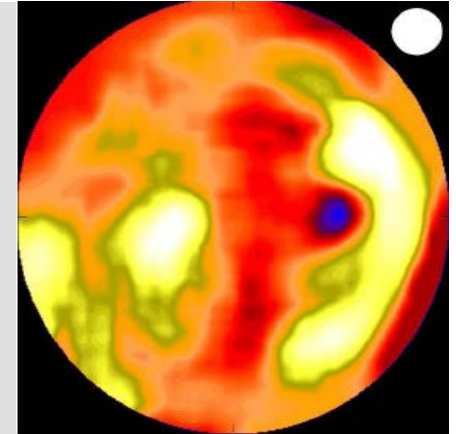


Interaction of the CMB with Astrophysical Plasma: high-E



Sergio Colafrancesco

Wits University - DST/NRF SKA Research Chair
INAF-OAR

Email: Sergio.Colafrancesco@wits.ac.za

Outline

📍 Lecture 1

- ❏ CMB photon interaction
- ❏ LSS: plasma content
- ❏ Spectral and spatial properties
- ❏ Plasma – CMB photon interaction: basic mechanisms
- ❏ ICS, Pair production, Primakov effect

📍 Lecture 2

- ❏ The SZ effect: thermal, non-th, kinetic, polarization
- ❏ General description
- ❏ Galaxy clusters
- ❏ RGs and other cases
- ❏ Experimental outline

📍 Lecture 3

- ❏ IC-CMB and high energy phenomena
- ❏ X-rays
- ❏ Gamma-rays
- ❏ Multi-frequency studies
- ❏ An experimental outline

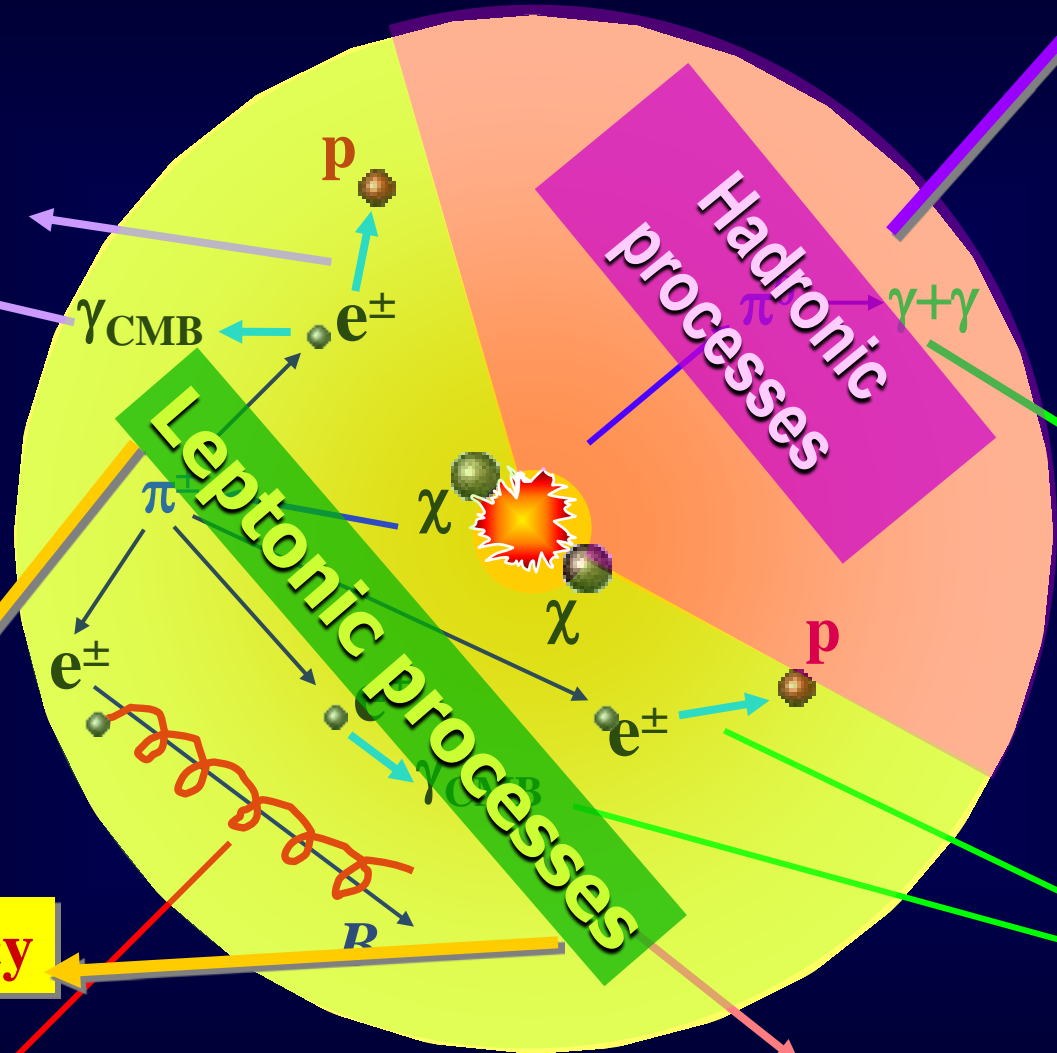
Inverse Compton Scattering

- Interaction of secondary electrons from DM annihilation with the CMB
 - Hard X-Rays
 - Gamma-rays
- Interaction of Cosmic Rays in cluster (with Radio Halos) with the CMB
 - Hard X-Rays
 - Gamma-rays
 - The WR model
- Interaction of Cosmic Rays in cavities with the CMB
 - HXR
 - Gamma-rays
- Interaction of Cosmic Rays in radiogalaxy lobes with the CMB
 - X-rays
 - Gamma-rays

Dark Matter annihilation

High frequency

X-rays
bremsstrahlung
ICS
 γ_{CMB}



Low frequency

Radio emission
Synchrotron

SZ effect
ICS

Gamma rays
(π^0 decay)

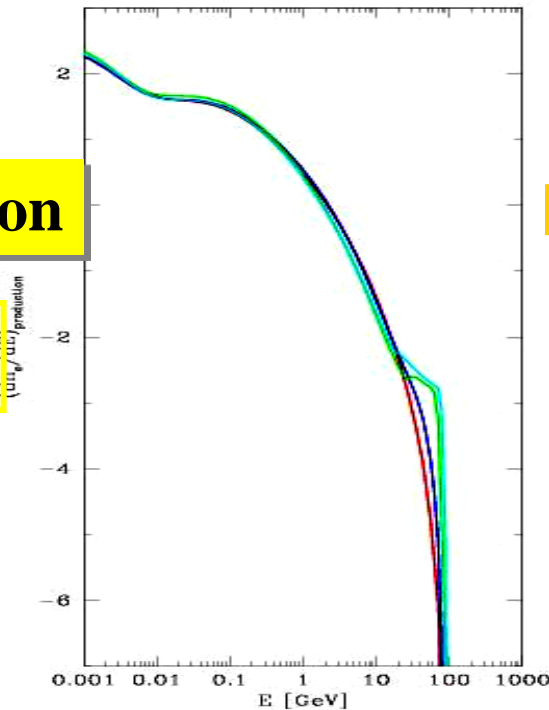
Gamma rays
bremsstrahlung
ICS

Leptons: e^\pm equilibrium spectrum

$$\frac{\partial n_e(E, r)}{\partial t} - \nabla [D(E) \nabla n_e(E, r)] - \frac{\partial}{\partial E} [b_e(E) n_e(E, r)] = Q_e(E, r)$$

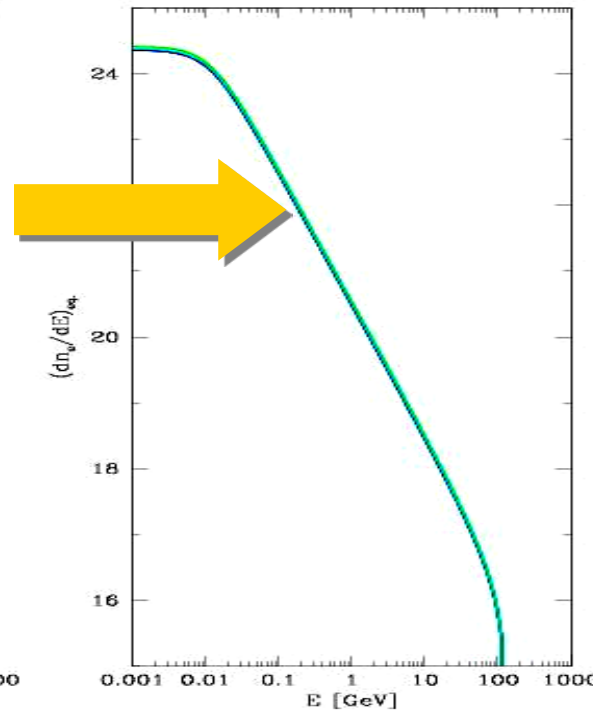
Production

$$Q_e(E, r)$$



Equilibrium

$$n_e(E, r)$$



Diffusion

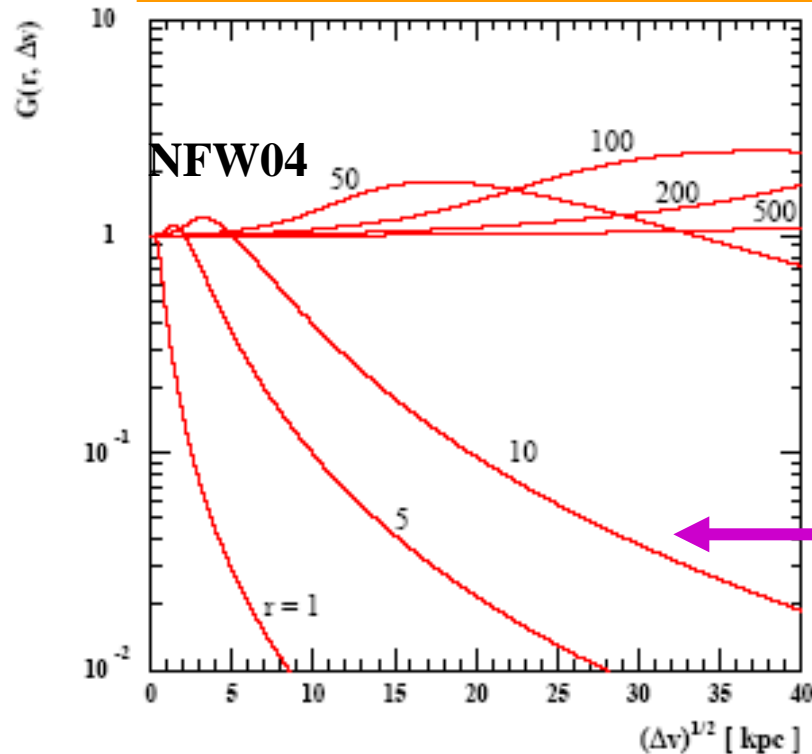
$$D(E) = D_0 E^\gamma B^{-\gamma}$$

E losses

$$b_e(E) = b_{IC} + b_{sync} + b_{Coul} + b_{brem}$$

Solution: complete

$$n_e(E, r) = \frac{1}{b(E)} \int_E^{M_\chi} dE' \hat{G}(r, \lambda - \lambda') Q_e(E, r)$$



Galaxy clusters

Galaxies

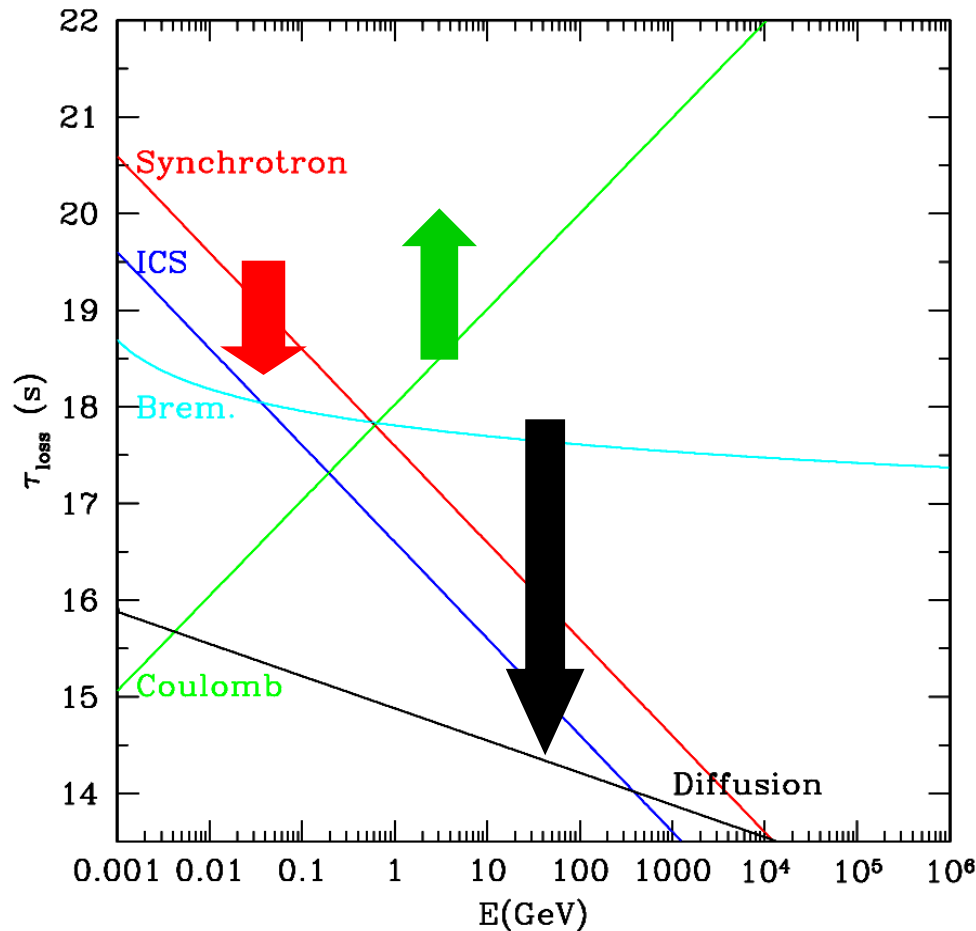
$$\hat{G} = \frac{1}{[4\pi\Delta\lambda]^{1/2}} \sum_{n=-\infty}^{+\infty} (-1)^n \int_0^{R_h} dr' \frac{(r')^2}{r'_n r} \left[\exp\left(-\frac{(r'-r_n)^2}{4\Delta\lambda}\right) - \exp\left(-\frac{(r'_n+r)^2}{4\Delta\lambda}\right) \right] \frac{n_\chi^2(r')}{n_\chi^2(r)}$$

