Institute for Space Astrophysics and Planetology
Istituto di Astrofisica e Planetologia Spaziali

Session II

On numerical helio/asteroseimic inversions and the properties of the mode set

Outline

Can we infer interior of stars?

Which tools do we have? → **Inversion techniques**

- Brief summary of the technique
- Results obtained for the Sun
- Results obtained for other stars
- Contributed talks (H. Schunker, A. Eff-Darwich)
- Discussion and Problems

Inverse Analysis

$$d_i = \int_0^R \mathcal{K}(r) f(r) \mathrm{dr} + \varepsilon_i \qquad i = 1,2, \ldots . M$$

Kernel

Errors

ILL POSED PROBLEM

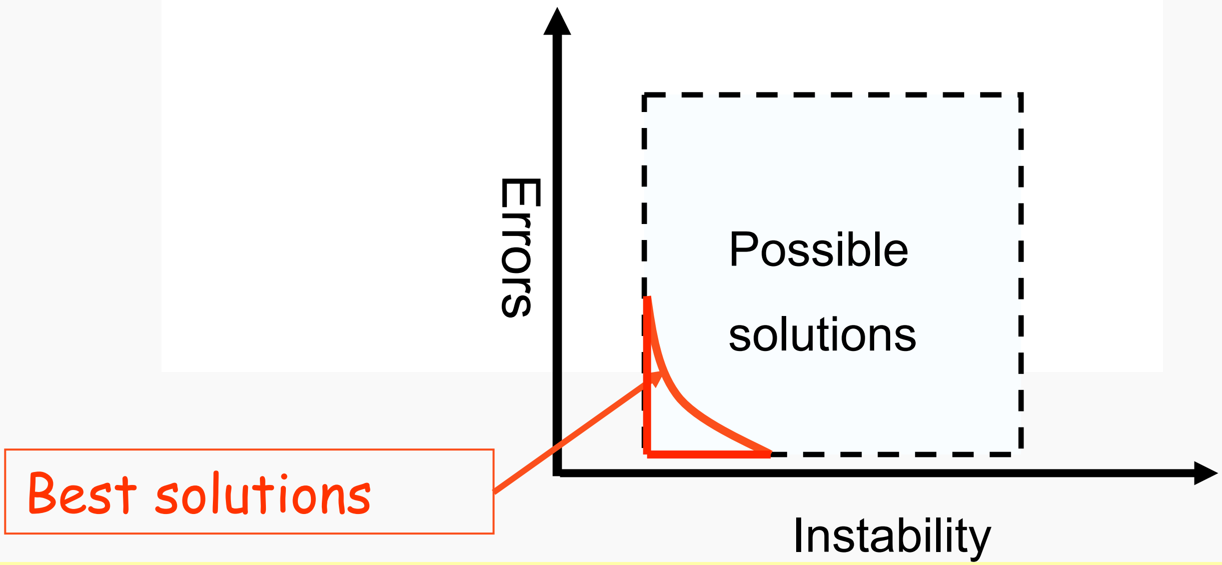
- NUMBER OF DATA ➡ FINITE SET
- DATA ➡ affected by ERRORS

Existence, Uniqueness, stability of solution

Inverse Analysis

ILL POSED PROBLEM

Existence, Uniqueness, stability of solution



- **Analytical techniques** -Using the asymptotic dispersion relation of oscillation frequencies
- **Numerical techniques** -Use of parameters: Regularization

Numerical inversions

- OLA, Optimally Localized averages
  (Backus & Gilbert 1968,1970)
- RLS, Regularized least-squares fitting method
  (Phillips 1962,Tikhonov 1963)

- Observed Data+errors
- Model of the observed star

Optimally Localized Averages (OLA)

Backus & Gilbert 1970

$$\delta\nu_{n,l} = \int_0^R \mathcal{K}_{n,l}(r)\, f(r)\mathrm{dr} + \sigma_{n,l}$$

Solution: a linear combination of the data that is a localized average near $r = r_0$

$$\overline{f(r_0)} = \sum_i^M c_i(r_0) 2\pi\delta\nu_i$$

Averaging kernel

$$K(r_0, r) = \sum_{i=1}^M c_i(r_0)\mathcal{K}_i(r)$$

Find coefficients as to minimize:

$$\int_0^{R_\odot} J(r_0, r) K(r_0, r)^2 dr + \mu \sum_{i=1}^M \sigma_i^2 c_i^2(r_0)$$

Localization function

SOLA, Subtractive OLA

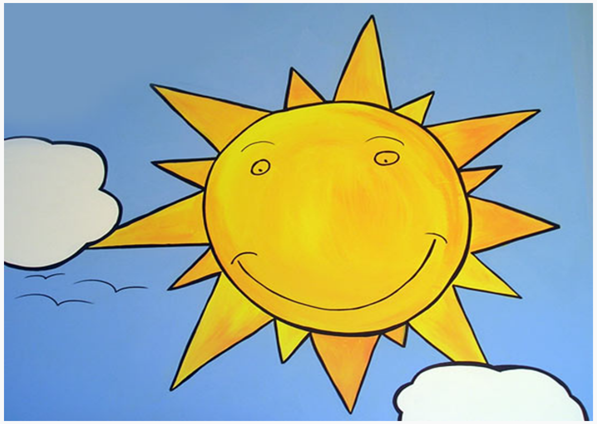
Pijpers & Thompson 1992

Choose the coefficients c_i so as to minimize

$$\int_0^{R_\odot} [\sum_{i=1}^{M} [K(r_0,r) - G(r_0,r)]^2\, dr + \mu \sum_{i=1}^{M} \sigma_i^2 c_i^2(r_0)\ .$$

E.g. $G=A \exp(-(r-r_0)^2/\ \delta^2)$.
the trade off parameter is rescaled at each r_0 to keep constant the width of the aver. kernel

For the Sun we can infer both rotation and internal structure



Inversion for solar structure

$$\frac{\nu_{obs} - \nu_{mod}}{\nu_{mod}} = \frac{\delta\nu_{n,l}}{\delta\nu_{n,l}} \Rightarrow \frac{Sun - model}{model}$$

Variational principle
(Chandrasekhar 1964)

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{R_\odot} K^i_{c^2,\varrho} \left(\frac{\delta\Gamma_1}{\Gamma_1}\right)_{\rm int} \mathrm{d}\, r + \int_0^{R_\odot} K^i_{u,Y} \frac{\delta u}{u} \,\mathrm{d}r + \int_0^{R_\odot} K^i_{Y,u}\, \delta Y \,\mathrm{d}r + \frac{F_{\rm surf}(\nu)}{Q_i} + \varepsilon_i \quad i = (n,l) = 1....M$$

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{R_\odot} K^i_{\Gamma_1,\varrho} \left(\frac{\delta\Gamma_1}{\Gamma_1}\right) \mathrm{d}\, r + \int_0^{R_\odot} K^i_{\varrho,\Gamma_1} \frac{\delta\rho}{\rho} \,\mathrm{d}r + \frac{F_{\rm surf}(\nu)}{Q_i} + \varepsilon_i$$

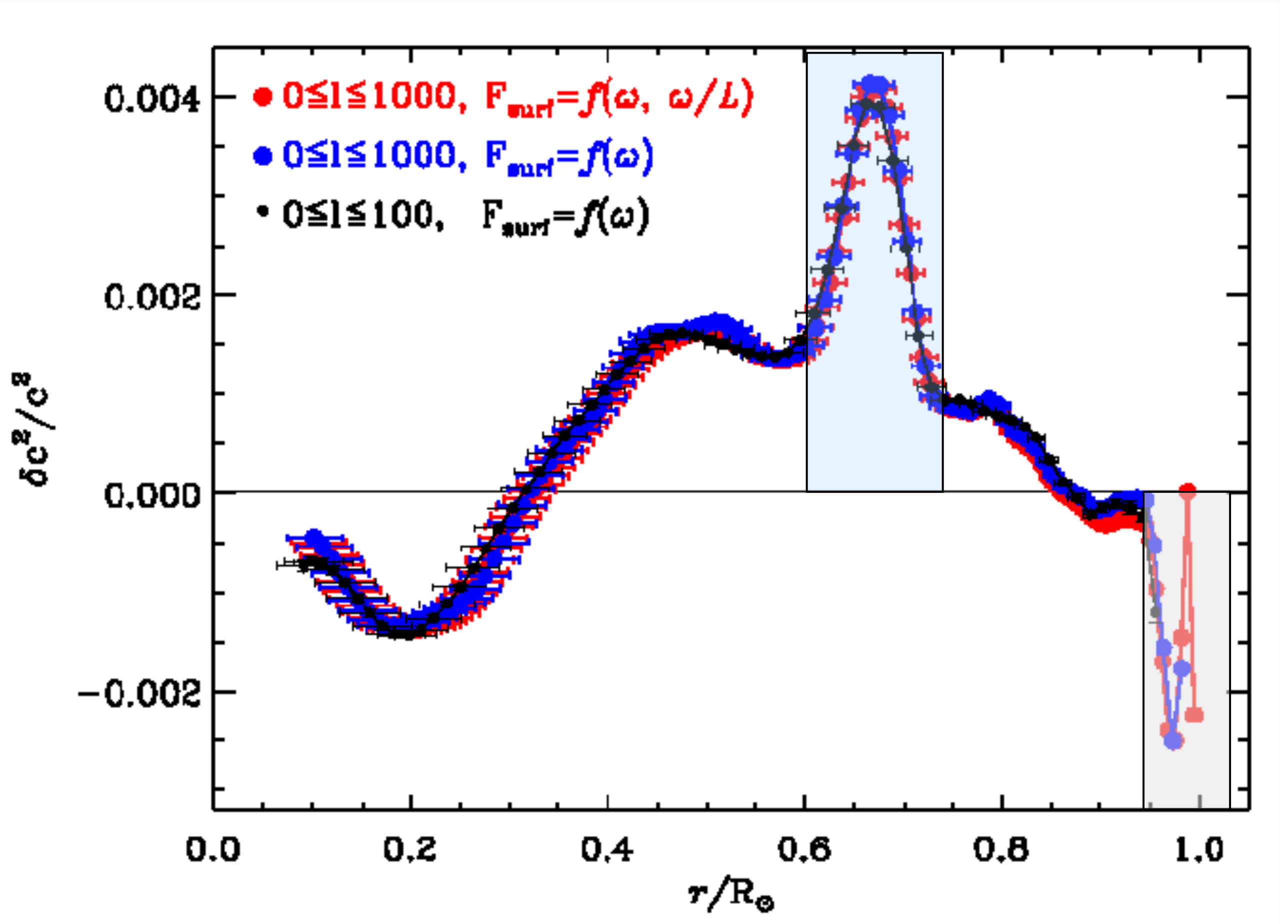
$$\frac{\delta\nu_i}{\nu_i} = \int_0^{R_\odot} K^i_{\Gamma_1,u} \left(\frac{\delta\Gamma_1}{\Gamma_1}\right) \mathrm{d}\, r + \int_0^{R_\odot} K^i_{u,\Gamma_1} \frac{\delta u}{u} \,\mathrm{d}r + \frac{F_{\rm surf}(\nu)}{Q_i} + \varepsilon_i \quad u = p/\rho$$

Notable successes for the Sun

- Depth of the solar convection zone (Christensen-Dalsgaard 1985)
- Diffusion of helium and heavy elements (Basu et al. 1996)
- Helium abundance (e.g. Gough 1984)
- Relativistic effect in the core (Elliot & Kosovichev 1998)
- Internal Dynamics (Schou et al. 1998….etc)
- Equation of state

Correctness of the standard solar model!!!

