High energy physics and inflation as a tool to see it

Lecture 1

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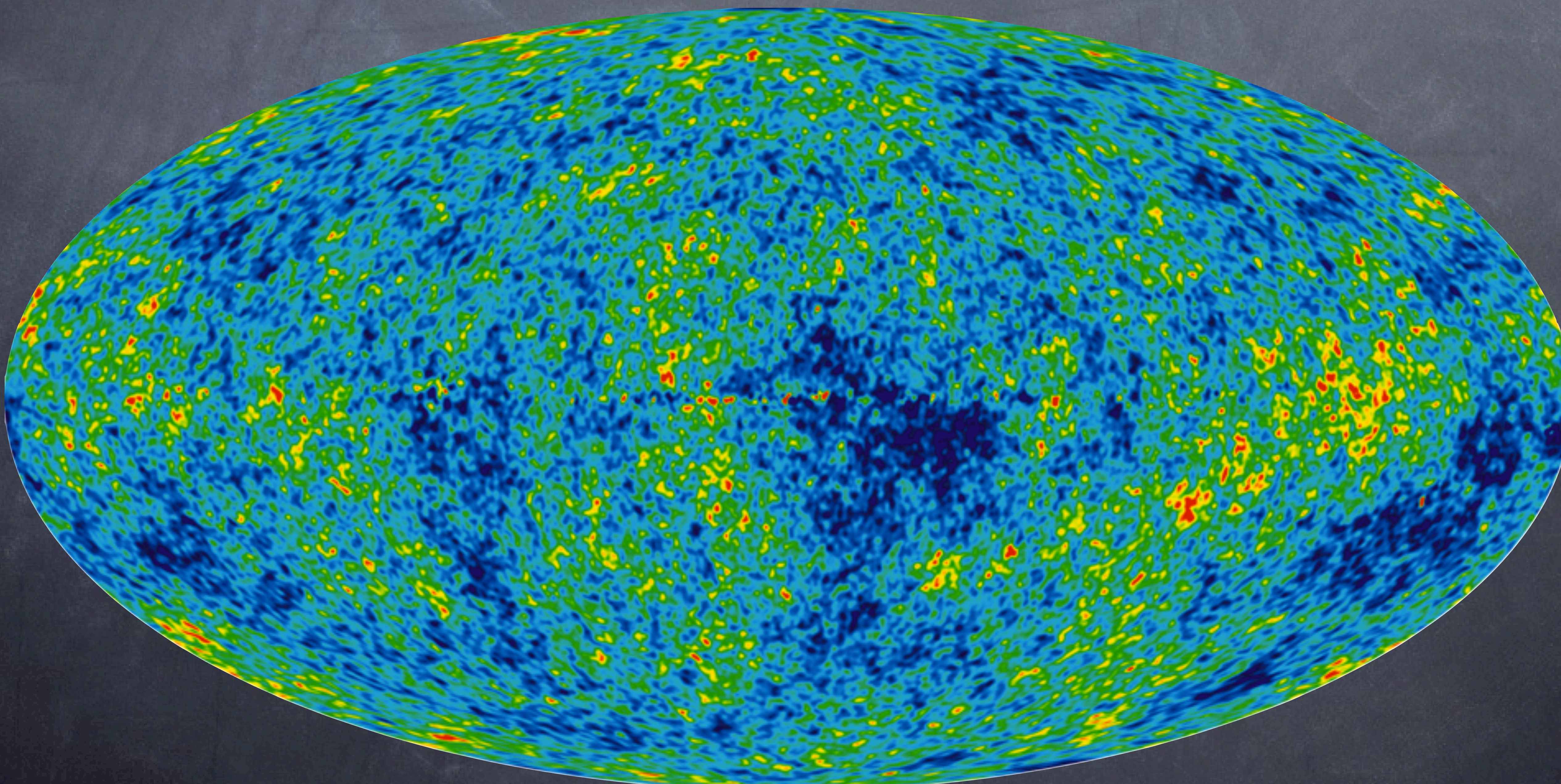
What is inflation and why do we care?

We have the difficult problem of setting initial conditions for the hot, dense phase before our present matter/Λ era



Why work on inflation?

Historically, inflation was thought of as a solution to the flatness, horizon and monopole problems of the conventional hot big bang.

Opinions differ, but (to me) it is not absolutely clear to what extent this is true - at least for the horizon and monopole problems.

It's quite possible to have phase transitions after inflation which would reintroduce topological defects.

Also, it's not clear under what conditions inflation can get under way. If we need some fine-tuned initial conditions at the beginning of the inflationary era, maybe all we have done is push the problem earlier in time but not remove it.

For me, the real reason to work on inflation is the fluctuations it produces.

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\boldsymbol{x}^2$$

Line element on spacetime

"Cosmic time"

Scale factor

I have neglected curvature, but its influence scales away very fast

The Hubble parameter is

$$H = \frac{\dot{a}}{a}$$

Inflation is an era when $\ddot{a} > 0$

so

$$\dot{H}a + H\dot{a} = a\left(\dot{H} + H^2\right) = aH^2\left(\frac{\dot{H}}{H^2} + 1\right) > 0$$

or

$$\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$

Rather than cosmic time, we often measure the duration of inflation in terms of "e-folds"

$$\exp N = \frac{a(\text{now})}{a(\text{then})}$$

N is number of e-folds between then and now

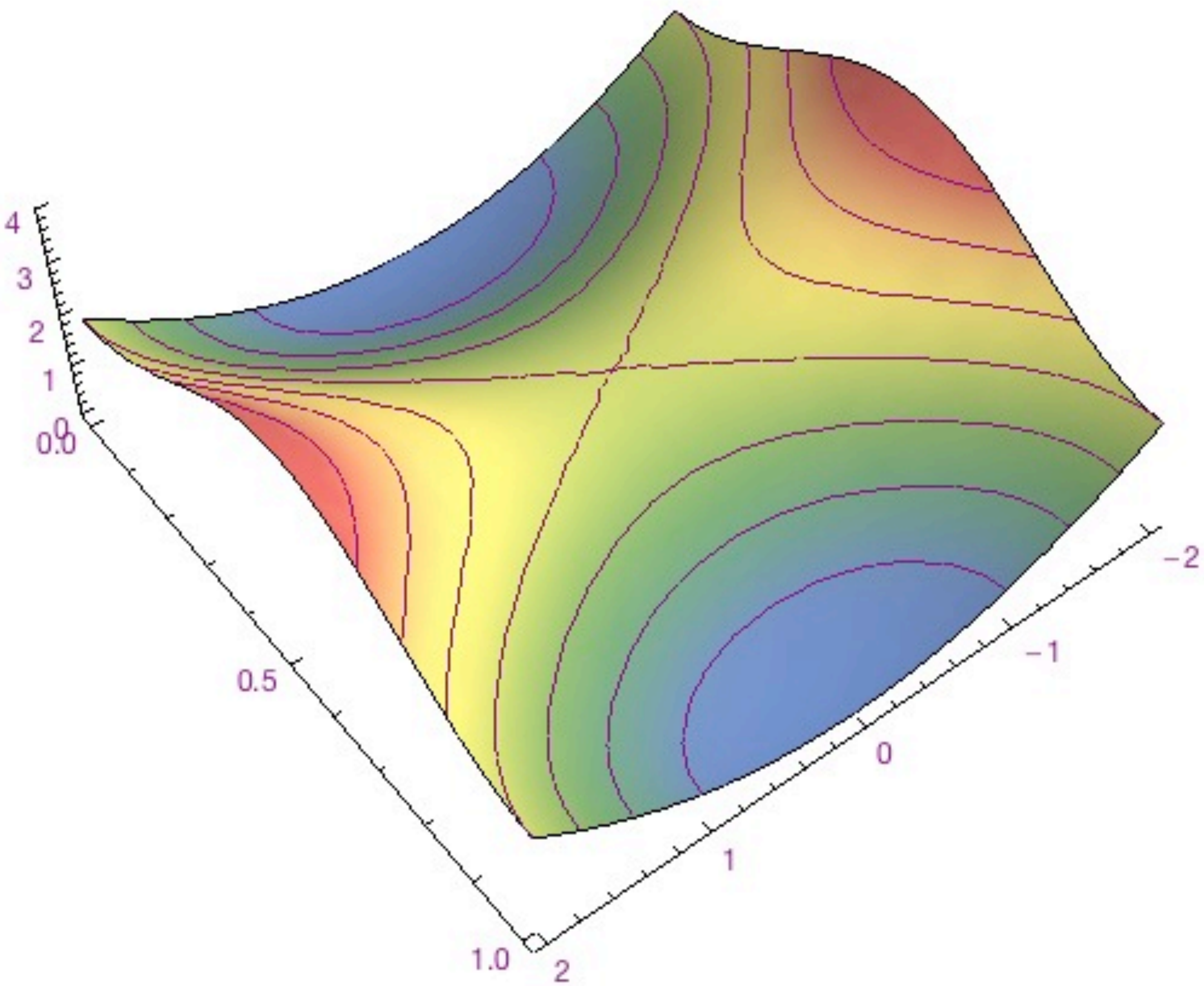
so

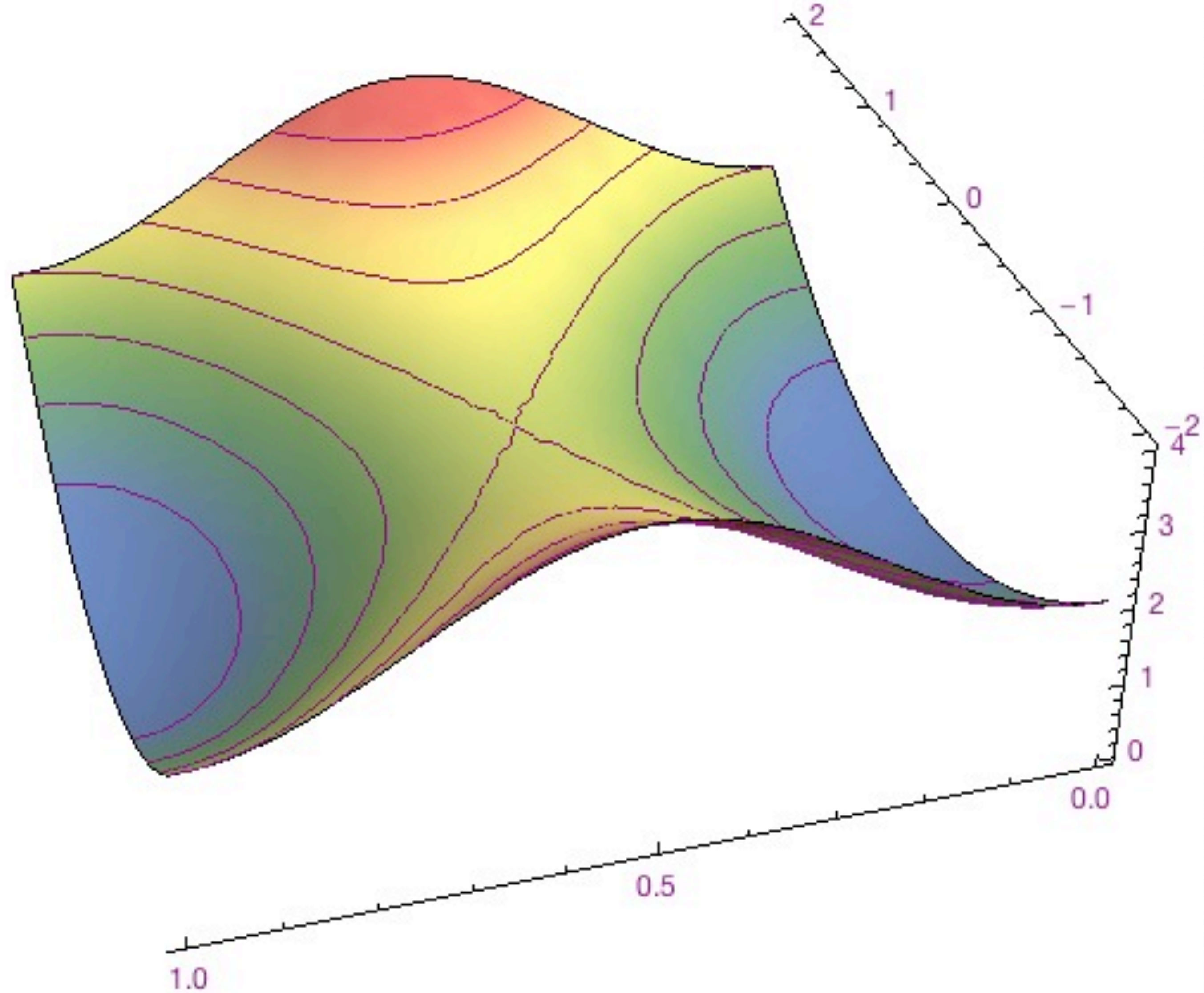
$$\exp(N)\,\mathrm{d}N = \frac{\dot{a}}{a(\text{reference})}\,\mathrm{d}t = \frac{a}{a(\text{reference})}\,H\,\mathrm{d}t$$

or $$\mathrm{d}N = H\,\mathrm{d}t$$

What ingredients are needed?