Energy losses vs. Diffusion

$$\tau_{loss} = \frac{E}{b(E, B, n_{th})}$$

$$\tau_D = \frac{R_h^2}{D(E)}$$

B increase



n_{th} decrease

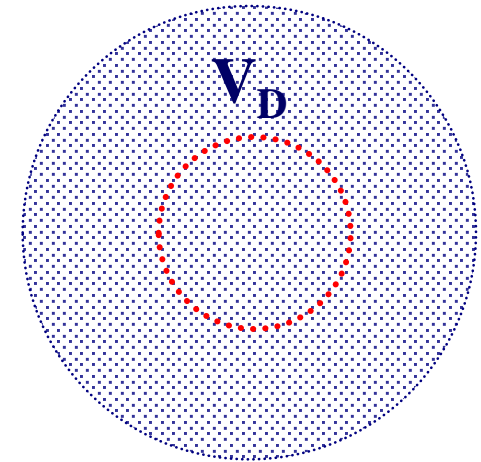
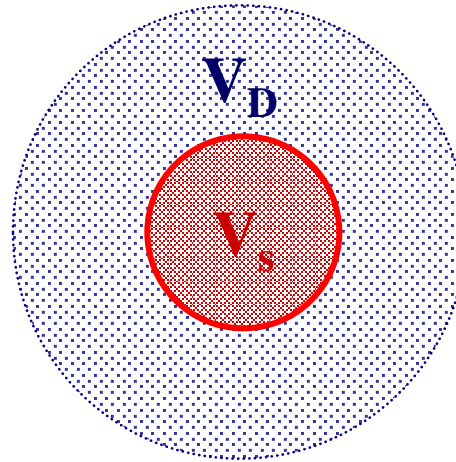
R_h decrease

Solution: qualitative

$$n_e(E, r) = [Q_e(E, r)\tau_{loss}] \cdot \frac{V_{source}}{V_{source} + V_{diffusion}} \cdot \frac{\tau_D}{\tau_D + \tau_{loss}}$$



$\tau_{loss} \ll \tau_D$



$\tau_{loss} \gg \tau_D$

$$n_e(E, r) = [Q_e(E, r)\tau_{loss}]$$

Galaxy clusters

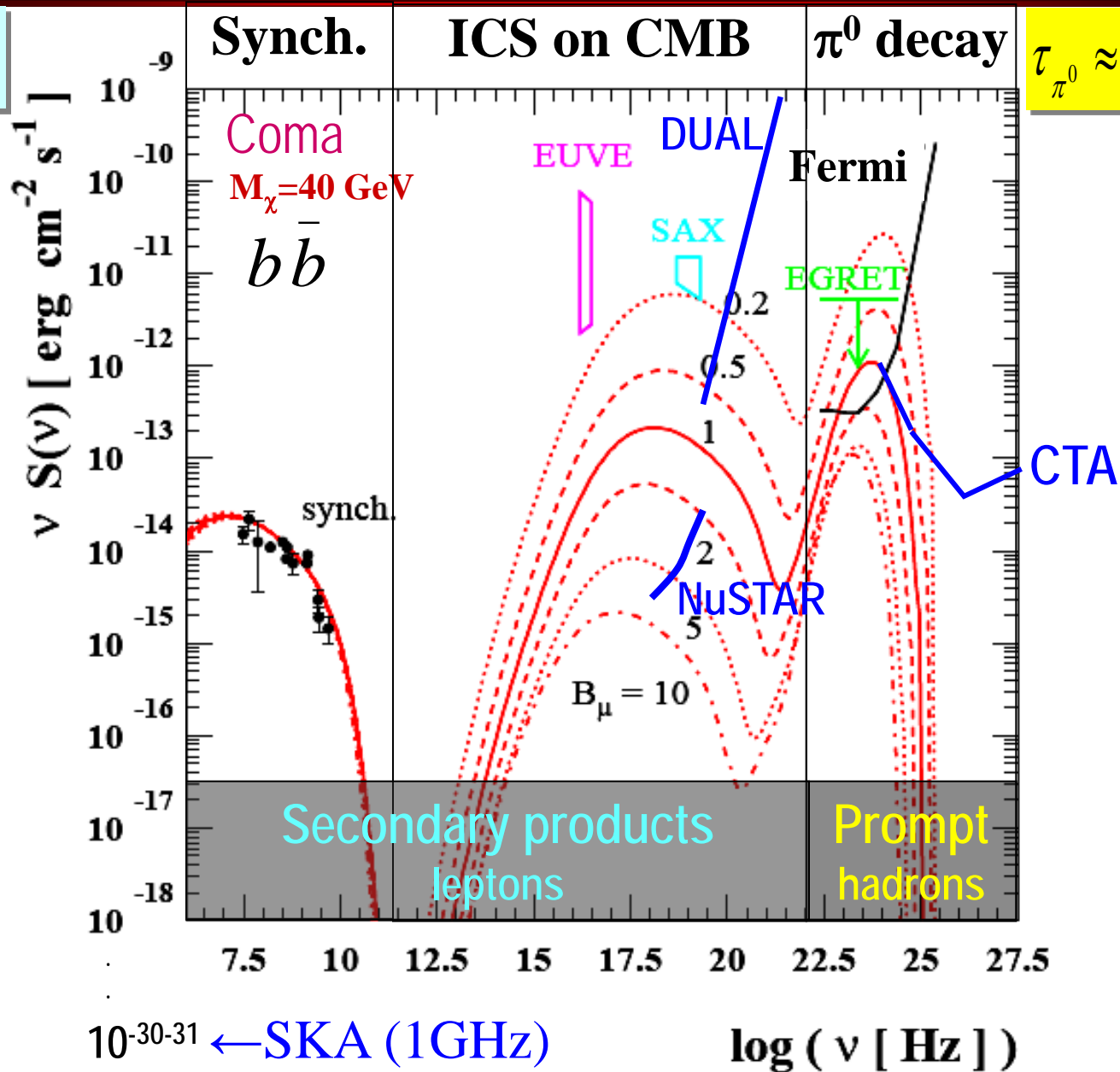
$$n_e(E, r) = [Q_e(E, r)\tau_{loss}] \cdot \frac{V_{source}}{V_{diffusion}} \cdot \frac{\tau_D}{\tau_{loss}}$$

Galaxies

Neutralino DM: SED

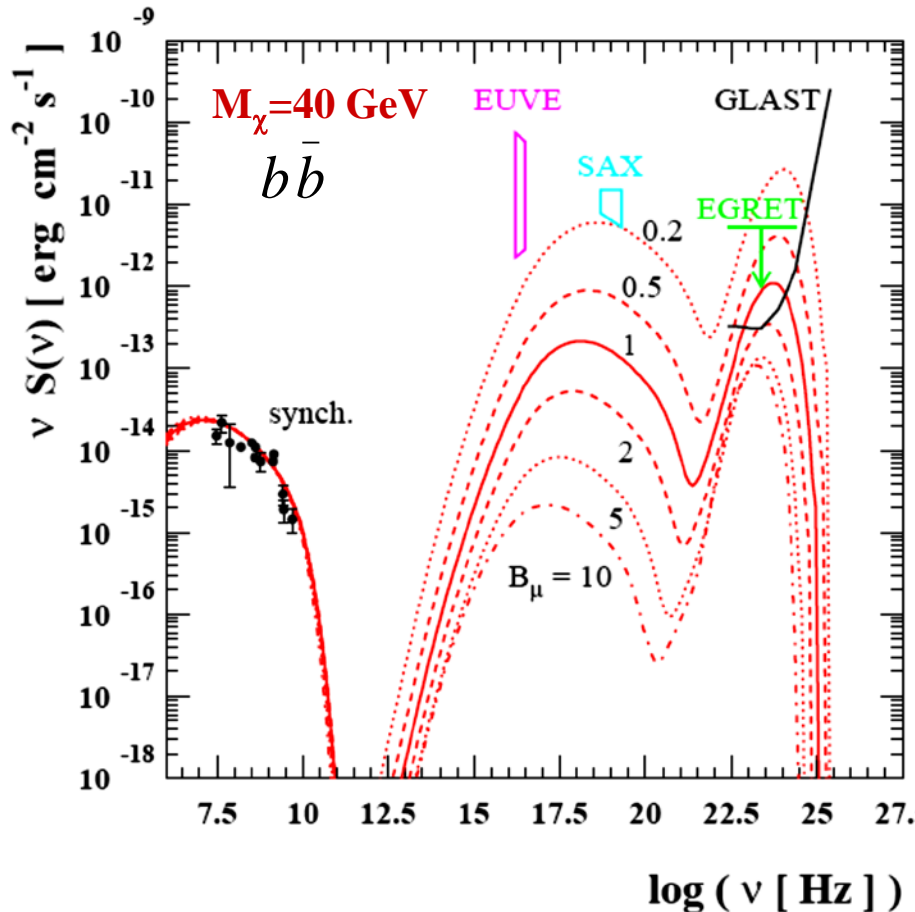
$$\tau_{\pi^\pm} \approx 2.6 \cdot 10^{-8} \text{ s}$$

