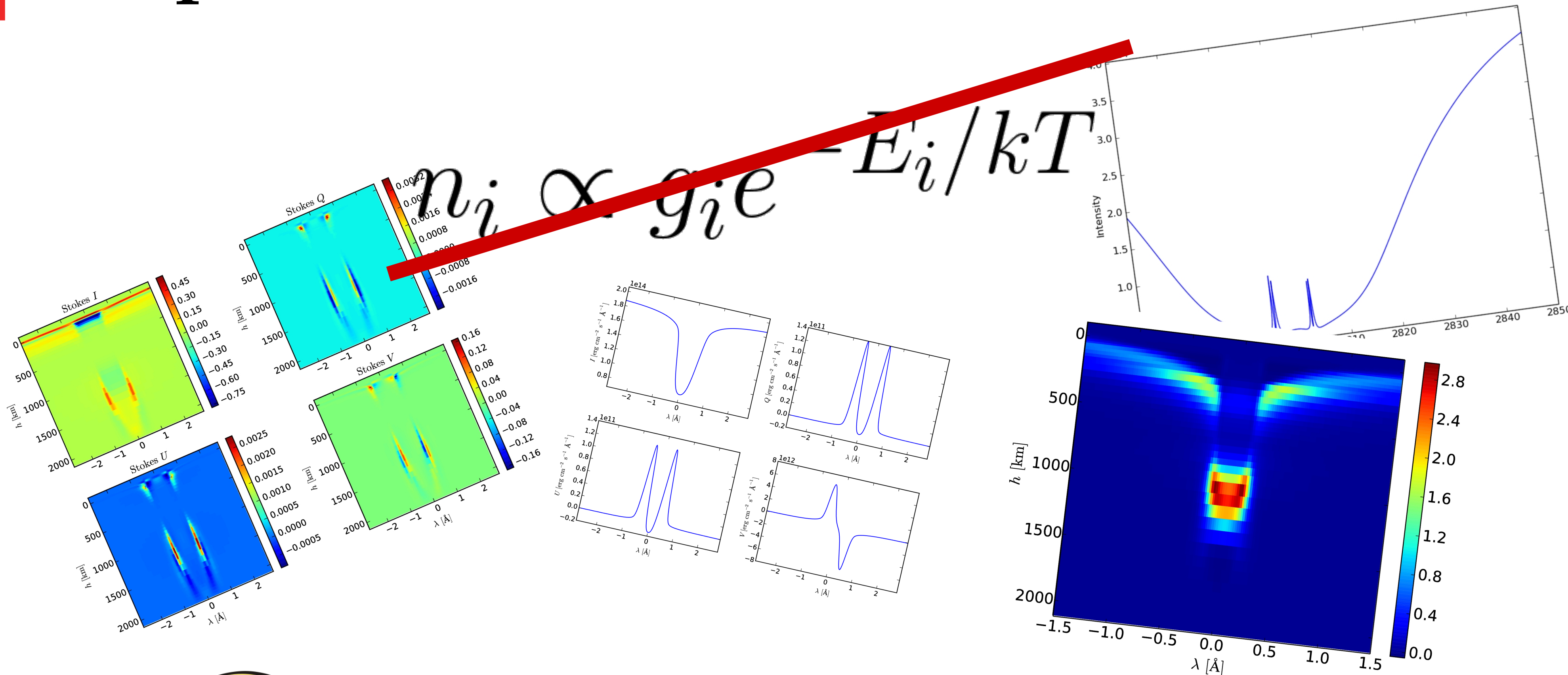
Response functions for NLTE lines

$$n_i \propto g_i e^{-E_i/kT}$$



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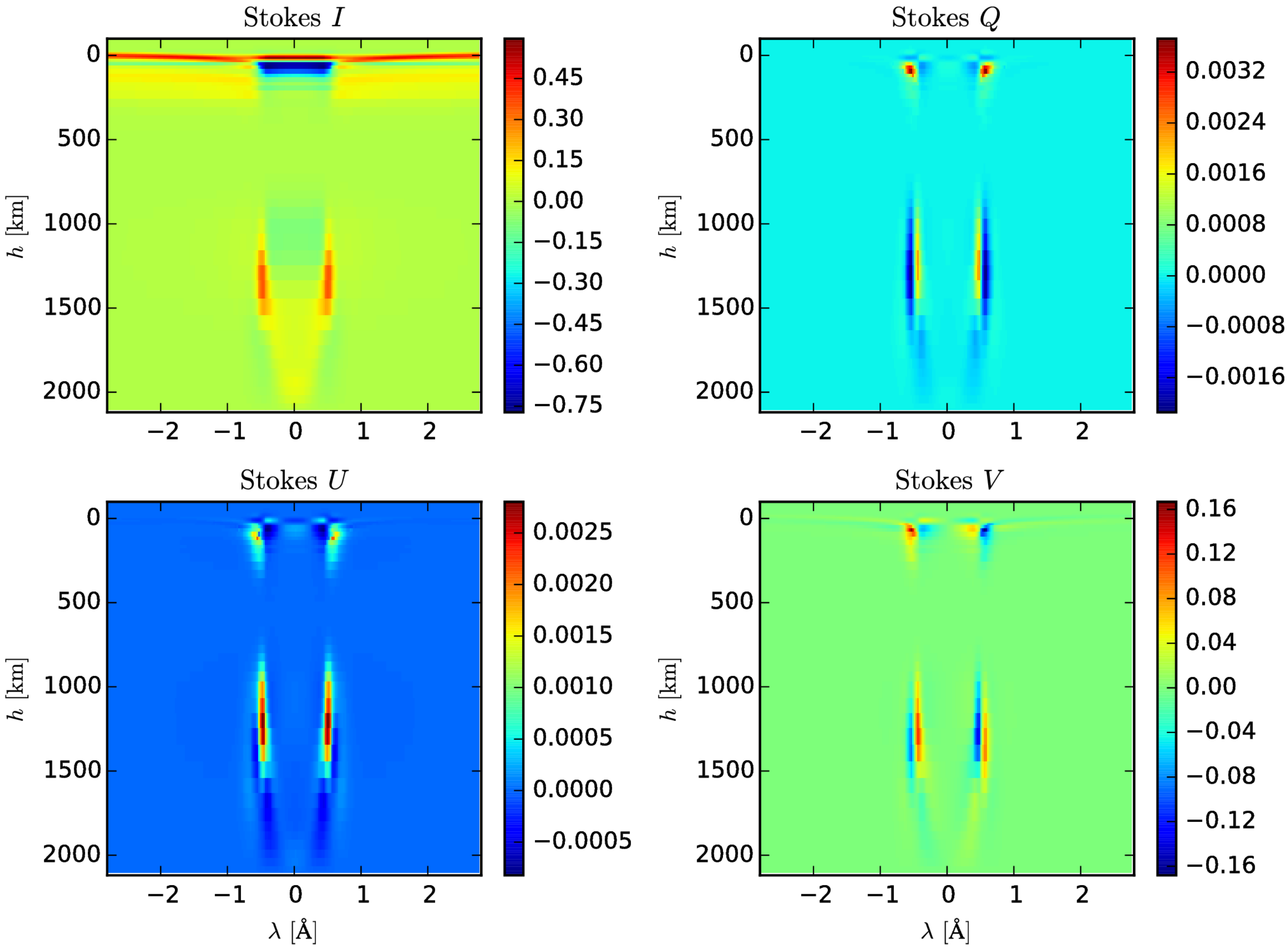
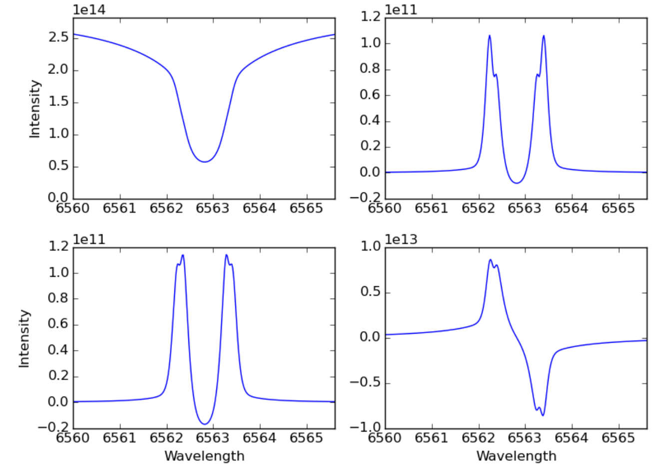
Main messages:

- We present an accurate and fast method for computing level population responses in NLTE
- This method could speed up the inversion process by an order of magnitude
- *Idea: A s*imple form of Hanle inversion can be easily devised from scalar NLTE inversion
- Anisotropy response function gives us better height span than the one for intensity alone

Response functions





Response function to temperature for a prototype 2-level NLTE line. 1000 G magnetic field.

