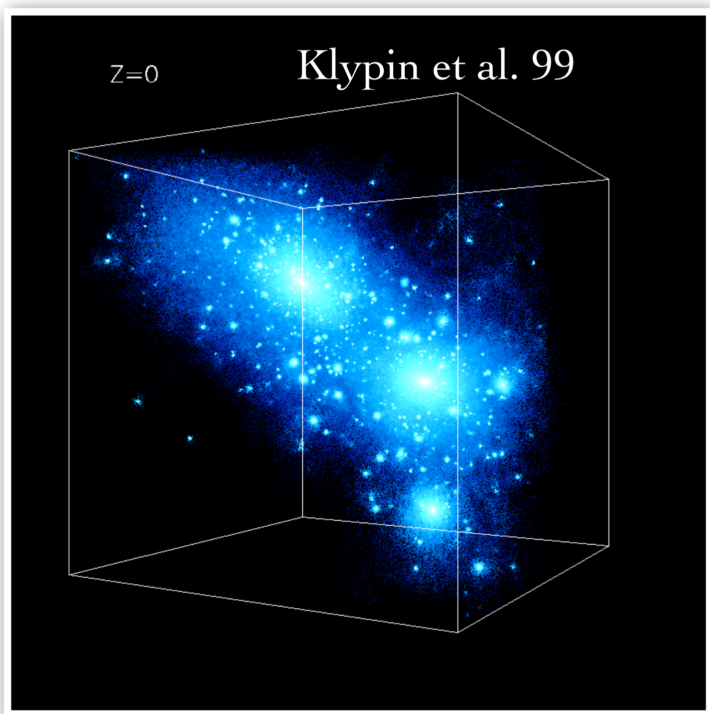

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