Comparison of damping mechanisms for transverse waves in coronal loops

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1. Introduction

Damping of transverse waves in different coronal structures is a commonly observed property. A number of mechanisms have been proposed to explain it. We carried out a Bayesian model comparison analysis to quantify the plausibility of three of them.

2. Modelling

The considered damping mechanisms are: resonant absorption, phase mixing and lateral wave leakage. Approximate expressions for the damping rate predicted by these models are:

$$r_0 = \frac{2}{\pi}\left(\frac{l}{R}\right)^{-1}\left(\frac{\zeta+1}{\zeta-1}\right);\ r_1 = \left(\frac{3}{\nu\pi^2}\right)^{1/3} l^{2/3}P^{-1/3};\ r_2 = \frac{4}{\pi^4}\left(\frac{R}{L}\right)^{-2},$$

with l/R the transverse density inhomogeneity, z the density contrast, ν the coefficient of kinematic viscosity, P the oscillation period and R/L the loop minor radius to length ratio. In the following, the subscripts 0, 1 and 2 will represent resonant, phase mixing and wave leakage, respectively.

3. Methodology

Our analysis is based on the application of Bayes' Rule which allows us to use all our available knowledge to compute the probability of a model (M), conditional on observed data (d) using the expression:

$$p(M|d) = \frac{p(d|M)p(M)}{p(d)},$$

where $p(M|d)$ is the posterior, p(M) the prior, p(d|M) the likelihood function and p(d) evidence.

Considering Bayes' Rule in terms of the parameters (Θ) of a model (M), the marginal likelihood permits us to measure the plausibility of the observed data given that the model M is true

$$p(d|M) = \int_\theta p(\theta|M)p(d|\theta,M)d\theta.$$

Taking posterior ratios in a one-to-one comparison between models, and considering that all models are equally probable a priori $p(M_0) = p(M_1) = p(M_2)$, we can obtain the Bayes' factor:

$$B_{ij} = \frac{p(M_i|d)}{p(M_j|d)} = \frac{p(d|M_i)}{p(d|M_j)},$$

where i, j=0, 1, 2 with i ≠ j (B_{01}, B_{02}, B_{12}). The magnitude of the Bayes factor enables us to quantify the relative plausibility of alternative models which can be classified in levels of evidence according to the following table from Kass & Raftery (1995):

$2\ln B_{ij}$	Evidence
0-2	Not Worth more tan a bare Mention (NWM)
2-6	Positive Evidence (PE)
6-10	Strong Evidence (SE)
>10	Very Strong Evidence (VSE)

