From Inflation to Nucleosynthesis

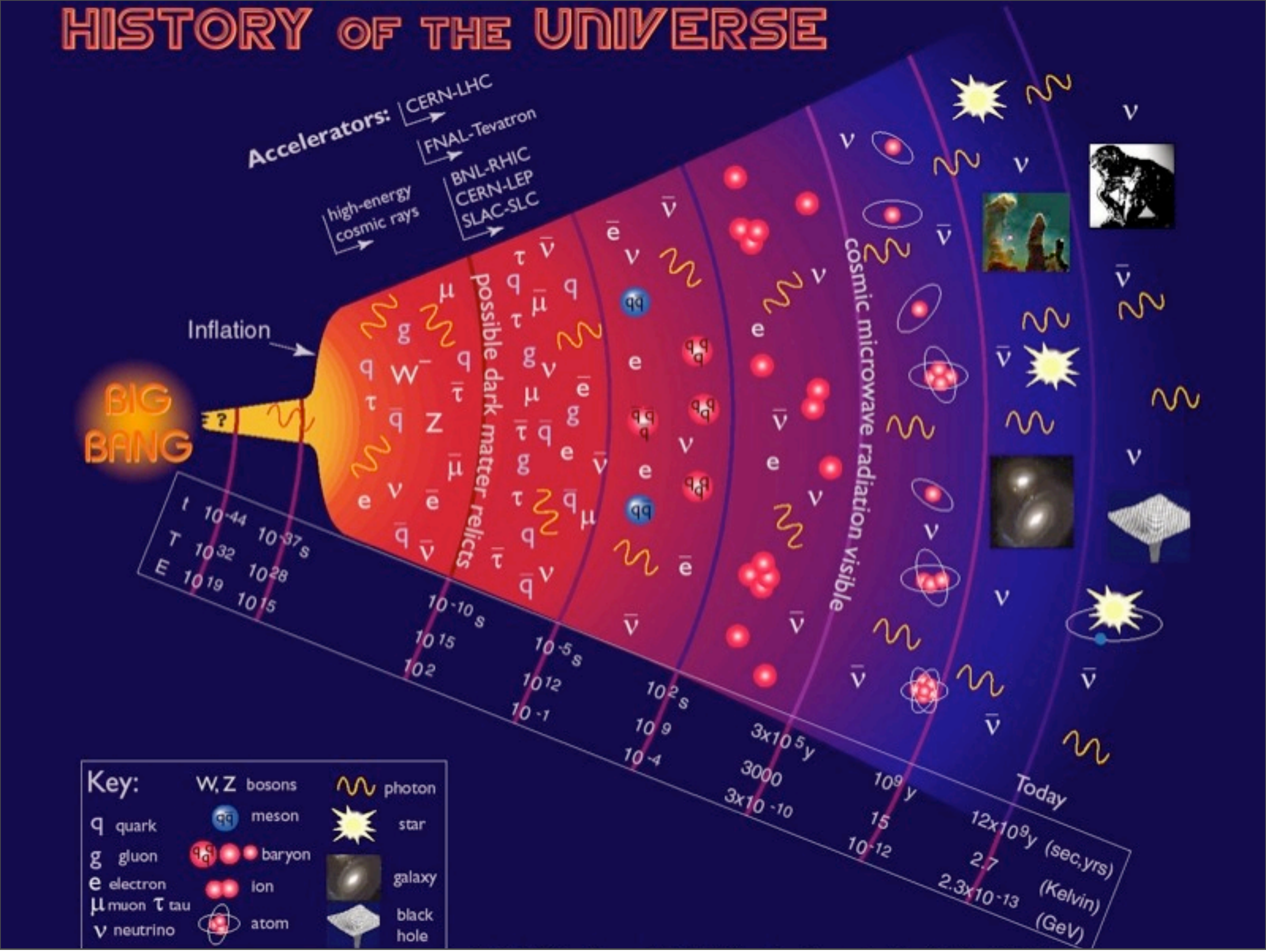
FRW Cosmology

Inflation

Baryogenesis

Big Bang Nucleosynthesis

Comparison between Theory and Observations


HISTORY OF THE UNIVERSE
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
12x109 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole

Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

R(t) is the scale factor

k is curvature constant :

k = -1, 0, +1 for spatially open, flat or closed Universes

with perfect-fluid source

$$T^{\mu\nu} = -pg^{\mu\nu} + (\rho + p)u^\mu u^\nu$$

and solve Einstein's equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

The (00) component gives:

$$H^2 \equiv \frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}$$

The (ii) components give:

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N(\rho + 3p)}{3}$$

In addition $T^{\mu\nu}{}_{;\nu} = 0$ gives:

$$\dot{\rho} = -3H(\rho + p)$$

Consider k = Λ = 0

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} \qquad \dot{\rho} = -3H(\rho + p)$$

i) Radiation dominated Universe: p = ϱ/3

$\varrho \sim R^{-4}$ and $R \sim t^{1/2}$

ii) Matter dominated Universe: p = 0

$\varrho \sim R^{-3}$ and $R \sim t^{2/3}$

The Universe today

Define the deceleration parameter : $$q_0 = -\frac{\ddot{R}_0 R_0}{\dot{R}_0^2}$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N(\rho + 3p)}{3}$$ becomes (with p << ϱ)

$$-2q_0 H_0^2 = \frac{2\Lambda}{3} - \frac{8\pi G_N \rho_0}{3} = \Lambda - H_0^2 - \frac{k}{R_0^2}$$

or $$\frac{k}{R_0^2} = \Lambda + H_0^2(2q_0 - 1)$$

or $$\frac{k}{R_0^2} = H_0^2(\frac{3}{2}\Omega_0 - q_0 - 1)$$

where $\Omega = \rho/\rho_c$ $\quad \rho_c = 3H^2/8\pi G_N = 1.88 \times 10^{-29} h^2$ g cm^{-3}

and h = H/100 (km/Mpc/s)