$$3H^2 M_{\rm P}^2 = \sum_\alpha \frac{1}{2}\dot{\phi}_\alpha \dot{\phi}_\alpha + V(\phi)$$





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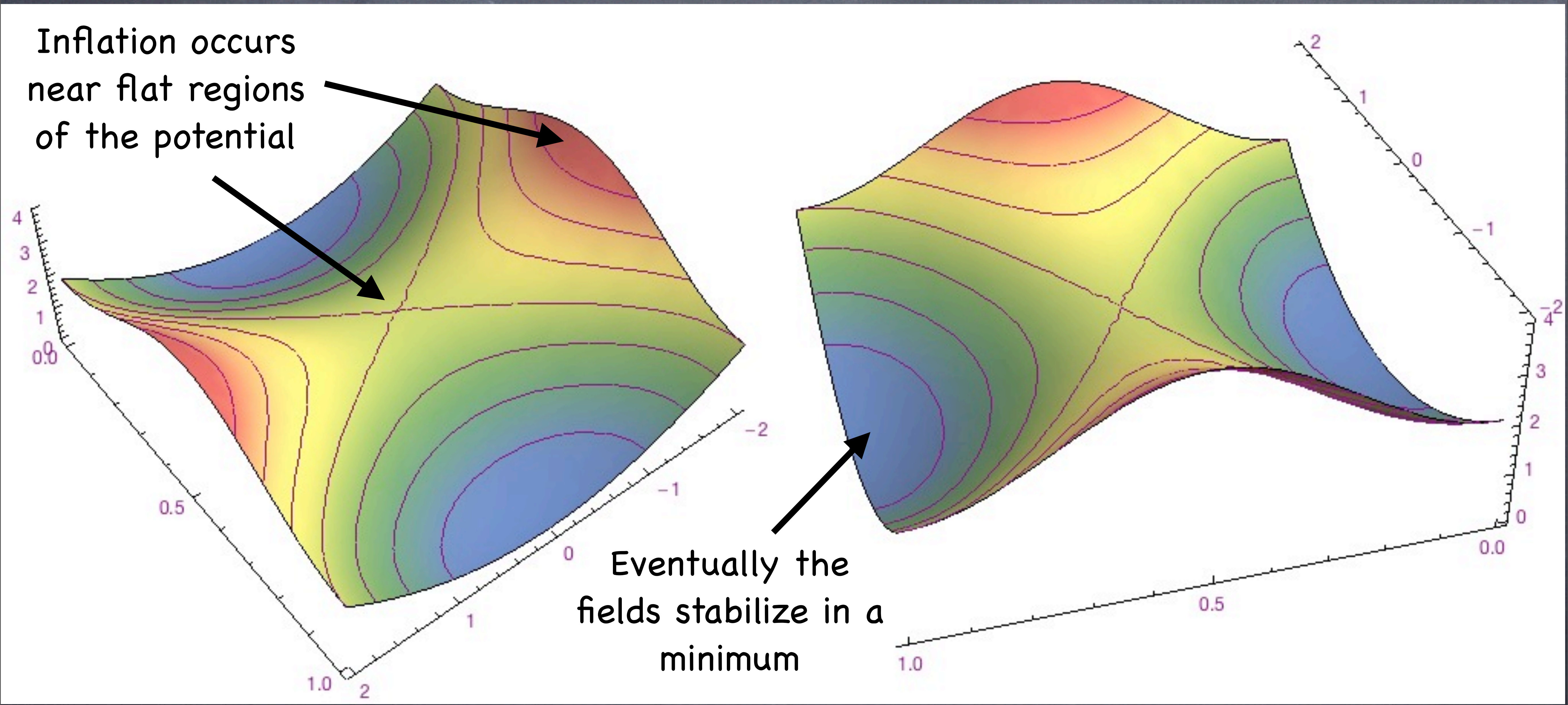
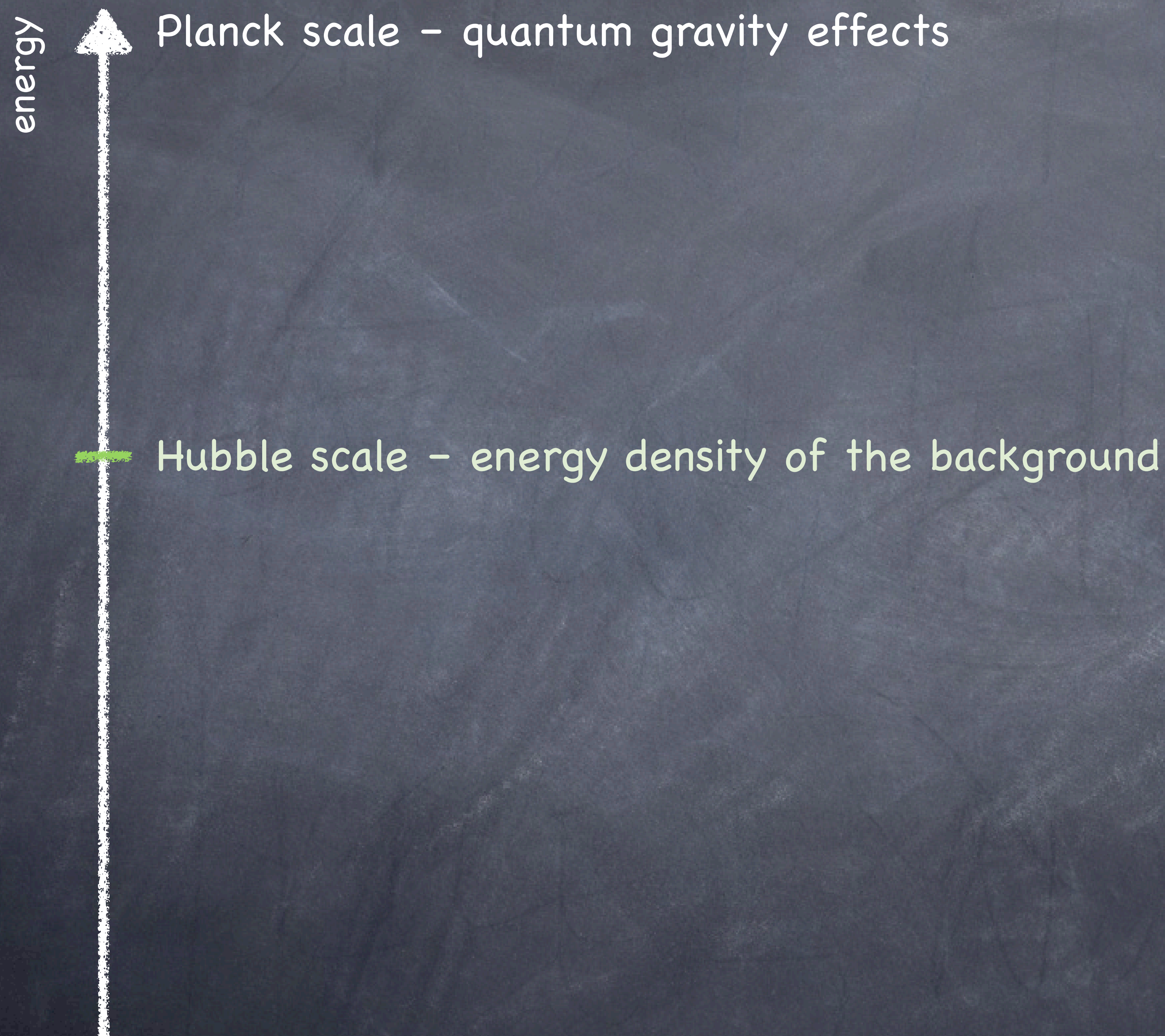


energy

Planck scale – quantum gravity effects


energy
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Hubble scale - energy density of the background

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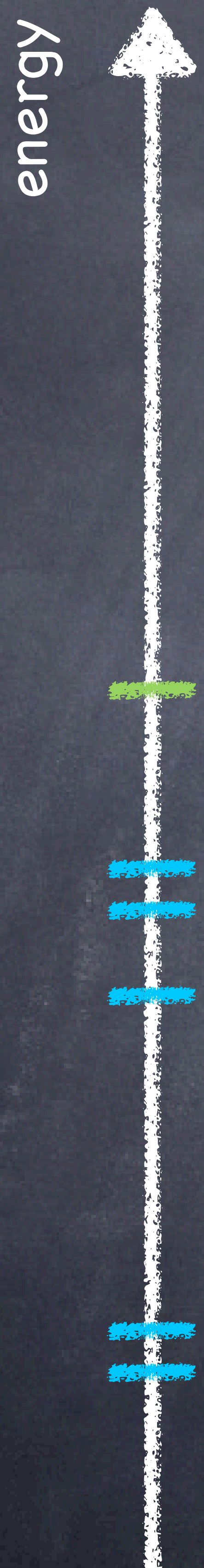
Planck scale - quantum gravity effects

Hubble scale - energy density of the background

At least one fluctuation which is light compared to the Hubble scale



Planck scale - quantum gravity effects

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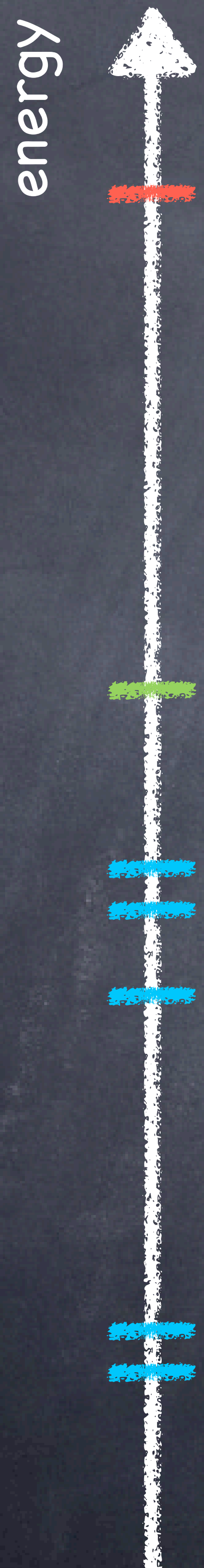
At least one fluctuation which is light compared to the Hubble scale

Possibly more light fluctuations



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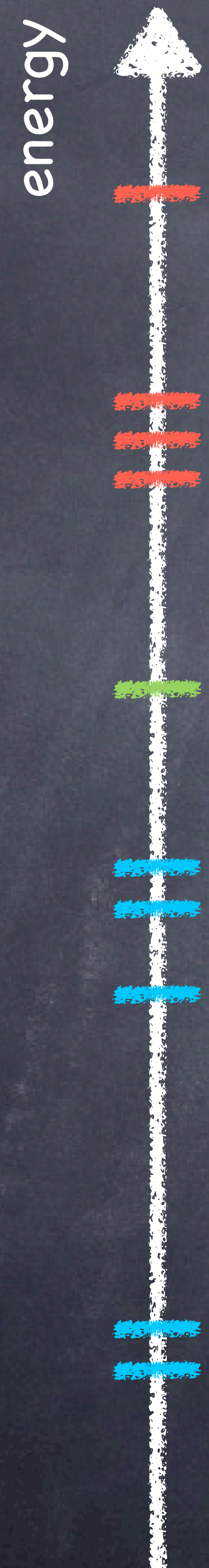

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In some theories, possibly interesting modes near the Hubble scale

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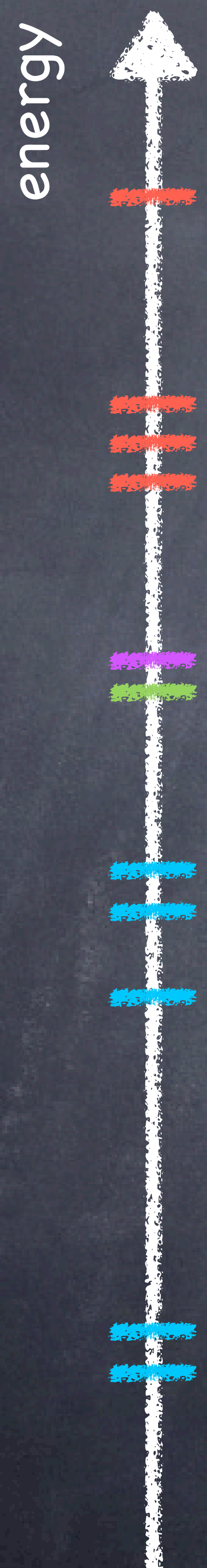

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Planck scale - quantum gravity effects

What is the physics which generates the background?
This is the province of detailed model-building and is notoriously difficult to get right. Typically, quantum effects spoil everything. There is not yet any successful approach.