Difference SUN-model



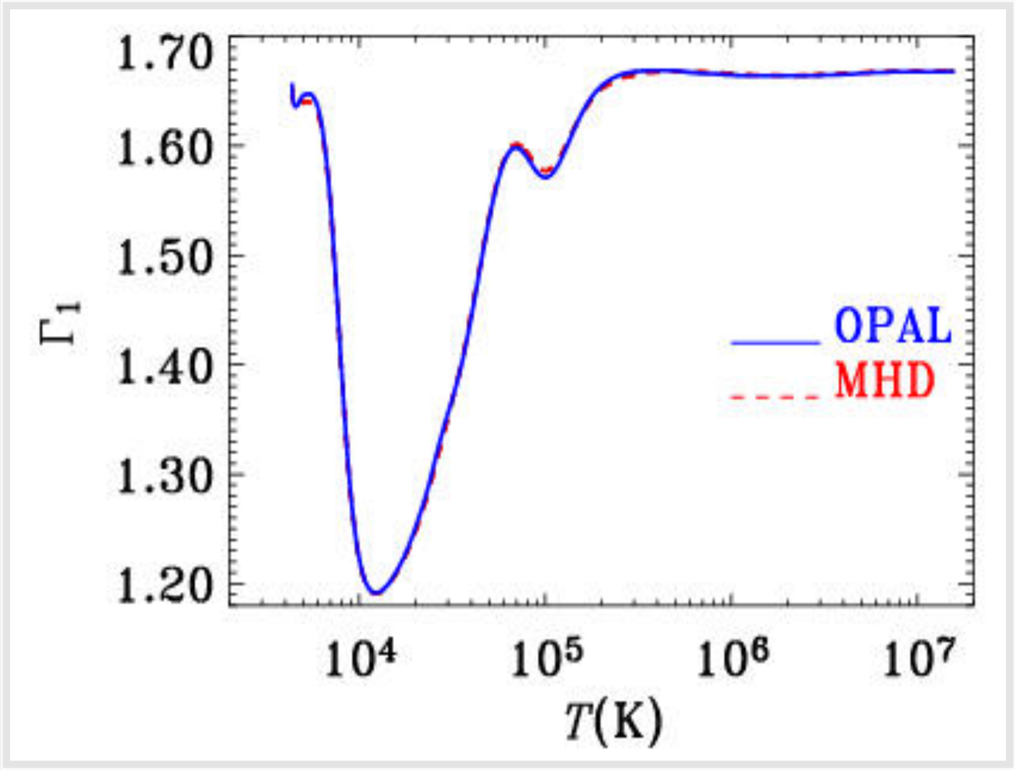
PROBING EOS IN THE STARS

First adiabatic exponent

$$\Gamma_1 = \left(\frac{\partial \ln p}{\partial \ln \rho} \right)_{ad}$$

In the SUN

$\Gamma_1 \approx 5/3$ in the interior except in the H and He ionization zones



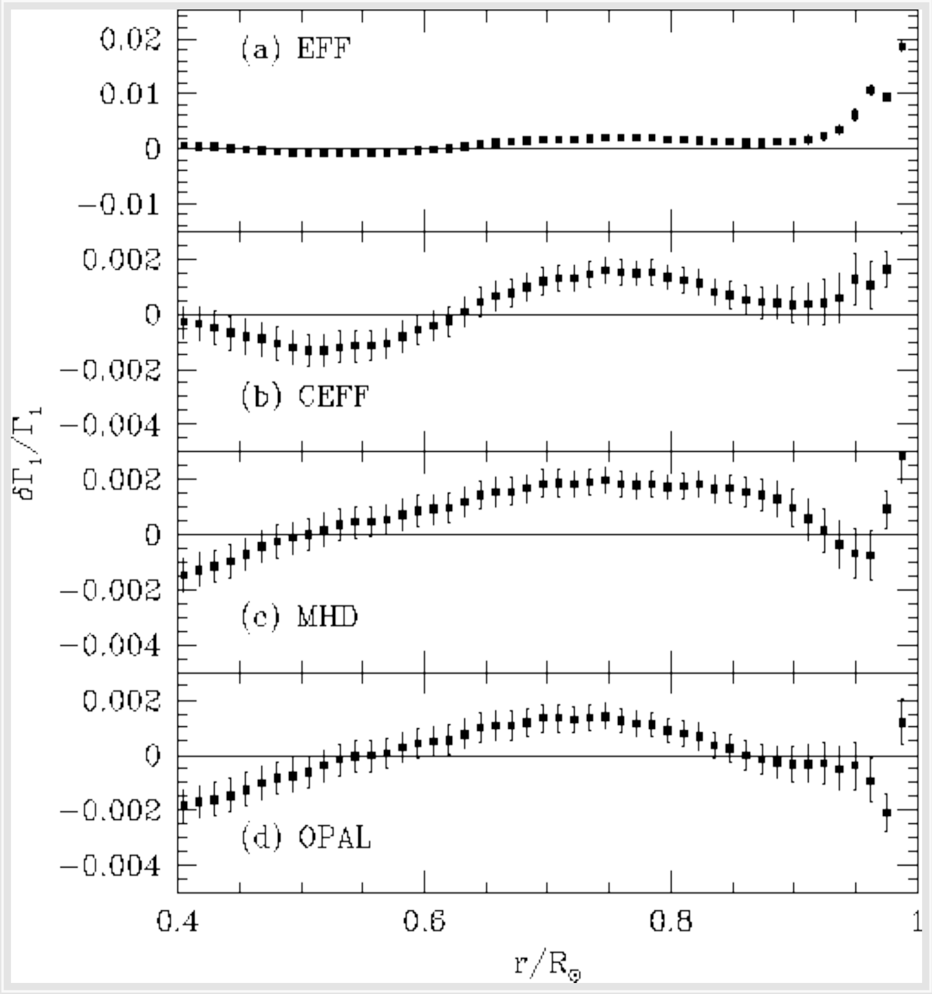
First Adiabatic Exponent

Inversion of data with $l \leq 100$

Basu & Christensen-Dalsgaard 1997;

Elliott & Kosovichev 1998;

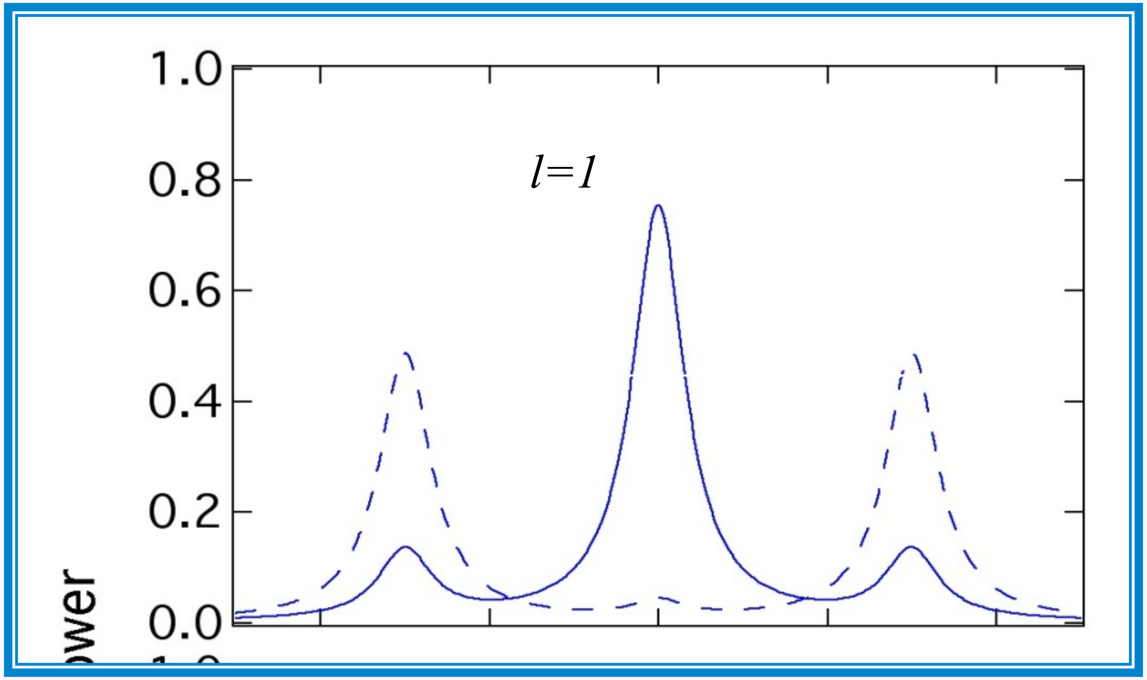
Di Mauro & Chrisensen-Dalsgaard 2001



Rotational splittings

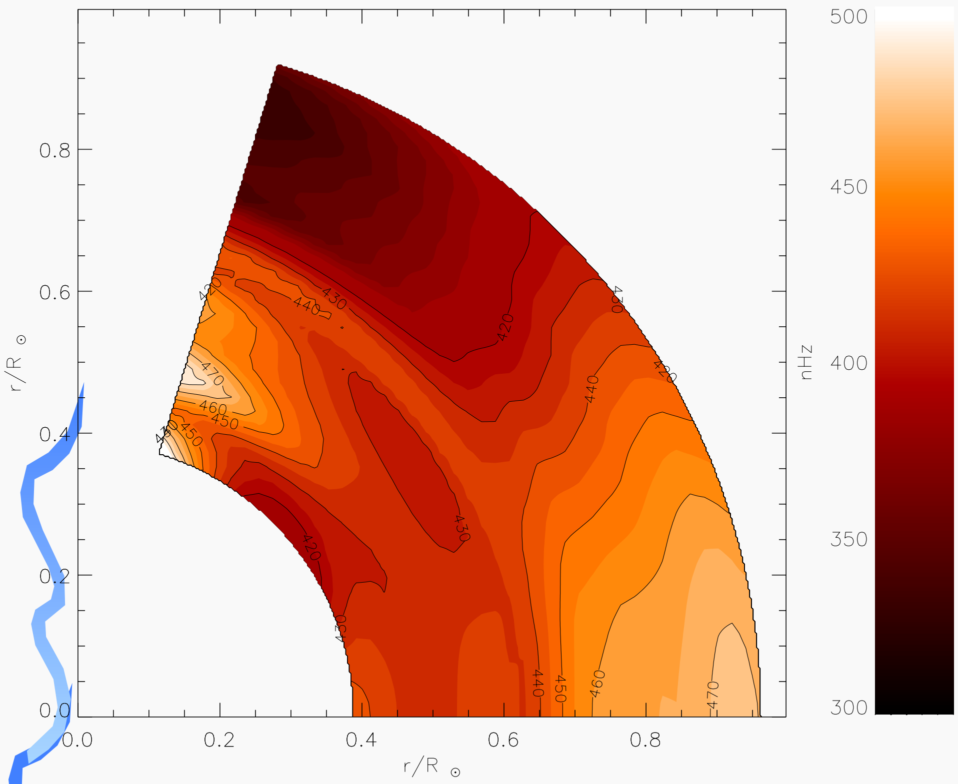
Rotation breaks spherical simmetry and splits the frequency of oscillations

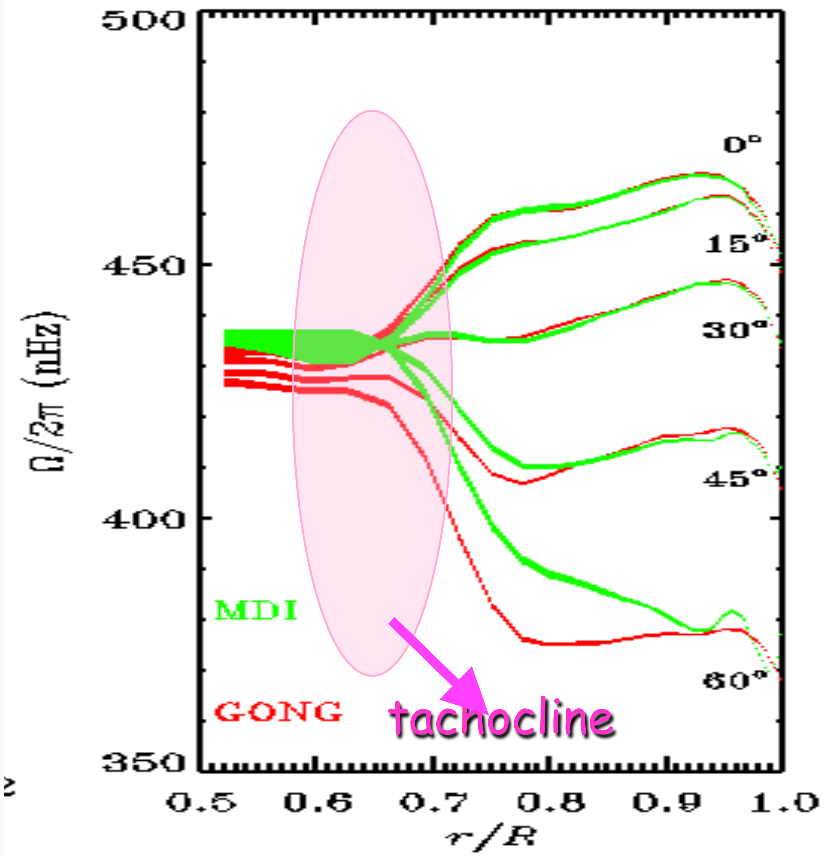
$$\delta\omega_{nl} = (1 - C_{nl})\Omega$$



l=1 mode seen under inclination: i=30°-80°
for a star of $R=5R_{\odot}$, rotating with $v_{eq}=3$ km/s

Internal Rotation





r_c/R = 0.7133 +/- 0.005 (Basu & Antia 2004)

