Peak-bagging in Helio- and Asteroseismology
SpaceInn Workshop 4.1, March 11, 2014, Tenerife, Spain

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Bayesian Nested Sampling as a tool for Peak Bagging of solar-like oscillations observed by Kepler

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Outlook

- Nested Sampling
  Bayes theorem, model selection, Nested Sampling Monte Carlo, ellipsoidal sampling

- The new code DIAMONDS
  Prior distributions, sampling efficiency tests

- Application: Peak Bagging of Punto (KIC 9139163)
  Background fitting, mode fitting, peak significance, tackling rotation

Nested Sampling

Bayes Theorem

$$\mathbf{D} = \{d_1, d_2, \ldots, d_m\}$$ Dataset (observations)

$$\mathcal{M} = \mathcal{M}\left(\boldsymbol{\theta}\right)$$ Model to be tested

$$\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_k\}$$ k free parameters (parameter vector)

$$\mathcal{L}\left(\boldsymbol{\theta}\right) = \mathcal{L}\left(\boldsymbol{\theta} \mid \mathbf{D}, \mathcal{M}\right)$$ Likelihood function

$$\pi\left(\boldsymbol{\theta}\right) = \pi\left(\boldsymbol{\theta} \mid \mathcal{M}\right)$$ Prior PDF

Bayes Theorem

$$p\left(\boldsymbol{\theta}\right) = p\left(\boldsymbol{\theta} \mid \mathbf{D}, \mathcal{M}\right)$$ Posterior PDF

$$p\left(\boldsymbol{\theta}\right) = \frac{\mathcal{L}\left(\boldsymbol{\theta}\right)\pi\left(\boldsymbol{\theta}\right)}{\mathcal{E}}$$ Bayes Theorem

$$p\left(\theta_1\right) = \int p\left(\boldsymbol{\theta}\right) d\theta_2 \ldots d\theta_k$$ Marginal PDF

θ_1

Mean
Mode
Median
Variance
Credible Intervals

Why is Evidence important?

$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\mathcal{E}}$$

Bayes Theorem

$$\mathcal{M}_i \quad \mathcal{M}_j$$

Two different competing models

$$B_{ij} = \frac{\mathcal{E}_i}{\mathcal{E}_j}$$

Bayes' factor

Why is Evidence important?

$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\mathcal{E}}$$

Bayes Theorem

$\mathcal{M}_i$ $\mathcal{M}_j$

Two different competing models

$$B_{ij} = \frac{\mathcal{E}_i}{\mathcal{E}_j}$$

Bayes' factor

$$B_{ij} \sim 150$$

Strong Evidence (Jeffreys' scale)

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Why is Evidence important?

$$p\left(\boldsymbol{\theta}\right) = \frac{\mathcal{L}\left(\boldsymbol{\theta}\right)\pi\left(\boldsymbol{\theta}\right)}{\mathcal{E}}$$

Bayes Theorem

A solution to model selection problem!

$\mathcal{M}_i$ $\mathcal{M}_j$

Two different competing models

$$B_{ij} = \frac{\mathcal{E}_i}{\mathcal{E}_j}$$

Bayes' factor

$$B_{ij} \sim 150$$

Strong Evidence (Jeffreys' scale)

Nested Sampling

Bayes Theorem

$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\mathcal{E}}$$

- Evidence is a k-dimensional integral

$$\mathcal{E} = \int \mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})\,d\boldsymbol{\theta}$$

- Convert evidence into a one-dimensional integral

$$\mathcal{E} = \int_0^1 \mathcal{L}(X)\,dX$$

$$dX = \pi(\boldsymbol{\theta})\,d\boldsymbol{\theta}$$

small portion of prior volume (prior mass)

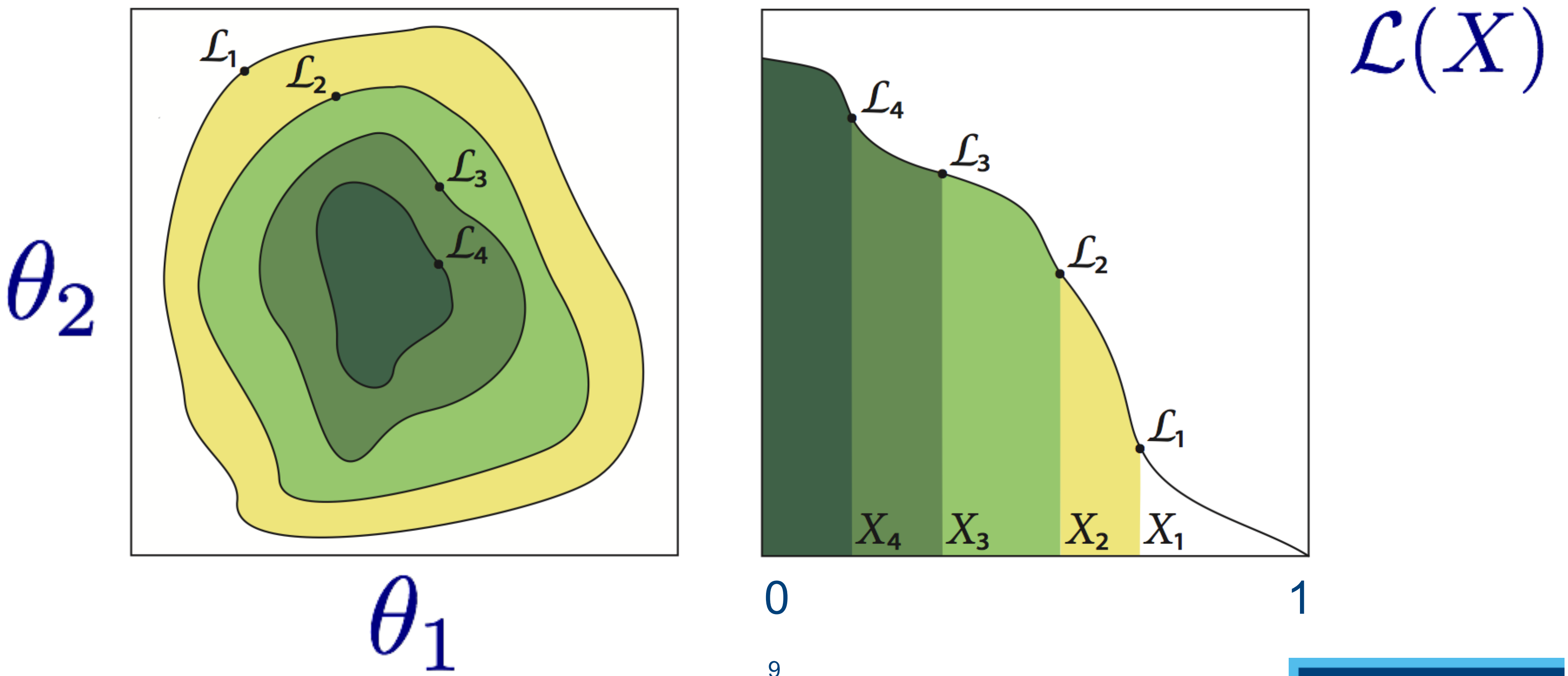
Nested Sampling

Bayesian Evidence

$$\mathcal{E} = \int_0^1 \mathcal{L}(X)\, dX$$

$$X(\lambda) = \int_{\mathcal{L}(\boldsymbol{\theta})>\lambda} \pi(\boldsymbol{\theta})\, d\boldsymbol{\theta}$$

$$X = 1 \to 0$$



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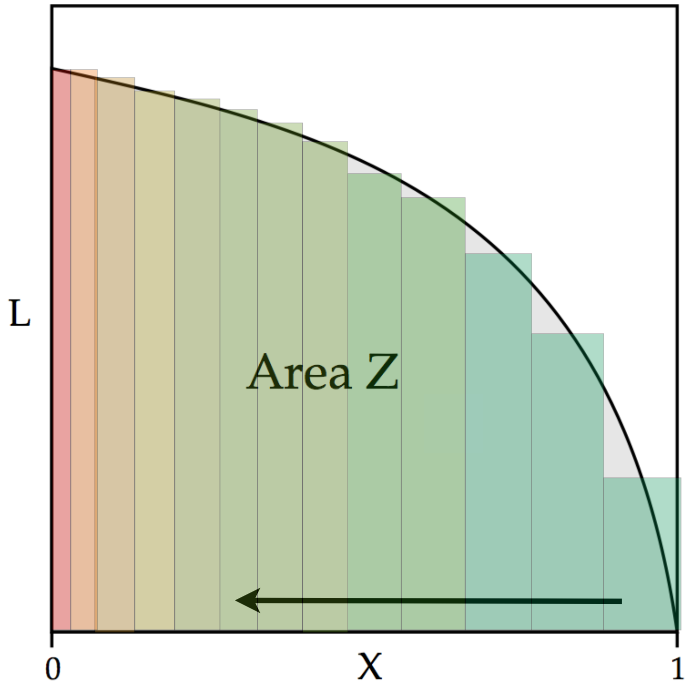
Nested Sampling

Bayesian Evidence

$$\mathcal{E} = \int_0^1 \mathcal{L}(X)\, dX$$

- Suppose we collect L_i for $0 < X_M < \ldots < X_2 < X_1 < 1$
- Evidence can be estimated by simple numerical method

$$\mathcal{E} = \sum_{i=0}^{M} \mathcal{L}_i \Delta X_i$$

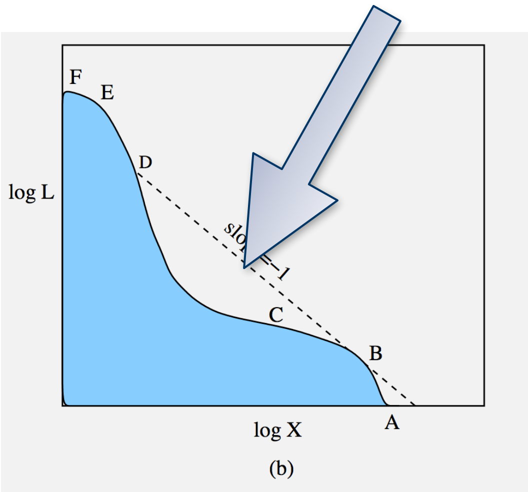


- Final posterior probability

$$P_i = \frac{\mathcal{L}_i \Delta X_i}{\mathcal{E}}$$

Nested Sampling

- **ADVANTAGES:**
  1. Typically requires **~100 times fewer** samples than thermodynamic integration to calculate **evidence** to same accuracy + error bar
  2. No troubles with phase changes (e.g. multi modal distributions)



- **BONUS:**
  Easy posterior probabilities as by-product

Nested Sampling

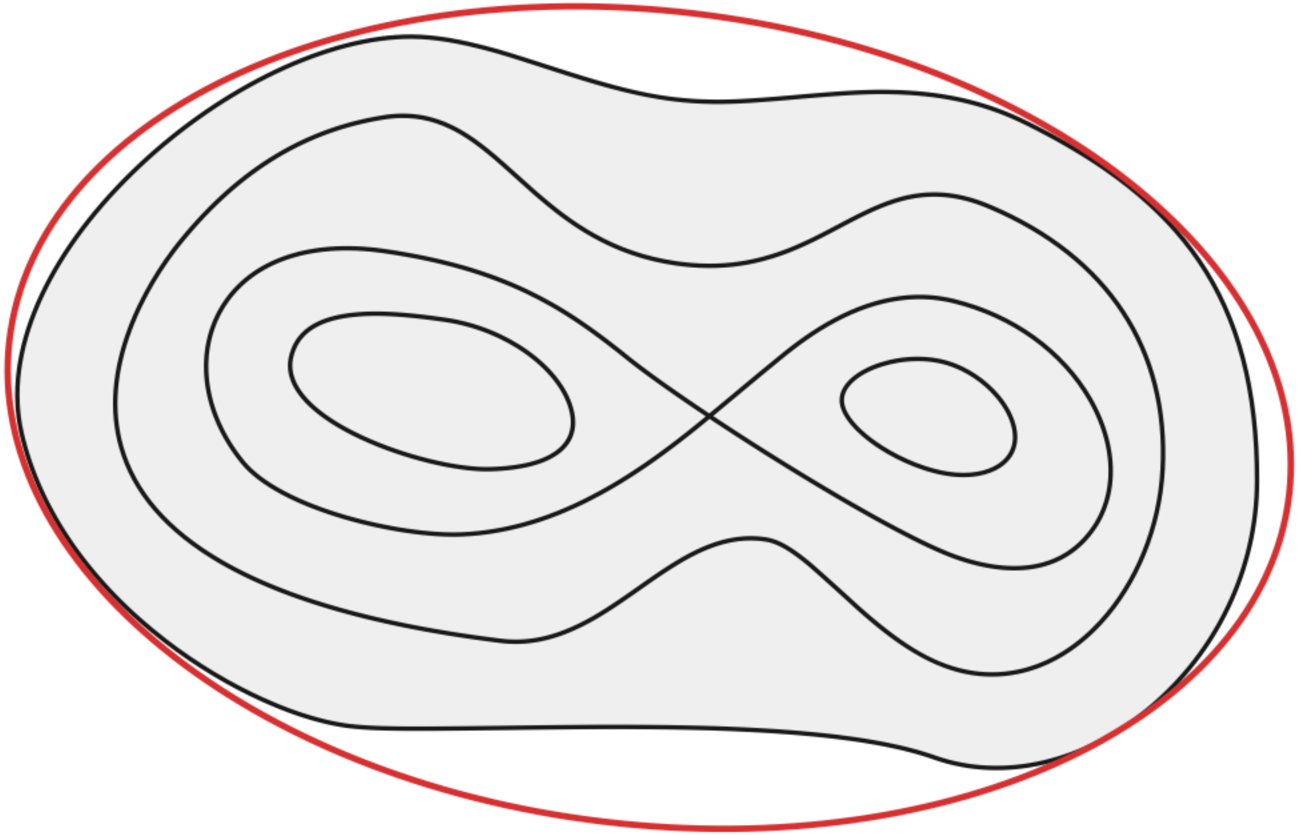
- **DISADVANTAGES:**
  Problematic drawing of a new point within hard likelihood constraint

