Radial Rotation Inversions Solar-like Stars

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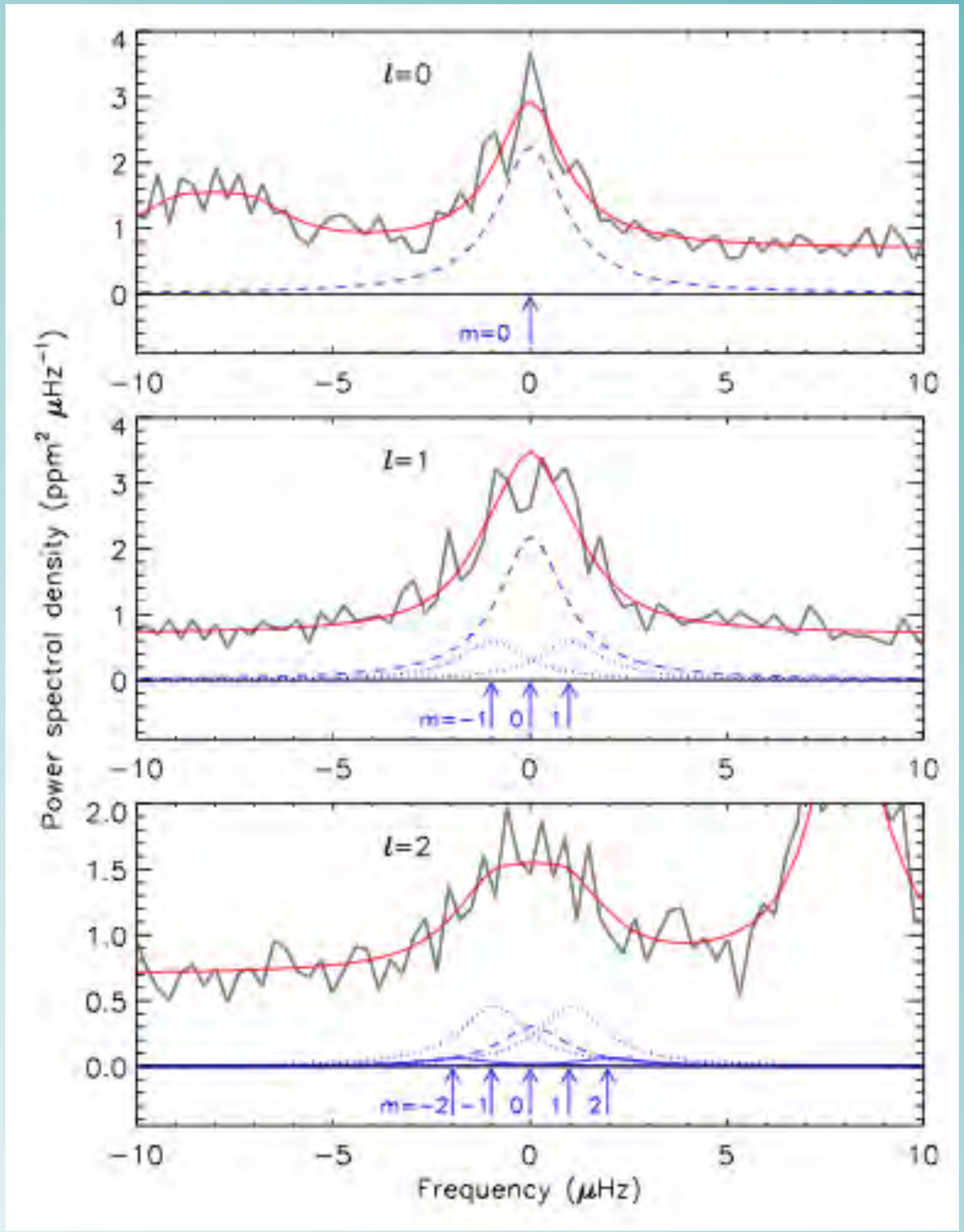
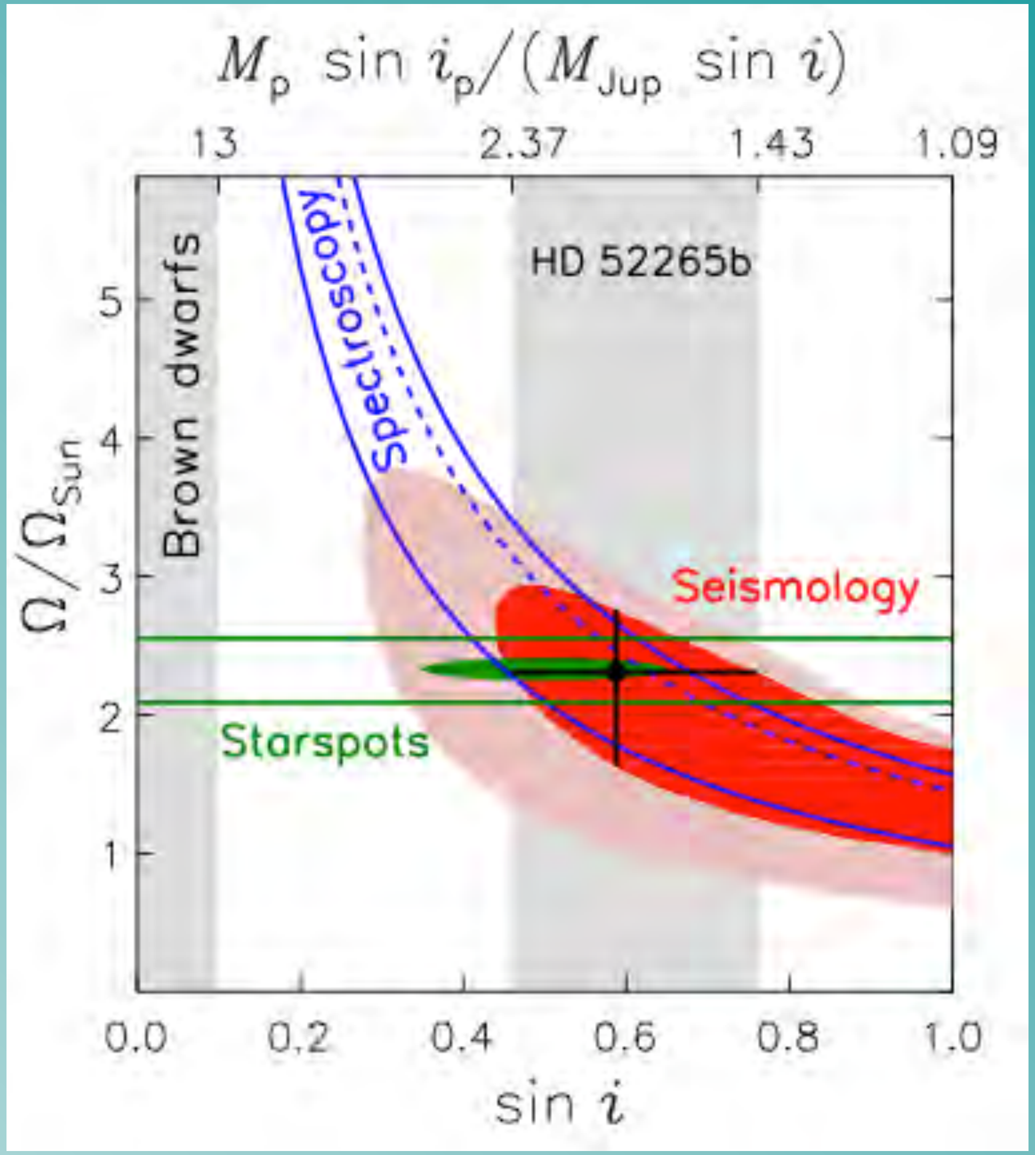
MPS