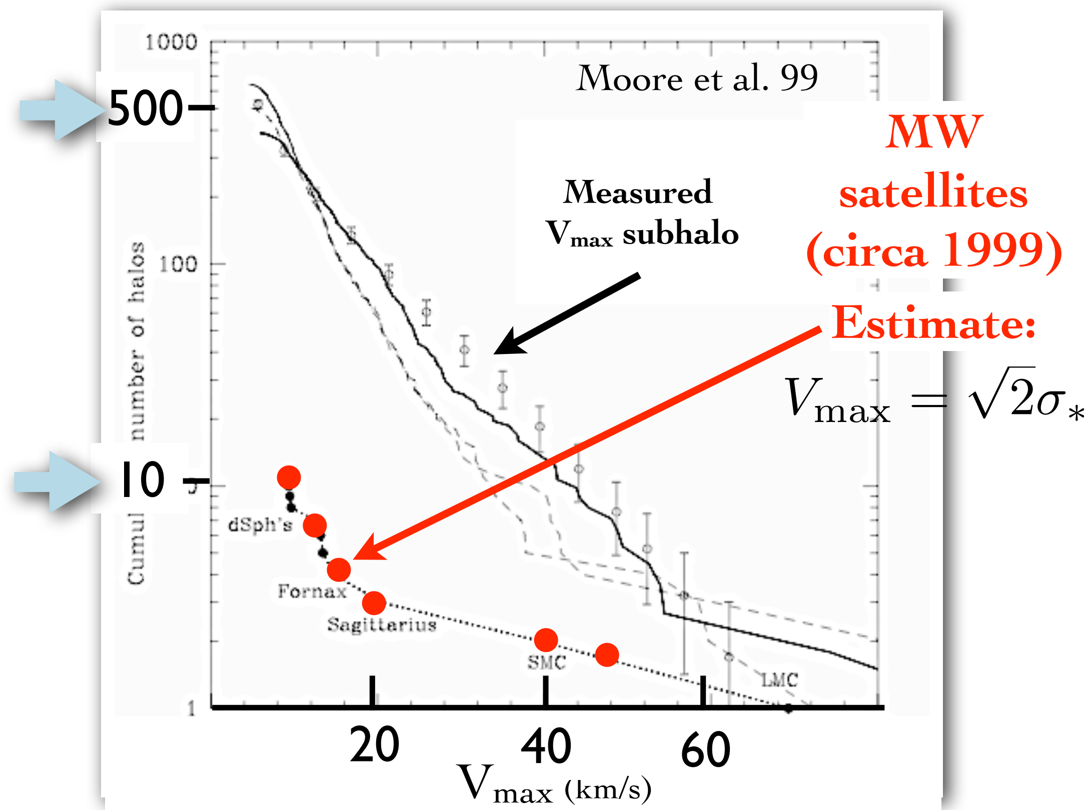
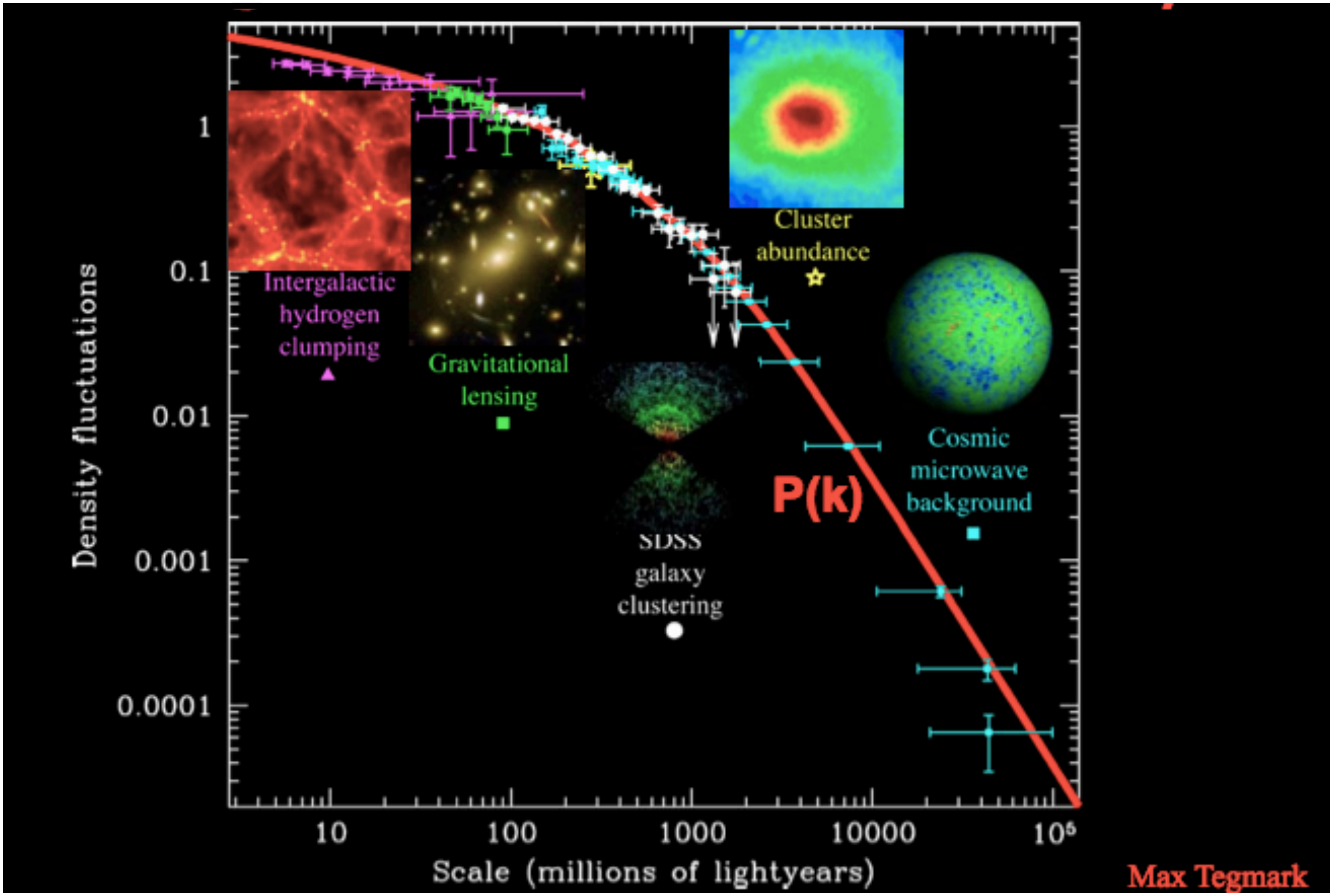


Large Scales: looks like CDM + Dark Energy



