



Abstract:

The study of the Sun is developing through the combination of observational data and numerical simulation which clarify physical mechanism. The bundles of loops were observed in active regions led to study the collective oscillations and internal fine structures of the loops. We studied the MHD oscillations of a system of magnetized loops under the zero- β condition and the stratification of density along the radial (step function) and the loops axis (z). A single partial differential equation as the wave equation, for z component of the perturbed magnetic field, is numerically solved based on finite element method by employing the appropriate boundary conditions. Eigenfunctions and eigenfrequencies are extracted for a system of loops with various number of multi-stranded loops inside the hypothetical monolithic loop. Results show that the interactions of multi-stranded loops are roughly correlated with their spatial configuration and density topology. The ratios of frequencies $\tilde{\omega}_{sys}/\tilde{\omega}_{mono}$ are extracted in order to studying the interaction influence of the loops on their collective oscillations. It is inferred that for a system of loops, the ratios of frequencies are found in large quantities than a system of loops with many tubes inside the equivalent monolithic tube. It is also concluded that for a system with a few number of loops, the density configuration possesses asymmetric topology inside the monolithic tube. While, by adding the loops inside the hypothetical tube and notifying the equivalent density are kept in the same quantity, the density structure tends to have symmetric topology. In this case the, the ratios of frequencies are lower [1].

Introduction

The aim is to study the effect of longitudinal variation of a density-stratified loop. We reduce the MHD equations to a wave equation with variable Alfvén speed for the z -component of magnetic field. The Eigenfunctions of the Sausage, Kink, torsional modes and their differences due to the variation scale length parameter (ϵ) are studied. Waves are very important in solar coronal plasma heating as waves are used for recognition of the internal structure of the earth in geophysics likewise it is employed to the internal structure of the Sun and coronal oscillations in Helioseismology and Coronal Seismology. Generally, waves and instabilities are originated by the disturbances of the system. For magnetized plasma the gas pressure, magnetic pressure and magnetic tension act as restoring force. Then, there will be a variety of the waves in the system. Alfvén, Slow and Fast Magneto-acoustic waves are identified in an infinite homogeneous by the ideal MHD equations (1-4).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \delta v) = 0, \quad \text{Continuity Eq,} \quad (1)$$

$$\frac{\partial \delta v}{\partial t} = \frac{1}{4\pi\rho} (\nabla \times \delta \mathbf{B}) \times \mathbf{B}, \quad \text{Momentum Eq,} \quad (2)$$

$$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\delta v \times \mathbf{B}), \quad \text{Induction Eq,} \quad (3)$$

$$\nabla \cdot \delta \mathbf{B} = 0, \quad \text{Solenoidal Constraint,} \quad (4)$$

If the plasma be inhomogeneous these three types of waves are coupled, for instance it is possible to have both Alfvén and magneto-acoustic waves in plasma. Magnetic flux tube have difference oscillations and these oscillations are expressed by the wave numbers (n, m, l) in cylindrical coordinates ($\hat{r}, \hat{\phi}, \hat{z}$). $m = 0$, $m = 1$ and $m = 2$ are respectively the wave numbers of Sausage, Kink and Fluting modes.