Solar-like stars: bulk rotation



CoRoT 4 months

HD52265: bulk rotation



Ballot et al 2006
Stahn 2011
Gizon et al 2013

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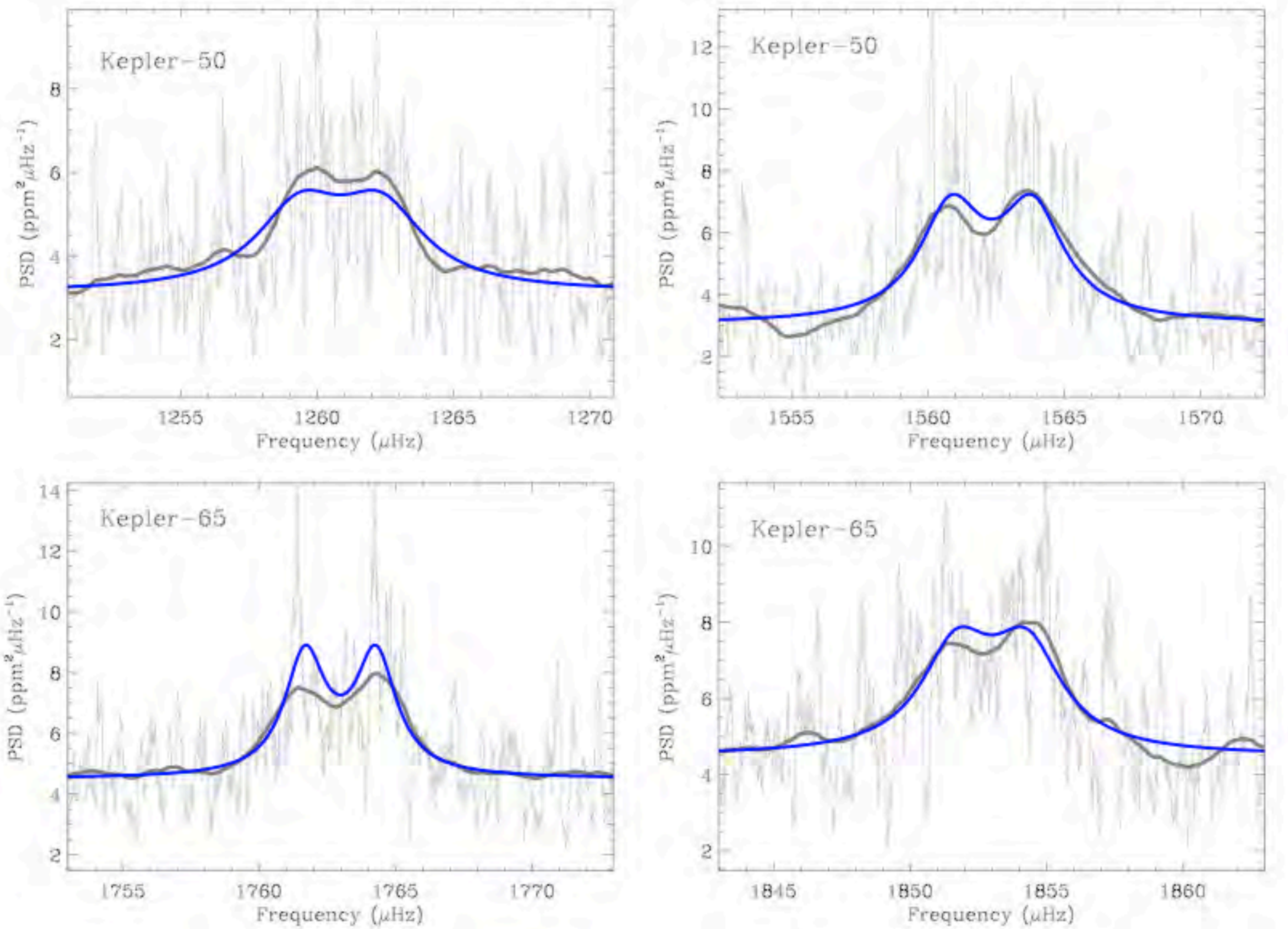
Solar-like stars: bulk rotation

solar-like stars: bulk rotation

The Astrophysical Journal, 766:101 (19pp), 2013 April 1



Kepler 18/27 months

Chaplin et al 2013

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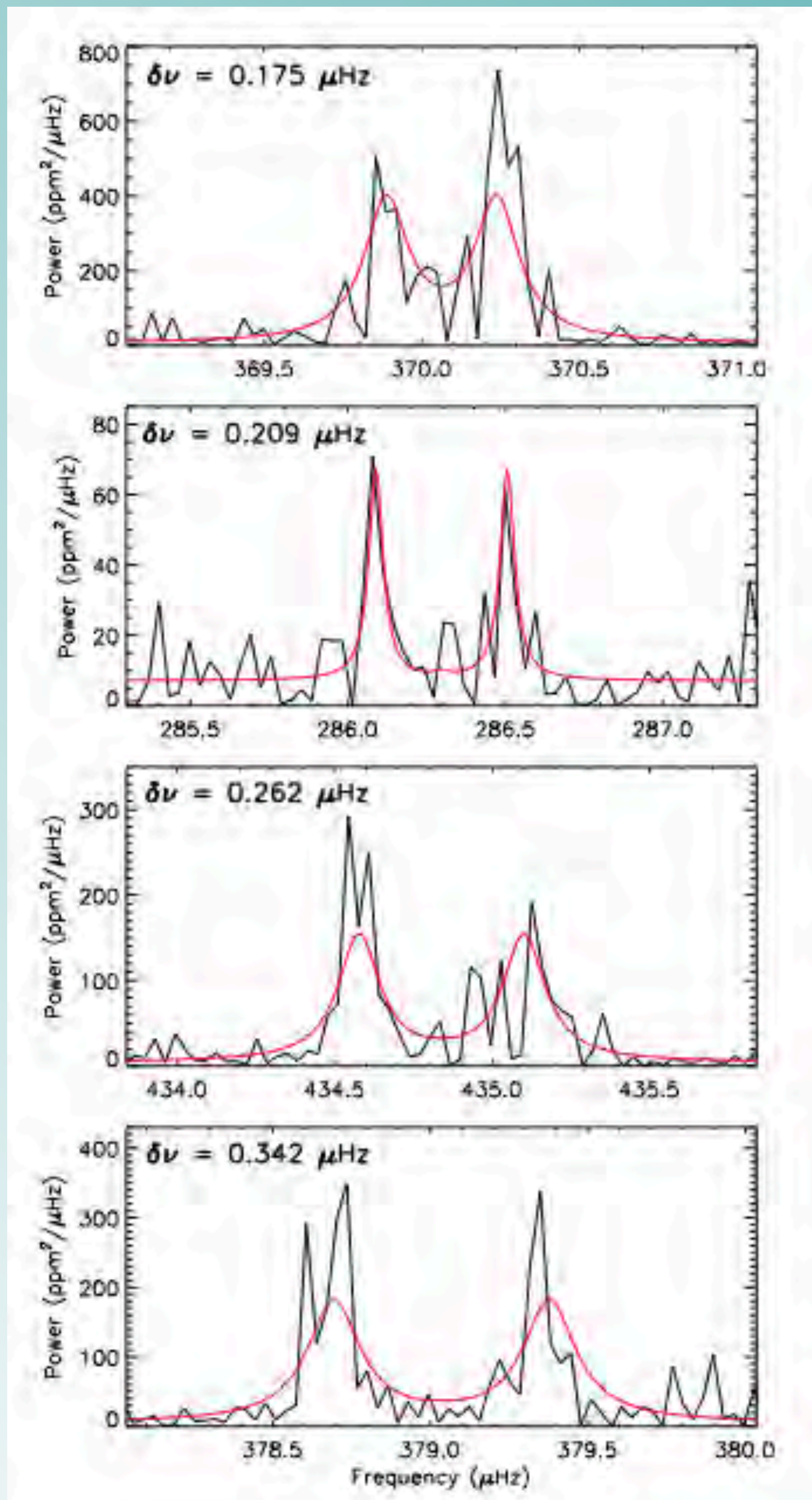
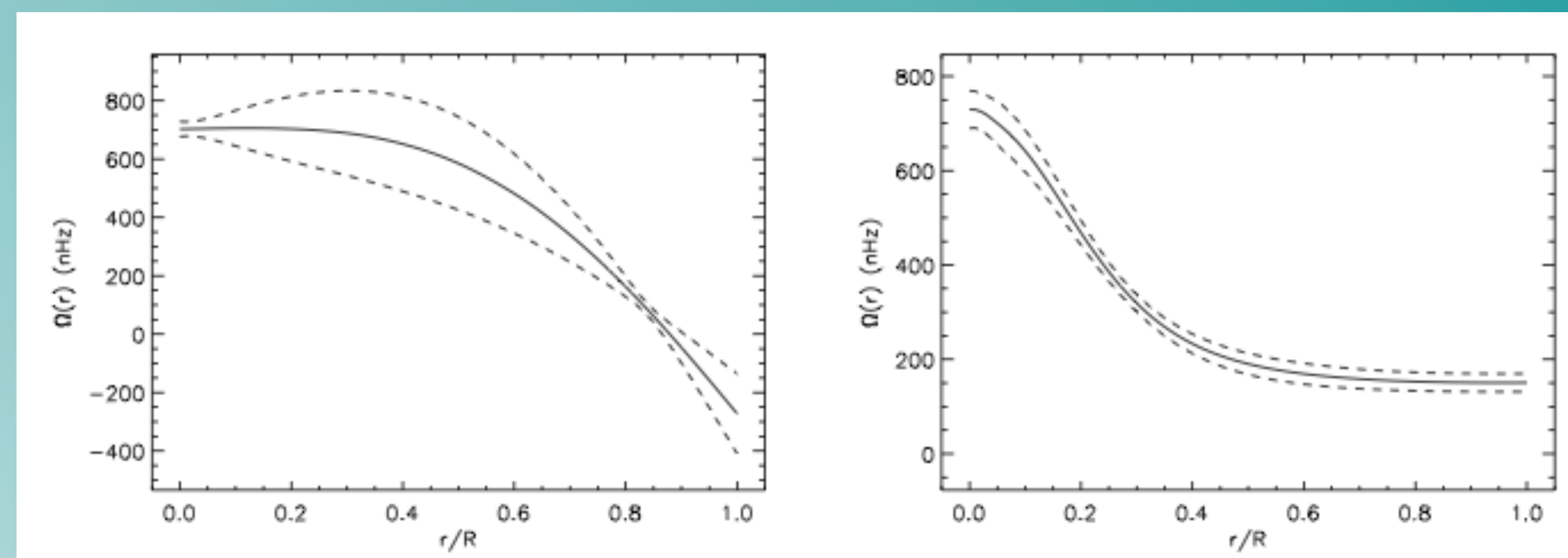
Sub-giants: radial rotation