↓

k-dimensional **ellipsoids** to approximate likelihood iso-contours and draw points more efficiently

Mukherjee P. et al. (2006; ApJ, 638, L51)

Ellipsoidal Sampling (ES)



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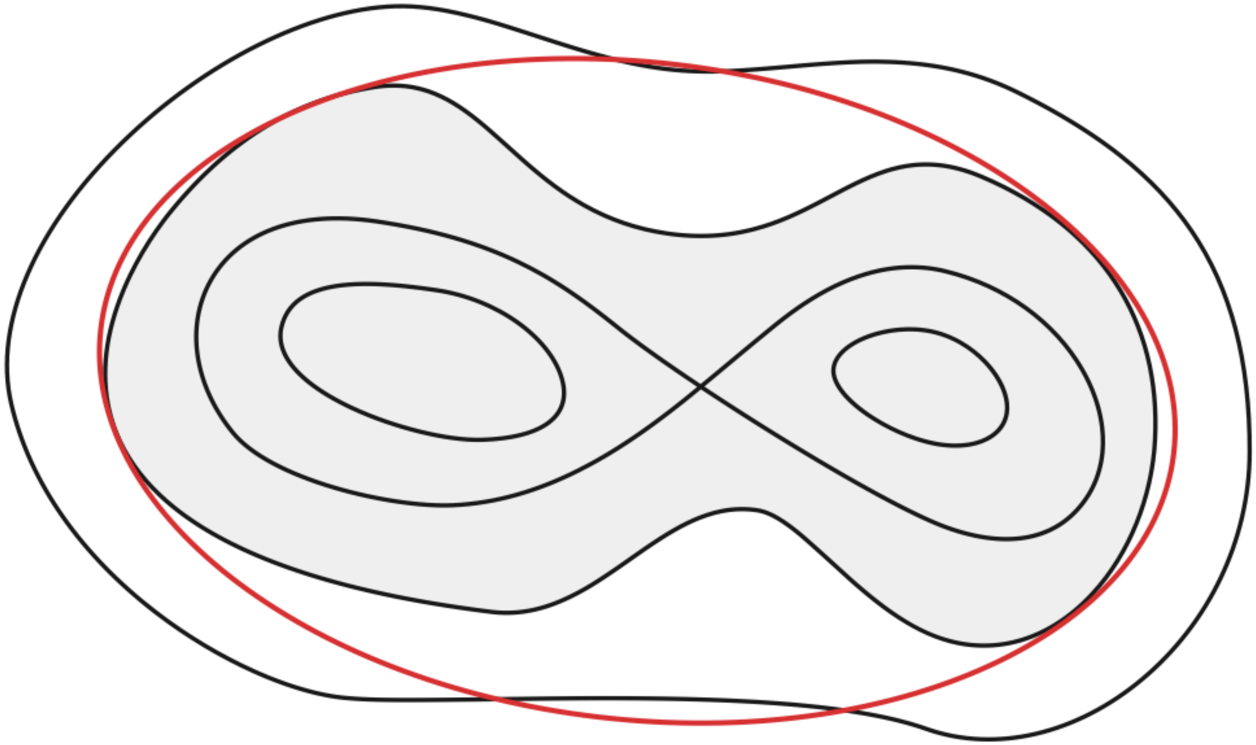
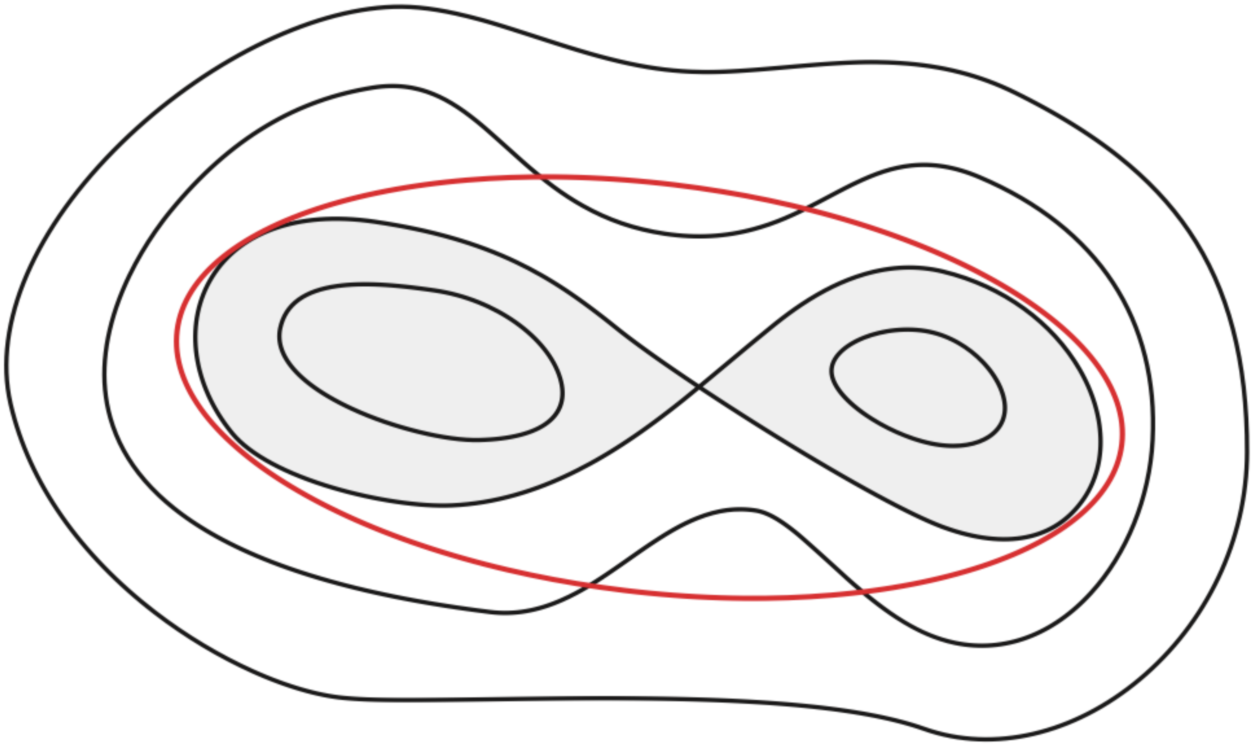


Ellipsoidal Sampling (ES)



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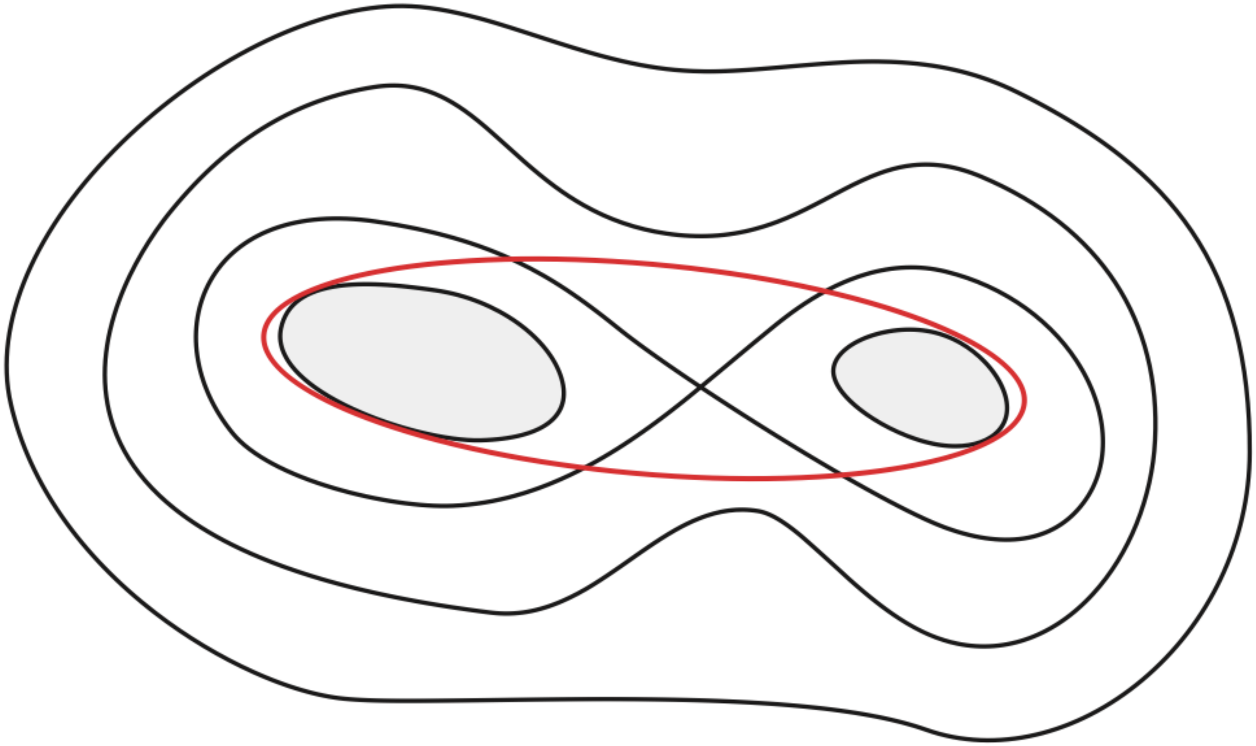
Ellipsoidal Sampling (ES)



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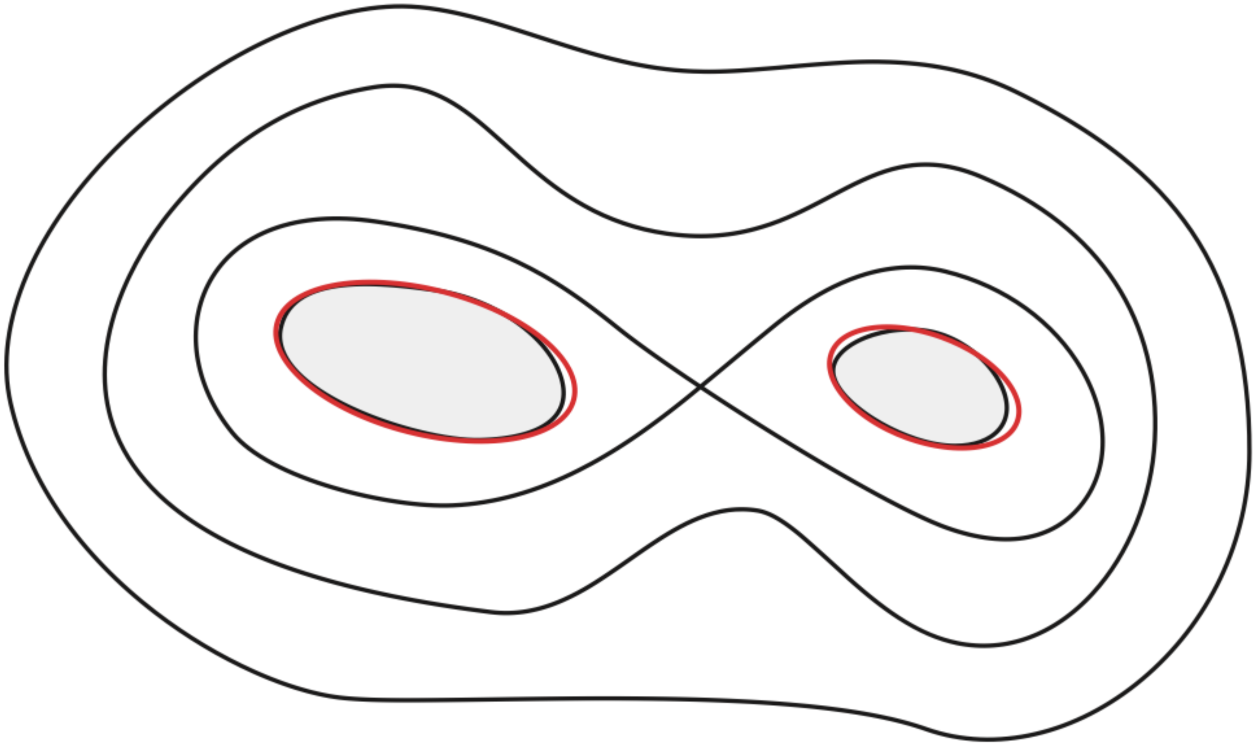
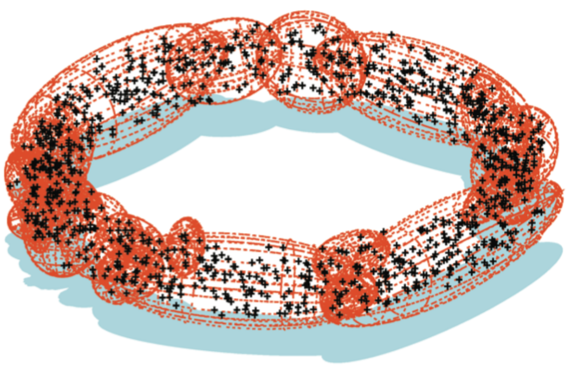


The DIAMONDS code

MultiNest (MULTI-modal NESTed sampling)

- New algorithm proposed by Feroz et al. (2008) and refined at later stage (2009)

- More efficient method for sampling multi-modal posteriors using ellipsoids



Feroz F., Hobson M. P (2008; MNRAS, 384, 449)
Feroz F. et al. (2009; MNRAS, 398, 1601)

DIAMONDS (high-DImensional And multi-MOdal NesteD Sampling)