INFERRING THE SUN's CORE

MDI l < 100 (Schou et al. 1998)+

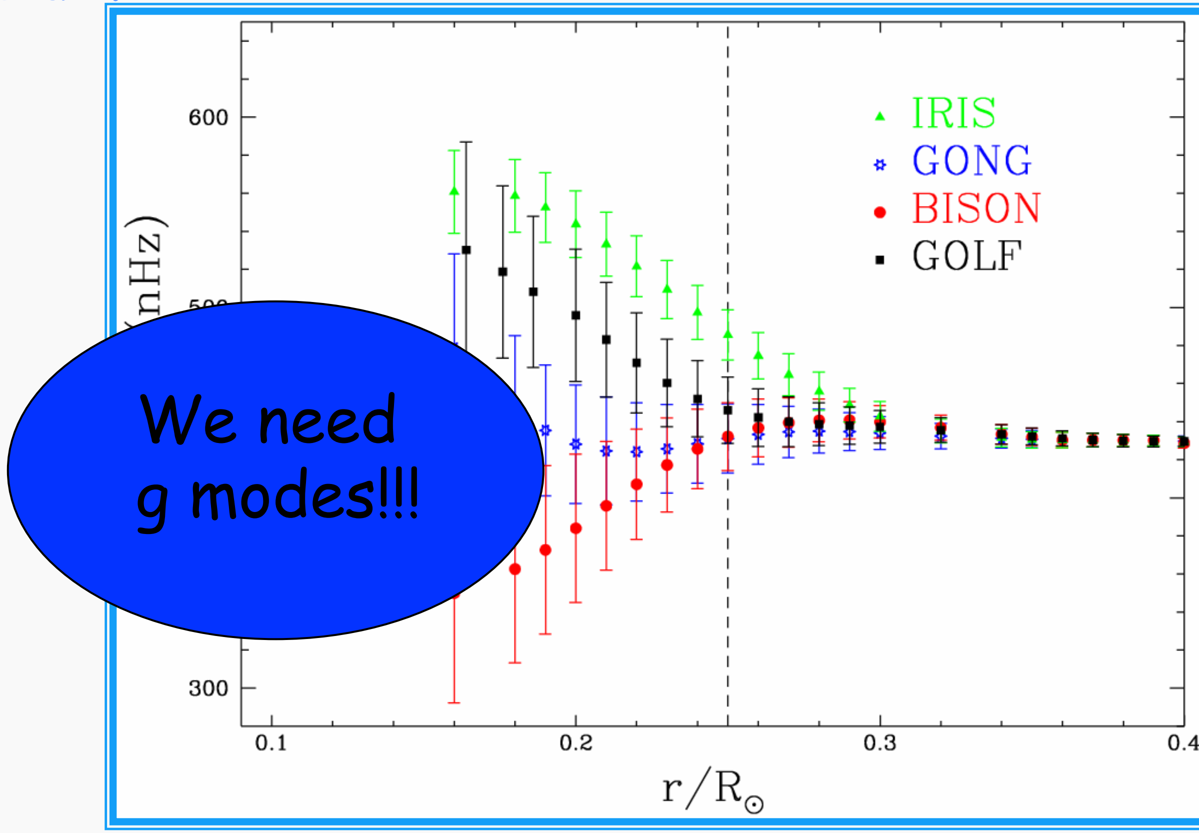
IRIS l=1-3 (Lazreck et al. 1996; Gizon et al 1997, Fossat 1998)

GONG l=1-3 (Gavryuseva & Gavryuseva 1998)

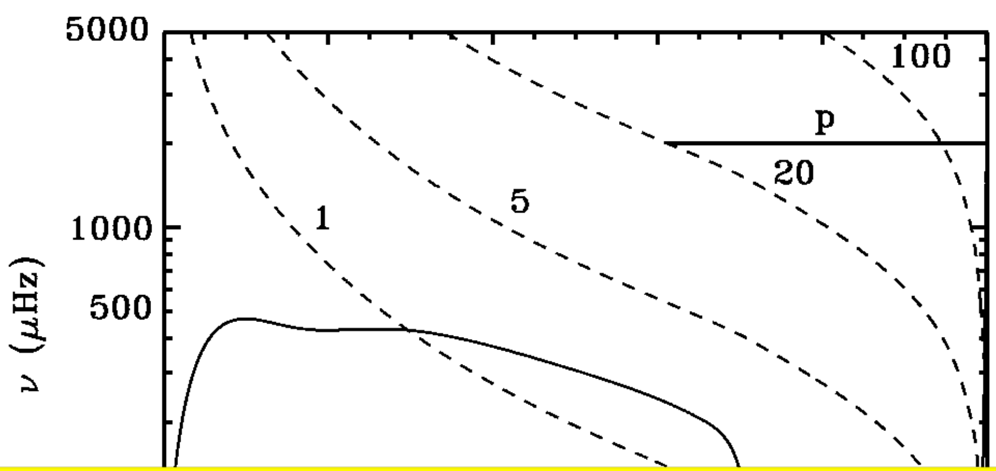
BISON +LOWL l=1-4 (Chaplin et al. 1999)

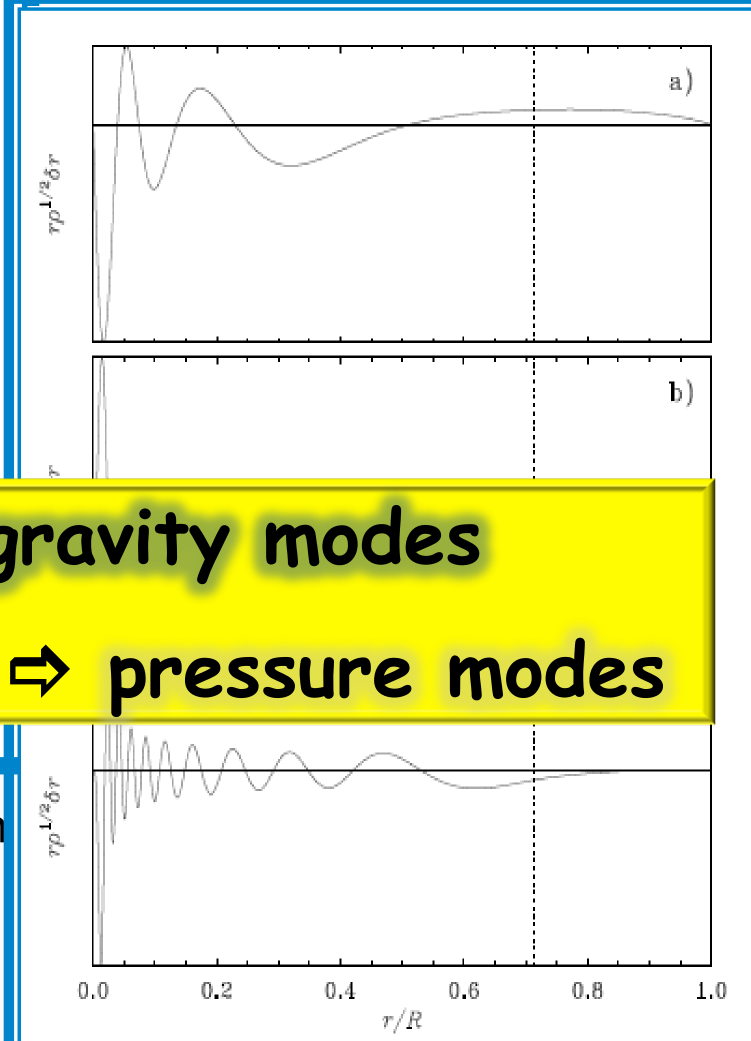
GOLF l=1-2 (Corbard et al. 1998)

Di Mauro et al. 1998



Trapping of the modes in MS star





- Condition of the core ⇨ **gravity modes**
- Condition in the envelope ⇨ **pressure modes**

Eigenfunction oscillates as function of *r* when

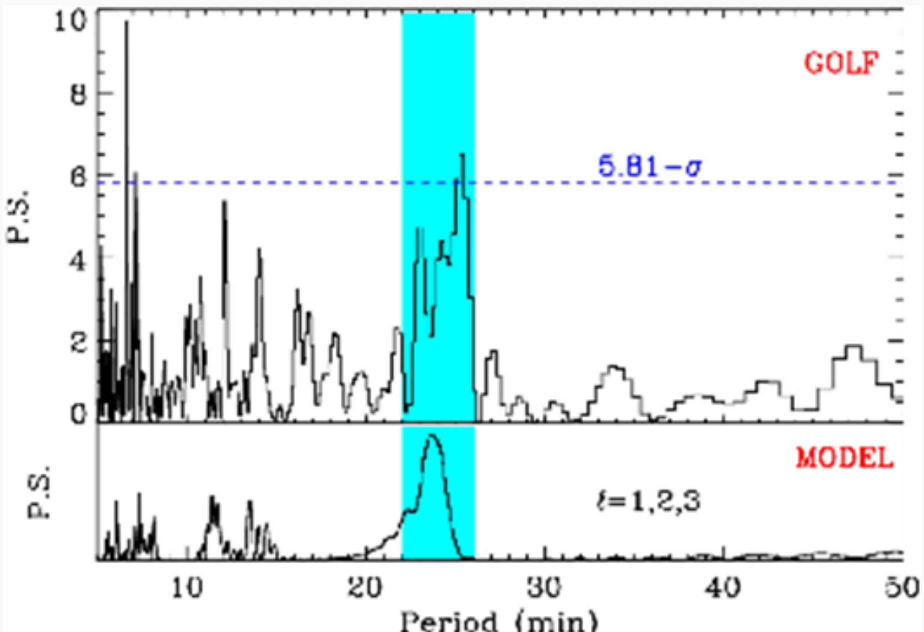
$\omega^2 > S_l^2, N^2$ **p modes**

$\omega^2 < S_l^2, N^2$ **g modes**

Gravity modes

Solar Gravity Modes detected with GOLF!!!

Garcia et al. 2007



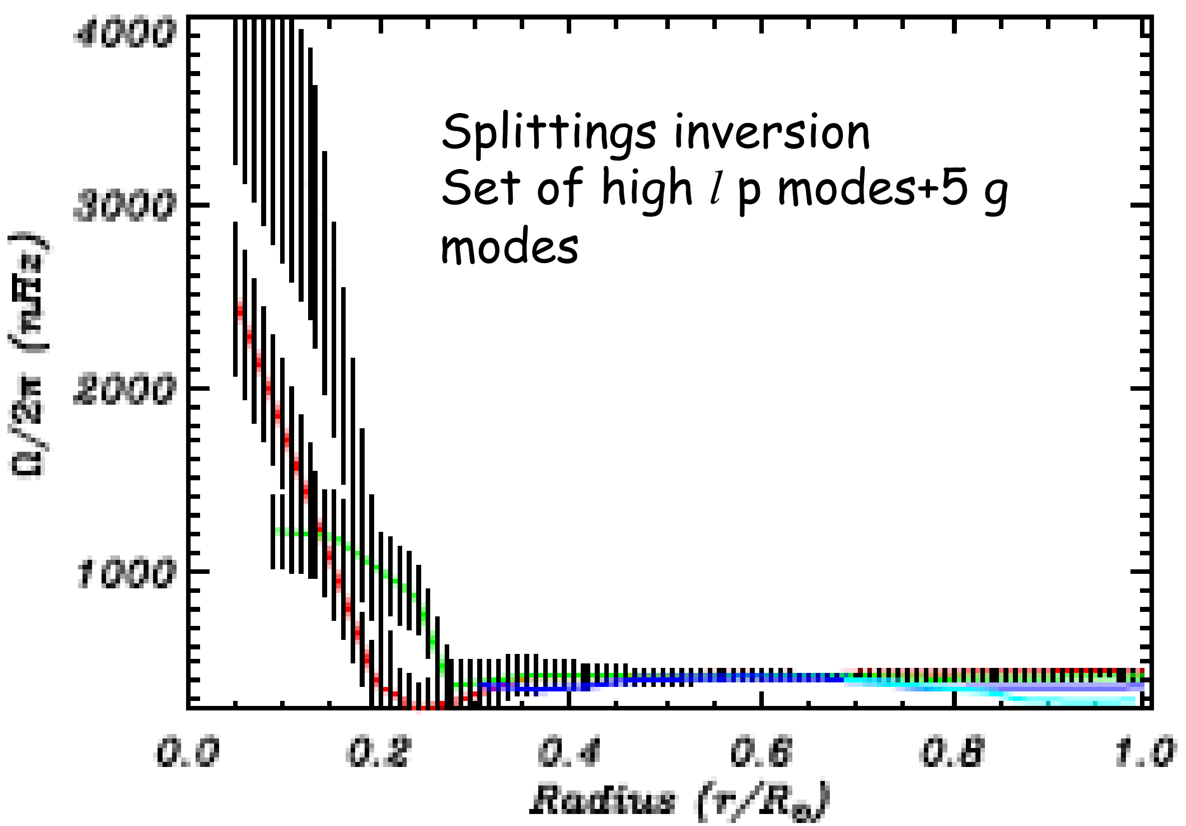
10 years of observations from GOLF

Gravity waves can be excited by convective plumes into radiative zone!!!

Dintrans et al. 2005

In the core

Eff-Darwich et al 2008, Garcia et al. 2011



From the Sun to the other stars…….

Can we extend helioseismic tools to other stars?

Helio- vs Asteroseismology

- Large distance
- Point-source character of target
- Stellar constraints

Small set of only low harmonic degree modes ➡ $l < 4$

What about the internal structure of stars?

