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Interpreting a millennium solar-like dynamo with the test-field method

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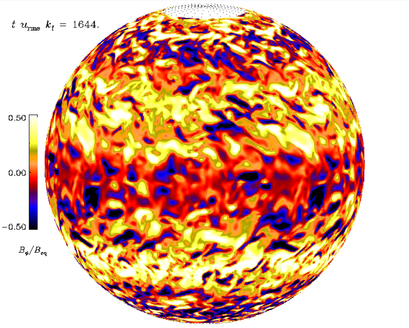
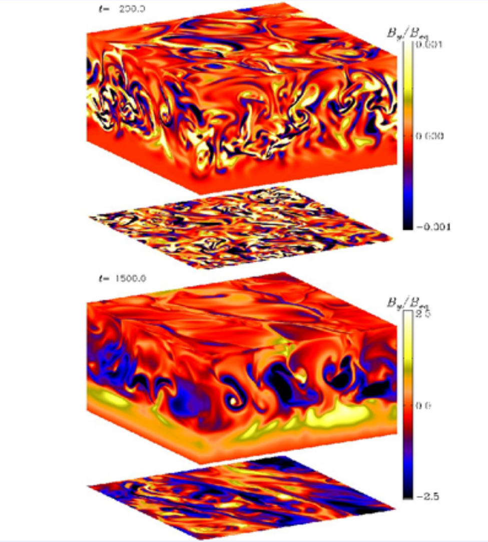
”millenium” solar-like dynamo





Global spherical convection dynamo e.g. (Käpylä et al. 2013, Käpylä et al. 2015)

”millenium” solar-like dynamo

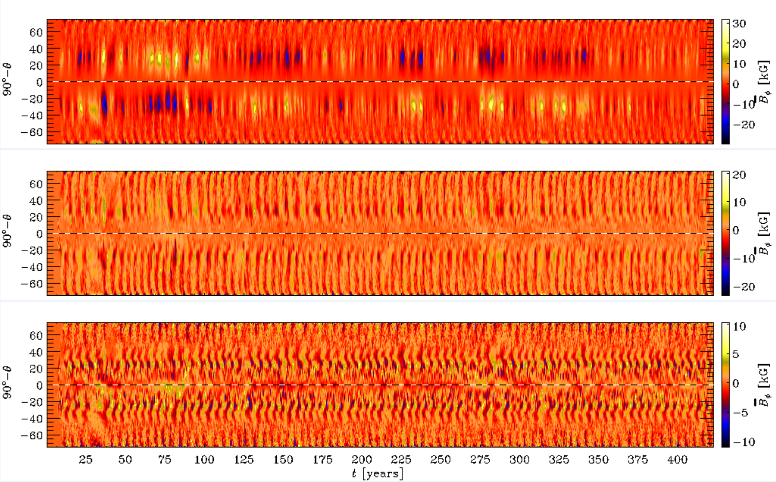


Figure: B_ϕ averaged azimuthally as function of latitude over time - layers near the base, middle and surface of the convection zone. Time derived by $5\Omega_\odot / R_\odot$, for a solar size star rotating 5x solar rate

model MHD equations

$$\frac{D\ln\rho}{Dt} = -\nabla\cdot\boldsymbol{U}, \tag{1}$$

$$\frac{D\boldsymbol{U}}{Dt} = \boldsymbol{g} - 2\Omega_0\times\boldsymbol{U} + \frac{1}{\rho}\left(\boldsymbol{J}\times\boldsymbol{B} - \nabla p + \nabla\cdot 2\nu\rho\mathbf{S}\right), \tag{2}$$

$$T\frac{Ds}{Dt} = \frac{1}{\rho}\left[-\nabla\cdot\left(\boldsymbol{F}^{\mathrm{rad}} + \boldsymbol{F}^{\mathrm{SGS}}\right) + \mu_0\eta\boldsymbol{J}^2\right] + 2\nu\mathbf{S}^2, \tag{3}$$

$$\boldsymbol{F}^{\mathrm{rad}} = -K\nabla T \quad \text{and} \quad \boldsymbol{F}^{\mathrm{SGS}} = -\chi_{\mathrm{SGS}}\rho T\nabla s \tag{4}$$

are heat fluxes, radiative and SGS (sub grid scale - numerical stability)

