Data assimilation as a tool to better understand the solar magnetism

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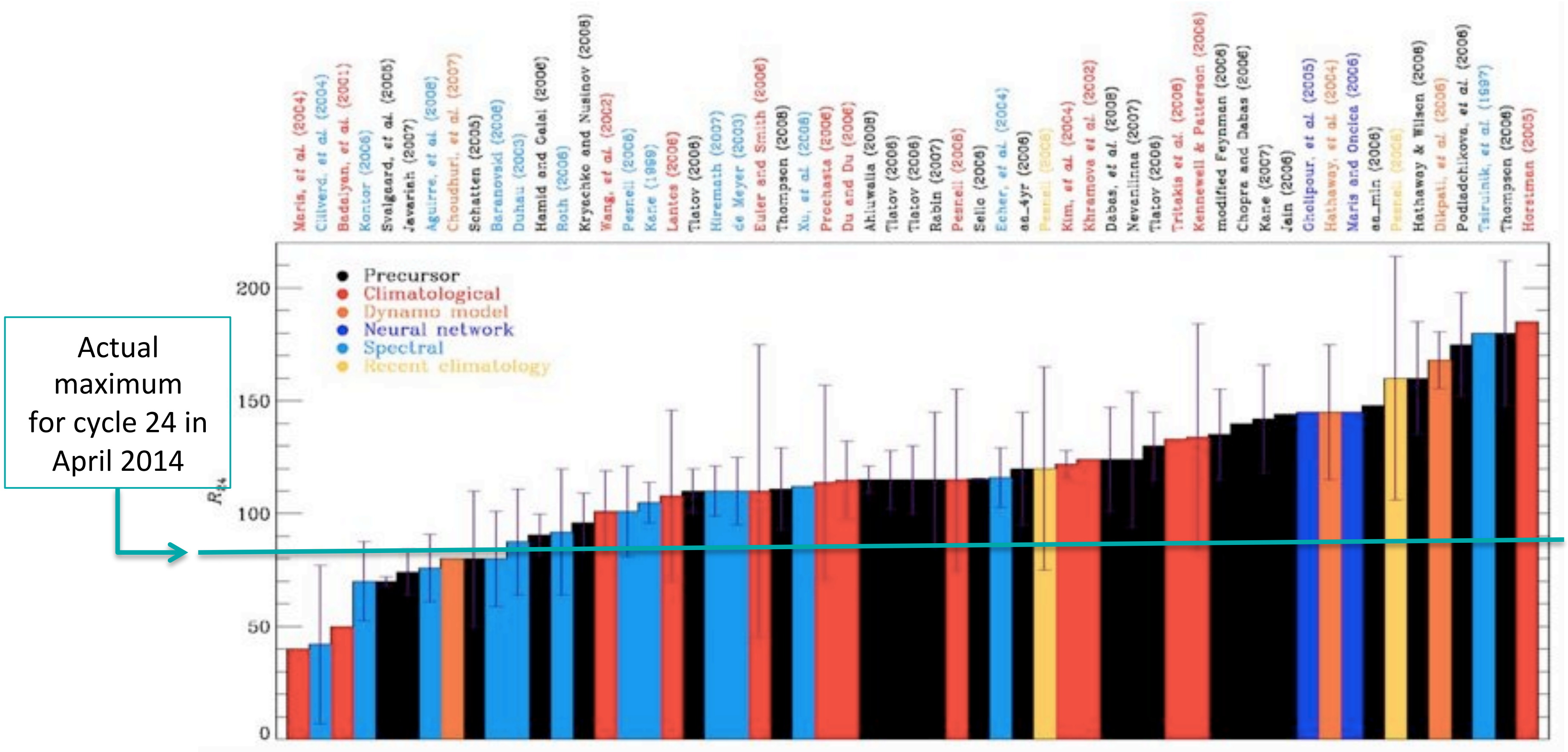
Open question:
Predicting future solar activity?



❑ Why not trying to combine models of solar magnetism and observational data?

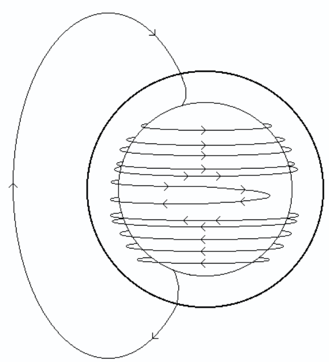
Physics-based predictions: simple mean-field dynamo models

Dynamo mechanism: **process through which motions of a conducting fluid can permanently regenerate and maintain a magnetic field against its ohmic dissipation**

It consists of the regeneration of both poloidal and toroidal fields

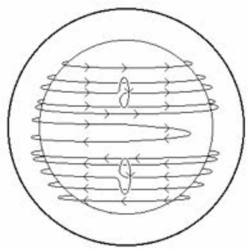
Sources of magnetic field

Poloidal → Toroidal

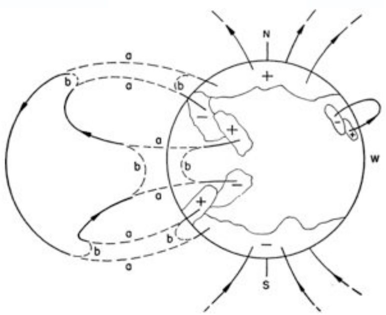


✓ Ω effect

Toroidal → Poloidal



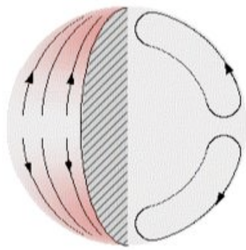
✓ α effect



✓ Babcock-Leighton effect

Transport of magnetic field

✓ Large-scale flows (meridional circulation)