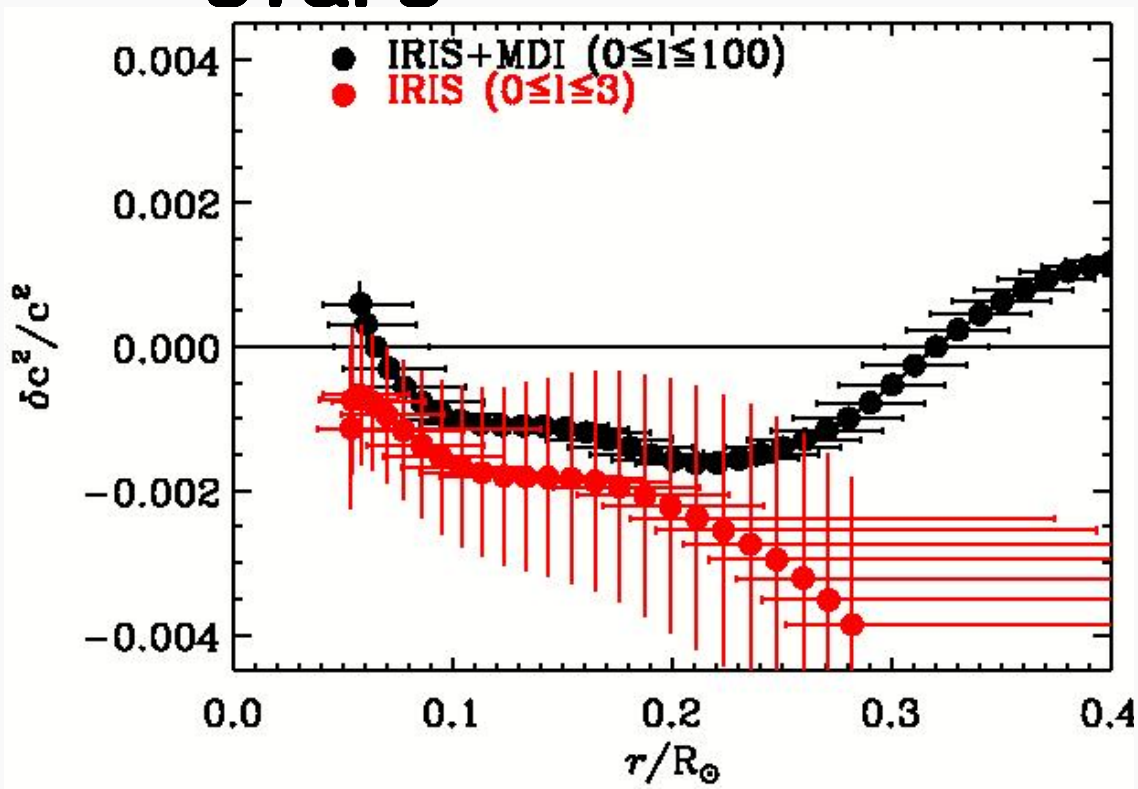
Gough & Kosovichev 1983
Roxburgh et al. 1998
Berthomieu et al. 2001
Basu 2002
Lorchard et al. 2004
Goupil et al. 2004

Inversion of artificial data has been successful…….but reality is different.

THE SUN AS A STAR

We can use the Sun as laboratory to learn how to deal with other stars

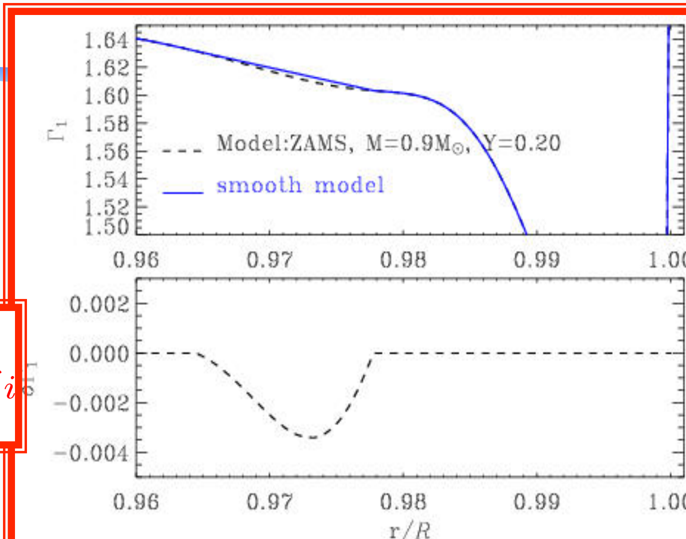
83 p-modes	
$l=0$	$9 \le n \le 32$
$l=1$	$7 \le n \le 32$
$l=2$	$8 \le n \le 28$
$l=3$	$11 \le n \le 22$

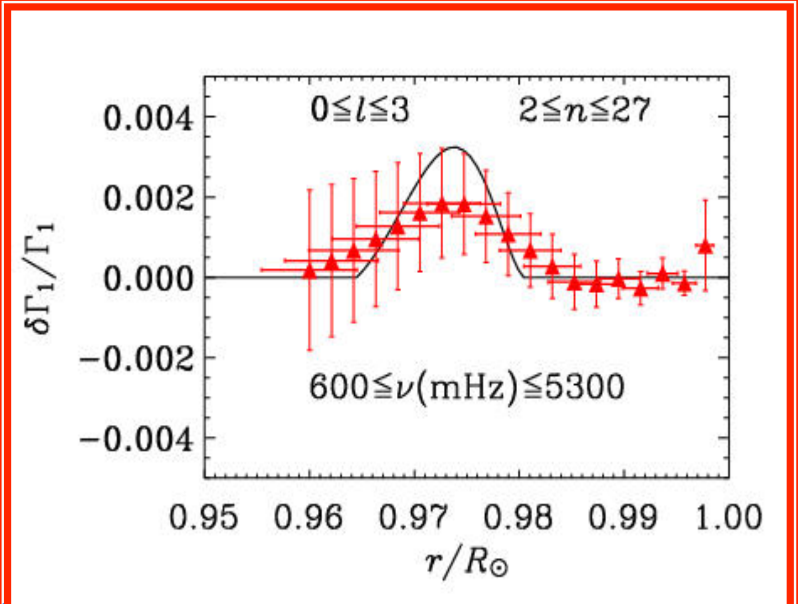


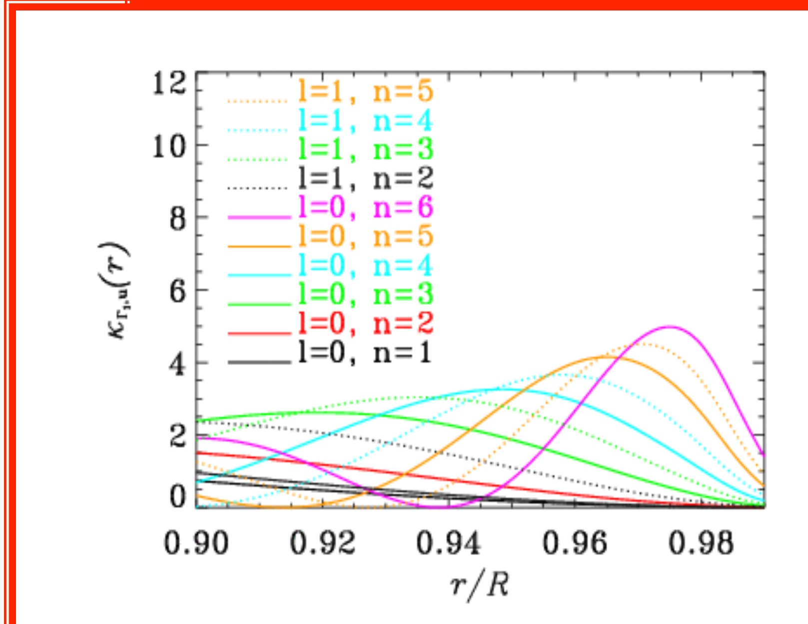
Probing the EOS

With artificial data

$$\frac{\delta\nu_i}{\nu_i} = \int_0^R K^i_{\Gamma_1,u}\left(\frac{\delta\Gamma_1}{\Gamma_1}\right) \mathrm{d}\,r + \int_0^R K^i_{u,\Gamma_1}\,\frac{\delta u}{u}\,\mathrm{d}r + \varepsilon_i$$





