High energy physics and inflation as a tool to see it

Lecture 3

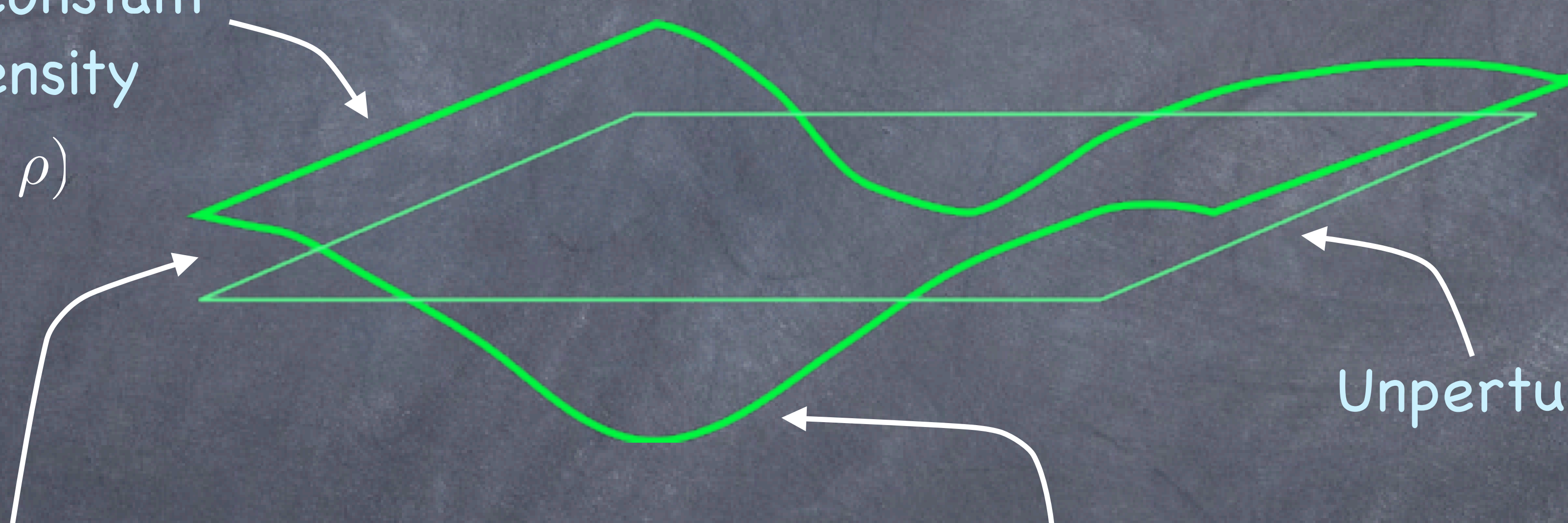
David Seery
University of Sussex

ISAPP 2012 La Palma

The conclusion is that, to detect light modes, we should look at departures from Gaussian statistics

But in which observable?

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \mathrm{e}^{2\zeta} \mathrm{d}\boldsymbol{x}^2$$



$$a(t) \equiv \exp \int^t H(t')\,\mathrm{d}t' = \exp N(t) \qquad \Rightarrow \qquad a(t)\mathrm{e}^{\zeta} \equiv \exp\{N(t) + \delta N(t)\}$$

Currently, what we can observe are the correlation functions of ζ, because these seed the density fluctuation

Spectrum

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)P(k)$$

Bispectrum

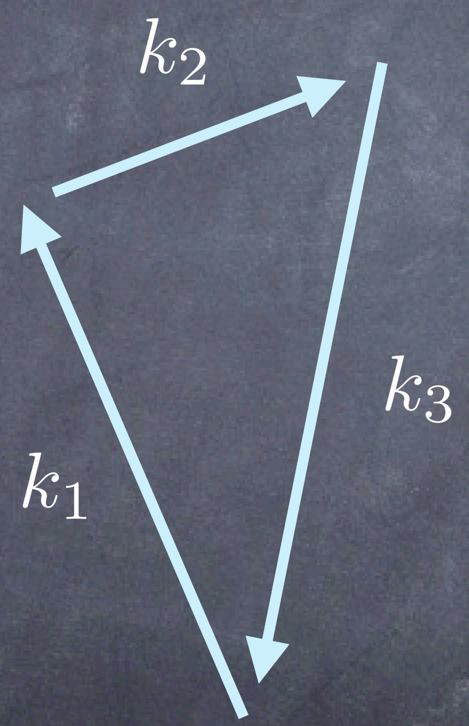
$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(k_1,k_2,k_3)$$

Trispectrum

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\zeta(\boldsymbol{k}_4)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)T(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$

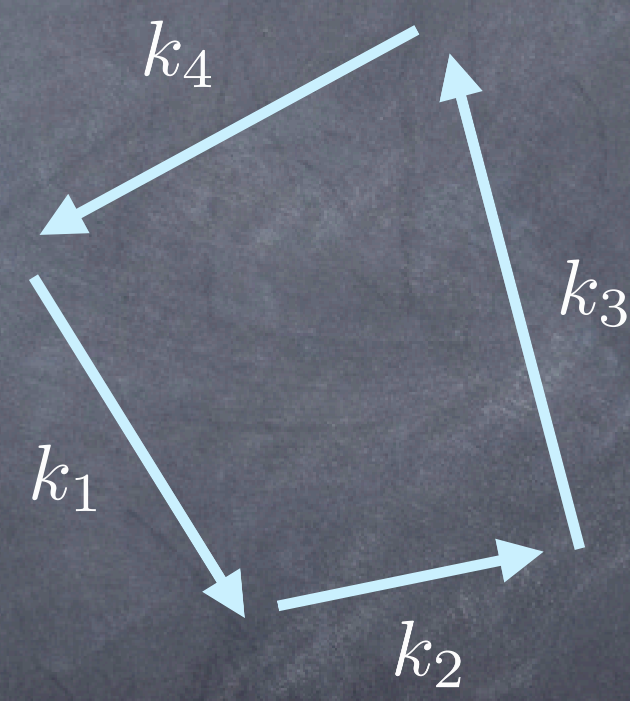
We have seen that the 2pf depends weakly on k and evolves in time.

The bispectrum and trispectrum are much more complicated functions. They depend on time and the momentum configuration



3pf triangle

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$$



4pf quadrilateral

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4 = 0$$

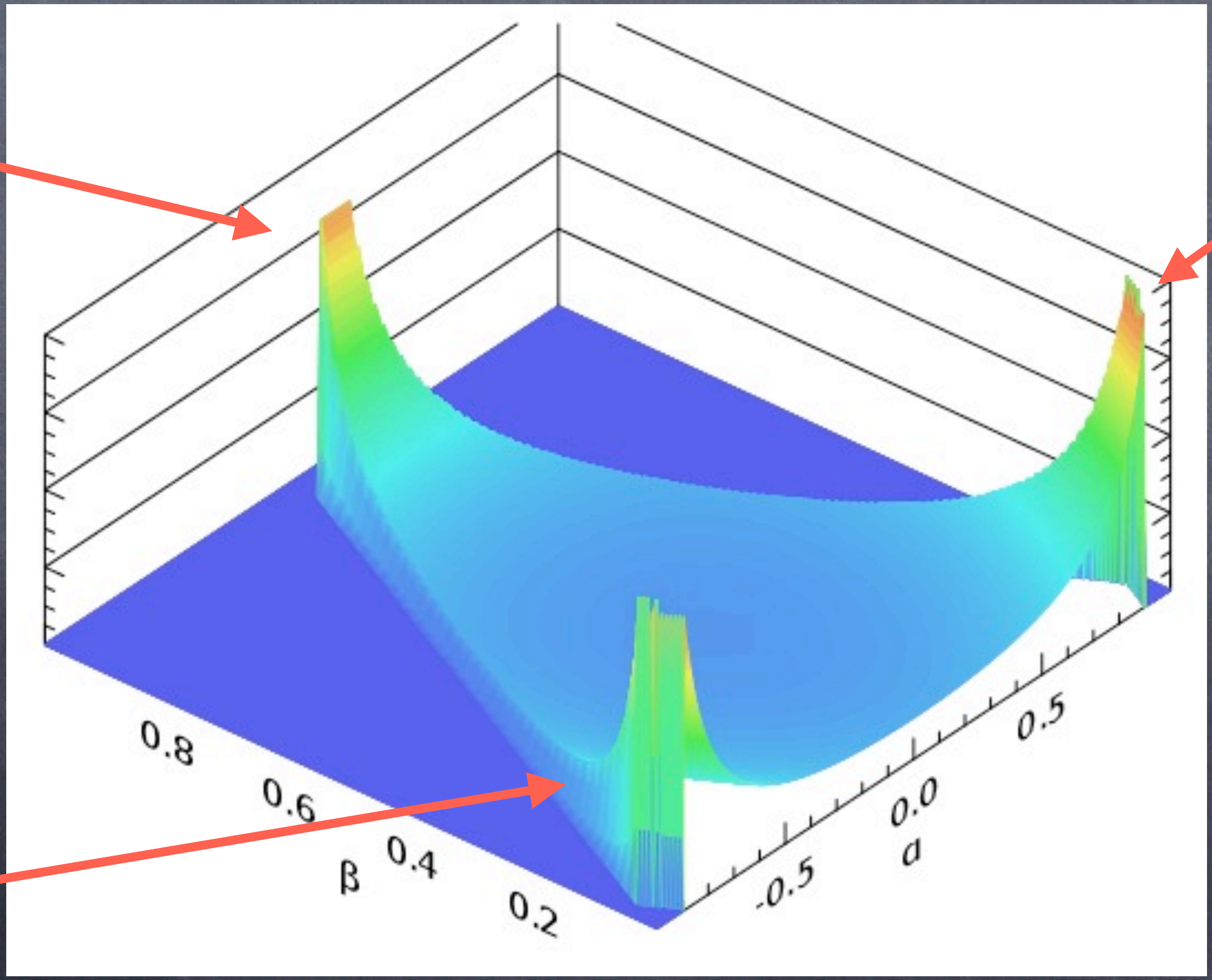
In principle, these functions can be arbitrarily complicated
But usually they turn out to be fairly simple