- C++11
- Possibility to choose different priors
- Improved sampling speed for ES
- Fully flexible and configurable for any problem

Corsaro E. & De Ridder J. in preparation

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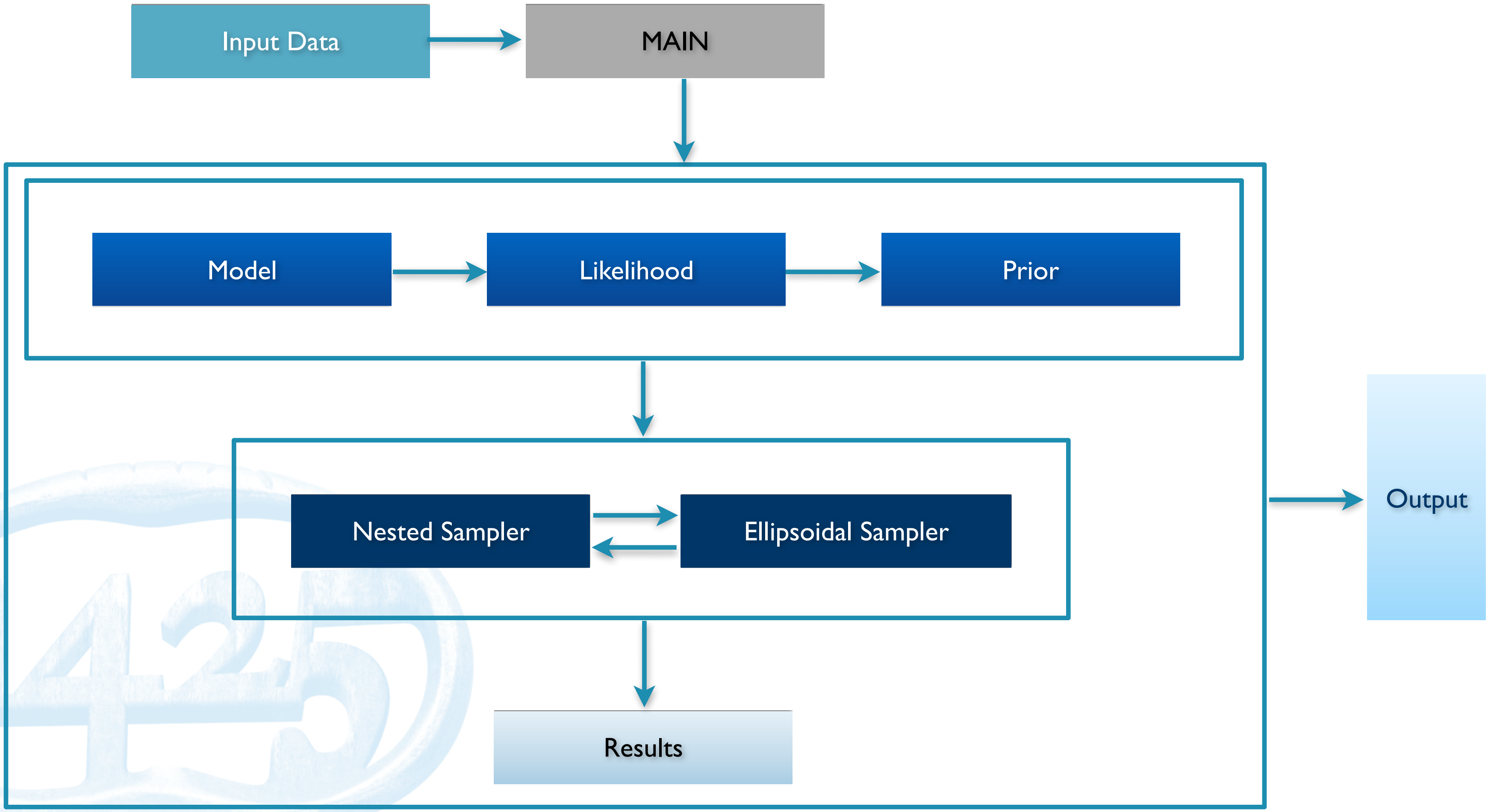
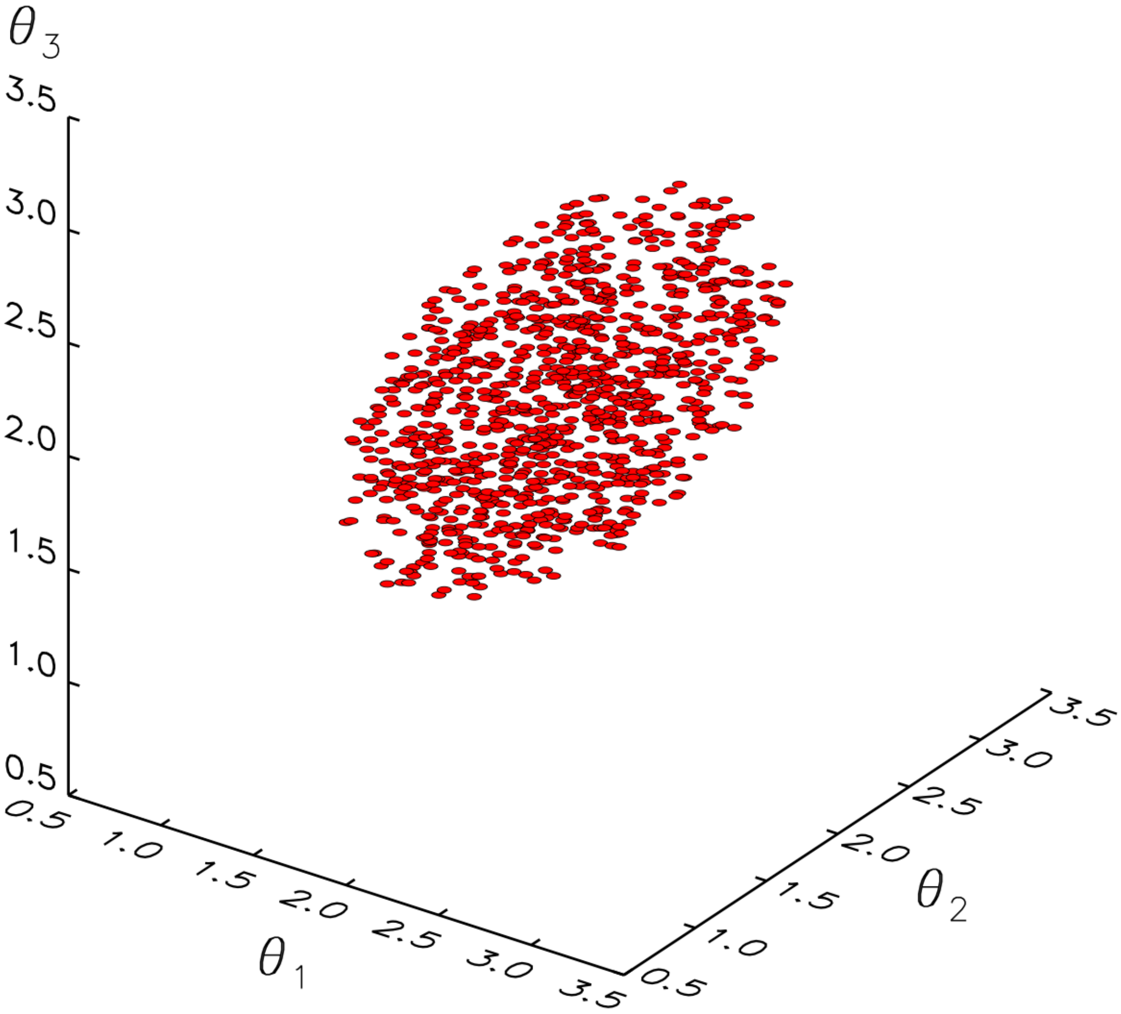
Working Scheme



Prior PDFs

3D Uniform



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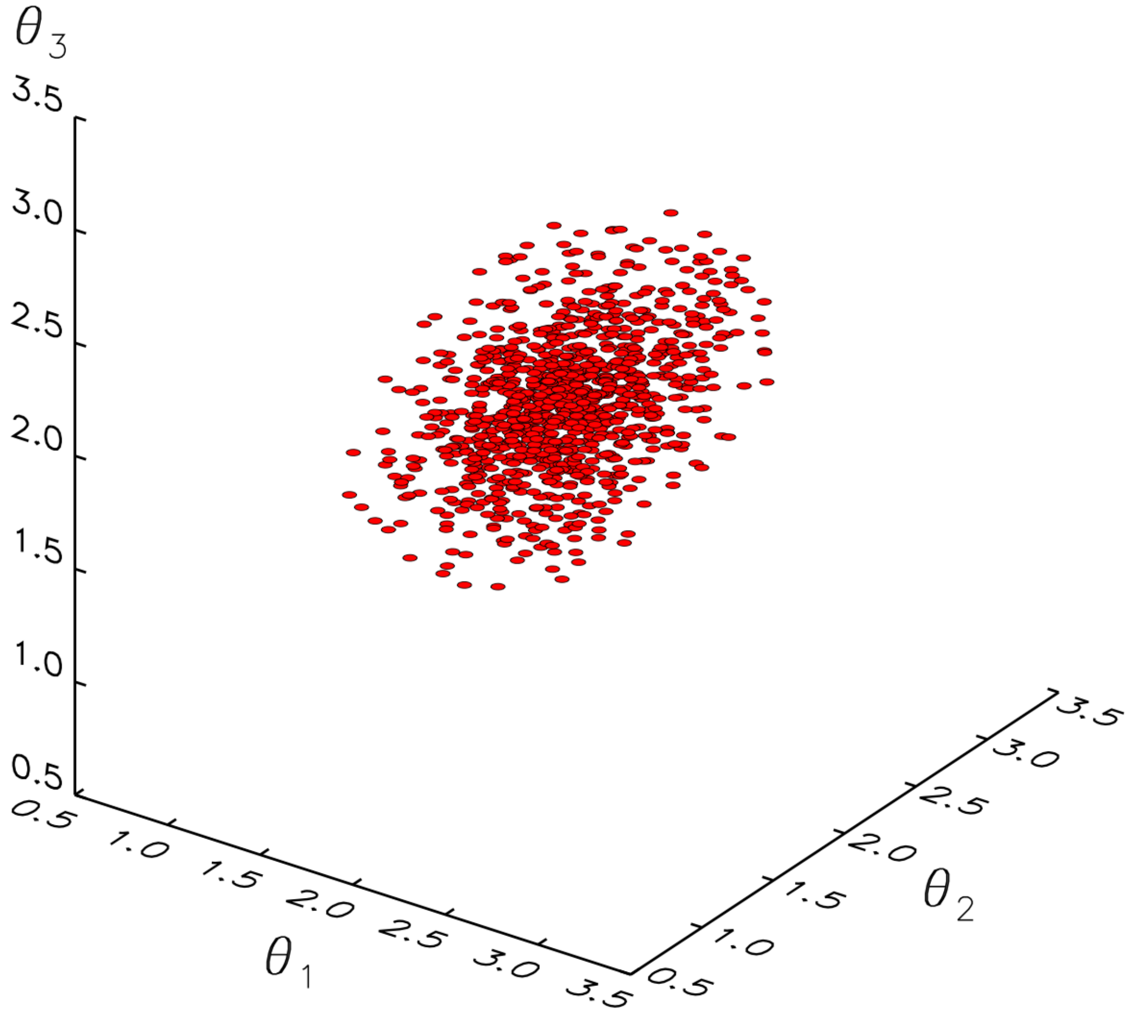
Prior PDFs