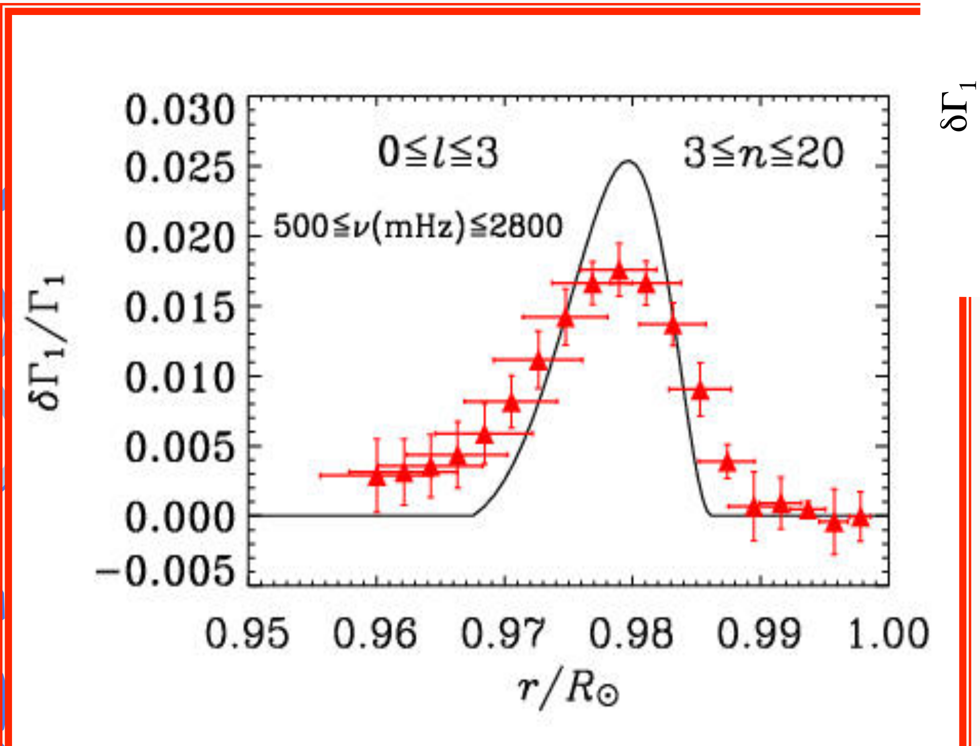


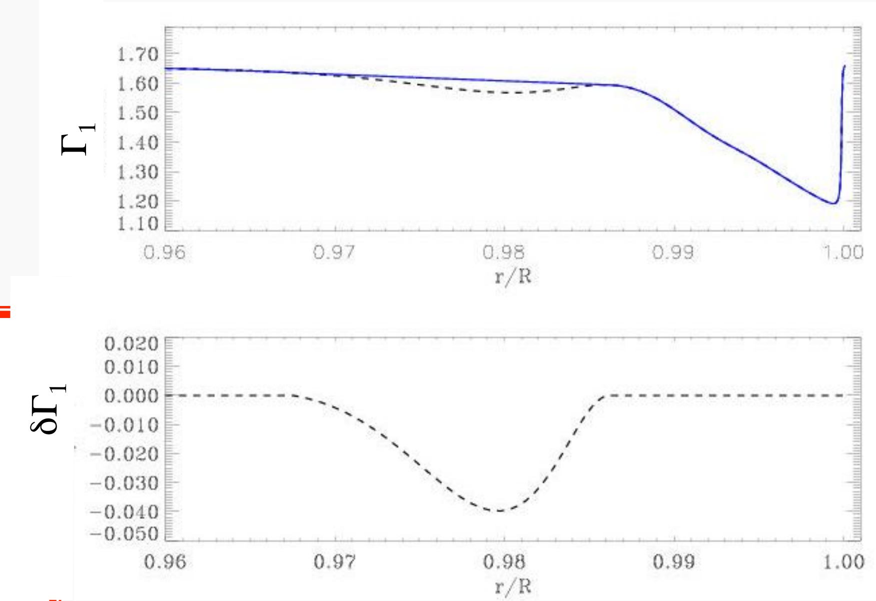
For high mass stars

Model $M=1.2M_{\odot}$

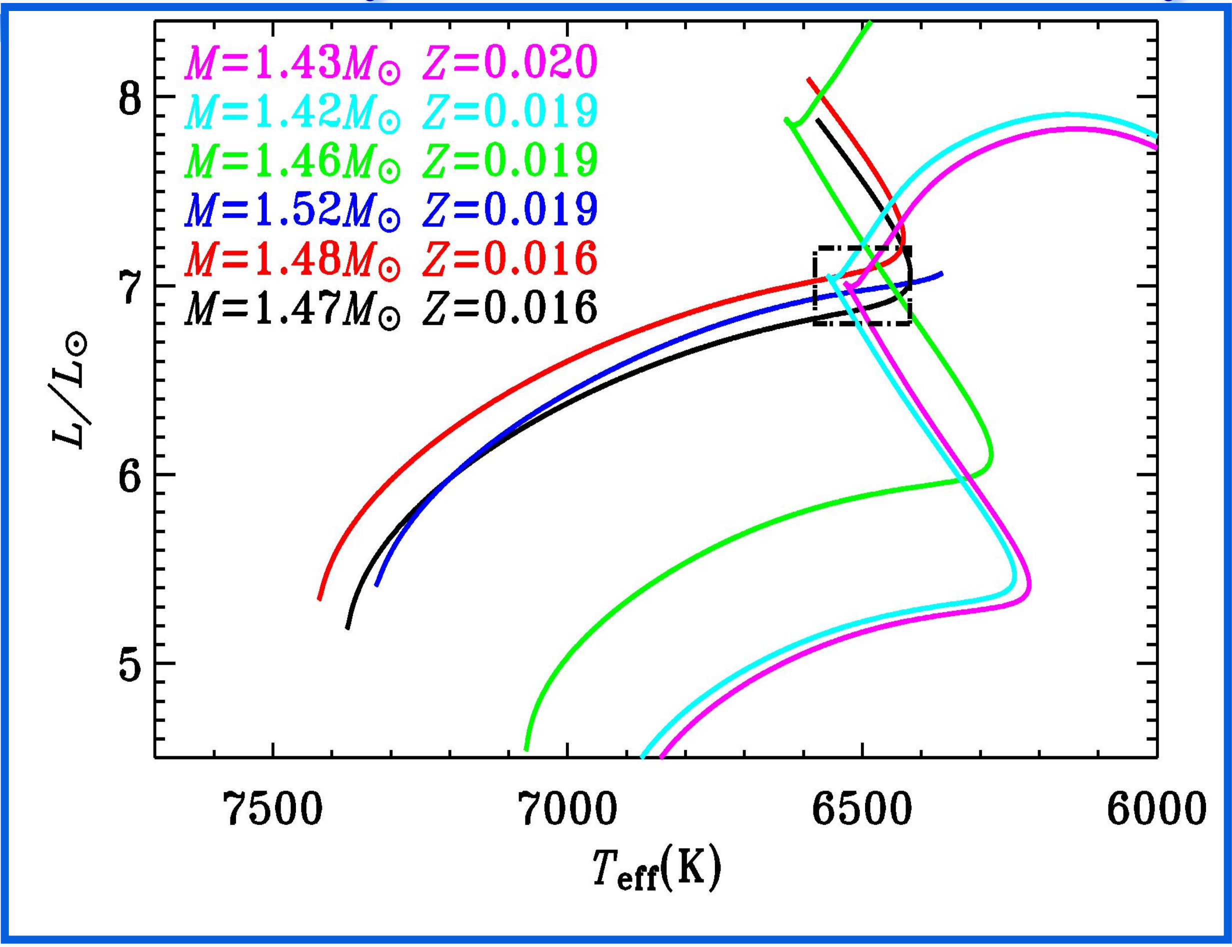
$Y=0.23$

Age=ZAMS





Evolutionary state of Procyon A

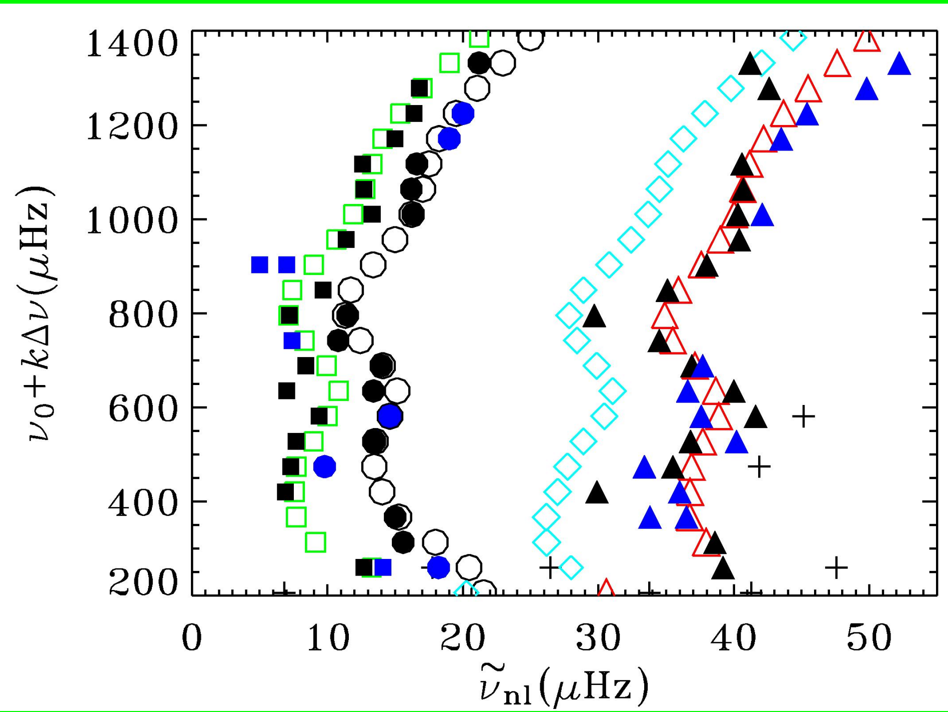


EOS OPAL 2001, diffusion of heavy elements

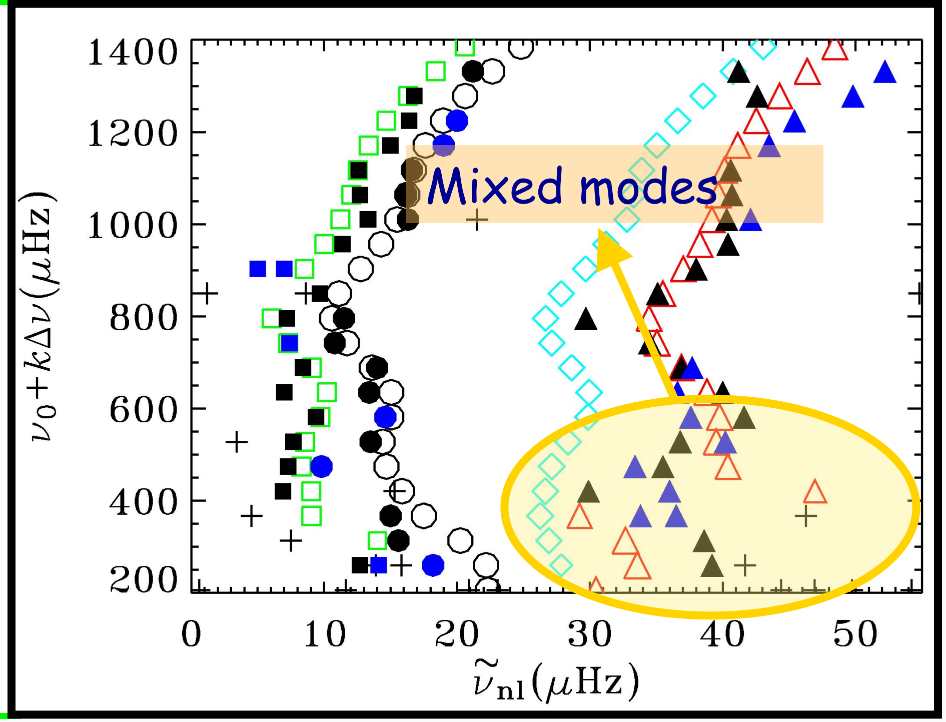
Di Mauro 2004

Echelle diagrams

Main Sequence



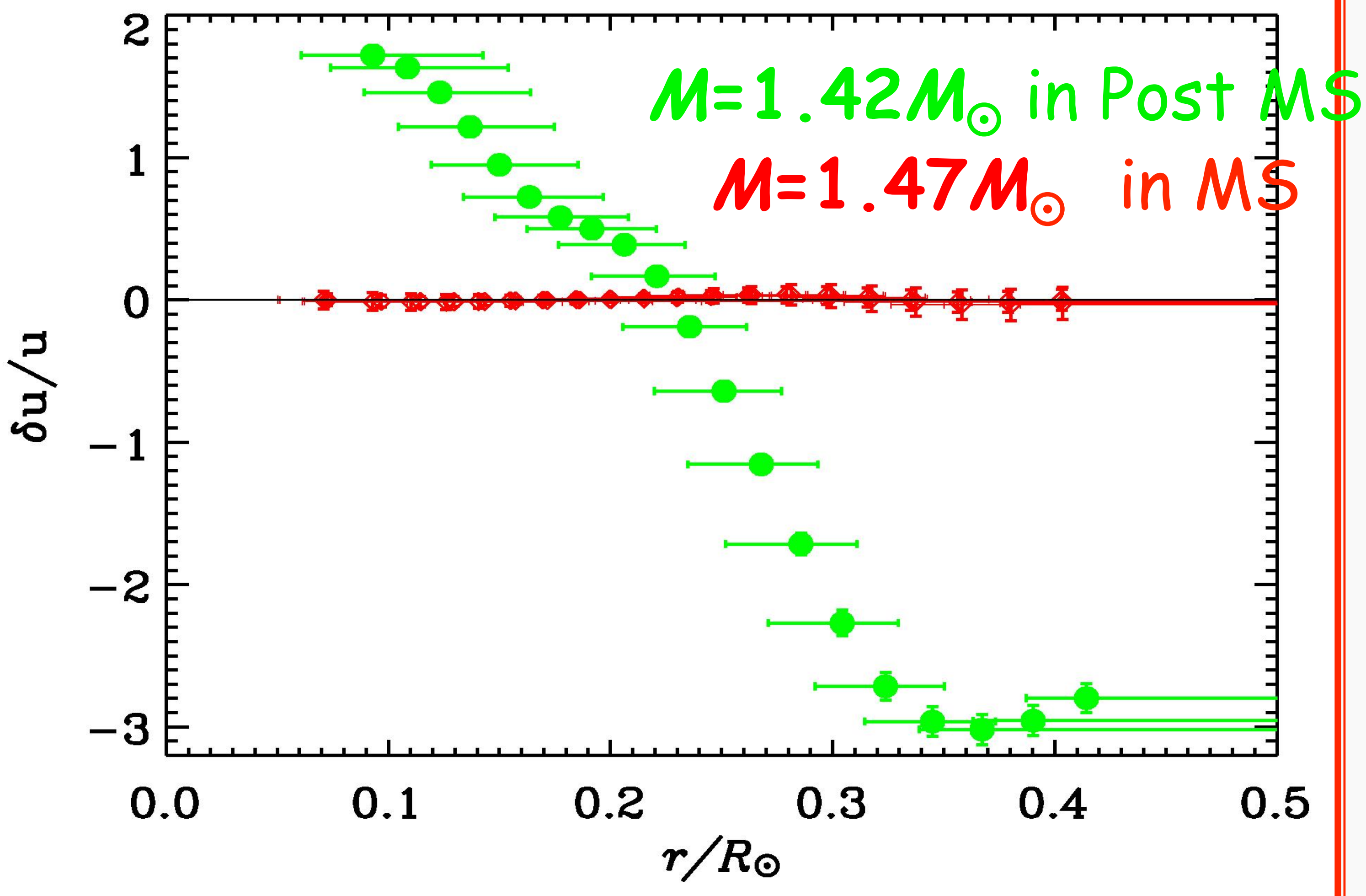
Post Main Sequence



Model	$M/M_\odot$	Age (Gyr)	Z	$L/L_\odot$	T_{eff} (K)	$R/R_\odot$	$\delta\nu_0$ (μHz)	$\Delta\nu$ (μHz)
MS	1.47	1.78	0.016	6.88	6501	2.07	4.2	53.6
PMS	1.42	2.51	0.020	6.72	6481	2.05	4.2	53.6

Inversion for Procyon

$$\frac{\delta\nu_i}{\nu_i} = \int_0^R K^i_{u,Y} \frac{\delta u}{u} \mathrm{d}r + \int_0^R K^i_{Y,u} \frac{\delta Y}{Y} \mathrm{d}\, r + \varepsilon_i$$