The Universe today

When $\Lambda = 0$,

$$q_0 = \frac{4\pi G_N \rho_0}{3H_0^2} = \frac{\Omega_0}{2}$$

and $$\frac{k}{R_0^2} = H_0^2(\Omega_0 - 1)$$ (still true for $p \neq 0$)

Evolution of Ω

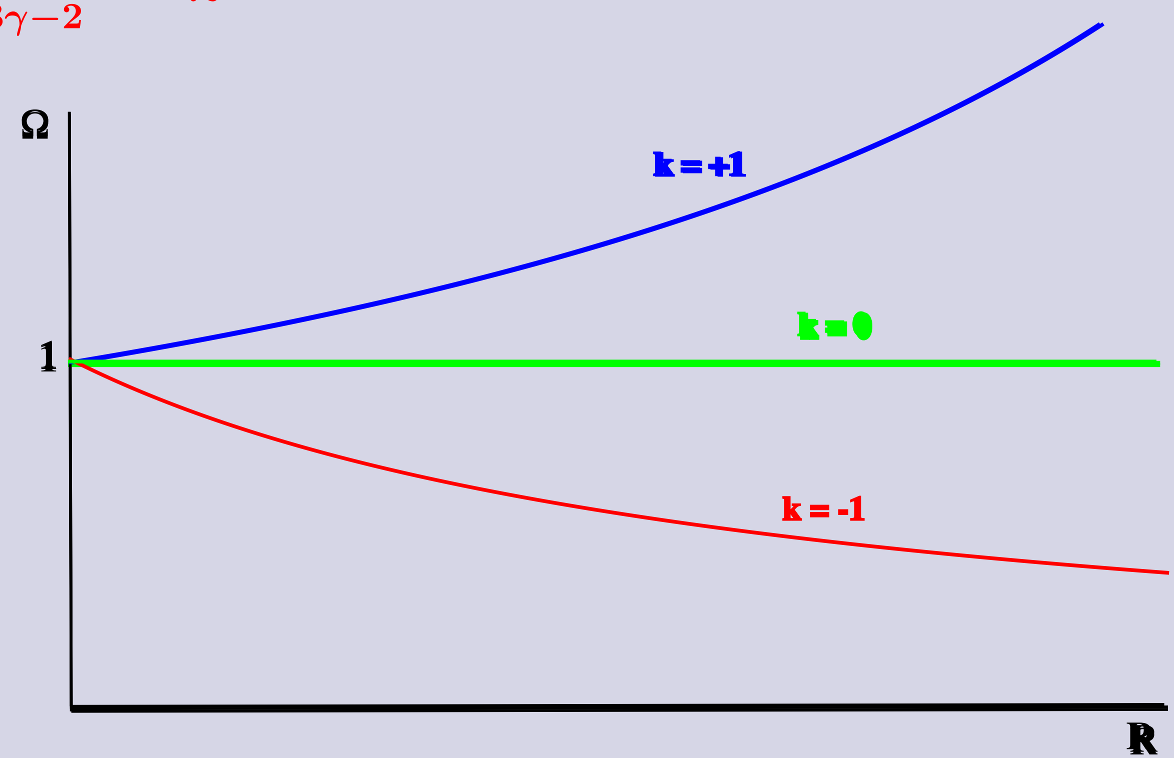
$(\Lambda = 0)$

$$\Omega = \frac{k}{R^2 H^2} + 1$$

$k = 0 \Rightarrow \Omega = 1$ always

write $$R^2 H^2 = \frac{8\pi G_N A}{3R^{3\gamma-2}} - k \qquad \rho = AR^{-3\gamma}$$

$$\Omega = \frac{k}{\frac{8\pi G_N A}{3R^{3\gamma-2}} - k} + 1$$



Age of the Universe

$(\Lambda = 0)$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2}$$

$$\rho = \rho_0 \left(\frac{R_0}{R}\right)^{3\gamma} \qquad \frac{k}{R_0^2} = H_0^2(\Omega_0 - 1) \qquad x = R/R_0$$

$$\dot{x}^2 = \Omega_0 H_0^2 (\frac{R_0}{R})^{3\gamma-2} - (\Omega_0 - 1) H_0^2$$

$$\dot{x} = H_0 \left[1 - \Omega_0 + \Omega_0 x^{2-3\gamma}\right]^{1/2}$$

$$H_0 t = \int_0^1 \frac{dx}{[1 - \Omega_0 + \Omega_0 x^{2-3\gamma}]^{1/2}}$$

Age of the Universe

$(\Lambda = 0)$

Special cases: $\Omega_0 = 1$

$\gamma = 1$

$$t = \frac{2}{3H}$$

$\gamma = 4/3$

$$t = \frac{1}{2H}$$

The Hot Thermal Universe

The energy density in photons:

$$\rho_\gamma = \int E_\gamma dn_\gamma$$

with density of states ($g_\gamma = 2$)

$$dn_\gamma = \frac{g_\gamma}{2\pi^2}[exp(E_\gamma/T) - 1]^{-1}q^2dq$$

giving

$$\rho_\gamma = \frac{\pi^2}{15}T^4 \quad p_\gamma = \frac{1}{3}\rho_\gamma \quad s_\gamma = \frac{4\rho_\gamma}{3T} \quad n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3$$

familiar blackbody relations

In general,

$$\boldsymbol{\rho_i = \int E_i dn_{q_i}}$$

with

$$\boldsymbol{dn_{q_i} = \frac{g_i}{2\pi^2}[exp[(E_{q_i} - \mu_i)/T] \pm 1]^{-1} q^2 dq}$$

and

$$\boldsymbol{E_{q_i} = \left(m_i^2 + q_i^2\right)^{1/2}}$$

$$s_i = \frac{1}{T}\left[E_{q_i} dn_{q_i} \mp \frac{kT}{(2\pi)^3}\int g_i \ln(1 \mp n_{q_i}) d^3 q_i\right]$$

μ is the chemical potential (μ → -μ for antiparticles)

Free energy density:

$$F = \rho - Ts = \mu n - p$$

Chemical potential related to particle-antiparticle asymmetry

Consider (for fermions),

$$n_f - n_{\bar{f}} = \frac{g_f}{2\pi^2}\int p^2 dp \left[\frac{1}{e^{\frac{E-\mu}{T}}+1} - \frac{1}{e^{\frac{E+\mu}{T}}+1}\right]$$

Define β=μ/T, and expand for small β

$$= \frac{g_f T^3}{6}\beta\left(1+\frac{\beta^2}{\pi^2}\right)$$

so, β is of order $\frac{n_f - n_{\bar{f}}}{n_\gamma} \sim 10^{-10}$

Non-relativistic number densities:

$$n_f = \frac{g_f}{2\pi^2} \int \frac{p^2 dp}{e^{\frac{(p^2+m^2)^{1/2}}{T}} + 1}$$

For $p \ll m$

$$n_f = \frac{g_f}{2\pi^2} e^{-m/T} \int e^{-p^2/2mT} p^2 dp$$

$$n_f \simeq \frac{g_f}{(2\pi)^{3/2}} (mT)^{3/2} e^{-m/T}$$

and

$$\rho = mn$$

For Radiation $m_i << T$:

$$\boldsymbol{\rho = \left(\sum_B g_B + \frac{7}{8}\sum_F g_F\right)\frac{\pi^2}{30}T^4 \equiv \frac{\pi^2}{30}N(T)\,T^4}$$

$$p = \frac{1}{3}\rho = \frac{\pi^2}{90}N(T)T^4$$

$$s = \frac{(\rho + p)}{T} = \frac{4}{3}\frac{\rho}{T} = \frac{2\pi^2}{45}N(T)T^3$$

Temperature	New Particles	$4N(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c{}^\dagger$	π's	69
$T_c < T < m_{\rm strange}$	π's + $u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{\rm charm}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\rm bottom}$	$\tau^\pm$	303
$m_b < T < m_{\rm W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\rm Higgs}$	$W^\pm, Z$	381
$m_H < T < m_{\rm top}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

$^\dagger T_c$ corresponds to the confinement-deconfinement transition between quarks and hadrons.

Grand Unified Theories:

Need to add:

12 Gauge Bosons; X, Y
24 Higgs bosons Σ to break SU(5)
6 more Higgses (to complete the SU(5) multiplets)

Grand Total: $N = 160.75$

Many more if theory is supersymmetric!

The quark-hadron transition

Consider a gas of pions:

$$n_\pi = \frac{3}{2} n_\gamma$$

Since $n_\gamma \sim 400\ \text{cm}^{-3}$

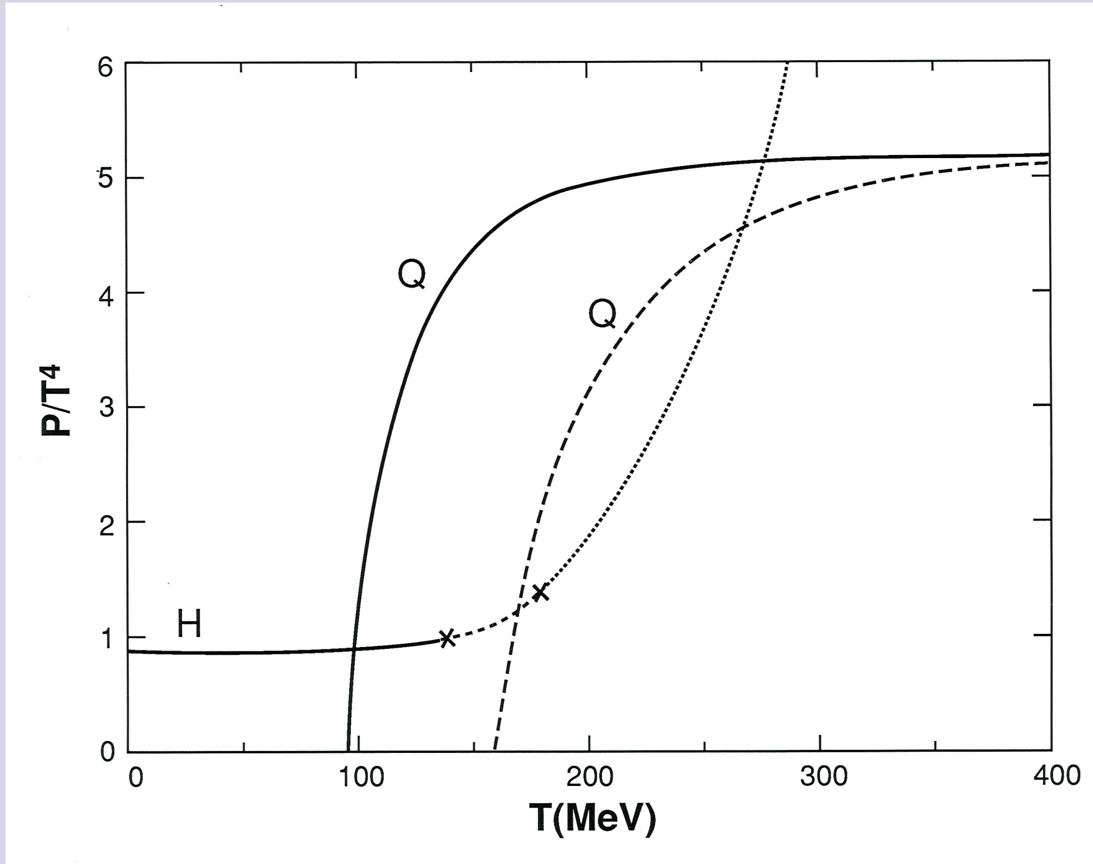
we can write

$$n_\pi = \frac{3}{2}(400\text{cm}^{-3})\left(\frac{T}{T_0}\right)^3$$

at 200 MeV, that's about .4 π's / fm^3

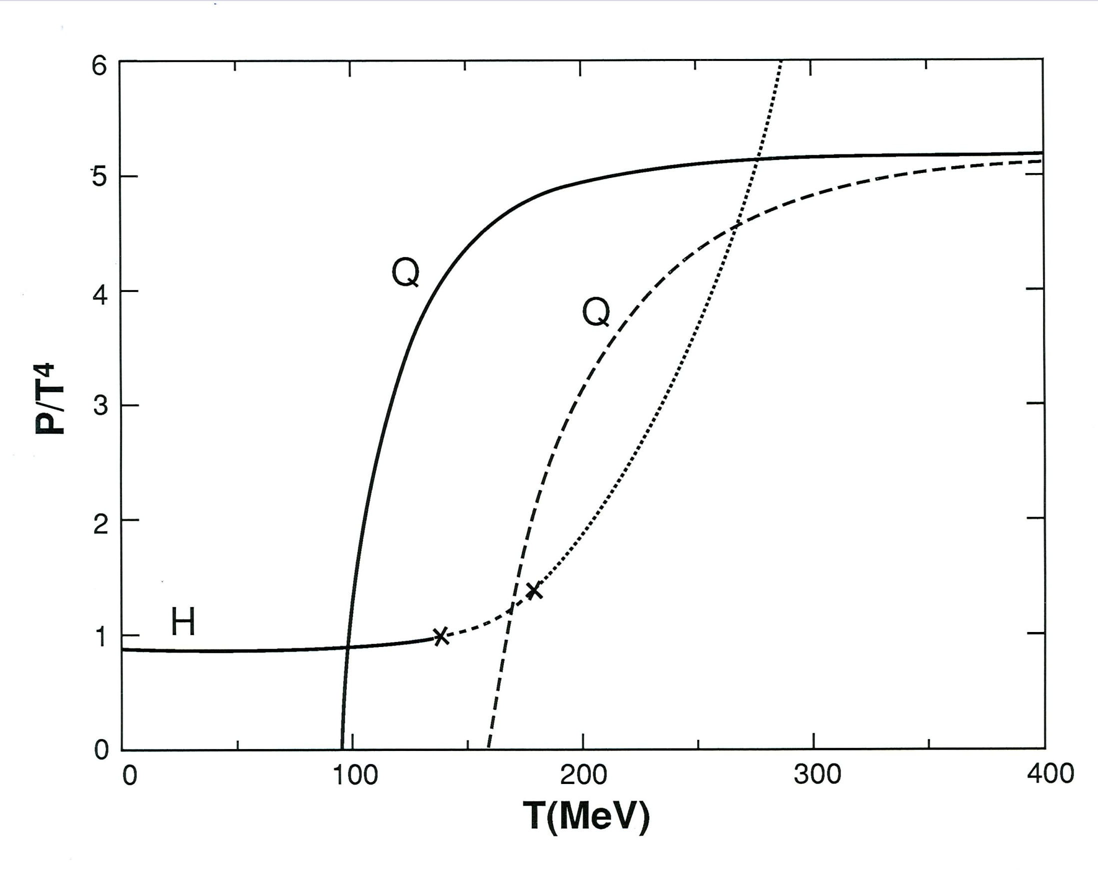
The quark-hadron transition

Consider the contributions of all known hadrons to the pressure, and the contribution of (free) quarks + a bag term (-B) to simulate confinement



The quark-hadron transition

Since F = -p (for μ=0), predict hadrons are present over quarks at high T!



