Fundamentals of Cosmology

(4) Observational Tests of Cosmological Models

- The Principal of Equivalence
- General Relativity
- Inhomogeneous Models
- Determination of Cosmological Parameters (excluding CMB)
- Comparison with Other Estimates

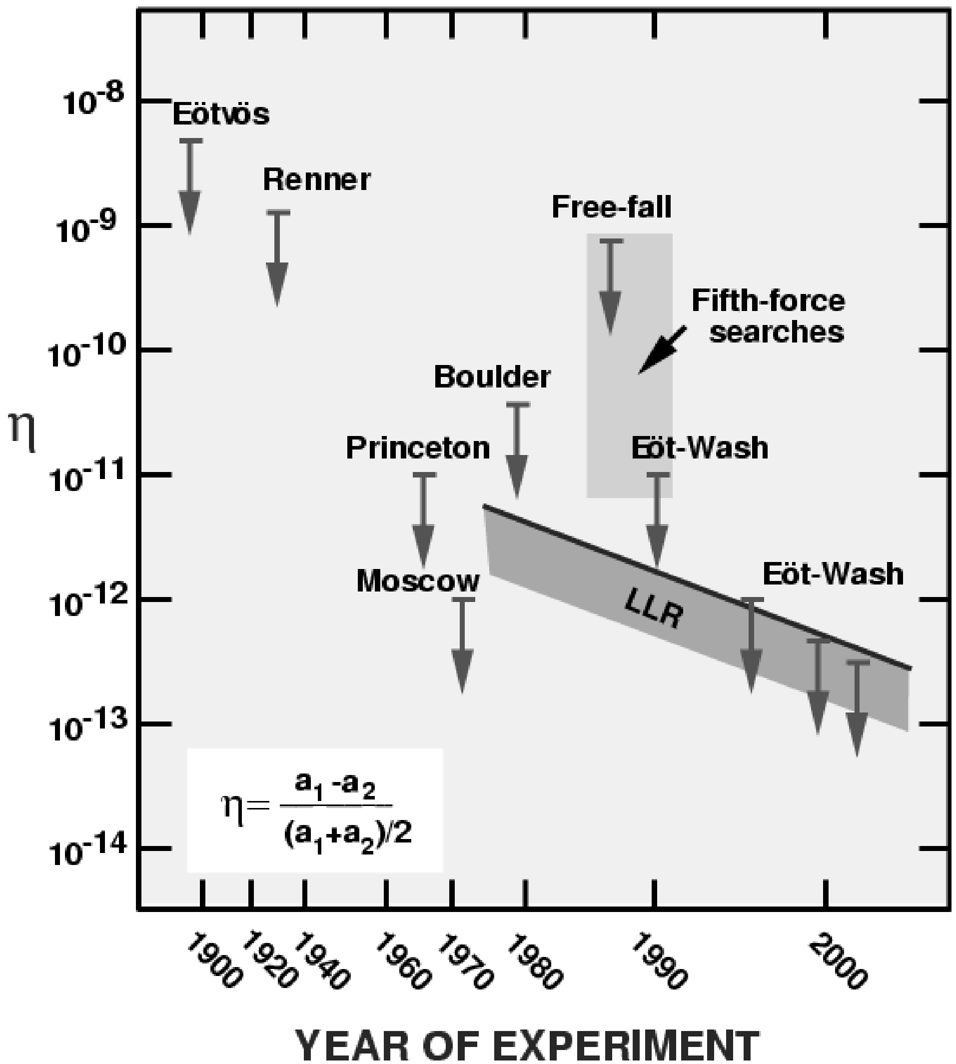
Testing General Relativity

First of all, how good is Einstein's *Equivalence Principle*? Following Will's exposition, the deviations from linearity can be written

$$m_{\rm g} = m_{\rm I} + \sum_{\rm A} \frac{\eta^{\rm A} E^{\rm A}}{c^2}\,. \tag{1}$$

$E^{\rm A}$ is the internal energy of the body generated by interaction A, η is a dimensionless parameter that measures the strength of the violation of the linearity of the relation between $m_{\rm g}$ and $m_{\rm I}$ induced by that interaction, and c is the speed of light. The internal energy terms include all the mass-energy terms which can contribute to the inertial mass of the body, for example, the body's rest energy, its kinetic energy, its electromagnetic energy, weak-interaction energy, binding energy and so on. If the inertial and gravitational masses were not exactly linearly proportional to each other, there would be a finite value $\eta^{\rm A}$ which would be exhibited as an difference in the accelerations of bodies of the same inertial mass composed of different materials.

Testing General Relativity



A measurement of, or limit to, the fractional difference in accelerations between two bodies yields the quantity known as the 'Eötvös ratio',

$$\eta = 2\frac{|a_1 - a_2|}{a_1 + a_2} = \sum_A \eta^A \left(\frac{E_1^A}{m_1 c^2} - \frac{E_2^A}{m_2 c^2} \right) , \quad (2)$$

where the subscript 'I' has been dropped from the inertial masses.

Will, C. M., 2006. The Confrontation between General Relativity and Experiment, *Living Reviews in Relativity*, **9**, Online article: cited on 21 June 2006 `http://www.livingreviews.org/lrr-2006-3`

Testing General Relativity

Examples of Parameterised Post-Newtonian (PPN) parameters and their significance.

Parameter	What it measures relative to General Relativity	Value in General Relativity	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature is produced by unit rest mass?	1	γ	γ
β	How much “nonlinearity” in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation			
ζ_1	of total momentum?	0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

Testing General Relativity

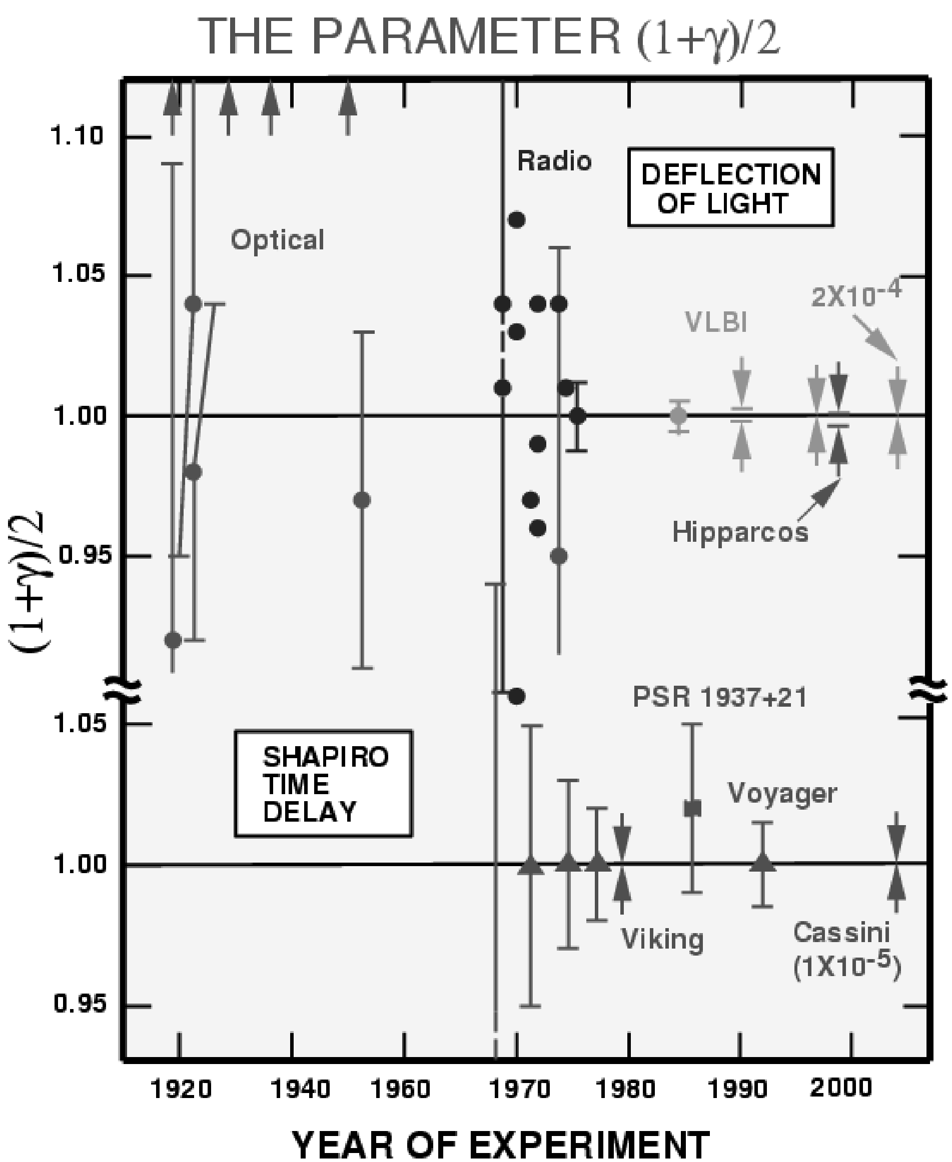
The Parameterised Post-Newtonian (PPN) parameters and their significance.

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^2} T_{\mu\nu} \, .$$

$$\begin{aligned}
g_{00} = \quad & -1 + 2U - 2\beta U^2 - 2\xi \Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\
& +2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 \\
& -(\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3) w^2 U - \alpha_2 w^i w^j U_{ij} \\
& +(2\alpha_3 - \alpha_1) w^i V_i + \mathcal{O}(\epsilon^3) \\
g_{0i} = \; - \; & \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi) V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi) W_i \\
& -\frac{1}{2}(\alpha_1 - 2\alpha_2) w^i U - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}) \\
g_{ij} = \quad & (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2)
\end{aligned}$$

The quantities U, U_{ij}, Φ_W, Φ_1, Φ_2, Φ_3, Φ_4, $\mathcal{A}$, V_i, W_i are various metric potentials which can be interpreted in terms of Newtonian gravity.

Testing General Relativity



Measurements of the quantity $(1+\gamma)/2$ from light deflection and time delay experiments. The value of γ according to General Relativity is unity. The time-delay measurements from the Cassini spacecraft yielded agreement with General Relativity at the level of 10^{-3} percent. VLBI radio deflection measurements have reached 0.02 percent accuracy. The *Hipparcos* limits were derived from precise measurements of the positions stars over the whole sky and resulted in a precision of 0.1 percent in the measurement of γ.

Testing General Relativity

Traditionally, there are four tests of General Relativity

- The *Gravitational Redshift* of electromagnetic waves in a gravitational field. Hydrogen masers in rocket payloads confirm the prediction at the level of about 5 parts in 10^5.
- The *Advance of the Perihelion of Mercury*. Continued observations of Mercury by radar ranging have established the advance of the perihelion of its orbit to about 0.1% precision with the result $\dot{\omega} = 42.98(1 \pm 0.001)$ arcsec per century. General Relativity predicts a value of $\dot{\omega} = 42.98$ arcsec per century.
- The *Gravitational Deflection of Light by the Sun* has been measured by VLBI and the values found are $(1+\gamma)/2 = 0.99992 \pm 0.00023$.
- The *Time Delay of Electromagnetic Waves* propagating through a varying gravitational potential. While en route to Saturn, the Cassini spacecraft found a time-delay corresponding to $(\gamma - 1) = (2.1 \pm 2.3) \times 10^{-5}$. Hence the coefficient $\frac{1}{2}(1+\gamma)$ must be within at most 0.0012 percent of unity.