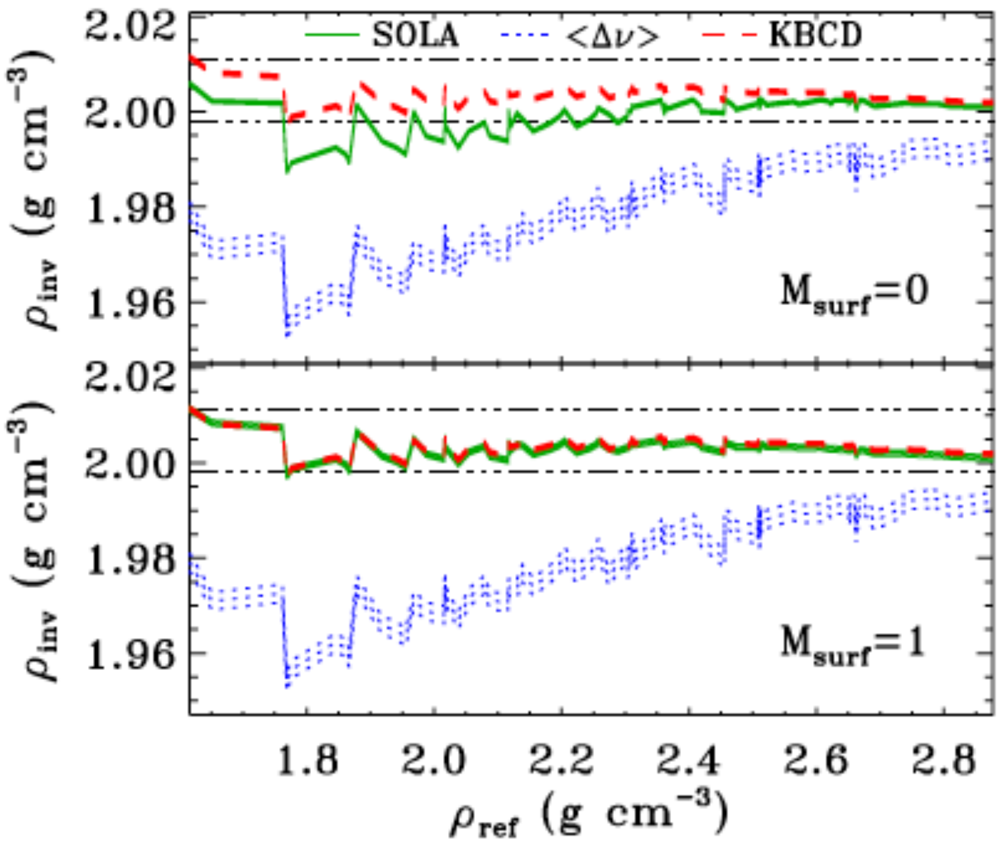
Stellar mean density

Reese et al. 2012

A method to find the mean density of stars: case of αCen B, HD49933, HD 49385

Binary sistem: known R and parallax

$\overline{\rho}$=2.046 g/cm^3



For the internal structure

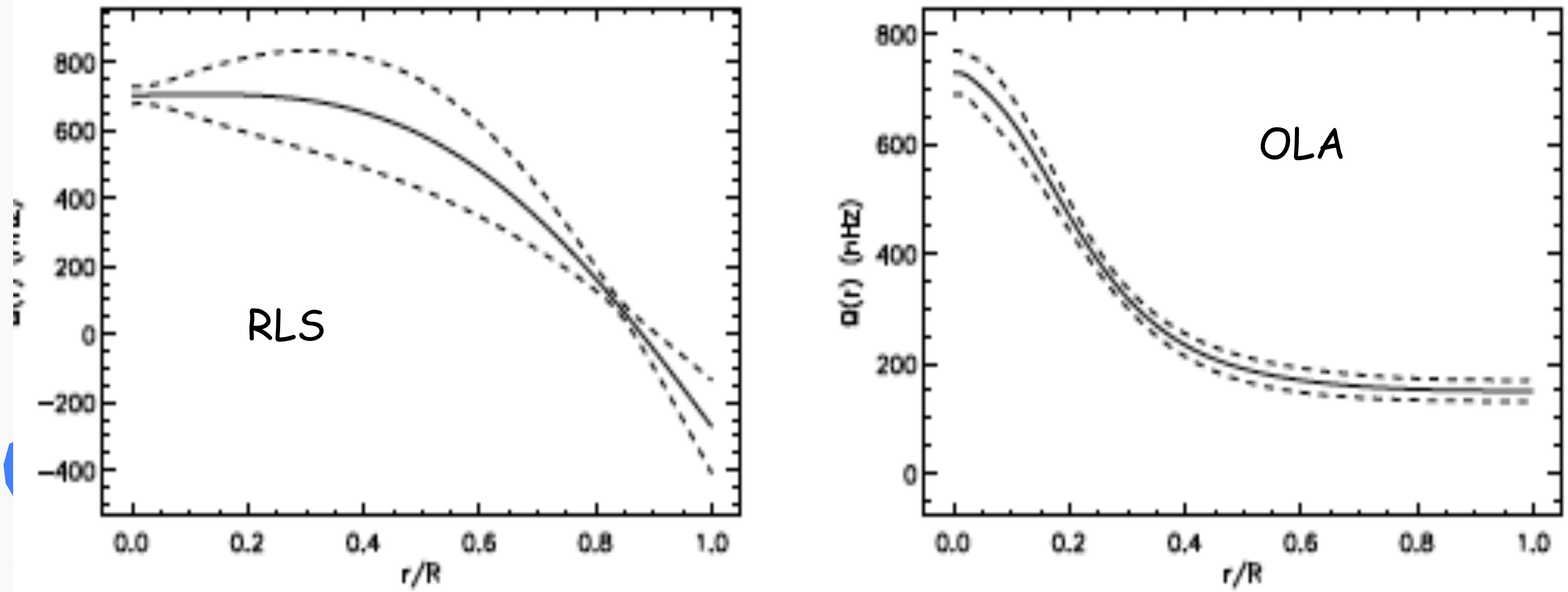
Under construction but feasible

Internal rotation of stars.....

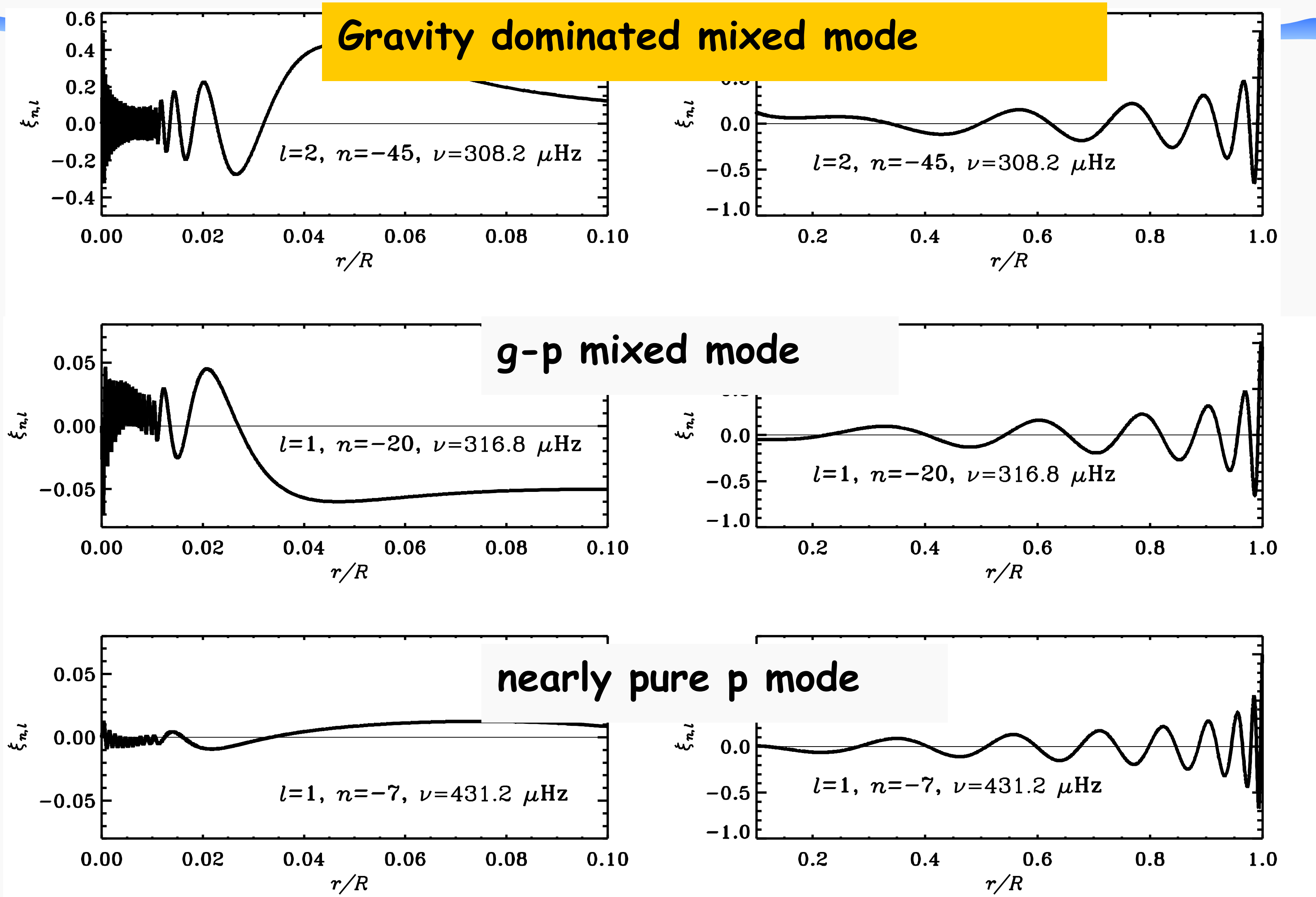
Inversion for a red giant

Deheuvels et al. 2012

KIC7341231 17 splittings with l=1



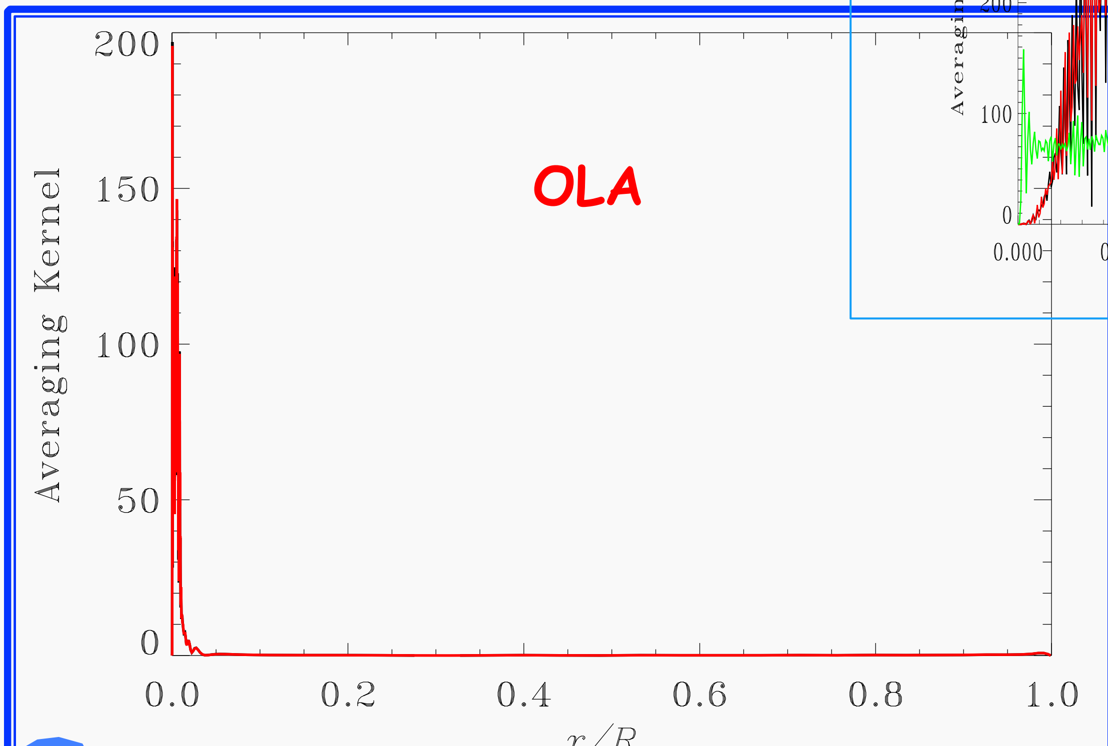
Mixed Modes

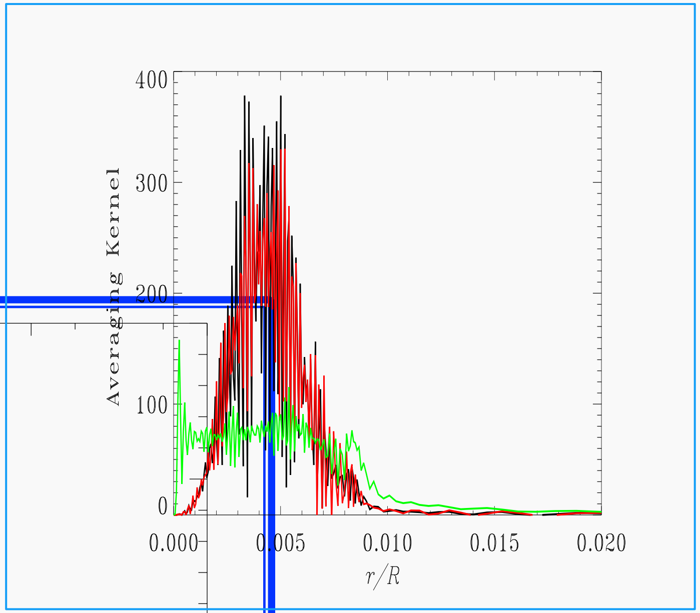


Gravity dominated mixed mode
l=2, n=−45, ν=308.2 μHz
l=2, n=−45, ν=308.2 μHz
g-p mixed mode
l=1, n=−20, ν=316.8 μHz
l=1, n=−20, ν=316.8 μHz
nearly pure p mode
l=1, n=−7, ν=431.2 μHz
l=1, n=−7, ν=431.2 μHz
r/R