Hubble scale - energy density of the background

At least one fluctuation which is light compared to the Hubble scale

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energy

Planck scale - quantum gravity effects

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Planck scale - quantum gravity effects

First, ignore any possible heavy or near-Hubble modes.

Where there is one remaining light mode, we have single-field inflation. At present, this is the best-understood case. Predictions decouple from the infrared behaviour of the theory.

At least one fluctuation which is light compared to the Hubble scale

Possibly more light fluctuations

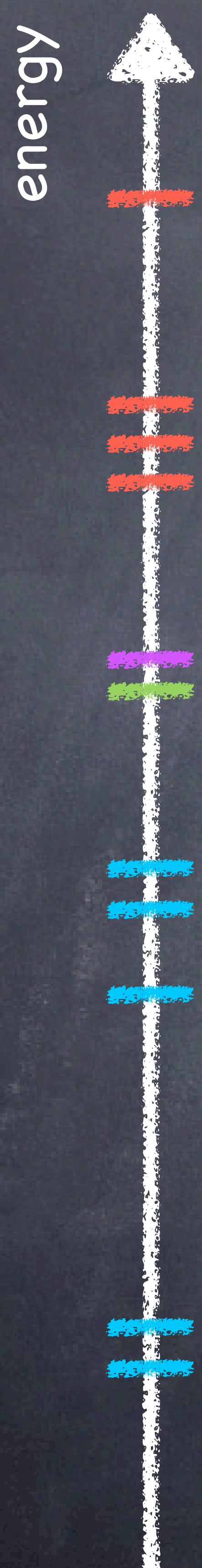
energy

Planck scale - quantum gravity effects

If there are multiple light modes then the situation is more complicated. Predictions are now sensitive to the infrared dynamics of the theory.
(We will see how this happens later.)

At least one fluctuation which is light compared to the Hubble scale

Possibly more light fluctuations



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Perhaps many heavy modes

Ideally, we would also like to detect the presence of heavy modes.

In flat-space quantum field theory, the Appelquist-Carazzone decoupling theorem is an obstruction to this.
But it does not apply in quite the same way with a dynamical background.

This means we can ask about the sensitivity of our predictions to the ultraviolet content of the model, as well as its infrared dynamics.

energy

TeV scale - energy scale of electroweak symmetry breaking

energy

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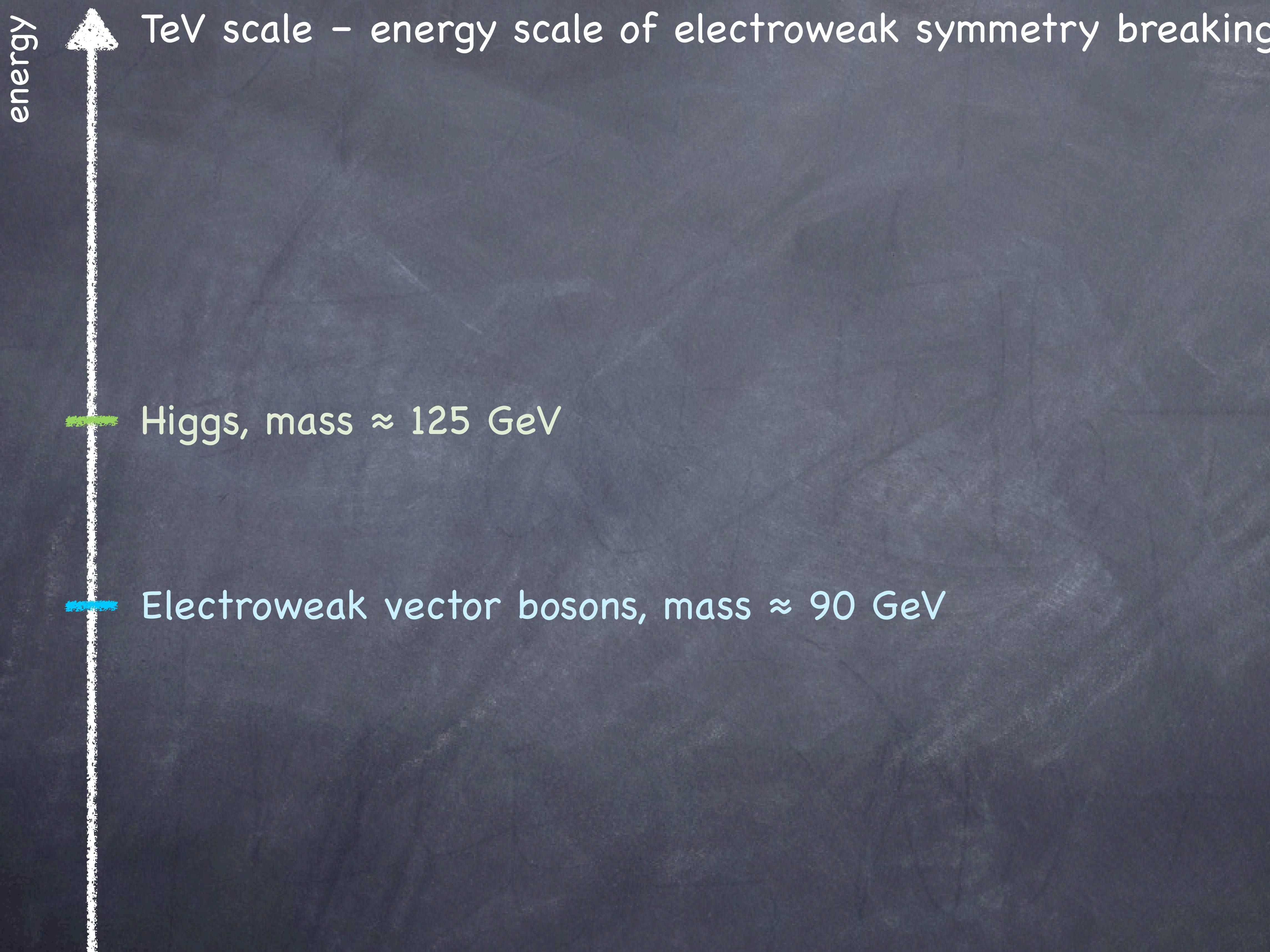
Higgs, mass ≈ 125 GeV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Higgs, mass ≈ 125 GeV
Electroweak vector bosons, mass ≈ 90 GeV

energy

TeV scale – energy scale of electroweak symmetry breaking

Higgs, mass ≈ 125 GeV

Electroweak vector bosons, mass ≈ 90 GeV

Lots of light stuff, electrons, neutrinos, …

energy

TeV scale - energy scale of electroweak symmetry breaking

Top quark, mass ≈ 170 GeV

Higgs, mass ≈ 125 GeV

Electroweak vector bosons, mass ≈ 90 GeV

Lots of light stuff, electrons, neutrinos, ...

energy

TeV scale - energy scale of electroweak symmetry breaking

Top quark, mass ≈ 170 GeV

Perhaps many heavy modes in a model like technicolour?

Higgs, mass ≈ 125 GeV

S, T, U parameters

Electroweak vector bosons, mass ≈ 90 GeV

Lots of light stuff, electrons, neutrinos, ...

energy

TeV scale – energy scale of electroweak symmetry breaking

Top quark, mass ≈ 170 GeV

~~Perhaps many heavy modes in a model like technicolour?~~ NO

Higgs, mass ≈ 125 GeV

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Electroweak vector bosons, mass ≈ 90 GeV

Lots of light stuff, electrons, neutrinos, ...

the mass of the top quark could be predicted, using high precision data from the accelerator LEP (Large Electron Positron) at the Laboratory CERN, Switzerland, several years before it was discovered in 1995 at the Fermi National Laboratory in USA.

... Similarly, comparison of theoretical values of quantum corrections involving the Higgs Boson with precision Measurements at LEP gives information on the mass of this as yet undiscovered particle.

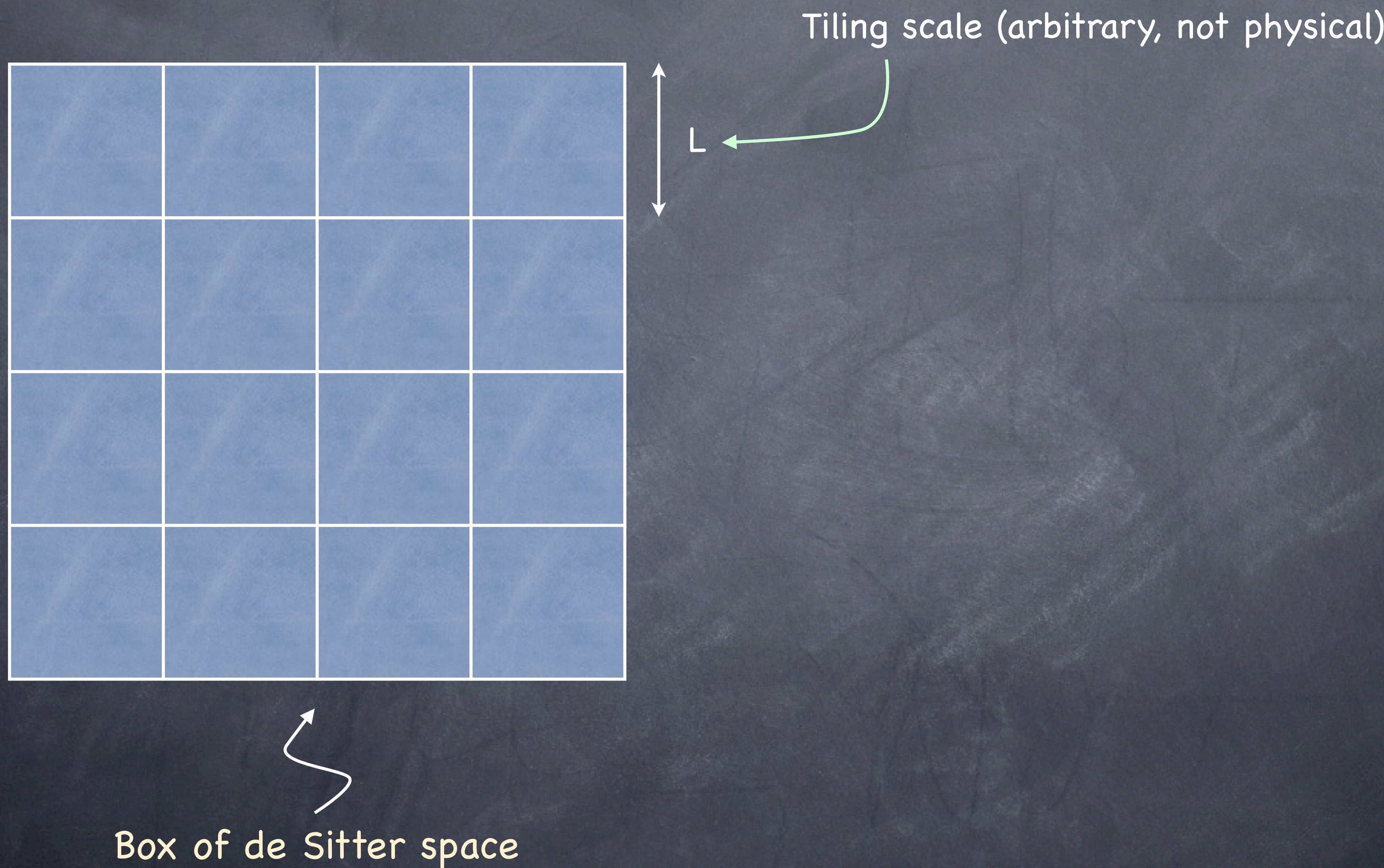
Nobel Prize citation, t'Hooft & Veltman 1999

This kind of sensitivity could let us rule out models based on the UV or IR content of the fluctuation spectrum, in the same way that precision electroweak measurements allowed us to rule out technicolour.

Of course, we always retain the option to go back to the "traditional" way of thinking about sensitivity to the UV – by searching for a consistent theory to describe the background.

It's not really clear how much success we can hope for with either route. But searching for the fluctuation spectrum is a much simpler first step, and anyway we can try it with data.