$$\tau_{\pi^0} \approx 8.4 \cdot 10^{-17} \text{ s}$$

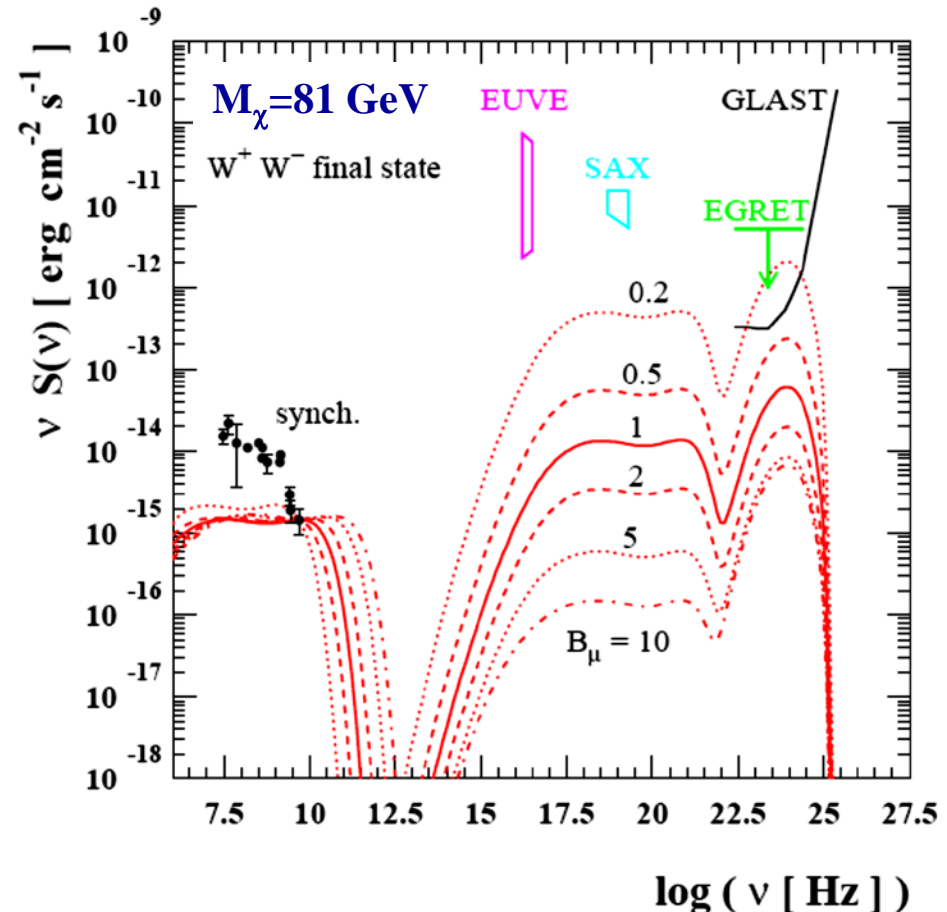


WIMP (neutralino) composition

Soft WIMP model

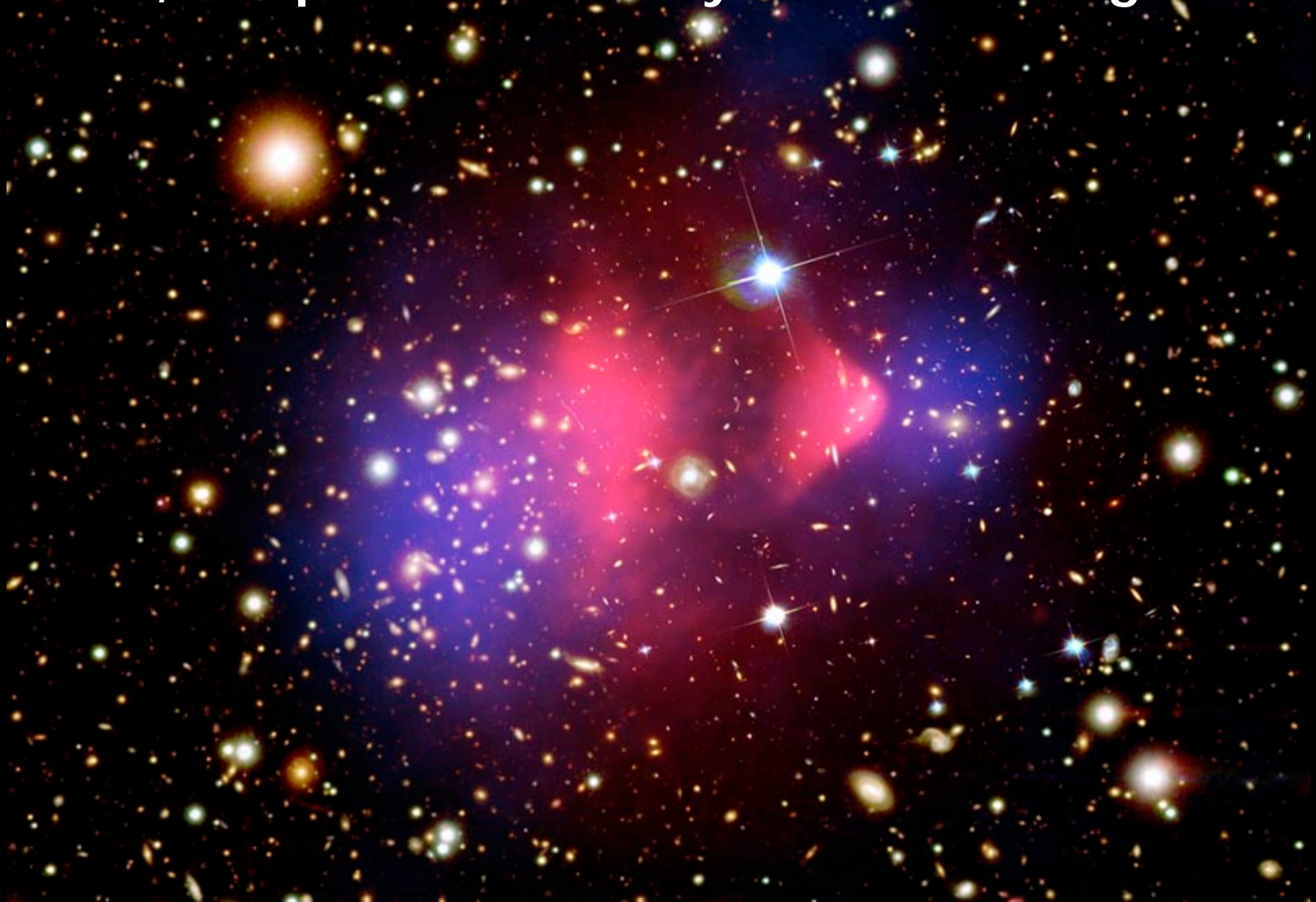


Hard WIMP model



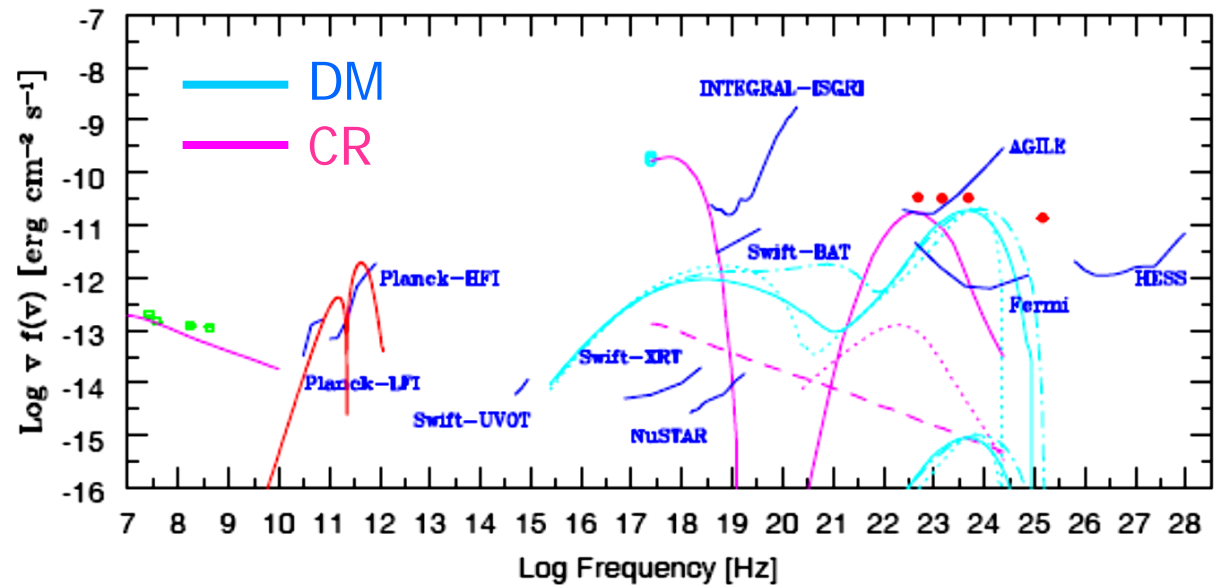
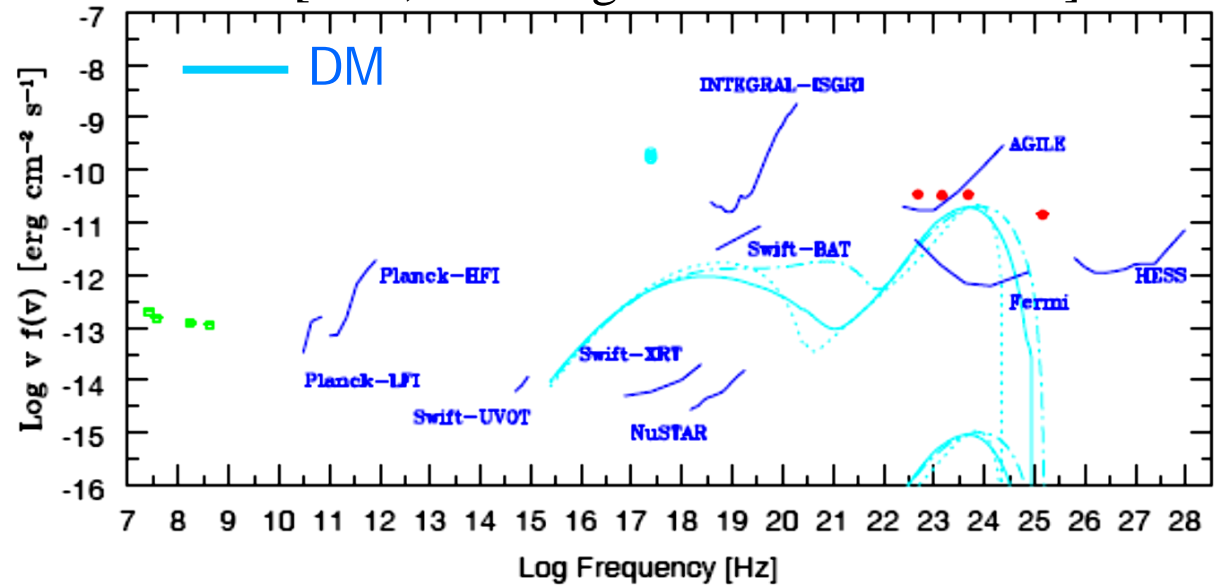
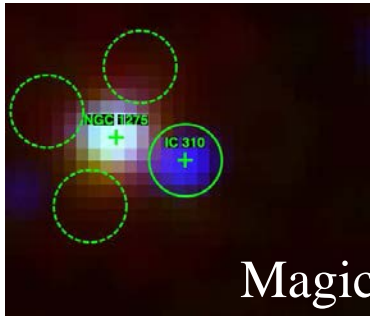
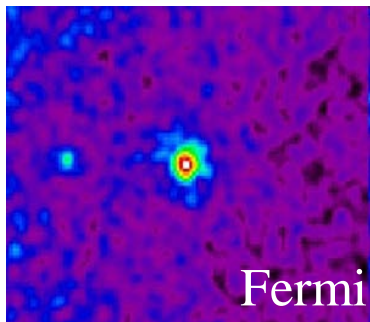
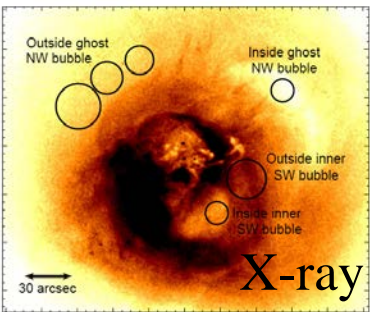
Galaxy Clusters DM Challenge

Large-size, co-spatial DM & baryons... but few good cases !

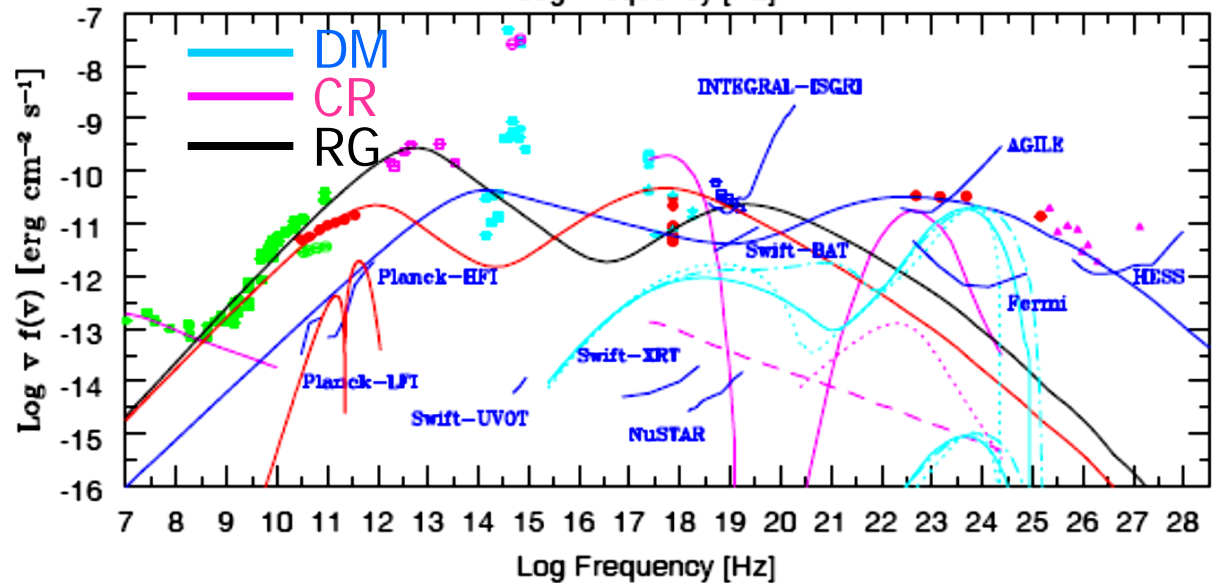
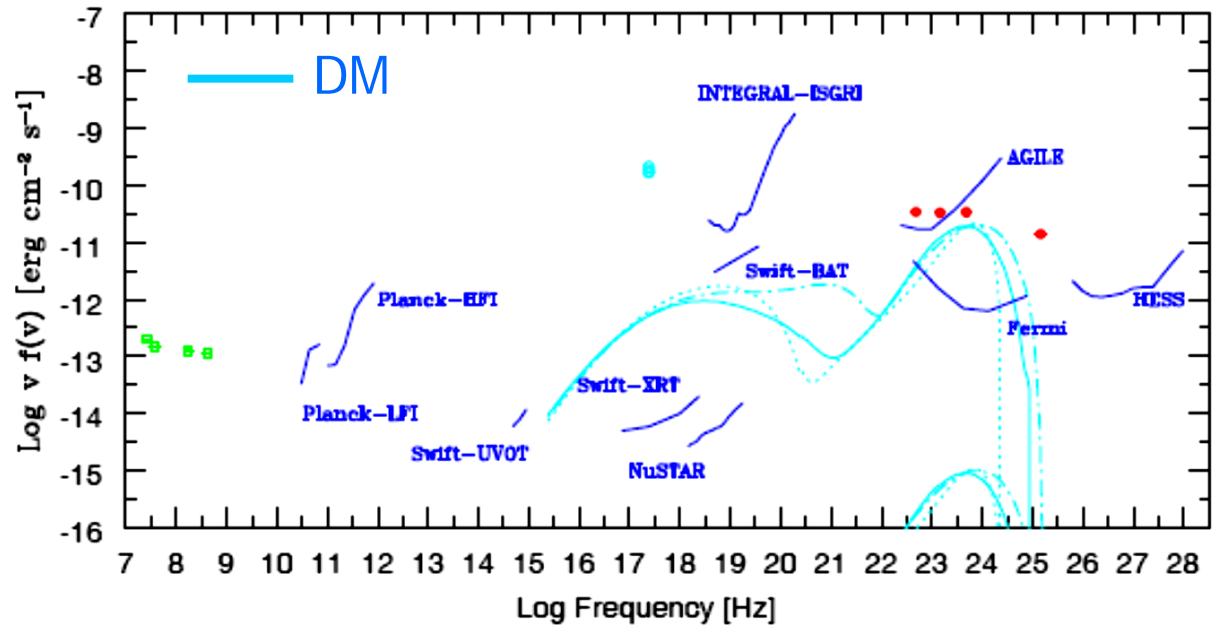
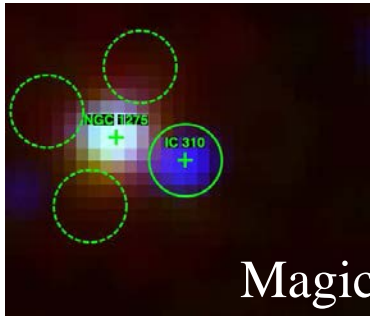
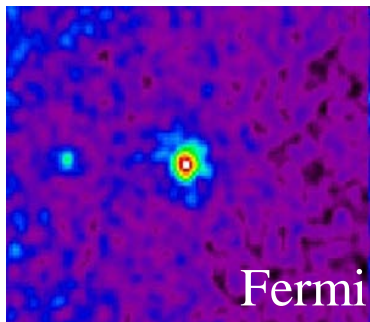
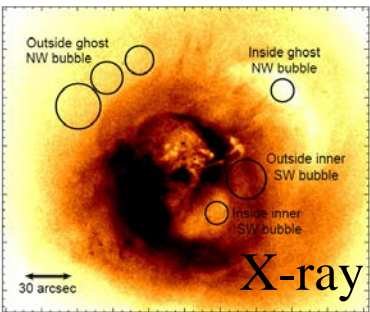


DM signals: the case of Perseus

[S.C., Marchegiani & Giommi 2010]

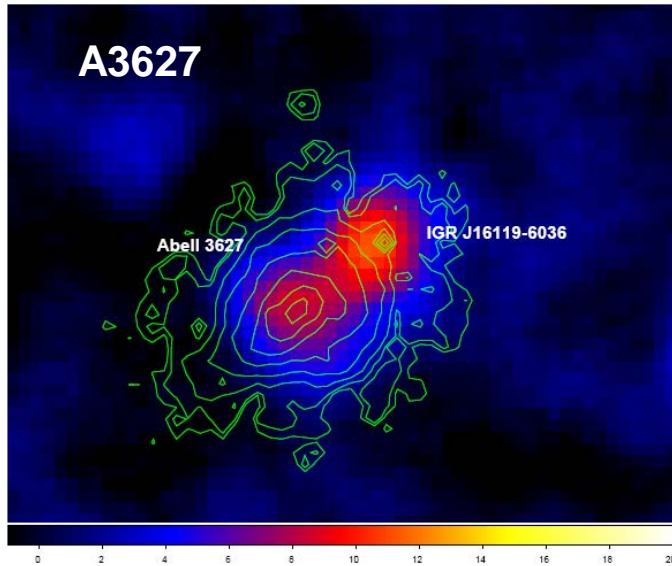


DM signals: the case of Perseus



A Dark Temptation

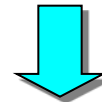
Explain HXR in cluster as DM annihilation signals



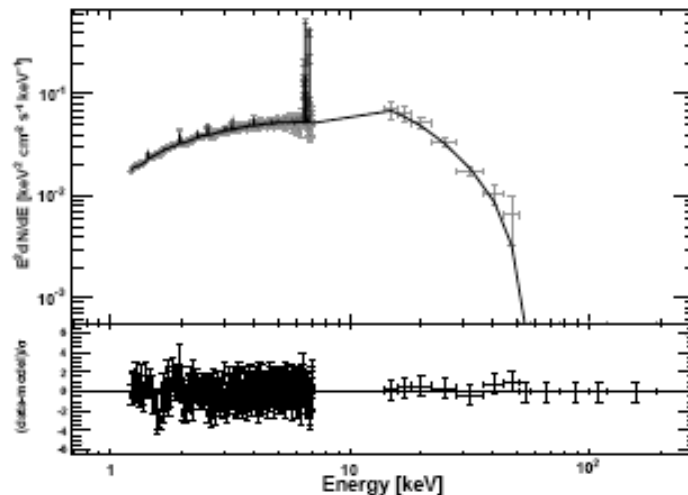
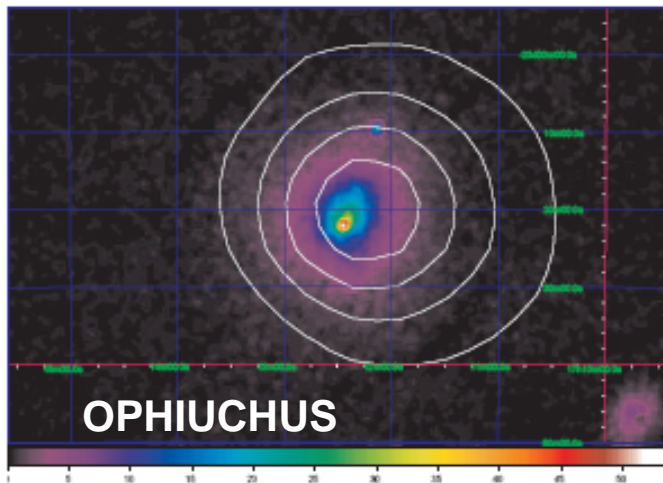
More than 20 clusters with Hard X-ray excess at $E > 20$ keV (Swift-BAT data, BeppoSAX data)

Equally fit with:

- Two temperature (thermal) plasma
- Thermal plasma + non-thermal power-law

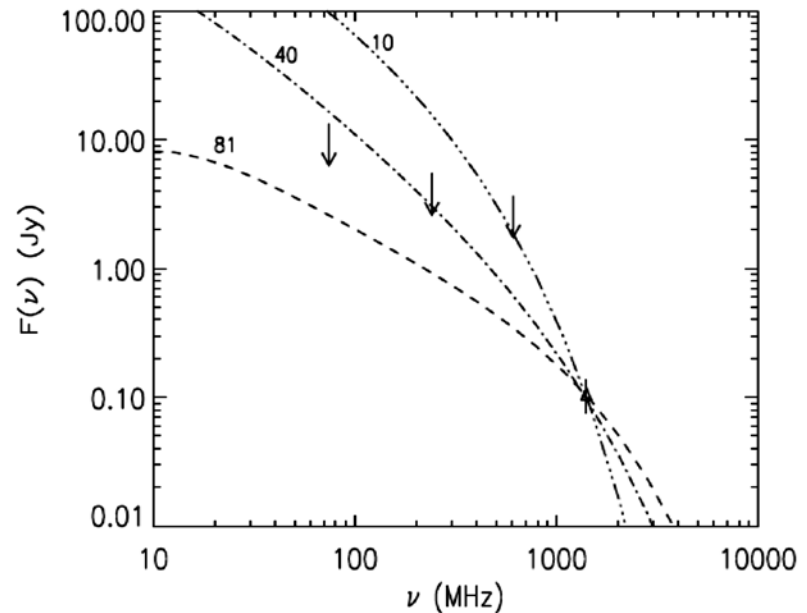
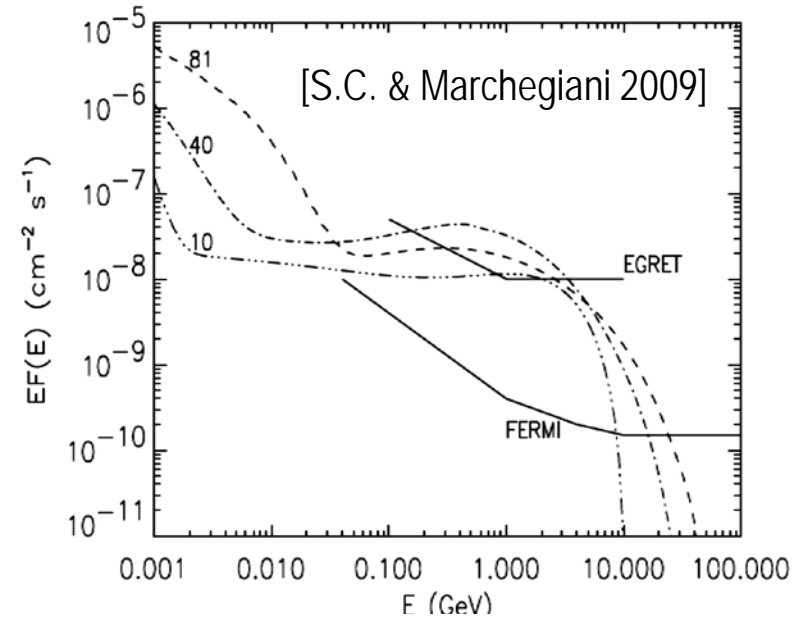
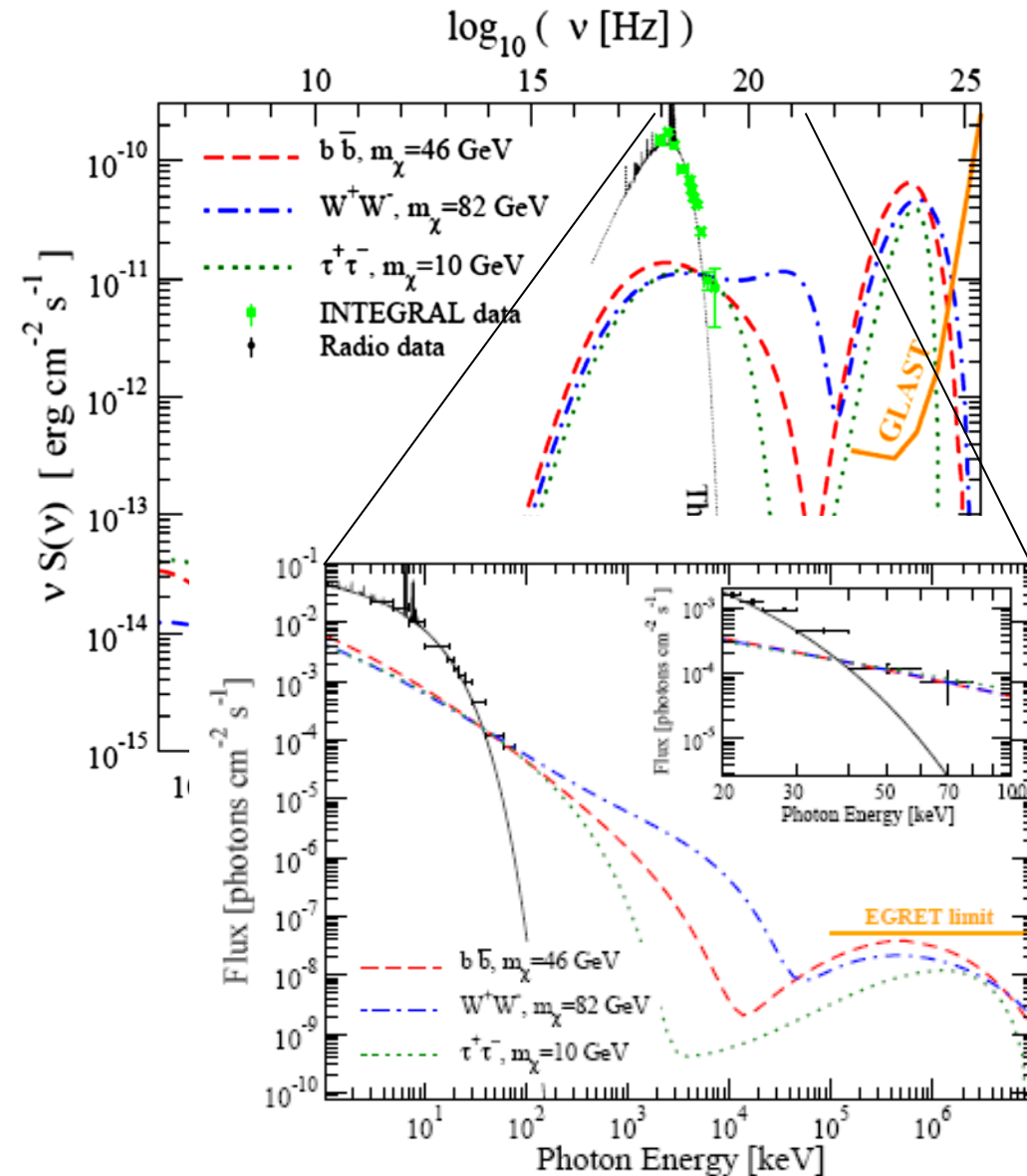


AGN emission or ICS from DM / CR interaction

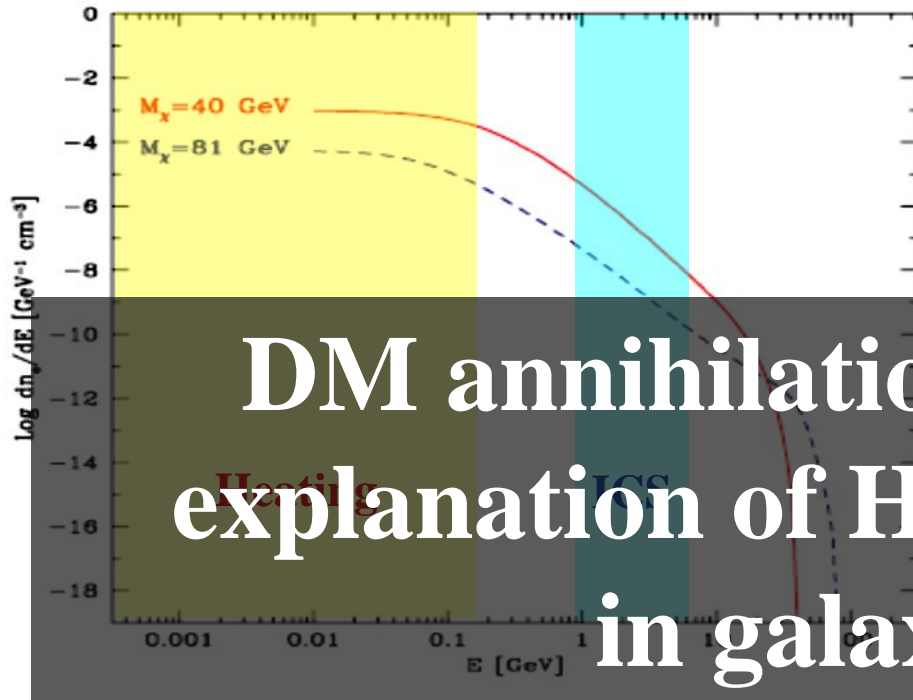


Hard X-ray excess

Consequences

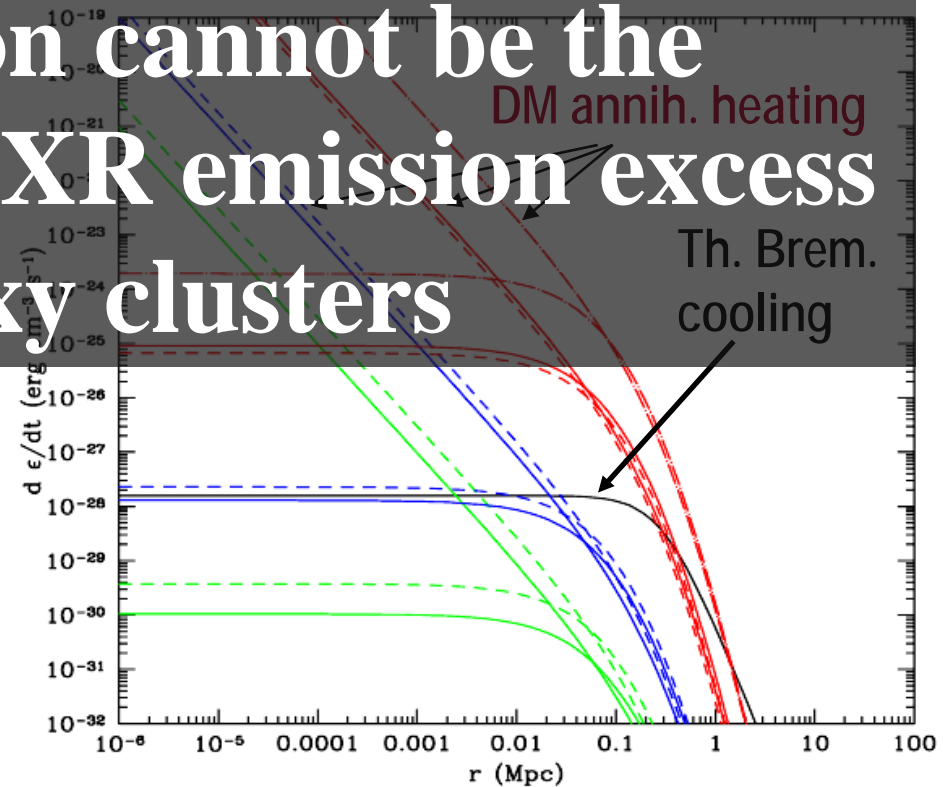


Consequences: DM & gas heating



DM models that fit the HXR flux of galaxy clusters produce also an excess heating of the gas.

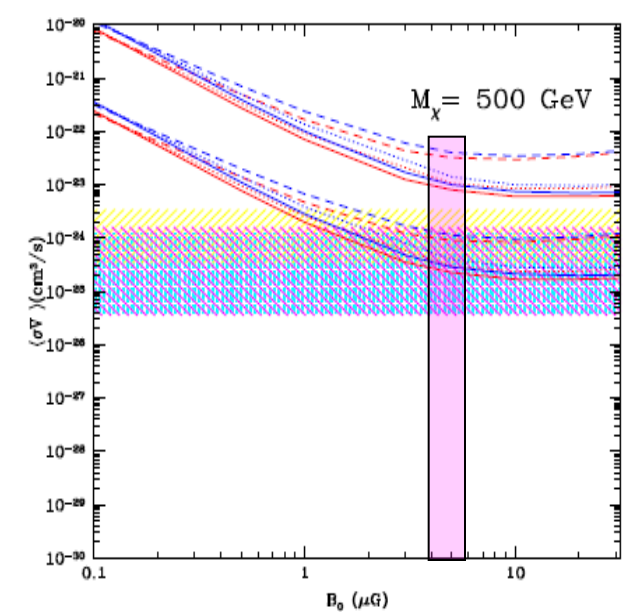
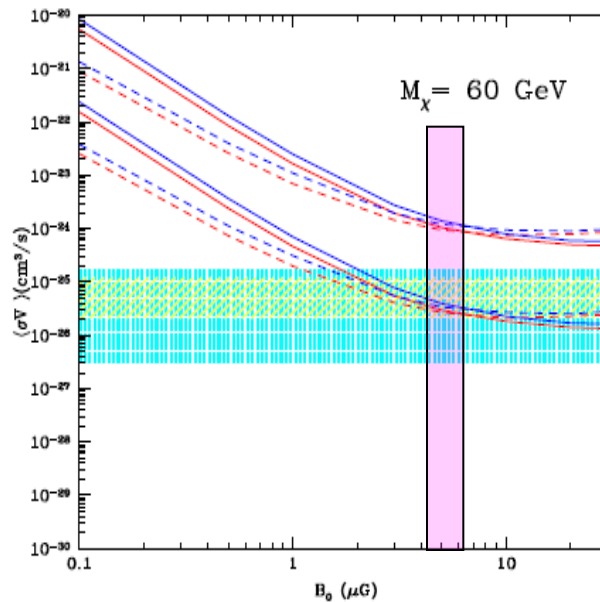
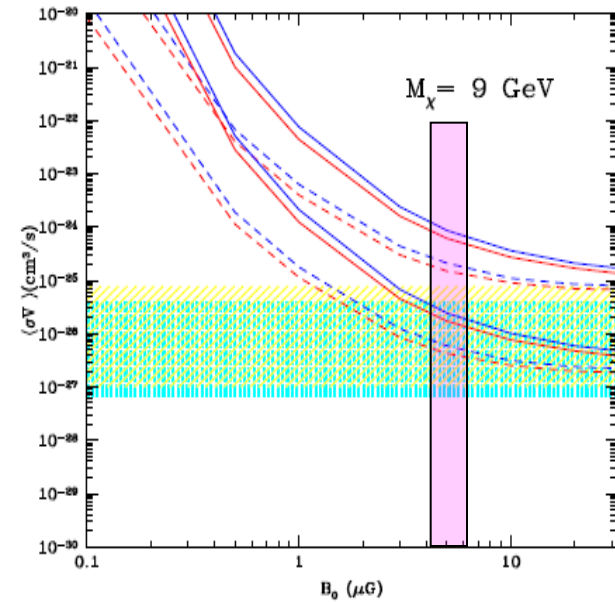
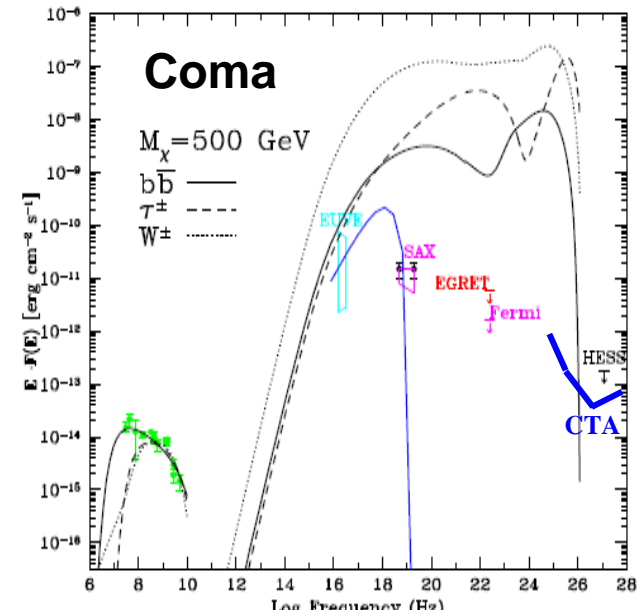
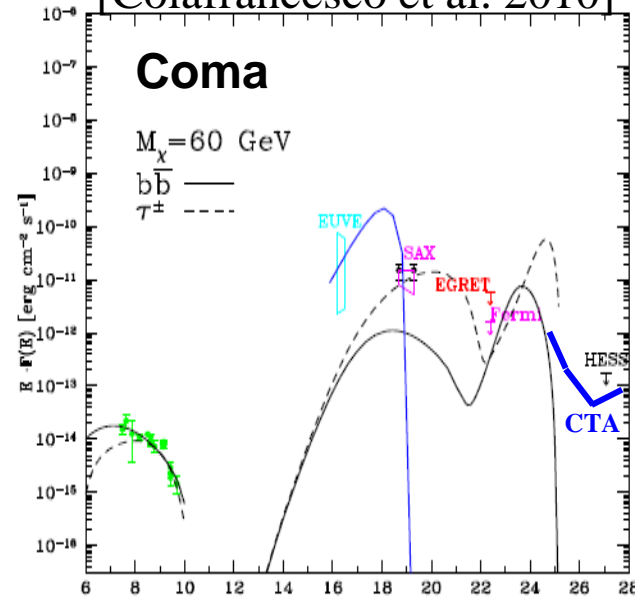
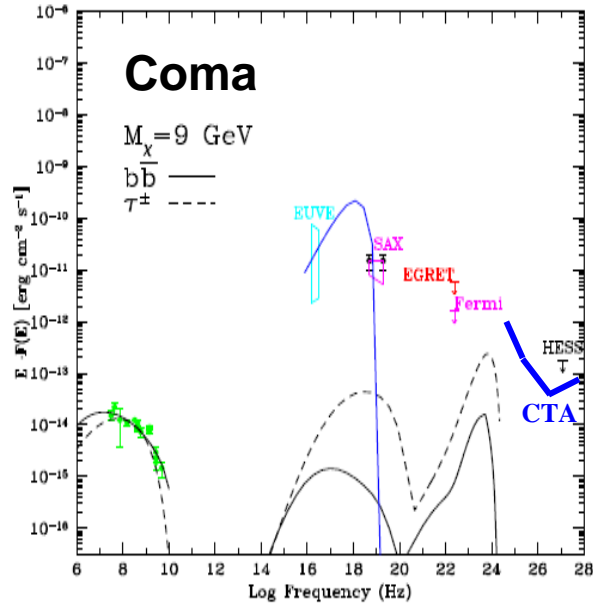
DM annihilation cannot be the explanation of HXR emission excess in galaxy clusters