$$k_1 = \frac{k_t}{4}(1+\alpha+\beta) \qquad k_2 = \frac{k_t}{4}(1-\alpha+\beta) \qquad k_3 = \frac{k_t}{2}(1-\beta)$$

$$k_t = \text{perimeter} = k_1 + k_2 + k_3 \qquad 0 \leq \beta \leq 1 \qquad \beta - 1 \leq \alpha \leq 1 - \beta$$

$k_3 \to 0$

$k_2 \to 0$



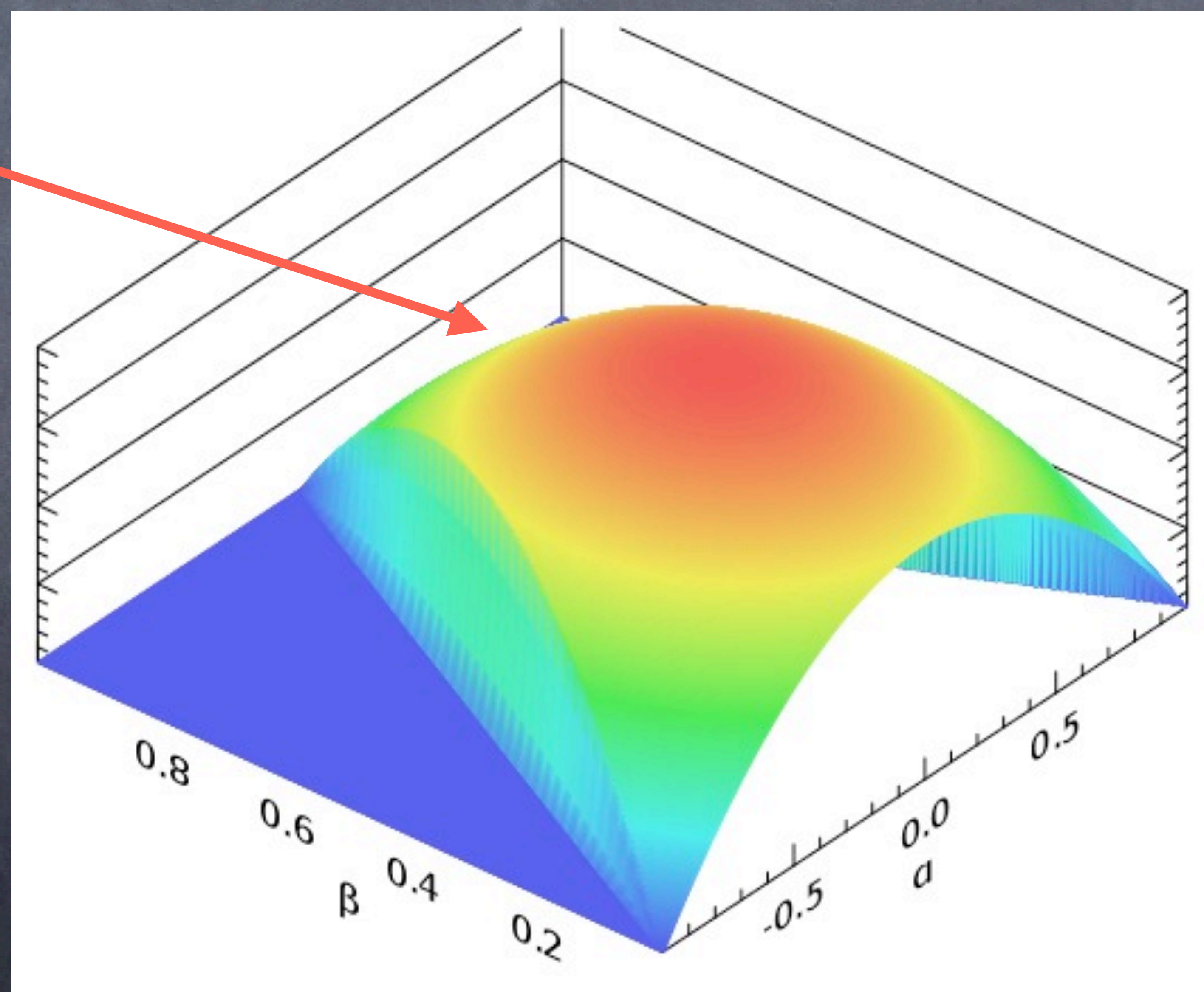
$k_1 \to 0$

In principle, these functions can be arbitrarily complicated
But usually they turn out to be fairly simple

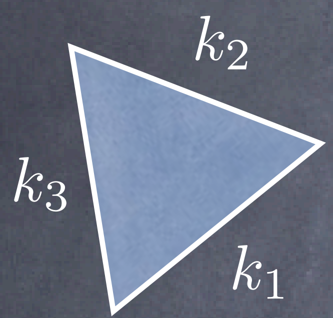
$$k_1 = \frac{k_t}{4}(1+\alpha+\beta) \qquad k_2 = \frac{k_t}{4}(1-\alpha+\beta) \qquad k_3 = \frac{k_t}{2}(1-\beta)$$

$$k_t = \text{perimeter} = k_1 + k_2 + k_3 \qquad 0 \leq \beta \leq 1 \qquad \beta - 1 \leq \alpha \leq 1 - \beta$$

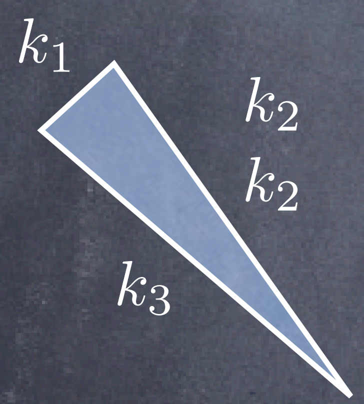
$k_1 \approx k_2 \approx k_3$



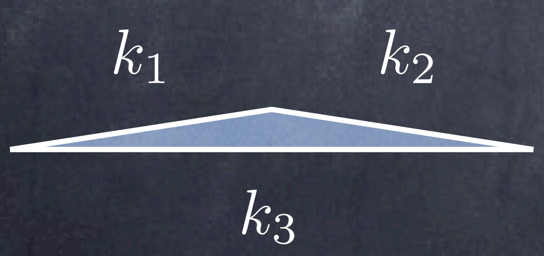
Where the bispectrum peaks is a signal of the microphysics underlying the fluctuations



Equilateral. Indicates that the fluctuations have exotic structure, such as nontrivial kinetic energy.
Favours stringy or supergravity scenarios.