The quark-hadron transition

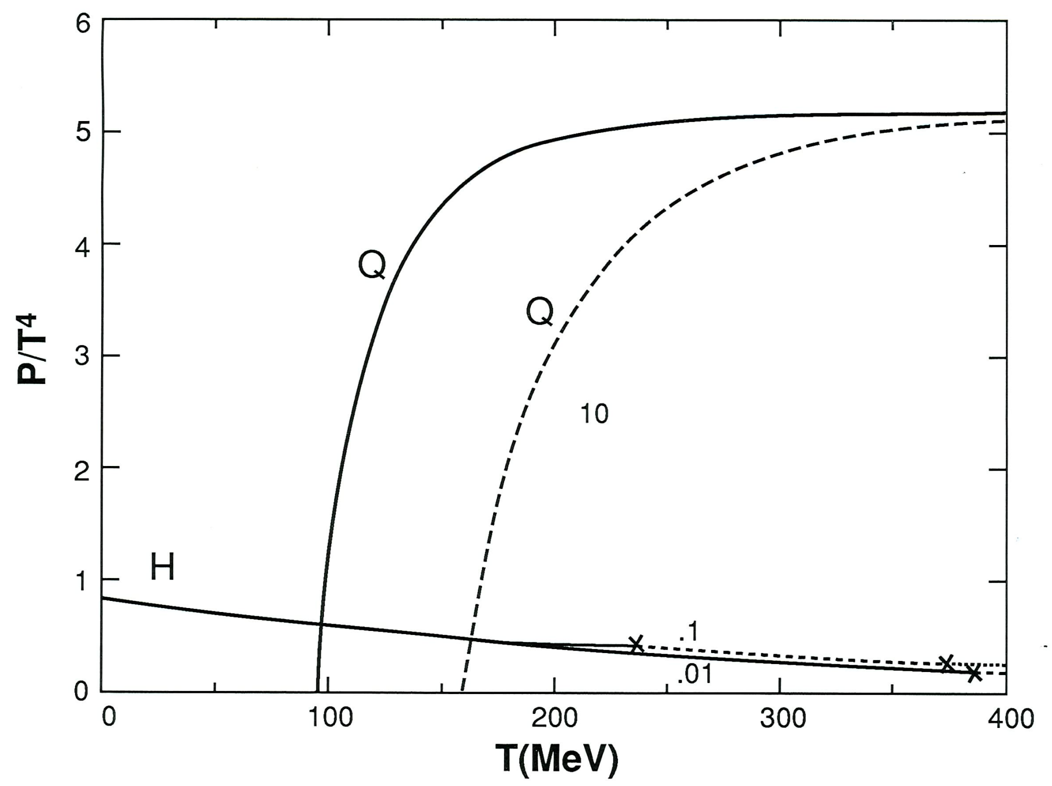
Problem cured by taking into account strong repulsive interactions among hadrons.

e.g. N-N (Reid) potential

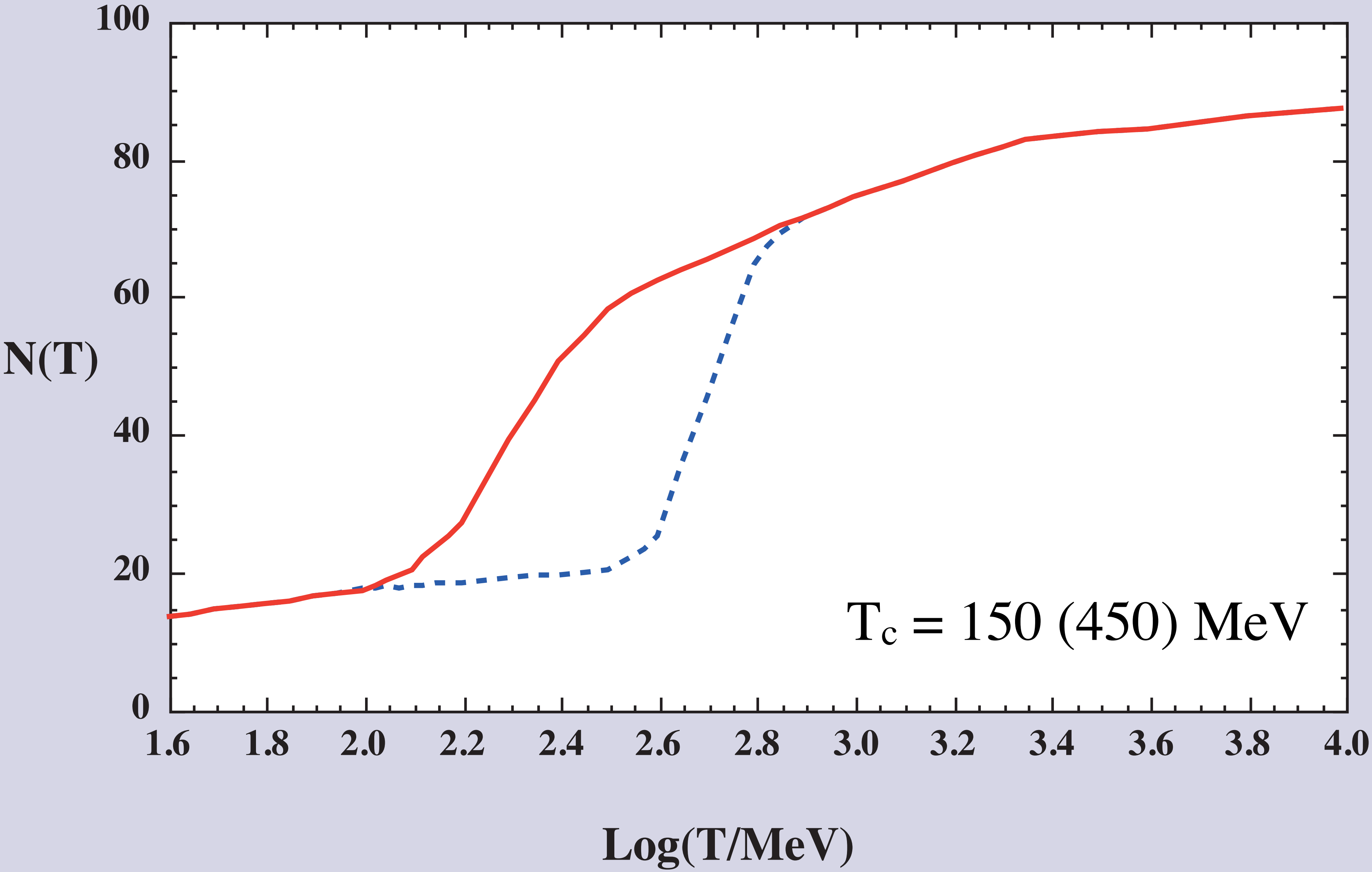
$$U(r) = (-10.46\, e^{-\mu r} - 1650.6\, e^{-4\mu r} + 6484.3\, e^{-7\mu r})/\mu r\,,$$