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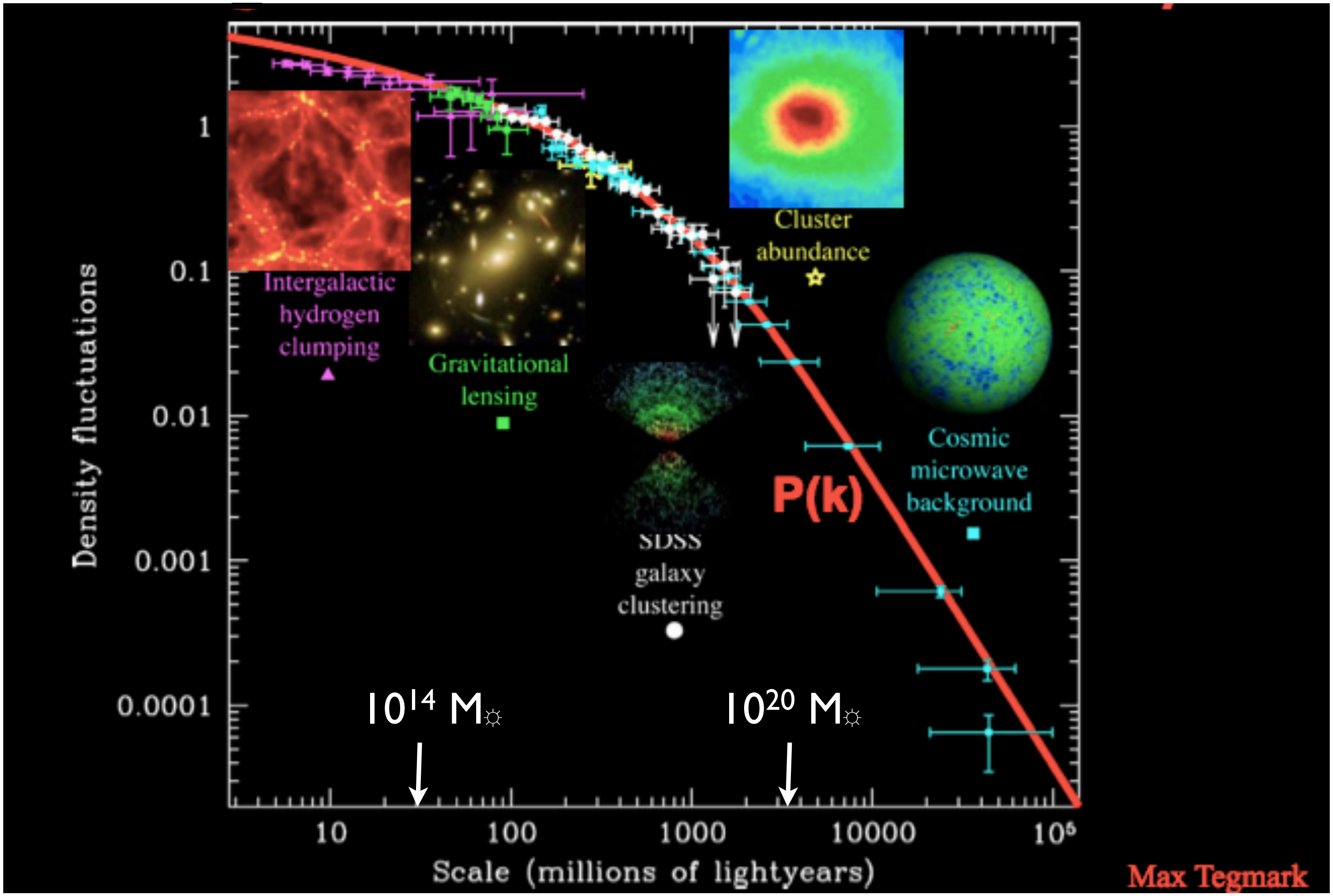