The Model and Equations of Motions

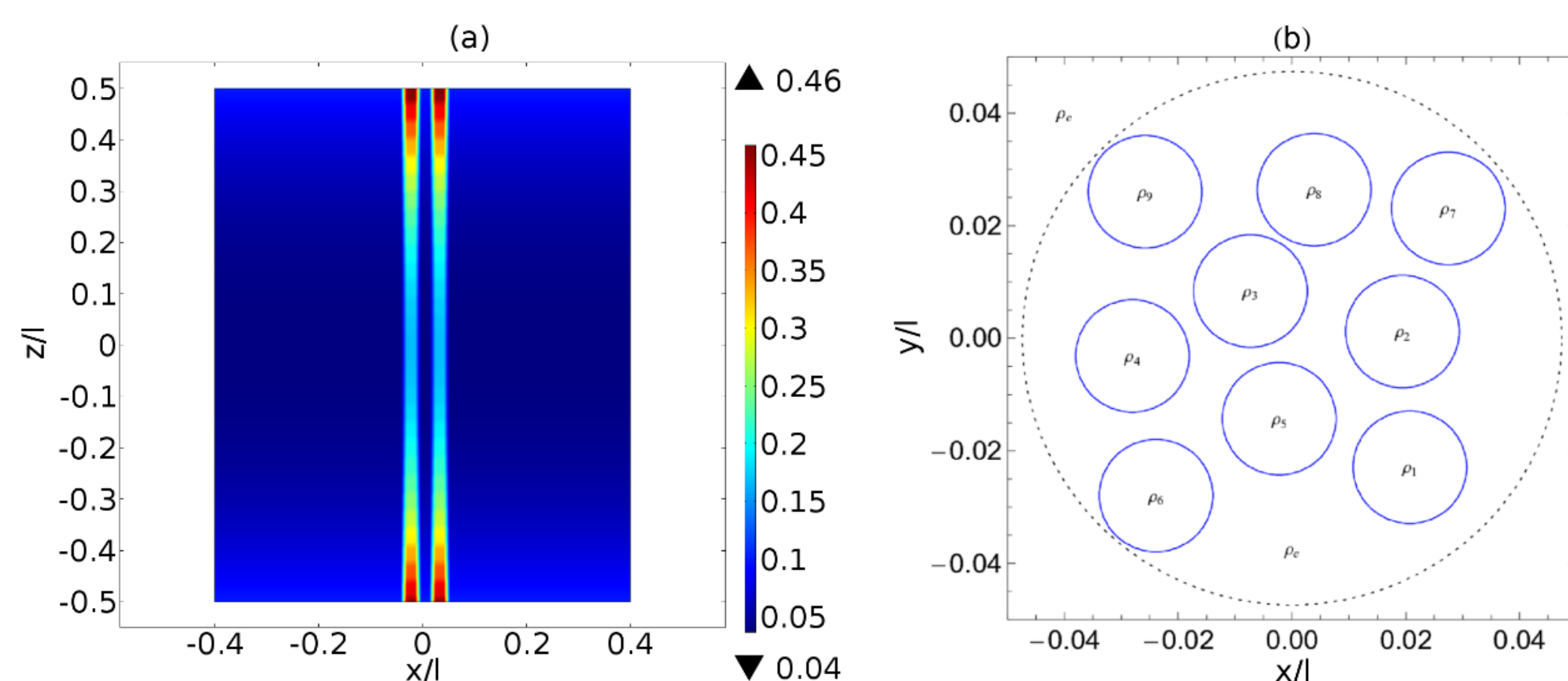
In this study by the solving of the three dimensional of MHD equations, the extracted wave equation has been solved numerically by the Finite Element method and the Sausage modes are simulated in the presence of longitudinally stratification density profile. The cylinder is assumed to have negligible gas pressure (zero- β approximation) and neglected gravity pressure.

With these assumptions the MHD equations are lead to this wave equation

$$\nabla^2 \mathbf{b}(\vec{r}, t) + \frac{\omega^2}{v_A^2} \mathbf{b}(\vec{r}, t) = 0, \quad v_A = \frac{B_0}{\sqrt{4\pi\rho}} \quad (5)$$

Where v_A is Alfvén speed and \mathbf{B} is magnetic field. The density profile is defined by a step function in lateral surface of a cylinder with the radius a , (Fig.1)

$$\rho(x, y) = \sum_{j=1}^N \left[(\rho_{i,j} - \rho_e) \theta \left(a - \sqrt{x_j^2 + y_j^2} \right) + \rho_e \right] \times f(\epsilon, z) \quad (6)$$



Boundary Conditions

Equation (5) by imposing appropriate boundary conditions for a system of flux tube(s) is an eigenvalue problem and can be solved in cylindrical coordinates. Regarding that the tube ends are frozen in the photosphere, the perturbed quantities (v_r and b_z) are considered as

$$v_r^{int}|_{r=a} = v_r^{ext}|_{r=a} = 0, \quad b_z^{int}|_{r=a} = b_z^{ext}|_{r=a} = 0 \quad \text{at} \quad z = \pm l/2$$

The boundary conditions at the lateral surface of flux tube ($r = a$) are give by

$$v_r^{int}|_{r=a} = v_r^{ext}|_{r=a}, \quad b_z^{int}|_{r=a} = b_z^{ext}|_{r=a}$$

At the tube axis, $r = 0$, we assume

$$b_z(r = 0, \theta, z) = \text{finite}, \quad -l/2 < z < l/2$$

At the large distance from the tube axis we suppose the evanescent condition

$$b_z(r \rightarrow \infty, \theta, z) \rightarrow 0$$

The relation between radial component of velocity and b_z are given by

$$\frac{v_{A_{int}}^2 l^2}{\omega^2 l^2 - 8v_{A_{int}}^2} \frac{\partial b_z^{int}}{\partial r} \Big|_{r=a} = \frac{v_{A_{ext}}^2 l^2}{\omega^2 l^2 - 8v_{A_{ext}}^2} \frac{\partial b_z^{ext}}{\partial r} \Big|_{r=a}$$

Method

The method we choice is the “Finite element method” (FEM) for computing the primitive variables, Eigenfrequencies and Eigenfunctions of our extracted partial differential equation. A typical work out of the (FEM) involves dividing the domain of the problem into a collection of sub-domains, with each sub-domain represented by a set of element equations to the original problem. The domain discretization is the “Free Tetrahedral” (Fig.2).