$$U_N(n) = 18.7\left(\frac{4\pi}{\mu^3}\right) n_N$$

$$= 680\, n_N\,,$$



Effective numbers of degrees of freedom



Time-temperature Relation

Recall $\gamma = 4/3$

$$t = \frac{1}{2H} \qquad H^2 = \frac{8\pi G_N \rho}{3}$$

$$t = (\frac{3}{32\pi G_N \rho})^{1/2} = (\frac{90}{32\pi^3 G_N N(T)})^{1/2} T^{-2}$$

or

$$t_s T^2_{MeV} = \frac{2.41}{\sqrt{N(t)}}$$

Equilibrium

- Particles will be in equilibrium if there is a reaction rate which is fast enough: $\Gamma > H$

  - interaction rate Γ
  - mean time between interactions $\tau \sim \Gamma^{-1}$
  - expansion rate H
  - age of the universe $t \sim H^{-1}$

$$\Gamma > H \;\; \Rightarrow \;\; t > \tau$$

Neutrinos

kept in thermal equilibrium by processes such as

$$e^+ + e^- \leftrightarrow \nu + \bar{\nu} \text{ or } e + \nu \leftrightarrow e + \nu \text{ etc.,}$$

expansion rate (again)

$$H = \left(\frac{8\pi G_{\rm N}\rho}{3}\right)^{1/2} = \left(\frac{8\pi^3}{90}N(T)\right)^{1/2} T^2/M_{\rm P}$$
$$\sim 1.66 N(T)^{1/2} T^2/M_{\rm P},$$

and $\Gamma = n\langle\sigma v\rangle$ and $\langle\sigma v\rangle \sim 0(10^{-2})T^2/M_W^4$

$$\Gamma_{wk} \sim 0(10^{-2})T^5/M_W^4$$

Neutrinos in equilibrium when

$$T > (500 M_W^4)/M_P)^{1/3} \sim 1 MeV.$$

Entropy Conservation

Energy conservation: $T^{\mu\nu}{}_{;\nu} = 0$

$$\dot{\rho} = -3H(\rho + p)$$

equivalent to

$$\dot{p}R^3 = \frac{d}{dt}(R^3(\rho + p)) \quad = \frac{d}{dt}(R^3 T s)$$

Now,

$$\dot{p} = \frac{dp}{dT}\frac{dT}{dt} = s\frac{dT}{dt}$$

so

$$s\frac{dT}{dt}R^3 = \frac{d}{dt}(R^3 T s) = s\frac{dT}{dt}R^3 + T\frac{d}{dt}(R^3 s)$$

$$\Longrightarrow \quad \frac{d}{dt}(R^3 s) = 0$$

Neutrino Temperature

- At T ~ 1 MeV neutrinos decouple
- At T ~ 1/2 MeV e^+ e^- annihilate to photons
- Entropy of "γ's" and ν's conserved speparately
- Prior to annihilation, $T_\gamma = T_\nu = T_>$

$$s_> = \frac{4}{3}\frac{\rho_>}{T_>} = \frac{4}{3}(2 + \frac{7}{2})(\frac{\pi^2}{30})T_>^3$$

- After annihilation, $T_\gamma = T_<$ but, $T_\nu = T_>$

$$s_< = \frac{4}{3}\frac{\rho_<}{T_<} = \frac{4}{3}(2)(\frac{\pi^2}{30})T_<^3$$

$$T_\nu = (4/11)^{1/3} T_\gamma \simeq 1.9K$$

Another example of Freeze-out

The Relic Density

At high temperatures $T \gg m_\chi$;