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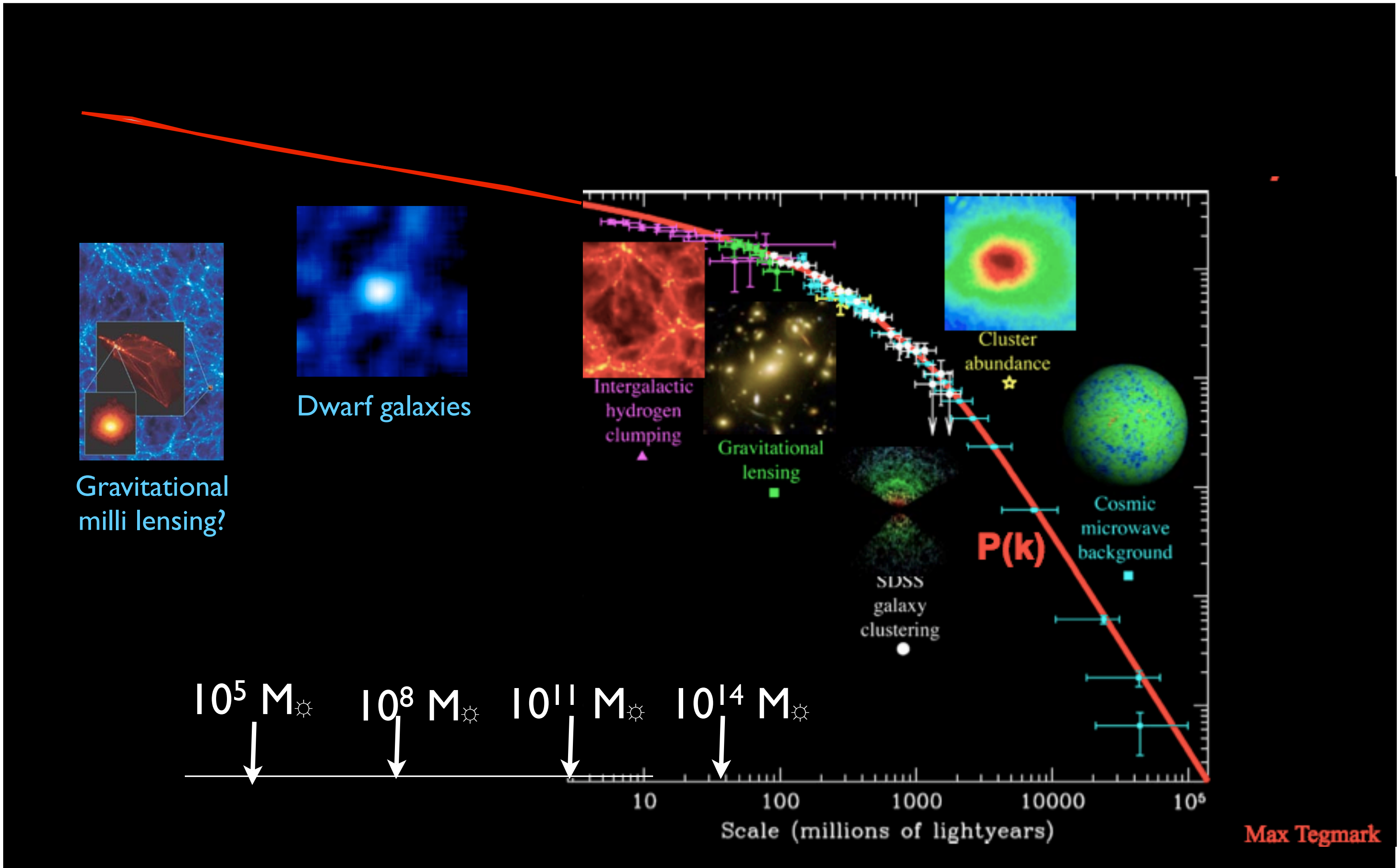
What about smaller scales?