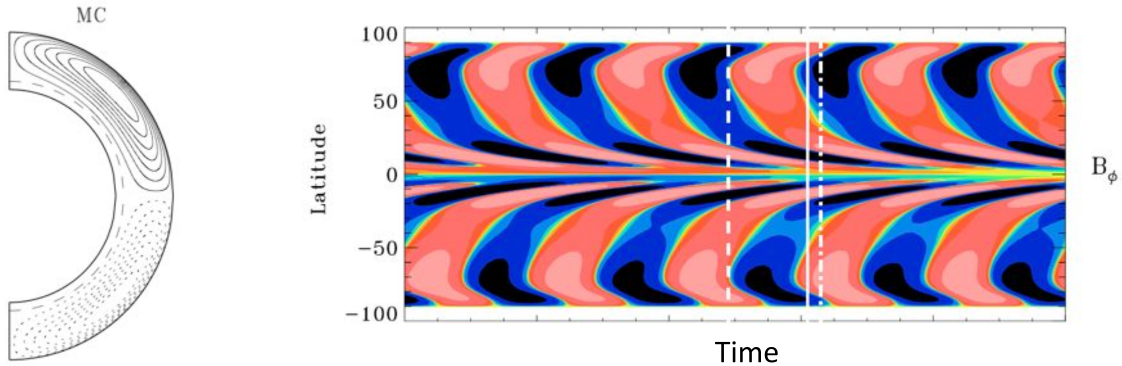


✓ Downward pumping by penetrative convection

✓ Transport from the base of the convection zone to the surface

The Sun: meridional flow internal profile

- Some dynamo models (Babcock-Leighton flux transport) using 1 single cell per hemisphere produce butterfly diagrams in agreement with observations

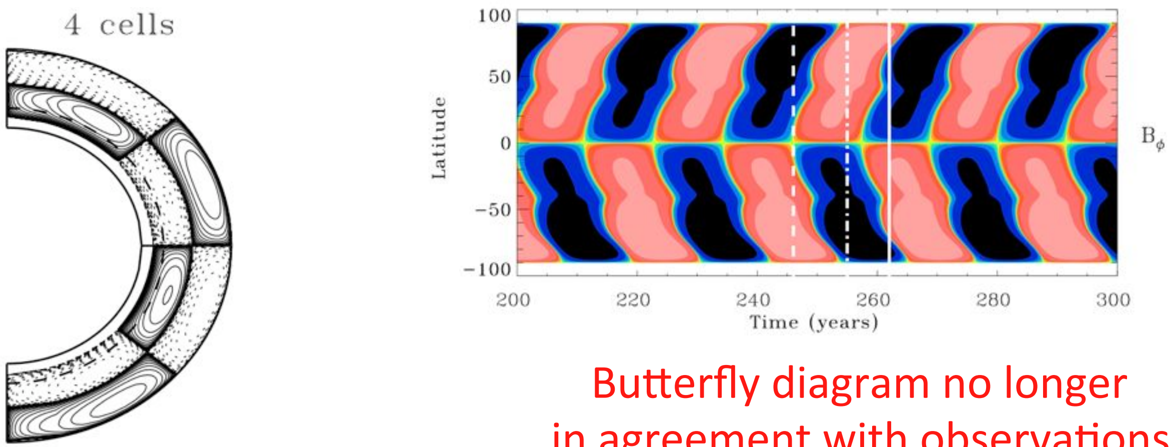


- BUT from observations and simulations, the MC may be multicellular

- If a complex profile persists for the whole cycle, the effect on the magnetic field may be dramatic

Jouve & Brun, 2007

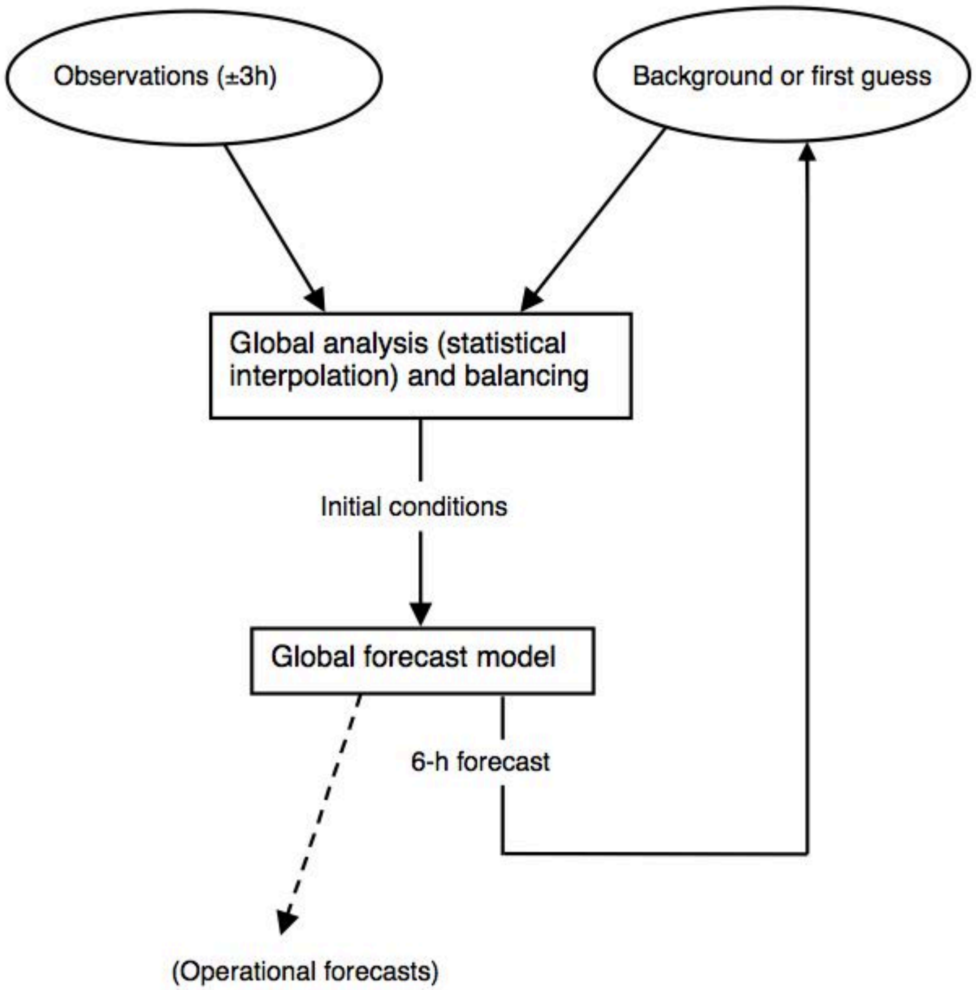
BL model:
MC with 4 cells per hemisphere