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$$\frac{\partial\boldsymbol{A}}{\partial t} = \boldsymbol{U}\times\boldsymbol{B} - \mu_0\eta\boldsymbol{J}, \tag{5}$$

model MHD equations - symbols

$\boldsymbol{A}$	magnetic vector potential
$\boldsymbol{U}$	velocity
$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$	magnetic field
$\boldsymbol{J} = \mu_0^{-1} \boldsymbol{\nabla} \times \boldsymbol{B}$	current density
μ_0	vacuum permeability
$D/Dt = \partial/\partial t + \boldsymbol{u} \cdot \boldsymbol{\nabla}$	material derivative
$\mathbf{S}$	rate of strain tensor
ρ	density
ν	kinematic viscosity
η	magnetic diffusivity
K	radiative heat conductivity
$\chi_{\rm SGS}$	turbulent heat conductivity (unresolved convective transport of heat)
s	specific entropy
T	temperature
p	pressure

Ideal gas law: $p = (c_{\rm P} - c_{\rm V})\rho T$, where adiabatic index $\gamma = c_{\rm P}/c_{\rm V} = 5/3$.

long term variation of magnetic cycle

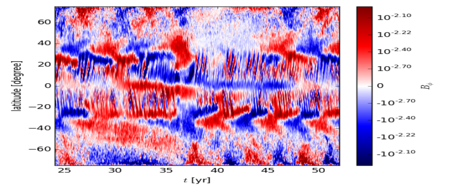

Figure: $\langle B_\phi \rangle_\phi$ near surface of the convection zone during grand minima south then north.

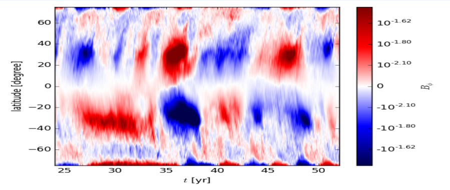

Figure: $\langle B_\phi \rangle_\phi$ near base of the convection zone during grand minima south then north.

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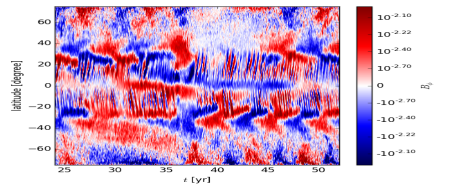

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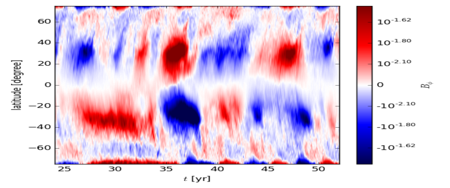

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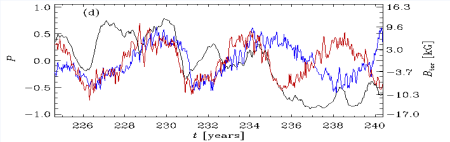

Figure: Parity (black) and $\langle B_\phi \rangle_\phi$ near surface (N:blue, S:red) at $\pm 25^\circ$ latitude, during high state of base toroidal mode

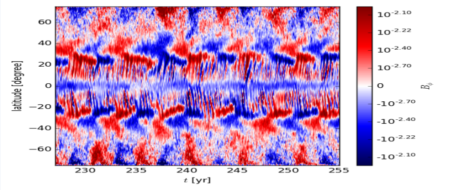

Figure: $\langle B_\phi \rangle_\phi$ near the surface of the convection zone during switch from N-S symmetry to asysmmetry.

mean field concept

(Krause & Rädler 1980) expressed the induction equation in terms of the *mean field* (e.g. azimuthal average) such that $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$ and $\boldsymbol{U} = \overline{\boldsymbol{U}} + \boldsymbol{u}$.