3D Gaussian



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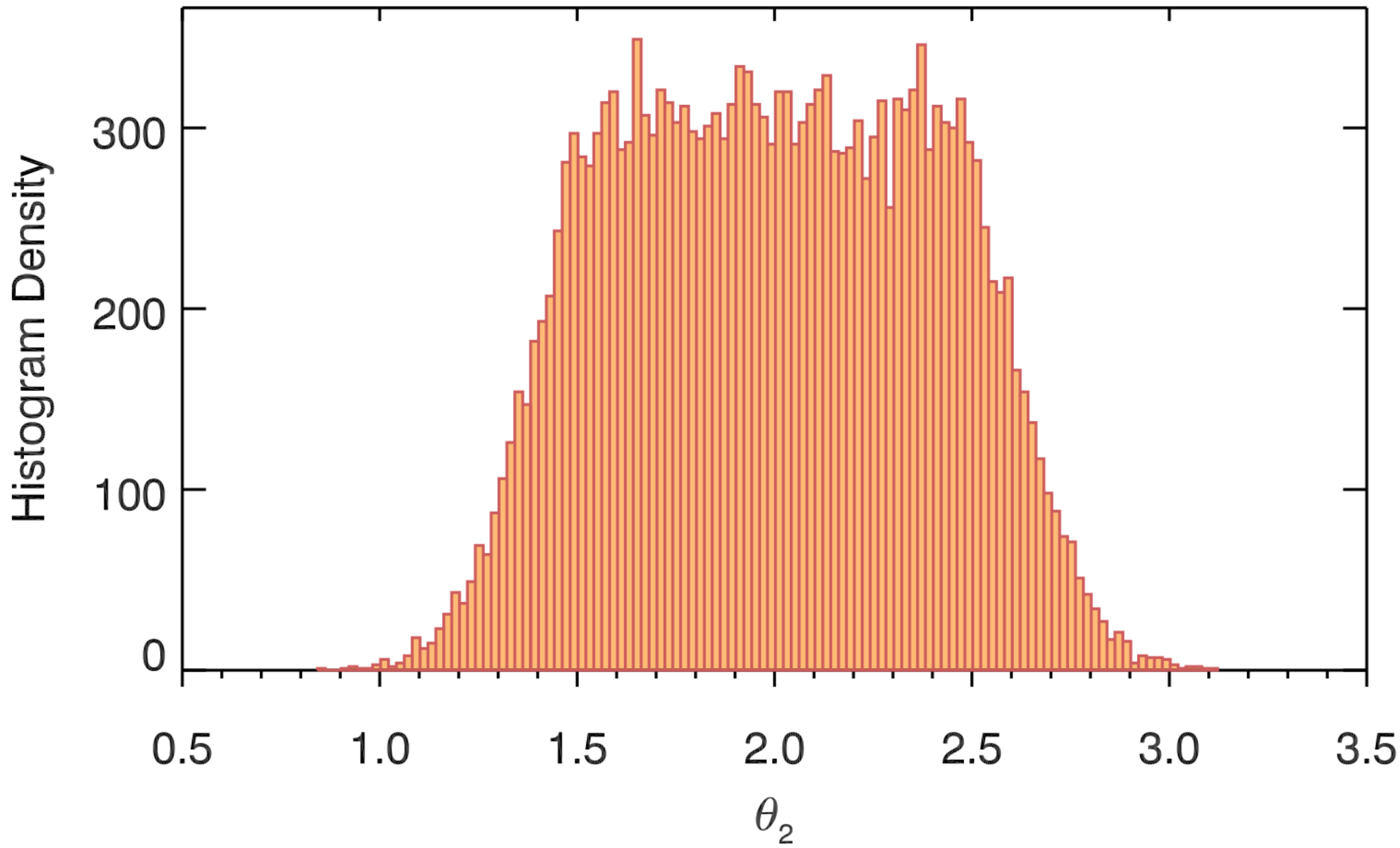
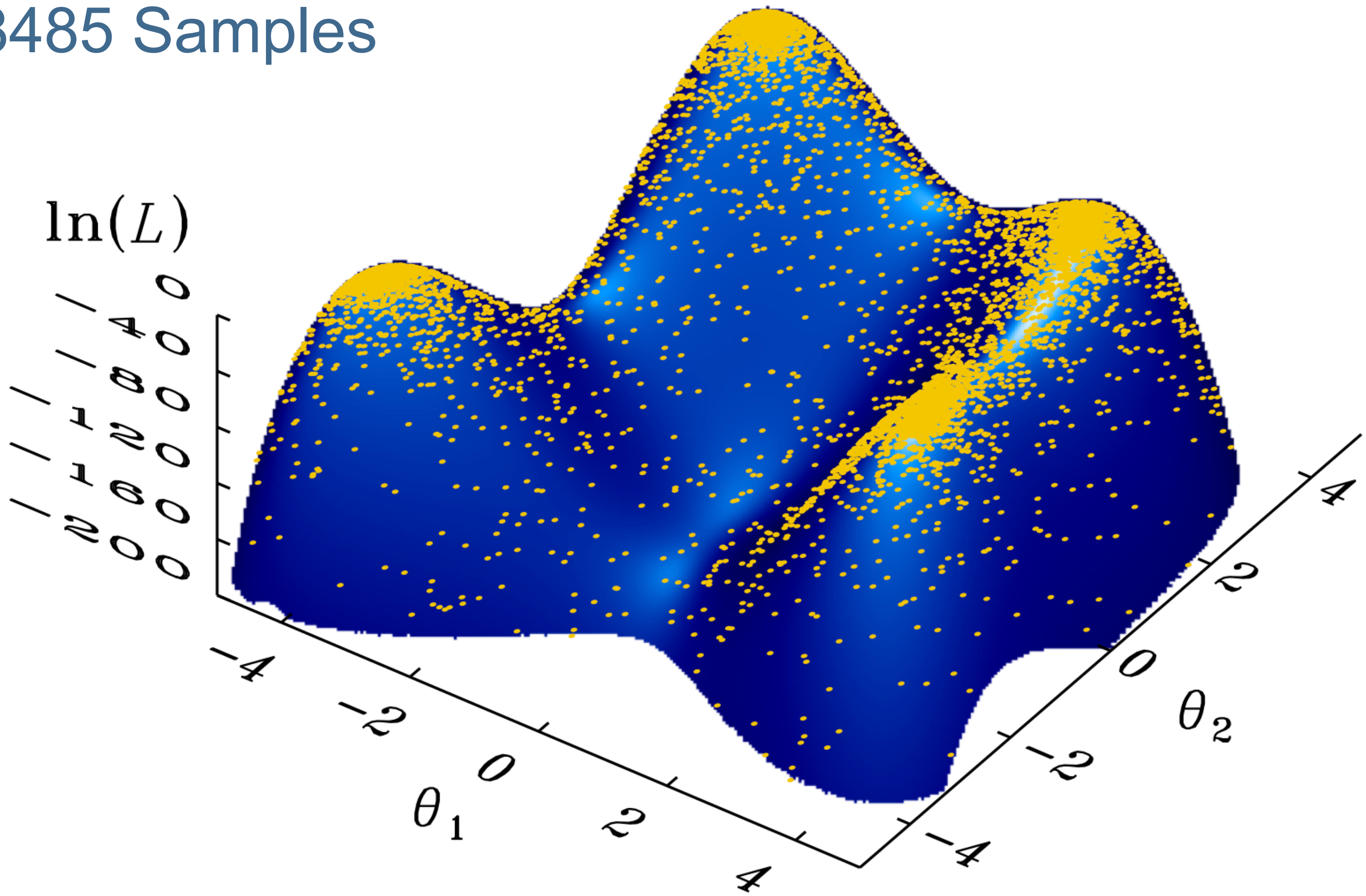
Prior PDFs



3D Super Gaussian

Sampling efficiency demos

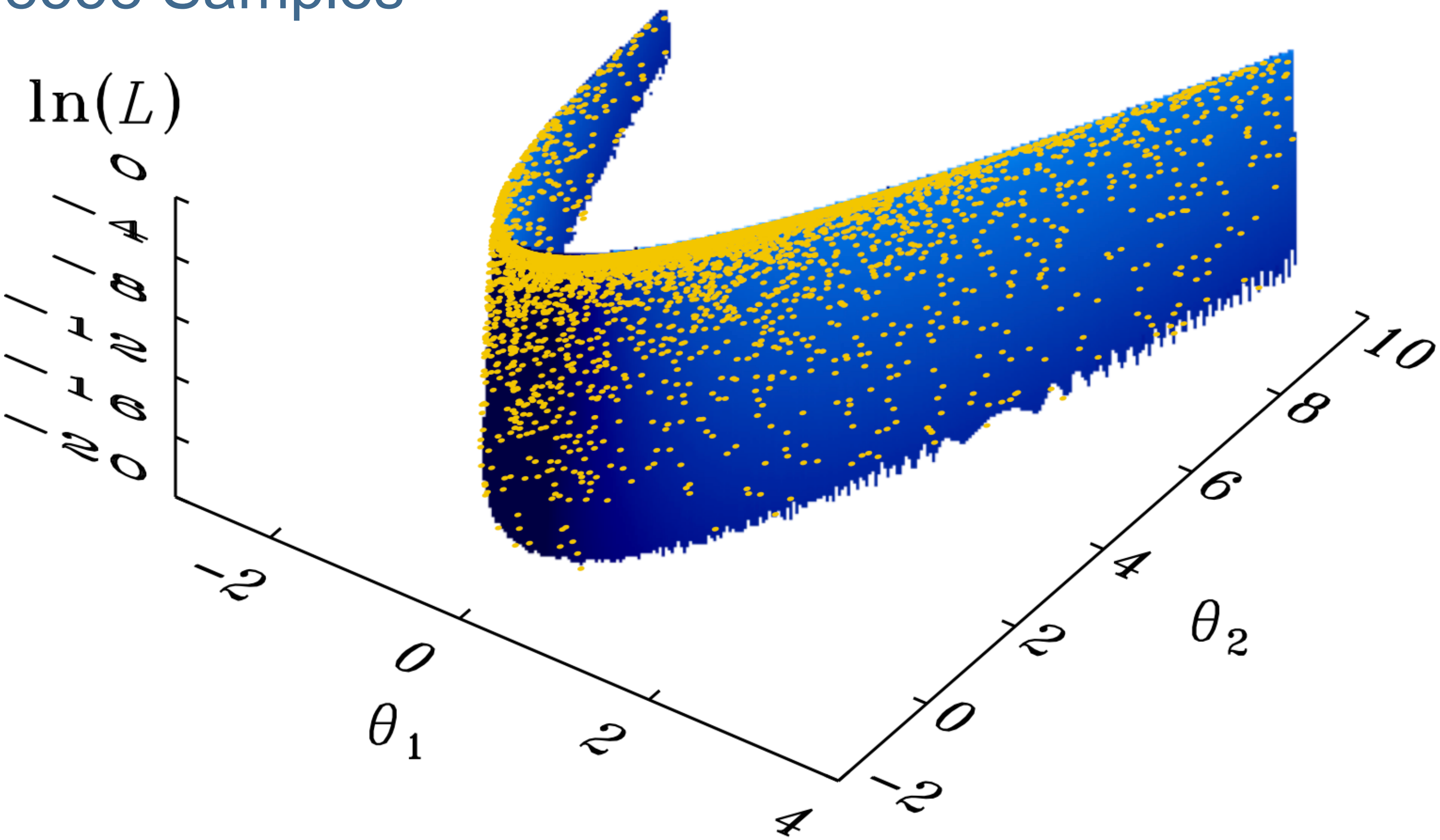
N = 8485 Samples



Himmelblau's Function

Sampling efficiency demos

N = 8558 Samples



Rosenbrock's Function

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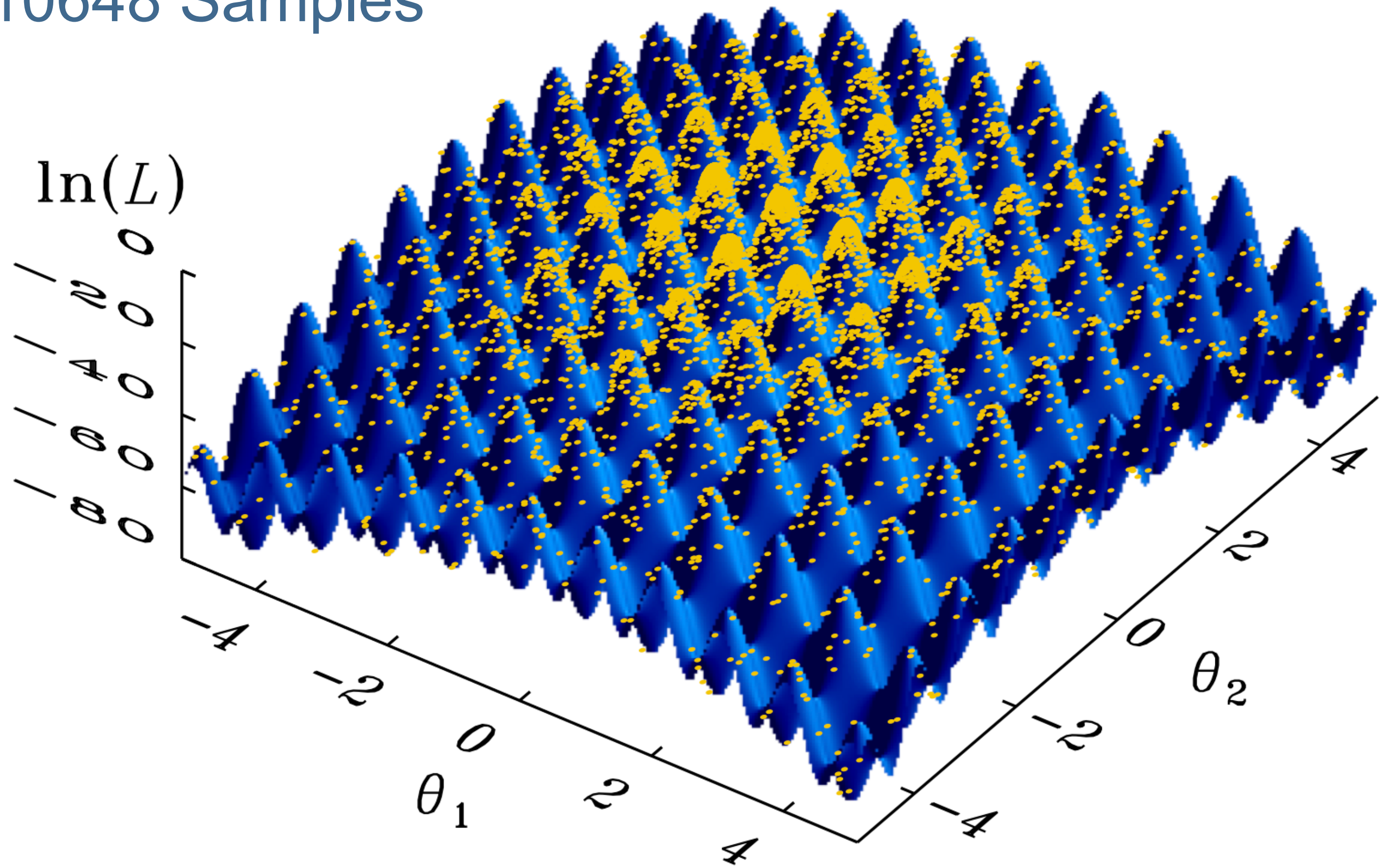
Sampling efficiency demos