Testing General Relativity - Gravity Probe-B

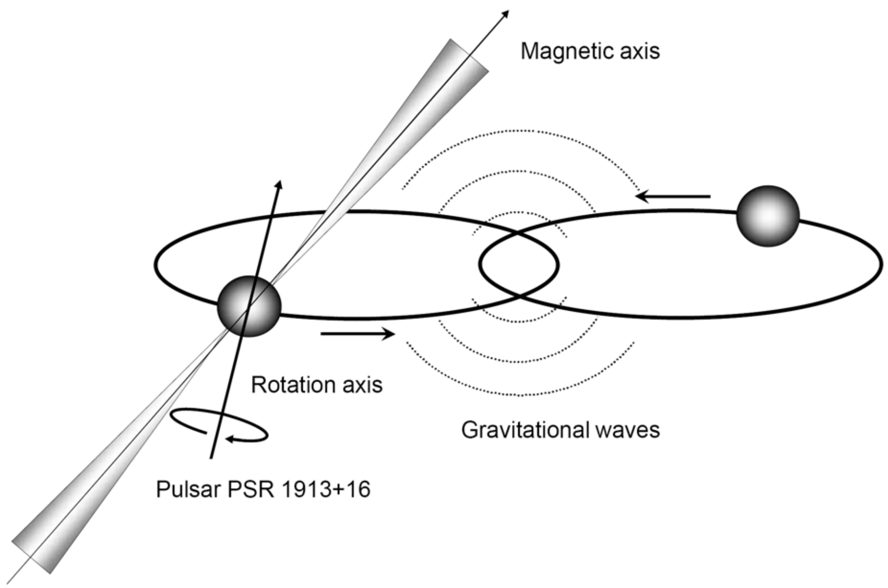
Gravity Probe B was launched 20 April 2004. It was a space experiment designed to test two fundamental predictions of Einstein's theory of General Relativity, the geodetic and frame-dragging effects, by means of cryogenic gyroscopes in Earth orbit. Data collection started 28 August 2004 and ended 14 August 2005.

Analysis of the data from all four gyroscopes results in the estimates shown in the Table:

	Prediction of GR	Measured value
Geodetic drift rate	$-6,606.1$ mas year^{-1}	$-6,601.8 \pm 18.3$ mas year^{-1}
Frame-dragging drift rate	-39.2 mas year^{-1}	-37.2 ± 7.2 mas year^{-1}

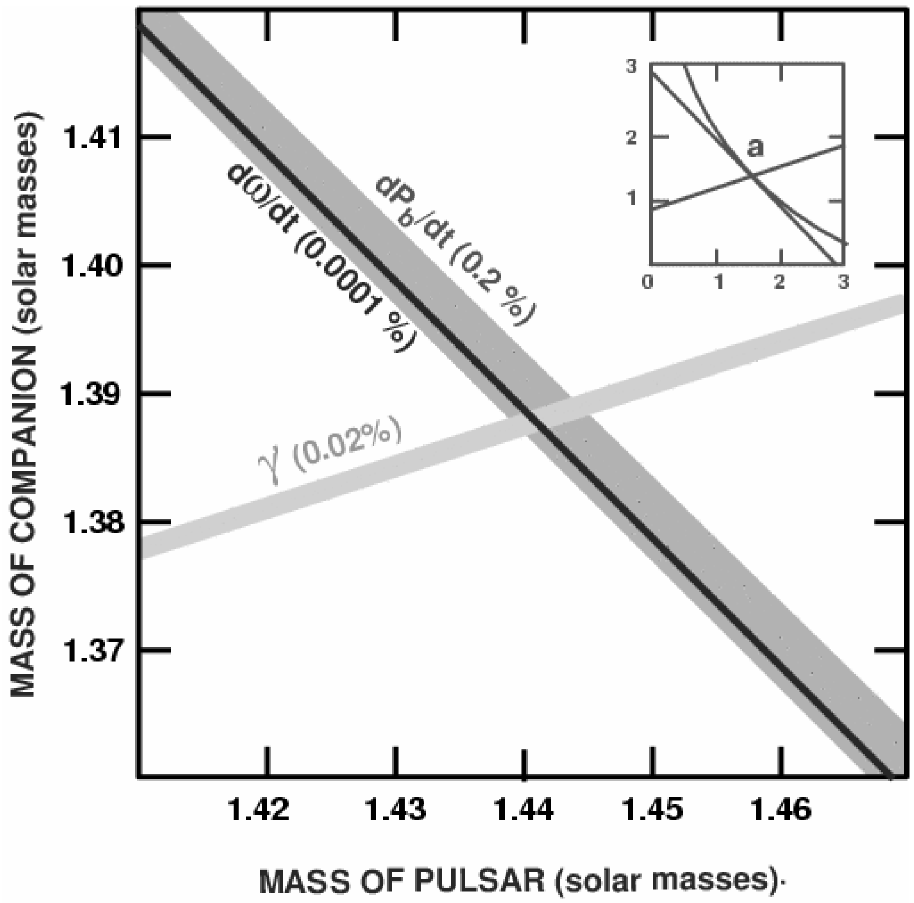
Another approach to measure frame dragging was carried out by Ciufolini and Pavlis who combined nodal precession data from LAGEOS I and II satellites with improved models for the Earths multipole moments. The latter were provided by two recent orbiting geodesy satellites, the European CHAMP and NASAs GRACE satellites. They found a $5 - 10$ percent confirmation of General Relativity.

Testing General Relativity – the Binary Pulsar



A schematic diagram of the orbit of the binary pulsar PSR 1913+16. The pulsar is one of a pair of neutron stars in binary orbits about their common centre of mass. There is displacement between the axis of the magnetic dipole and the rotation axis of the neutron star. The radio pulses are assumed to be due to beams of radio emission from the poles of the magnetic field distribution. Many parameters of the binary orbit and the masses of the neutron stars can be measured with very high precision by accurate timing measurements.

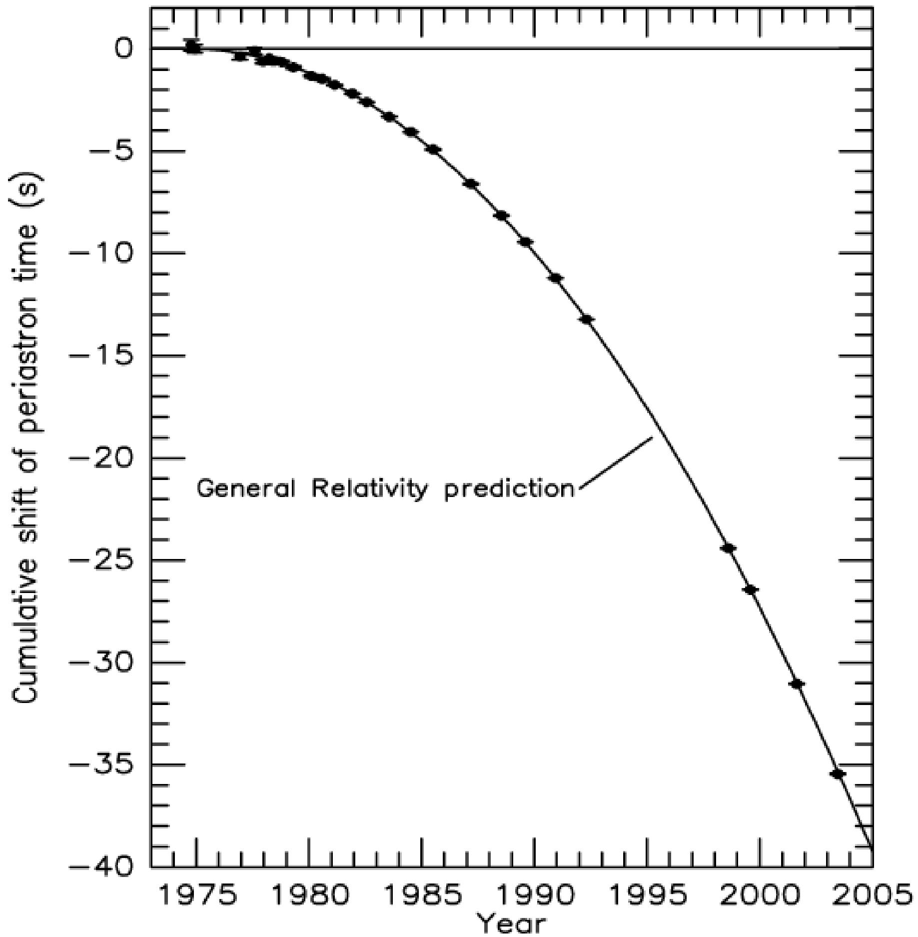
The Masses of the Neutron Stars in the Binary Pulsar



The width of each strip reflects the observational uncertainties in the timing measurements, shown as a percentage. The inset shows the same three most accurate constraints on the full mass plane; the intersection region has been magnified 400 times in the full figure.

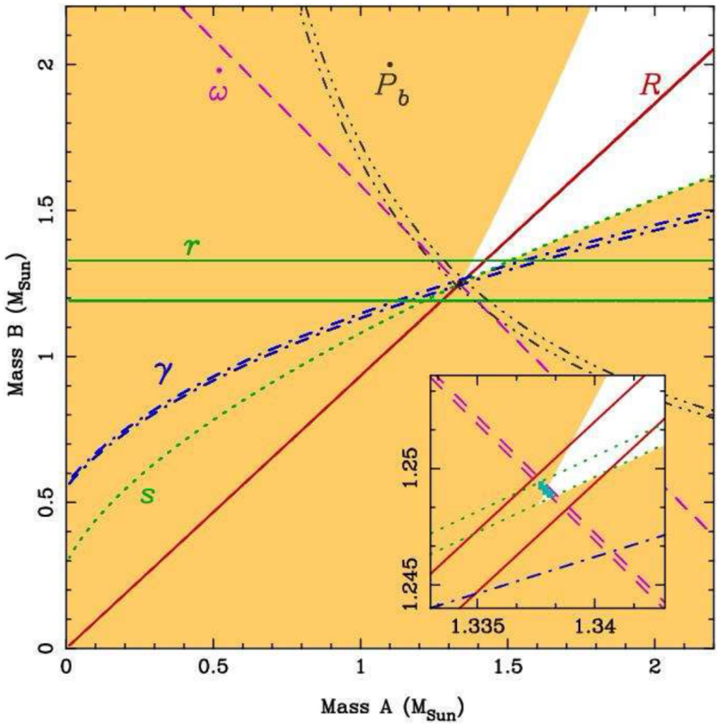
If General Relativity were not the correct theory of gravity, the lines would not intersect at one point.

Gravitational Radiation of the Binary Pulsar



The binary pulsar emits gravitational radiation and so leads to a speeding up of the stars in the binary orbit. The diagram shows the change of orbital phase as a function of time for the binary neutron star system PSR 1913+16 compared with the expected changes due to gravitational radiation energy loss by the binary system. These observations enable many alternative theories of gravity to be excluded.

J0737-3039



J0737-3039 is an even closer binary neutron star system in which both stars are pulsars. Therefore, their orbits can be determined more precisely. In 2.5 years, the precision with which General Relativity can be tested has approached that of PSR 1913+16.

PK parameter	Observed	GR expectation	Ratio
$\dot{P}_b$	1.252(17)	1.24787(13)	1.003(14)
γ (ms)	0.3856(26)	0.38418(22)	1.0036(68)
s	0.99974(−39,+16)	0.99987(−48,+13)	0.99987(50)
$r(\mu s)$	6.21(33)	6.153(26)	1.009(55)

But, pulsar B faded and disappeared in March 2008, probably due to precession of the pulsar beam.