Otto sub-giant
local fits; many methods MLE, MAP

RLS

OLA



Deheuvels et al 2012

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Sub-giants: radial rotation

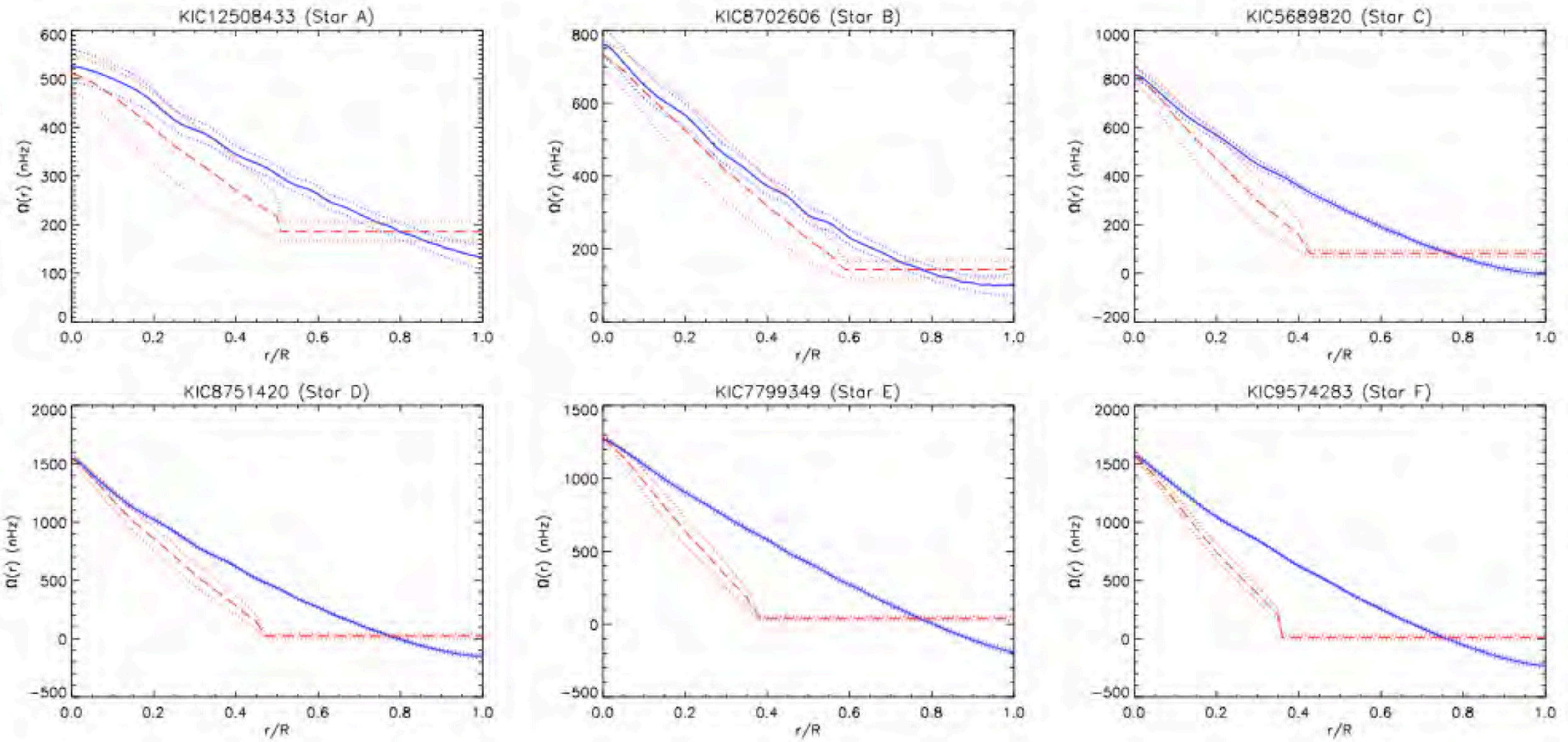

Fig. 10. Optimal rotation profiles obtained by applying the RLS method with a smoothness condition on the rotation profile on the entire star (blue solid lines) or only in the radiative interior while the convective envelope is assumed to rotate as a solid-body (red long-dashed lines). The dotted lines indicate the 1-σ error bars for both types of inversions.

Deheuvels et al 2014

Inversions

RLS Inversion

$$\sum_{i \in M} \left[\delta\omega_i - \sum_{j}^{N} \overline{\Omega}_j B_{ij} \right]^2 + \mu F(\overline{\Omega})$$

$$\delta\omega_i = \int_0^R K_i(r)\Omega(r)dr$$

$$B_{ij} = \int_0^R K_{i\star}(r)\phi_j(r)dr$$

$$\boxed{\overline{\Omega}(r_0) = \sum_{i=1}^{M} c_i(r_0)\delta\omega_i}$$

Questions

1) How does the uncertainty in the stellar models affect the inverted rotation profiles?

2) Do surface constraints help?

Inversions

RLS Inversion

minimise:

$$\sum_{i\in M}\left[\delta\omega_i - \sum_j^N \overline{\Omega}_j B_{ij}\right]^2 + \mu F(\overline{\Omega})$$

smoothness

$$\delta\omega_i = \int_0^R K_i(r)\Omega(r)dr$$

perturbed model

$$B_{ij} = \int_0^R K_{i\star}(r)\phi_j(r)dr$$

reference model

$$\overline{\Omega}(r_0) = \sum_{i=1}^M c_i(r_0)\delta\omega_i$$

$$\sigma_\Omega(r_0) = \sqrt{\sum_{i=1}^M [c_i(r_0)\sigma_i]^2}$$

reference model

perturbed model

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1) How does the uncertainty in the stellar models affect the inverted rotation profiles?

- HD52265

  -- best-fit stellar model

  age $T_\star = 2.37 +/- 0.39$ Gyr

  mass $M_\star = 1.27 + / - 0.03 M_\odot$

  metallicity $Z_\star = 0.03$

  helium abundance $Y_\star = 0.28$

  mixing length parameter $\alpha_\star = 1.8$

  -- reference model

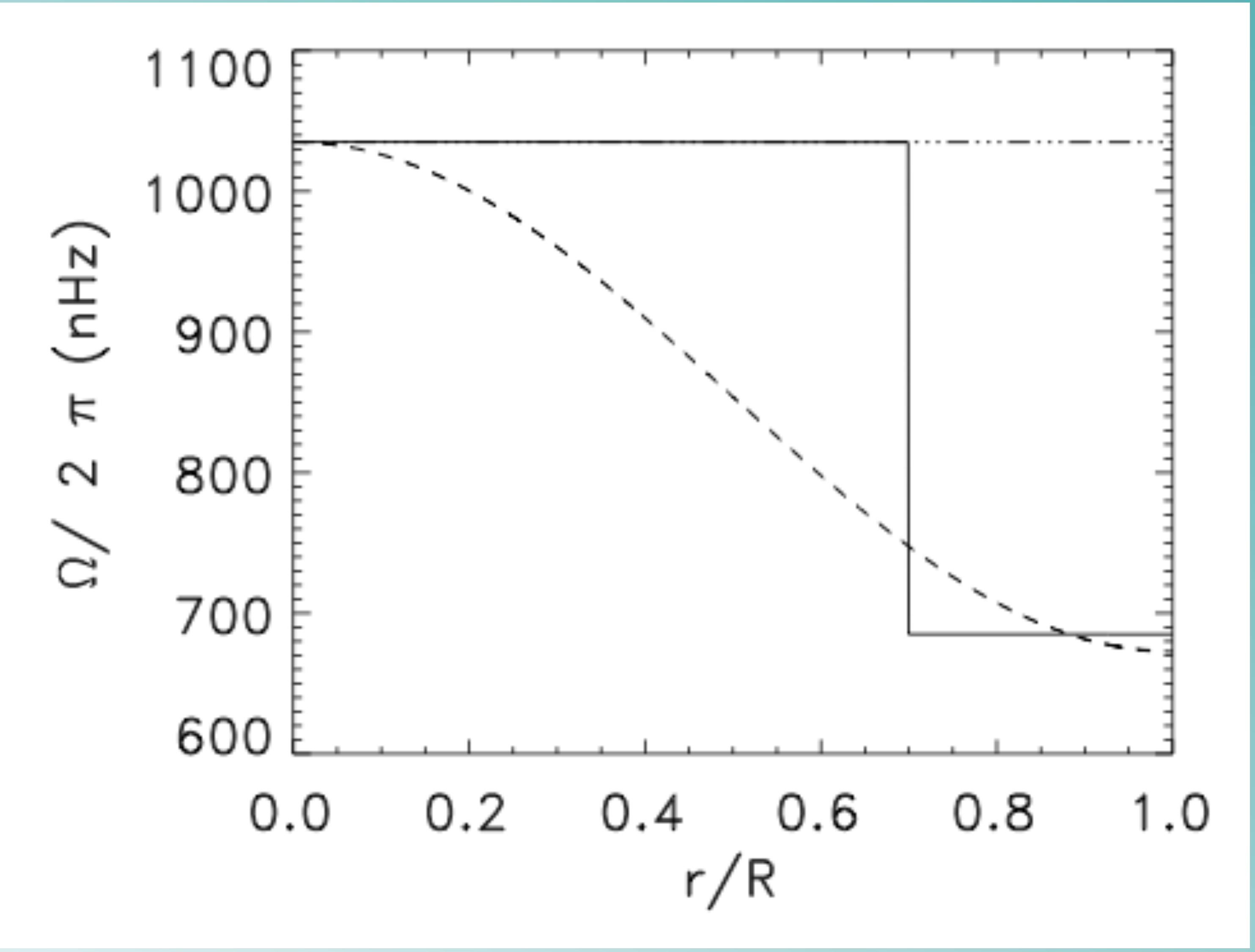
  -- perturbed models

  +/- 5-sigma in age and mass

  +/- 1-sigma in other quantities

1) How does the uncertainty in the stellar models affect the inverted rotation profiles?

- HD52265
  -- ref + perturbed stellar models
  -- synthetic rotation profiles



1) How does the uncertainty in the stellar models affect the inverted rotation profiles?