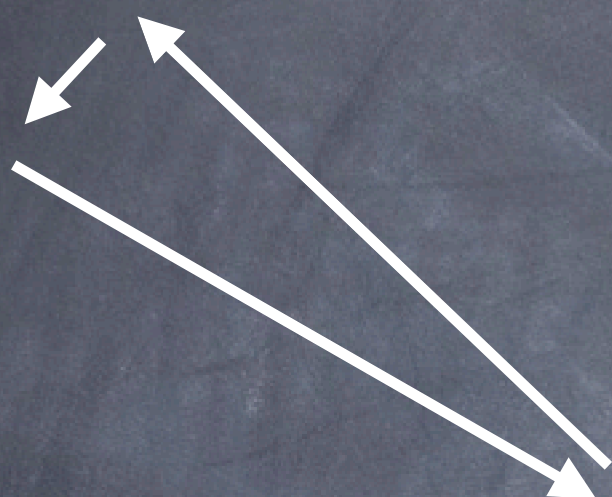


Squeezed. Indicates that the fluctuations have time evolution. Since that is forbidden in single-field models, this implies multiple light modes.



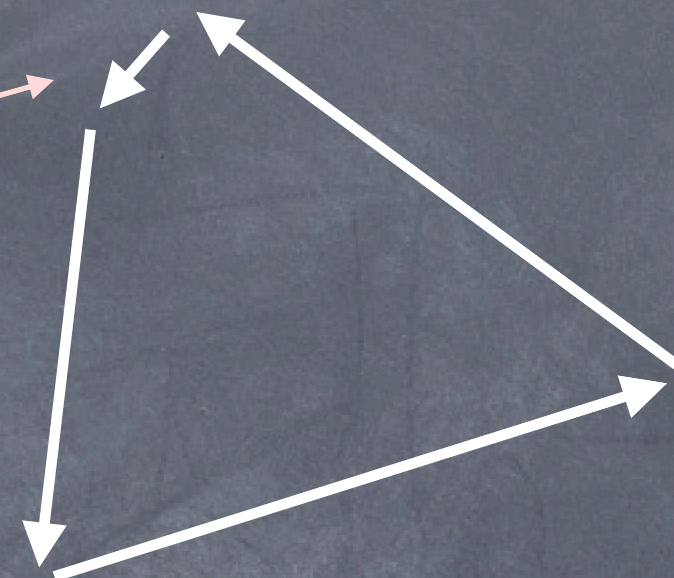
Folded. Indicates a near zero-energy "resonance" between positive and negative energy modes.
Favours non-vacuum initial conditions.

When the Planck data arrive next year, there will probably be some talk about the "squeezed" and "collapsed" limits of the n-pfs



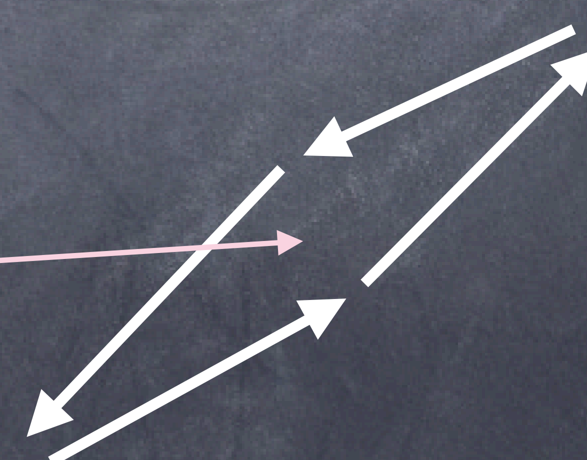
3pf squeezed to a line

one side becoming very small



4pf squeezed to a triangle

no side becomes small, but the diagonal does



4pf collapsed to a line

Why are these limits useful? Let's go back to the separate universe expansion

$$\delta\phi_\alpha(\text{now}) = \frac{\partial\phi_\alpha(\text{now})}{\partial\phi_i(\text{then})}\delta\phi_i(\text{then}) + \frac{1}{2}\frac{\partial^2\phi_\alpha(\text{now})}{\partial\phi_i(\text{then})\partial\phi_j(\text{then})}\delta\phi_i(\text{then})\delta\phi_j(\text{then}) + \cdots$$

We are interested in k-space correlation functions. Fourier transforming and using the index convention that Greek = now, Latin = then,

$$\delta\phi_\alpha(\boldsymbol{k}) = \frac{\partial\phi_\alpha}{\partial\phi_i}\delta\phi_i(\boldsymbol{k}) + \frac{1}{2}\frac{\partial^2\phi_\alpha}{\partial\phi_i\partial\phi_j}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\delta\phi_i(\boldsymbol{k}-\boldsymbol{q})\delta\phi_j(\boldsymbol{q}) + \cdots$$

(Full disclosure: no-one else uses this convention. But it is helpful to shorten the notation)

$$\delta\phi_\alpha(\boldsymbol{k}) = \frac{\partial\phi_\alpha}{\partial\phi_i}\delta\phi_i(\boldsymbol{k}) + \frac{1}{2}\frac{\partial^2\phi_\alpha}{\partial\phi_i\partial\phi_j}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\delta\phi_i(\boldsymbol{k}-\boldsymbol{q})\delta\phi_j(\boldsymbol{q}) + \cdots$$

Computing the three-point function, we find a contribution

$$\langle\delta\phi_\alpha(\boldsymbol{k}_1)\delta\phi_\beta(\boldsymbol{k}_2)\delta\phi_\gamma(\boldsymbol{k}_3)\rangle \supseteq$$
$$\frac{1}{2}\frac{\partial\phi_\alpha}{\partial\phi_i}\frac{\partial\phi_\beta}{\partial\phi_j}\frac{\partial\phi_\gamma}{\partial\phi_m\partial\phi_n}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\langle\delta\phi_i(\boldsymbol{k}_1)\delta\phi_j(\boldsymbol{k}_2)\delta\phi_m(\boldsymbol{k}_3-\boldsymbol{q})\delta\phi_n(\boldsymbol{q})\rangle$$
$$+ \text{permutations}$$

(there are others, but this one wins)

$$\delta\phi_\alpha(\boldsymbol{k}) = \frac{\partial\phi_\alpha}{\partial\phi_i}\delta\phi_i(\boldsymbol{k}) + \frac{1}{2}\frac{\partial^2\phi_\alpha}{\partial\phi_i\partial\phi_j}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\delta\phi_i(\boldsymbol{k}-\boldsymbol{q})\delta\phi_j(\boldsymbol{q}) + \cdots$$

Computing the three-point function, we find a contribution

$$\langle\delta\phi_\alpha(\boldsymbol{k}_1)\delta\phi_\beta(\boldsymbol{k}_2)\delta\phi_\gamma(\boldsymbol{k}_3)\rangle \supseteq$$

$$\frac{1}{2}\frac{\partial\phi_\alpha}{\partial\phi_i}\frac{\partial\phi_\beta}{\partial\phi_j}\frac{\partial\phi_\gamma}{\partial\phi_m\partial\phi_n}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\langle\delta\phi_i(\boldsymbol{k}_1)\delta\phi_j(\boldsymbol{k}_2)\delta\phi_m(\boldsymbol{k}_3-\boldsymbol{q})\delta\phi_n(\boldsymbol{q})\rangle$$

$$+ \text{ permutations}$$

(there are others, but this one wins)

Then we pair up all the fields using Wick's theorem

$$\langle\delta\phi_i(\boldsymbol{k}_1)\delta\phi_j(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{H_*^2}{2k^3}\delta_{ij}$$

$$\langle\delta\phi_\alpha(\boldsymbol{k}_1)\delta\phi_\beta(\boldsymbol{k}_2)\delta\phi_\gamma(\boldsymbol{k}_3)\rangle \supseteq (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{\partial\phi_\alpha}{\partial\phi_i}\frac{\partial\phi_\beta}{\partial\phi_j}\frac{\partial^2\phi_\gamma}{\partial\phi_i\partial\phi_j}\frac{H_*^2}{2k_1^3}\frac{H_*^2}{2k_2^3}$$