Variation of the Gravitational Constant with Cosmic Epoch

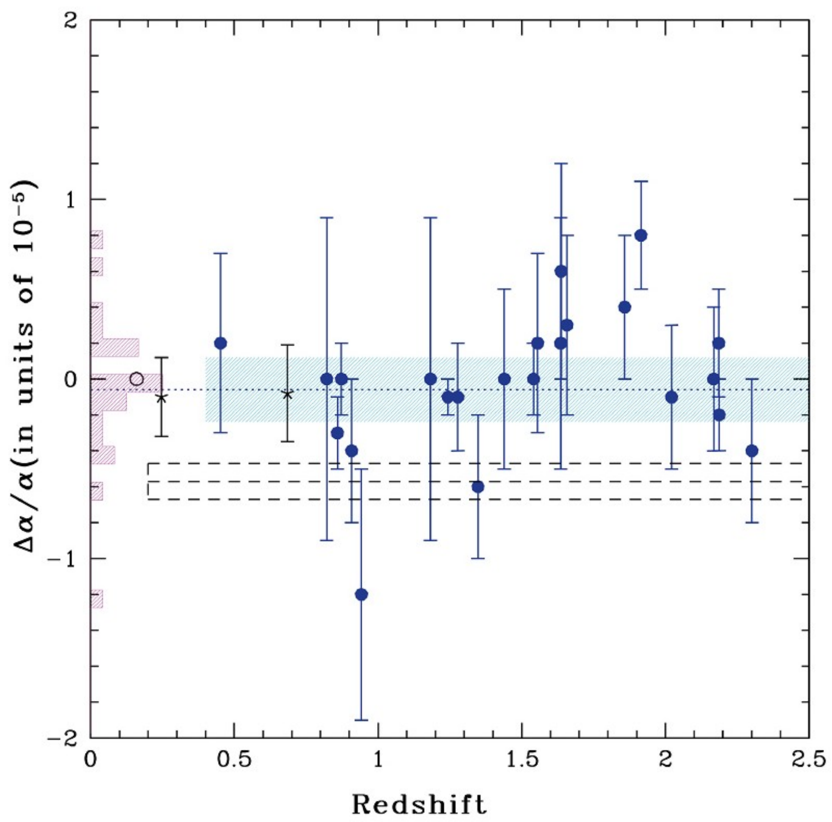
Various solar system, astrophysical and cosmological tests can be made to find out if the gravitational constant has varied over cosmological time-scales.

Method	$(\dot{G}/G)/10^{-13}$ year^{-1}
Lunar laser ranging	4 ± 9
Binary pulsar PSR 1913+16	40 ± 50
Helioseismology	0 ± 16
Big Bang nucleosynthesis	0 ± 4

For binary pulsar data, the bounds are dependent upon the theory of gravity in the strong-field regime and on the neutron star equation of state. Big-bang nucleosynthesis bounds assume specific form for time dependence of G.

There can have been little variation in the value of the gravitational constant over the last 10^{10} years.

Variation of the Fine Structure Constant with Cosmic Epoch



There has been some debate about whether or not the fine-structure constant α has changed very slightly with cosmic epoch from observations of fine-structure lines in large redshift absorption line systems in quasars. The Australian observers reported a small decrease in the value of α, shown by the open rectangle. The ESO observers found little evidence for changes.

The Determination of Cosmological Parameters

First of all, let us list the traditional set of cosmological parameters.

- *Hubble's constant*, H_0 – the present rate of expansion of the Universe,

$$H_0 = \left(\frac{\dot{a}}{a}\right)_{t_0} = \dot{a}(t_0)\ . \tag{3}$$

- The *deceleration parameter*, q_0 – the present dimensionless deceleration of the Universe.

$$q_0 = -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t_0} = -\frac{\ddot{a}(t_0)}{H_0^2}\ . \tag{4}$$

- The *density parameter*, Ω_0 – the ratio of the present mass-energy density of the Universe ϱ_0 to the critical density $\varrho_c = 3H_0^2/8\pi G$,

$$\Omega_0 = \frac{\varrho_0}{\varrho_c} = \frac{8\pi G\varrho_0}{3H_0^2}\ . \tag{5}$$

The Determination of Cosmological Parameters

For many aspects of astrophysical cosmology, it is important to determine separately the density parameter in baryonic matter Ω_B and the overall density parameter Ω_0, which includes all forms of baryonic and non-baryonic dark matter.

- The density parameter of the vacuum fields, or the dark energy,

$$\Omega_\Lambda = 8\pi G \varrho_v / 3H_0^2 = \Lambda / 3H_0^2 \; , \tag{6}$$

  where Λ is the cosmological constant.

- The *curvature of space*,

$$\kappa = c^2 / \Re^2 \; . \tag{7}$$

- The *age of the Universe*, T_0,

$$T_0 = \int_0^1 \frac{\mathrm{d}a}{\dot{a}} \; . \tag{8}$$

The Determination of Cosmological Parameters

Within the context of the Friedman world models, these are not independent parameters. Specifically, from Einstein's field equations for a uniformly expanding Universe,

$$\ddot{a} = -\frac{4\pi G}{3} a \left(\varrho + \frac{3p}{c^2} \right) + \tfrac{1}{3} \Lambda a \; ; \qquad (9)$$

$$\dot{a}^2 = \frac{8\pi G \varrho}{3} a^2 - \frac{c^2}{\Re^2} + \tfrac{1}{3} \Lambda a^2 \; , \qquad (10)$$

it is straightforward to show that

$$\kappa \left(\frac{c}{H_0} \right)^2 = (\Omega_0 + \Omega_\Lambda) - 1 \; , \qquad (11)$$

and

$$q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda \; . \qquad (12)$$

We should attempt to measure all the quantities independently and find out if these indeed describe our Universe.

The Determination of Deceleration Parameter

At small redshifts the differences between the world models depend only upon the deceleration parameter and *not* upon the density parameter and the curvature of space. In order to relate observables to intrinsic properties, we need to know how the distance measure D depends upon redshift and this involved two steps. First, we work out the dependence of the comoving radial distance coordinate r upon redshift z and then form the distance measure $D = \Re \sin(r/\Re)$.

Let us first carry out this calculation in terms of the *kinematics* of a world model decelerating with deceleration parameter q_0. We can write the variation of the scale factor a with cosmic epoch in terms of a Taylor series as follows

$$\begin{aligned} a &= a(t_0) + \dot{a}(t_0)\,\Delta t + \tfrac{1}{2}\ddot{a}(t_0)(\Delta t)^2 + \ldots \\ &= 1 - H_0\tau - \tfrac{1}{2}q_0 H_0^2\tau^2 + \ldots\ , \qquad (13) \end{aligned}$$

where we have introduced H_0, q_0 and the look-back time $\tau = t_0 - t = -\Delta t$; t_0 is the present epoch and t is some earlier epoch.

The Determination of Deceleration Parameter

It is straightforward to show that

$$\frac{1}{1+z} = 1 - x - \frac{q_0}{2}x^2 + \ldots \qquad ; \qquad z = x + \left(1 + \frac{q_0}{2}\right)x^2 + \ldots . \tag{14}$$

The dependence upon the curvature only appears in third-order in z and so to second-order, we find the kinematic result

$$D = \left(\frac{c}{H_0}\right)\left[z - \frac{z^2}{2}(1+q_0)\right] . \tag{15}$$

The distance measure D is the quantity which appears in the relation between observables and intrinsic properties, $L = 4\pi D^2(1+z)^2$ for bolometric luminosities.

From the full solution of the dynamical field equations, we obtain a similar result (see *Galaxy Formation* for details). Preserving quantities to third order in z, we find

$$D = \frac{c}{H_0}\left[z - \frac{z^2}{2}(1+q_0) + \frac{z^3}{6}\left(3 + 6q_0 + 3q_0^2 - 3\Omega_0\right)\right] . \tag{16}$$

The Modern List of Cosmological Parameters

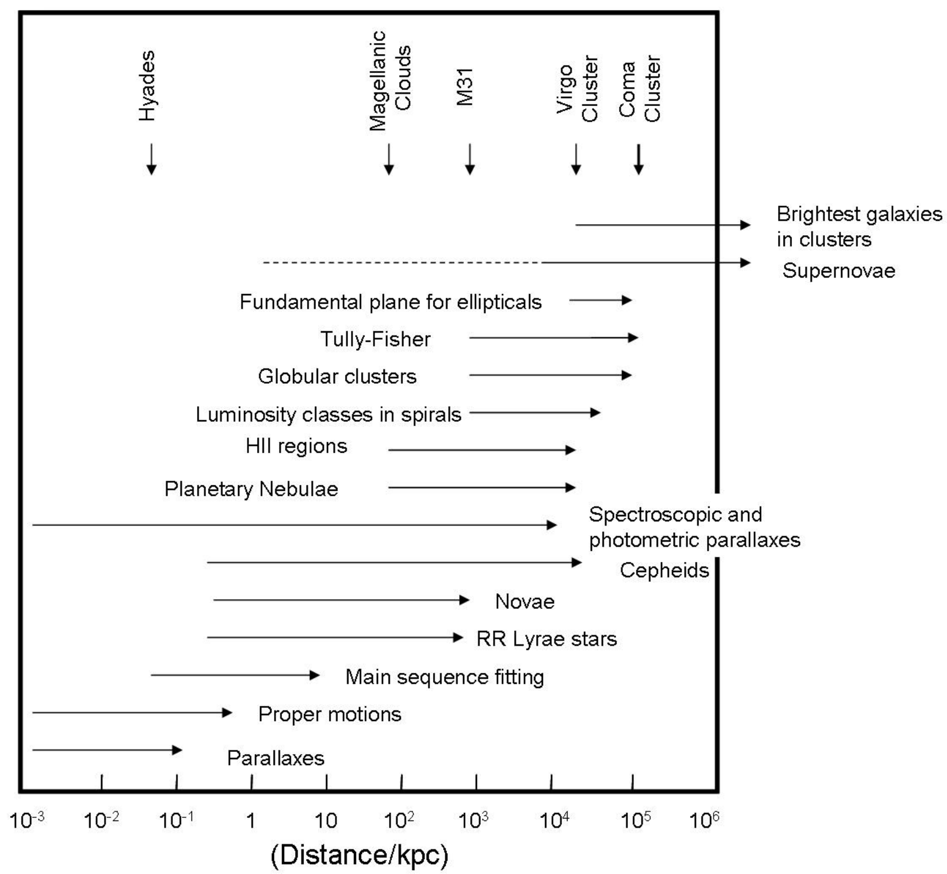
Parameter	Definition
$\omega_{\mathrm{B}} = \Omega_{\mathrm{B}} h^2$	baryon density parameter
$\omega_{\mathrm{D}} = \Omega_{\mathrm{D}} h^2$	cold dark matter density parameter
h	Hubble's constant
Ω_{Λ}	dark energy density parameter
n_{S}	scalar spectral index
τ	reionisation optical depth
σ_8	density variance in 8 Mpc spheres
w	dark energy equation of state
Ω_k	curvature density parameter
$f_{\nu} = \Omega_{\nu}/\Omega_{\mathrm{D}}$	massive neutrino fraction
N_{ν}	number of relativistic neutrino species
$\Delta^2_{\mathcal{R}}$	amplitude of curvature perturbations
r	tensor-scalar ratio
A_{S}	amplitude of scalar power-spectrum
$\alpha = \mathrm{d}\ln K/\mathrm{d}\ln k$	running of scalar spectral index
A_{SZ}	SZ marginalization factor
b	bias factor
z_{S}	weak lensing source redshift

Confrontation with Observation

The revolution of the last 10 years has been that there are now convincing independent approaches to estimating basic cosmological parameters. These include:

- (Fluctuation spectrum and polarisation of the Cosmic Microwave Background Radiation) - forbidden
- The value of Hubble's constant from the HST Key Project and other methods.
- $m - z$ relation for supernovae of Type 1A
- Mass density of the Universe from the infall velocities of galaxies into large scale structures.
- The formation of the light elements by primordial nucleosynthesis.
- Cosmic time-scale from the theory of stellar evolution and nucleocosmochronology.
- The power spectrum of galaxies from the Sloan Digital Sky Survey and the AAO 2dF galaxy survey.