- HD52265
  -- ref + perturbed stellar models
  -- synthetic rotation profiles
  -- compute splittings

$$\delta\omega_i = \int_0^R K_i(r)\Omega(r)dr$$

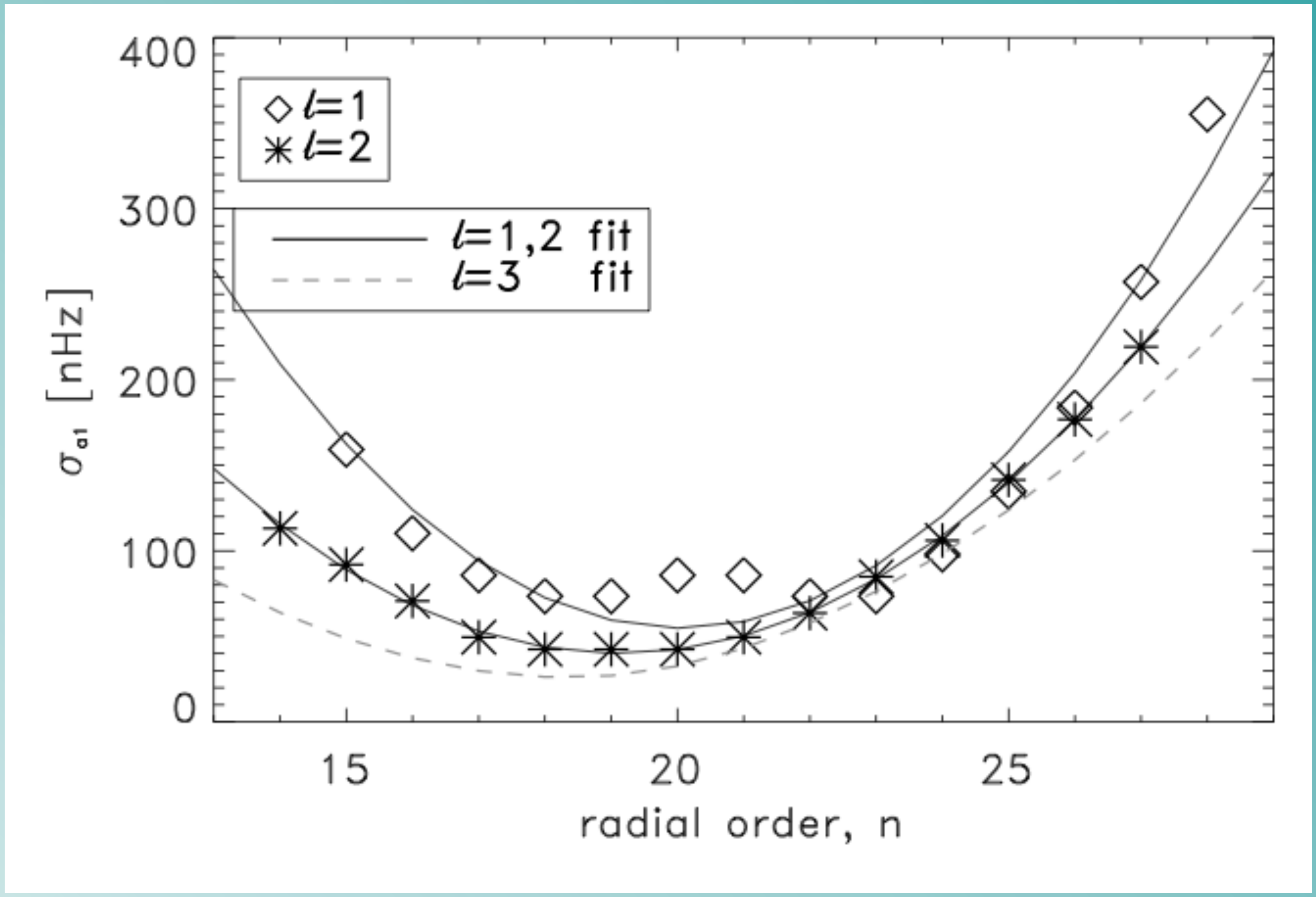
↓ perturbed model ($K_i(r)$)

1) How does the uncertainty in the stellar models affect the inverted rotation profiles?

- HD52265
  -- ref + perturbed stellar models
  -- synthetic rotation profiles
  -- compute splittings
  -- add noise

$$\sigma_{\delta\omega} = \frac{\sigma_\omega}{\sqrt{1/3\ell(\ell+1)}}$$

$$n = 16, \ldots, 25$$
$$\ell = 1, 2$$
$$\langle \sigma \rangle = 80 \text{ nHz}$$



1) How does the uncertainty in the stellar models affect the inverted rotation profiles?

- HD52265
  -- ref + perturbed stellar models
  -- synthetic rotation profiles
  -- compute splittings (perturbed model)
  -- add noise
  -- invert the splittings: RLS, step function fit

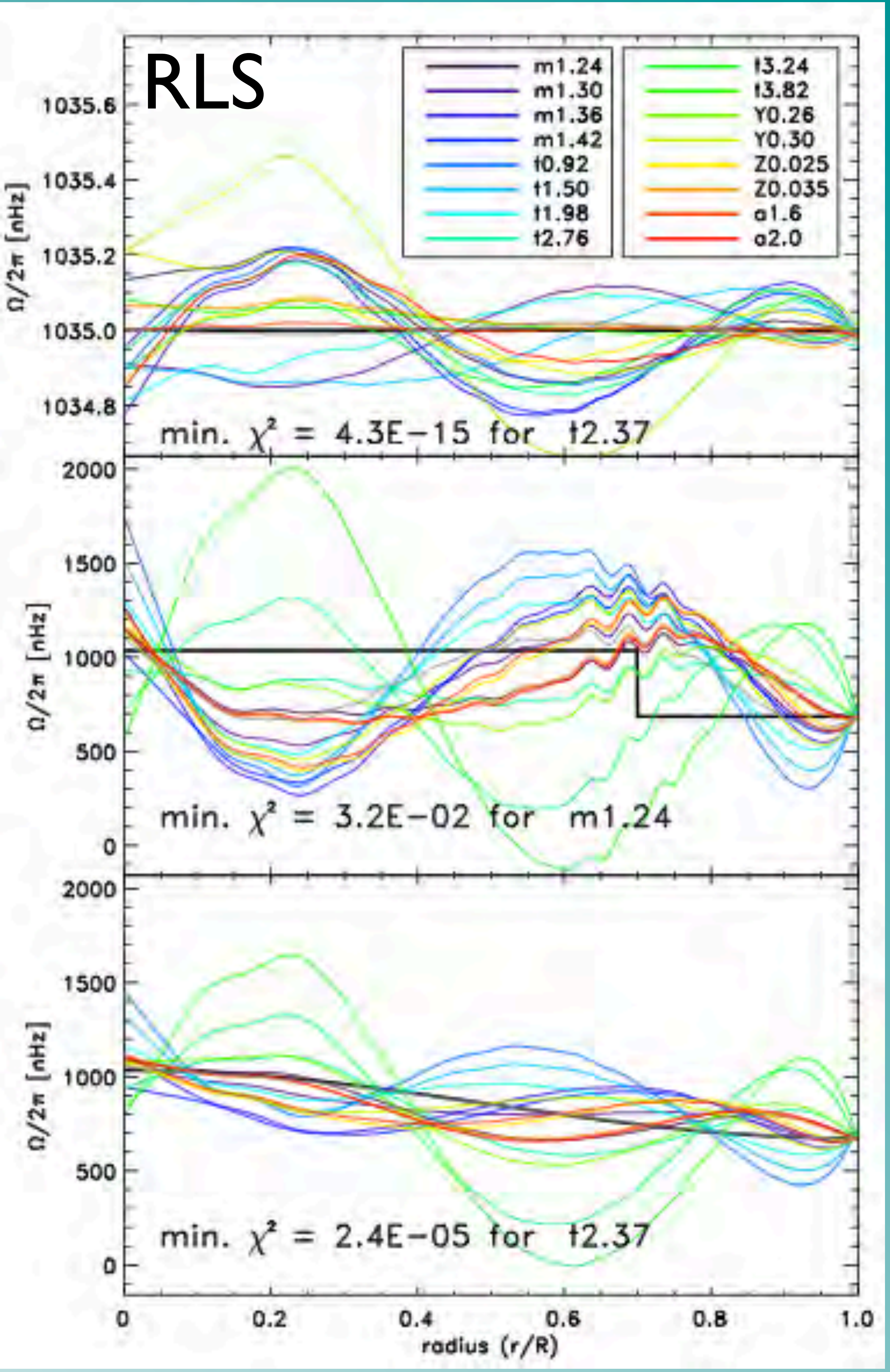
$$\chi^2_{\rm red} = \frac{1}{M}\sum_{i=1}^{M}\frac{\left[\delta\omega_i - \int_0^R K_\star(r)\overline{\Omega}(r)\right]^2}{\sigma_i^2}$$

No noise

Reference model (t2.37)

age T⋆ = 2.37 Gyr
mass M⋆ = 1.27 M⊙
metallicity Z⋆ = 0.03
helium abundance Y⋆ = 0.28
mixing length parameter α⋆ = 1.8

Also did this by
fitting a piece-wise discontinuous function
but differences are
even smaller than for RLS