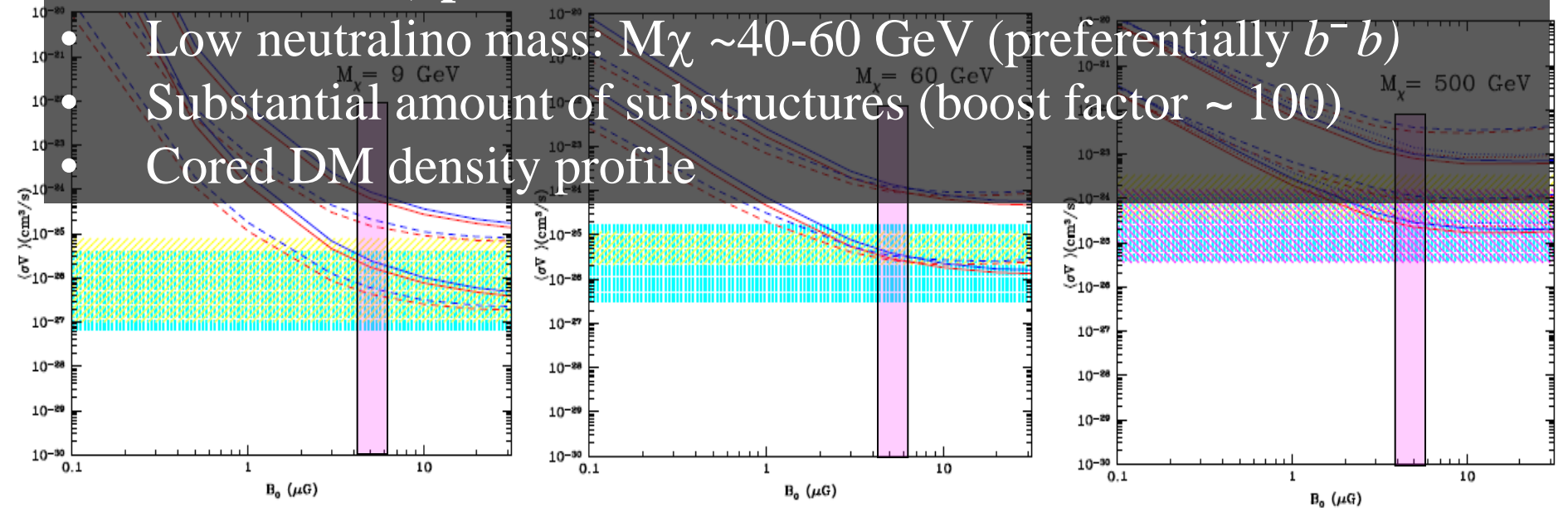
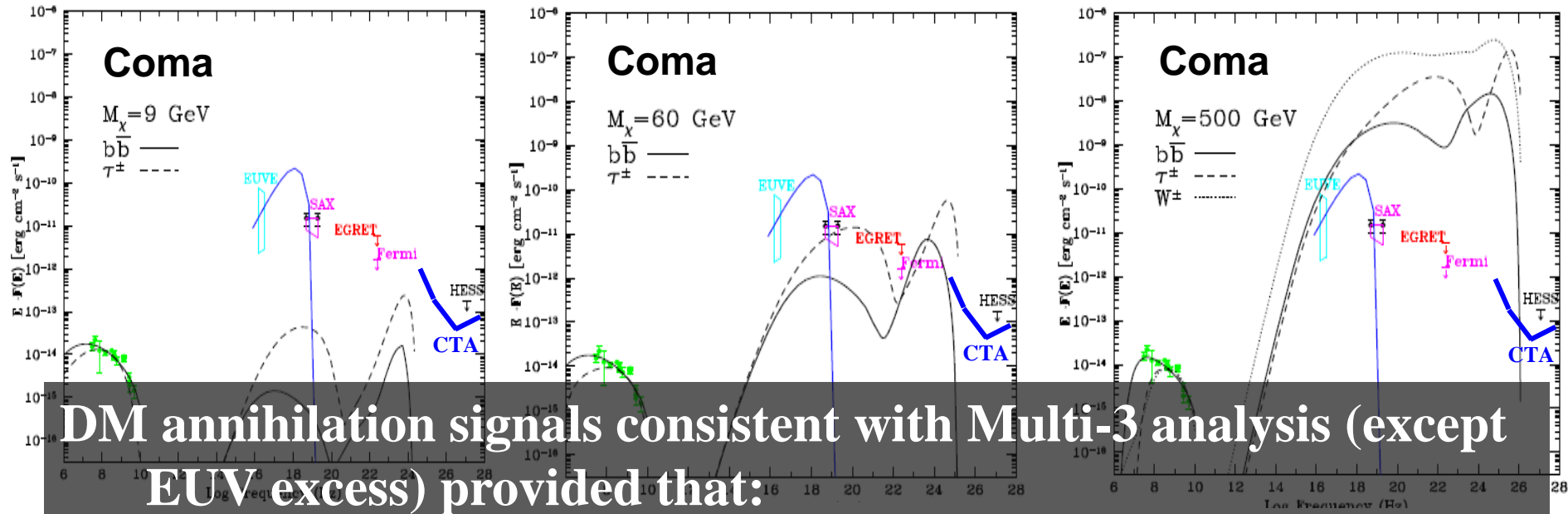
[Colafrancesco & Marchegiani 2009]

DM models & non-thermal phenomena


[Colafrancesco et al. 2010]

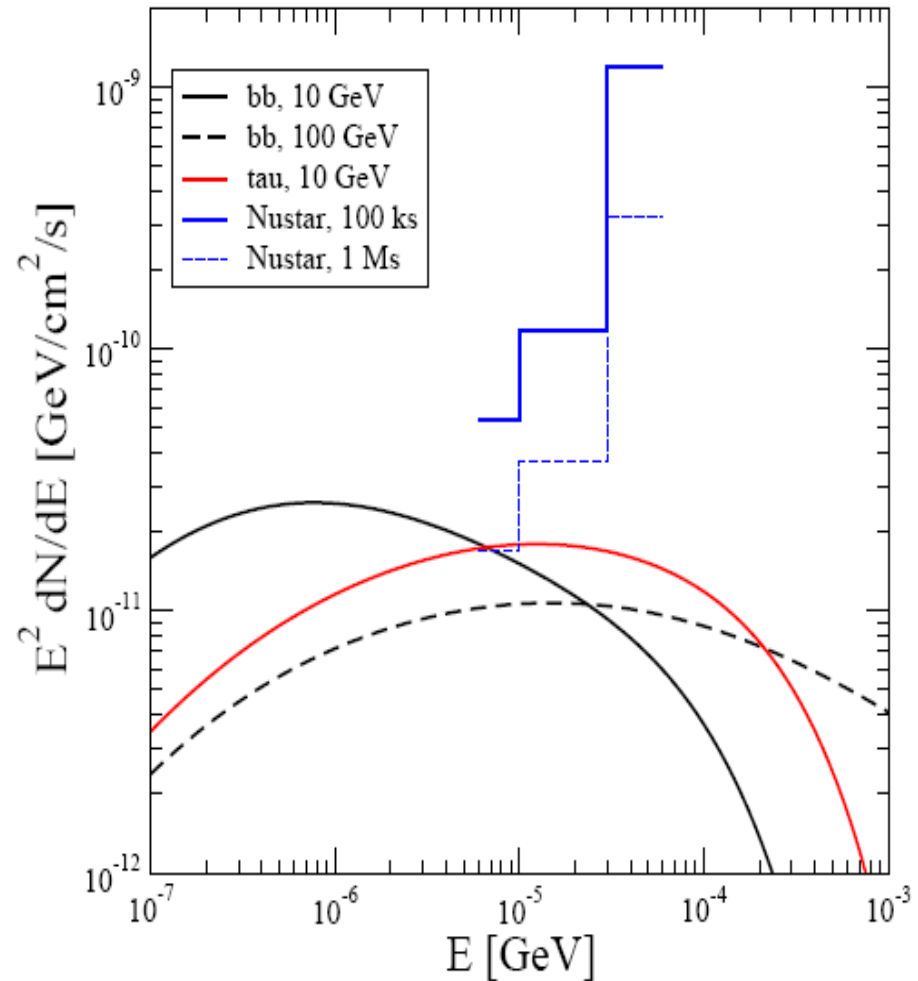
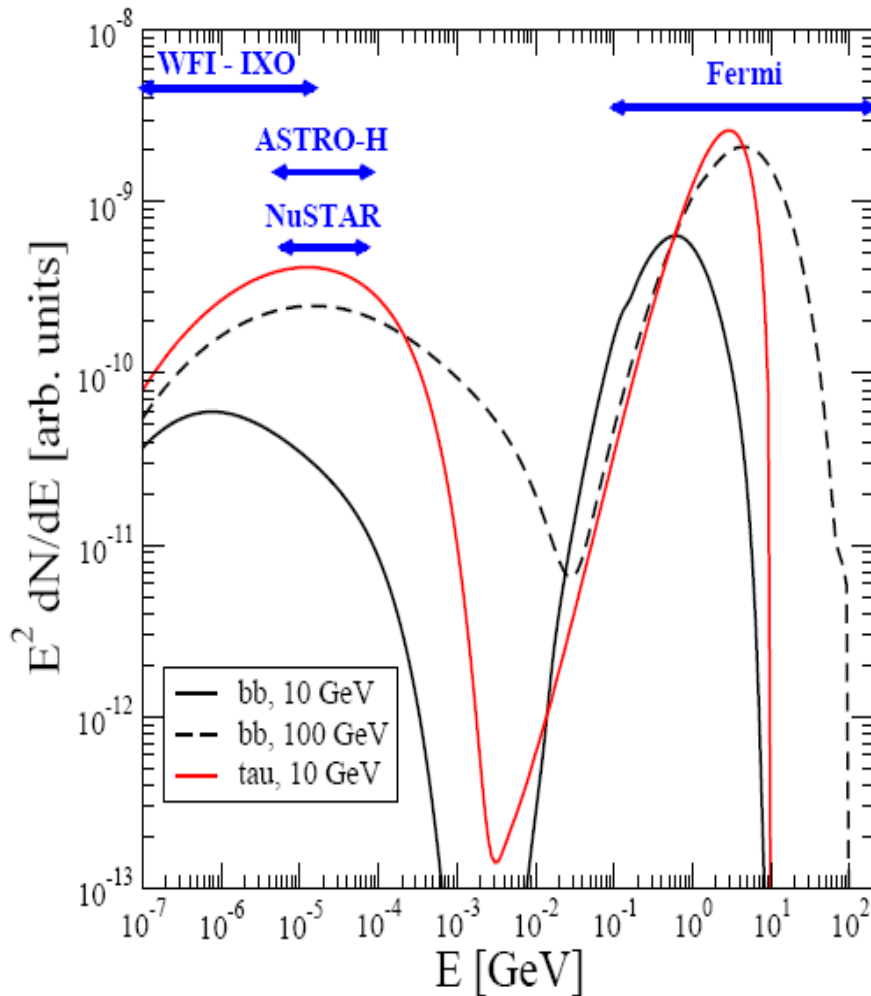


DM models & non-thermal phenomena



Dark Temptations never go away ...