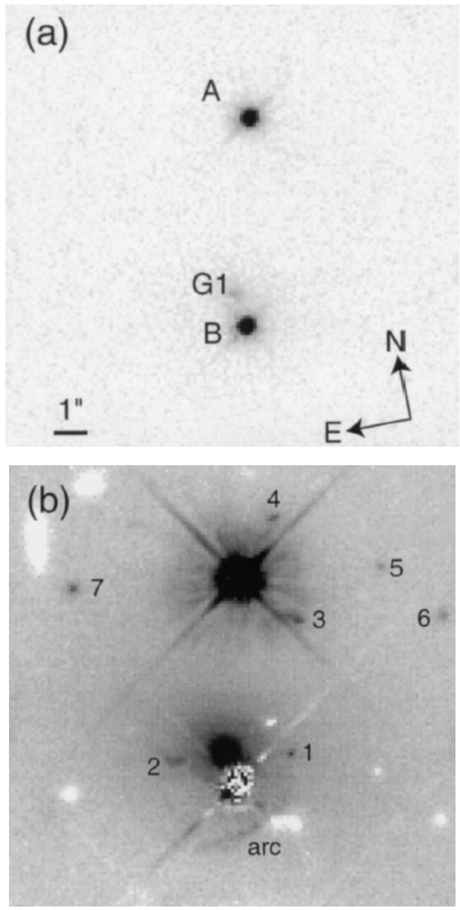
Hubble's Constant - the HST Key Project



The controversies of the 1970s and 1980s have been resolved thanks to a very large effort by many observers to improve knowledge of the distances to nearby galaxies. This approach can be thought of as the ultimate in the traditional 'distance indicator' approach to the determination of cosmological distances.

- Final result of HST key project $H_0 = 72 \pm 7$ (1-σ) km s^{-1} Mpc^{-1}.

Hubble's Constant - Gravitational Lensing



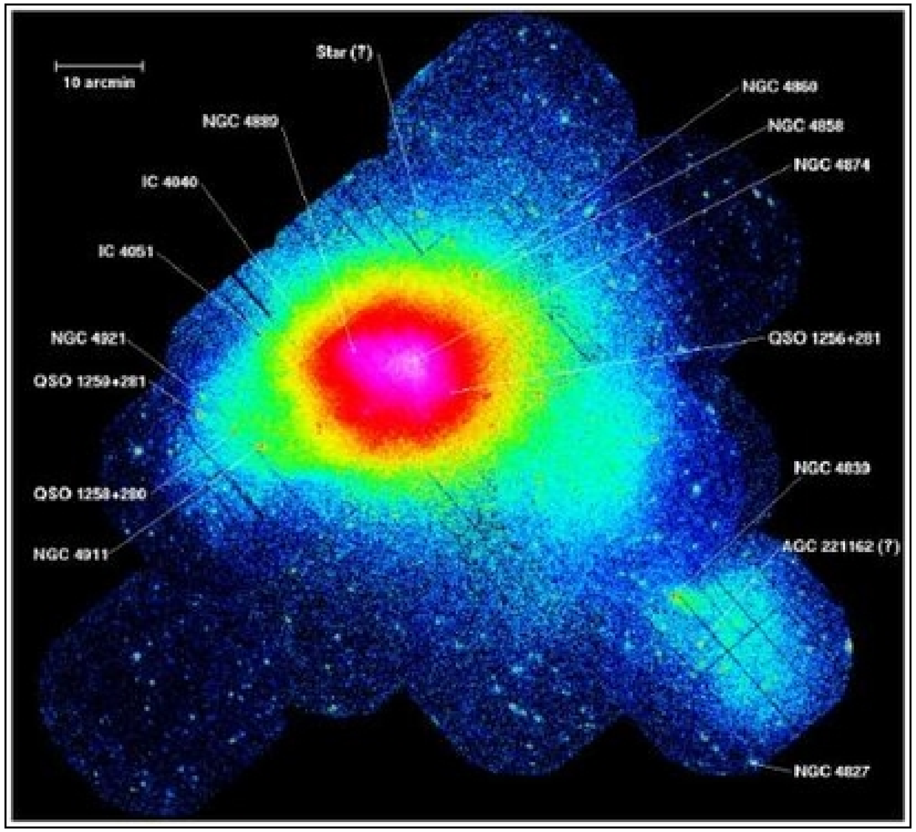
Optical images of the double quasar 0957+561 observed by the Hubble Space Telescope
(top) 80 s (bottom) 37,000 s.

A time-delay of 418 days has been measured for the two components of the double quasar 0957+561. This observation enables physical scales at the lensing galaxy to be determined, the main uncertainty resulting from the modelling of the mass distribution in the lensing galaxy. They derived a value of Hubble's constant of $H_0 = 64 \pm 13$ km s^{-1} Mpc^{-1} at the 95% confidence level.

A statistical analysis of 16 multiply imaged quasars by Oguri (2007) found a value of Hubble's constant of:

- $H_0 = 68 \pm 6\,\mathrm{(stat)} \pm 8\,\mathrm{(syst)}$ km s^{-1} Mpc^{-1} Mpc^{-1}.

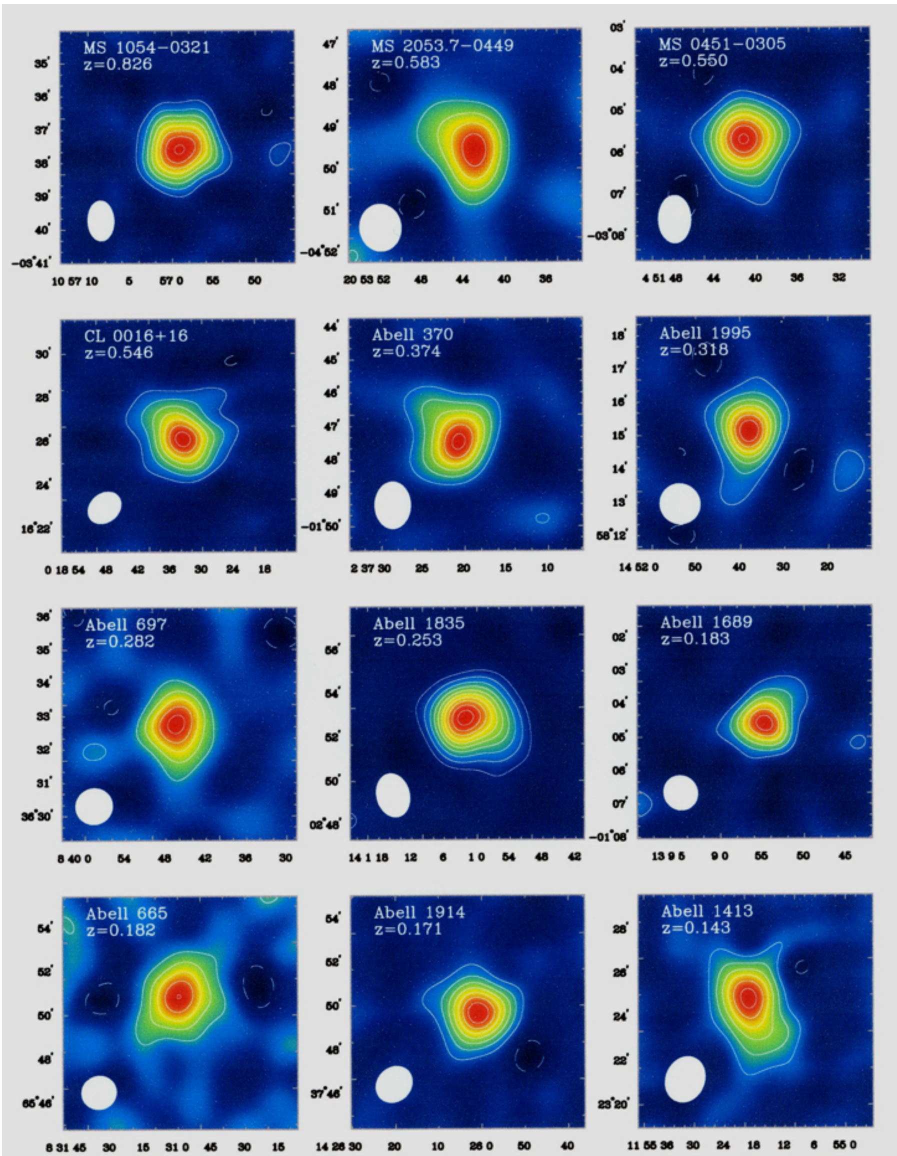
Hubble's Constant - the Sunyaev-Zeldovich Effect



The Coma cluster of galaxies in X-rays.

- The Sunyaev-Zeldovich effect determines the quantity $N_{\rm e}T_{\rm e}L$, where L is the dimension of the volume of hot gas.
- The bremsstrahlung X-ray emission of the cluster determines the quantity $L^3N_{\rm e}^2T^{-1/2}$.
- The temperature T can be estimated from the shape of the bremsstrahlung spectrum.
- Hence $N_{\rm e}$ can be eliminated between these two relations and an estimate of L found.
- By measuring the angular size θ of the emitting volume, the distance of the cluster can be found. Once the redshift of the cluster has been measured, Hubbles constant can be estimated.

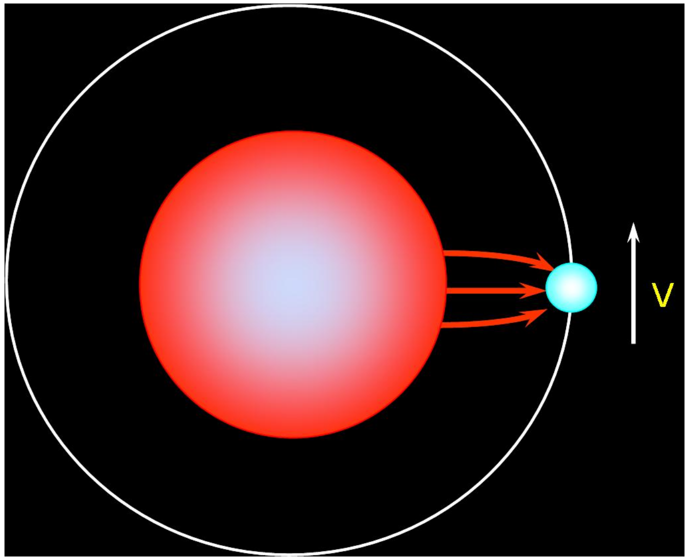
Hubble's Constant - the Sunyaev-Zeldovich Effect

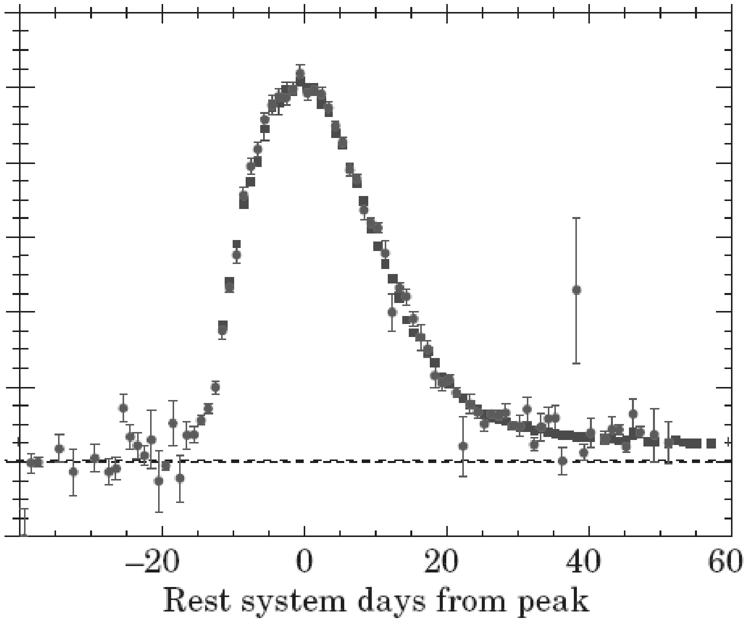


Bonamente and his colleagues studied 38 clusters of galaxies in the redshift interval $0.14 \leq z \leq 0.89$ using X-ray data from the Chandra X-ray Observatory and measurements of the corresponding Sunyaev-Zeldovich decrements from the Owens Valley Radio Observatory and the Berkeley-Illinois-Maryland Association interferometric arrays (Bonamente et al. 2006).