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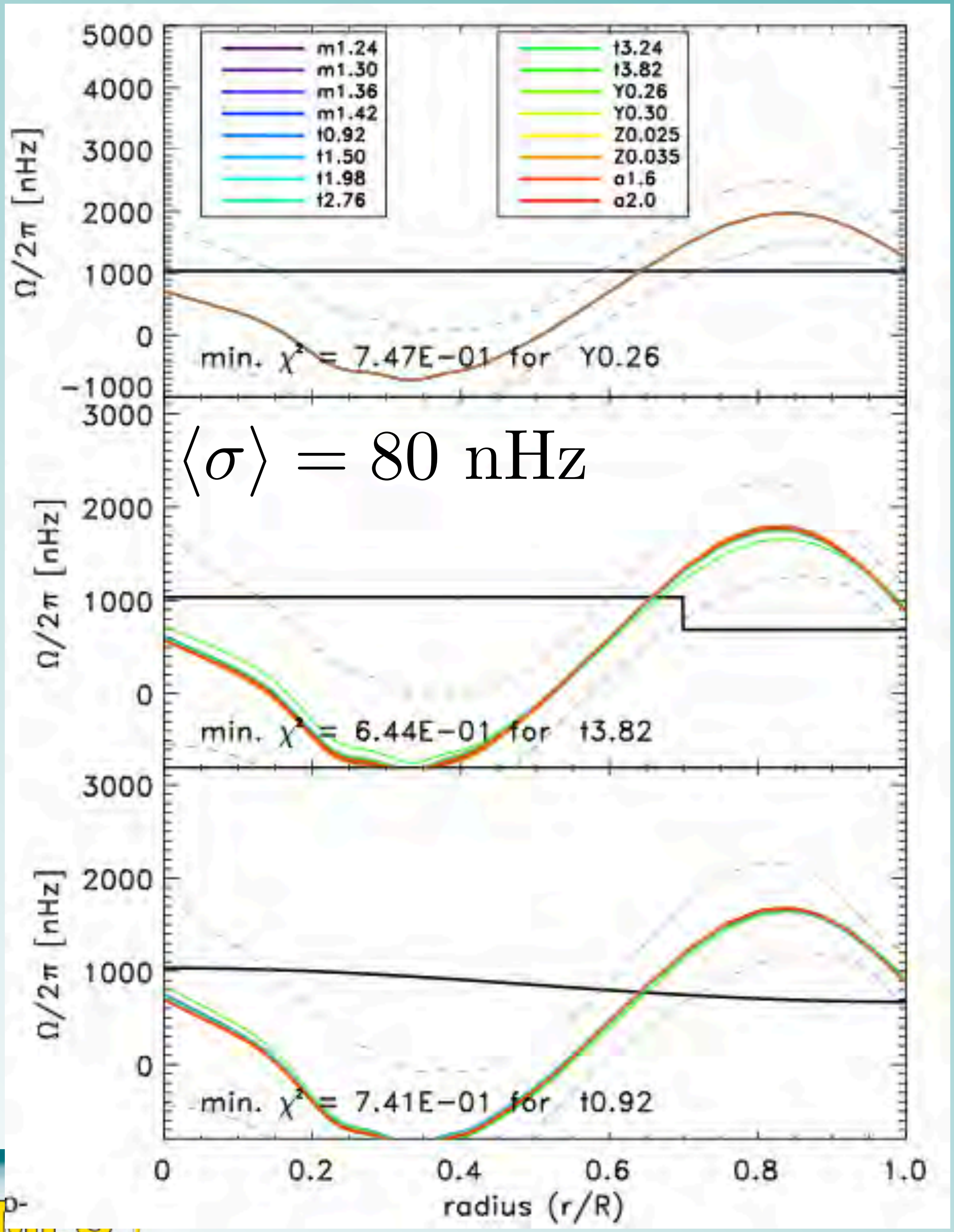
With noise

uncertainties on splitting measured from Kepler are +/- 80nHz (27 months, 3 x solar rotation rate; Chaplin et al 2013)



$$\sigma_\Omega(r_0) = \sqrt{\sum_{i=1}^{M} [c_i(r_0)\sigma_i]^2}$$

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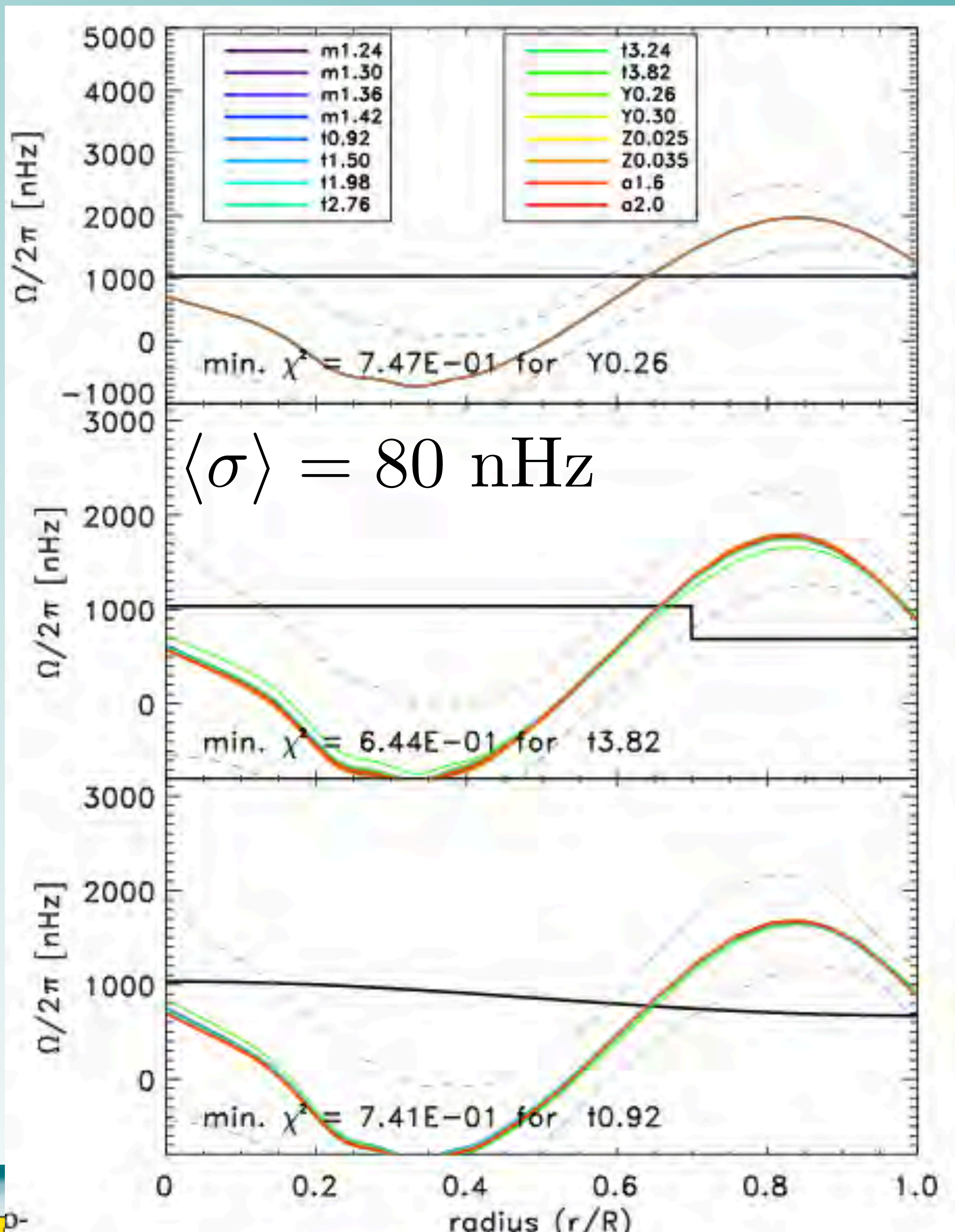
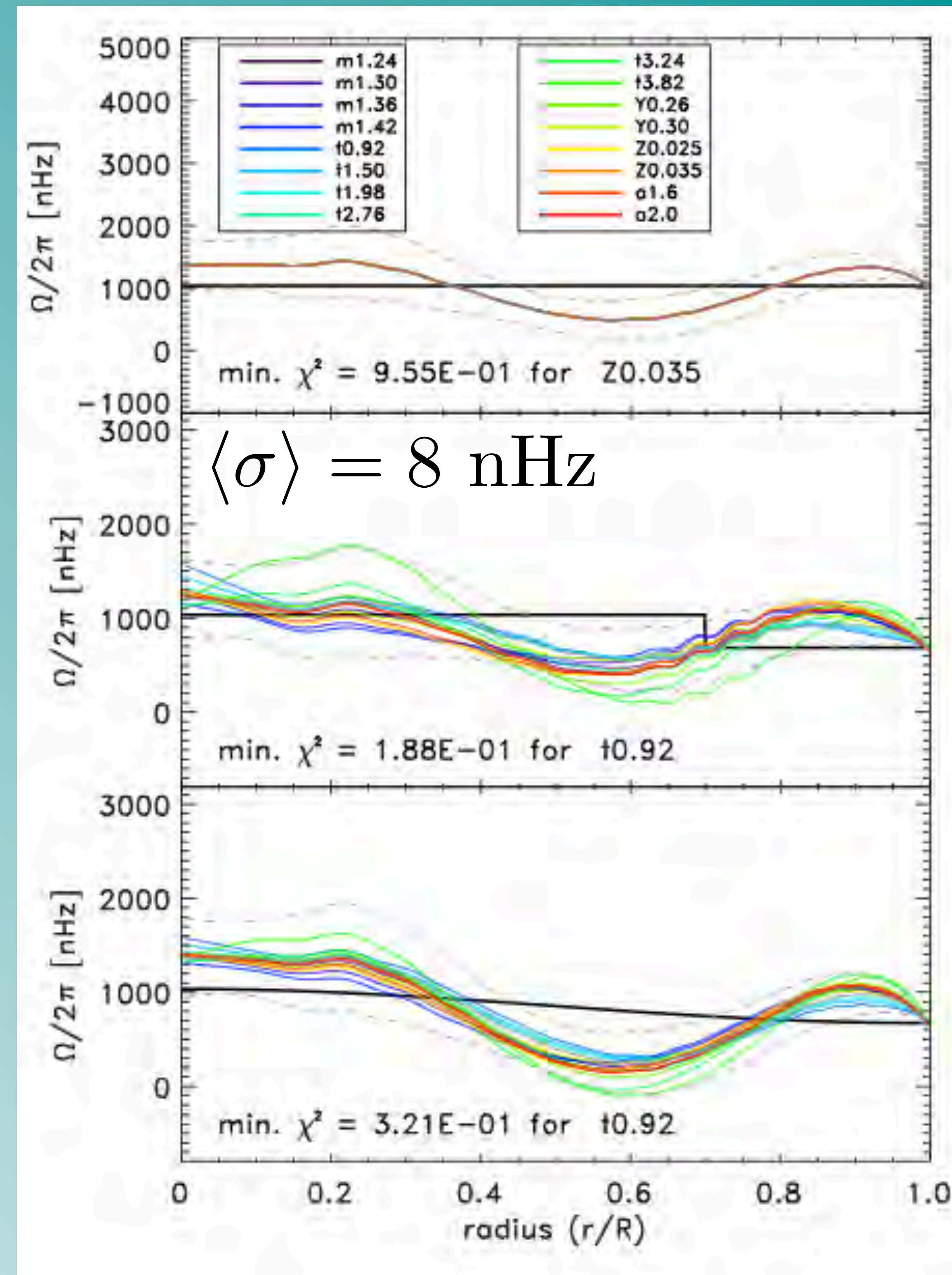
Reduced noise

uncertainties on splitting measured from Kepler are +/- 80nHz (27 months, 3 x solar rotation rate; Chaplin et al 2013)