$$+ \text{ permutations}$$

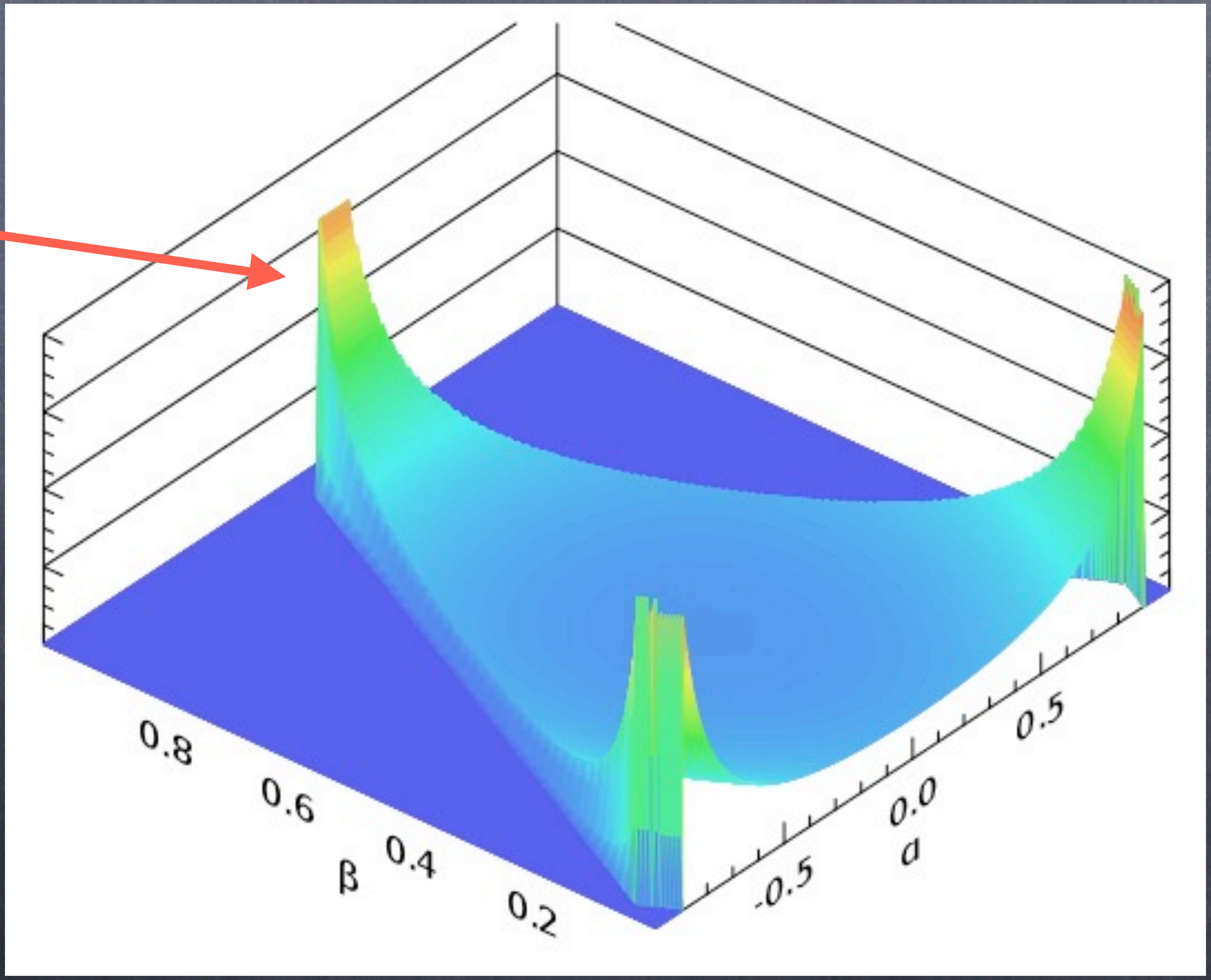
$$\langle \delta\phi_\alpha(\boldsymbol{k}_1)\delta\phi_\beta(\boldsymbol{k}_2)\delta\phi_\gamma(\boldsymbol{k}_3)\rangle \supseteq (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{\partial\phi_\alpha}{\partial\phi_i}\frac{\partial\phi_\beta}{\partial\phi_j}\frac{\partial^2\phi_\gamma}{\partial\phi_i\partial\phi_j}\frac{H_*^2}{2k_1^3}\frac{H_*^2}{2k_2^3}$$

$$+ \text{permutations}$$

It's easy to see that this bispectrum peaks when one of the $k_i \to 0$

[In fact, this bispectrum is often called the "local shape" because it comes from the local separate universe expansion.]

In this limit, the bispectrum is growing like $1/k^3$



But we made this estimate using massless mode functions

f_{NL} and all that

When we have a bispectrum of this type, it is usually written as

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$$
$$\times \frac{6}{5} f_{\mathrm{NL}} \left[P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_2) \right]$$

$$\frac{1}{k_1^3 k_2^3} \qquad \frac{1}{k_1^3 k_3^3} \qquad \frac{1}{k_2^3 k_3^3}$$

In fact, people often use this definition of f_{NL} even for bispectra which are not local. This makes f_{NL} a complex function of the momenta, but it's not very useful to the observers

f_{NL} and all that

When we have a bispectrum of this type, it is usually written as

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$$

$$\times\frac{6}{5}f_{\rm NL}\left[P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_2)\right]$$

$$\frac{1}{k_1^3k_2^3} \qquad \frac{1}{k_1^3k_3^3} \qquad \frac{1}{k_2^3k_3^3}$$

Our only freedom is in the amplitude, which is conventionally parametrized using f_{NL}

In fact, people often use this definition of f_{NL} even for bispectra which are not local. This makes f_{NL} a complex function of the momenta, but it's not very useful to the observers

Currently, we have some bounds from WMAP and galaxy surveys

$-10 < f_{\rm NL} < 74$ WMAP7 (Komatsu et al. 2010)

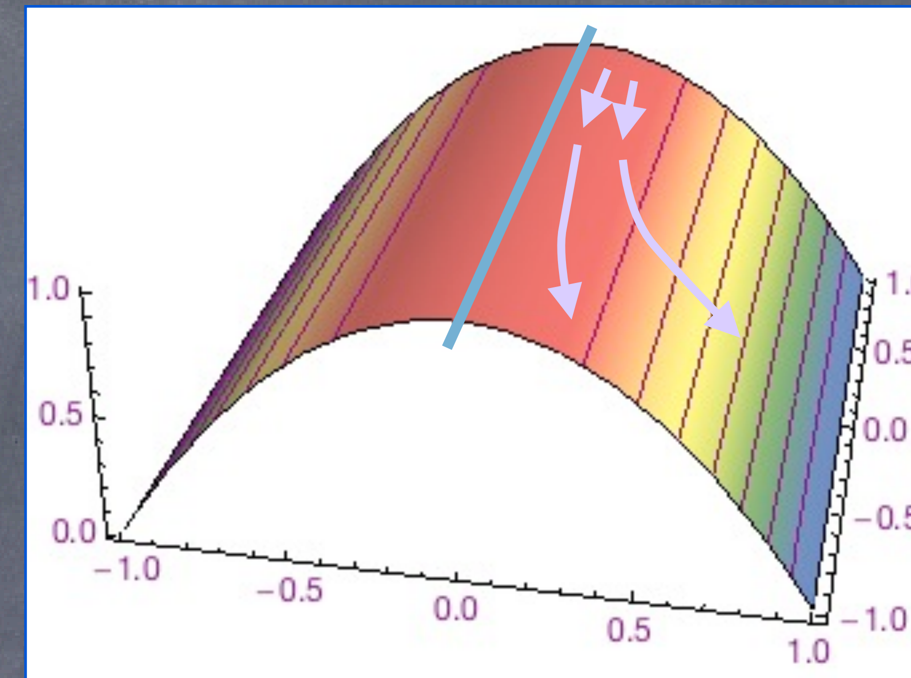
$-29 < f_{\rm NL} < 70$ SDSS (Slosar et al. 2008)