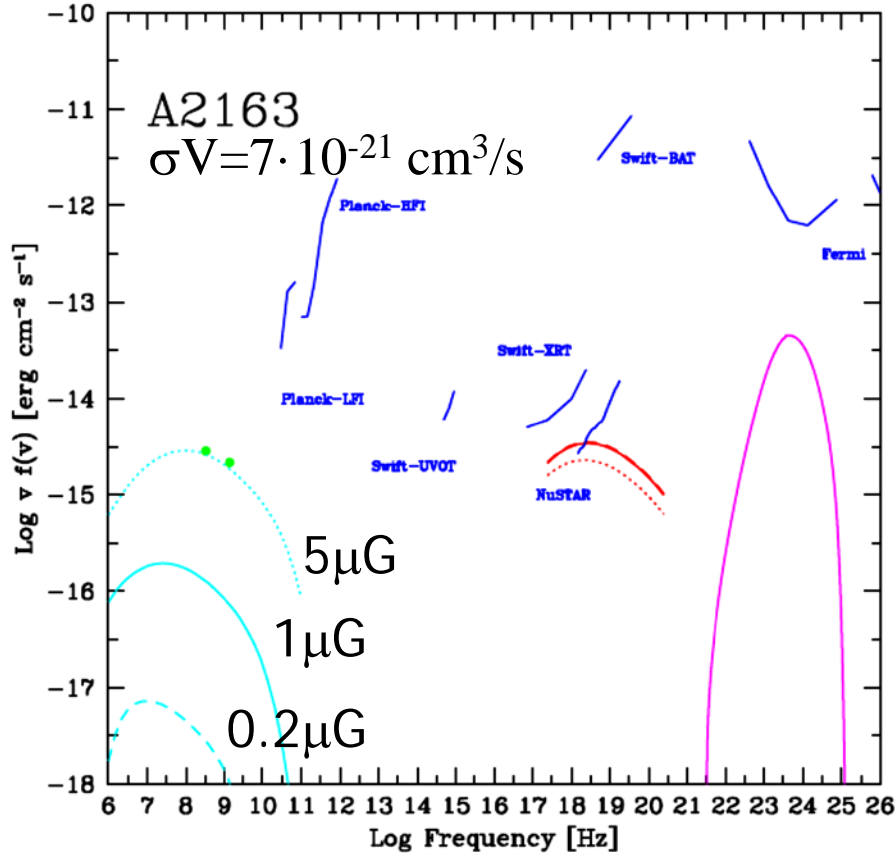
Normalized to $F(E > 0.1 \text{ GeV})$  Possible detection for $t_{\text{exp}} > 4\text{Msec}$



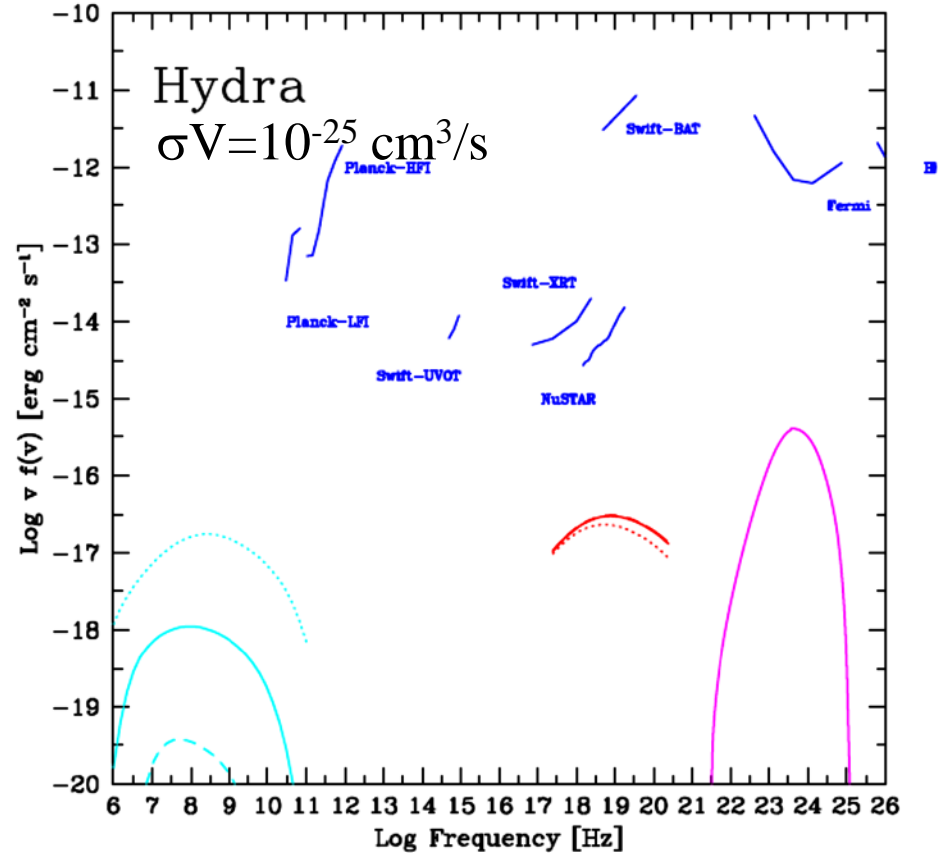
[Jeltema & Profumo arXiv:1108.1407]

HXR – gamma vs. HXR - Radio

Normalized to $F(\nu=1.4\text{GHz})$
With known $B=5\mu\text{G}$

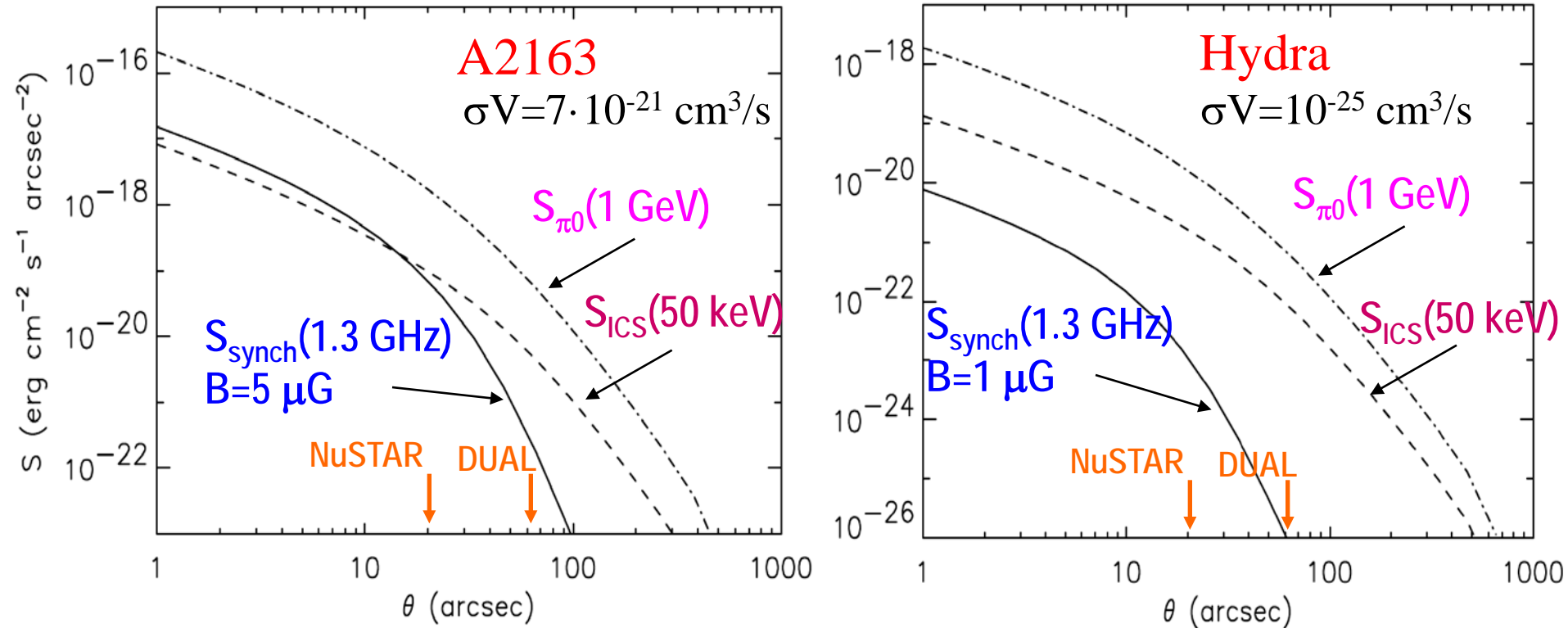


GeV experiments are far from
DM signal detections



HXR – Radio correlation provides stronger constraints on DM
(MeerKAT/SKA vs. NuSTAR/DUAL combined observations)

DM signal profiles HXR-Radio-gamma

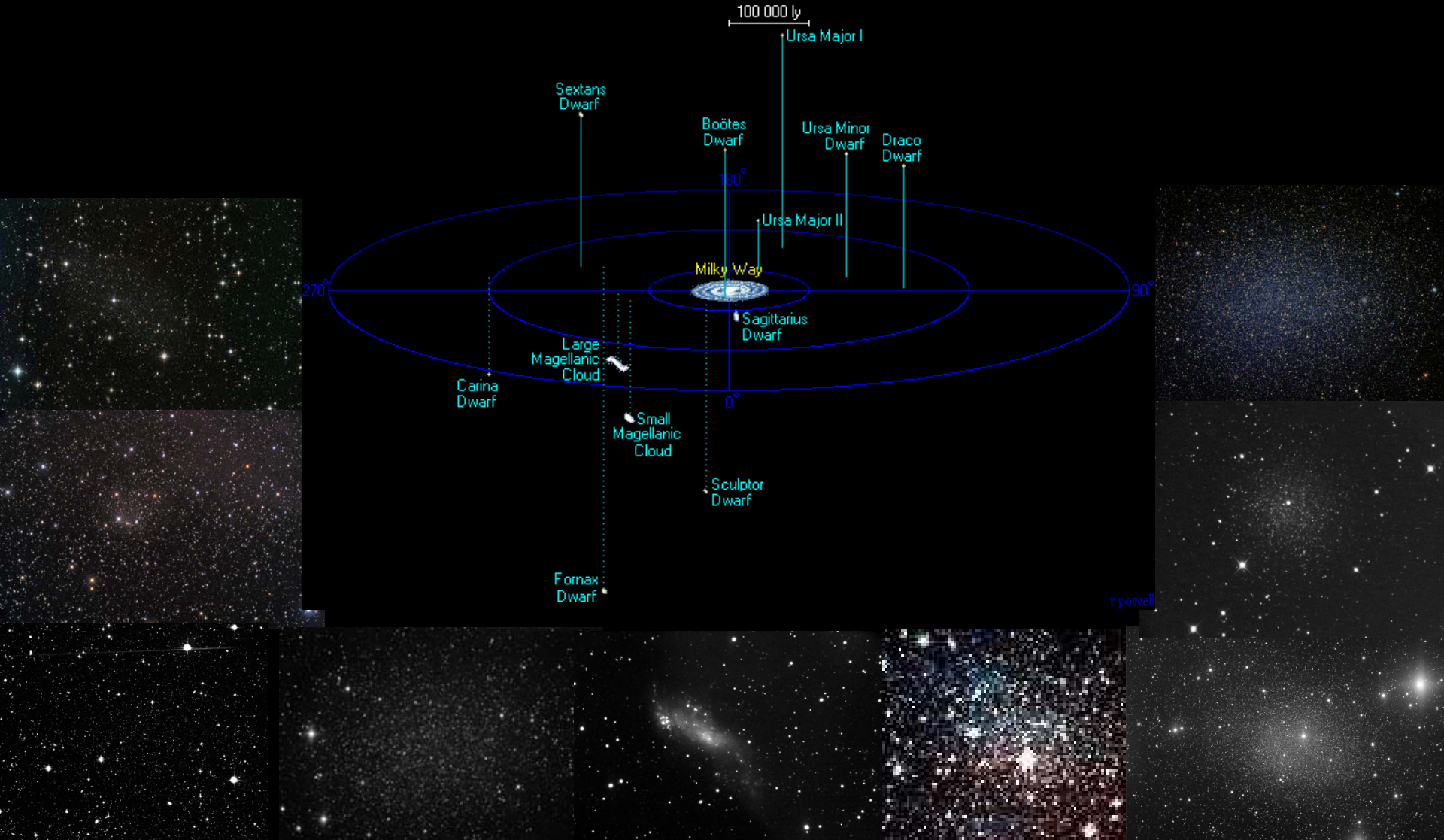


There is a clear spatial signature of DM signals visible in the HXR

-  Clear HXR-radio correlations at large angular scales (> 1 arcmin)
-  No clear HXR-gamma correlation at all angular scales

Dwarf Spheroidals DM challenge

Small-size, dynamically un-relaxed... but few good cases !



The darkest galaxies in the universe

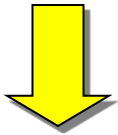


Segue 1 dwarf galaxy $\rightarrow M/L_V \sim 3400 M_\odot/L_\odot$

The Dwarf Galaxies DM challenge

Sub-galactic size systems

- R ~ kpc
- No gas
- Little dust
- No Crs
- 1 (or 2) stellar populations
- M/L ~ 500 - 3500

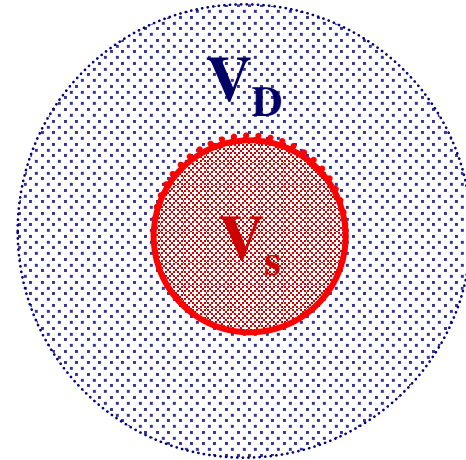


- + Ideal systems to probe DM
- + Clean multi- ν features

but...

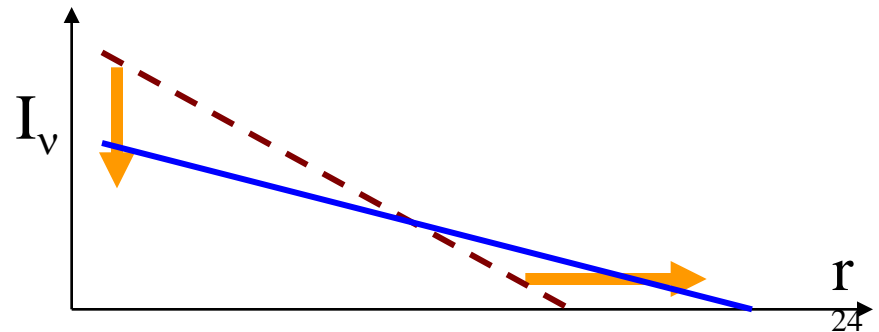
- Strong diffusion effects
- Low signals

$$n_e(E, r) = [Q_e(E, r)\tau_{loss}] \cdot \frac{V_{source}}{V_{source} + V_{diffusion}} \cdot \frac{\tau_D}{\tau_D + \tau_{loss}}$$