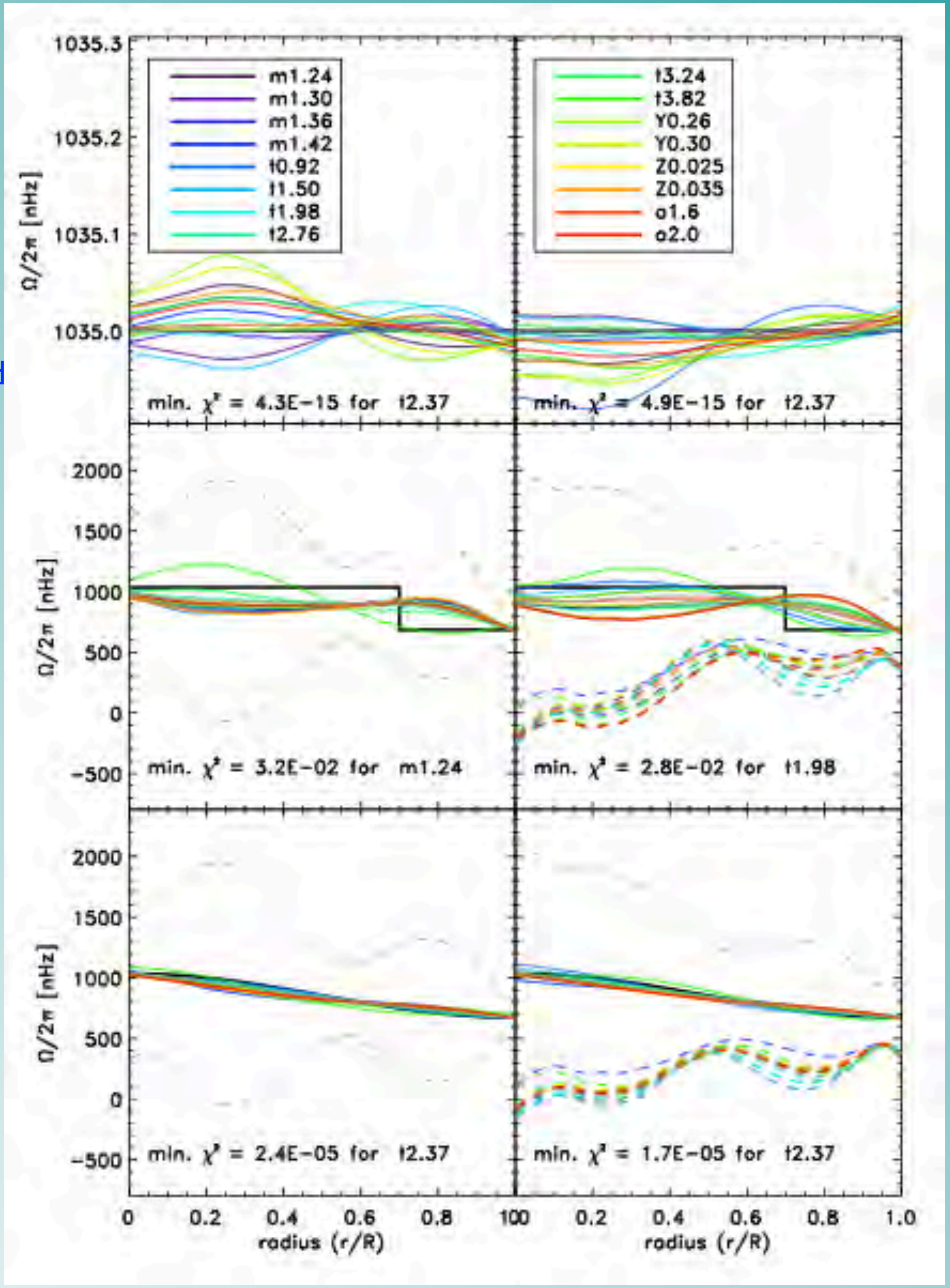


Reverse the experiment

Exp 1

$$\overline{\Omega}(r_0) = \sum_{i=1}^{M} c_i(r_0)\delta\omega_i$$

reference model — perturbed model



Exp 2

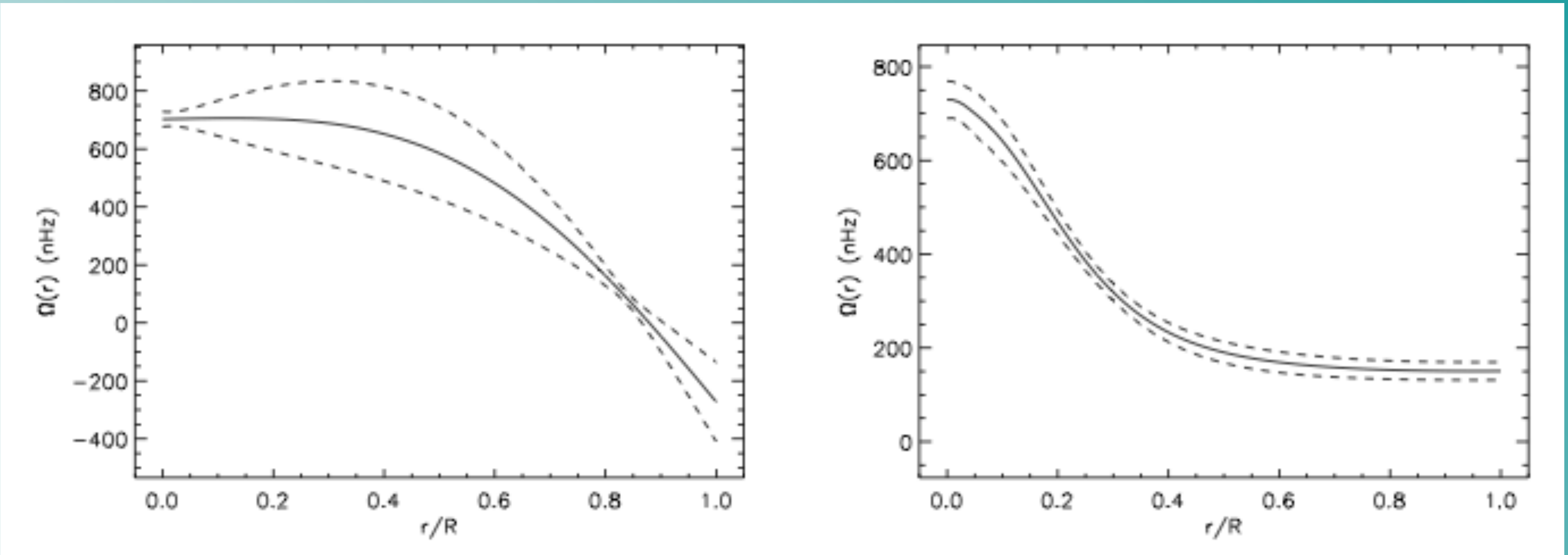
$$\overline{\Omega}(r_0) = \sum_{i=1}^{M} c_i(r_0)\delta\omega_i$$

perturbed model — reference model

$$\sigma_\Omega(r_0) = \sqrt{\sum_{i=1}^{M} [c_i(r_0)\sigma_i]^2}$$

2) Do surface constraints help?