Fluctuations from inflation



Fluctuations from inflation

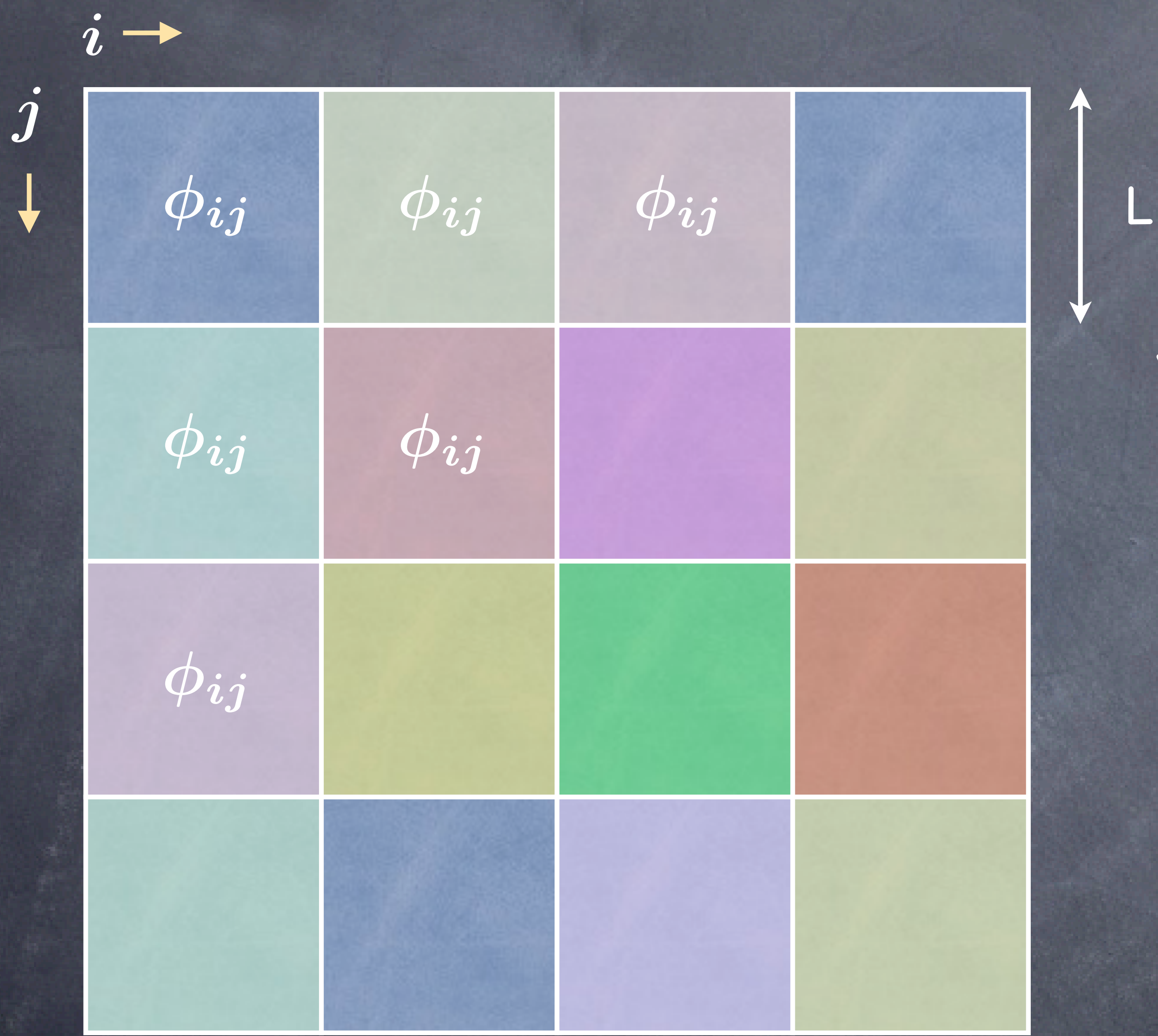
Tiling scale (arbitrary, not physical)

We need a nearly smooth background field Φ_{ij} in each "tile" or "box," which evolves coherently up to small gradient corrections

Box of de Sitter space

Fluctuations from inflation

Tiling scale (arbitrary, not physical)



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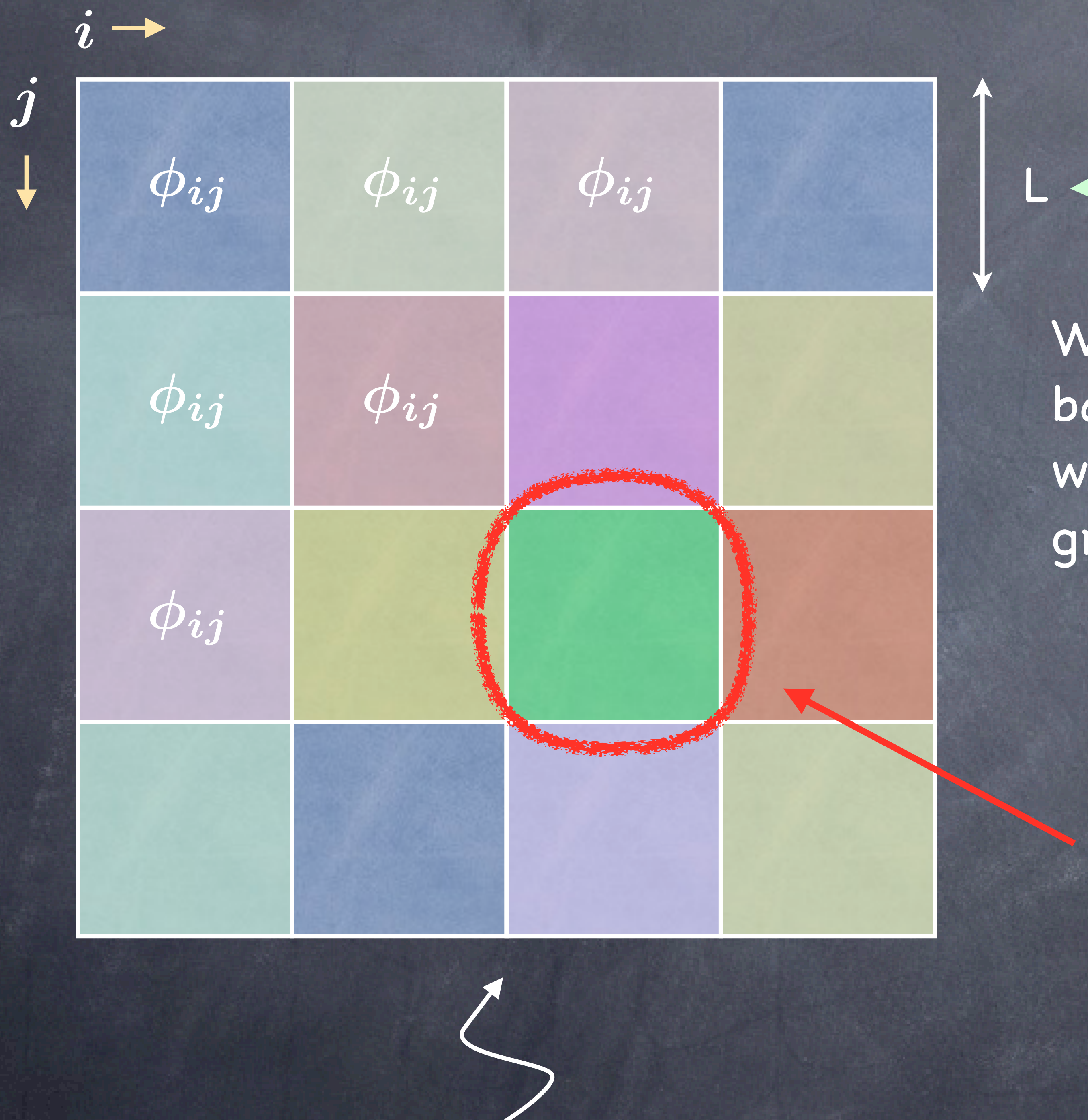

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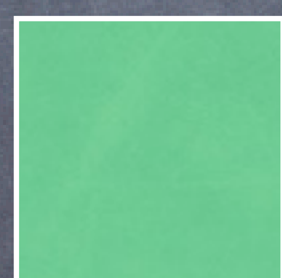
Focus on a single tile

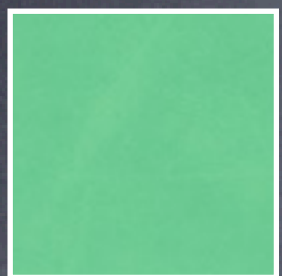
Box of de Sitter space



time

e-folds N

active scalars Φ



time

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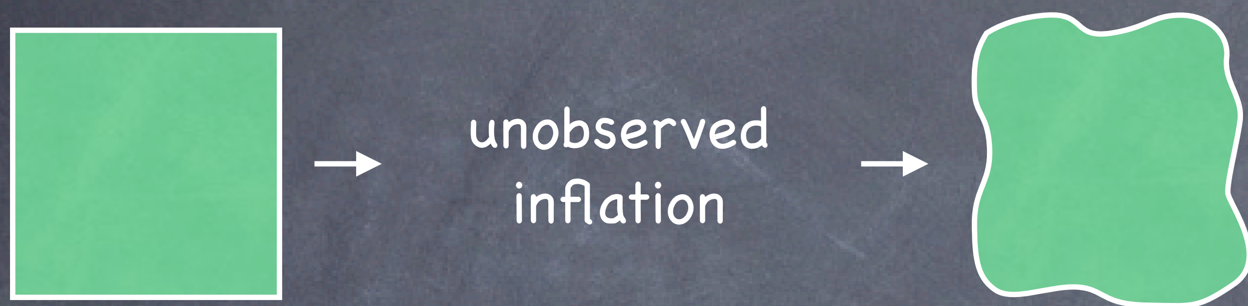
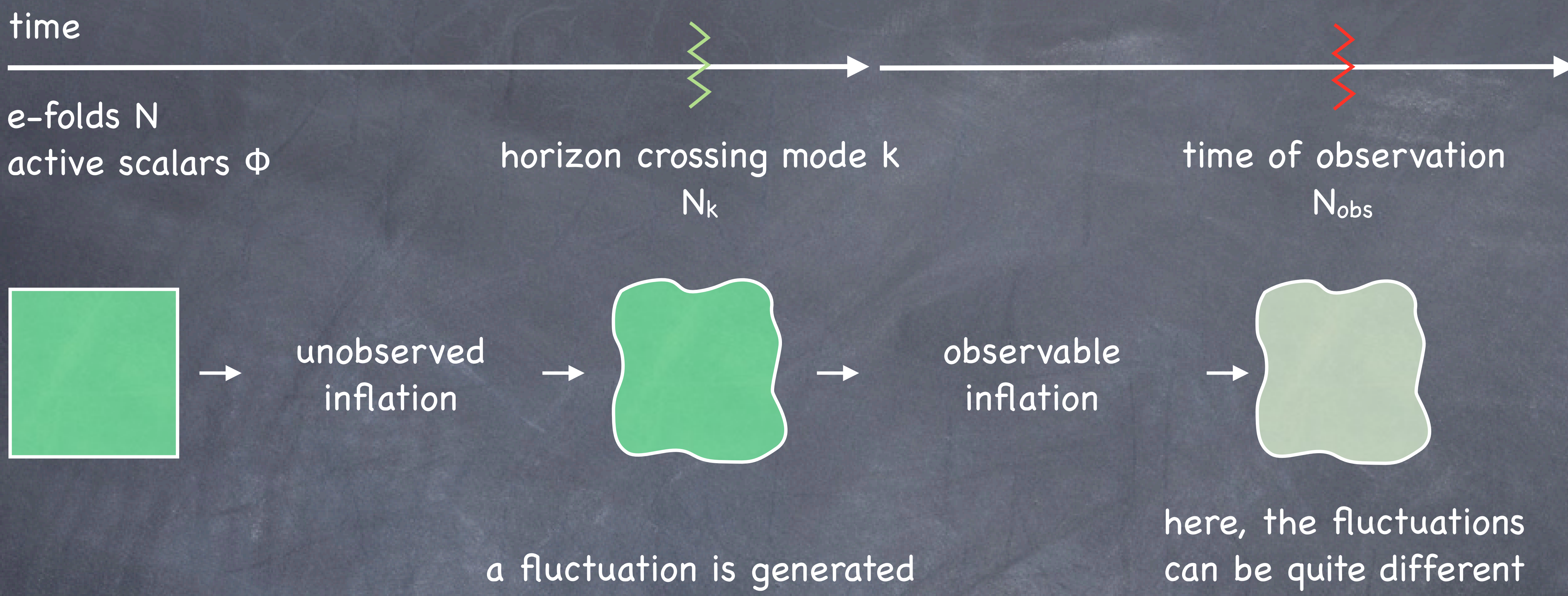
horizon crossing mode k