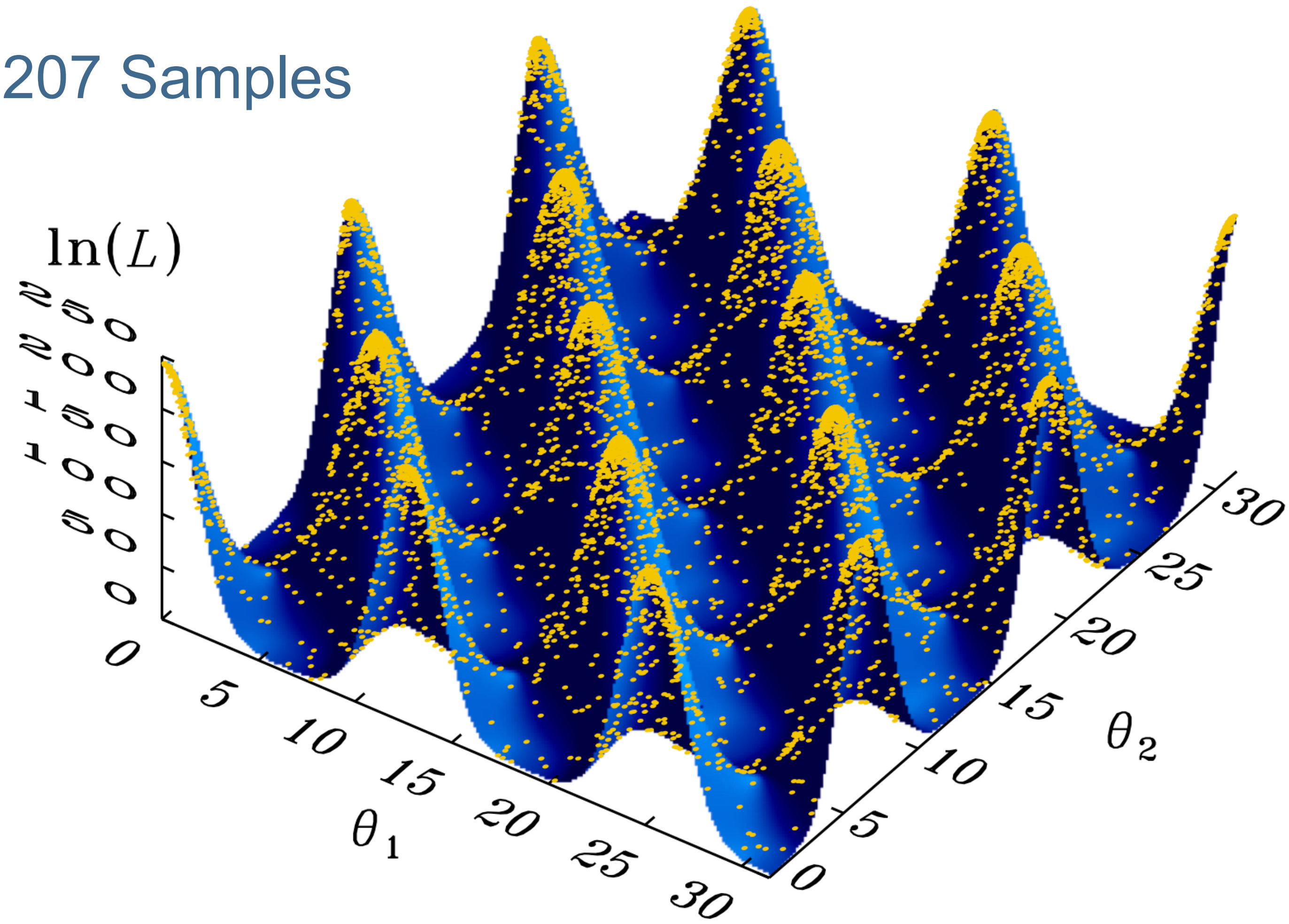
N = 10648 Samples



Rastrigin's Function

Sampling efficiency demos

N = 8207 Samples



Eggbox Function

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Peak Bagging

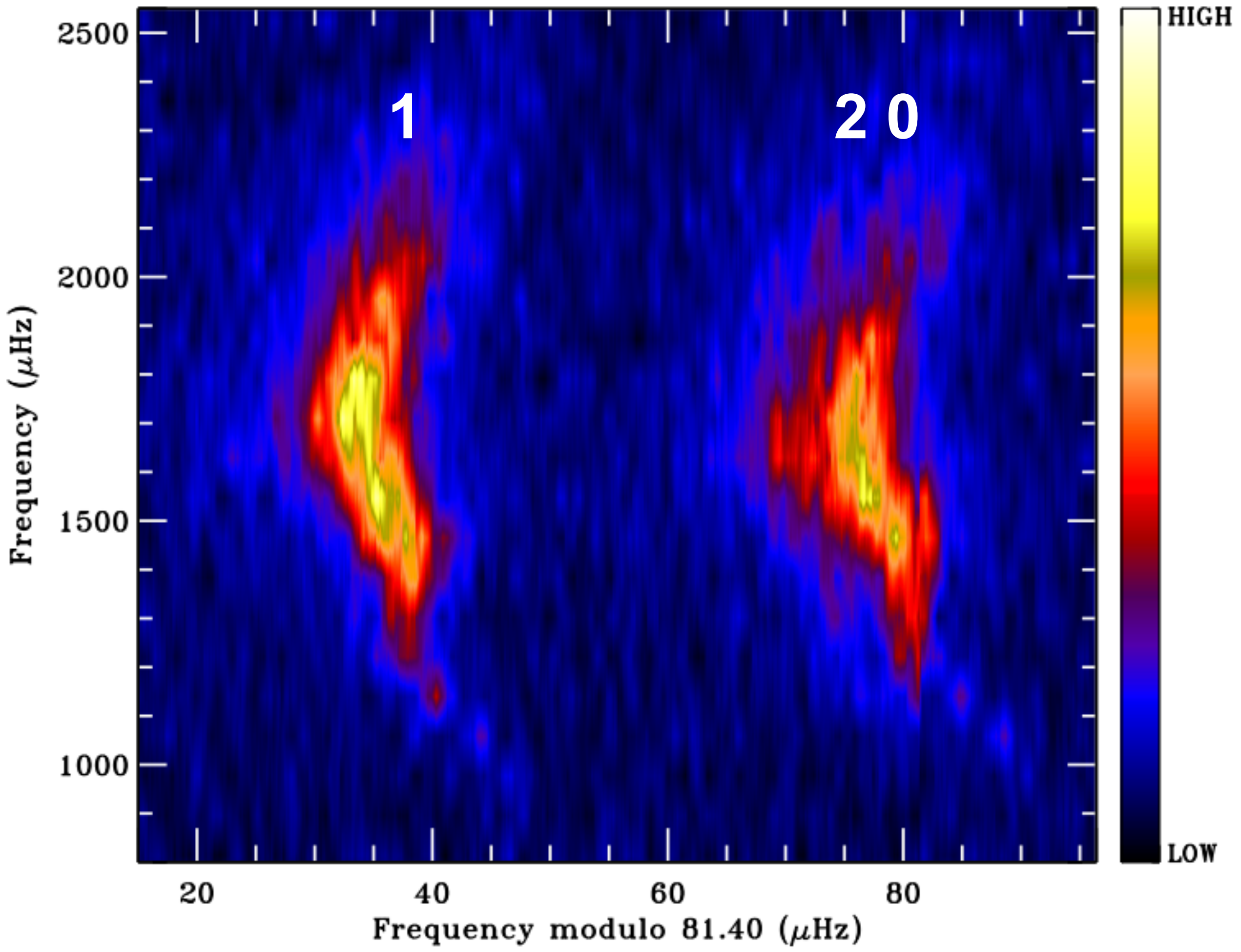
Punto (KIC 9139163)

Q5-Q17.2

1147.5 days

$$T_{\text{eff}} \simeq 6405K$$
$$\nu_{\text{max}} \simeq 1712\,\mu\text{Hz}$$
$$\Delta\nu \simeq 81.4\,\mu\text{Hz}$$

$$M \simeq 1.57 M_{\odot}$$
$$R \simeq 1.41 R_{\odot}$$



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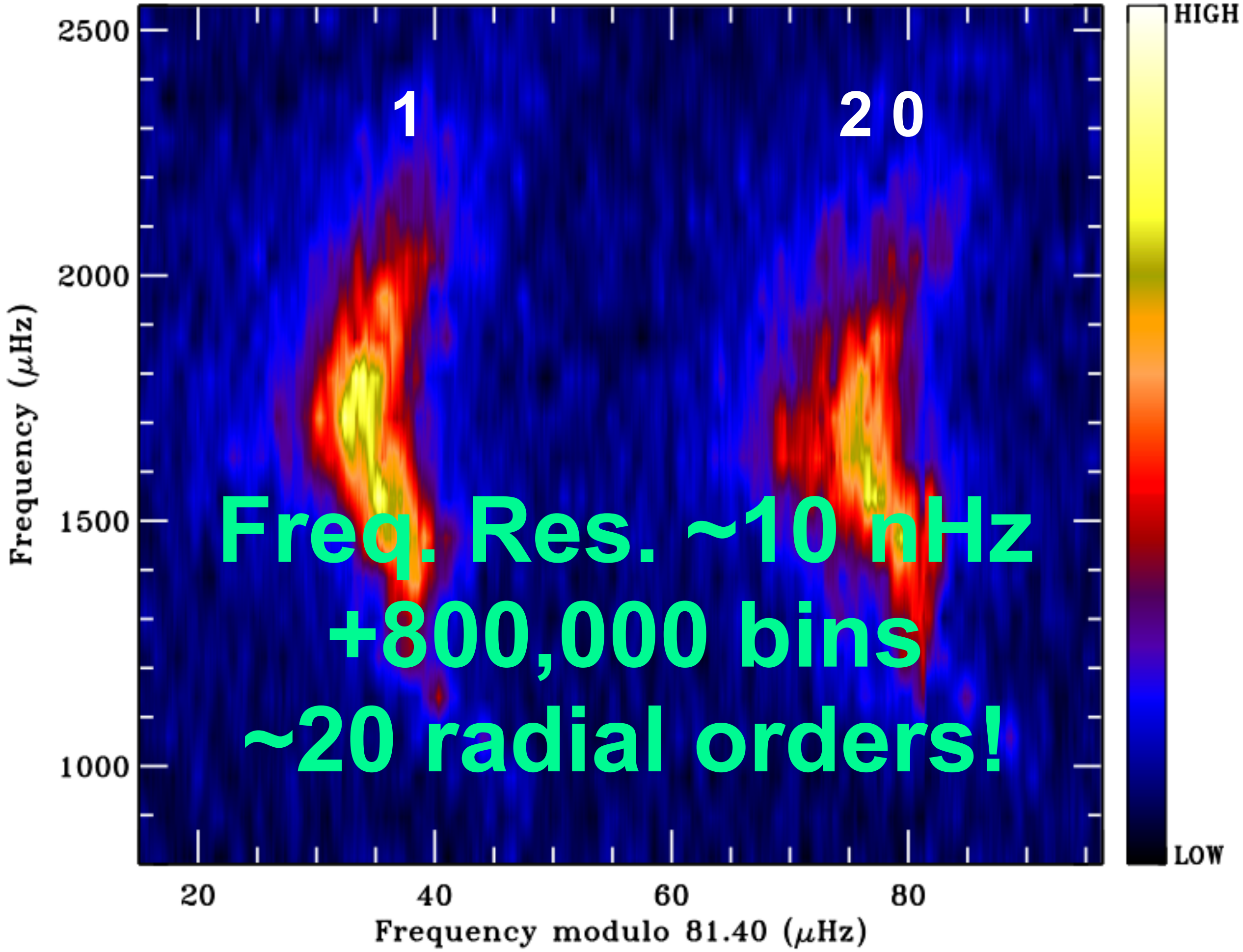

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Background

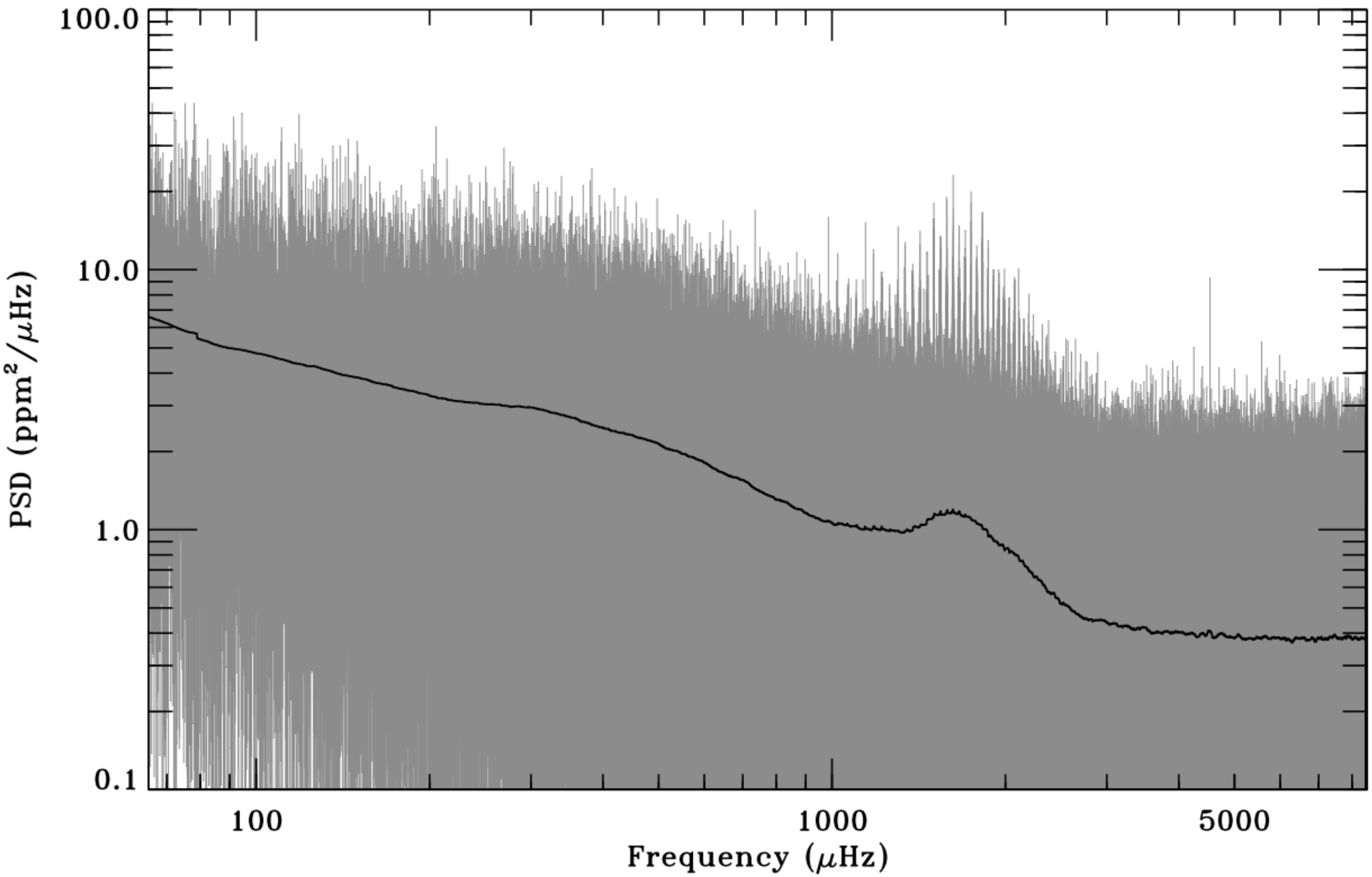
$$B(\nu) = W + a\nu^{-b} + \sum_{i=1}^{m} \frac{4\tau_i \sigma_i^2}{1 + (2\pi\nu\tau_i)^{c_i}} + H_{\mathrm{osc}} \exp\left[-\frac{(\nu - \nu_{\mathrm{max}})^2}{2\sigma_{\mathrm{env}}^2}\right]$$

1 or 2 Harvey-like profiles?