RLS OLA



Otto
sub-giant

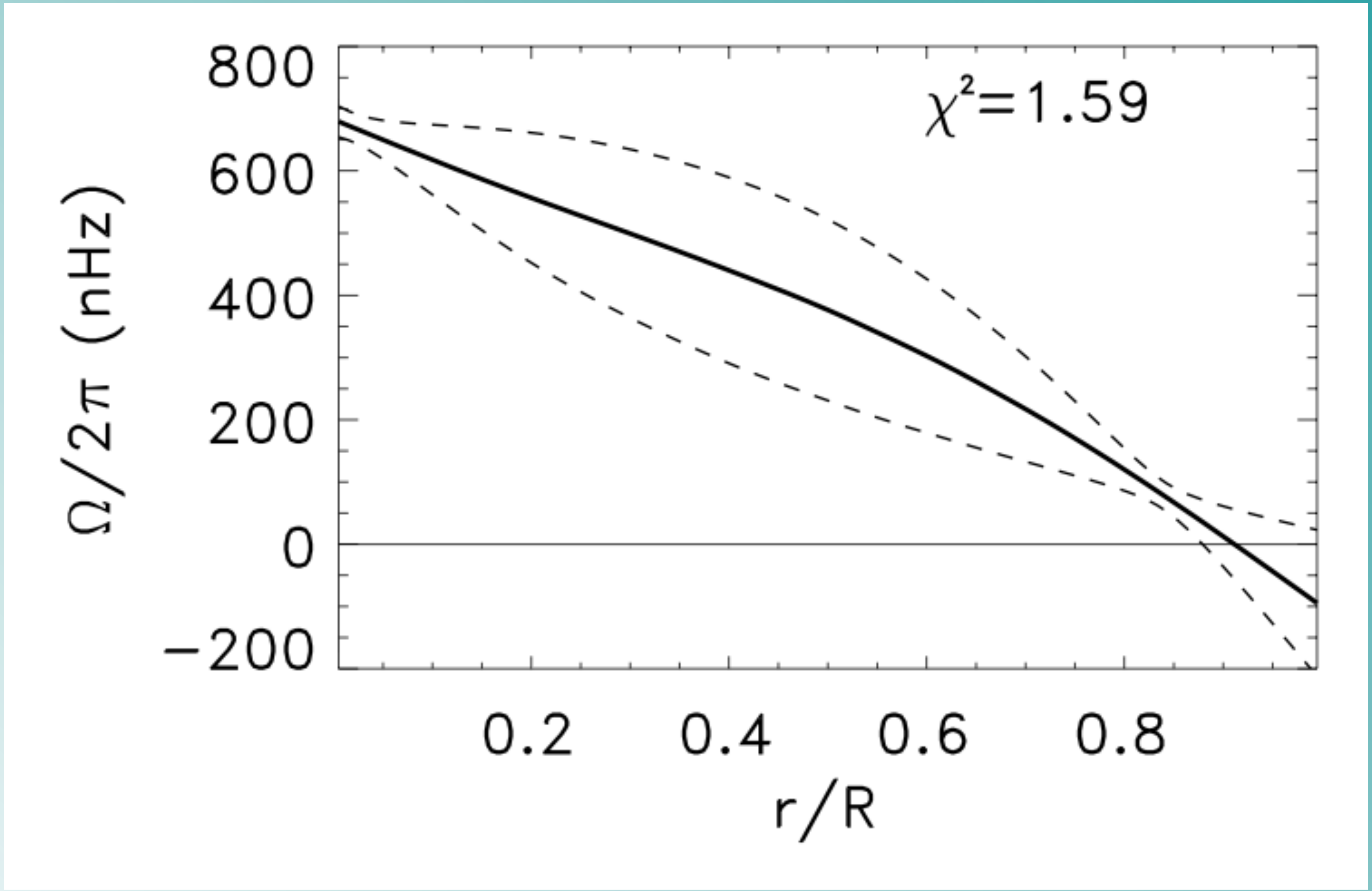
2) Do surface constraints help?

minimise: $$\sum_{i\in M}\left[\delta\omega_i - \sum_{j}^{N}\overline{\Omega}_j B_{ij}\right]^2 + \mu F(\overline{\Omega}) + \nu(\Omega_S - \overline{\Omega}_N)^2$$

$$\boxed{\overline{\Omega}(r_0) = \tilde{c}(r_0)\Omega_S - c_i(r_0)\delta\omega_i}$$

$$\sigma_\Omega = \sqrt{\tilde{c}^2\sigma^2_{\Omega_S} + c_i^2\sigma_i^2}$$

2) Do surface constraints help?



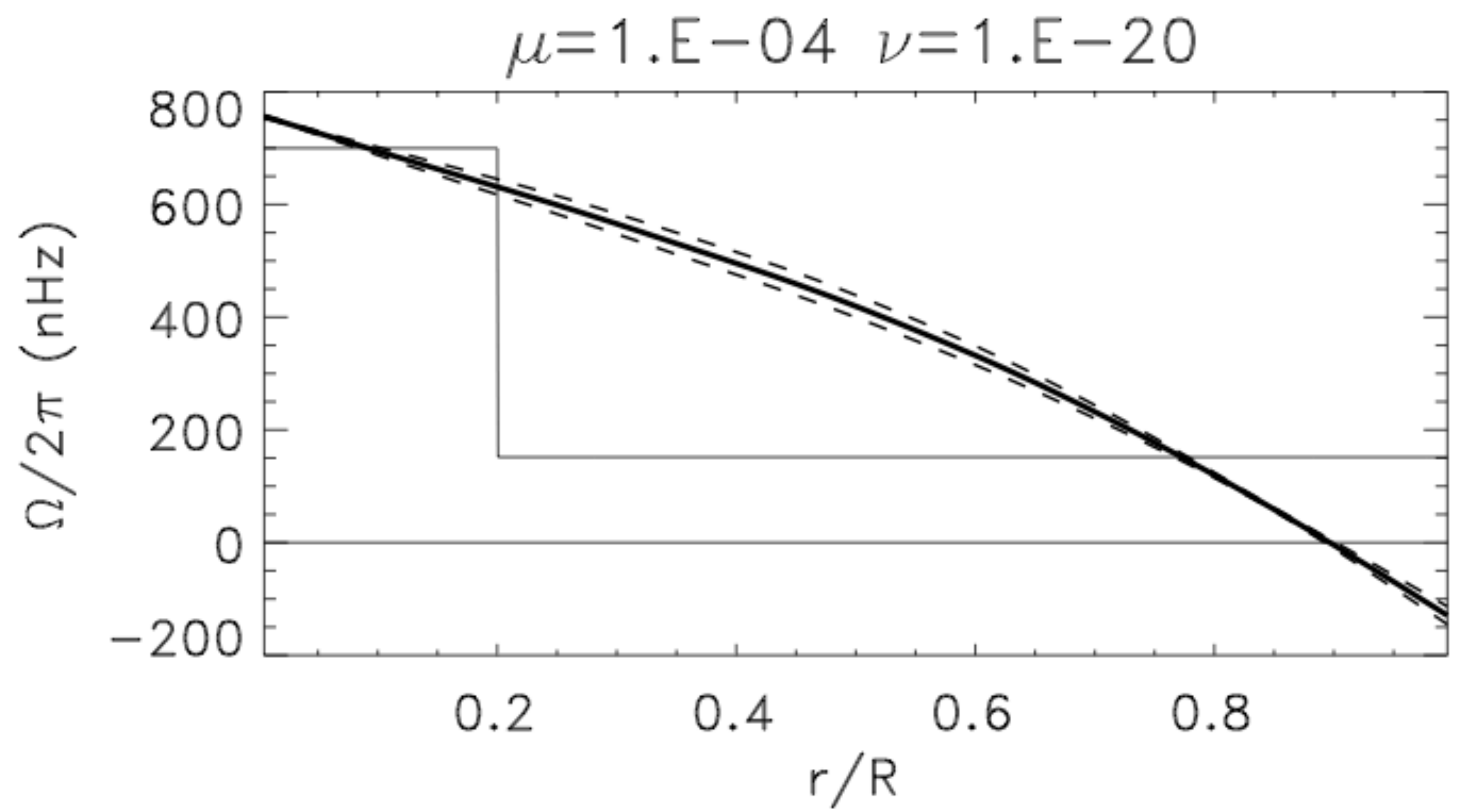
Uncertainties (one year of data) given by Deheuvels
RLS done in exactly the same way

synthetic case

$$\sum_{i\in M}\left[\delta\omega_i - \sum_{j}^{N}\overline{\Omega}_j B_{ij}\right]^2 + \mu F(\overline{\Omega}) + \nu(\Omega_S - \overline{\Omega}_N)^2$$

no (starspot) surface constraint available for Otto (!)

$v \sin i$ ($\mathrm{km\,s^{-1}}$)	$< 1.0 \pm 0.5$



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synthetic case

$$\sum_{i\in M}\left[\delta\omega_i - \sum_j^N \overline{\Omega}_j B_{ij}\right]^2 + \mu F(\overline{\Omega}) + \nu(\Omega_S - \overline{\Omega}_N)^2$$

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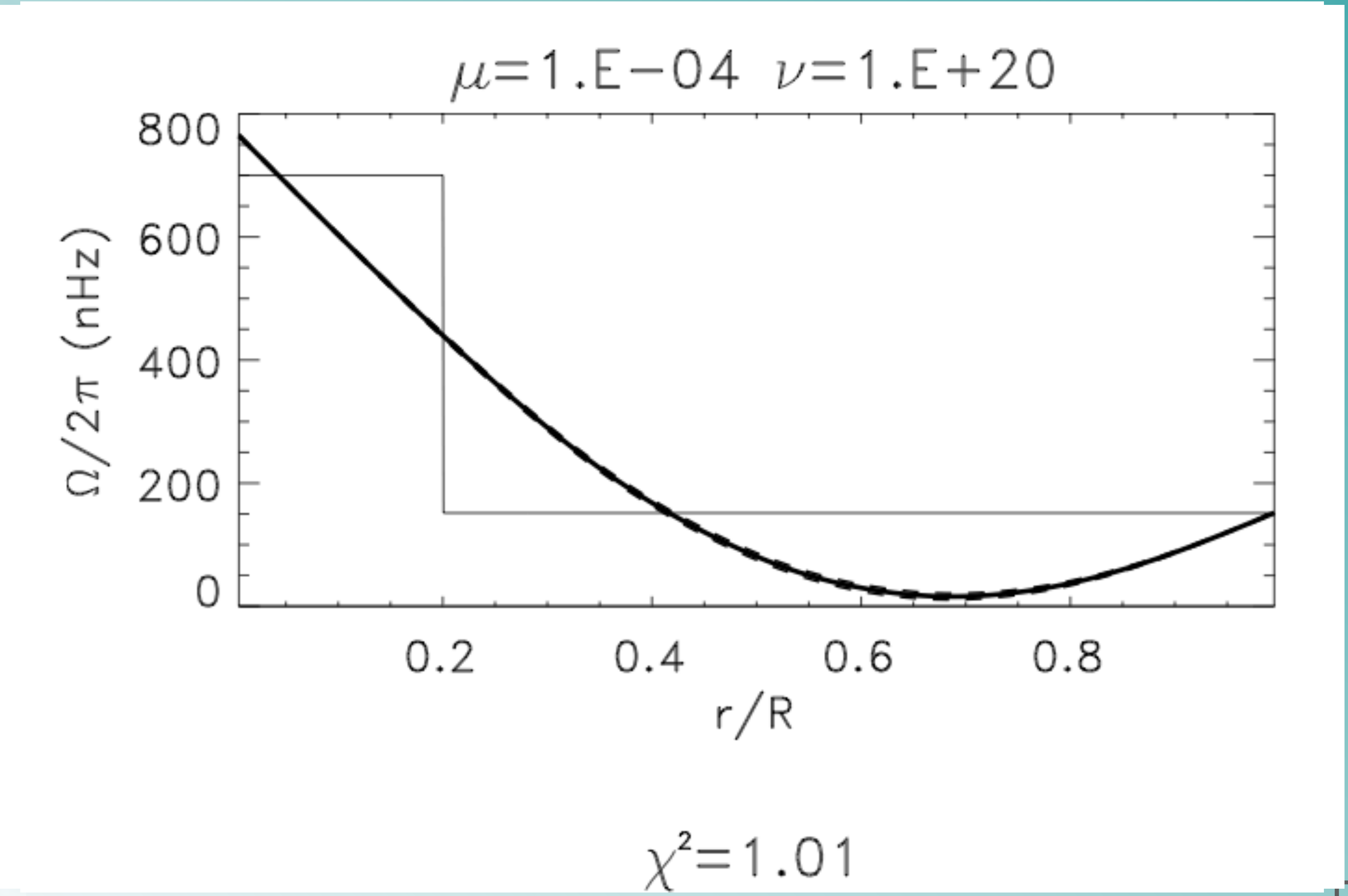