N_k



a fluctuation is generated


time
e-folds N
active scalars Φ
horizon crossing mode k
N_k
time of observation
N_{obs}
unobserved inflation
observable inflation
a fluctuation is generated
here, the fluctuations can be quite different

time

e-folds N
active scalars Φ

horizon crossing mode k
N_k

time of observation
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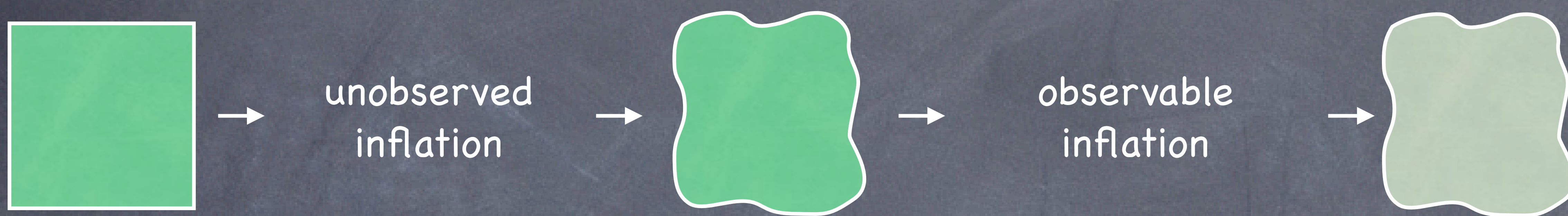

a fluctuation is generated

here, the fluctuations can be quite different

at horizon-crossing, we have two clear, separated scales which help control the calculation

time scales
(slow roll scales)

$$\epsilon \sim \frac{V'^2}{V^2} \qquad \eta \sim \frac{V''}{V} \qquad \xi \sim \frac{V'''V'}{V^2} \qquad \mathbf{10^{-2}}$$

quantum scale

$$\frac{H^2}{M_{\mathrm{P}}^2} \qquad \mathbf{10^{-10}} \text{ ish}$$

Fluctuations in de Sitter space

First, let's forget about interactions and study the free theory.
Later, we can go back and deal with interactions if we want.

$$S_2 = \frac{1}{2}\int \mathrm{d}^4x \, \sqrt{-g}\,\Big[(\partial\phi)^2 + V(\phi)\Big]$$

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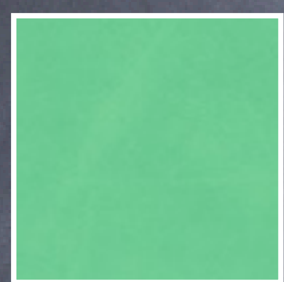

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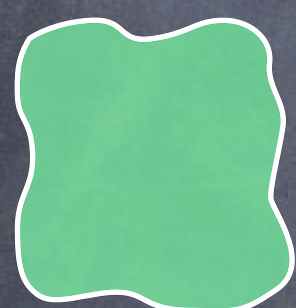

$$S_2 = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,\Big[(\partial\delta\phi)^2 + m^2\delta\phi^2\Big]$$

mass m comes from the
details of the background theory (V),
plus the background which is chosen,
plus mixing with gravity

What do we actually want to compute?

(Remember this is nontrivial - we need all that LSZ trauma for scattering)

In a quantum field theory, almost the only objects we have are correlation functions, so let's compute the simplest - the 2-point function.

In any case, we will need this for the propagator when we include interactions. I'm dropping δ, but this is for fluctuations.

$$G_2(x, y) = \langle \phi(x)\phi(y) \rangle$$

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$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \mathrm{d}\boldsymbol{x}^2 = a(\eta)^2 \Big[-\mathrm{d}\eta^2 + \mathrm{d}\boldsymbol{x}^2 \Big]$$

It is simplest to use a 3+1 split of spacetime $\longrightarrow$ $x = (\eta, \boldsymbol{x})$

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The expectation value means "expectation taken in the vacuum state".

$$\langle \text{vac} \mid \phi(\eta_1, \boldsymbol{x}_1)\phi(\eta_2, \boldsymbol{x}_2) \mid \text{vac} \rangle$$

But which vacuum are we talking about? Remember this implicitly includes assumptions about the UV behaviour.

For the time being, let's keep calculating

To leave our options open, let's work in an arbitrary mixed state described by a density matrix ρ, rather than the vacuum

$$\langle \phi(\eta_1, \boldsymbol{x}_1)\phi(\eta_2, \boldsymbol{x}_2)\rangle_\rho = \mathrm{tr}\left[\phi(\eta_1, \boldsymbol{x}_1)\phi(\eta_2, \boldsymbol{x}_2)\rho\right]$$

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$$= \int [\mathrm{d}^3\phi^*] \, \langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1)\phi(\eta_2, \boldsymbol{x}_2)\rho | \phi^* \rangle$$

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$$= \int [\mathrm{d}^3 \phi^*] \, \langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) \phi(\eta_2, \boldsymbol{x}_2) \rho | \phi^* \rangle$$

this means an integral over a 3-dimensional field $\Phi(x)$

we are thinking in a Schrödinger picture for field theory, with the field in a state with configuration $\Phi^*(x)$ at a time η_* later than either η_1 or η_2.

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$$= \int [\mathrm{d}^3\phi^*]\ \langle\phi^*|\phi(\eta_1,\boldsymbol{x}_1)\phi(\eta_2,\boldsymbol{x}_2)\rho|\phi^*\rangle$$

Inserting two more resolutions of unity, thought of as states of the field at an early time η_0, we get

$$= \int [\mathrm{d}^3\phi^*\ \mathrm{d}^3\phi^+\ \mathrm{d}^3\phi^-]\ \langle\phi^*|\phi(\eta_1,\boldsymbol{x}_1)|\phi^+\rangle\langle\phi^+|\rho|\phi^-\rangle\langle\phi^-|\phi(\eta_2,\boldsymbol{x}_2)|\phi^*\rangle$$

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integrals over 3d field configurations

Monday, 16 July 12

This says that the two-point expectation value is built up out of bits we can identify

$\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle$ amplitude for transition from state Φ^+ to Φ^* with emission of an extra Φ

The LSZ formula would tell us that the rate for this transition involves

$$\int_{\text{phase space}} |\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle|^2 \times \text{propagator factors}$$

here we are not calculating rates, but expectation values

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$$|\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle|^2 = \text{probability of transition } \phi^+ \to \phi^* \text{ with emission of } \phi$$

integrate over all field configurations

$$\int [\mathrm{d}^3\phi^* \, \mathrm{d}^3\phi^+] \, |\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle|^2 = \text{probability of all transitions} + \text{emission of } \phi$$

$$\int [\mathrm{d}^3\phi^* \, \mathrm{d}^3\phi^+] \, |\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle|^2 = \text{probability of all transitions + emission of } \phi$$

what we actually have is very nearly the same

$$\int [\mathrm{d}^3\phi^* \, \mathrm{d}^3\phi^+ \, \mathrm{d}^3\phi^-] \, \langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle \underbrace{\langle \phi^+ | \rho | \phi^- \rangle}_{\text{weighting}} \langle \phi^- | \phi(\eta_2, \boldsymbol{x}_2) | \phi^* \rangle$$

(δ-function in a pure state, eg., vacuum)

What we have computed, if we begin in the vacuum, is the probability for all transitions from the vacuum which are accompanied by emission of a Φ "particle"

$$\int [\mathrm{d}^3\phi^*\ \mathrm{d}^3\phi^+]\ |\langle\phi^*|\phi(\eta_1,\boldsymbol{x}_1)|\phi^+\rangle|^2 = \text{probability of all transitions + emission of } \phi$$

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$$\int [\mathrm{d}^3\phi^* \; \mathrm{d}^3\phi^+] \; |\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle|^2 = \text{probability of all transitions + emission of } \phi$$

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In the general case, this formula computes the correctly weighted average of all transition probabilities from states in the statistical ensemble described by ρ which are accompanied by emission of a Φ

$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+] \ |\langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle|^2 = \text{probability of all transitions} + \text{emission of } \phi$$

what we actually have is very nearly the same

$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+ \ \mathrm{d}^3\phi^-] \ \langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle \langle \phi^+ | \rho | \phi^- \rangle \langle \phi^- | \phi(\eta_2, \boldsymbol{x}_2) | \phi^* \rangle$$

In the general case, this formula computes the correctly weighted average of all transition probabilities from states in the statistical ensemble described by ρ which are accompanied by emission of a Φ

Because we don't insist that the transition is to vacuum + Φ, we can find some interesting effects.

However, if we start with the vacuum, the probability is dominated by the amplitude for transition to vacuum + Φ

What we have arrived at is the “Schwinger” or “in-in” formulation of expectation values

It is very closely related to the Cutkosky rules (QCD, nuclear physics), finite density physics (condensed matter, critical phenomena, dynamical critical phenomena and phase transitions, ...) and finite temperature physics (in the real-time formulation)

It can also be thought of as an initial-value formulation of QFT, compared to the usual Feynman boundary-value formulation

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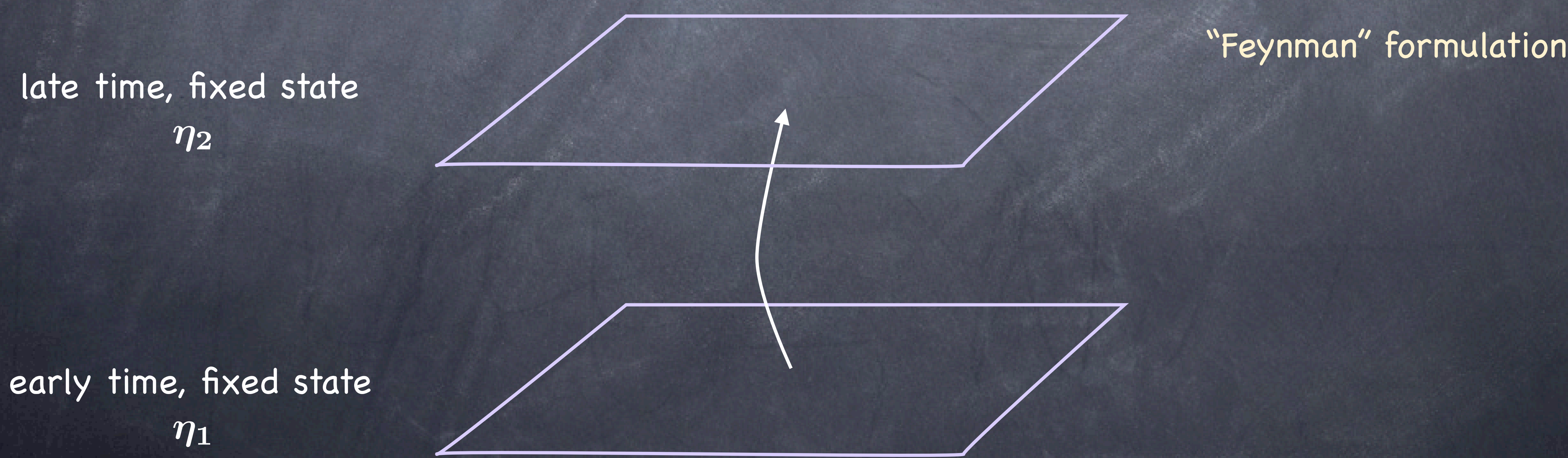

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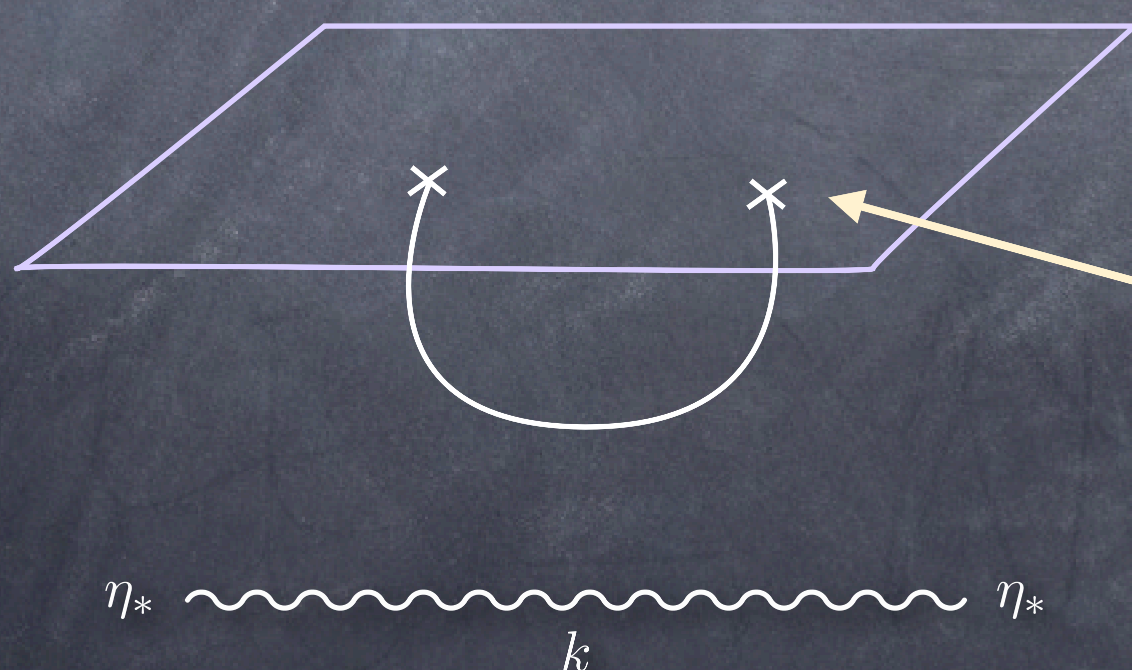
~~late time, fixed state~~