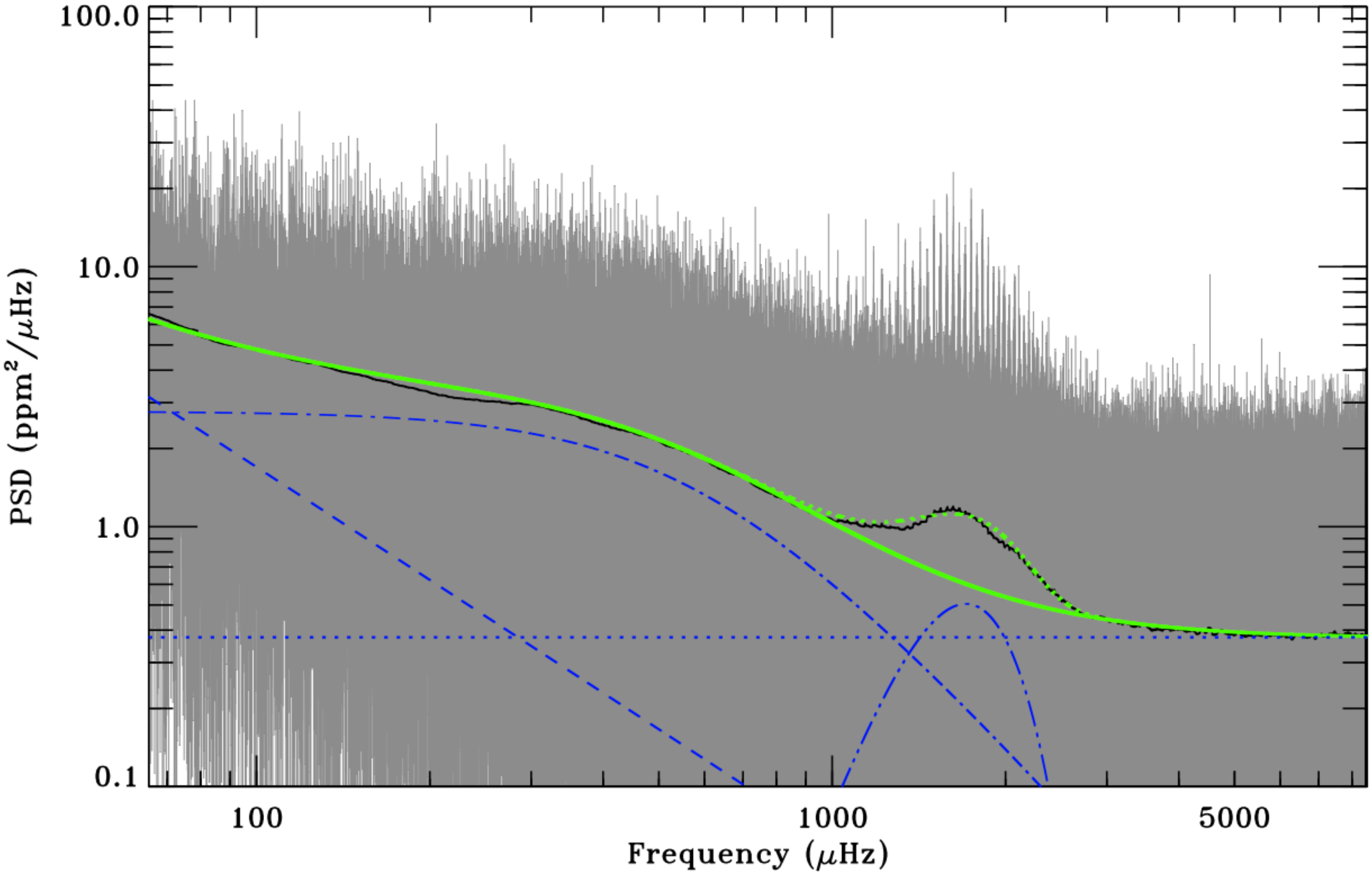
$$B_{12} = \frac{\mathcal{E}_1}{\mathcal{E}_2} \gg 150$$

ONLY GRANULATION DETECTED

Background



Background



Peak Bagging

$$B(\nu) = W + a\nu^{-b} + \sum_{i=1}^{m} \frac{4\tau_i \sigma_i^2}{1 + (2\pi\nu\tau_i)^{c_i}} + H_{\mathrm{osc}} \exp\left[-\frac{(\nu - \nu_{\mathrm{max}})^2}{2\sigma_{\mathrm{env}}^2}\right]$$

3 free parameters per mode

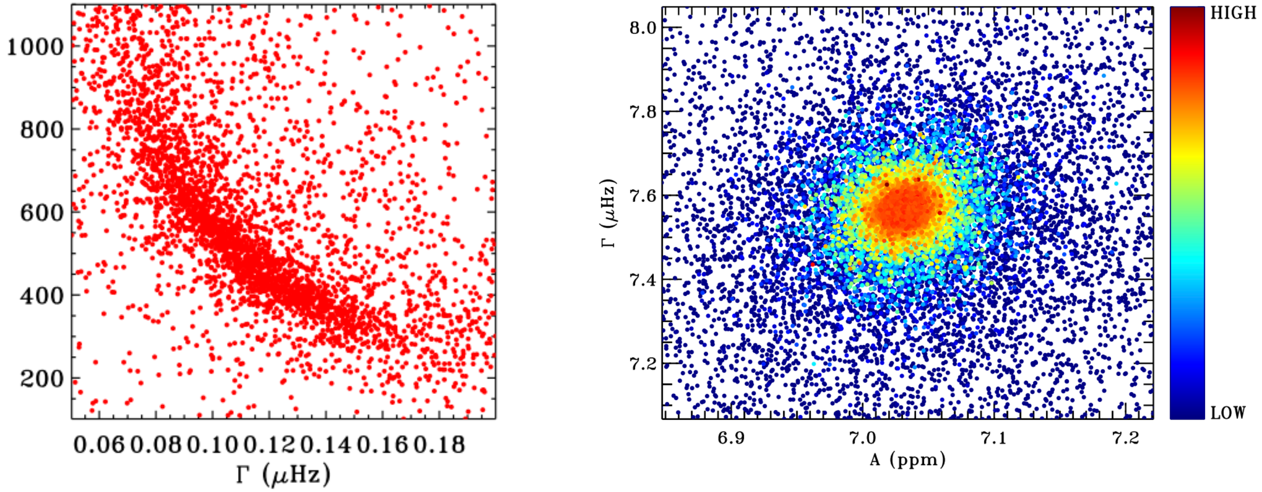
$$(\nu_i, A_i, \Gamma_i)$$

$$P_{\mathrm{osc}}(\nu) = \sum_{i=1}^{N} \frac{A_i^2/(\Gamma_i \pi)}{1 + 4(\nu - \nu_i)^2/\Gamma_i^2}$$

Fitting one Lorentzian profile

Height versus Amplitude

$$A^2 = \pi H \Gamma$$



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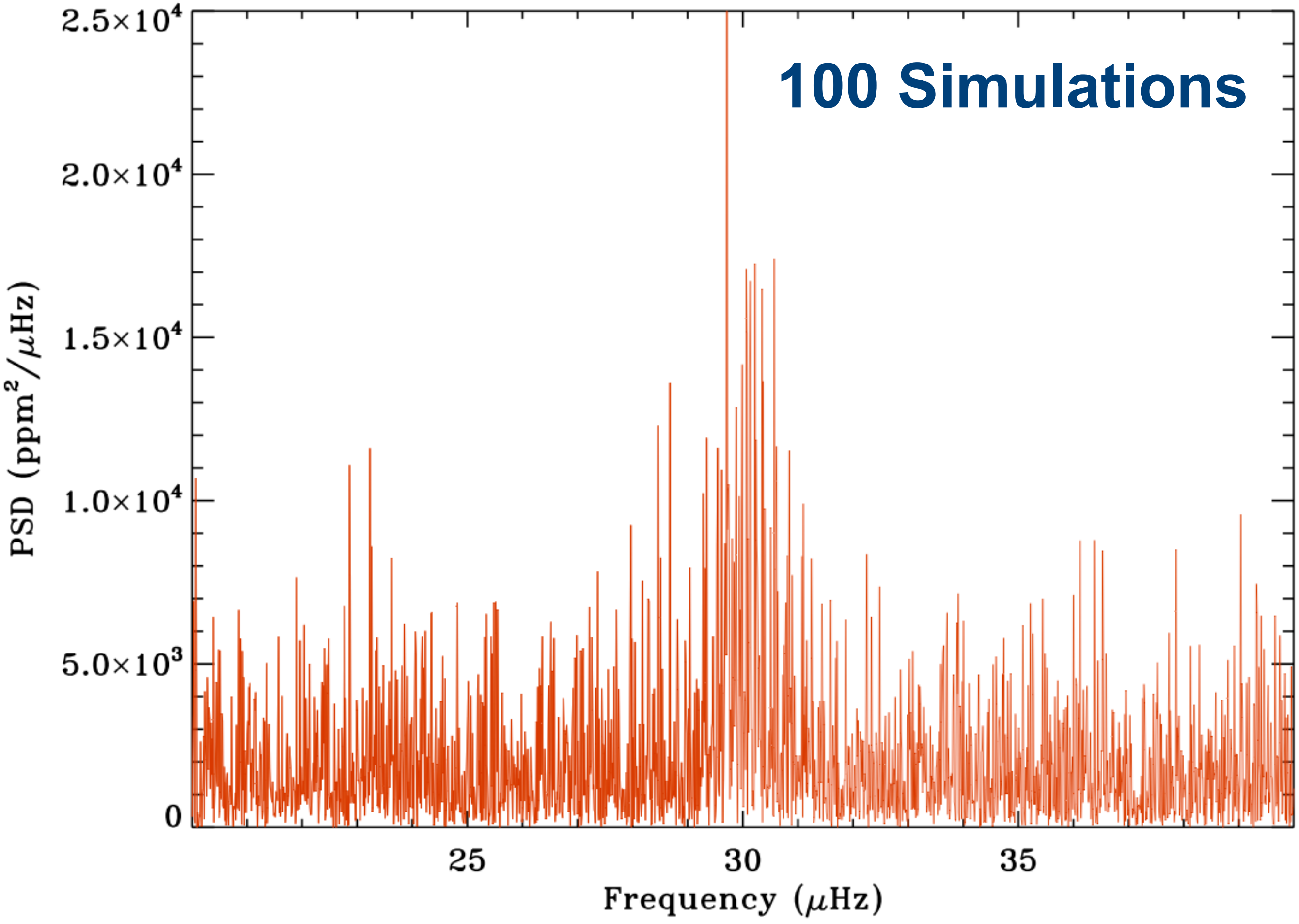
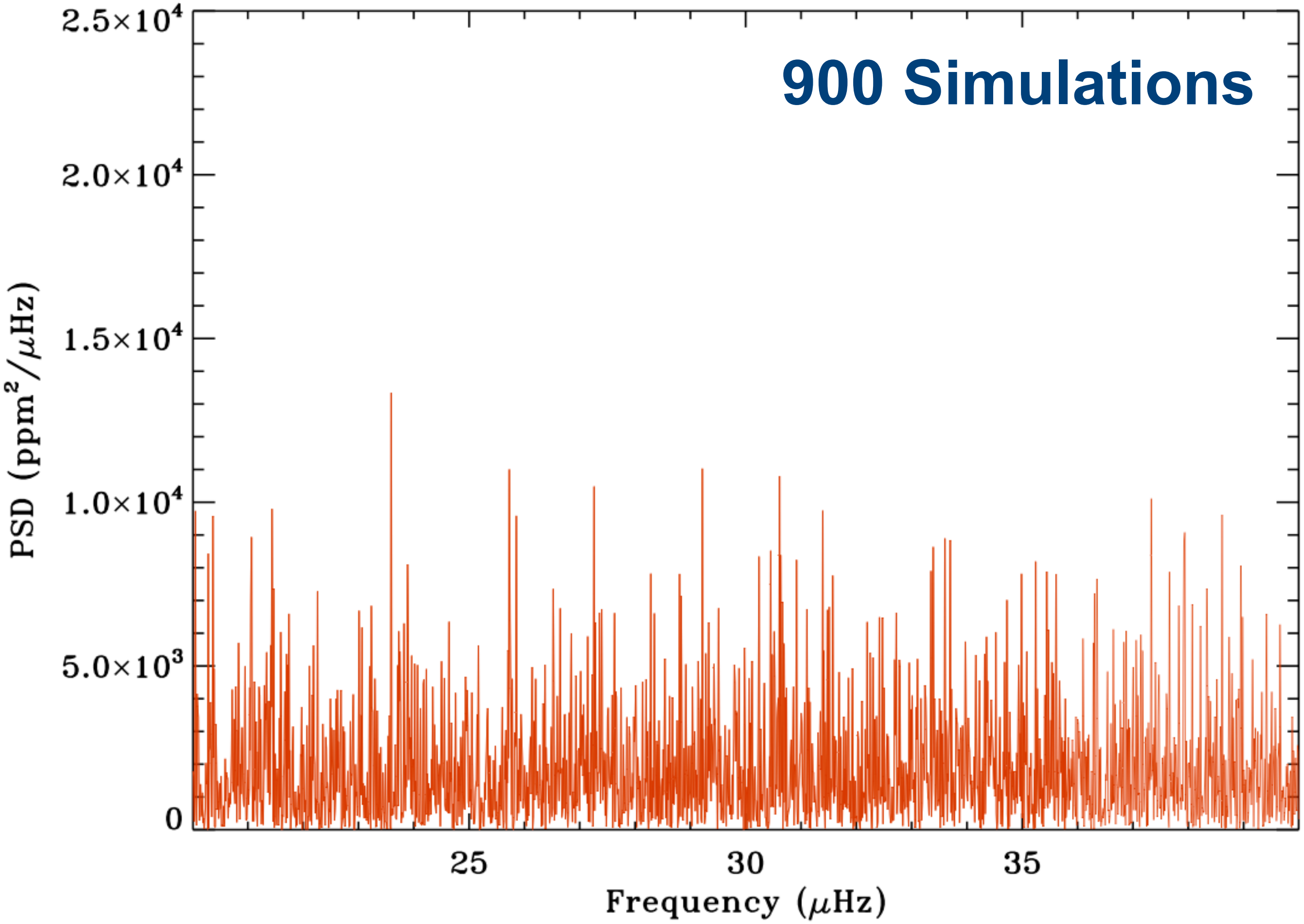
Peak Significance - 1 Peak