- Hubble's constant $H_0 = 76.9^{+3.9}_{-3.4}\,(\text{stat})^{+10.0}_{-8.0}\,(\text{syst})$ $\text{km s}^{-1}\,\text{Mpc}^{-1}$,
  assuming $\Omega_0 = 0.3$ and $\Omega_\Lambda = 0.7$.

Supernovae of Type 1a

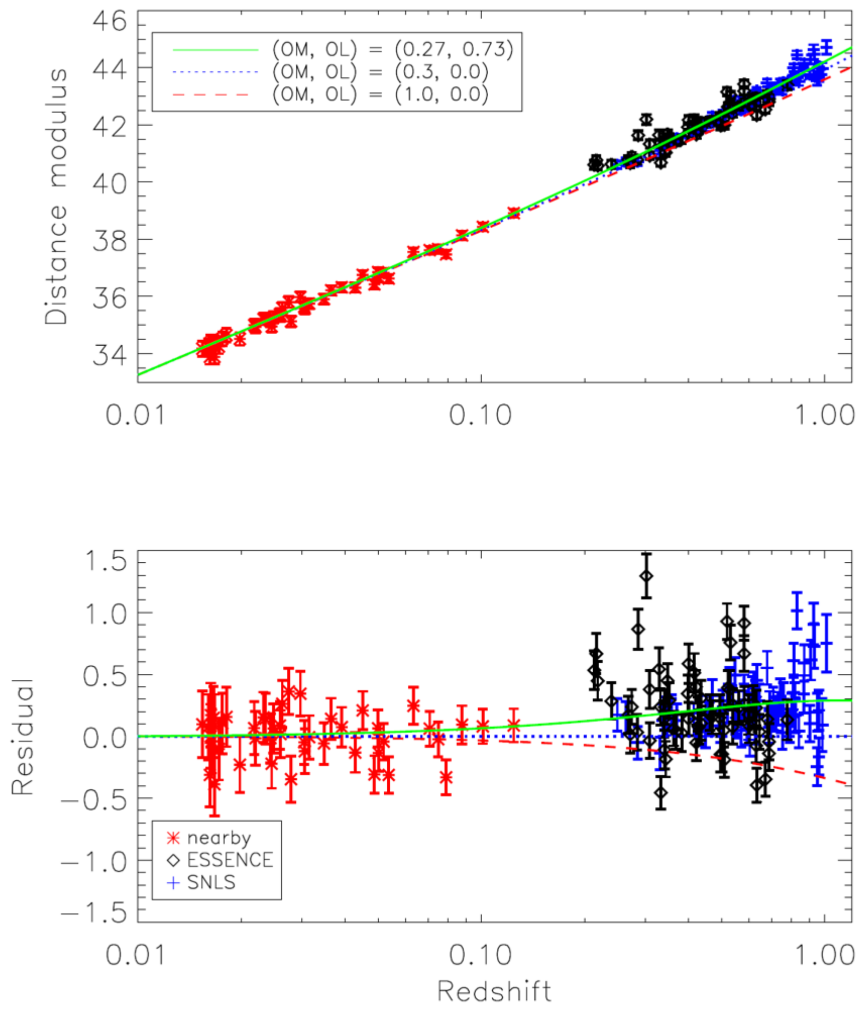




The Type 1a supernovae are associated with the explosions of white dwarfs in binary systems. The explosion mechanism is probably nuclear deflagration associated with mass transfer from the companion onto the surface of the white dwarf which destroys the star. This process is expected to result in a uniform class of explosions.

Type 1a supernovae have dominated methods for extending the redshift-distance relation to large redshifts. They are very bright explosions and are observed to have remarkably standard properties, particularly when corrections are made for the luminosity-width relation.

Results of the combined ESSENCE and SNLP Projects



The ESSENCE project has the objective of measuring the redshifts and distances of about 200 supernovae. The Supernova Legacy Project aims to obtain distances for about 500 supernovae. The observations are consistent with a finite and positive value of the cosmological constant. Luminosity distance modulus vs. redshift for the ESSENCE, SNLS, and nearby SNe Ia (shown in red).

For comparison the overplotted green line and residuals are for a ΛCDM model with $w = -1$, $\Omega_0 = 0.27$ and $\Omega_\Lambda = 0.73$ (Wood-Vasey et al., 2007).

Redshift Distortions due to Infall into Large-scale Density Perturbations

Galaxies in the vicinity of a supercluster are accelerated towards it, thus providing a measure of the mean density of gravitating matter within the system. The velocities induced by large-scale density perturbations depend upon the *density contrast* $\delta\varrho/\varrho$ between the system studied and the mean background density. A typical formula for the infall velocity u of test particles into a density perturbation is

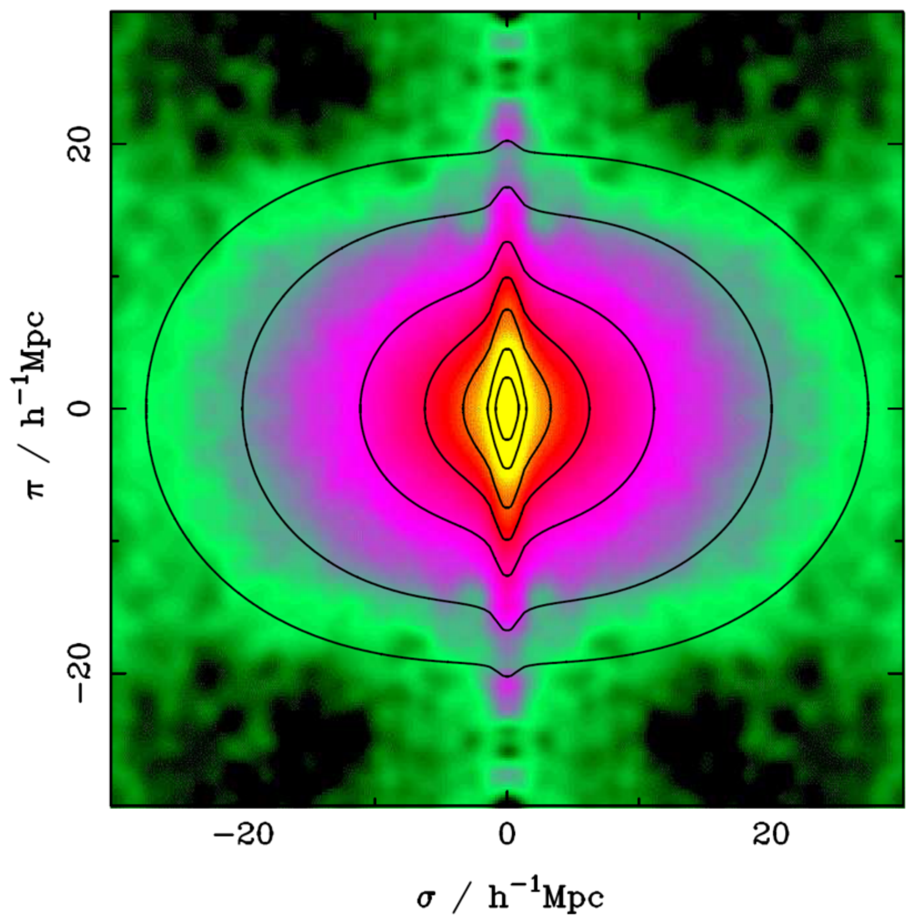
$$u \propto H_0 r \Omega_0^{0.6} \left(\frac{\delta\varrho}{\varrho}\right)_0 \tag{17}$$

(Gunn 1978). In the case of small spherical perturbations, a result correct to second order in the density perturbation was presented by Lightman and Schechter (1990).

$$\frac{\delta v}{v} = -\frac{1}{3}\Omega_0^{4/7}\left(\frac{\delta\varrho}{\varrho}\right)_0 + \frac{4}{63}\Omega_0^{13/21}\left(\frac{\delta\varrho}{\varrho}\right)_0^2 . \tag{18}$$

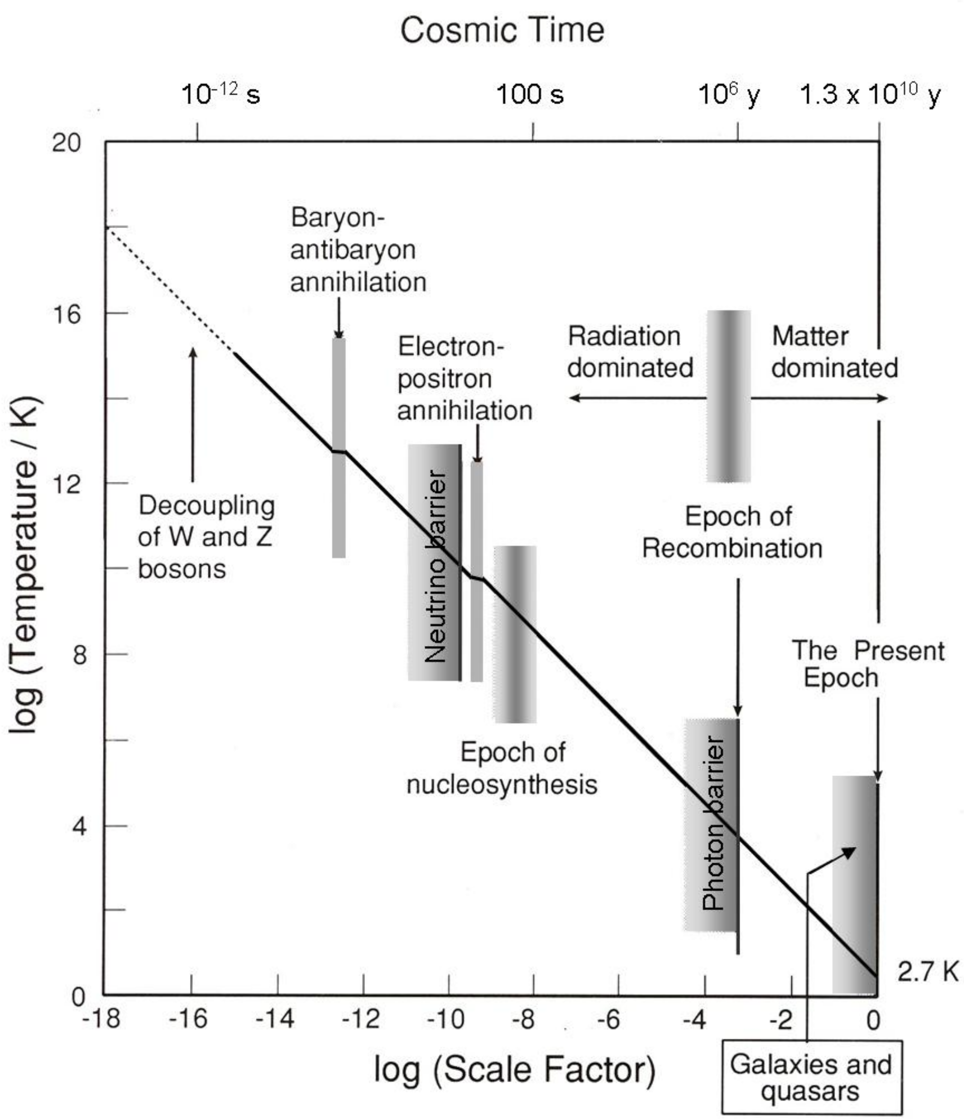
In Gunn's analysis, values of $\Omega_0 \sim 0.2 - 0.3$ were found, similar to the values found in clusters of galaxies (Bahcall 2000).