~~η_2~~

early time, fixed state

η_1



~~"Feynman" formulation~~

"Schwinger" formulation

both external legs at late time, so no quanta enter the diagram

instead, they are nucleated like an instanton

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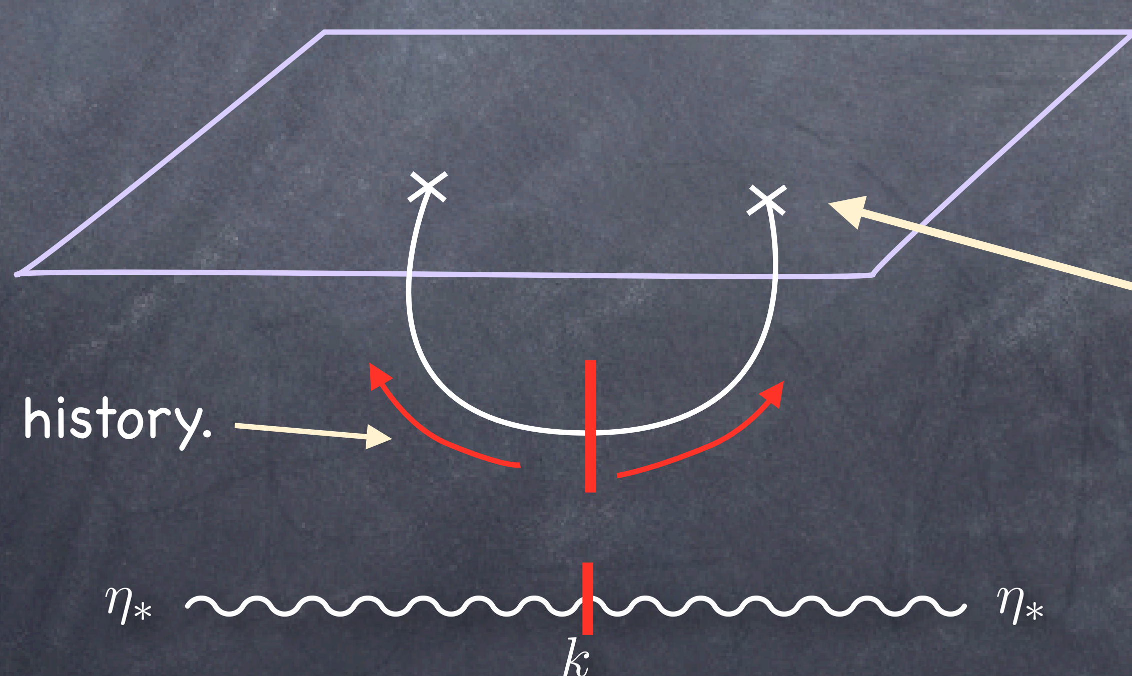

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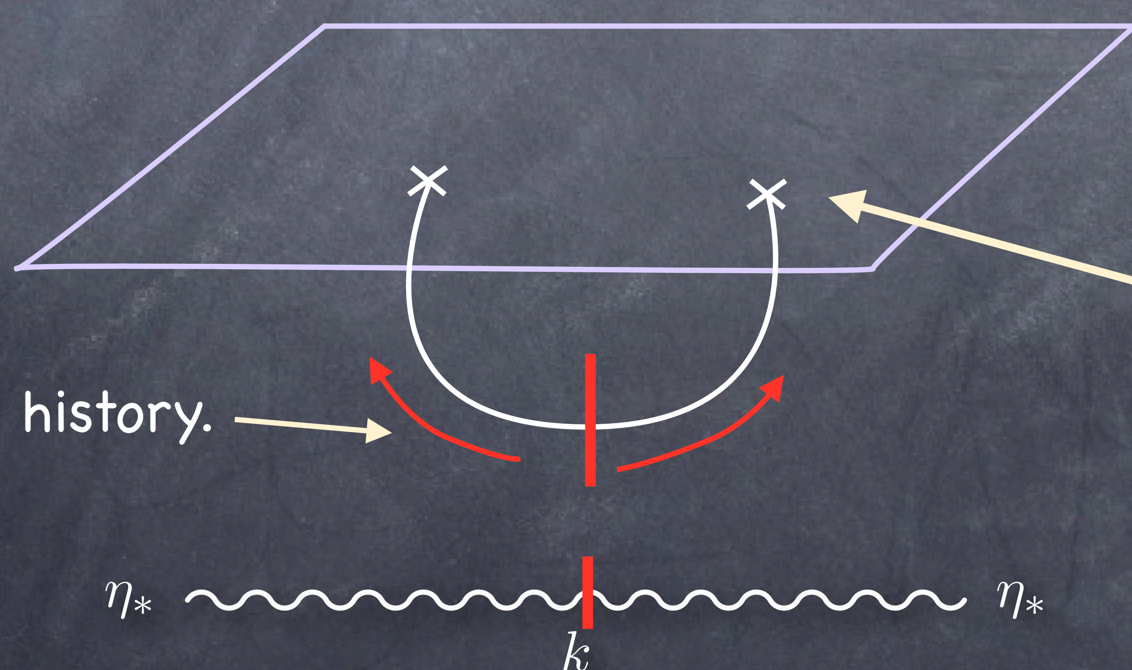

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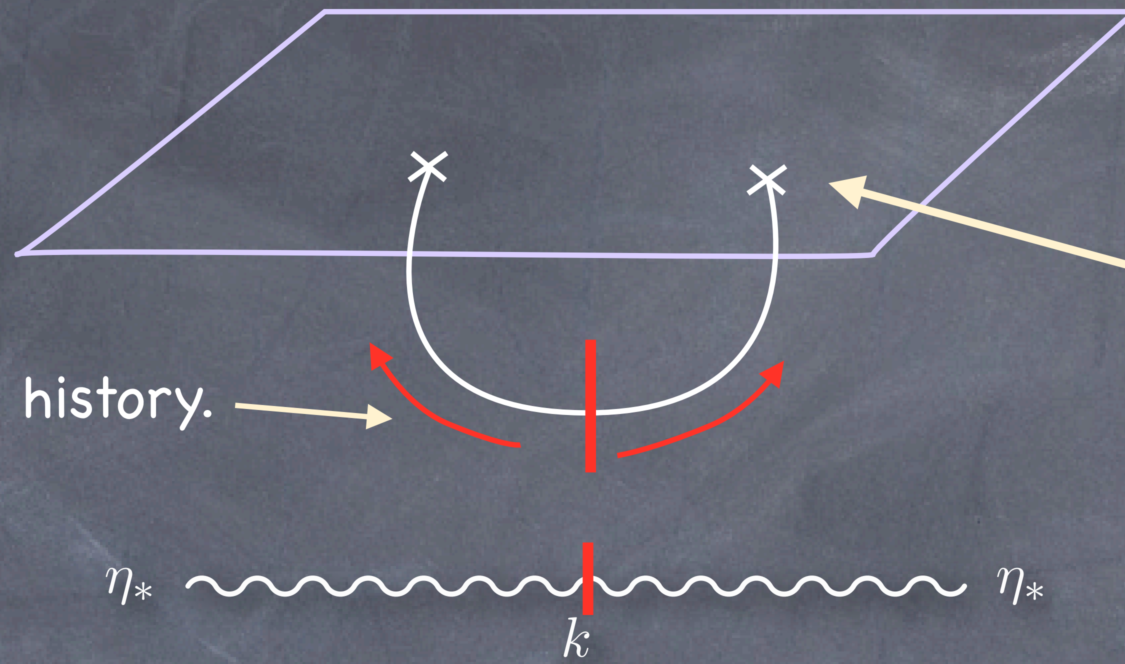

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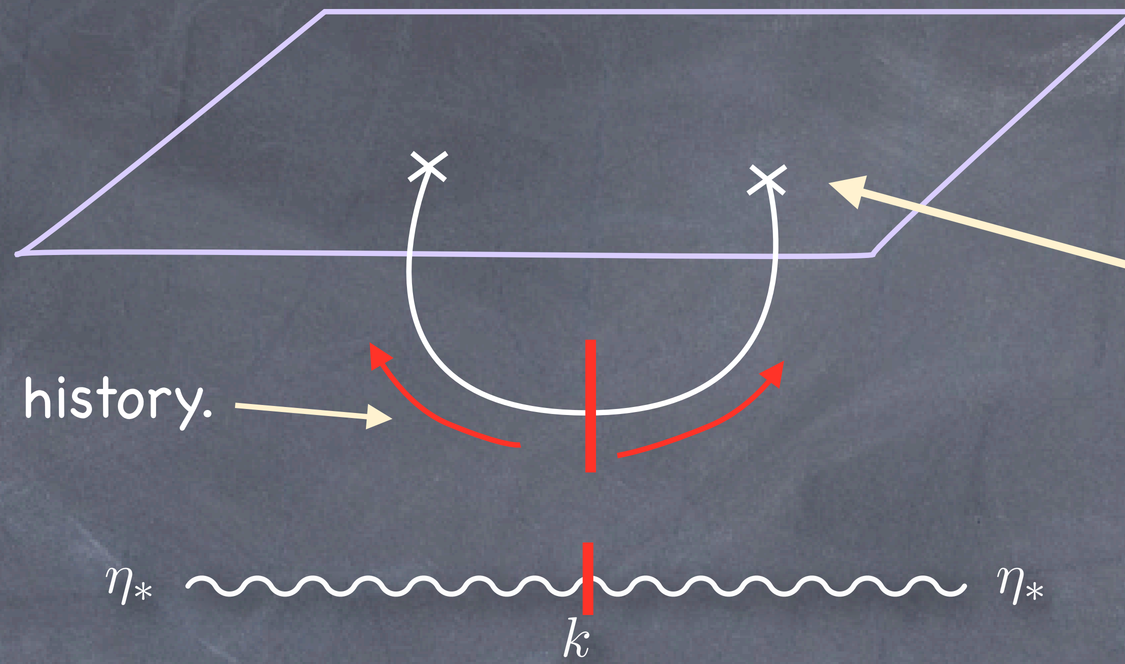

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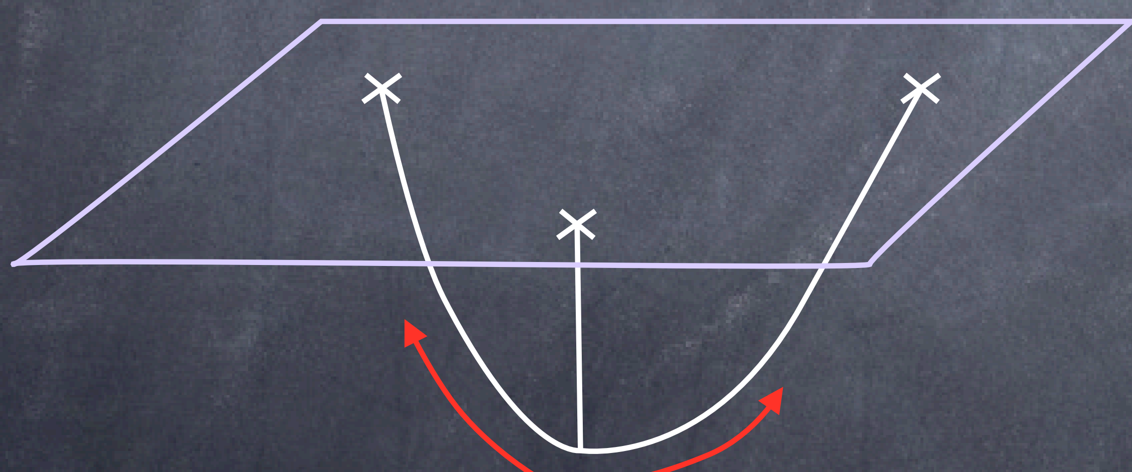
precisely the same thing happens for higher n-point functions



3 quanta nucleate and separate

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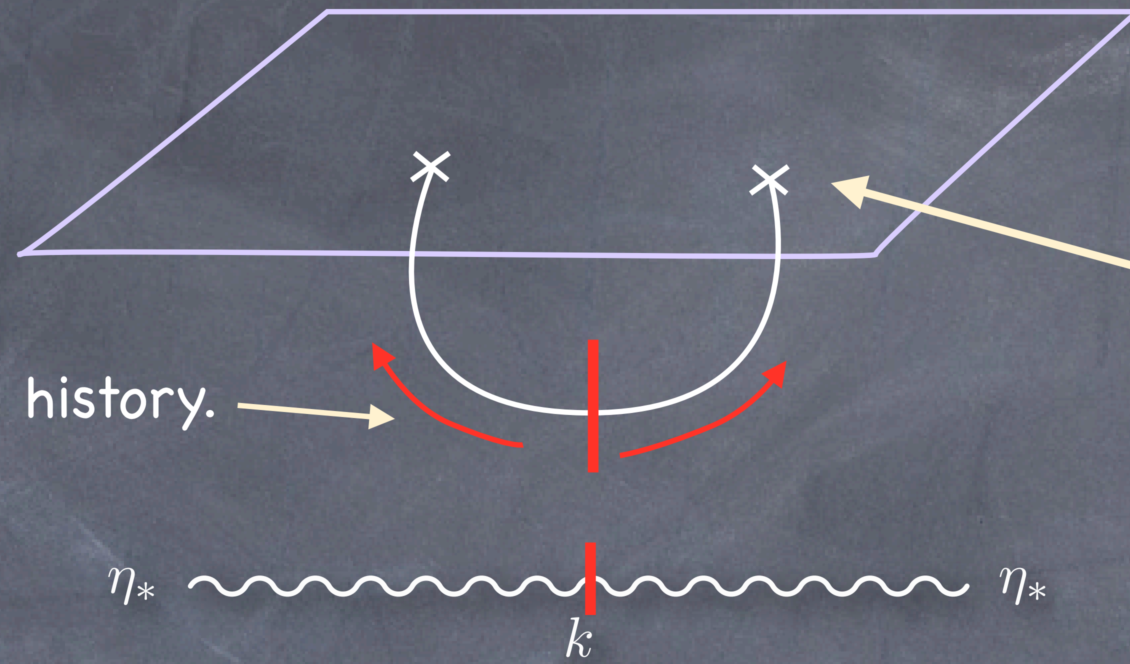