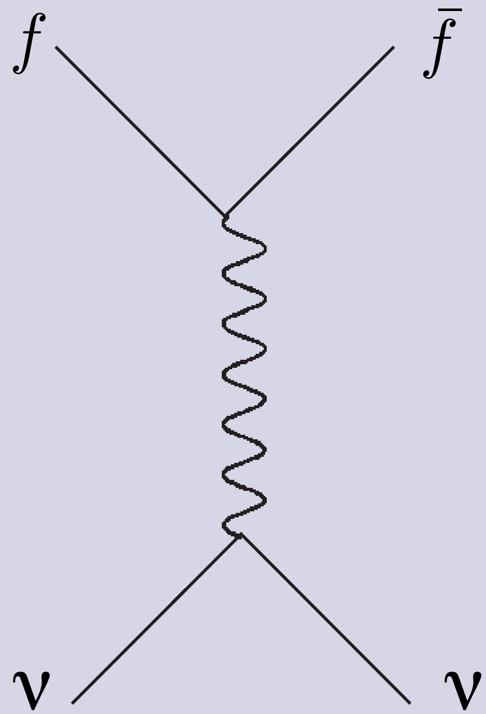
χ's in equilibrium $\Gamma > H$ $n_\chi \sim n_\gamma$

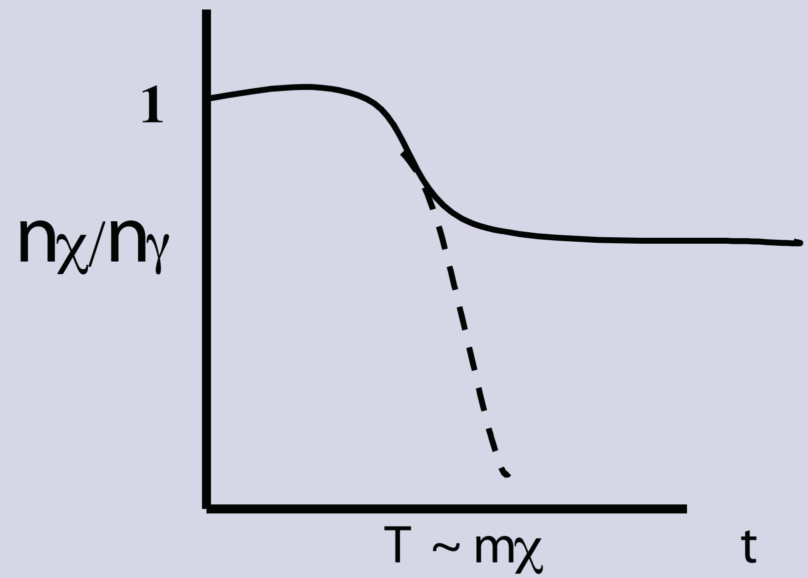
$\Gamma \sim n\sigma v \sim T^3 \sigma v$; $HM_p \sim \sqrt{\rho} \sim T^2$

As $T < m_\chi$; annihilations drop n_χ

$n_\chi \sim e^{-m_\chi/T}\, n_\gamma$

Until freeze-out, $\Gamma < H$ $n_\chi/n_\gamma \sim$ constant





Annihilation Cross sections:

$$\nu\nu \to f\bar{f}$$

$$\langle \sigma v_{rel} \rangle = \sum_f \frac{1}{M^2}\left(1 - \frac{m_f^2}{M^2}\right)^{1/2} [a_f + b_b x + \ldots]$$

$$a_f = C_0 + C_1 + C_2$$

$$b_f = -\frac{3}{2}(2C_0 + C_1) + \frac{3}{4}\beta_f(C_0 + C_1 + C_2)$$

$$\beta_f = m_f^2/(M^2 - m_f^2)$$

$$\xi_f = m_f^2/M^2$$

$$C_{0f} = \frac{2}{3\pi}M^4\left[7\xi_f A_f^2 - 2\xi_f V_f^2\right]$$

$$C_{1f} = \frac{4}{3\pi}M^4\left[-(2 + 2\xi_f + 12\xi_f\eta)A_f^2 - (2 - \xi_f)V_f^2\right]$$

$$C_{2f} = \frac{8}{3\pi}M^4\left[(1 + 12\xi_f\eta^2)A_f^2 + V_f^2\right]$$

$$V_f = \sqrt{\frac{\lambda}{2}}G(T_f^{3L} - 2Q_f \sin^2\theta_W)$$

$$A_f = \sqrt{\frac{\lambda}{2}}G(-T_f^{3L})$$

The Relic Density:

$$\frac{dn}{dt} = -3\frac{\dot{R}}{R}n - \langle\sigma v\rangle(n^2 - n_0^2)$$

$$\frac{df}{dx} = m_\chi \left(\frac{1}{90}\pi^2\kappa^2 N\right)^{1/2} (f^2 - f_0^2)$$

$f = n/T^3$

$$\Omega_\chi h^2 \simeq 1.9 \times 10^{-11} \left(\frac{T_\chi}{T_\gamma}\right)^3 N_f^{1/2} \left(\frac{\text{GeV}}{ax_f + \frac{1}{2}bx_f^2}\right)$$

What is (T_χ/T_γ) ?