Redshift Distortions due to Infall into Large-scale Density Perturbations



The two-dimensional correlation function for galaxies selected from the 2dF Galaxy Redshift Survey. The flattening in the vertical direction is due to the infall of galaxies into large-scale density perturbations. The elongations along the central vertical axis are associated with the velocity dispersion in clusters of galaxies. The inferred overall density parameter is $\Omega_0 = 0.25$.

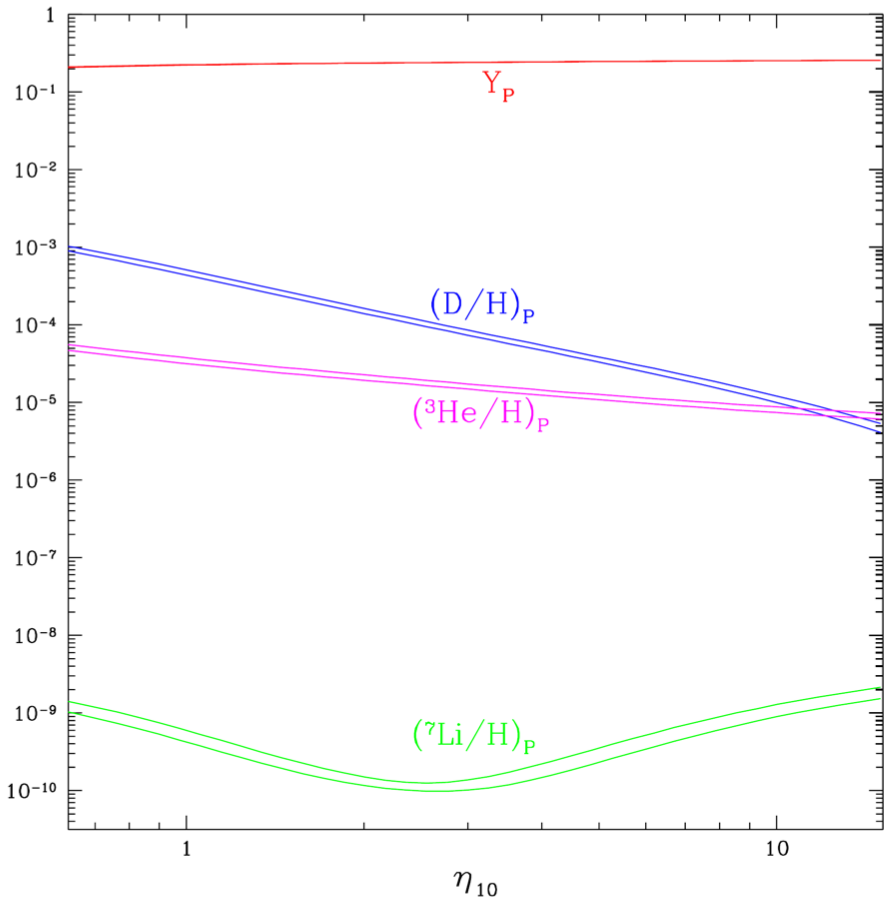
Summary of the Thermal History of the Universe



This diagram summarises the key epochs in the thermal history of the Universe. The key epochs are

- The epoch of recombination.
- The epoch of equality of matter and radiation.
- The epoch of nucleosynthesis

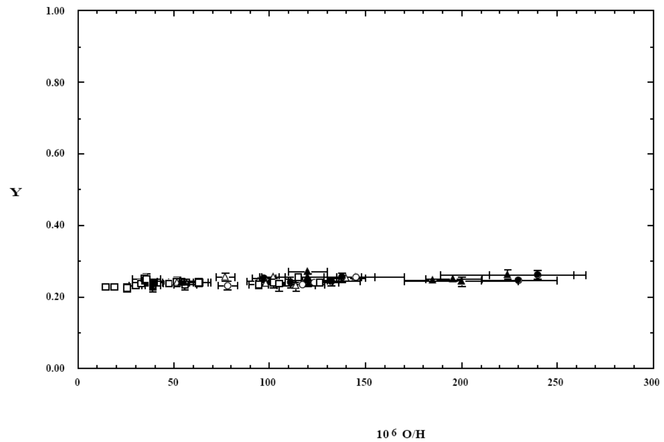
Formation of the Light Elements by Primordial Nucleosynthesis



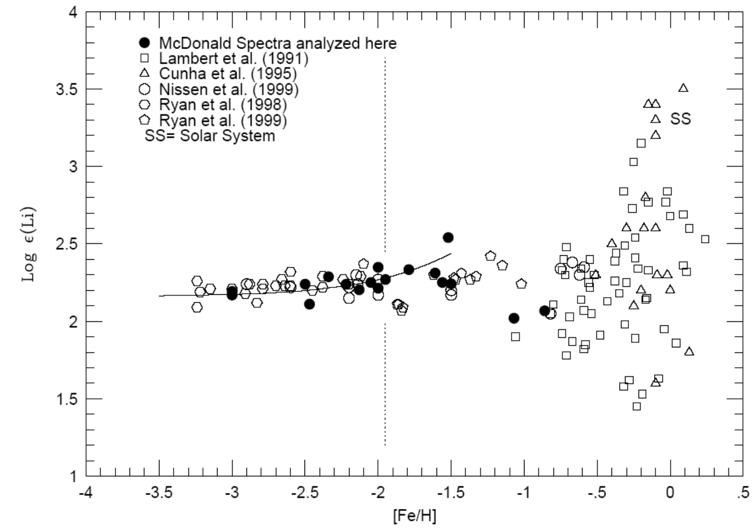
The predicted primordial abundances of the light elements as a function of the present baryon-to-photon ratio in the form $\eta = 10^{10} n_B / n_\gamma = 274\, \Omega_B h^2$. Y_p is the abundance of helium by mass, whereas the abundances for D, ^{3}He and ^{7}Li are plotted as ratios by number relative to hydrogen. The widths of the bands reflect the theoretical uncertainties in the predictions.

Observed and Predicted Light Element Abundances

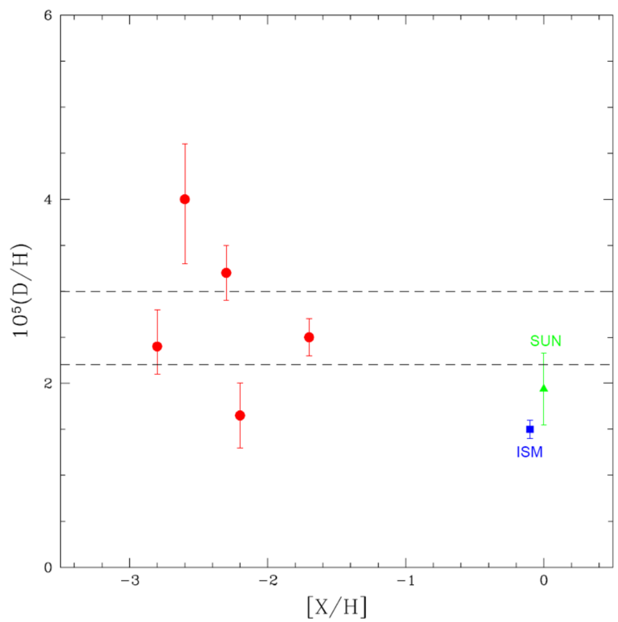
Helium



Lithium



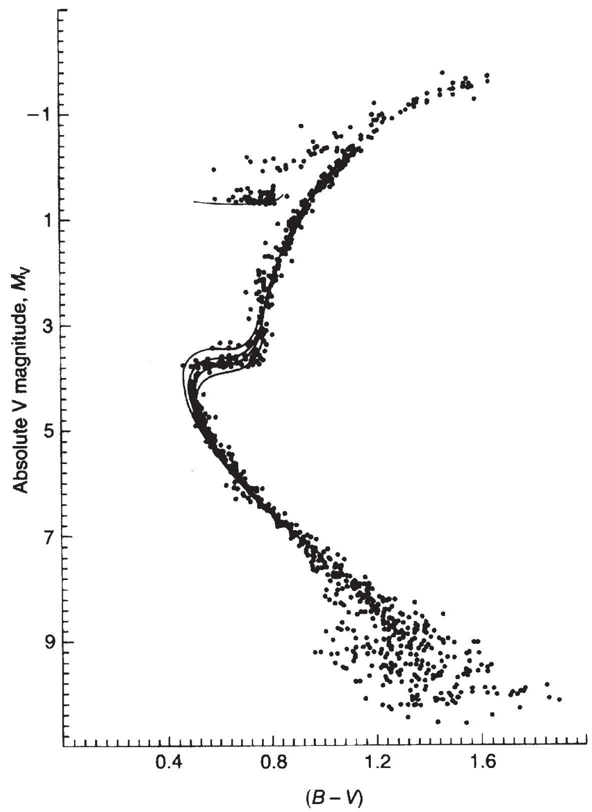
Deuterium



Steigman finds a best fitting value $\Omega_B h^2 = 0.022^{+0.003}_{-0.002}$. This can be compared with the independent estimate from the power-spectrum of fluctuations of the Cosmic Microwave Background Radiation $\Omega_B h^2 = 0.0224 \pm 0.0009$.