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$$\frac{\partial}{\partial t}(\overline{\boldsymbol{B}} + \boldsymbol{b}) = \nabla \times (\overline{\boldsymbol{U}} + \boldsymbol{u}) \times (\overline{\boldsymbol{B}} + \boldsymbol{b}) + \eta \nabla^2 (\overline{\boldsymbol{B}} + \boldsymbol{b}), \qquad (6)$$

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where $\boldsymbol{G} = \boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}$

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The EMF $\mathcal{E} = \overline{\boldsymbol{u} \times \boldsymbol{b}}$ in curvelinear coordinates can be expressed

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How might we determine these coefficients?

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$$\mathcal{E}^{(i)} = \tilde{a}_{jk}\overline{\boldsymbol{B}}_{T_k}^{(i)} + \tilde{b}_{jkr}\frac{\partial \overline{\boldsymbol{B}}_{T_k}^{(i)}}{\partial r} + \tilde{b}_{jk\theta}\frac{1}{r}\frac{\partial \overline{\boldsymbol{B}}_{T_k}^{(i)}}{\partial \theta} \tag{10}$$

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magnetoconvection-*B*

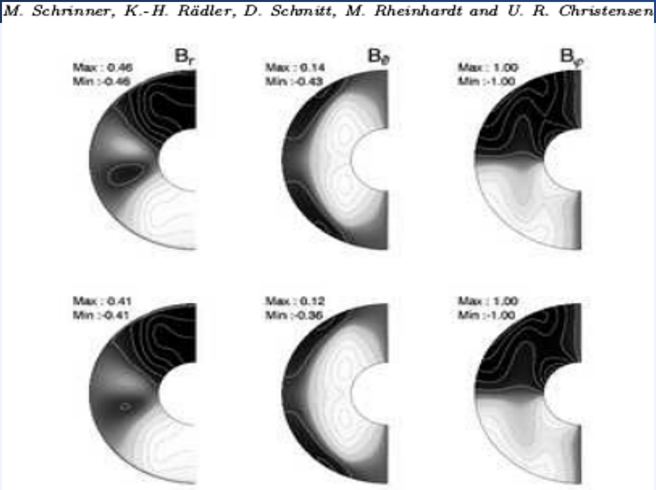

Figure: (Schrinner et al. 2007) magnetoconvection: azimuthally averaged magnetic field components resulting from DNS (upper), mean-field calculations derived from test field (lower). $[(\rho\mu_0\eta\Omega)^{1/2}]$

magnetoconvection-$\mathcal{E}$

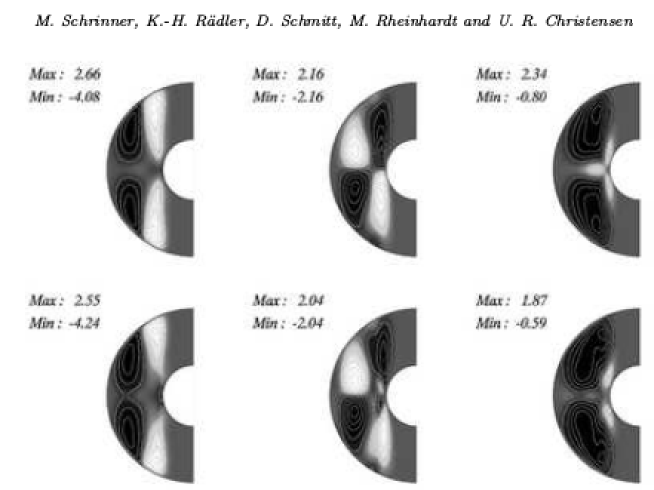

Figure: (Schrinner et al. 2007) electromotive forces in the magnetoconvection (top) $\mathcal{E}_r^{\rm MHD}$, $\mathcal{E}_\theta^{\rm MHD}$, $\mathcal{E}_\phi^{\rm MHD}$, and (bottom) $\mathcal{E}_r^{\rm MF}$, $\mathcal{E}_\theta^{\rm MF}$, $\mathcal{E}_\phi^{\rm MF}$. $[(\eta/D)(\rho\mu_0\eta\Omega)^{1/2}]$