Butterfly diagram no longer in agreement with observations

Combining data and models

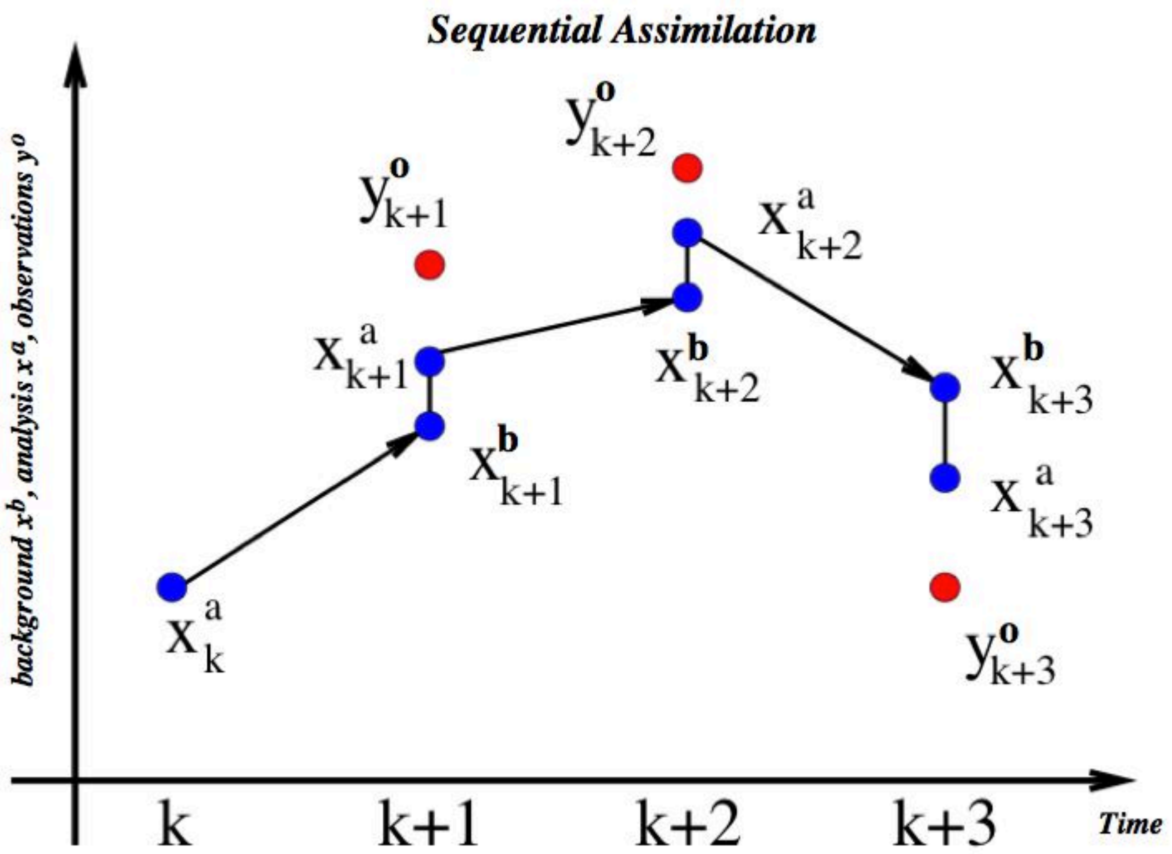
- Drive models with data:
  - Magnetic observations into dynamo models
    (Dikpati et al. 2006, Choudhuri et al. 2007)
  - Active regions into surface flux-transport
    (Schrijver & DeRosa 2003, Cheung & DeRosa 2012)



- Assimilate data into models:
  - Purpose: « using all available information to determine
    as accurately as possible the state
    of the atmospheric or oceanic flow »
    (Talagrand, 1997)
  - Operational for weather forecasting for decades