Cosmic Time-scales

The Ages of the Oldest Globular Clusters:



Bolte (1997) :

$T_0 = 15 \pm 2.4\ (\mathrm{stat})^{+4}_{-1}\ (\mathrm{syst})\ \mathrm{Gy}$

Chaboyer (1998) :

$T_0 = (11.5 \pm 1.3)\ \mathrm{Gy}$

Nucleocosmochronology

CS 22892-052 has iron abundance is 1000 times less than the solar value. A number of species never previously observed in such metal-poor stars were detected, as well a single line of thorium. A lower limit to the age of the star is

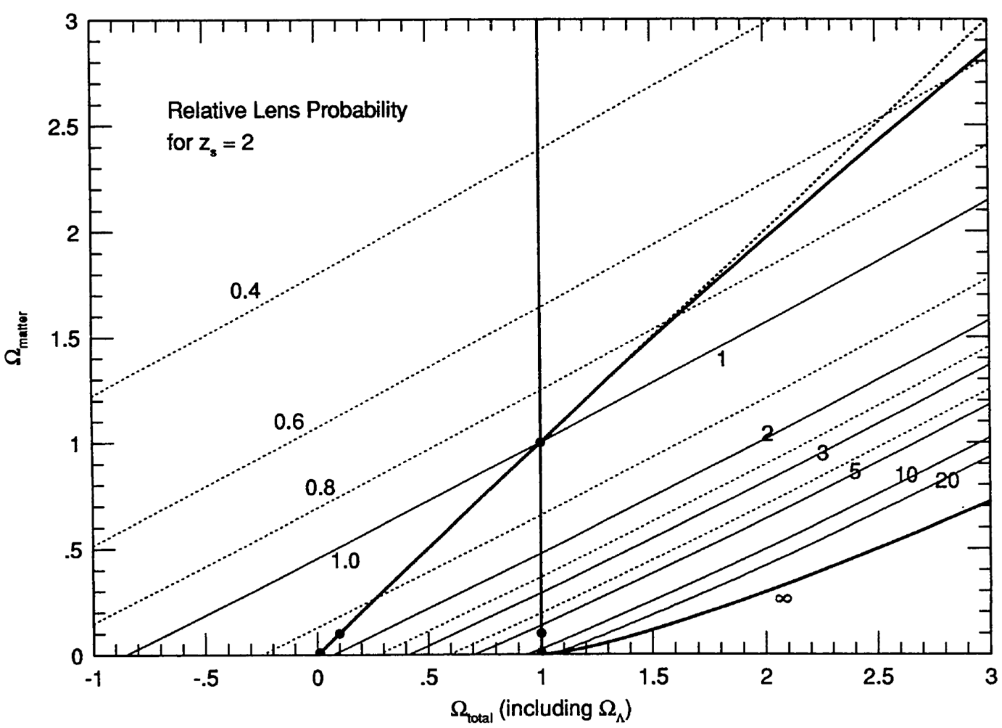
$$(15.2 \pm 3.7) \times 10^9 \text{ years .}$$

Schramm found a lower limit to the age of the Galaxy of 9.6×10^9 years and his best estimates of the age of the Galaxy are somewhat model-dependent, but typically ages of about $(12 - 14) \times 10^9$ years.

Statistics of Gravitational Lensing

Carroll and his colleagues present the probability of strong gravitational lensing relative to the case of the Einstein–de Sitter model. The probability for any other model becomes

$$p(z_S) = \frac{15H_0^2}{4c^2}\left[1 - \frac{1}{(1+z_S)}\right]^{-3} \int_0^{z_S} \left(\frac{D_L D_{LS}}{D_S}\right)^2 \times \frac{(1+z)^2\,dz}{[(1+z)^2(\Omega_0 z + 1) - \Omega_\Lambda z(z+2)]^{1/2}}\,. \quad (19)$$



Statistics of Gravitational Lensing

The largest survey to date designed specifically to address this problem has been the Cosmic Lens All Sky Survey (CLASS) in which a very large sample of flat spectrum radio sources were imaged by the Very Large Array (VLA), Very Long Baseline Array (VLBA) and the MERLIN long-baseline interferometer. The CLASS collaboration has reported the point-source lensing rate to be one per 690 ± 190 targets (Mitchell et al. 2005). The CLASS collaboration found that the observed fraction of multiply-lensed sources was consistent with flat world models, $\Omega_0 + \Omega_\Lambda = 1$, in which density parameter in the matter Ω_0 was

$$\Omega_0 = 0.31 {}^{+0.27}_{-0.14} (68\%) {}^{+0.12}_{-0.10} (\text{systematic}) \, . \tag{20}$$

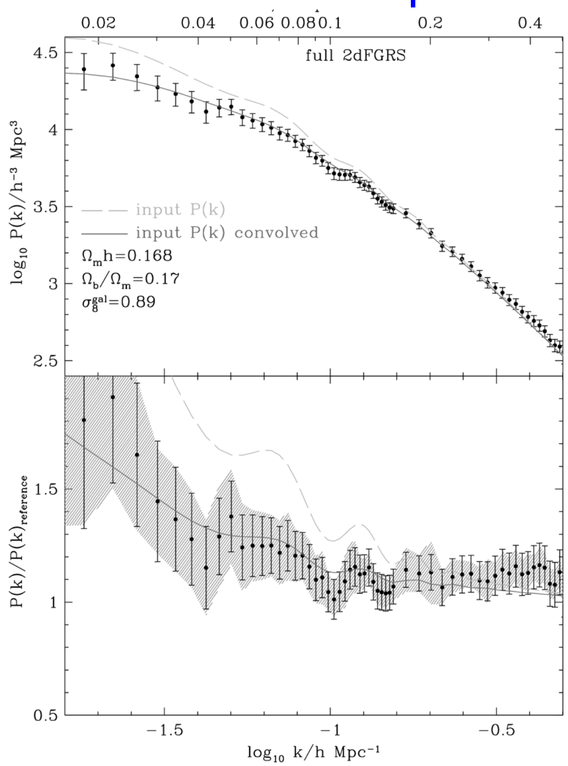
Alternatively, for a flat universe with an equation of state for the dark energy of the form $p = w \varrho c^2$, they found an upper limit to w,

$$w < -0.55 {}^{+0.18}_{-0.11} (68\%) \, , \tag{21}$$

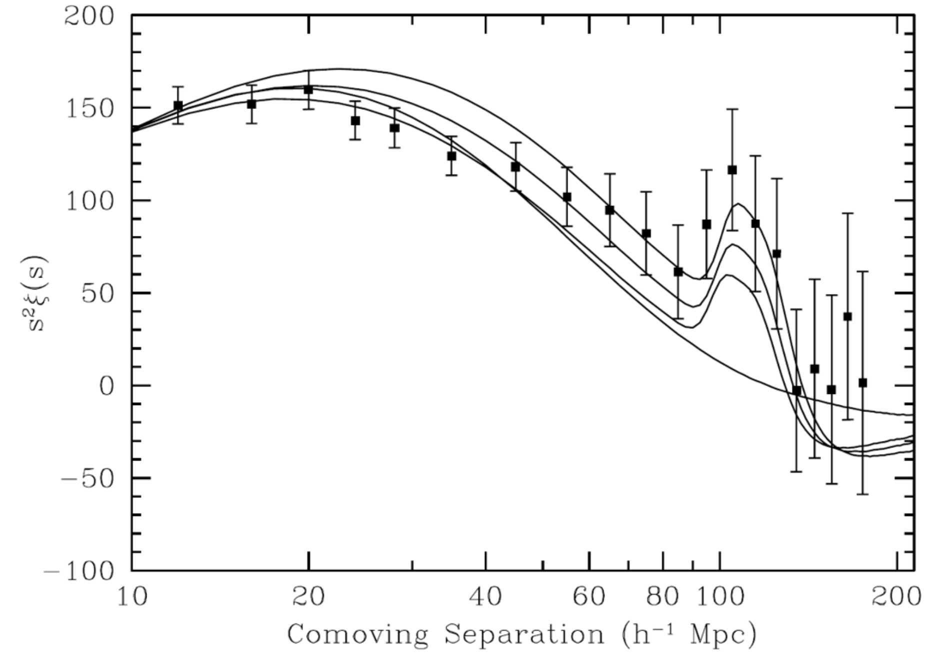
consistent with the standard value for the cosmological constant $w = -1$.

Acoustic Peaks in the Galaxy Power-Spectra

AAT 2dF Power-spectrum

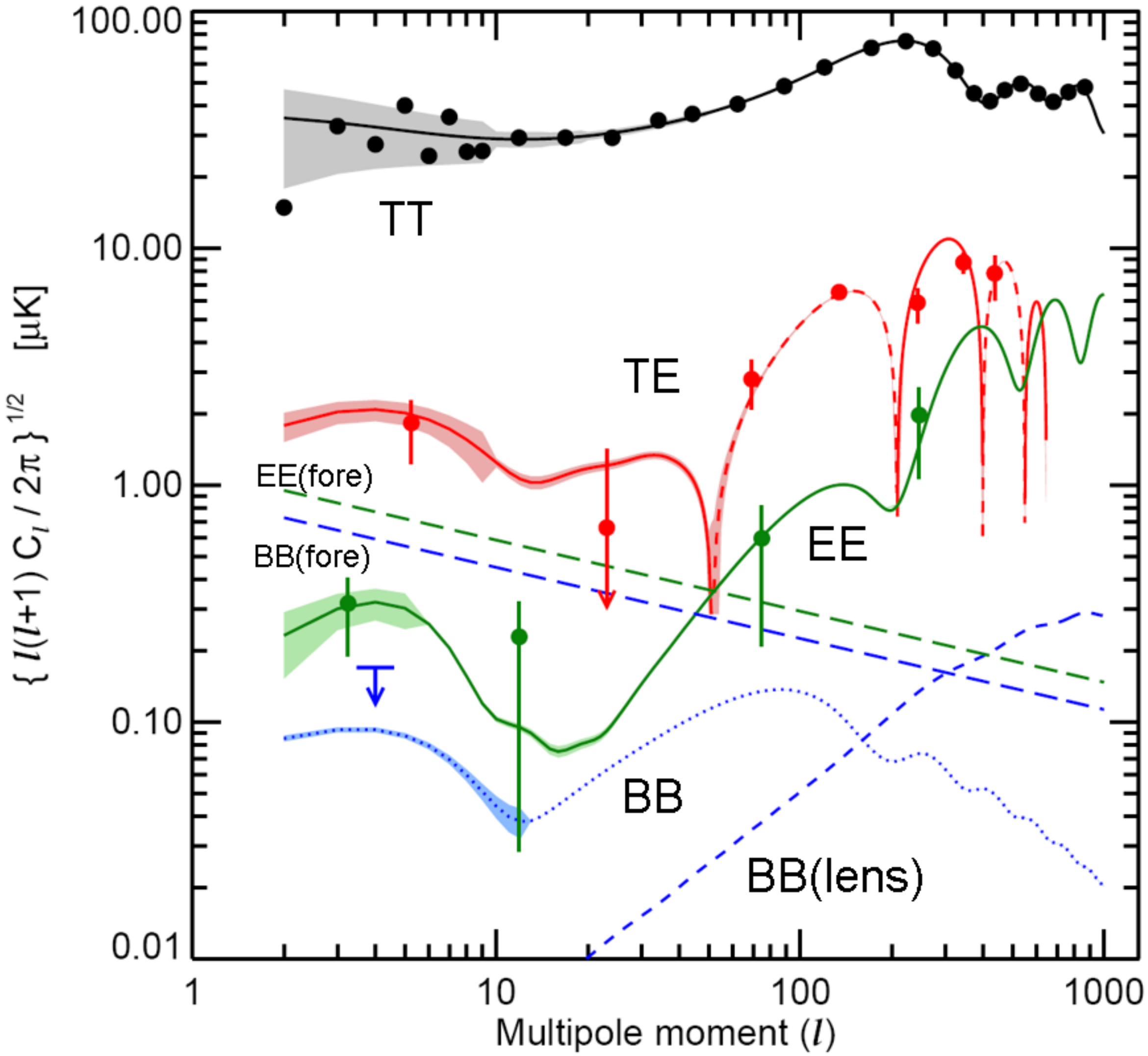


Sloan Digital Sky Survey



Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) galaxy sample plus the Luminous Red Galaxy (LRG) plus the 2-degree Field Galaxy Redshift comprises 893,319 galaxies over 9100 deg^2. Baryon acoustic oscillations are observed in all redshift slices out to $z = 0.5$. $\Omega_0 = 0.286 \pm 0.018$ and $H_0 = 68.2 \pm 2.2$ km s^{1} Mpc^{1}.

The Temperature Fluctuations and Polarisation of the Cosmic Microwave Background Radiation



This topic will be dealt with by the other lecturers in the course.