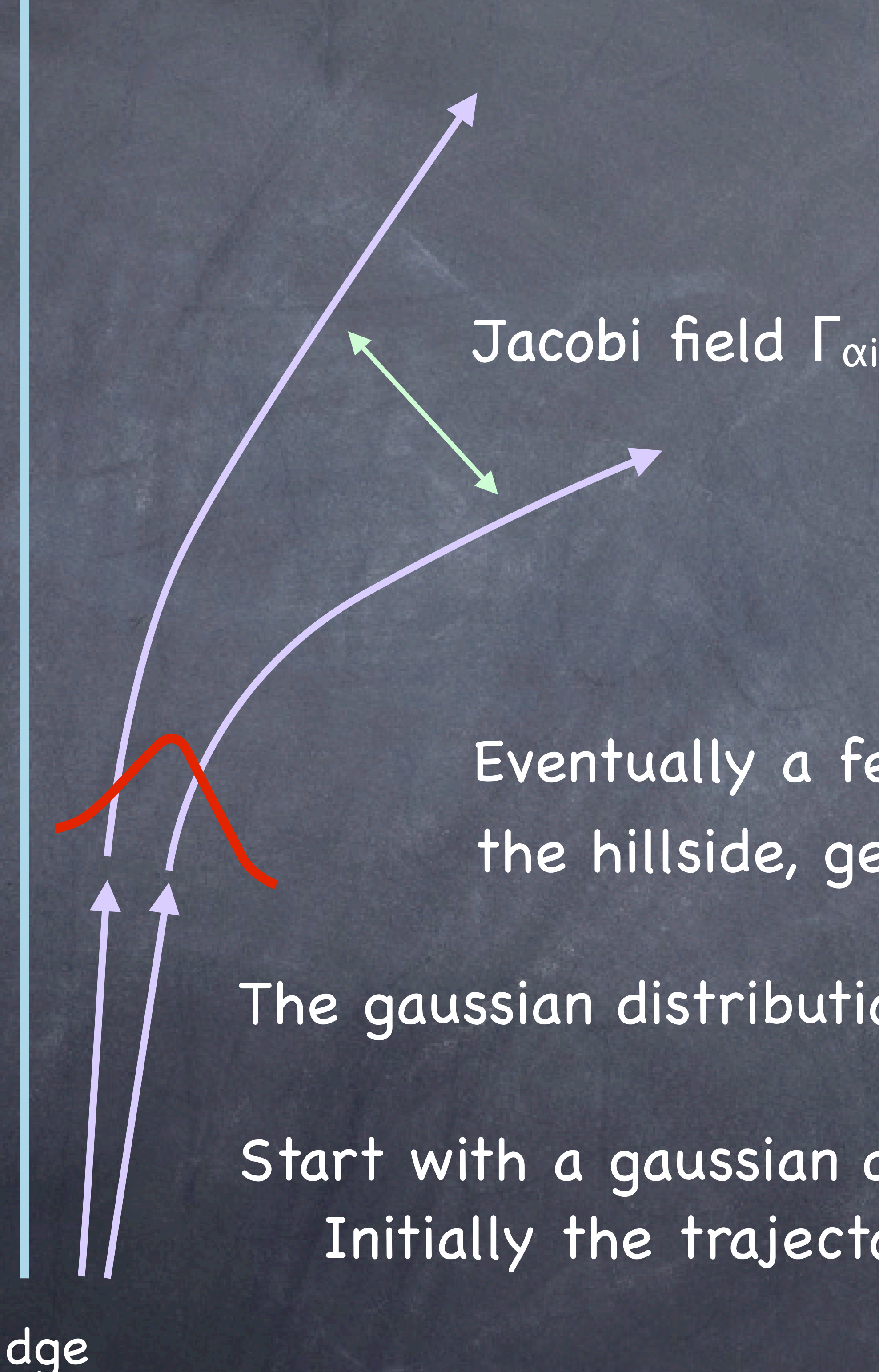
$-5 < f_{\rm NL} < 59$ WMAP7+SDSS combined

Planck may measure $\Delta f_{\rm NL} \approx 5$ (ish)

Future CMB satellites and large-scale galaxy surveys like
DES or EUCLID might do even better
(maybe in a decade or so for the galaxies)

Eventually they disperse nonlinearly away from the ridge





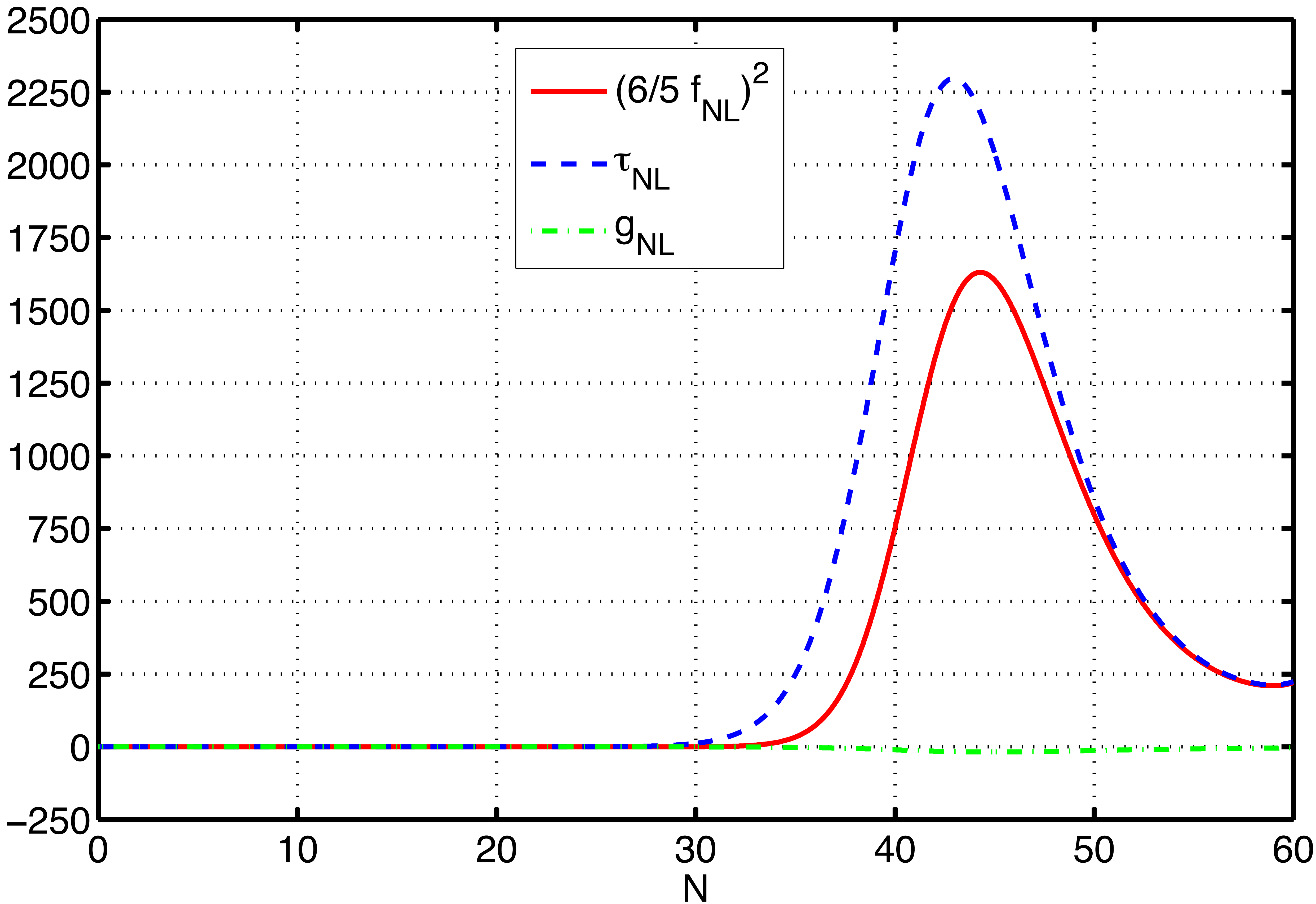
Eventually a few trajectories slide away down the hillside, generating a **heavy tail**