Credit: E. Kalnay

Sequential data assimilation



- Analysis step

$$x^a{}_k = x^b{}_k + W_k(y_k - H_k x^b{}_k)$$

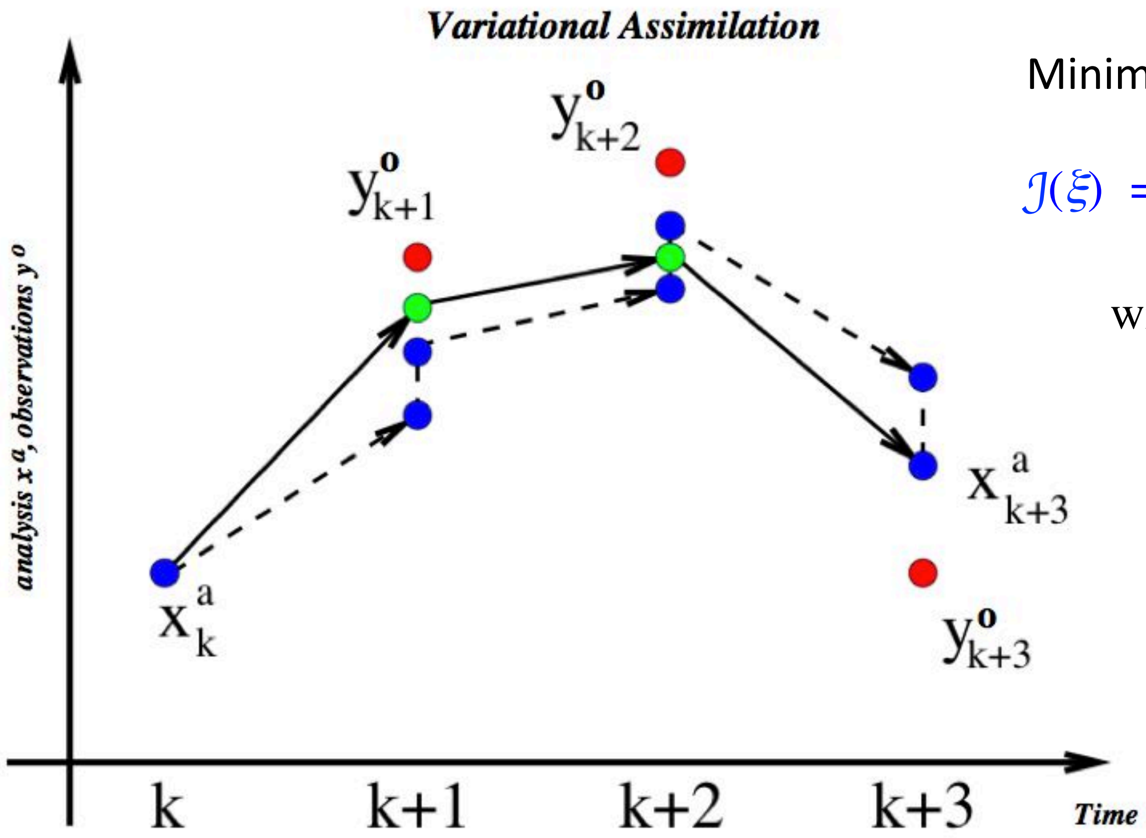
- Forecast step

$$x^b{}_{k+1} = M_k x^a{}_k$$

Propagates information forward in time

❑ Used recently in a simple $\alpha\Omega$ dynamo model (Kitiashvili et al., 2008) and on a BL dynamo model to reconstruct the amplitude of the surface MC by assimilating synthetic magnetic data (Dikpati et al., 2014)

Variational data assimilation



- Different analysis step

Minimize an objective function to get x^a_{K-1}

$$\mathcal{J}(\xi) = (1/2)\,(x^b - \xi)^T [P^b]^{-1} (x^b - \xi) + (1/2)\,\Sigma_k [y_k - H_k\xi_k]^T R_k^{-1} [y_k - H_k\xi_k]$$

with $\xi_{k+1} = M_k\xi_k, \quad k = 0, \ldots, K\text{-}1$

- Forecast step

$$x^b{}_K = M_k x^a{}_{K-1}$$

Propagates information both forward and backward in time

- Used recently in models of solar flares (Bélanger et al. 2007, Strugarek & Charbonneau 2014) and on a simple $\alpha\Omega$ dynamo model to reconstruct the α−effect (Jouve et al. 2011)

An example of Var. DA in dynamo models:

BLFT model in spherical geometry

Hung, Jouve, Brun, Fournier, Talagrand, 2015

- Model equations

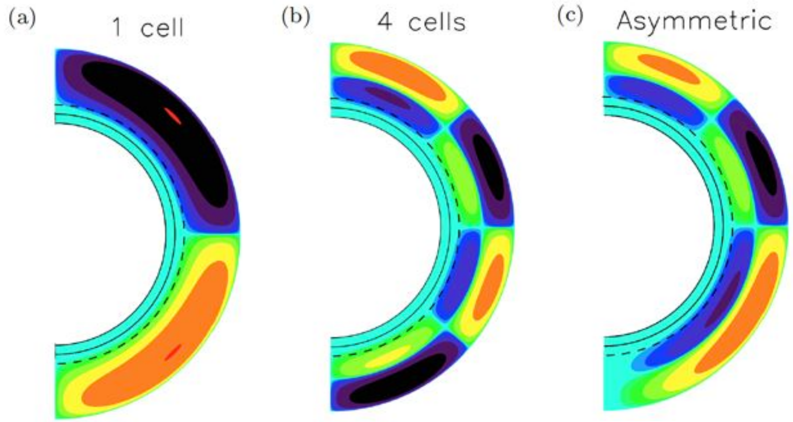
$$\partial_t A_\phi = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right) A_\phi - Re \frac{\vec{v}_p}{\varpi} \cdot \nabla(\varpi A_\phi) + C_s S(r,\theta,B_\phi),$$

$$\partial_t B_\phi = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right) B_\phi + \frac{1}{\varpi}\frac{\partial(\varpi B_\phi)}{\partial r}\frac{\partial(\eta/\eta_t)}{\partial r} - Re\varpi \vec{v}_p \cdot \nabla\left(\frac{B_\phi}{\varpi}\right) - Re B_\phi \nabla \cdot \vec{v}_p + C_\Omega \varpi \left[\nabla \times (A_\phi \hat{e}_\phi)\right] \cdot \nabla\Omega,$$

- Meridional circulation: the main ingredient

$$\vec{v}_p = \nabla \times (\psi \hat{e}_\phi)$$

$$\psi(r,\theta) = -\frac{2(r-r_{mc})^2}{\pi(1-r_{mc})} \times \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n} d_{i,j} \sin\left[\frac{i\pi(r-r_{mc})}{1-r_{mc}}\right] P_j^1(-\cos\theta) & \text{if } r_{mc} \le r \le 1 \\ 0 & \text{if } r_{bot} \le r < r_{mc} \end{cases}$$



m=2, n=4 => 8 coefs $d_{i,j}$

Case	$d_{1,2}$	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	$d_{2,4}$
1	3.33×10^{-1}	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	9.47×10^{-2}
3	0.00	-5.74×10^{-2}	-8.75×10^{-2}	-3.83×10^{-2}	5.47×10^{-2}