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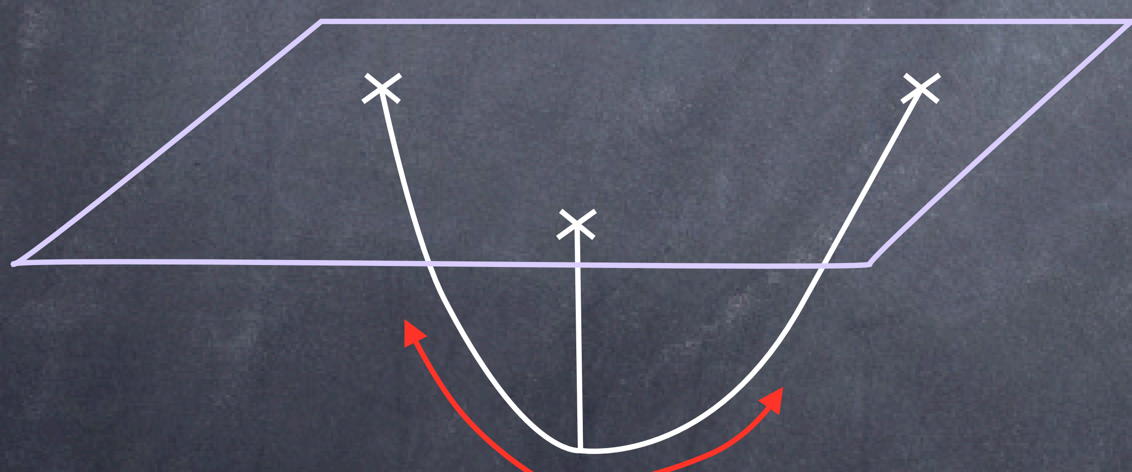

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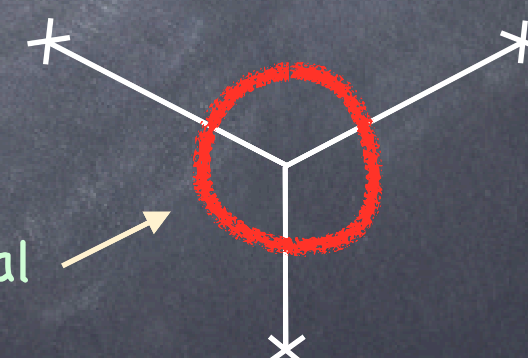

the Feynman rules always give an integral over all space

$$\int \mathrm{d}^4x\sqrt{-g}\cdots$$

$$\mathrm{d}^3x\,\mathrm{d}t\;a(t)^3$$

The two-point function - calculation

Now we understand what we are computing, we have to finish the job and calculate it

Our first problem is the statistical weight

$$\langle \phi^+ | \rho | \phi^- \rangle$$

Each state is a field configuration at the initial time η_0.

So we expect a functional of the fields evaluated at that time

$$\langle \phi^+ | \rho | \phi^- \rangle = \exp(-\mathcal{S})$$

(eg., by Euclidean time path integral)

$$\mathcal{S} = \frac{1}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[f_{\boldsymbol{k}} \Big(|\phi_0^+|^2 + |\phi_0^-|^2 \Big) - g_{\boldsymbol{k}} \Big(\phi_0^+ \phi_0^{-*} + \phi_0^{+*} \phi_0^- \Big) \right]$$

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depend on the initial state/ensemble (more complex if not Gaussian)

vacuum

$$f_{\boldsymbol{k}} = \omega_{\boldsymbol{k}} = \boldsymbol{k}^2 + m^2$$

$$g_{\boldsymbol{k}} = 0$$

finite temperature T = 1/β

$$f_{\boldsymbol{k}} = \omega_{\boldsymbol{k}} \coth \omega_{\boldsymbol{k}} \beta$$

$$g_{\boldsymbol{k}} = \frac{\omega_{\boldsymbol{k}}}{\sinh \omega_{\boldsymbol{k}} \beta}$$

Each transition amplitude can be calculated by a path integral

$$\langle \phi^* | \mathcal{O} | \phi^+ \rangle = \int [\mathrm{d}^4\phi]_{\phi^+}^{\phi^*} \; \mathcal{O} \; \exp(\mathrm{i}S[\phi])$$

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which interpolate between Φ^+ at the initial time η_0
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ie, 4-dimensional fields

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ie, 4-dimensional fields

The backwards transitions can be calculated by Hermitian conjugation

$$\langle\phi^-|\mathcal{O}|\phi^*\rangle = \left(\int[\mathrm{d}^4\phi]_{\phi^-}^{\phi^*}\ \mathcal{O}\ \exp(\mathrm{i}S[\phi])\right)^\dagger = \int[\mathrm{d}^4\phi]_{\phi^-}^{\phi^*}\ \mathcal{O}^\dagger\ \exp(-\mathrm{i}S[\phi])$$

Now we collect all the pieces!

$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+ \ \mathrm{d}^3\phi^-] \ \langle\phi^*|\phi(\eta_1, \boldsymbol{x}_1)|\phi^+\rangle\langle\phi^+|\rho|\phi^-\rangle\langle\phi^-|\phi(\eta_2, \boldsymbol{x}_2)|\phi^*\rangle$$

$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+ \ \mathrm{d}^3\phi^-]$$

Now we collect all the pieces!

$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+ \ \mathrm{d}^3\phi^-] \ \underline{\langle\phi^*|\phi(\eta_1,\boldsymbol{x}_1)|\phi^+\rangle}\langle\phi^+|\rho|\phi^-\rangle\langle\phi^-|\phi(\eta_2,\boldsymbol{x}_2)|\phi^*\rangle$$

$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+ \ \mathrm{d}^3\phi^-] \int [\mathrm{d}^4\phi_A]_{\phi^+}^{\phi^*} \ \phi_A(\eta_1,\boldsymbol{x}_1) \exp(\mathrm{i}S[\phi_A])$$

Now we collect all the pieces!

$$\int [\mathrm{d}^3\phi^* \; \mathrm{d}^3\phi^+ \; \mathrm{d}^3\phi^-] \; \langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle \underline{\langle \phi^+ | \rho | \phi^- \rangle} \langle \phi^- | \phi(\eta_2, \boldsymbol{x}_2) | \phi^* \rangle$$

$$\int [\mathrm{d}^3\phi^* \; \mathrm{d}^3\phi^+ \; \mathrm{d}^3\phi^-] \int [\mathrm{d}^4\phi_A]_{\phi^+}^{\phi^*} \; \phi_A(\eta_1, \boldsymbol{x}_1) \exp(\mathrm{i}S[\phi_A])$$

$$\times \; \exp\left(-\mathcal{S}\right)$$

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$$\times \; \exp\left(-\mathcal{S}\right)$$

$$\times \int [\mathrm{d}^4\phi_B]_{\phi_-}^{\phi^*} \; \phi_B(\eta_2, \boldsymbol{x}_2) \exp(-\mathrm{i}S[\phi_B])$$

Now we collect all the pieces!

$$\int [\mathrm{d}^3\phi^* \, \mathrm{d}^3\phi^+ \, \mathrm{d}^3\phi^-] \, \langle \phi^* | \phi(\eta_1, \boldsymbol{x}_1) | \phi^+ \rangle \langle \phi^+ | \rho | \phi^- \rangle \langle \phi^- | \phi(\eta_2, \boldsymbol{x}_2) | \phi^* \rangle$$

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$$\times \exp(-\mathcal{S})$$

$$\times \int [\mathrm{d}^4\phi_B]_{\phi_-}^{\phi^*} \, \phi_B(\eta_2, \boldsymbol{x}_2) \exp(-\mathrm{i}S[\phi_B])$$

We can merge these into one integral over all 4d fields ending in Φ*

$$\int [\mathrm{d}^4\phi_A]^{\phi_*}$$

unrestricted

Now we collect all the pieces!

$$\int [\mathrm{d}^3\phi^* \; \mathrm{d}^3\phi^+ \; \mathrm{d}^3\phi^-] \; \langle\phi^*|\phi(\eta_1, \boldsymbol{x}_1)|\phi^+\rangle\langle\phi^+|\rho|\phi^-\rangle\langle\phi^-|\phi(\eta_2, \boldsymbol{x}_2)|\phi^*\rangle$$

$$\int [\mathrm{d}^3\phi^* \; \mathrm{d}^3\phi^+ \; \mathrm{d}^3\phi^-] \int [\mathrm{d}^4\phi_A]_{\phi^+}^{\phi^*} \; \phi_A(\eta_1, \boldsymbol{x}_1) \exp(\mathrm{i}S[\phi_A])$$

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$$\int [\mathrm{d}^4\phi_A]^{\phi_*} \qquad \int [\mathrm{d}^4\phi_B]^{\phi_*}$$

unrestricted unrestricted

Now we collect all the pieces!

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$$\int [\mathrm{d}^3\phi^* \ \mathrm{d}^3\phi^+ \ \mathrm{d}^3\phi^-] \int [\mathrm{d}^4\phi_A]^{\phi^*}_{\phi^+} \ \phi_A(\eta_1, \boldsymbol{x}_1) \exp(\mathrm{i}S[\phi_A])$$

$$\times \ \exp\left(-\mathcal{S}\right)$$

$$\times \int [\mathrm{d}^4\phi_B]^{\phi^*}_{\phi_-} \ \phi_B(\eta_2, \boldsymbol{x}_2) \exp(-\mathrm{i}S[\phi_B])$$

We can merge these into one integral over all 4d fields ending in Φ*

$$\int [\mathrm{d}^4\phi_A]^{\phi_*}$$ unrestricted $$\int [\mathrm{d}^4\phi_B]^{\phi_*}$$ unrestricted

We can merge these into unrestricted integrals + δ-function

$$\int [\mathrm{d}^4\phi_A \ \mathrm{d}^4\phi_B] \ \delta[\phi_A(\eta_*, \boldsymbol{x}) - \phi_B(\eta_*, \boldsymbol{x})]$$ ⟵ for all x

After all that, we have

$$\int [\mathrm{d}^4\phi_A \; \mathrm{d}^4\phi_B] \; \delta[\phi_A(\eta_*, \boldsymbol{x}) - \phi_B(\eta_*, \boldsymbol{x})] \; \phi_A(\eta_1, \boldsymbol{x}_1)\phi_B(\eta_2, \boldsymbol{x}_2) \\ \times \exp\left(\mathrm{i}S[\phi_A] - \mathrm{i}S[\phi_B] - \mathcal{S}\right)$$

The δ-function can be implemented using

$$\delta[\phi_A(\eta_*, \boldsymbol{x}) - \phi_B(\eta_*, \boldsymbol{x}] \propto \lim_{\epsilon \to 0} \exp\left[-\frac{1}{\epsilon} \int \mathrm{d}^3x \; a^3 \Big(\phi_A(\eta_*, \boldsymbol{x}) - \phi_B(\eta_*, \boldsymbol{x}) \Big)^2 \right]$$

After all that, we have

$$\int [\mathrm{d}^4\phi_A\ \mathrm{d}^4\phi_B]\ \delta[\phi_A(\eta_*,\boldsymbol{x}) - \phi_B(\eta_*,\boldsymbol{x})]\ \phi_A(\eta_1,\boldsymbol{x}_1)\phi_B(\eta_2,\boldsymbol{x}_2) \\ \times \exp\left(\mathrm{i}S[\phi_A] - \mathrm{i}S[\phi_B] - \mathcal{S}\right)$$