The gaussian distribution is preserved in the early phases

Start with a gaussian distribution

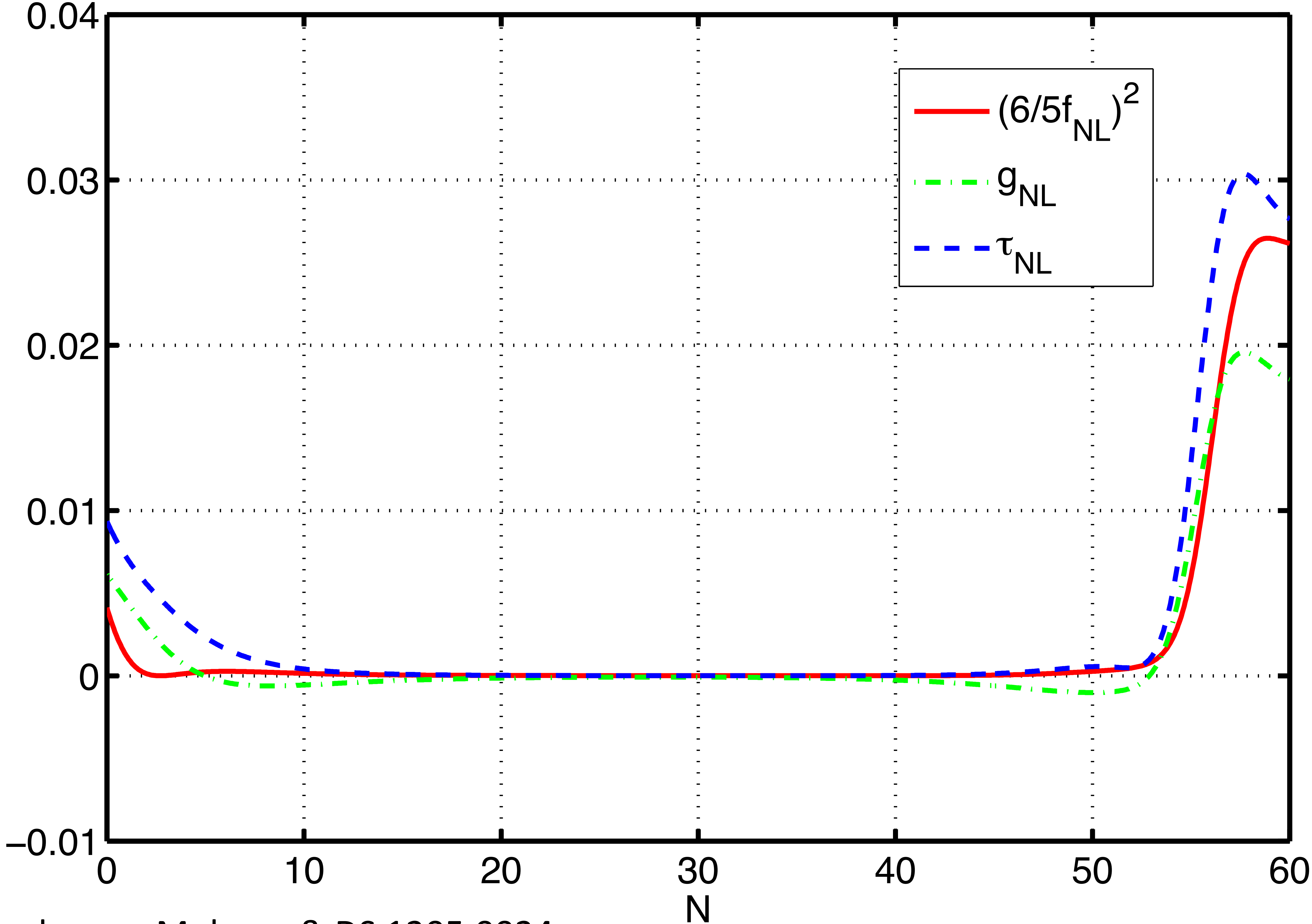
Initially the trajectories keep close to each other

(García-Bellido & Wands)



Anderson, Mulryne & DS 1205.0024

“quasi-realistic” D-brane model of Dias, Frazer & Liddle (2012)

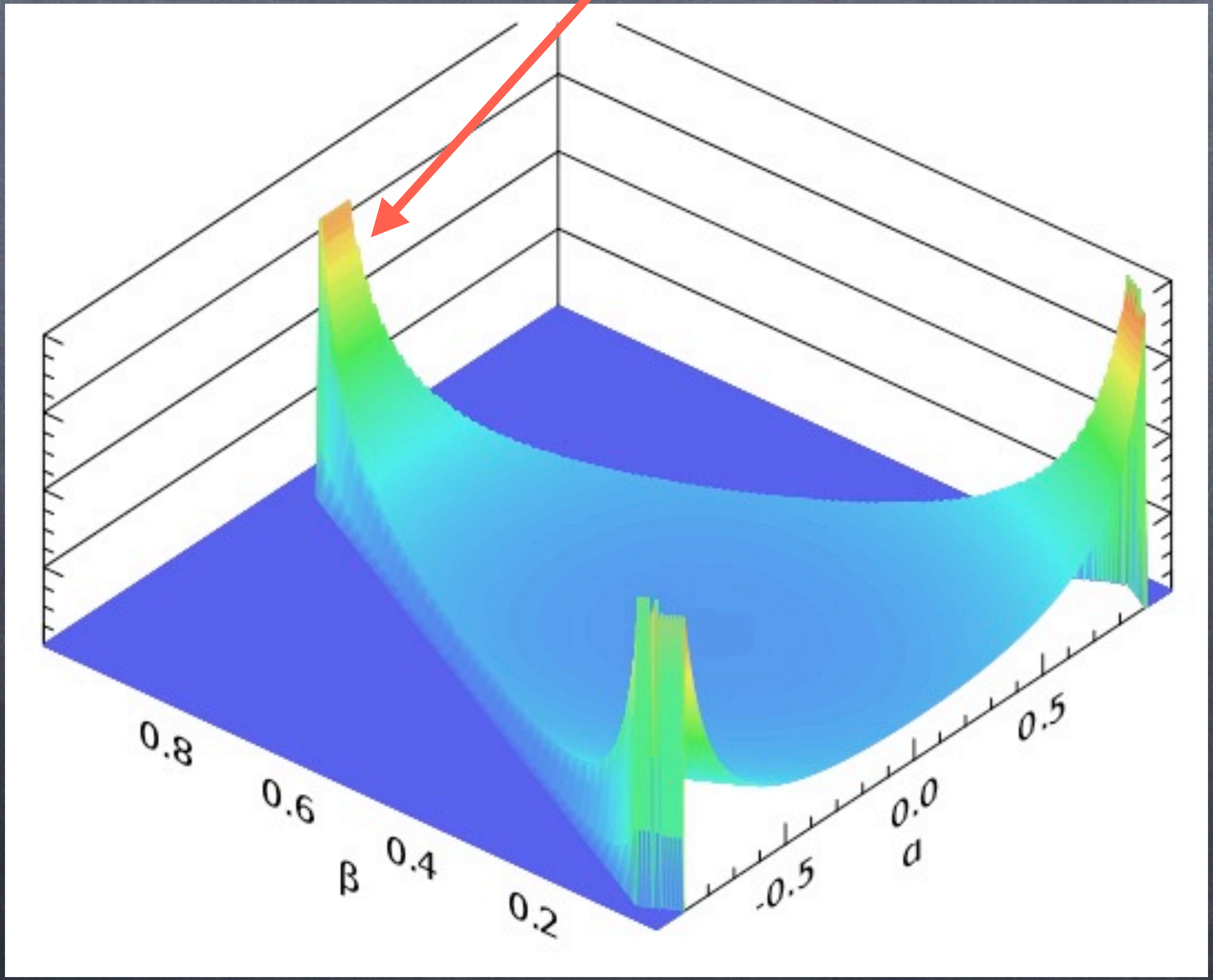


Anderson, Mulryne & DS 1205.0024

More generally, if there are some massive modes, the scaling of the spikes in the squeezed limit can be different from $1/k^3$

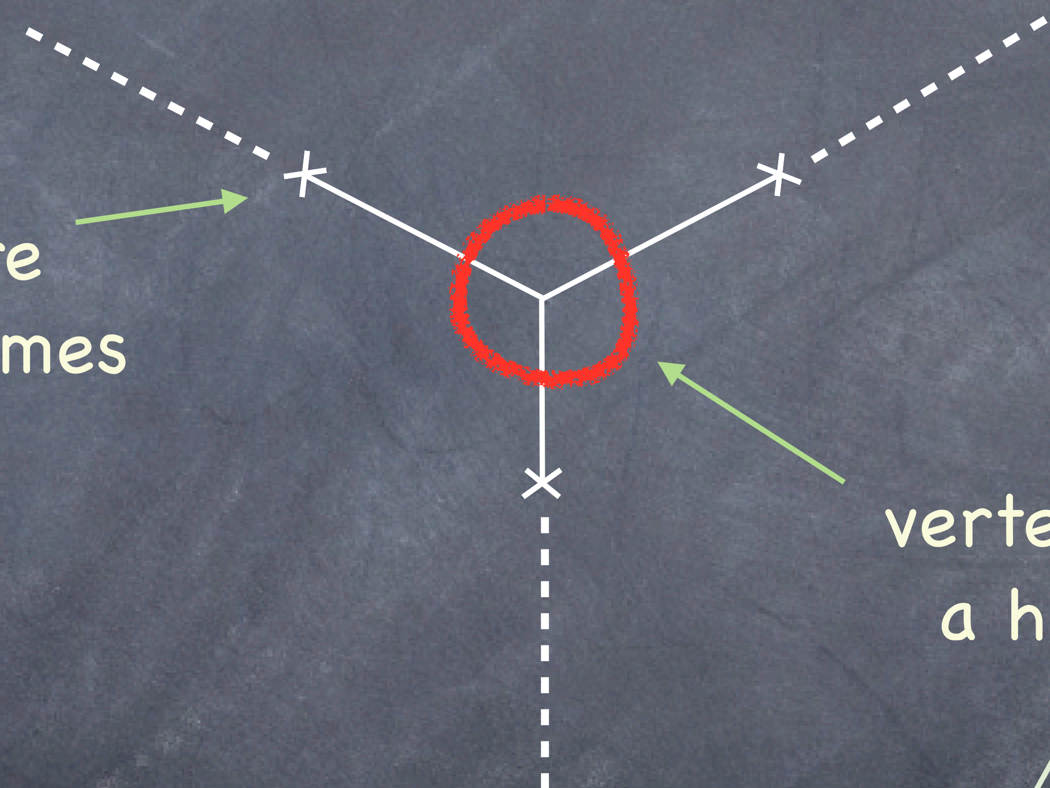
$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle \sim \frac{1}{k^{-3/2-\nu}}$$