Averaging kernels

Averaging kernels for $\Omega(r)$ built with 15 mixed-modes of $l=1$



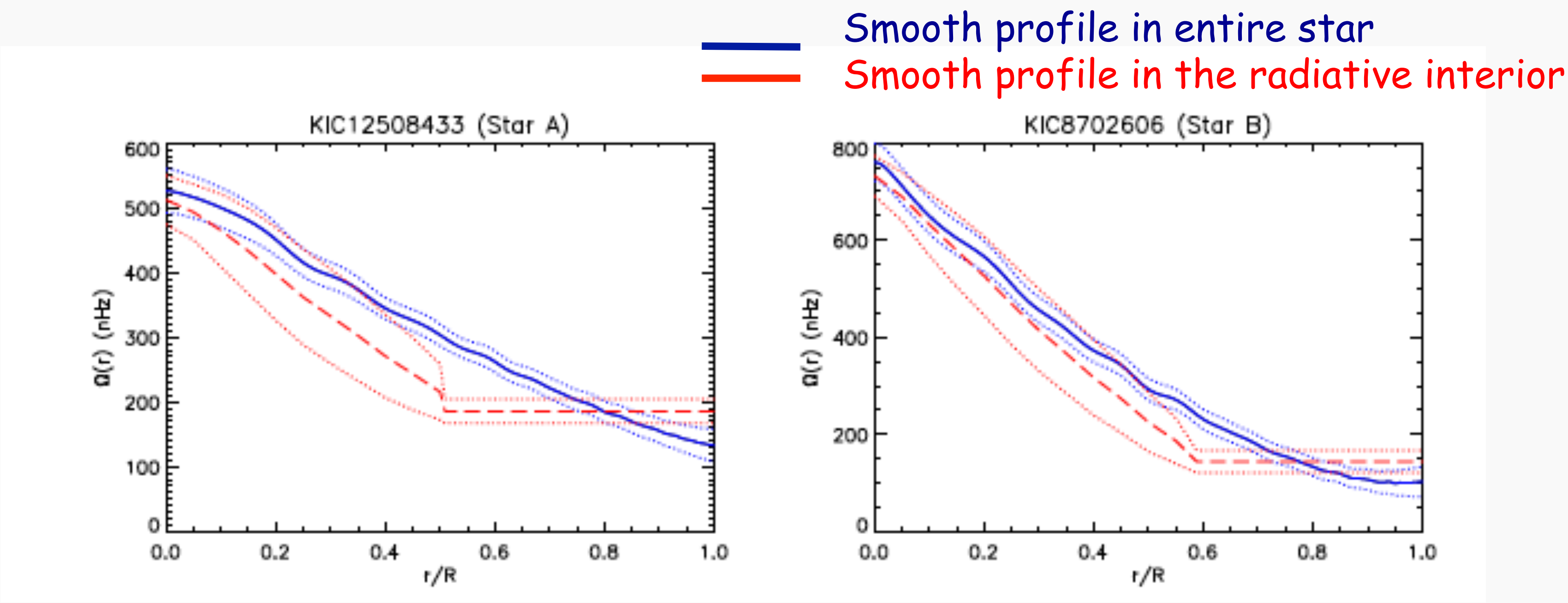


Subgiants

Deheuvels et al. 2014 submitted

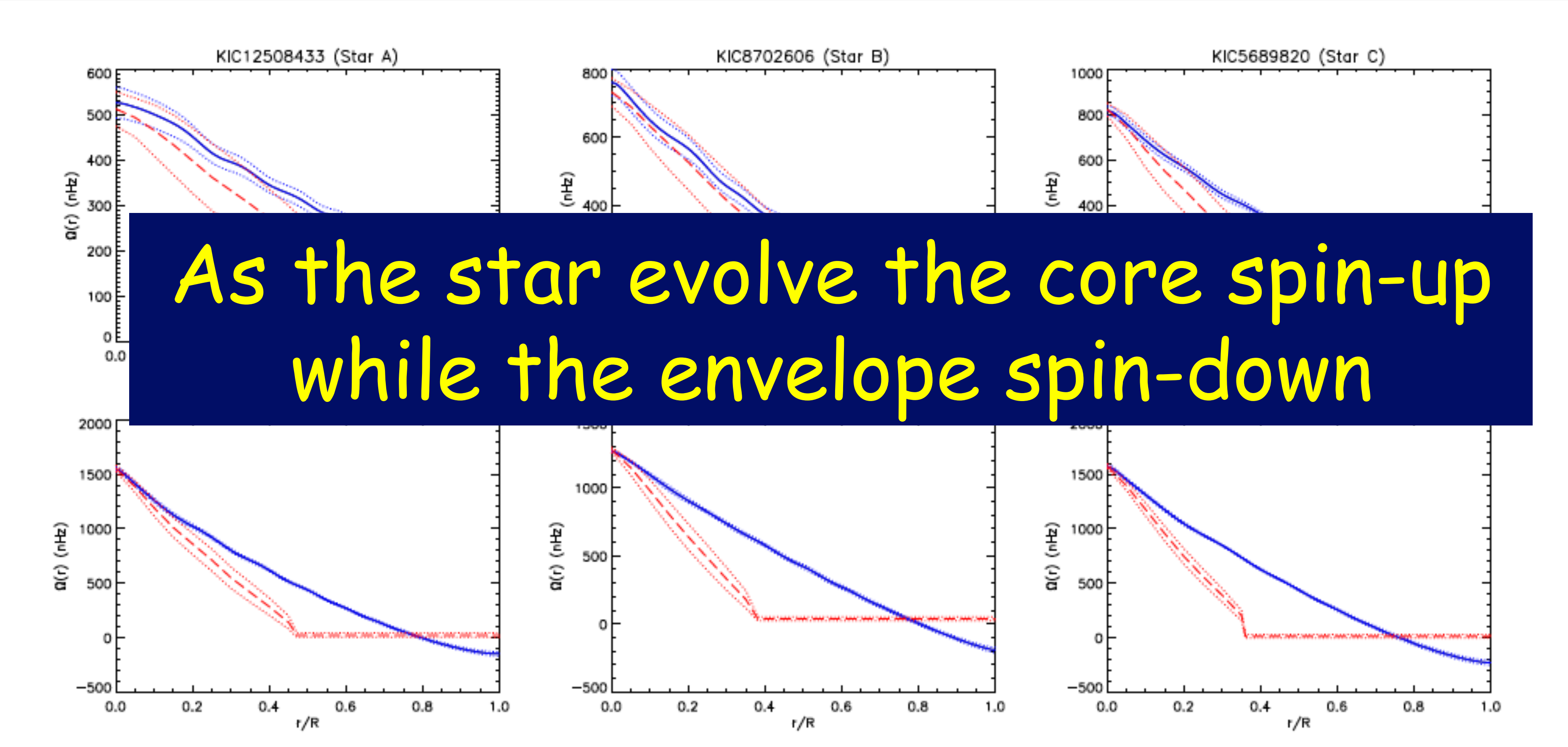
RLS inversion of l=1,2 splittings

Searching the rotation profile which closer match splittings



From subgiants to red giants

Deheuvels et al. 2014 submitted



As the star evolve the core spin-up while the envelope spin-down

Summary for rotation

- We can infer internal rotation of stars by using inversion techniques
  - MS stars: g modes+p modes
  - Post MS: mixed modes+p modes
  - Red giants: mixed modes
- We can extend helioseismic tools to other stars
- We can reconstruct the stellar rotation's history

Contribution talks

- Hannah Schunker → Inferring the internal rotation of solar-like stars by RLS
- Antonio Eff-Darwich → What we can learn from the heliosesmic inversion

Inverse Analysis

- Linearization might be questionable
  - Basic parameters are not so well known
  - Model can be very different from the observed star
- Statistical properties are well defined in linear inversions
- Very important the choice of the variables to be inverted
  - E.g:Pair of functions (u,Y), since the sensitivity of Y is small and confined in the outer layers

Basu, 2003
Roxburgh and Vorotsov 20

PROBLEMS

- Models

-Fundamental parameters are not so well known

-Model can be very different from the observed star

- Data

Fitting methods

Optimization algorithm

Observables: log g,[Fe/H], T_{eff}, $\Delta\nu$, $\delta\nu$, set of frequencies

Parameters: X_0, Z_0, α, M

Search for a set of parameters that minimizes:

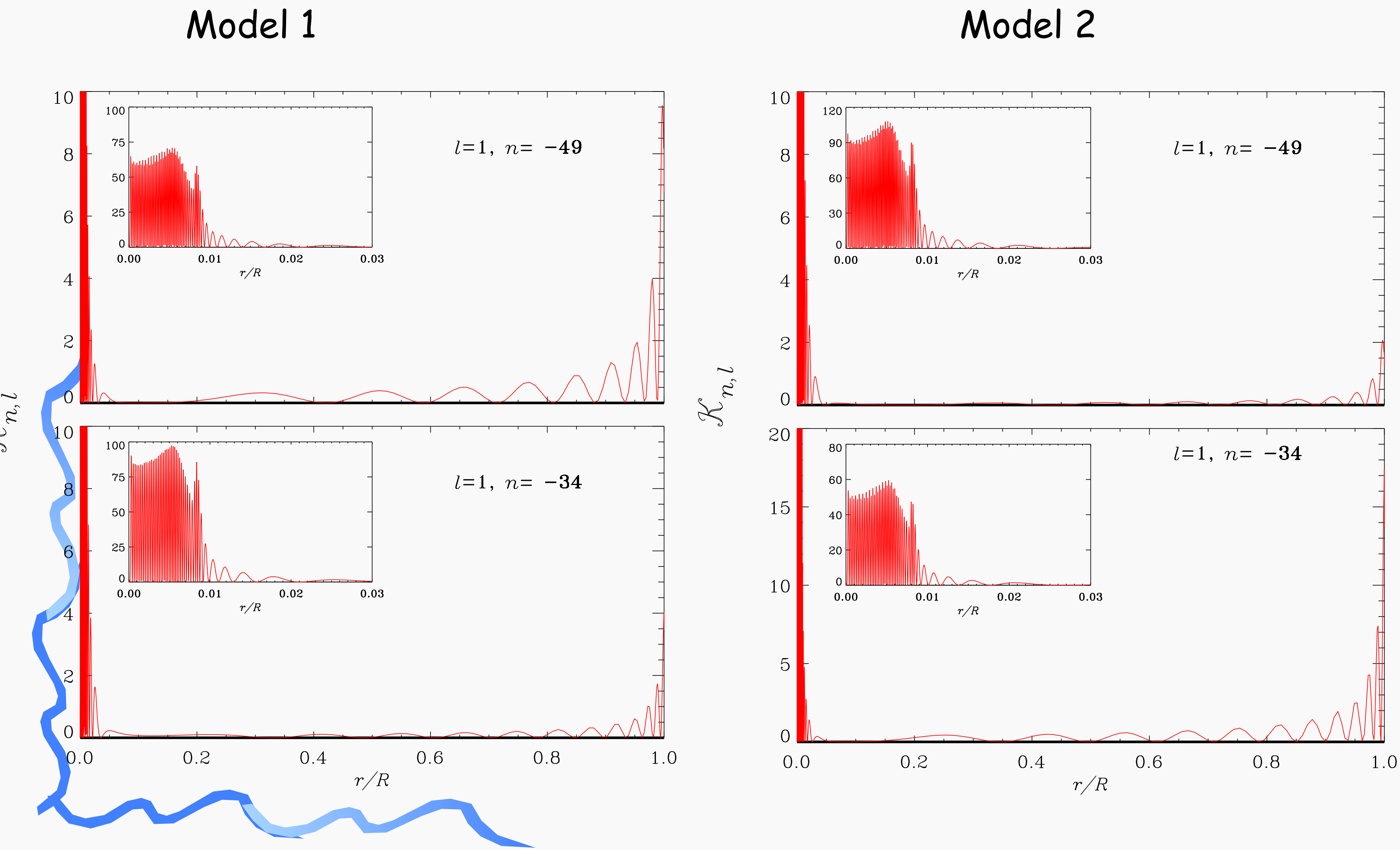
$$\chi^2 \equiv \sum_{A,B} \sum_i \left(\frac{O_i^{mod} - O_i^{obs}}{\sigma_{O_i^{obs}}} \right)^2$$

- Grids of models
- Pipelines (e.g. YB, SEEK, RADIUS)
  - Fits $\Delta\nu$
  - Fits $\Delta\nu$, $\delta\nu$, T_{eff}, log g, [Fe/H]
  - Fixed α, Y_i and fits $\Delta\nu$, T_{eff}, log g, [Fe/H]
- Genetic algorithms (Metcalfe et al 2004)