Response functions

$$R_{q_k} = \frac{dI^+}{dq_k}$$

They tell us how the emergent intensity responds to perturbations of different physical paramters at different depths

$$\frac{d\hat{I}}{ds} = -\hat{K}\hat{I} + \hat{j}$$

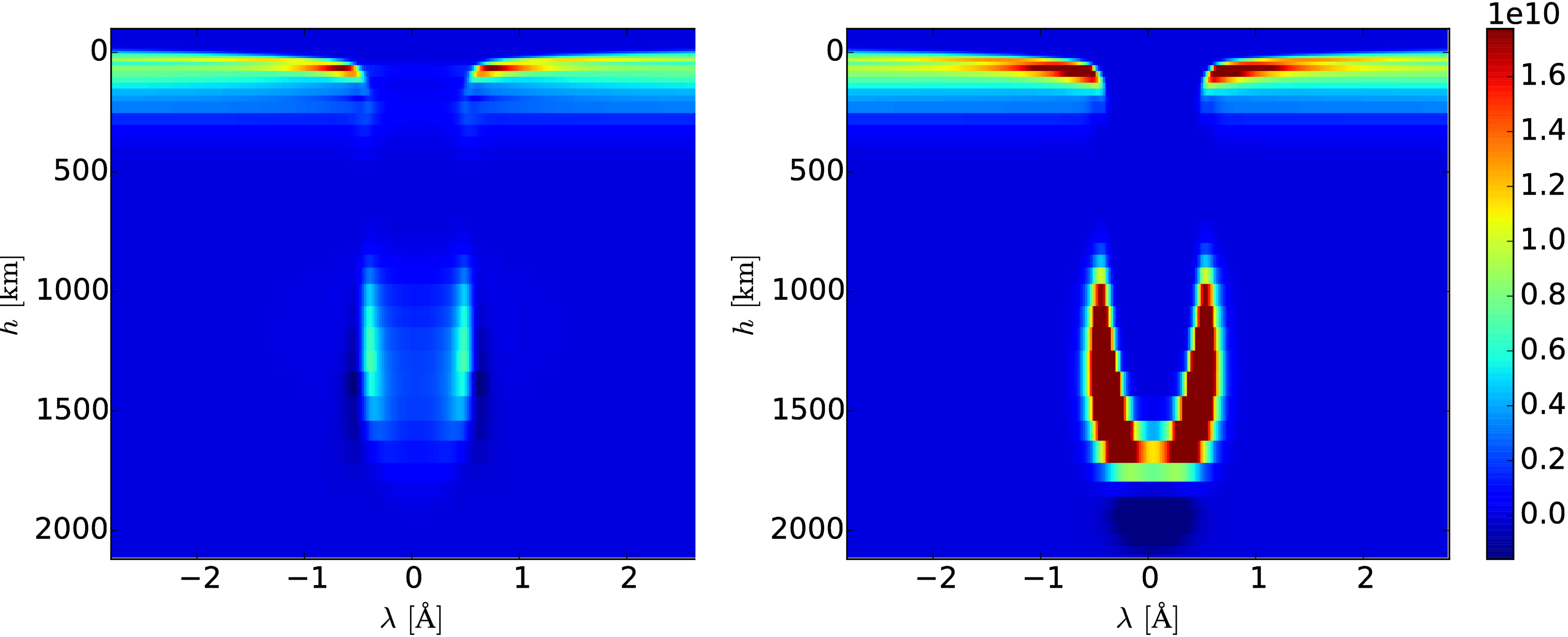
Usual approach (e.g. SIR, SPINOR):

- Compute perturbations in absorption/dispersion/emission terms
- Propagate them analyticaly to the top
- Done!