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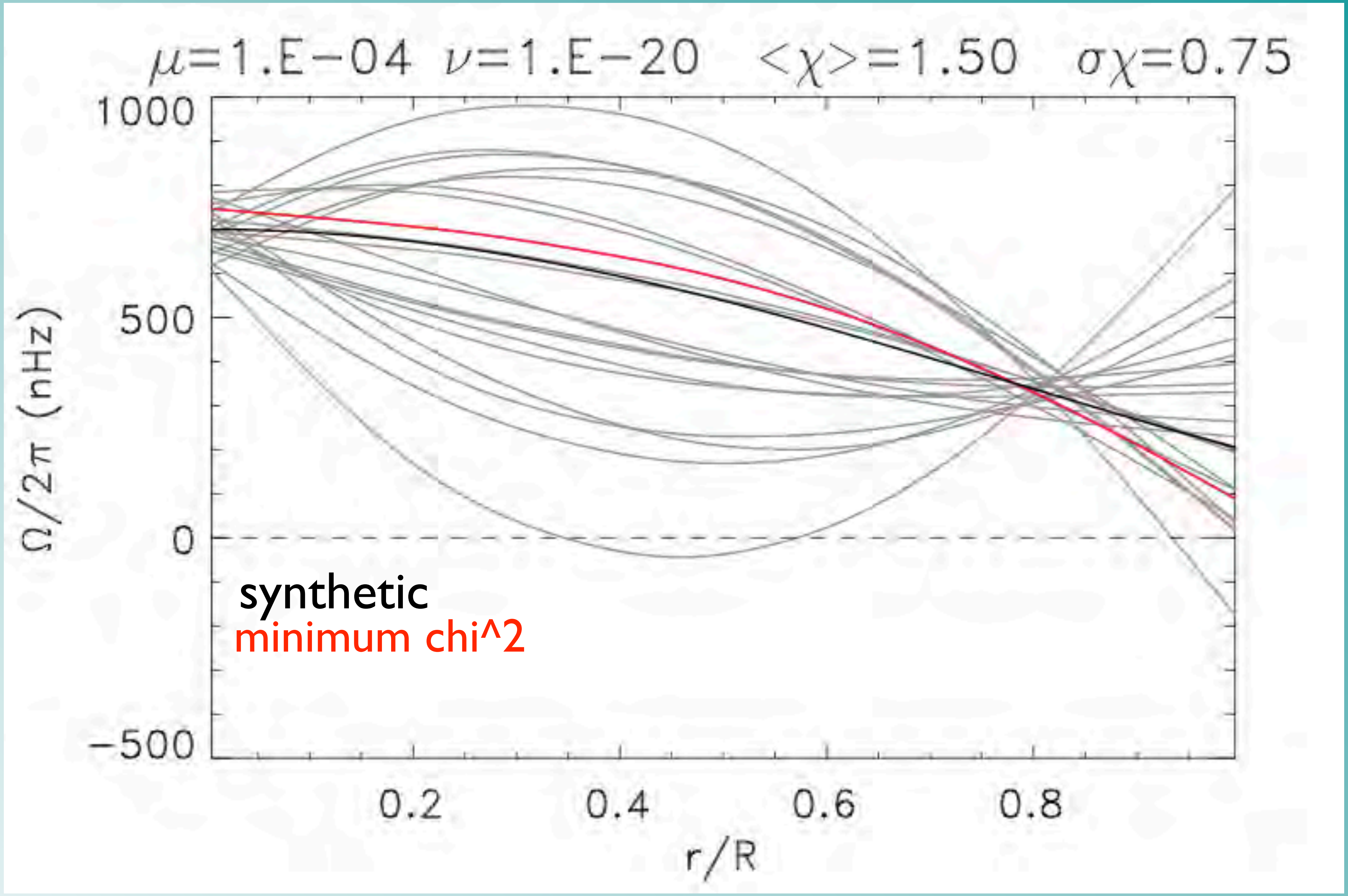
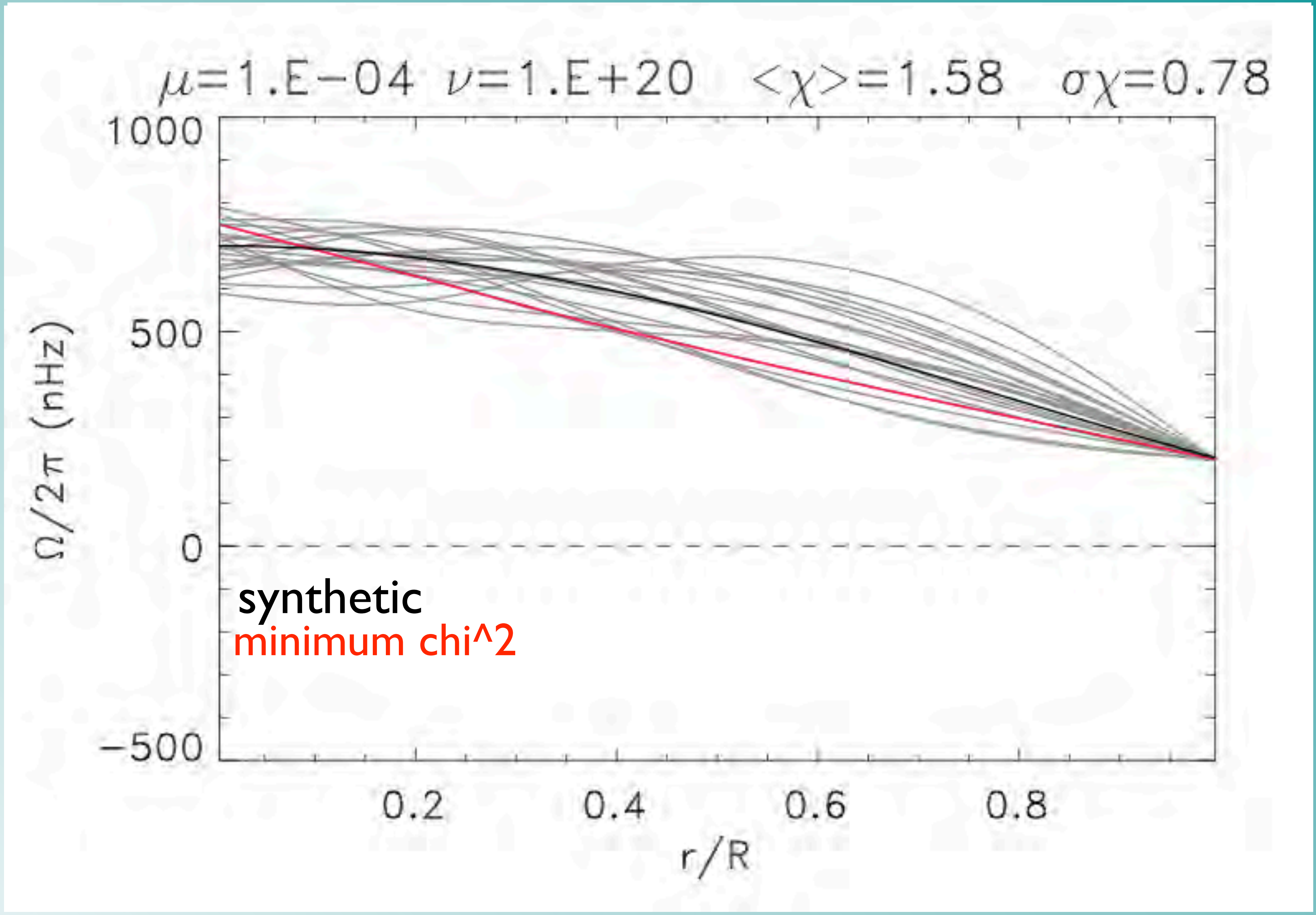
2) Do surface constraints help?



2) Do surface constraints help?



work in progress

- Uncertainties in models currently don't seem to matter - need to look further at different physical models
- Surface constraints will certainly help under certain conditions - need to determine exactly what those conditions are