$$x_f \approx 1/20 \qquad N_f \approx N(m_\chi/20)$$

e.g., for $m_\chi = 100\ GeV$, $\quad T_f \approx 5GeV \quad N_f \approx 345/4$

$$(T_\chi/T_\gamma)^3 = (43/4N_f) \times (4/11)$$

Neutrinos

- Relic Density limit on light ν masses:

$$\rho_\nu = \frac{3}{11}\frac{g_\nu}{2} m_\nu n_\gamma$$

$$\Omega_\nu h^2 \simeq 0.01 m_\nu(eV)\frac{g_\nu}{2}$$

- WMAP +2df + limit

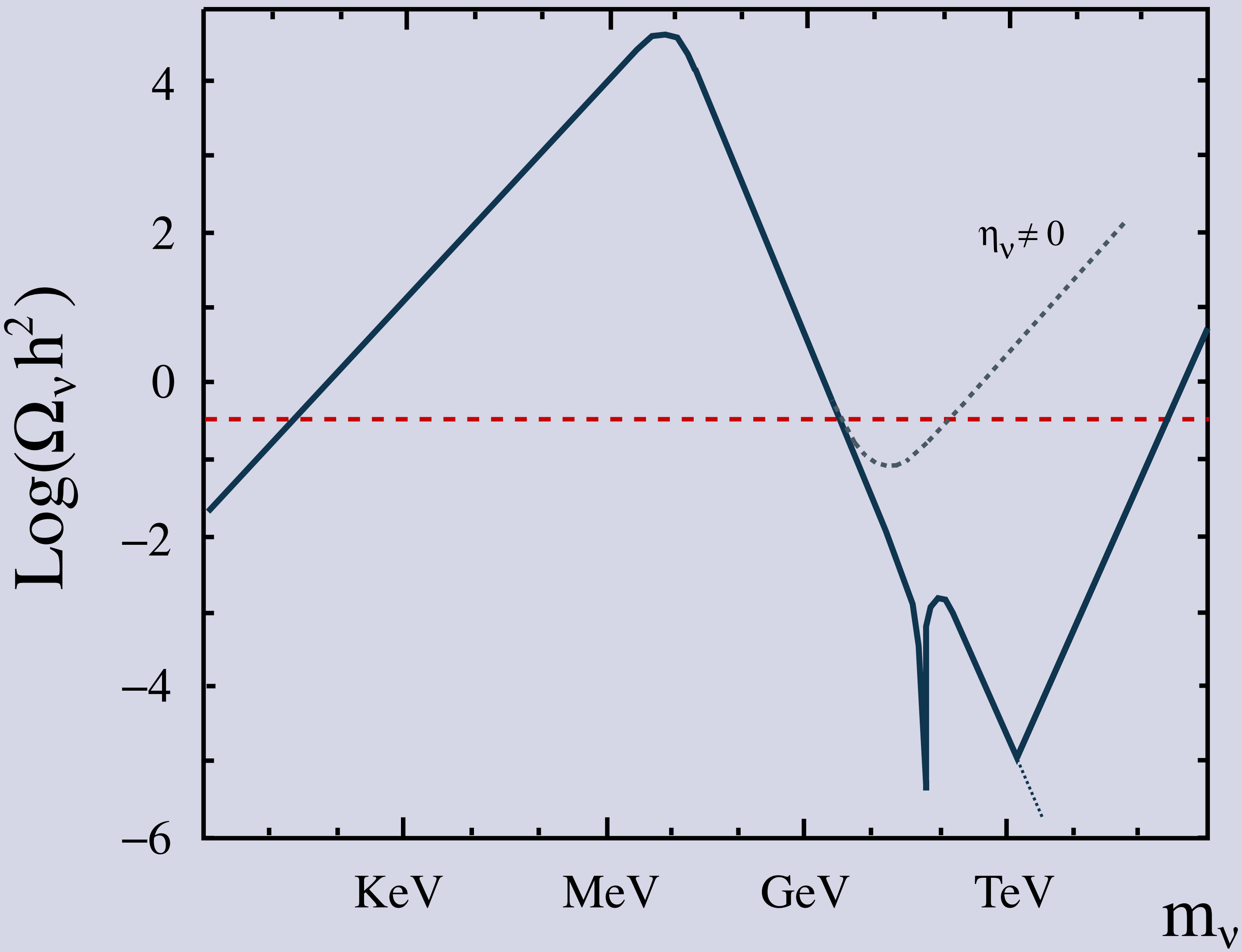
$$m_{tot} < 0.7eV \Longrightarrow \Omega h^2 < 0.0076$$

- Heavy neutrinos (m > GeV) excluded as dark matter

Neutrinos

Light ν's (m_ν < 1 MeV): Left over with $n_\nu \approx n_\gamma$

Heavy ν's (m_ν > 1 MeV): Left over from annihilations



Matter Domination

Radiation density:

$$\rho_r = \frac{\pi^2}{30}\left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right]T^4$$

Matter density:

$$\rho_m = m_{\rm N}\eta\, n_\gamma$$

$$\rho_r = \rho_m,$$

$$T_{eq} = 0.22\, m_{\rm N}\,\eta \qquad \text{or} \qquad \left(1 + z_{\rm eq}\right) = 0.22\,\eta\frac{m_{\rm N}}{T_0}$$

With Dark Matter

$$\rho_m = \Omega_{\rm m} \rho_c \left(\frac{T}{T_0} \right)^3$$

$$T_{eq} = 0.9 \frac{\Omega_{\rm m} \rho_c}{T_0^3} \qquad \text{or} \qquad (1 + z_{eq}) = 2.4 \times 10^4 \Omega_{\rm m} h^2$$