Problems in NLTE:

- Level populations are not explicitly given
- Dependencies are non-local and non linear
- So are the responses
- How to proceed?

NLTE vs LTE



Response function to temperature for a prototype 2-level NLTE line. Unpolarized (isotropic) case.

The key is computing the population responses

$$n_i \propto g_i e^{-E_i/kT}$$

Statistical equilibrium instead of Saha-Boltzmann equation!

$$\frac{dn_i}{dt} = \sum_j n_j R_{ji} - n_i R_{ij} = 0$$

$$\frac{dR_{ij}}{dq_k} = \frac{\partial R_{ij}}{\partial J_{ij}} \frac{dJ_{ij}}{dq_k} + \frac{\partial R_{ij}}{\partial q_k}$$

All the non-locallity and non-linearity is here.

$$J_{ij} = \int \phi_{ij}(\lambda)\, d\lambda \oint \frac{d\Omega}{4\pi} I(\Omega, \lambda)$$

And now (bear with me):

$$\frac{dR_{ij}}{dq_k} = \frac{\partial R_{ij}}{\partial J_{ij}} \frac{dJ_{ij}}{dq_k} + \frac{\partial R_{ij}}{\partial q_k} \qquad J_{ij} = \int \phi_{ij}(\lambda)\, d\lambda \oint \frac{d\Omega}{4\pi} I(\Omega, \lambda)$$

$$I_l = \int_0^\infty \mathcal{O}(\tau) S(\tau) d\tau = \Lambda[\chi, j]$$

$$\frac{dI_l}{dq_k} = \sum_{l'} \sum_{i'} \left[\frac{\partial I_l}{\partial \chi_{l'}} \frac{\partial \chi_{l'}}{\partial n_{l'}} + \frac{\partial I_l}{\partial j_{l'}} \frac{\partial j_{l'}}{\partial n_{l'}} \right] \frac{dn_{i'}}{dq_k} + r_{l',i'}$$

Which leads to a linear system of ND x NL equations:

Final linear system

$$\begin{pmatrix} T_{1,1,1,1} + N_{1,1,1,1} & \cdot & \cdot & \cdots & N_{1,N_D,1,N_L} \\ \cdot & & & & \\ \cdot & T_{l,l,i,i} + N_{l,l',i,i'} & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & N_{l,l',i,i'} & & & \\ \cdot & & & & \end{pmatrix} \hat{\mathcal{R}} = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ b_{i,l,k} \\ \cdot \\ \cdot \end{pmatrix}$$

Local terms (surviving from SE equation)