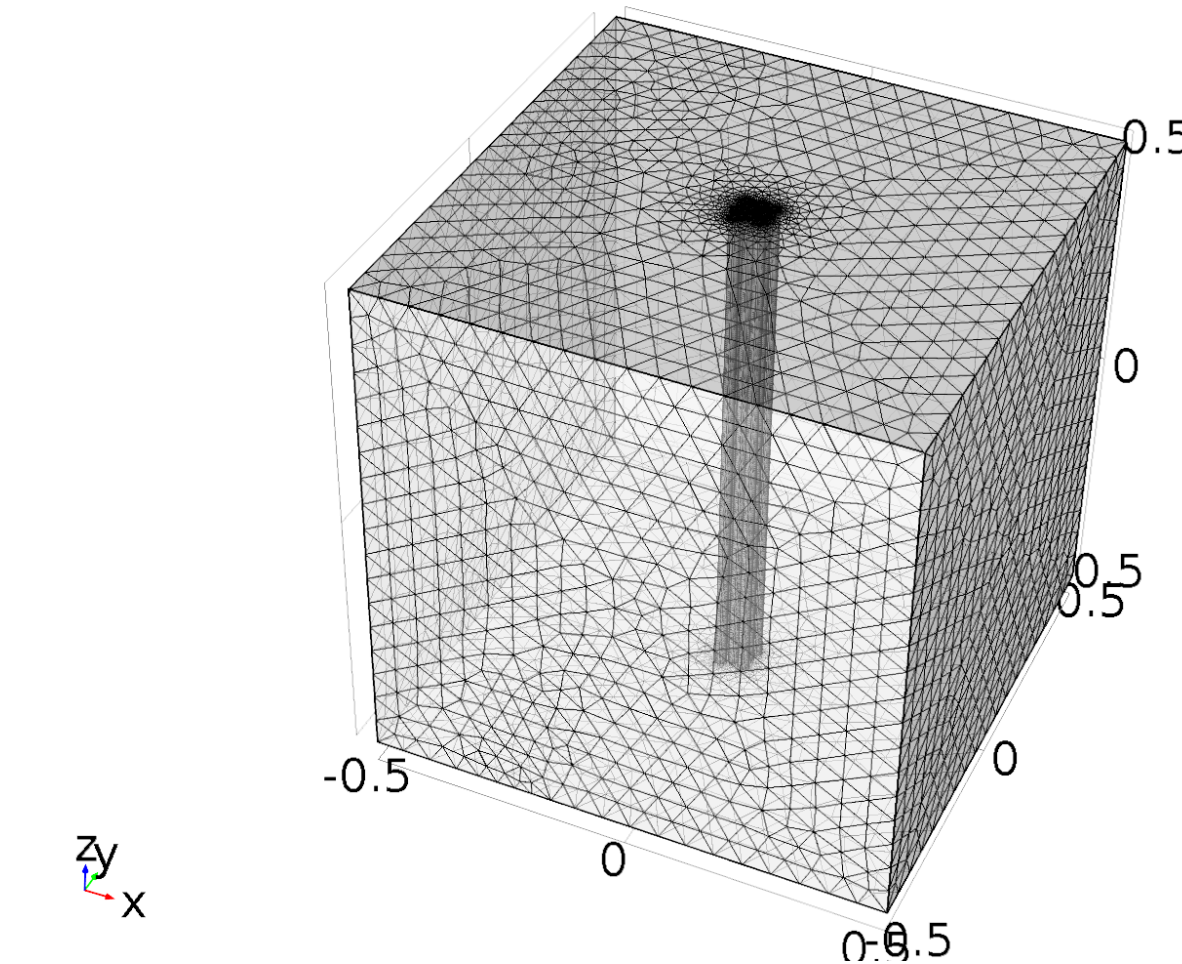


Fig.2. Meshed cube by the Finite Element Method with tetrahedral domain representing nine flux tubes bounded at $z = \pm 0.5$ inside it.