We have the best chance of seeing these effects if the masses are not too large, near Hubble



“Quasi-single field inflation” QSFI

In some models, the presence of heavier fields can manifest itself directly. This is very similar to using light particles to track the decays of heavier particles at the LHC.



mixes with curvature perturbation and becomes conserved

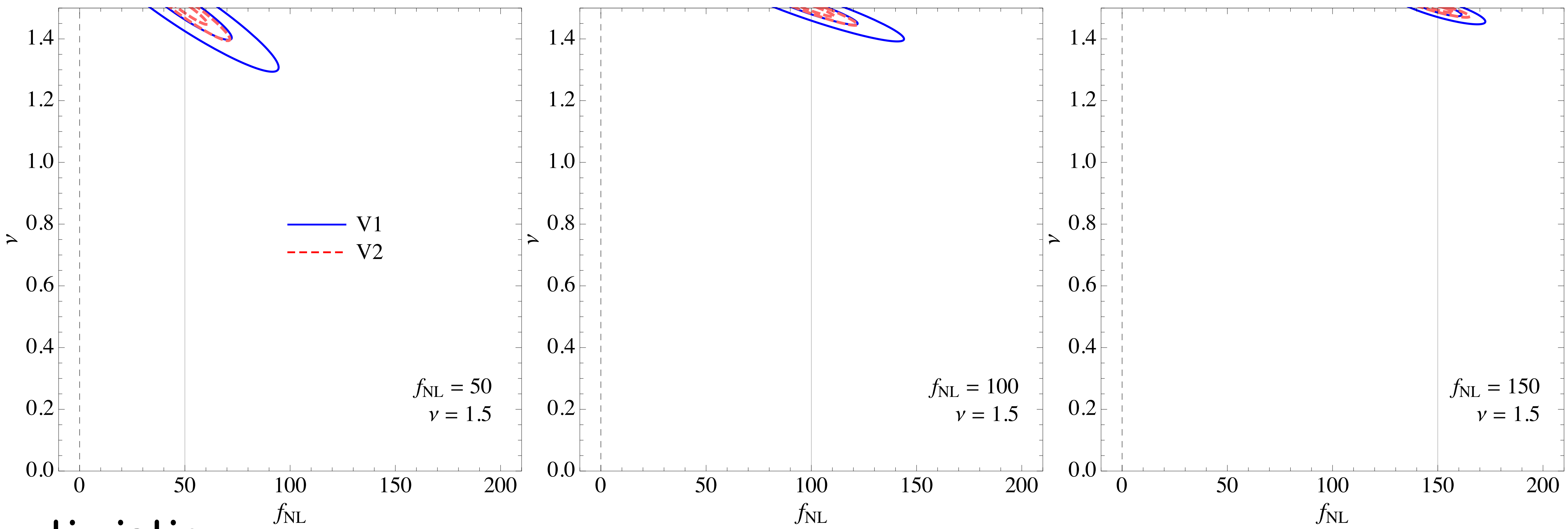
vertex comes from a heavyish field

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle \sim \frac{1}{k^{-3/2-\nu}}$$

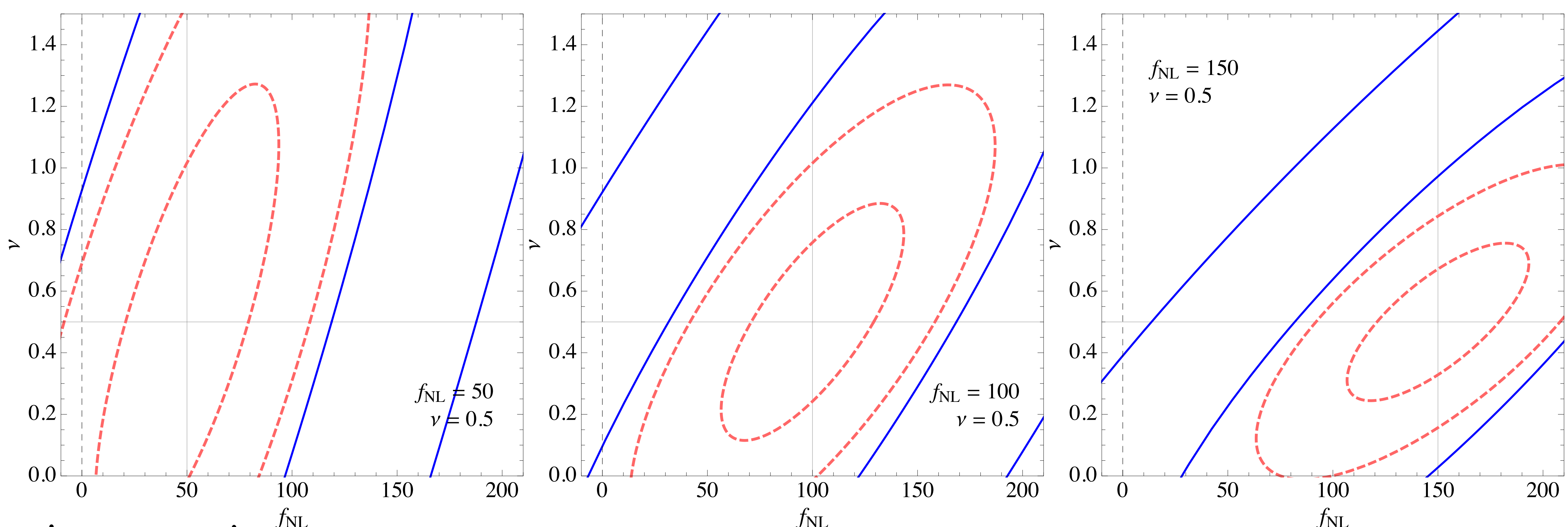
Effects and Detectability of Quasi-Single Field Inflation in the Large-Scale Structure and Cosmic Microwave Background

Emiliano Sefusatti*
Institut de Physique Théorique, CEA, IPhT, F-91191 Gif-sur-Yvette, France &
The Abdus Salam International Center for Theoretical Physics, strada costiera, 11, 34151 Trieste, Italy

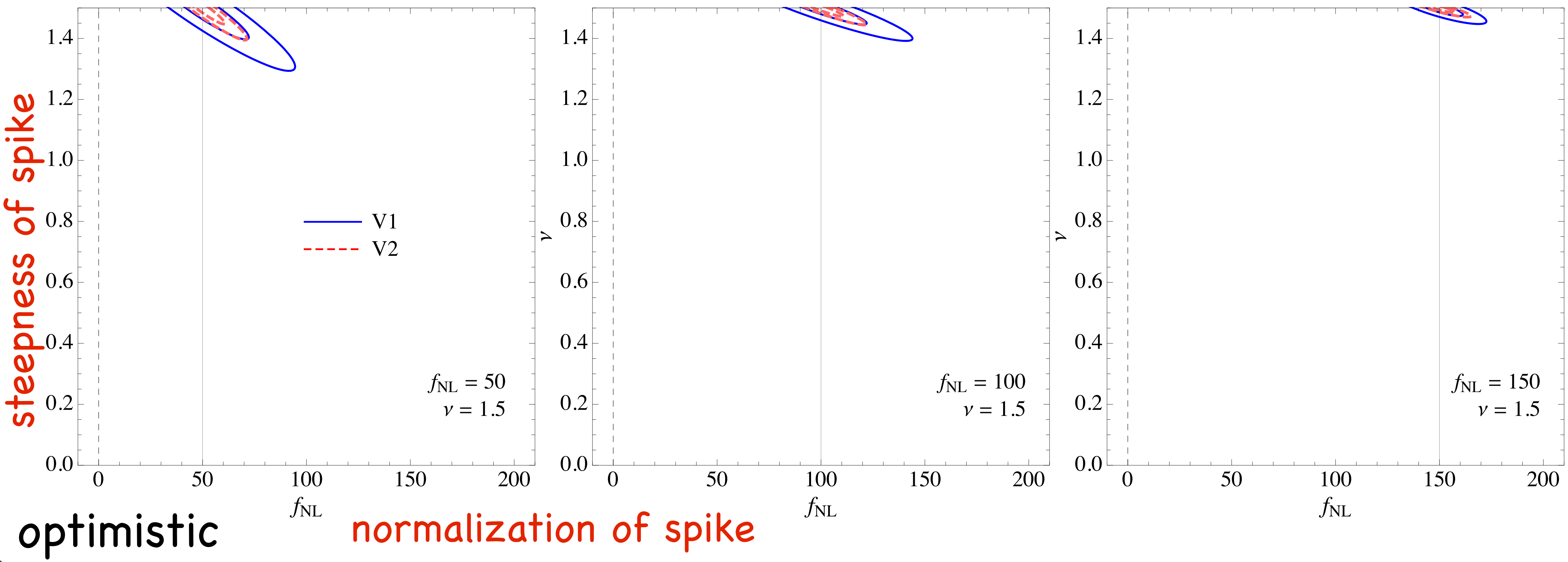
James R. Fergusson,† Xingang Chen,‡ and E.P.S. Shellard§
Center for Theoretical Cosmology, Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK



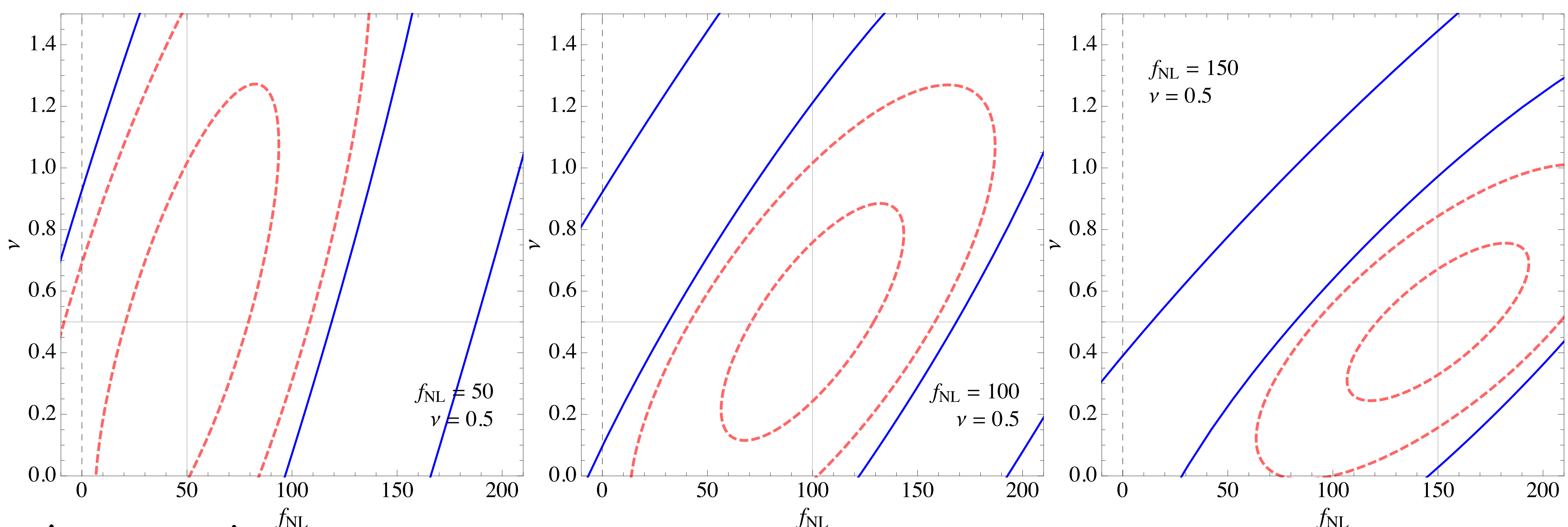
optimistic



not so much



optimistic

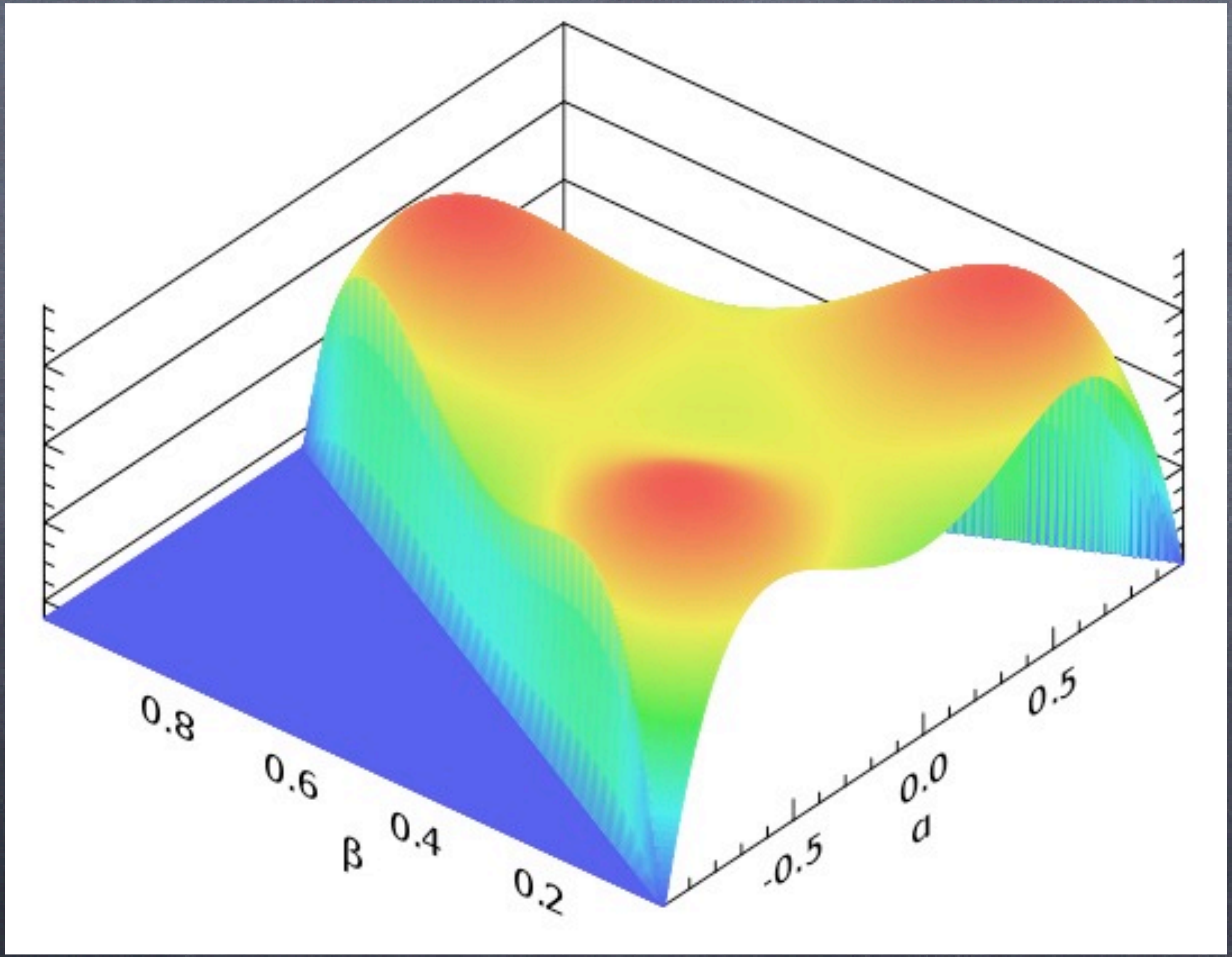


not so much

Sometimes there are even more delicate effects.
Some fashionable models ("Galileons") can produce other shapes

$$k_1 = \frac{k_t}{4}(1+\alpha+\beta) \qquad k_2 = \frac{k_t}{4}(1-\alpha+\beta) \qquad k_3 = \frac{k_t}{2}(1-\beta)$$

$$k_t = \text{perimeter} = k_1 + k_2 + k_3 \qquad 0 \leq \beta \leq 1 \qquad \beta - 1 \leq \alpha \leq 1 - \beta$$



Summary

- ★ A signal in the "squeezed" peaks of the bispectrum is strong evidence for time evolution, and therefore multiple light modes.
  The normalization depends on the detailed infrared dynamics.
- ★ The steepness of these peaks is a diagnostic for heavier modes, with masses around the Hubble scale.
  [For heavier masses we have to try the search in other ways, by looking for oscillations around the inflationary trajectory.]
- ★ The same is true for the "squeezed" and "collapsed" limits of the trispectrum. If we can measure the trispectrum well enough, we can cross-check.