The Concordance Model

Adopting $H_0 = 73$ km s^{-1} Mpc^{-1}, we find the following self-consistent set of parameters:

Hubble's constant	$H_0 = 73$ km s^{-1} Mpc^{-1}
Curvature of Space	$\Omega_\Lambda + \Omega_0 = 1$
Baryonic density parameter	$\Omega_B = 0.04$
Cold Dark Matter density parameter	$\Omega_D = 0.24$
Total Matter density parameter	$\Omega_0 = 0.28$
Density Parameter in Vacuum Fields	$\Omega_\Lambda = 0.72$
Optical Depth for Thomson Scattering on Reheating	$\tau = 0.09$

The remarkable feature of these figures is that they are now known to better than 10% accuracy. This is an extraordinary revolution. We live in the era of precision cosmology.

But there is also a huge challenge – we need to understand the physics to better than 10% to determine the cosmological parameters with improved precision.

The Properties of the Concordance Model

The scale factor-cosmic time relation for $\Omega_0 + \Omega_\Lambda = 1$ model is

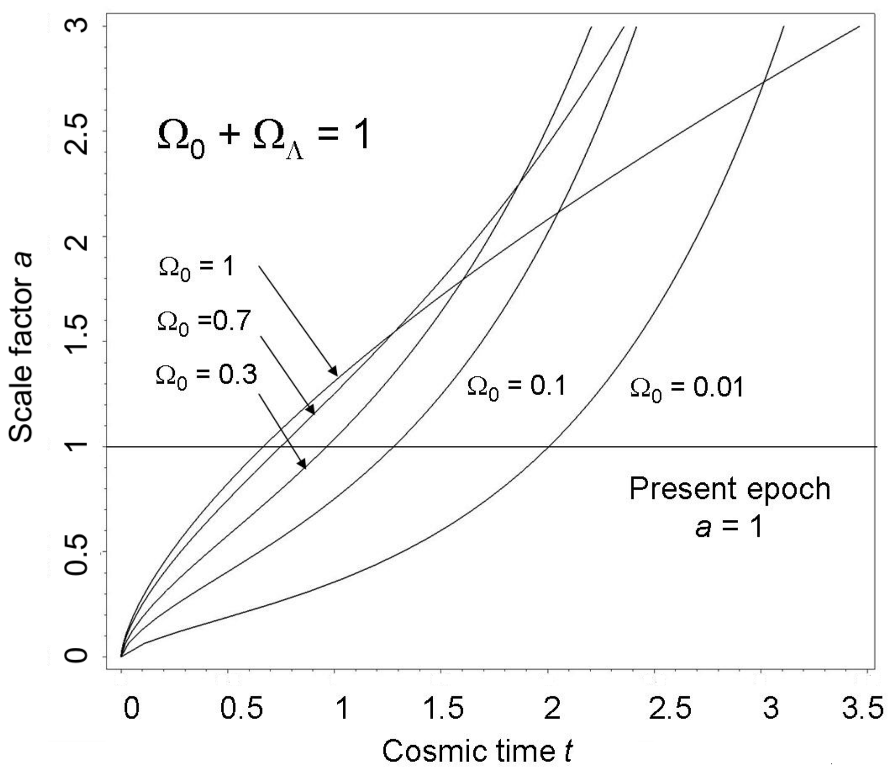
$$t = \int_{t_1}^{t_0} \mathrm{d}t = -\frac{1}{H_0} \int_{\infty}^{z} \frac{\mathrm{d}z}{(1+z)[\Omega_0(1+z)^3 + \Omega_\Lambda]^{1/2}} , \quad (22)$$

where $a = (1+z)^{-1}$. This results in an accelerating Universe at the present epoch.

If $\Omega_\Lambda = 0.72$ and $\Omega_0 = 0.28$, the age of the world model is

$$T_0 = 0.983 H_0^{-1} = 1.32 \times 10^{10} \text{ years.} \quad (23)$$

Thus, $T_0 \equiv H_0^{-1}$. Is this a coincidence? In the standard ΛCDM model, the answer is "Yes".



Inhomogeneous Universes

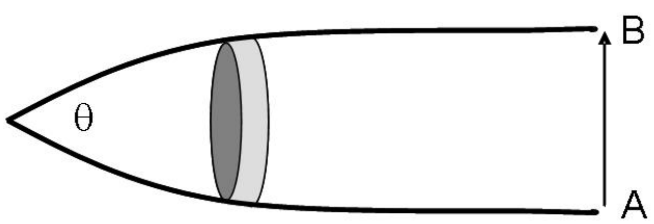
We consider the case of the critical Einstein–de Sitter world model, $\Omega_0 = 1, \Omega_\Lambda = 0$, for which the spatial geometry is flat, $\kappa = 0, \Re = \infty$.

Suppose the Universe were so inhomogeneous that all the matter was condensed into point-like objects. The following argument is due to Zeldovich. Then, there is only a small probability that there will be any matter within the light-cone subtended by a distant object of small angular size. Because of the long-range nature of gravitational forces. The background metric remains the standard flat Einstein–de Sitter metric

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}t^2 - \frac{a^2(t)}{c^2}\left[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right] \\ &= \mathrm{d}t^2 - \frac{a^2(t)}{c^2}\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right] , \end{aligned}$$

where $a(t) = (t/t_0)^{2/3}$ and $t_0 = \frac{2}{3}H_0$ is the present age of the Universe. r, x, y and z are co-moving coordinates referred to the present epoch t_0.

Inhomogeneous Universes



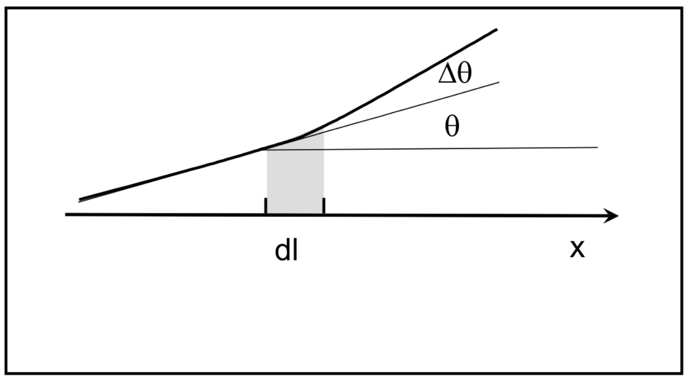
In the model of an inhomogeneous Universe, we consider the propagation of the light rays in this background metric, but include in addition the effect of the absence of matter within the light cone subtended by the source at the observer. The angular deflection of a light ray by a point mass, or by an axially symmetric distribution of mass at the same distance, is

$$\frac{4GM(< p)}{pc^2}\,, \tag{24}$$

where $M(< p)$ is the mass within 'collision parameter' p, that is, the distance of closest approach of the light ray to the point mass.

Inhomogeneous Universes

Density boundary



Density boundary

Because of the principle of superposition, the effect of the 'missing mass' within the light cone may be precisely found by supposing that the distribution of mass has negative density $-\varrho(t)$ within the light cone. The deviations of the light cones from the homogeneous result, $\mathrm{d}\theta = \mathrm{d}y/\mathrm{d}x =$ constant, are due to the influence of the 'negative mass' within the light cone. As a result, the light rays bend *outwards* rather than inwards, as in the usual picture.

Going through the mathematics, the resulting deflection, we find

$$L = \frac{2c\Theta}{5H_0}\left[1 - (1+z)^{-5/2}\right] \ . \qquad (25)$$

Inhomogeneous Universes

Corresponding results have been obtained for Friedman models with $\Omega_0 \neq 1$ by Dashevsky and Zeldovich and by Dyer and Roeder. In these cases, if $\Omega_0 > 1$,

$$L = \frac{3c\Omega_0^2\Theta}{4H_0(\Omega_0-1)^{5/2}}\left[\sin^{-1}\left(\frac{\Omega_0-1}{\Omega_0}\right)^{1/2} - \sin^{-1}\left(\frac{\Omega_0-1}{\Omega_0(1+z)}\right)^{1/2}\right]$$
$$-\frac{3c\Omega_0\Theta}{4H_0(\Omega_0-1)^2}\left[1-\frac{(1+\Omega_0 z)^{1/2}}{(1+z)}\right]+\frac{1}{2(\Omega_0-1)}\left[1-\frac{(1+\Omega_0 z)^{1/2}}{(1+z)^2}\right].$$

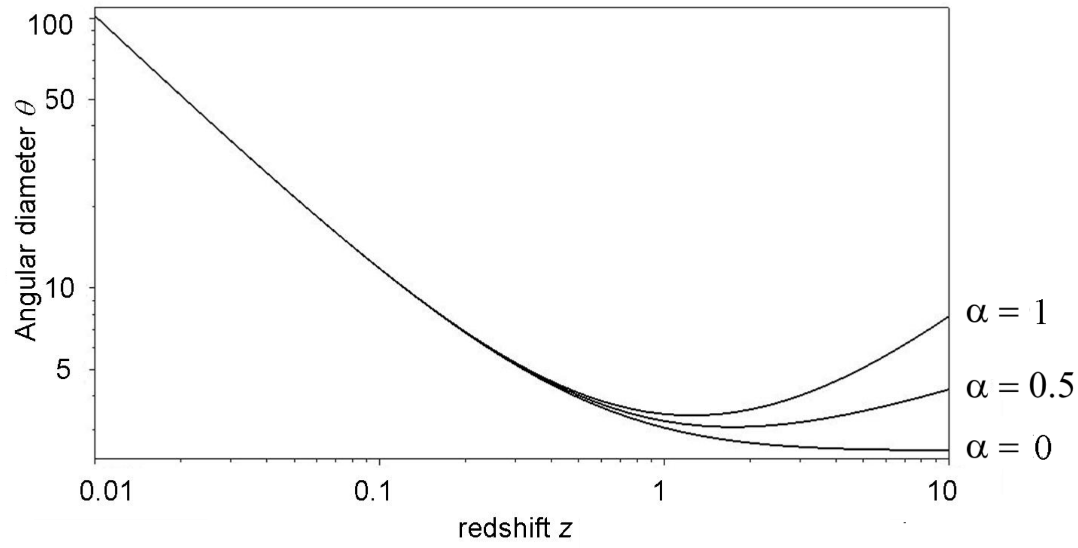
If $\Omega_0 < 1$, the inverse trigonometric functions are replaced by inverse hyperbolic functions according to the rule $\sin^{-1} \mathrm{i}x = \mathrm{i}\sinh^{-1}x$.

Inhomogeneous Universes

Dyer and Roeder have presented the analytic results for intermediate cases in which a certain fraction α of the total mass density is uniformly distributed within the light cone. For the Einstein de Sitter model, they find the simple result:

$$L = \Theta D_{\rm A} = \Theta \frac{2}{\beta}(1+z)^{(\beta-5)/4}[1-(1+z)^{-\beta/2}] \, , \qquad (26)$$

where $\beta = (25 - 24\alpha)^{1/2}$.



The minimum in the standard $\theta - z$ relation disappears in the maximally inhomogeneous model.

Inhomogeneous Universes

Thus, if no minimum is observed in the $\theta - z$ relation for a class of standard rods, it does not necessarily mean that the Universe must have $\Omega_0 \approx 0$. It might just mean that the Universe is of high density and is highly inhomogeneous.

If a minimum *is* observed in the $\theta - z$ relation, there must be matter within the light cone and limits can be set to the inhomogeneity of the matter distribution in the Universe. The effects upon the observed intensities of sources may be evaluated using the usual procedures, that is, the $\theta - z$ relation may be used to work out the fraction of the total luminosity of the source incident upon the observer's telescope using the reciprocity theorem. The end results are not so very different from those of the standard models, but remember, we are in the era of precision cosmology.