$\mathcal{M}_1$



Peak Significance - 1 Peak

$\mathcal{M}_2$



Peak Significance - 1 Peak

- Computed evidences

$$\mathcal{E}_1 \qquad \mathcal{E}_2$$

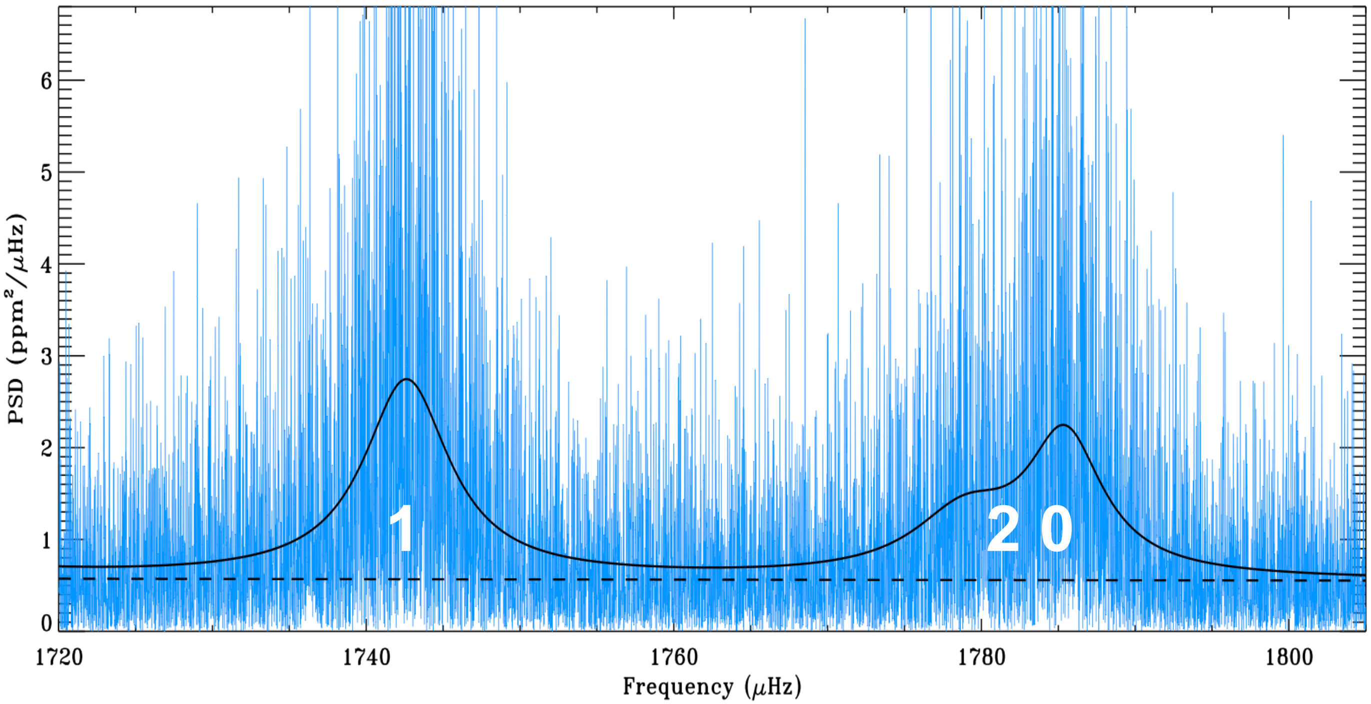
$$\mathcal{M}_1 \qquad \mathcal{M}_2$$

- Only strong evidence ratios $B_{12} = \mathcal{E}_1 / \mathcal{E}_2 \sim 150$

0 FALSE POSITIVES
1 FALSE NEGATIVE

Peak Significance - 2 Peaks

$\mathcal{M}_1$ **Both ℓ = 2 and ℓ = 0**



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Peak Significance - 2 Peaks

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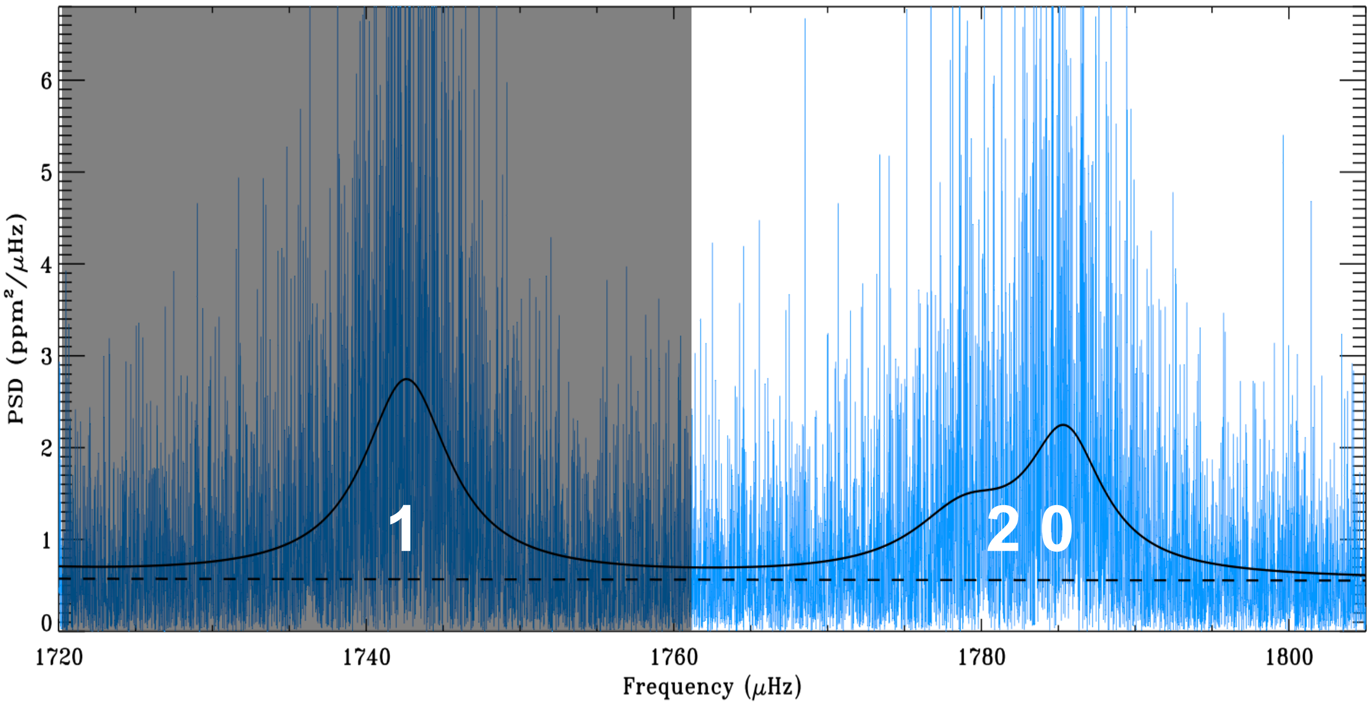
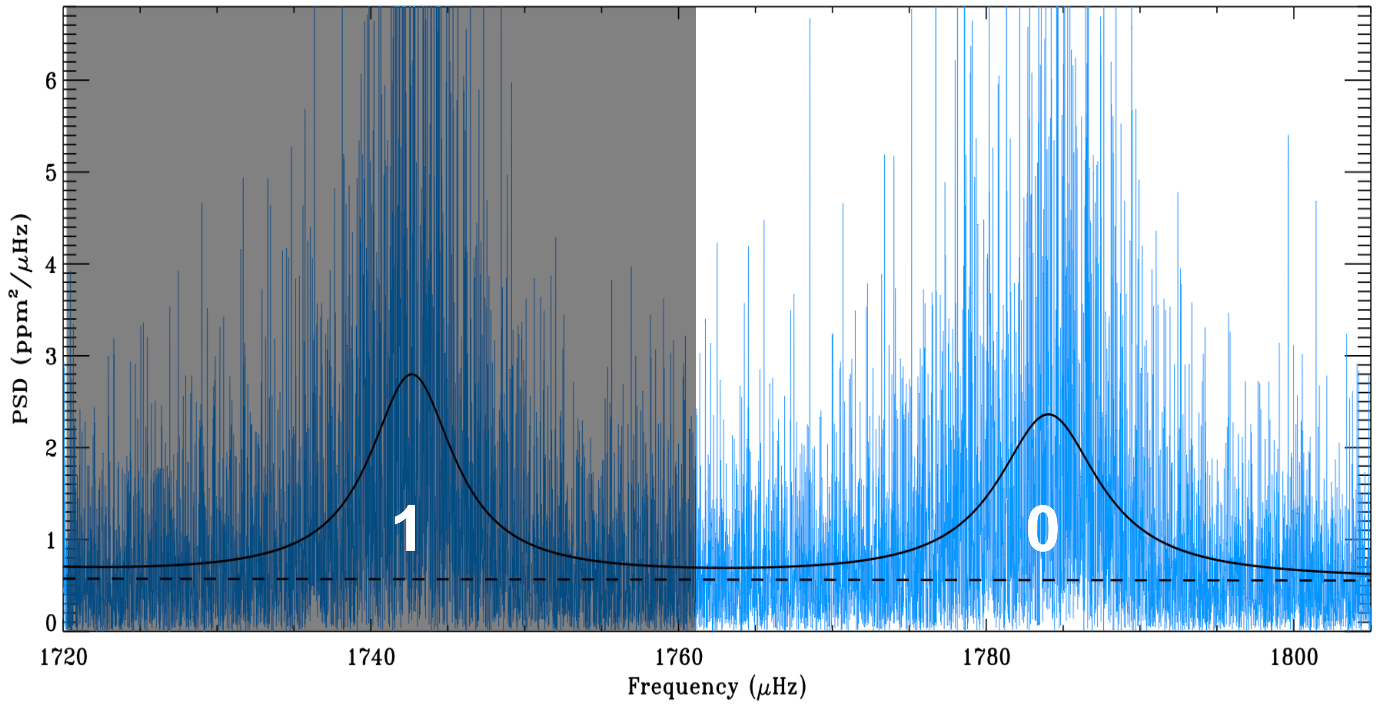