An example of Var. DA in dynamo models:

Twin experiments

- We produce synthetic observations with a given MC and input parameters

- We noise the data (normal distribution with std=percentage of σ)

- We choose a cost function to be minimized:

$$\mathcal{J}_A = \sum_{i=1}^{N_t^o} \sum_{j=1}^{N_\theta^o} \frac{\left[A_\phi(R_s, \theta_j, t_i) - A_\phi^o(R_s, \theta_j, t_i)\right]^2}{\sigma_{A_\phi}^2(R_s, \theta_j)}, \quad \mathcal{J}_B = \sum_{i=1}^{N_t^o} \sum_{j=1}^{N_\theta^o} \frac{\left[B_\phi(r_c, \theta_j, t_i) - B_\phi^o(r_c, \theta_j, t_i)\right]^2}{\sigma_{B_\phi}^2(r_c, \theta_j)},$$

- Initial guess for the minimization procedure: a 1 cell MC
- Initial conditions for direct code: magnetic field produced by this 1 cell model

- We minimize the cost function by adjusting the control vector $d_{i,j}$

- The diagnostic quantities:

$$\frac{\Delta p}{p} = \sqrt{\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (d_{i,j} - d_{i,j\,true})^2}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{i,j\,true}^2}}, \qquad \mathcal{J}/\mathcal{J}_o \quad \text{and} \quad \mathcal{J}_{norm} = \frac{1}{\epsilon}\sqrt{\frac{\mathcal{J}}{N}}, \quad \text{when noisy data}$$

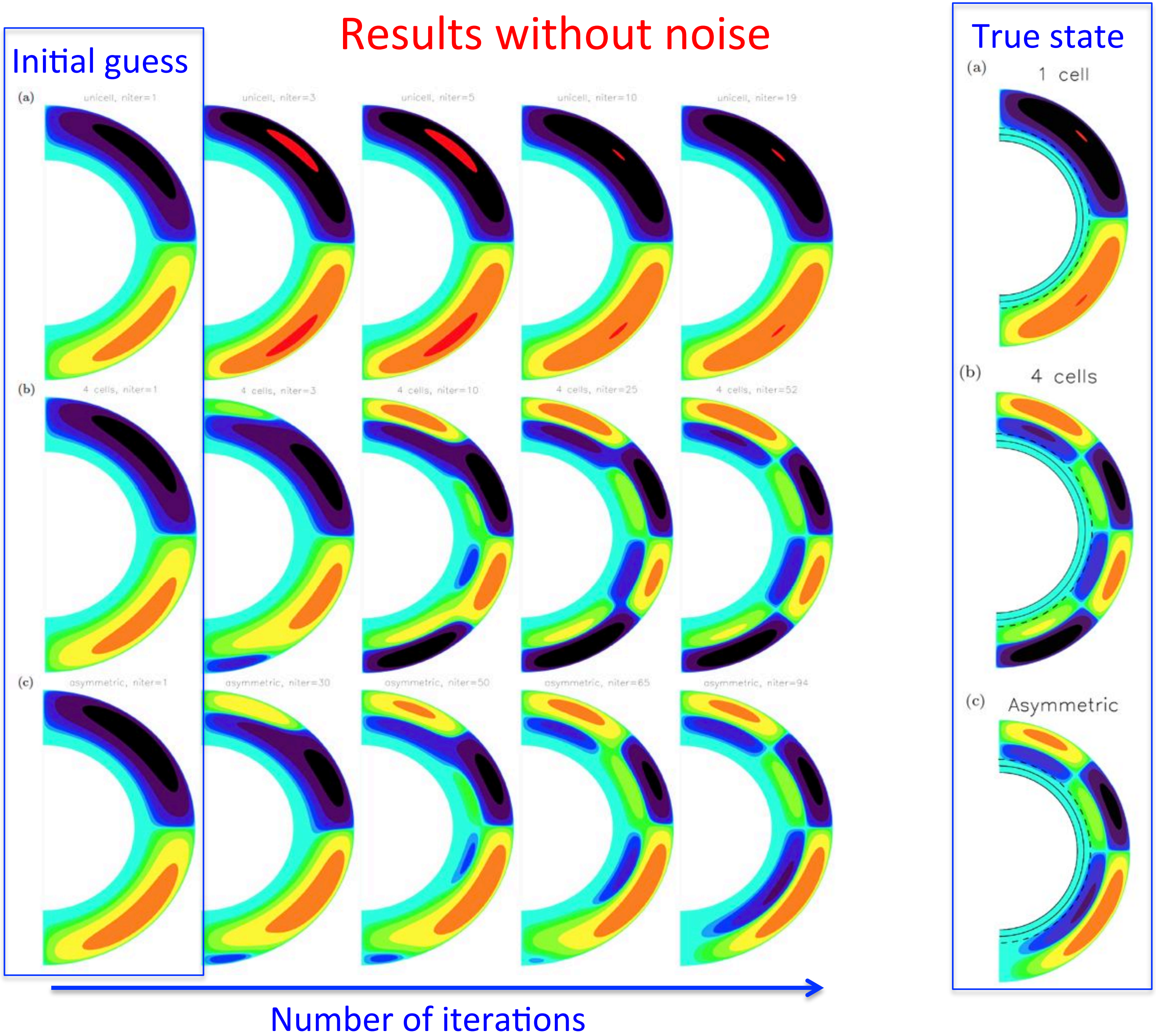
An example of Var. DA in dynamo models:

Results without noise

1 cell model

4 cell model

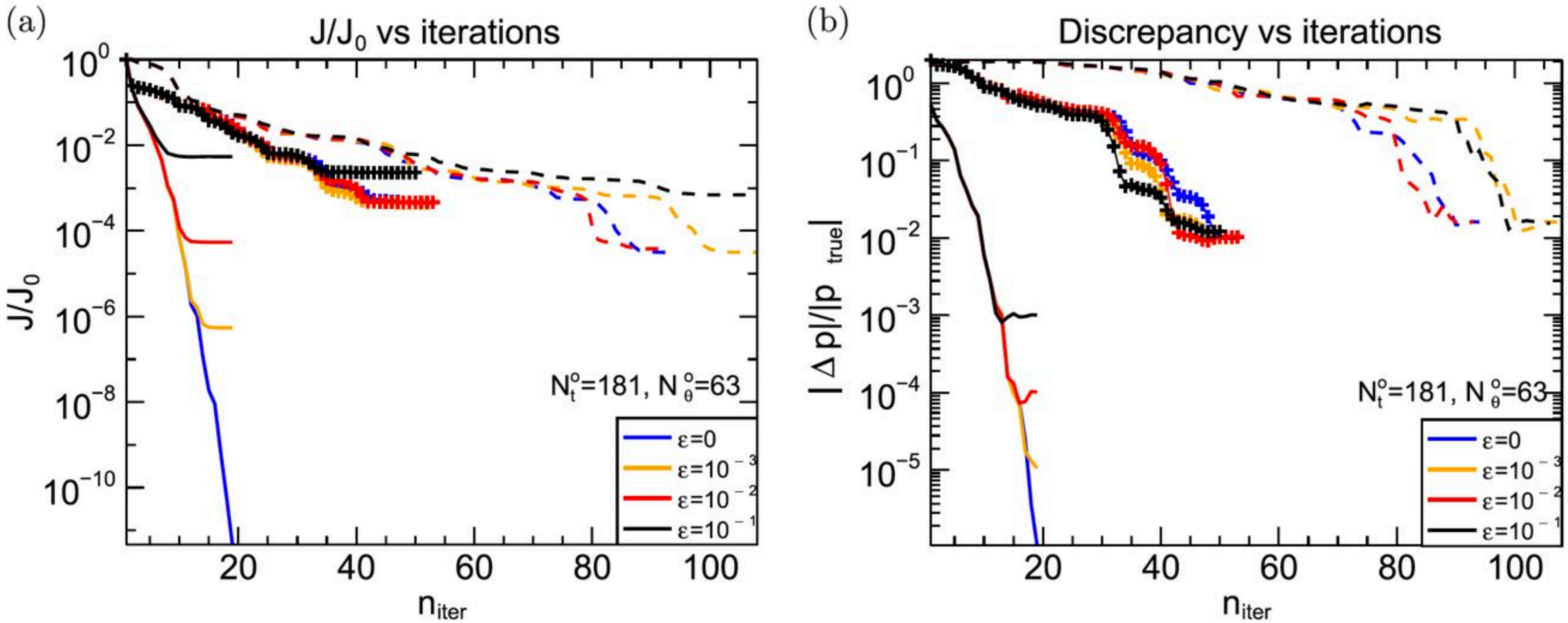
Asymmetric model



An example of Var. DA in dynamo models:

Results with noise

- ❑ Results for various noise levels, uniform sampling

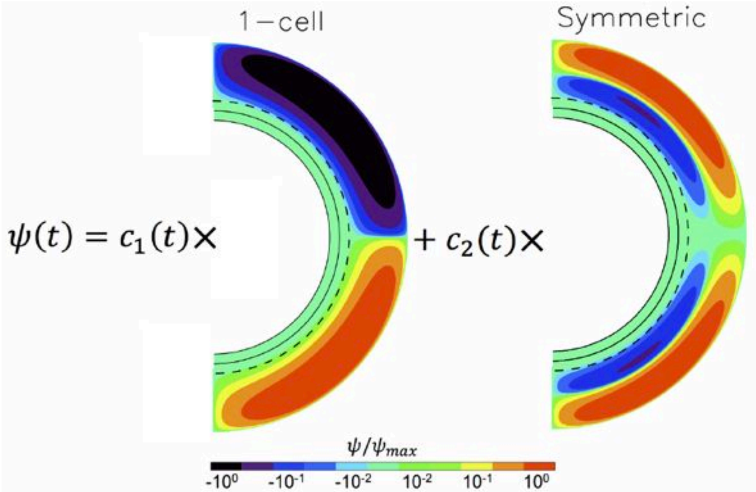


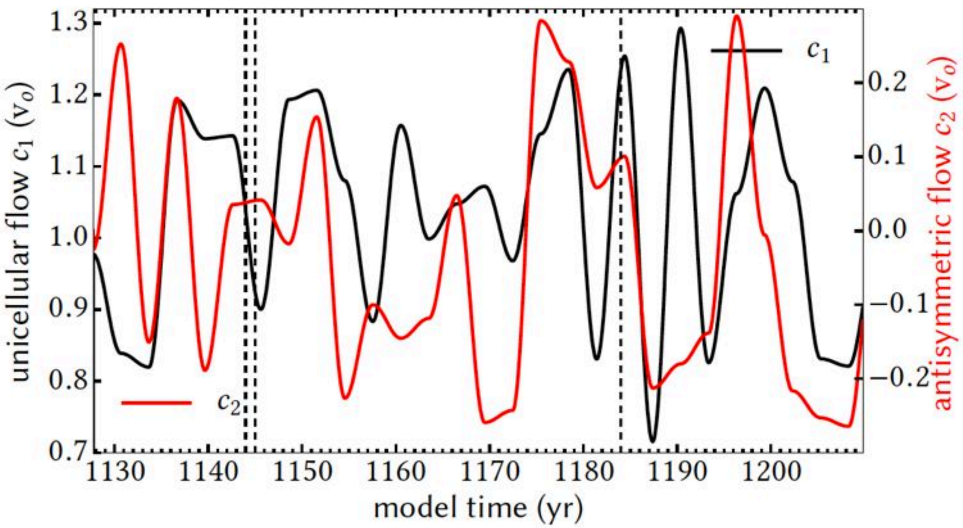
- ❑ Magnetic field recovery for 30% noise for the 4-cell and the asymmetric cases:

  - 4 cell: Aphi2
  - Asymmetric: Aphi3

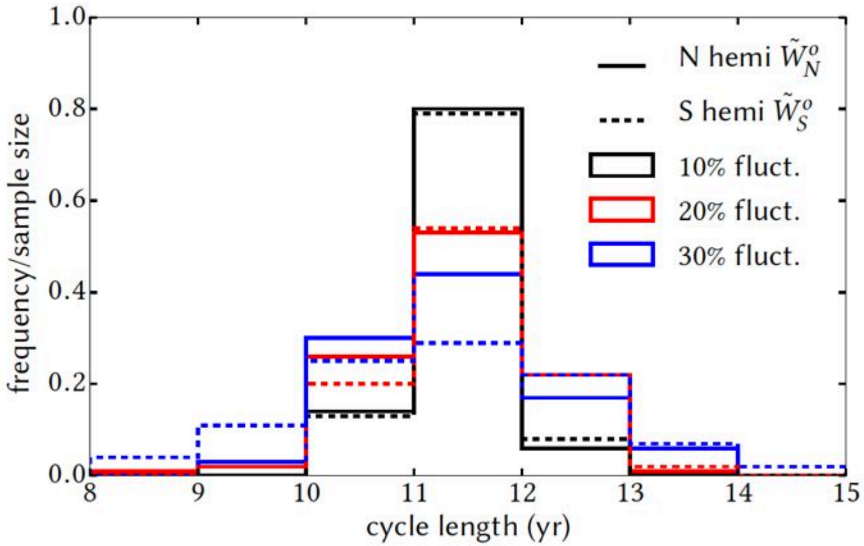
A more complete model:
time-varying meridional flow

- ❑ More challenging attempt: recover a MC with time-varying amplitude and profile





- ❑ Produces a modulation in the cycle period and amplitude



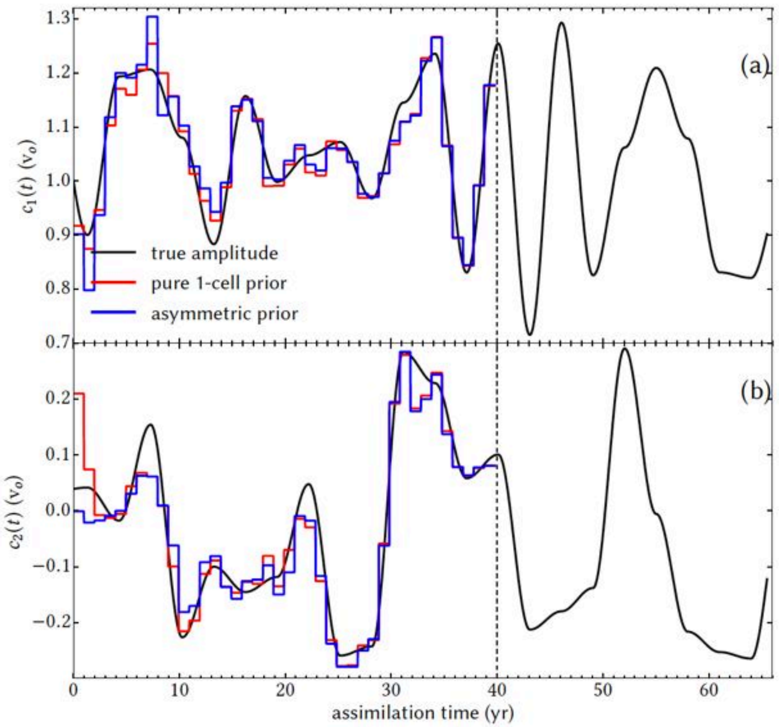
Hung, Brun, Fournier, Jouve, Talagrand, submitted

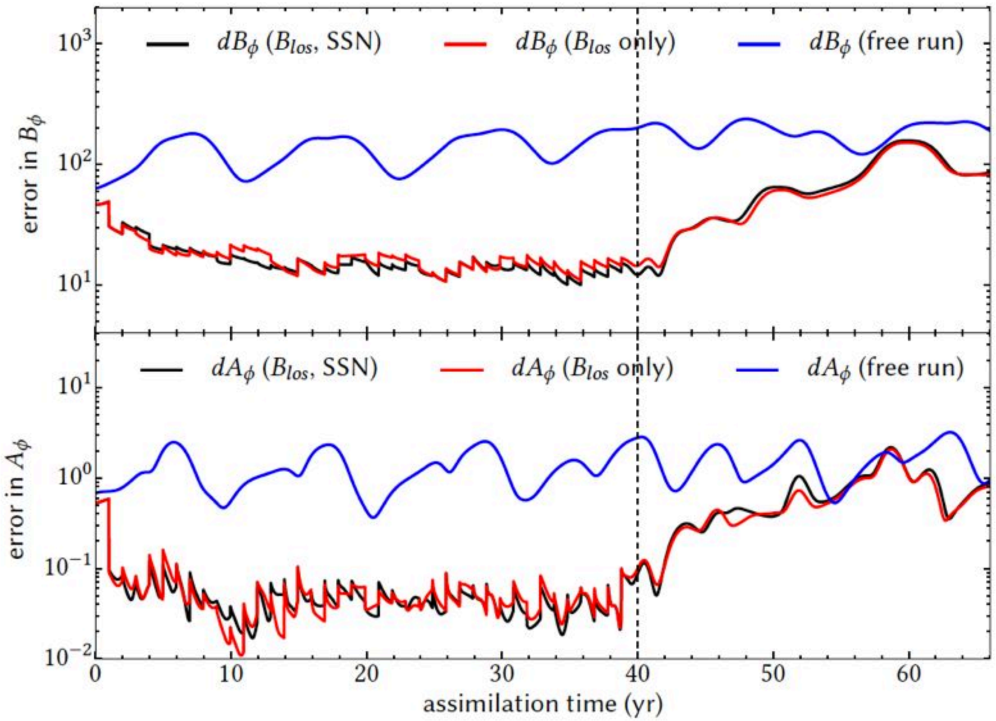
A more complete model:
analysis step

- ❑ Variational DA is performed every year for 40 years where data is available

- ❑ A 10% noise is added to the synthetic data (proxy for SSN and surface magnetic field)