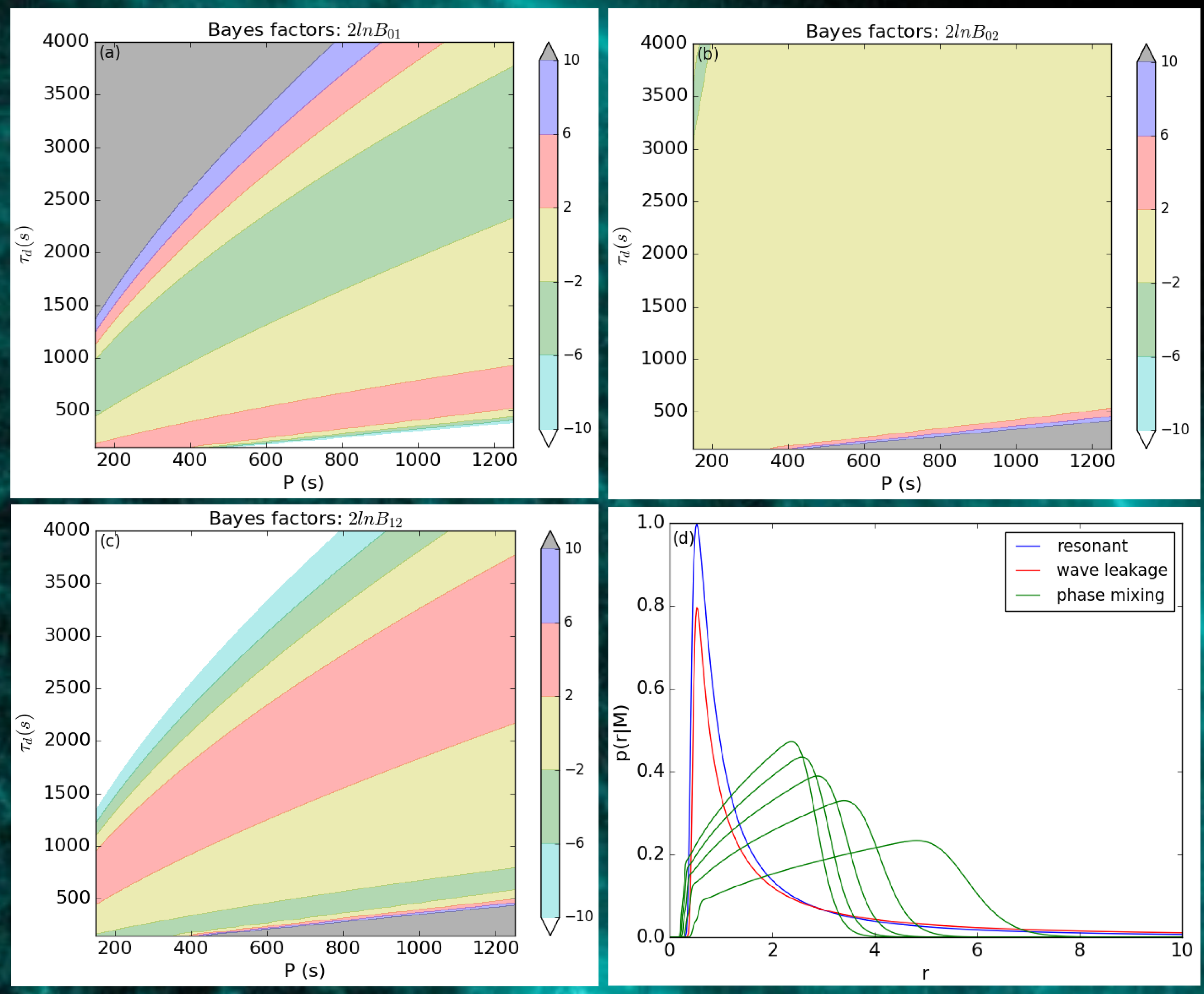

Figure 1. Bayes factors in one-to-one comparison between damping models (panels a, b and c), with uniform priors and Gaussian likelihoods . Different colours represent the levels of evidence: NW (yellow), PE (green/red), SE (blue/purple), VSE (white/grey). In panel d, marginal likelihoods are represented. Phase mixing results have been calculated for five fixed wave periods: P= 150, 425, 700, 975 and 1250 s from right to left.

4. Results

The outcomes from the computation of Bayes factors in the one to one comparison between alternative damping models, for a particular value of the measurement errors, are shown in Figure 1. Panels a, b and c display the distribution of this magnitude in the observable (P, τ_d)- plane.

The general result is that the evidence for one model against another depends on the particular combination of observed periods and damping times.

In particular:

Resonant vs. phase mixing (panel a): the upper-left corner corresponding to large values of τ_d/P, indicates strong (purple) and very strong (grey) evidence in favour of resonant absorption. The lower-right corner corresponding to low damping ratios shows very strong evidence (white) in favour of phase mixing model. In the remaining regions different coloured bands denote positive (pink for resonant and green for phase mixing) or insignificant evidence (yellow) depending on the observables (P, τ_d).