Results

In Figures below, the sets of the systems of nonidentical and identical tubes including one to nine tubes are demonstrated. The ratios of frequencies are studied. First, we consider a single tube with its associated density and the equivalent density which are fixed in $\rho_{eq} = 3.675\rho_e$ and $\rho_{eq} = 3.6037\rho_e$ for nonidentical and identical cases, respectively. The equivalent density and the ratios of frequencies for the oscillations of a system of tube(s) to their equivalent monolithic tube, $\tilde{\omega}_{sys}/\tilde{\omega}_{mono}$, are represented for both nonidentical and identical cases in Tables 1 and 2, respectively. We see that the ratios of frequencies are decreased by increasing the number of flux tubes in both nonidentical and identical system of flux tubes.

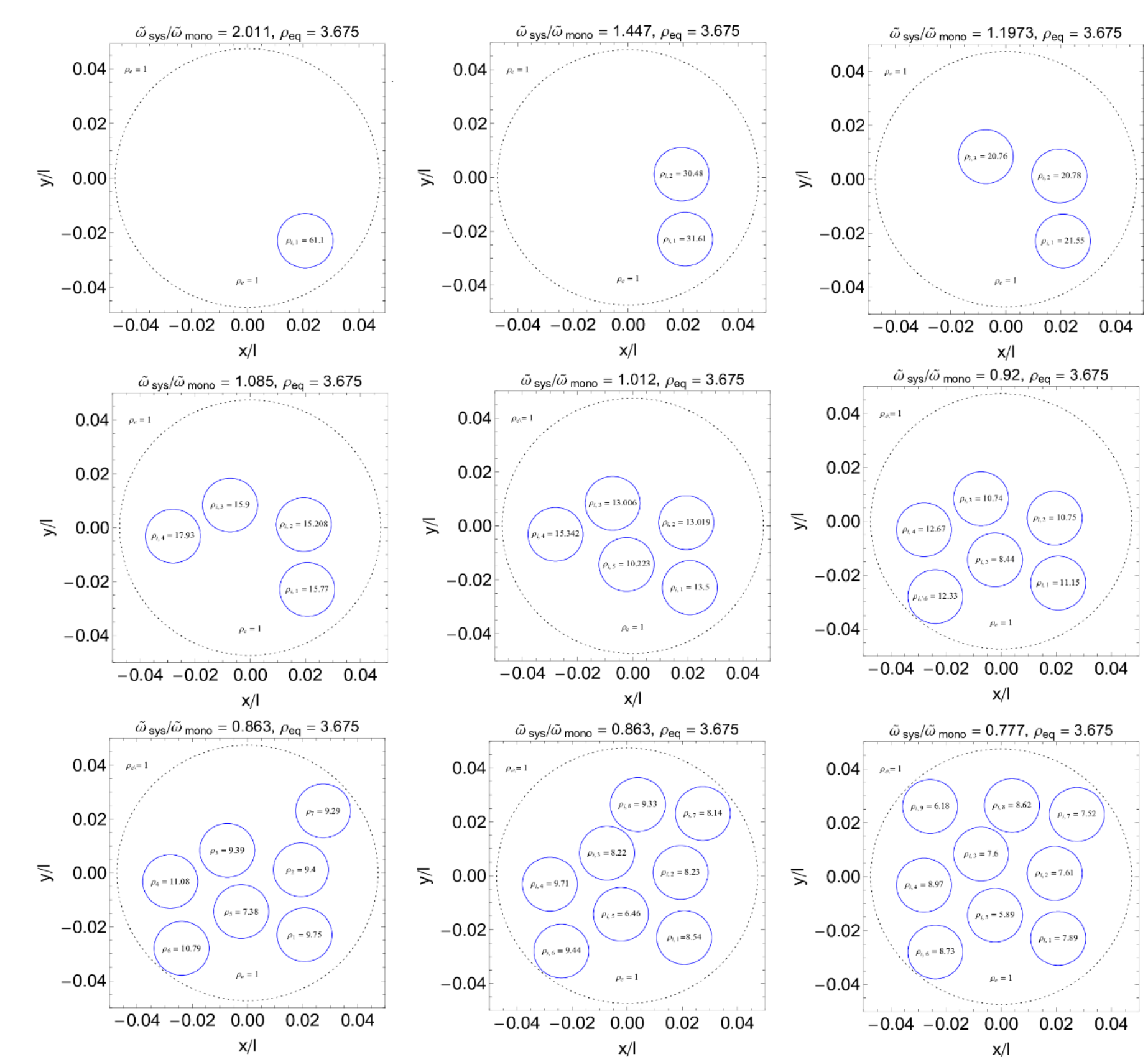


Fig.3. Systems of 1 to 9 nonidentical tube(s) inside the equivalent monolithic tube are shown.