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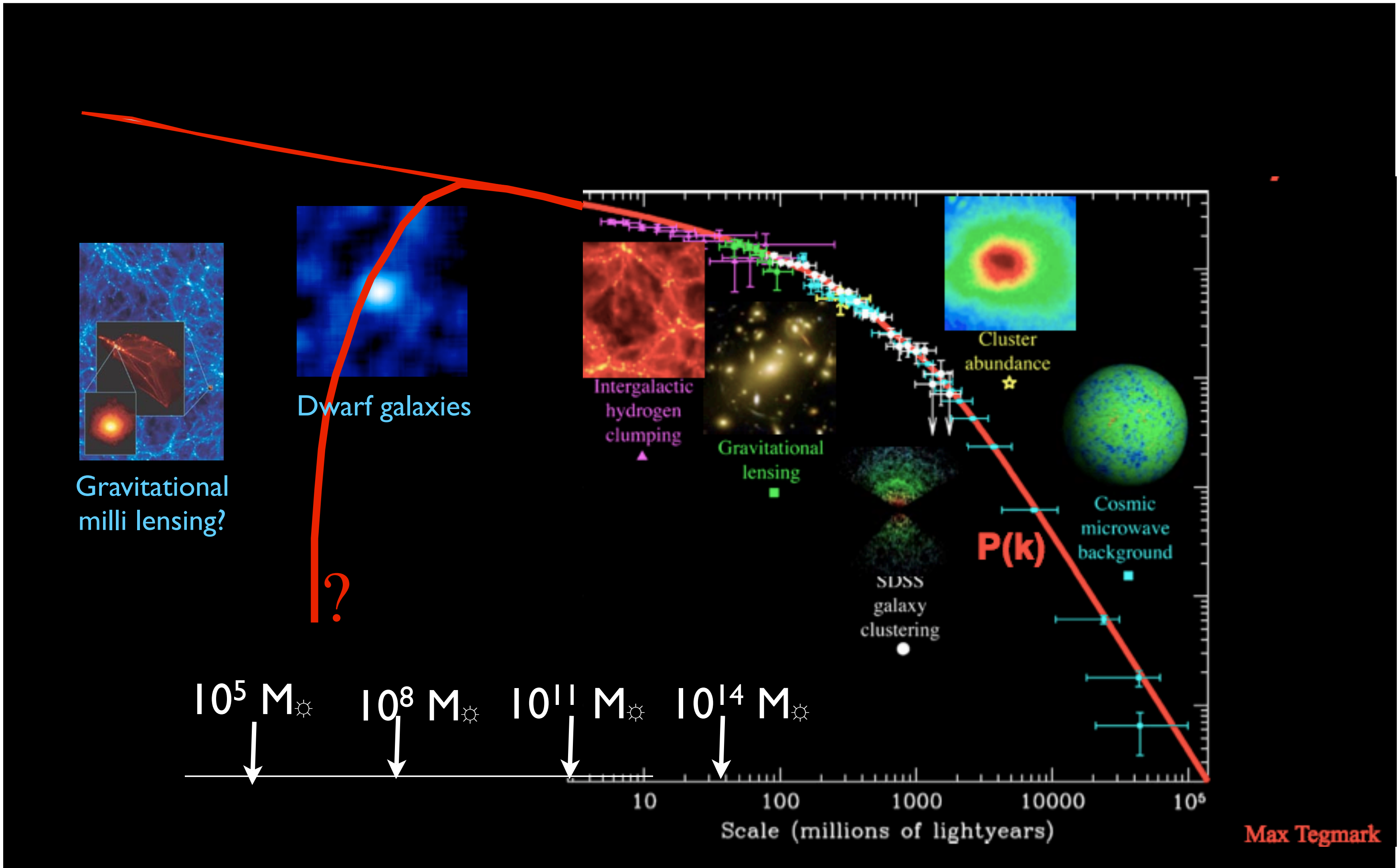

End Lecture 1