Peak Significance - 2 Peaks

$\mathcal{M}_2$ **Only ℓ = 0**



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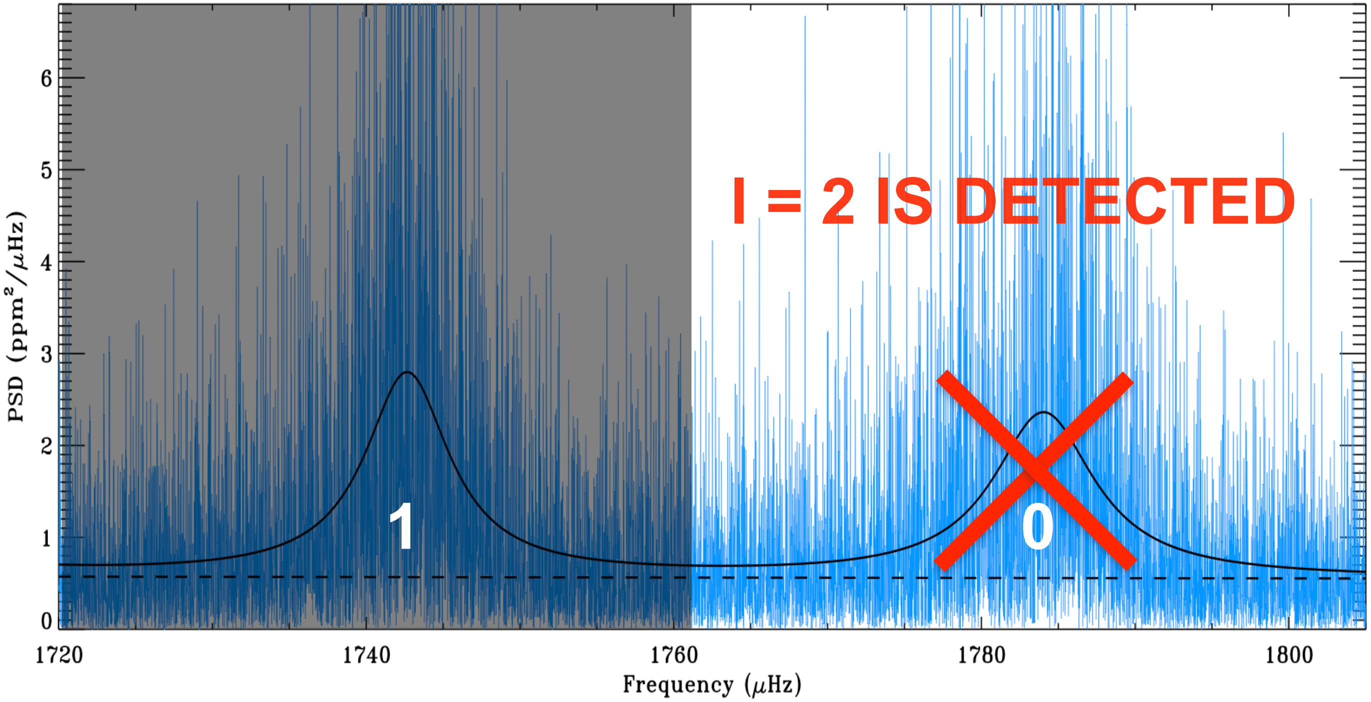
Peak Significance - 2 Peaks

$\mathcal{M}_2$

Only ℓ = 0

$$B_{12} = \frac{\mathcal{E}_1}{\mathcal{E}_2} \gg 150$$



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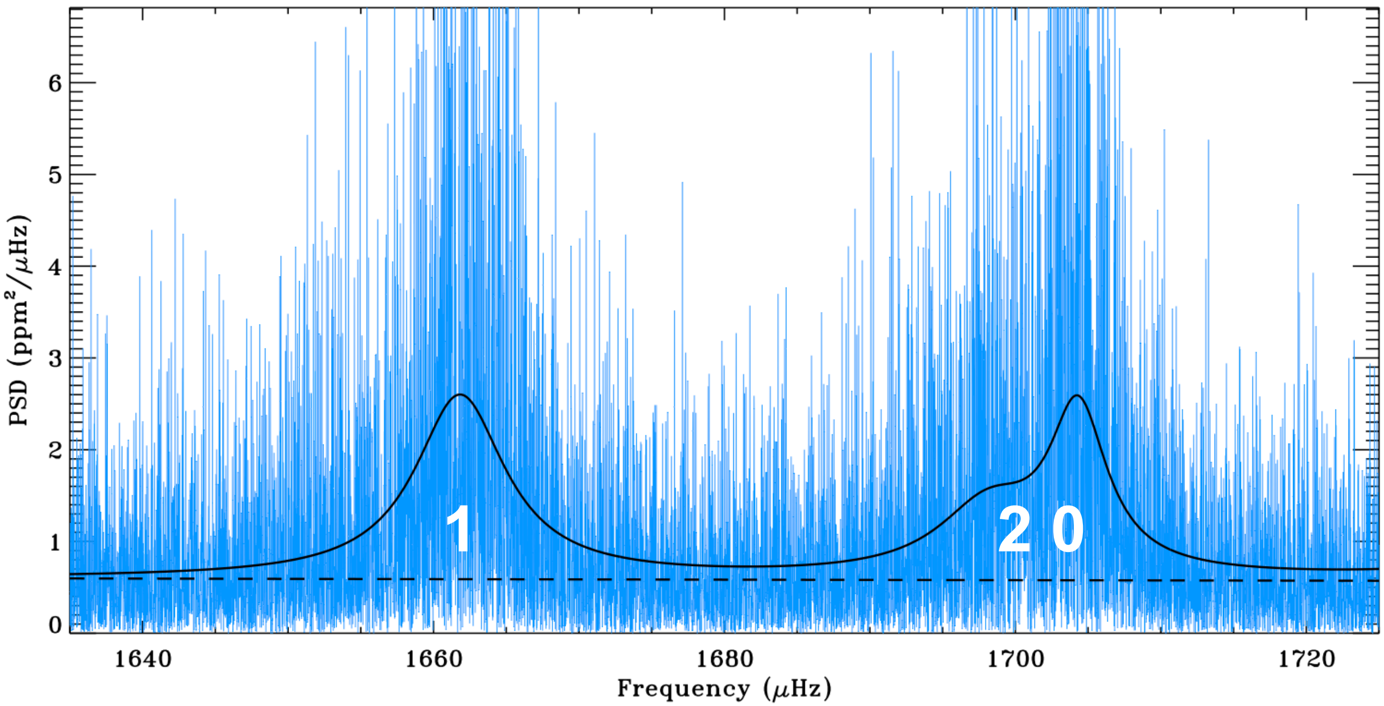
Tackling Rotation from ℓ = 1 modes

$\mathcal{M}_1$

$$\frac{A_n^2/\left(\Gamma_n \pi\right)}{1+4\left(\nu-\nu_n\right)^2/\Gamma_n^2}$$



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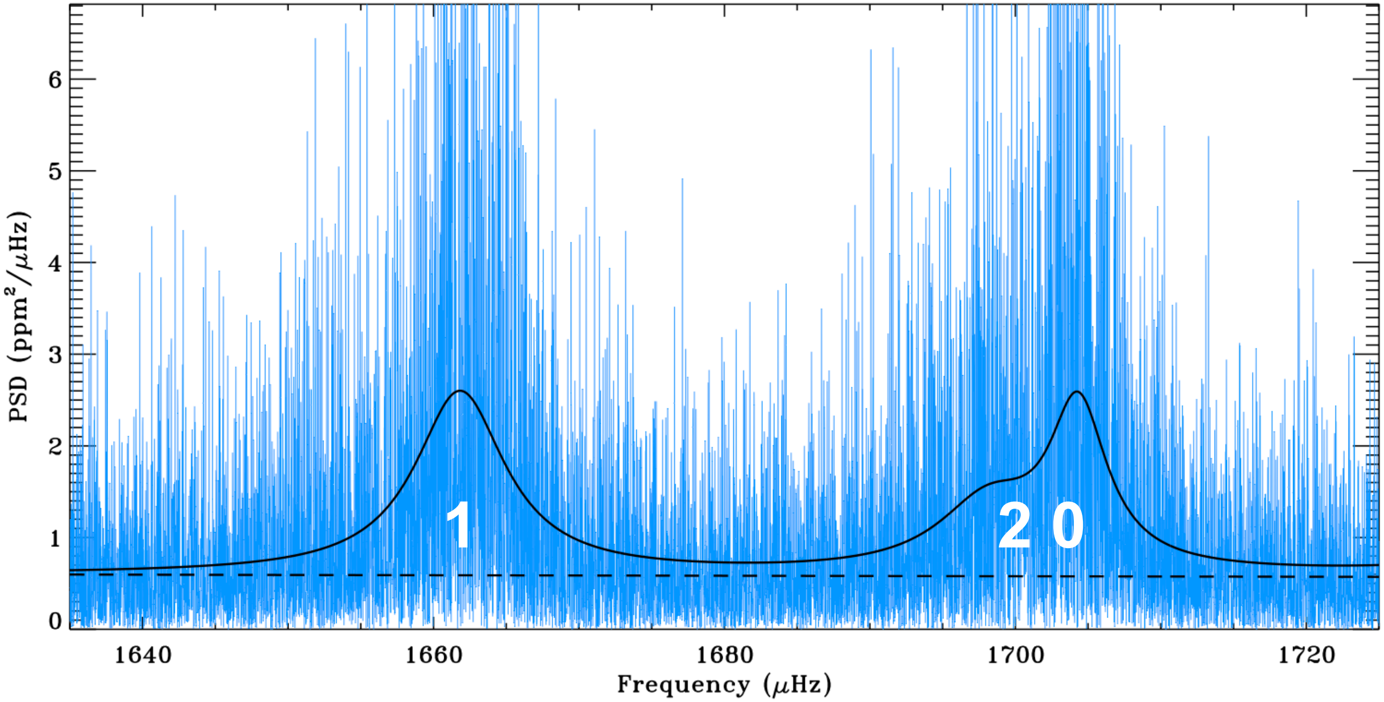
Tackling Rotation from ℓ = 1 modes

$\mathcal{M}_1$

$$\frac{A_n^2/\left(\Gamma_n\pi\right)}{1+4\left(\nu-\nu_n\right)^2/\Gamma_n^2}$$

$\mathcal{M}_2$

$$\sum_{m=-1}^{1}\frac{V_m\left(i\right)A_n^2/\left(\Gamma_n\pi\right)}{1+4\left(\nu-\nu_n+m\delta\nu_{\mathrm{rot}}\right)^2/\Gamma_n^2}$$



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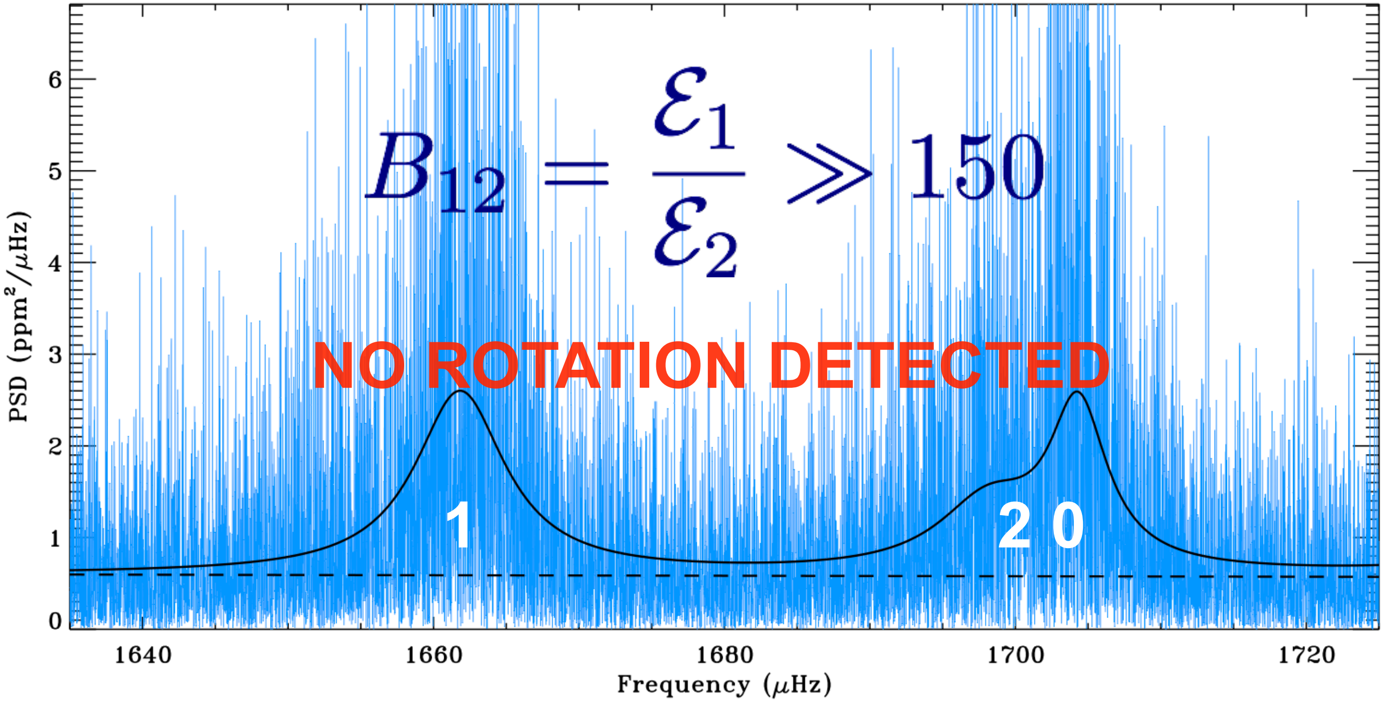
Tackling Rotation from $\ell = 1$ modes

$\mathcal{M}_1$

$$\frac{A_n^2/\left(\Gamma_n \pi\right)}{1+4\left(\nu-\nu_n\right)^2/\Gamma_n^2}$$

$\mathcal{M}_2$

$$\sum_{m=-1}^{1} \frac{V_m\left(i\right) A_n^2/\left(\Gamma_n \pi\right)}{1+4\left(\nu-\nu_n+m\delta\nu_{\mathrm{rot}}\right)^2/\Gamma_n^2}$$



$$B_{12} = \frac{\mathcal{E}_1}{\mathcal{E}_2} \gg 150$$

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Conclusions

- Nested Sampling offers a valuable way of performing Bayesian inferences in high-dimensions with more efficiency and speed than classical techniques as MCMC

- Bayesian evidence can be very useful:
  1. Peak significance (detection signal criterion) in either peak-to-noise or peak-to-peak
  2. Test different background models
  3. Tackling rotation

- DIAMONDS has potential in the Peak Bagging analysis of challenging datasets and targets: Parallelization? ES can be troublesome - something better?

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