Problems with grid fitting

- Dependence on evolution codes, stellar parameters
- Discrepancies between grid and AMP fits
- ν_{max} scaling??
- Sanity check: application to stellar clusters, eclipsing binaries

Individual kernels



PROBLEMS

- Models

-Fundamental parameters are not so well known

-Model can be very different from the observed star

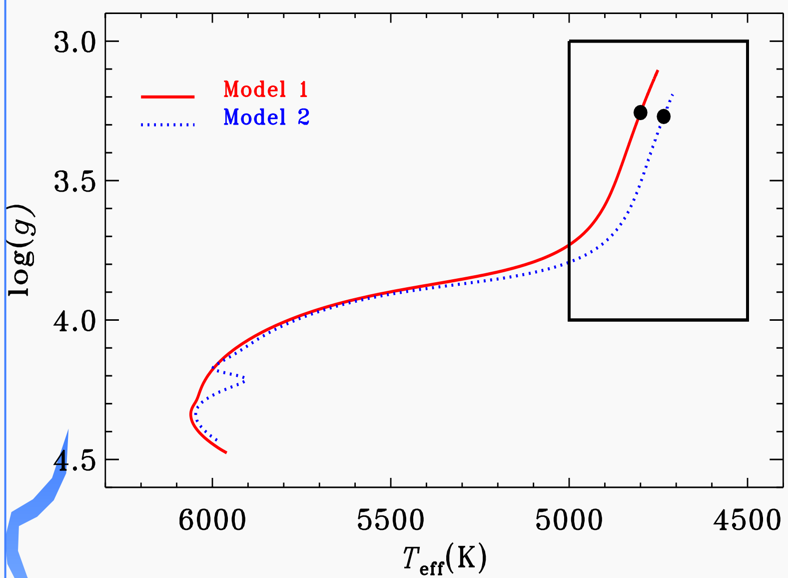
- Data

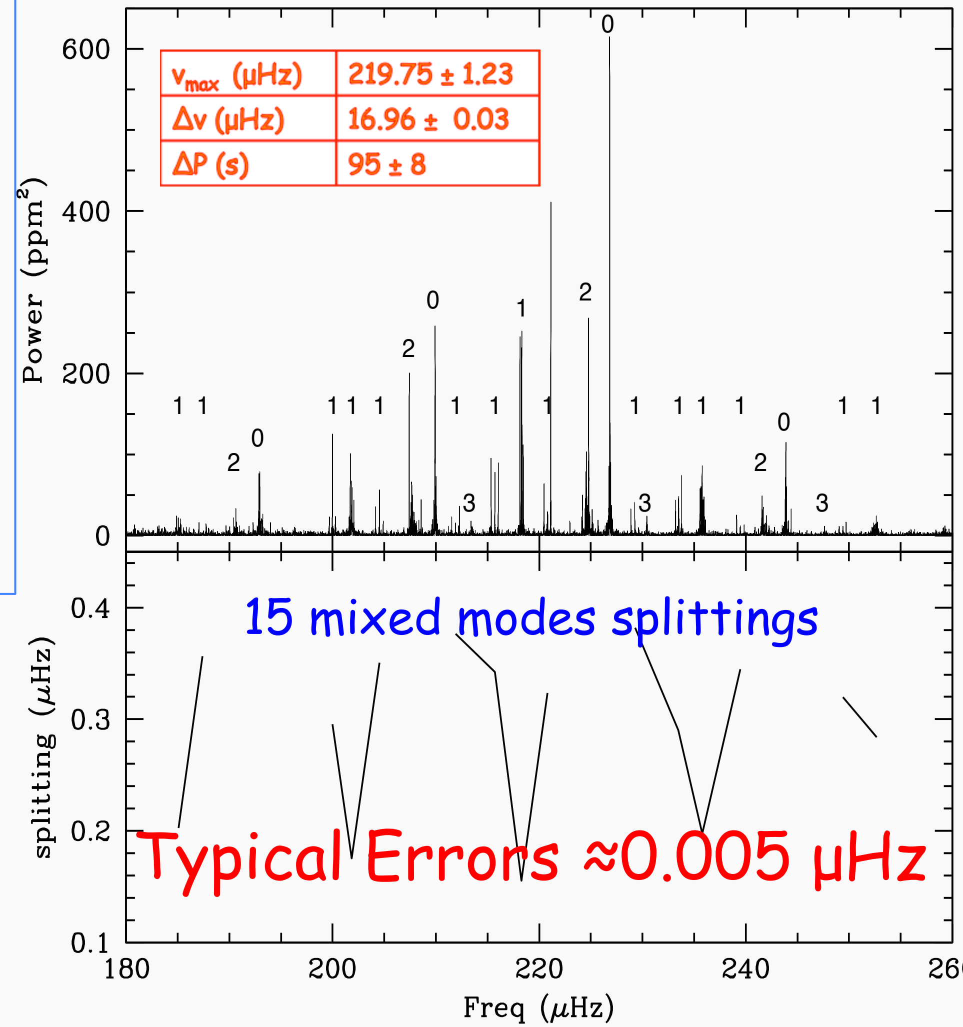
☆Very high accuracy of data in comparison to errors in model

☆Modes often sound same internal region

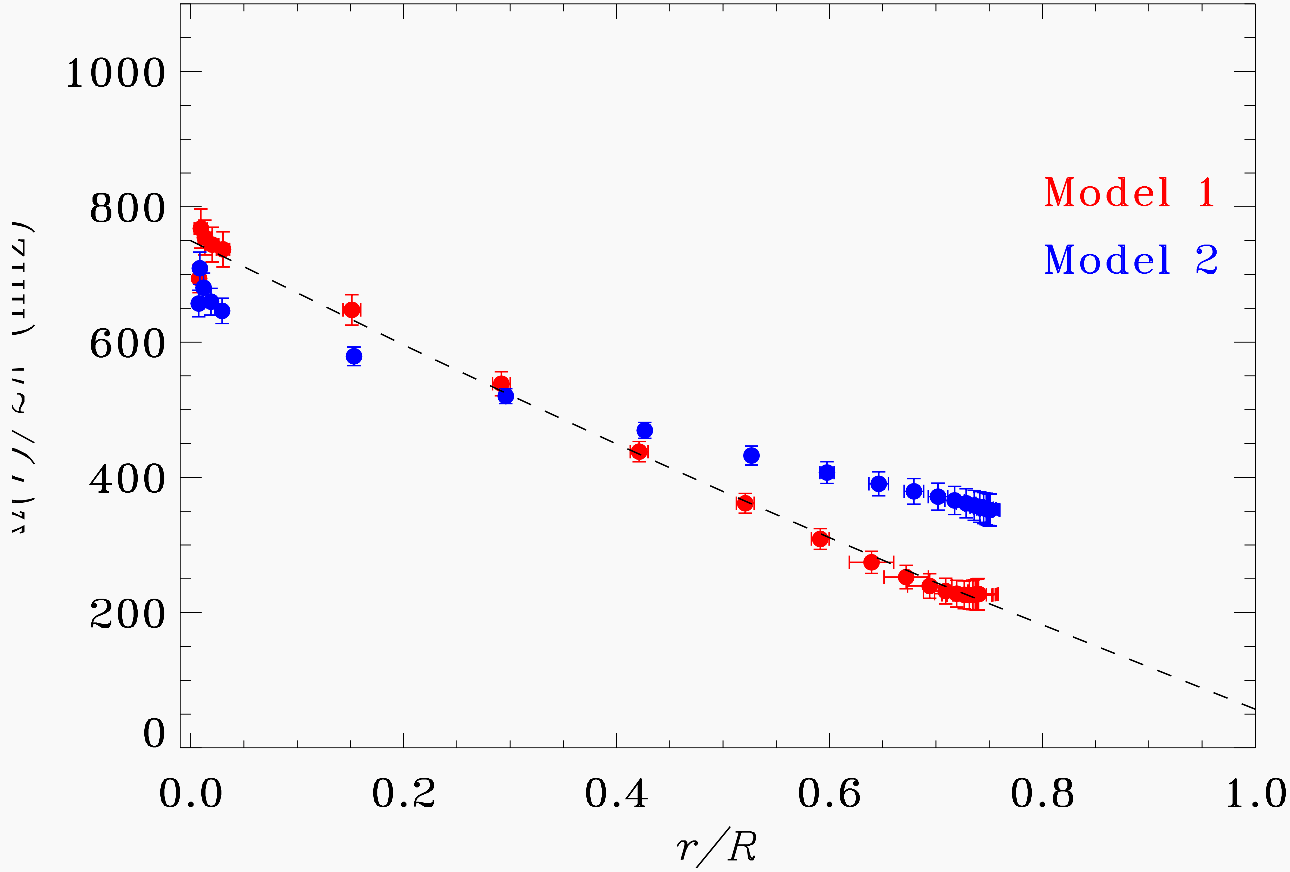
☆Data computed by different group are different

A red giant: KIC4448777



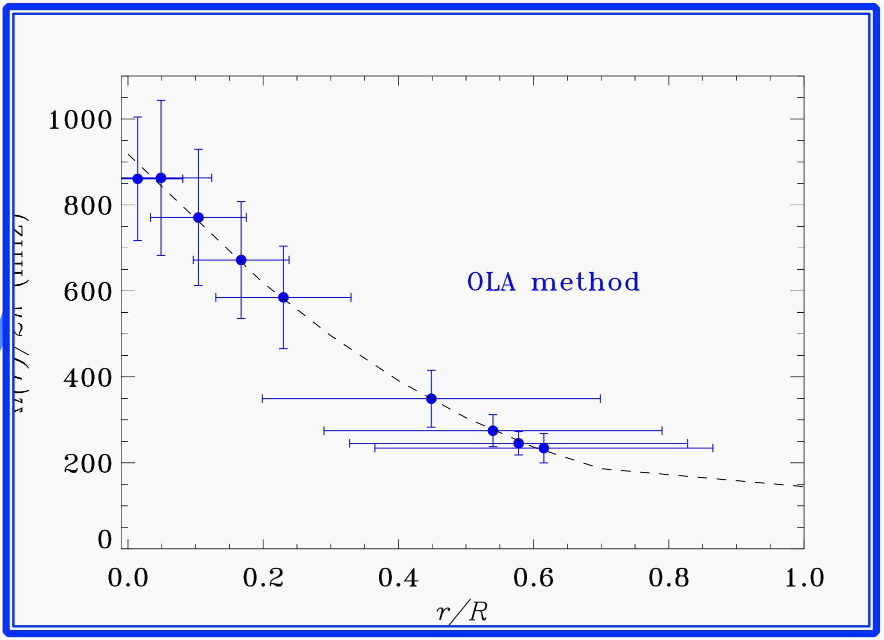


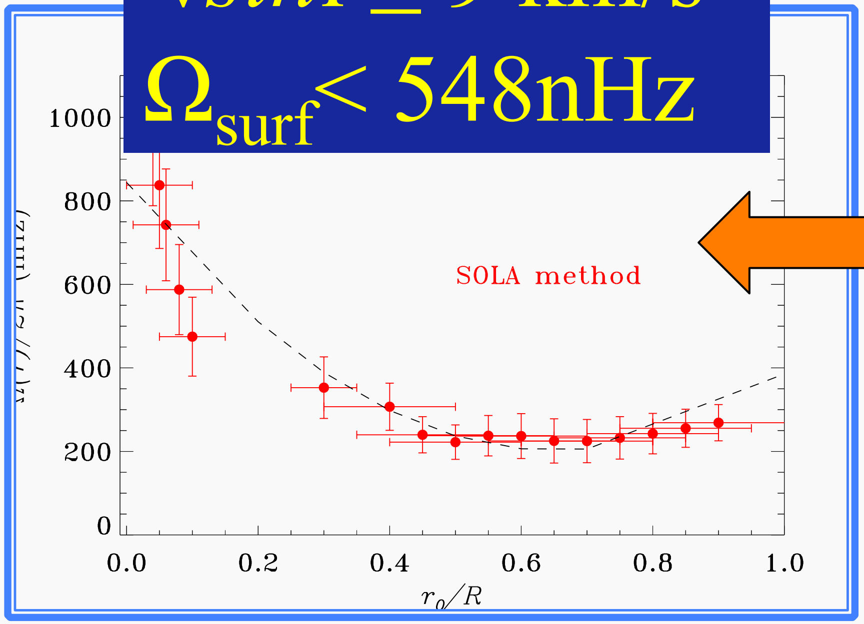
	Model 1	Model 2
$M/M_\odot$	1.02	1.13
T_{eff} (K)	4800	4735
log g (dex)	3.26	3.27
$R/R_\odot$	3.94	4.08
$L/L_\odot$	7.39	7.22
$(Z/X)_i$	0.022	0.032



Model 1
Model 2
1000
800
600
400
200
0
0.0
0.2
0.4
0.6
0.8
1.0
r/R

Rotation in red- giants





$v\sin i \leq 9$ km/s

$\Omega_{surf} < 548$nHz

Models with those measured splittings are consistent with an internal rotation of

$5 \leq \Omega_{core}/\Omega_{surf} \leq 10$

Only l=1 modes in the Sun