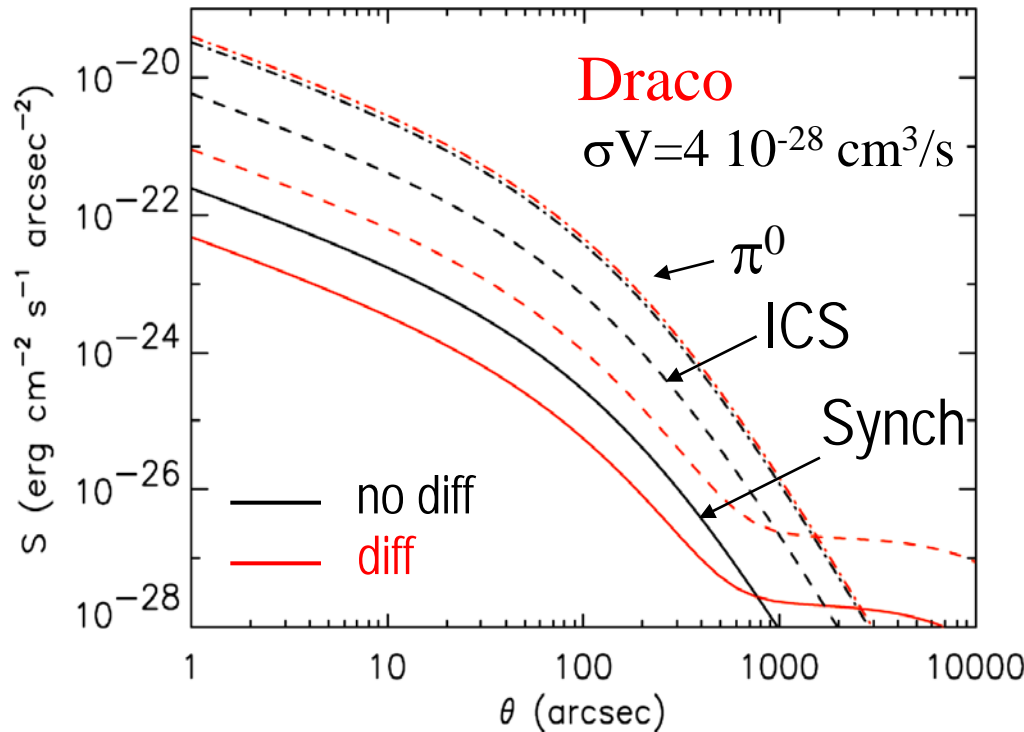
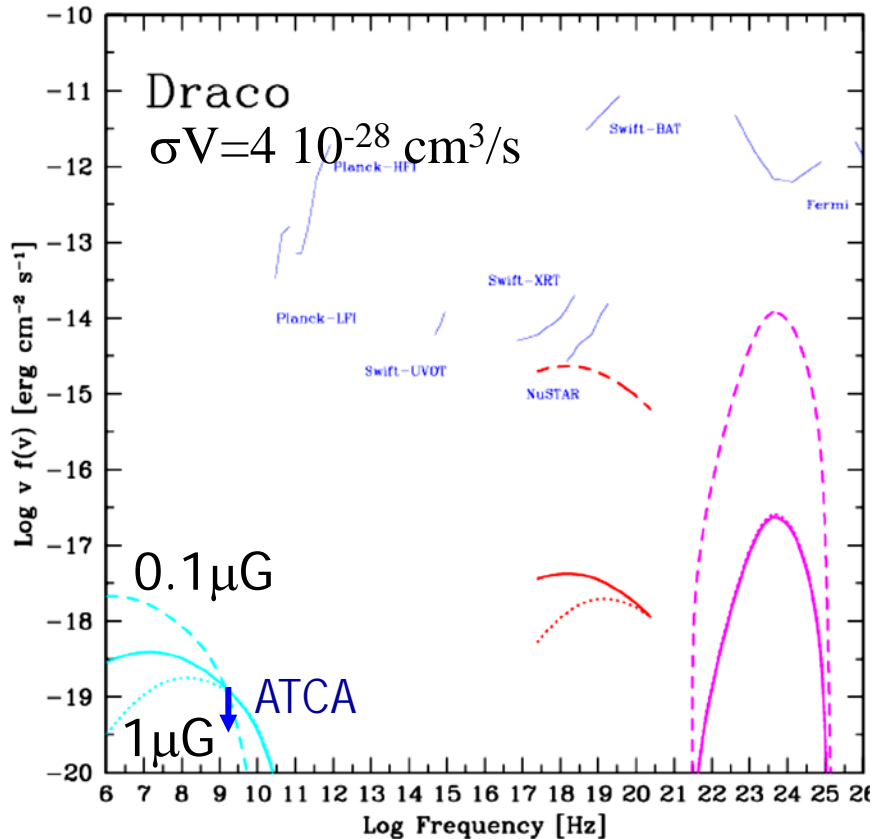
$$\tau_{loss} \gg \tau_D$$

$$n_e(E, r) = [Q_e(E, r)\tau_{loss}] \cdot \frac{V_{source}}{V_{diffusion}} \cdot \frac{\tau_D}{\tau_{loss}}$$

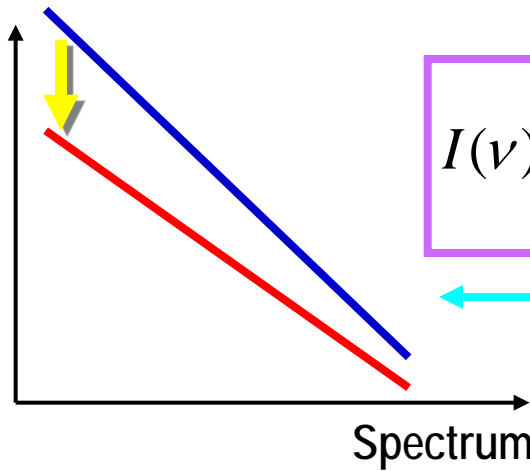


Expectations: the HXR range

Normalization fixed by lack of detection by ATCA ($F_{1.3\text{GHz}} < 10\mu\text{Jy}$)

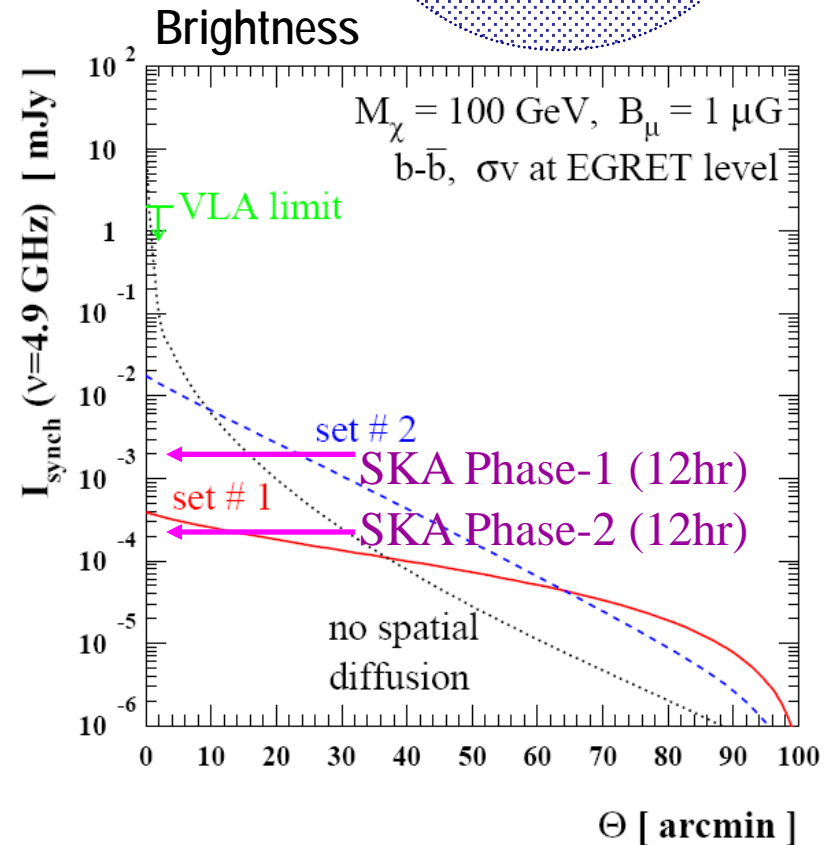
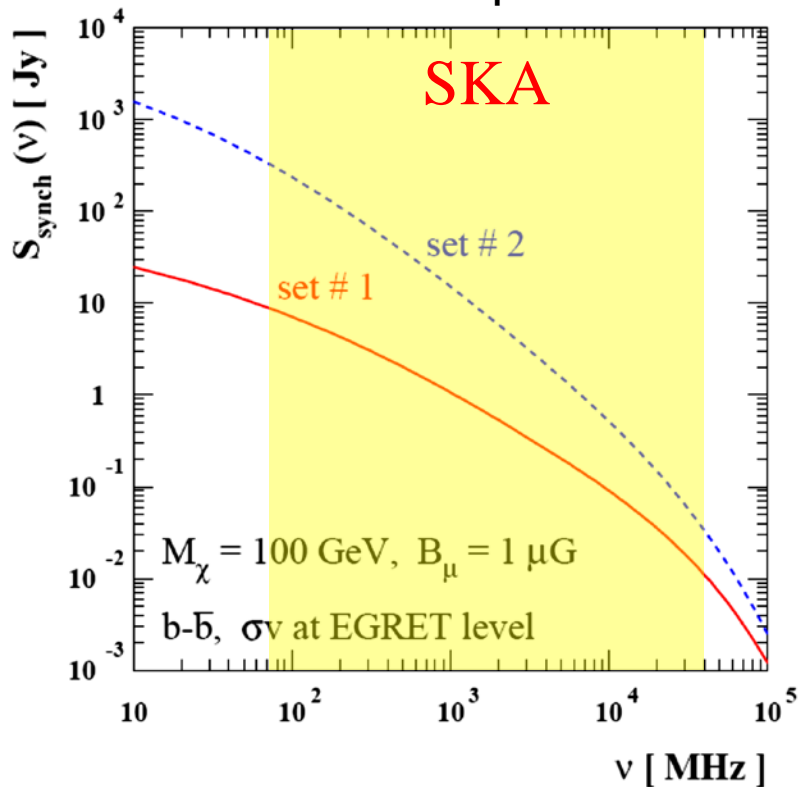
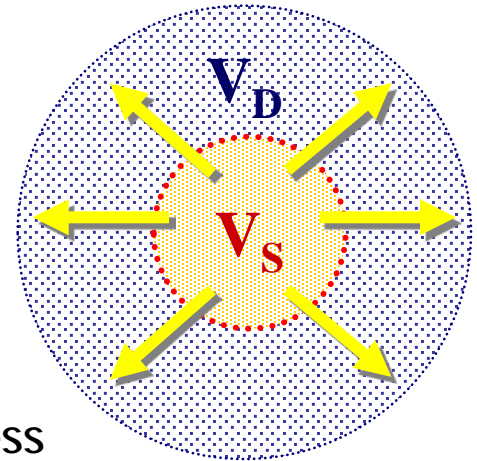


Dwarf Sph. Galaxies & DM

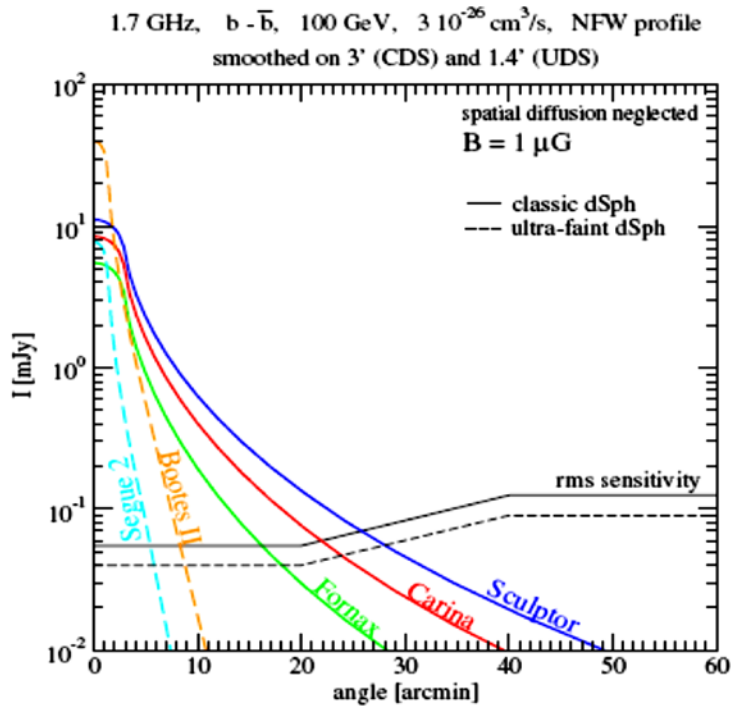


$$I(\nu) \propto B \otimes D_e \otimes n_e(E_e, \nu, r) \frac{\langle \sigma \nu \rangle}{M_\chi^2}$$

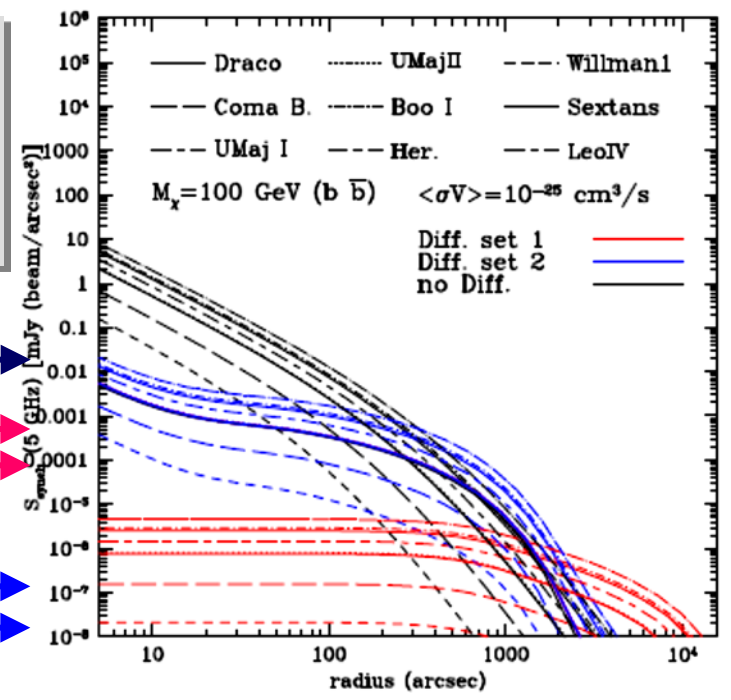
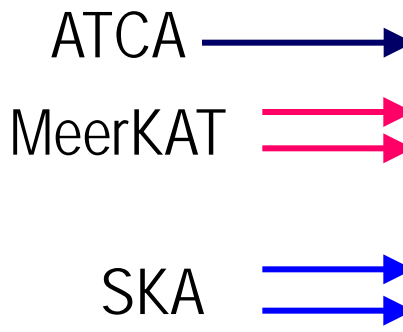
$$D_e = D_0 (E_e / B)^\gamma$$