test field application to millenium data

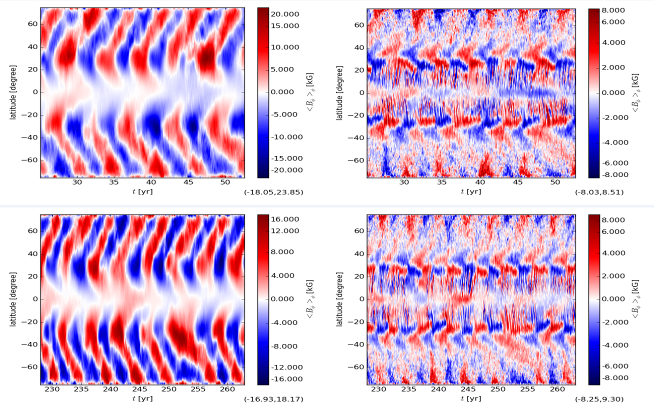

Figure: Butterfly diagrams for the B_ϕ anti-symmetric (upper) and symmetric (lower) epochs - base of convection zone (left) and surface (right)

Conversion between radial and azimuthal field

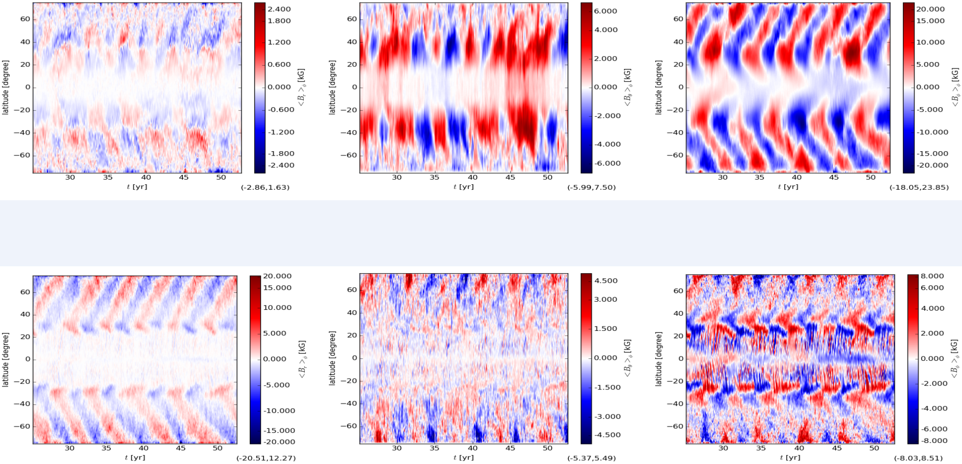

Figure: Butterfly diagrams for anti-symmetric epoch - base convection zone (upper) to surface (lower) B_r, B_θ, B_ϕ (left to right)

Highly chaotic dynamics

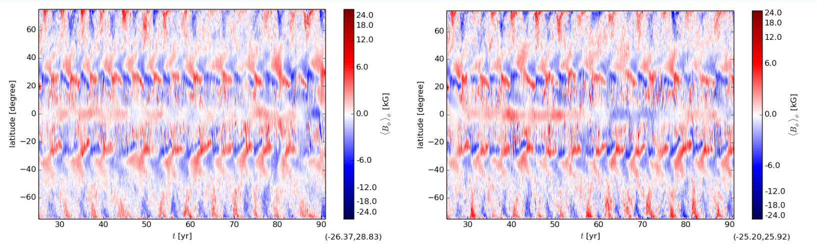

Figure: Time averages B_ϕ near surface with boundary matching millenium (left), and corrected perfect conducting boundary (right).

Käpylä M J, Käpylä P J, Olspert N, Brandenburg A, Warnecke J, Karak B B & Pelt J 2015 *ArXiv e-prints* .

Käpylä P J, Mantere M J, Cole E, Warnecke J & Brandenburg A 2013 *ApJ* **778**, 41.

Krause F & Rädler K H 1980 *Mean-field magnetohydrodynamics and dynamo theory*.

Schrinner M, Rädler K H, Schmitt D, Rheinhardt M & Christensen U R 2007 *Geophysical and Astrophysical Fluid Dynamics* **101**, 81–116.