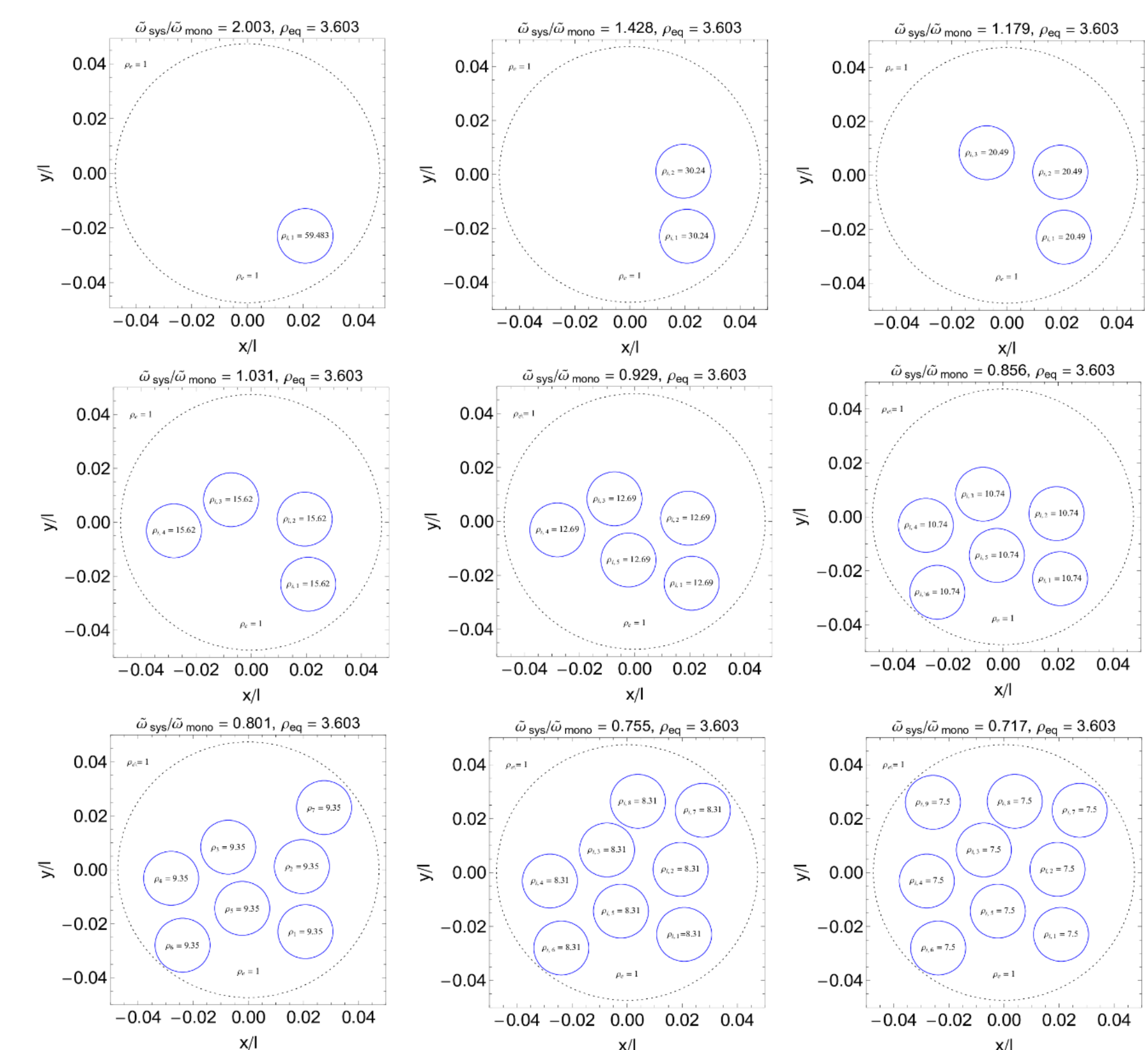


Fig.4. Systems of 1 to 9 identical tube(s) inside with the related ratios of frequencies $\tilde{\omega}_{sys}/\tilde{\omega}_{mono}$

Conclusion

It can be inferred that for a system of tube(s) with a few number of flux tubes, the ratios of frequencies are found in large quantities than a system of tubes with many tubes inside the equivalent monolithic tube. It is also concluded that for a system with a few number of flux tubes, the density configuration possesses asymmetric topology inside the monolithic tube. While, by adding the tubes inside the hypothetical tube and notifying the equivalent density are kept in the same quantity, the density structure tends to have symmetric topology. In this case the, the ratios of frequencies are lower.

References

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