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Collecting all terms in the exponential, we have an "effective action"
(note that the singular terms have no factor of i)

$$\exp\left\{\mathrm{i}S[\phi_A] - \mathrm{i}S[\phi_B] + \delta(\eta-\eta_0)\times(\mathcal{S}\ \text{terms}) + \delta(\eta-\eta_*)\times(\delta\text{-fn terms})\right\}$$

initial conditions

fields share the same final state

After all that, we have

$$\int [\mathrm{d}^4\phi_A \, \mathrm{d}^4\phi_B] \, \delta[\phi_A(\eta_*, \boldsymbol{x}) - \phi_B(\eta_*, \boldsymbol{x})] \, \phi_A(\eta_1, \boldsymbol{x}_1)\phi_B(\eta_2, \boldsymbol{x}_2) \times \exp\left(\mathrm{i}S[\phi_A] - \mathrm{i}S[\phi_B] - \mathcal{S}\right)$$

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$$\exp\left\{ \mathrm{i}S[\phi_A] - \mathrm{i}S[\phi_B] + \delta(\eta - \eta_0) \times (\mathcal{S} \text{ terms}) + \delta(\eta - \eta_*) \times (\delta\text{-fn terms}) \right\}$$

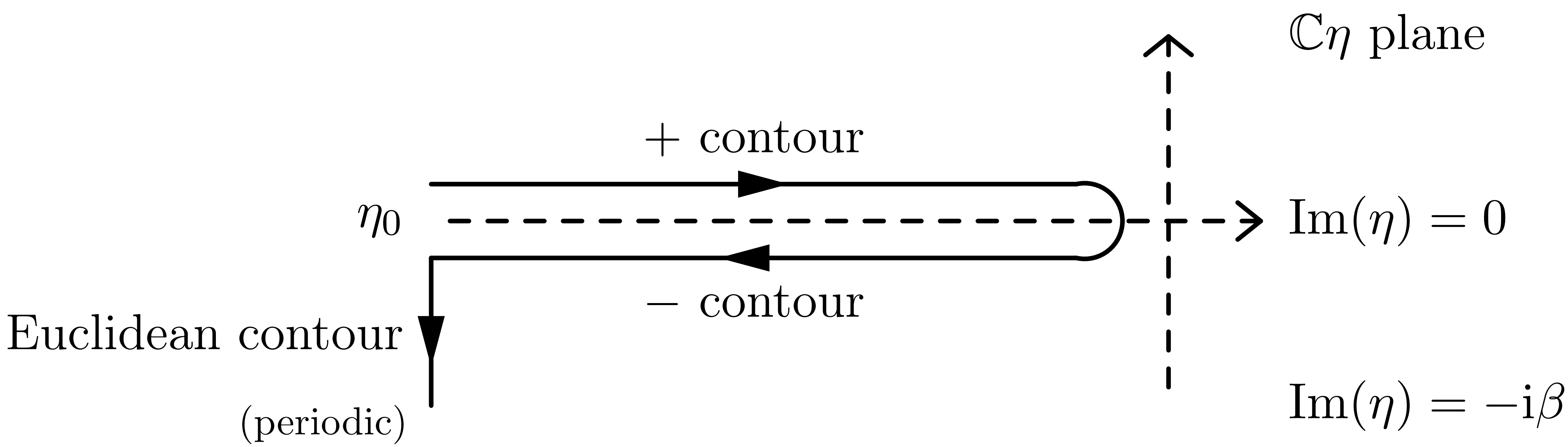
initial conditions

fields share the same final state

(remember we said this could be represented as a Euclidean path integral)

To simplify the notation, it is helpful to consolidate the + and - fields into a single integral over a contour.

We also relabel A → + and B → -



“Kadanoff-Baym” contour

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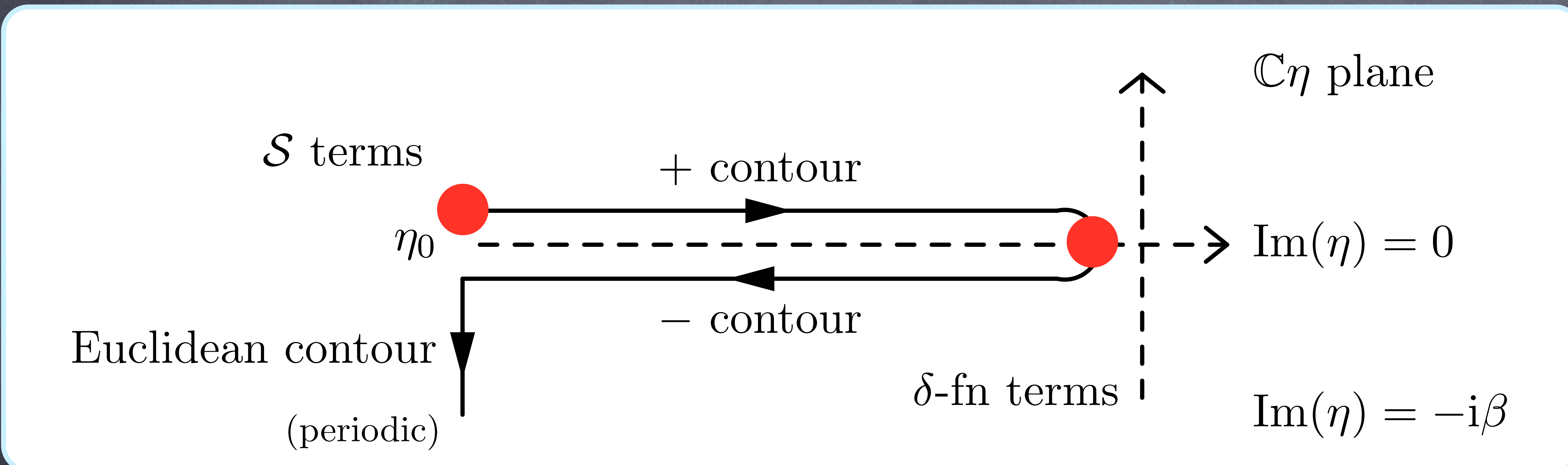

"Kadanoff-Baym" contour

The S-terms and the δ-function terms give boundary conditions

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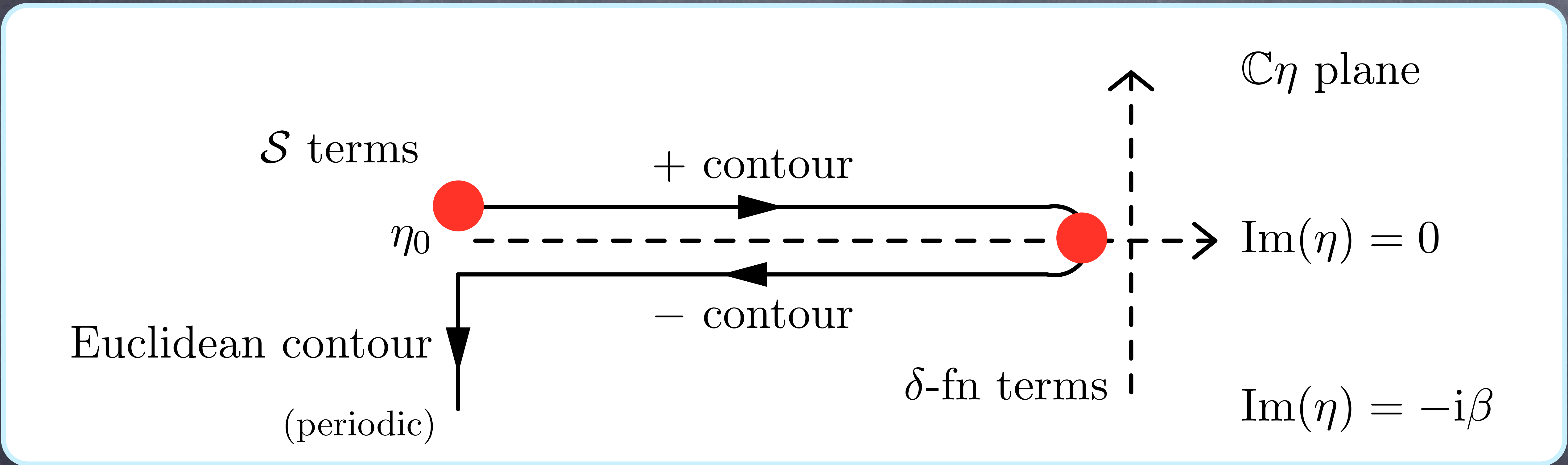

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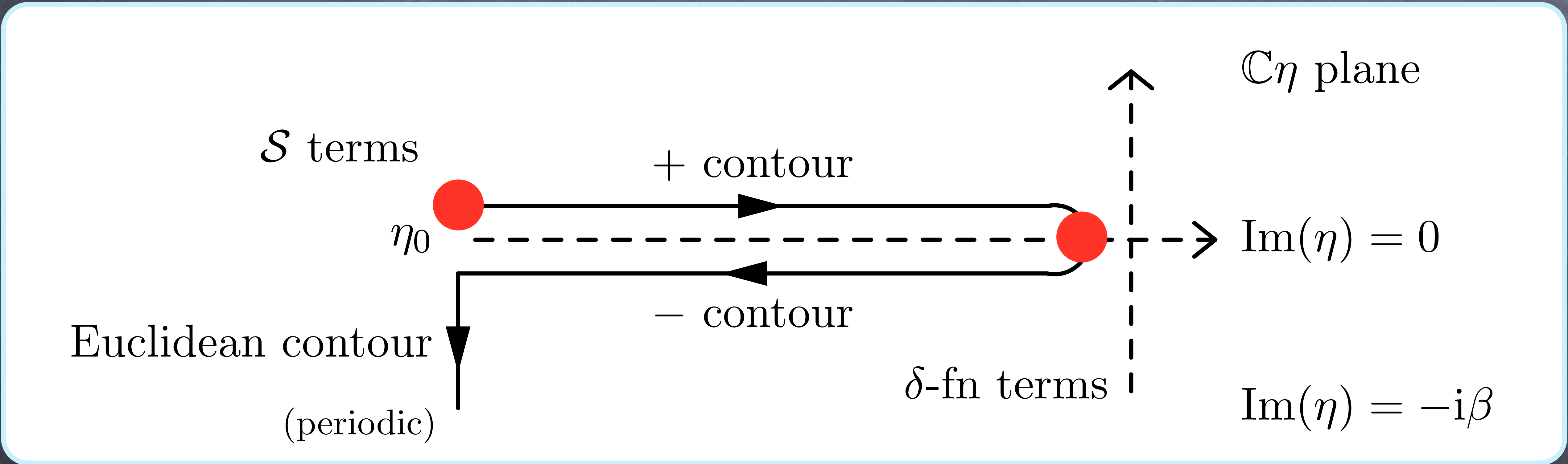

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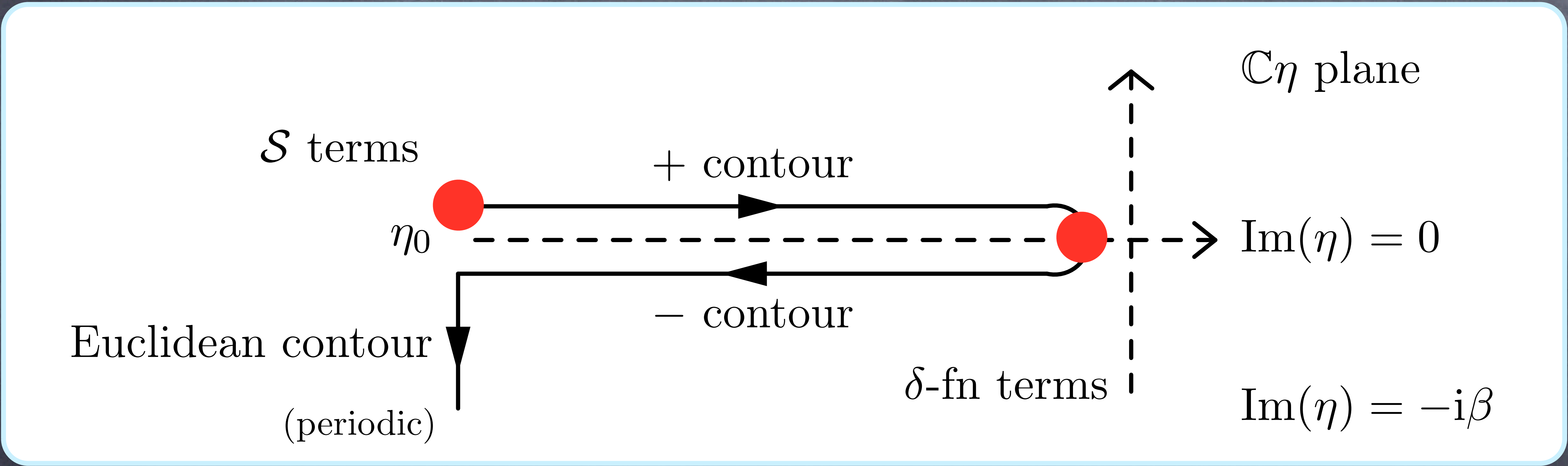

If we send $\eta_0 \to -\infty$, we get Schwinger's theory

If we send $\beta \to \infty$, we get the Gell-Mann / Low theorem.
This says we pick out the lowest energy state, ie., the true vacuum