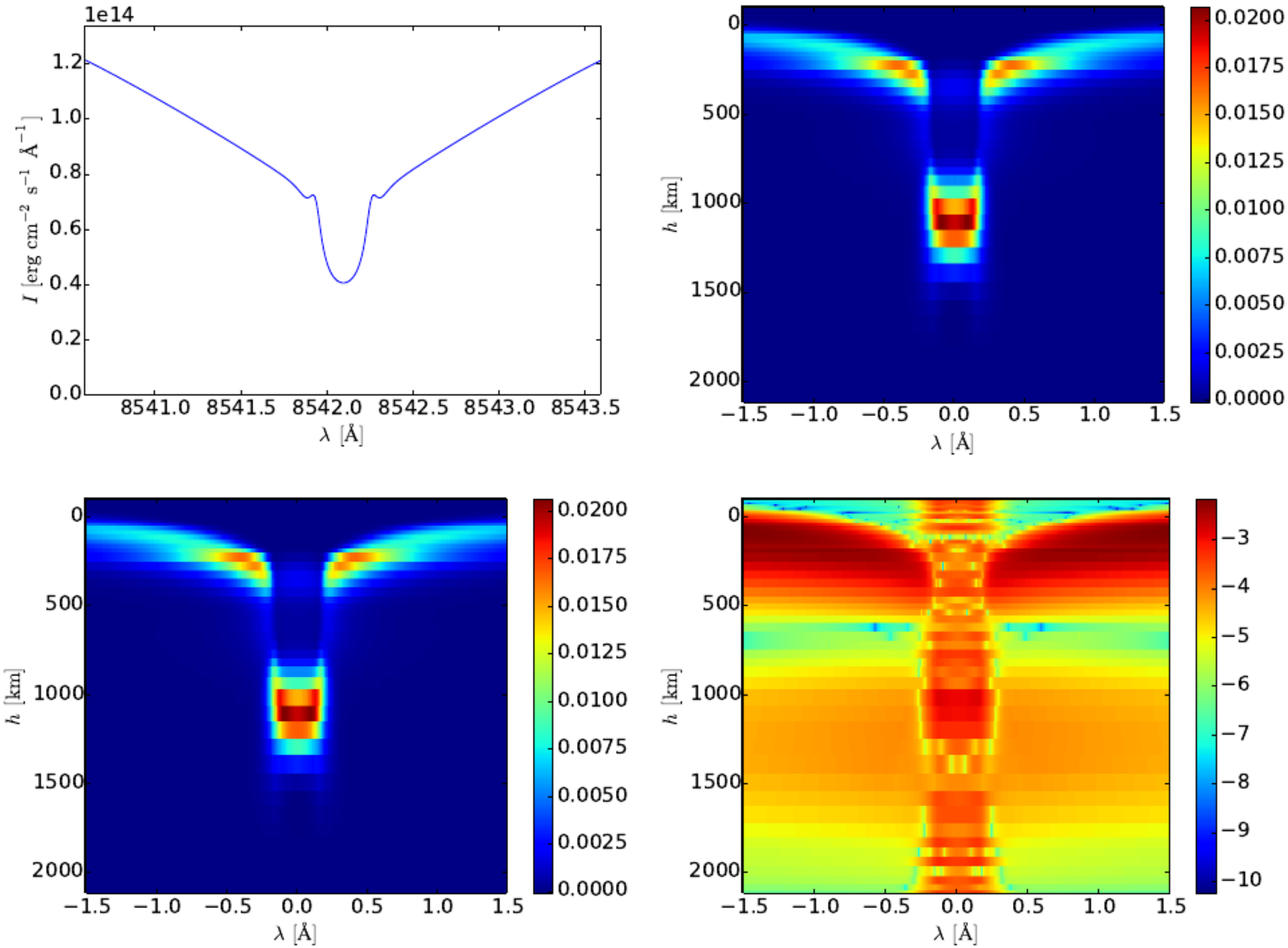
Level responses

Non-local, radiative (a la lambda operator) coupling

All perturbations which are **not** due to level perturbations (known stuff)

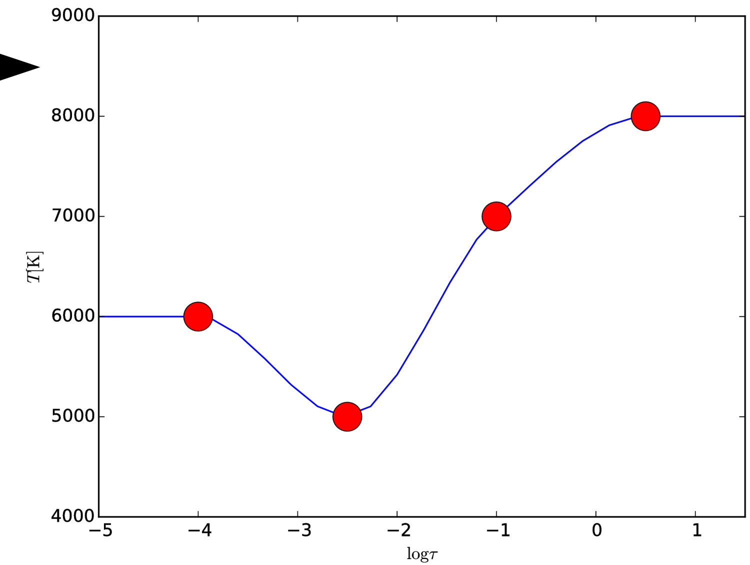
Does it work?