Resonant vs. wave leakage (panel b): most of the (P, τ_d)- plane is coloured in yellow resulting in a lack of evidence for a particular model. Grey and purple regions (very strong and strong evidence) indicate the dominance of the resonant mechanism for the lowest values of r and positive evidence (green for wave leakage and pink for resonant) is located in very small regions.

Phase mixing vs. wave leakage (panel c): the white (very strong evidence) and the blue (strong evidence) regions are located at large values of r, indicating the dominance of the wave leakage model over phase mixing. For low damping ratios, we have very strong evidence (grey) in favour of phase mixing. The remaining regions point to positive (pink for phase mixing and green for wave leakage) or negligible evidence (yellow) depending on the observables (P, τ_d).

Figure 1d shows the marginal likelihoods for the three considered damping models as a function of the damping ratio r. Resonant absorption and wave leakage are more plausible for low damping rate values, while the evidence for phase mixing attains larger values for low-intermediate values of r.

We applied this method to a set of 89 loop oscillation events observed by Verwichte et al (2013) and Goddard et al. (2016). Figure 2 shows the distribution of Bayes factors as a function of the damping rate for those events, with colours indicating the magnitude of the evidence in the three one-to-one model comparisons.

In all three panels the blue colour (NWM) dominates. There are some events coloured in red and yellow that show positive evidence. Panels b and c show one particular case with strong evidence (green) and some black points that represent very strong evidences.

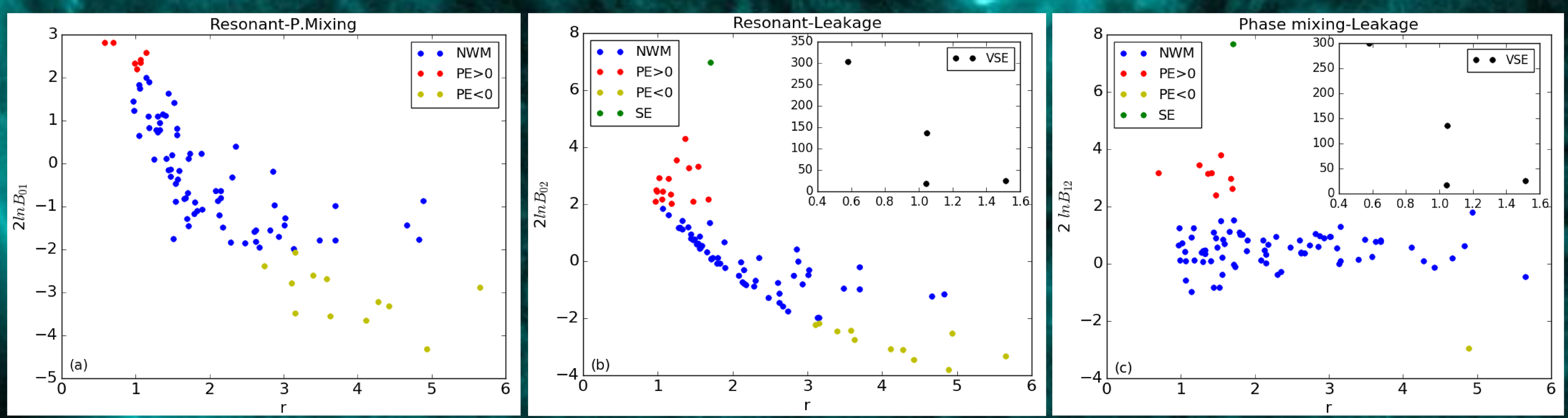

Figure 2. Bayes factors of 89 events selected from Verwichte et al. (2013, A&A 552, A138) and Goddard et al. (2016, A&A 585, A137)

5. Conclusions

We presented a method to compute relative probabilities between alternative damping mechanisms for transverse coronal loop oscillations. As a general rule, a single damping mechanism cannot explain the observed damping times and damping ratios. However, the method enables us to assign a level of evidence to each considered theoretical model in analytic cases and real observations with their associated uncertainty.