- ❑ The MC is recovered to an accuracy of more than 90%
- ❑ The error on the reconstructed magnetic field is less than 10%





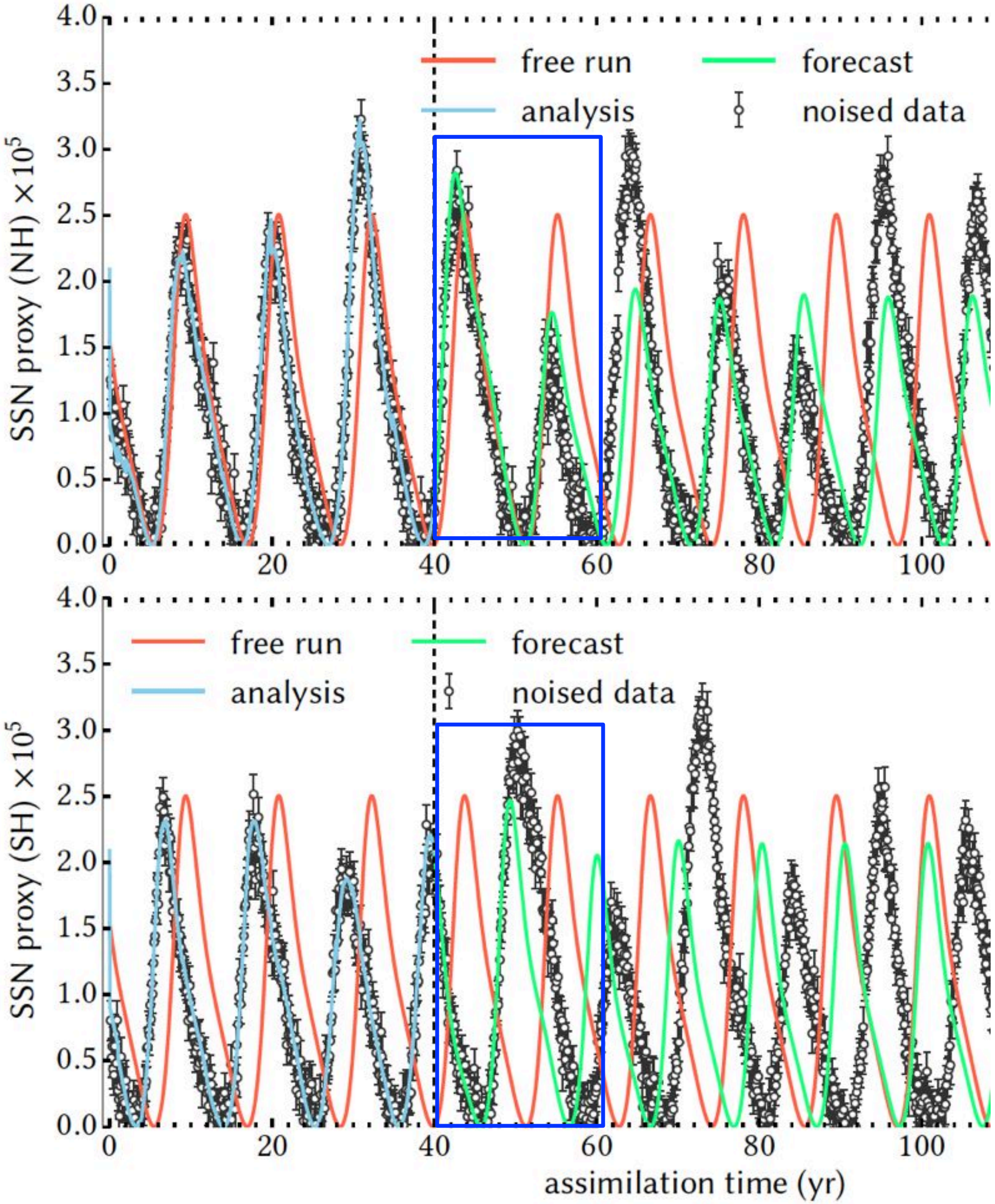
A more complete model:
forecast step

- We produce a forecast for the next solar cycles in our model after 40yrs of assimilation

- The predictability horizon is of the order of 1 to 2 cycles.

Northern hemisphere

Southern hemisphere



Conclusions and perspectives

- A proof of concept is established: we can make use of DA in solar physics, sequential as well as variational assimilation

- DA may be used to infer potentially important ingredients of dynamo models: Hung et al. use a polar coordinate model with a time-varying meridional circulation and recover both its amplitude and profile from noised magnetic data

- The model was used to produce the data (twin experiments), we now wish to move to real observations and actual predictions (for cycle 25?)

- Longer-term:
  Apply data assimilation techniques to a full spherical 3D MHD models
  (many get large scale regular magnetic cycles now, e.g. Ghizaru, Brown, Augustson, Gastine, Käpylä, Warnecke, Hotta, Fan,…)
  and automatic differentiation algorithms exist to get the adjoint code!
  http://www-tapenade.inria.fr:8080/