- A simplified example of CaII 8542 (5 levels, LTE continuum opacity/emissivity, no magnetic field)
- Fast: Same as one NLTE solution
- Accurate and, in the cases considered- robust



Advantages

- Main application are inversion codes. →

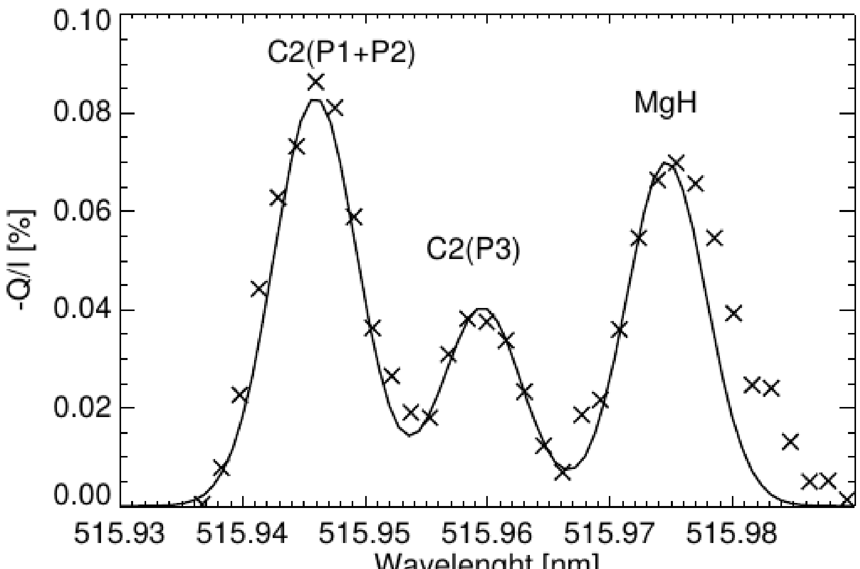


- Numerical responses to nodes take time of ~ 10 NLTE solutions → we save an order of magnitude
- Coupling matrix does not depend on perturbation in question → we can perturb and compute whatever we want
- Studying responses themselves is interesting and this way we can do it quickly (i.e. for MHD cubes)

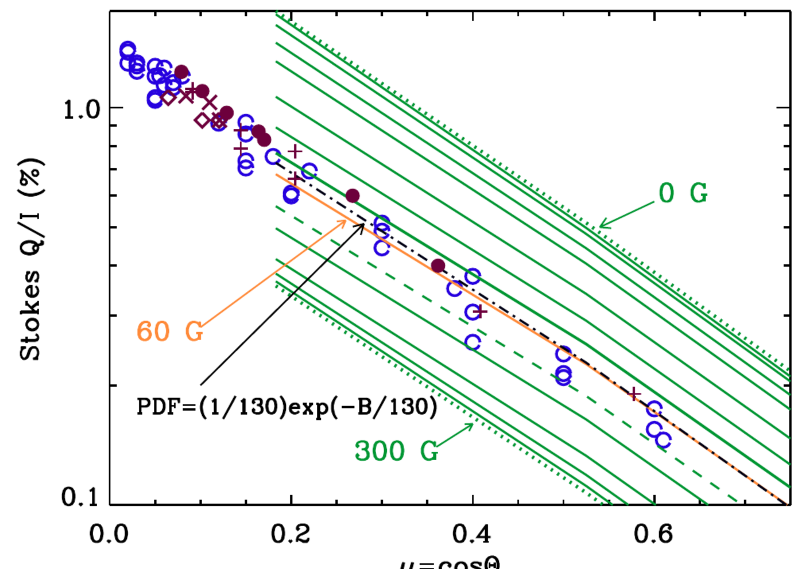
Application to scattering line polarization

Hanle diagnostics so far (unresolved observations):

- Compute anisotropy & collisional depolarization
- Fit to get magnetic field.



Milic & Faurobert, 2012



Trujillo Bueno et al, 2004

Application to scattering line polarization

Hanle diagnostics for resolved observations:

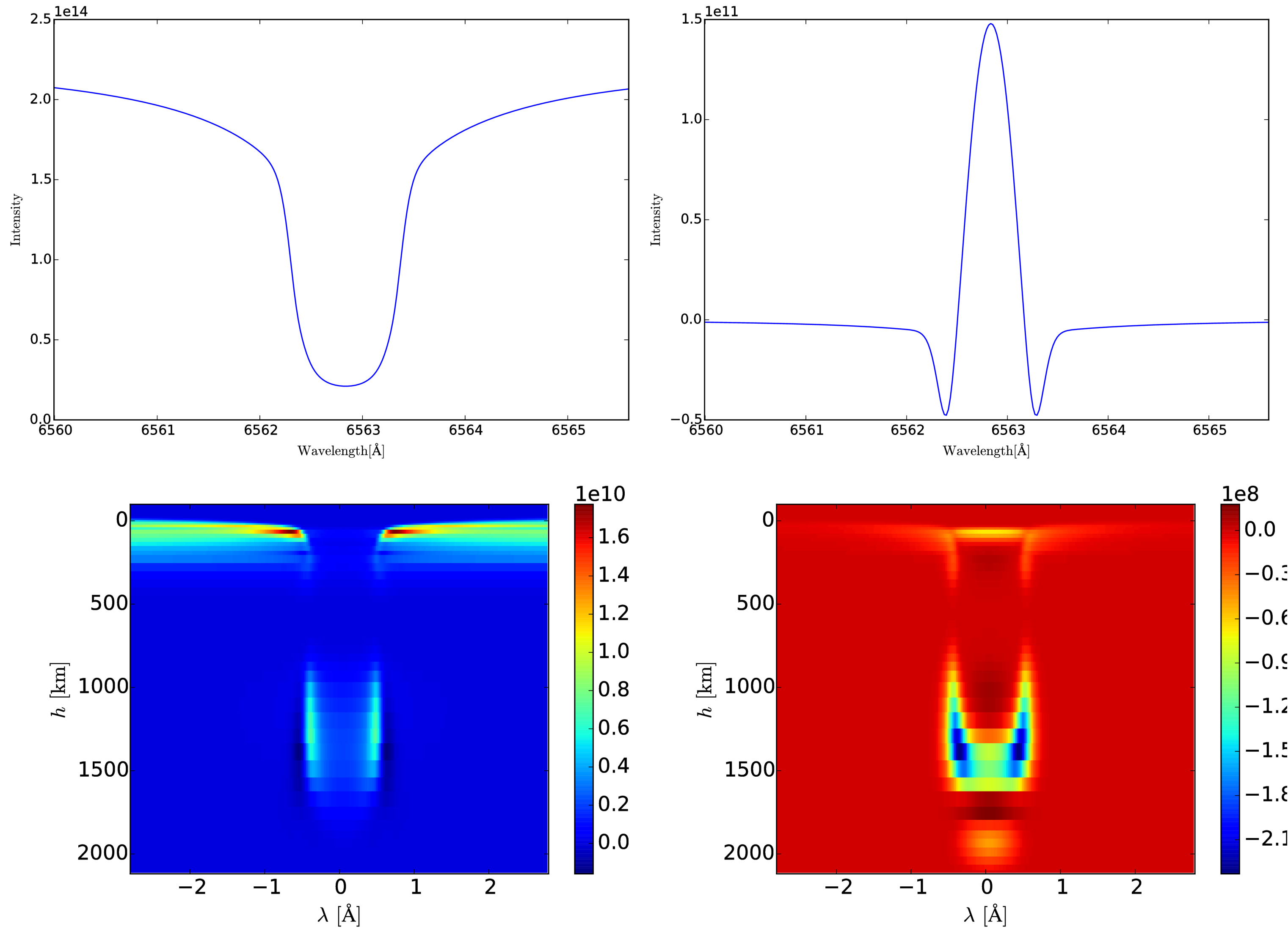
- **Invert, in NLTE, intensity profile.**
- Compute anisotropy & collisional depolarization
- Fit to get magnetic field.

Let's try and compute response functions for scattering polarization!

$$S_Q \approx (1-\mu^2)\frac{3}{8}\iint (3\mu'^2-1)I(\mu',\lambda)\phi(\lambda)d\lambda d\mu'$$

Scattering polarization responses

- Strong, scattering dominated line, formed high in the atmosphere.
- Polarization responses more nor local, and more „extended“
- Better diagnostics?



Main messages (again):

- We present an accurate and fast method for computing level population responses in NLTE
- These can speed up the inversion process by an order of magnitude, and in preliminary testing work well.
- This approach straightforwardly leads to anisotropy responses and thus to the first approximation of the response of scattering polarization.
- *Questions? Critics? Comments?*