ATCA → MeerKAT → SKA

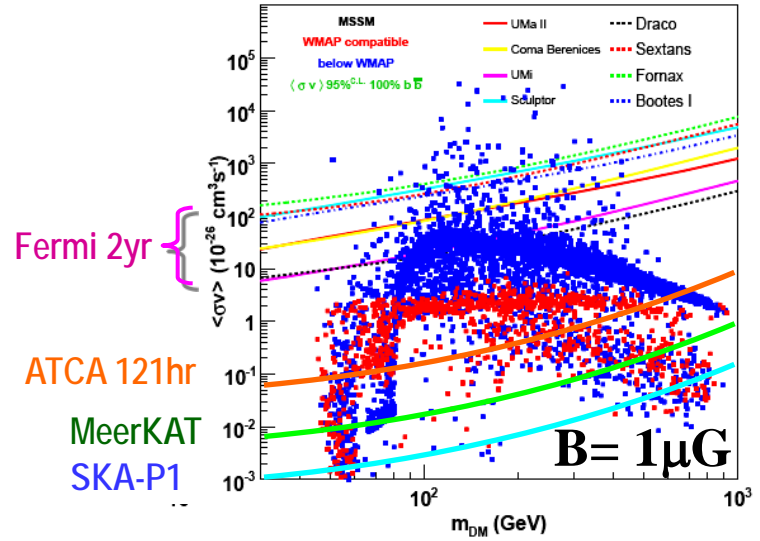
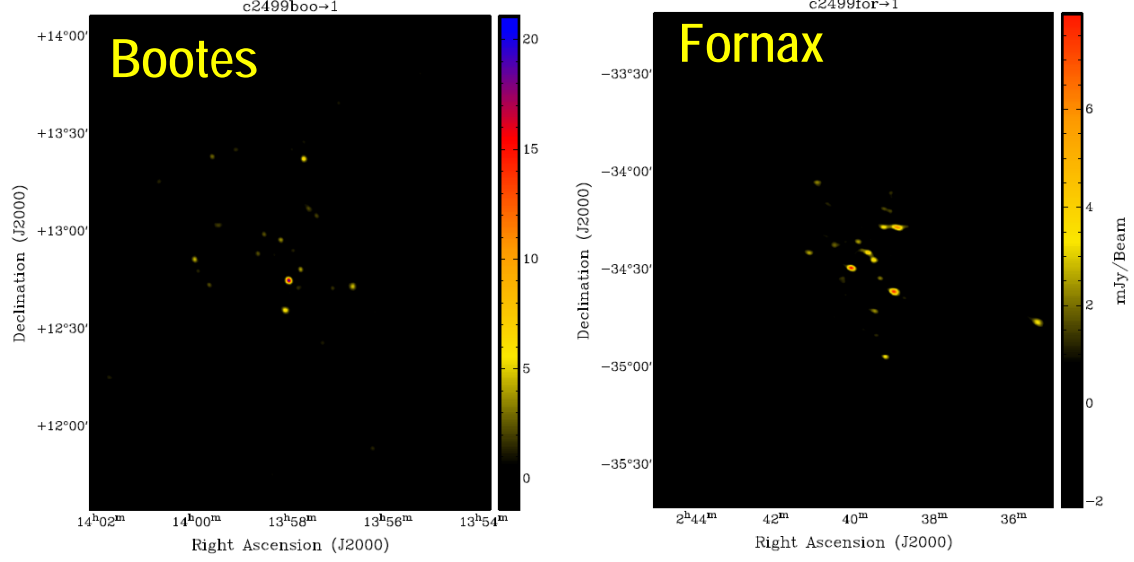
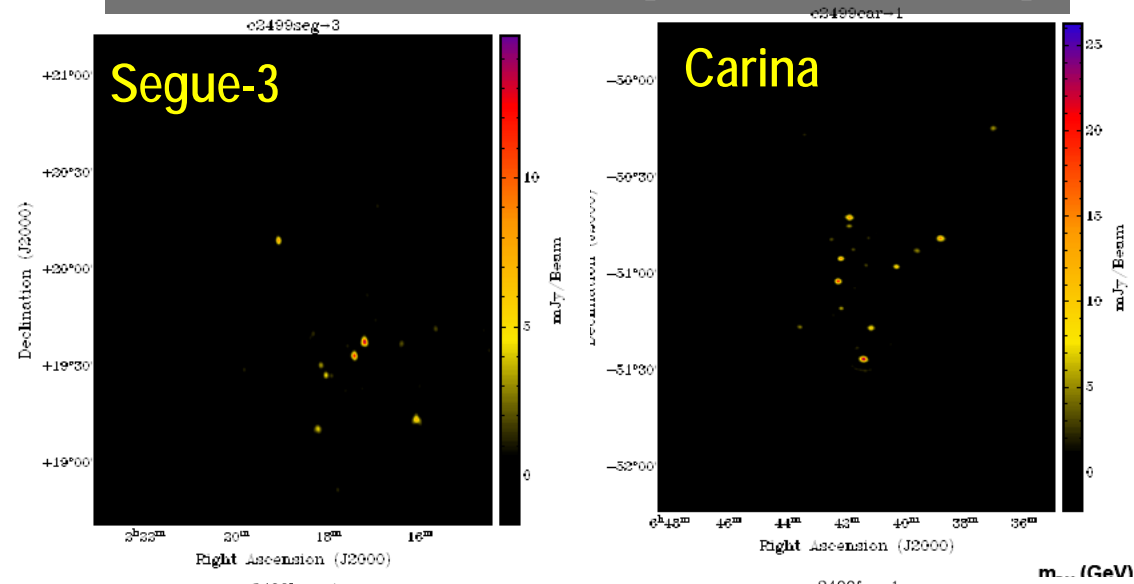


121.5 hours
ATCA
observations



Dark Matter search @ radio

121.5 hr ATCA [S.C. et al. 2011]

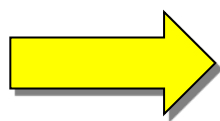


Improve current DM limits by a factor:

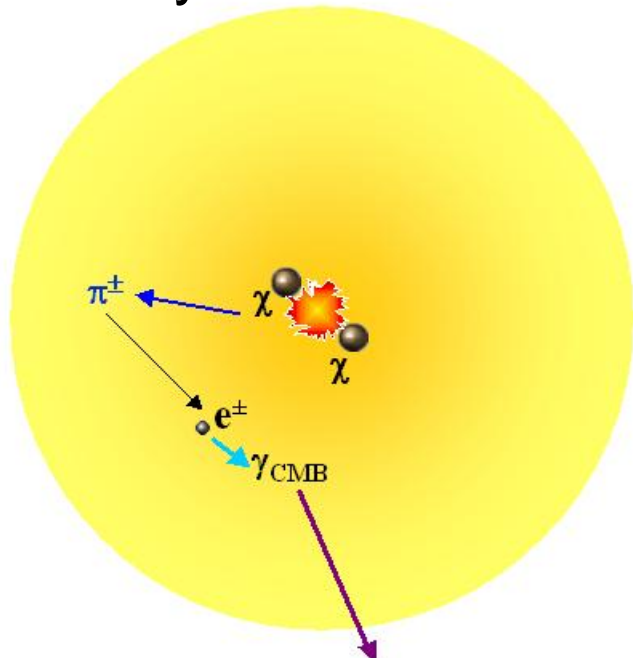
- ~100 ATCA (20 μJy)
- ~1000 MeerKAT (1 μJy)
- ~10000 SKA-P1 (0.1 μJy)

DM search @ radio: Synch+SZE_{DM}

Inverse Compton Scattering
of **CMB photons**
by secondary **DM electrons**

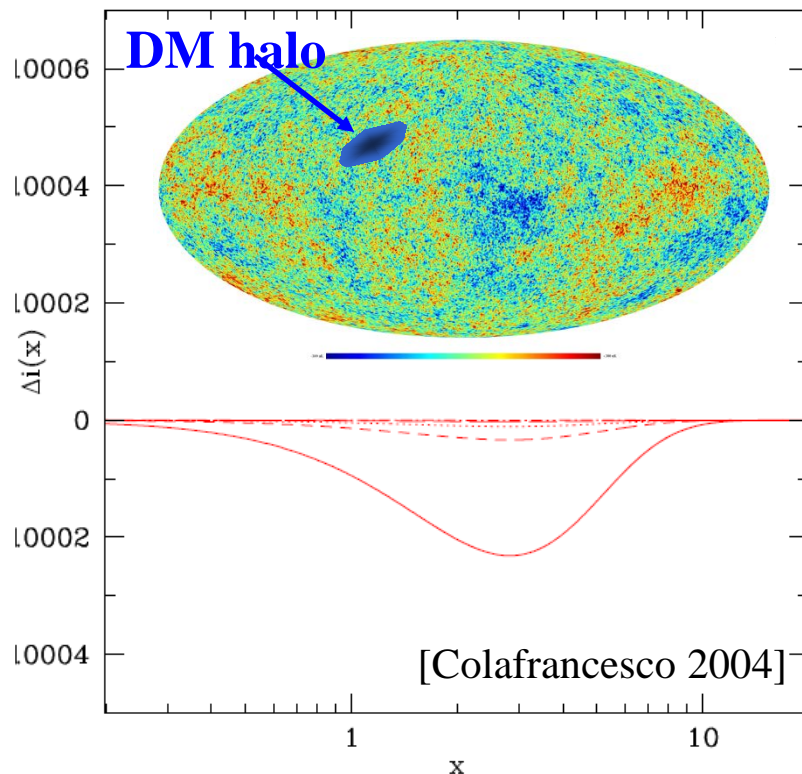


$$\frac{\Delta T_{CMB}}{T_{CMB}} \approx g(x; M_\chi) \cdot \int dl \cdot P_e$$



SKA-P2 (0.1-45 GHz)

MeerKAT (0.7-20 GHz)



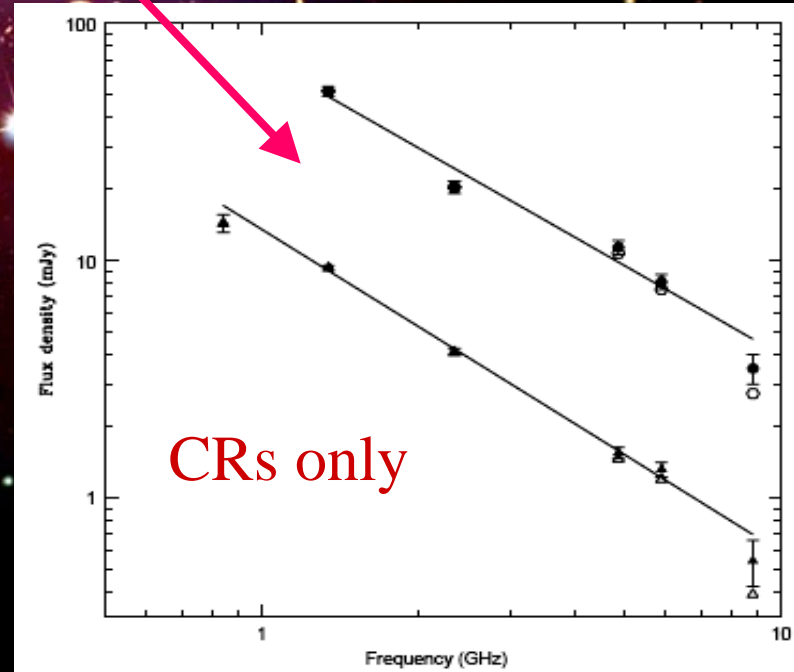
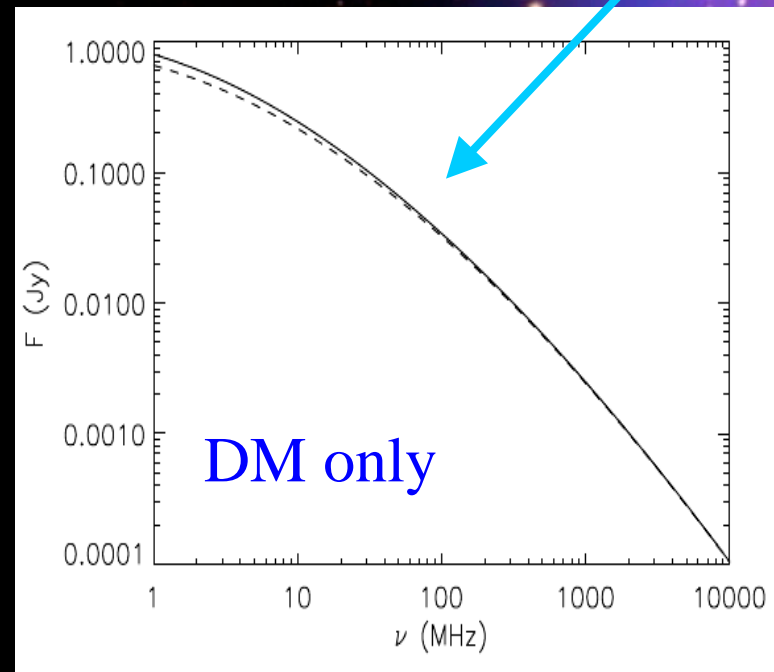
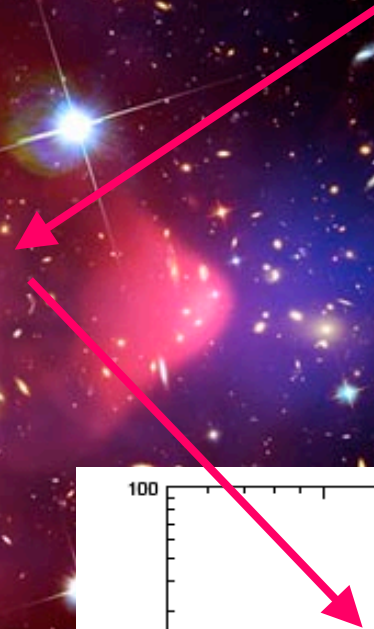
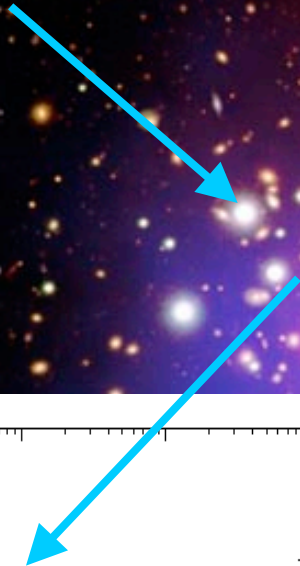
- Measure radio (low ν) & ICS emission (high ν)
- Disentangle electron population and B-field \rightarrow
- DM halo Cosmology: “purified” DM halo

$$\mathbf{F}_{\text{radio}}/\mathbf{F}_{\text{ICS}} = \mathbf{U}_{\text{B}}/\mathbf{U}_{\text{CMB}}$$

DM search @ radio: galaxy clusters

Baryons + Cosmic Rays

Dark Matter



Dark Matter & Radio Halos

Dark Matter annihilation can reproduce the spectral and spatial features of galaxy clusters Radio Halos

[S.C. et al. 2001, 2006, 2008, 2010, 2011]

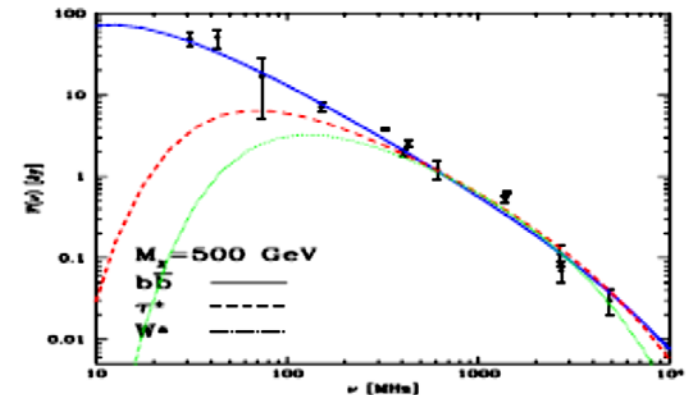
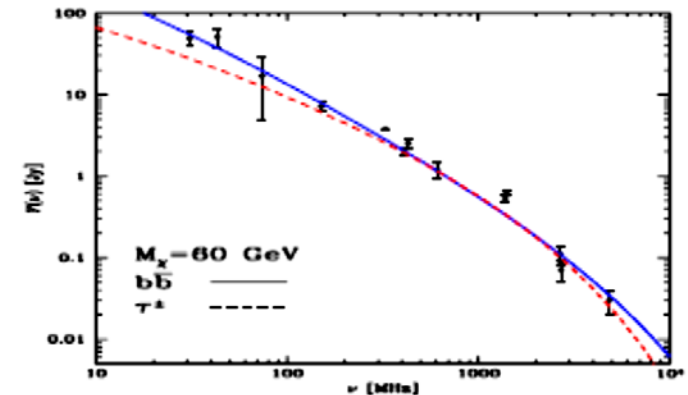
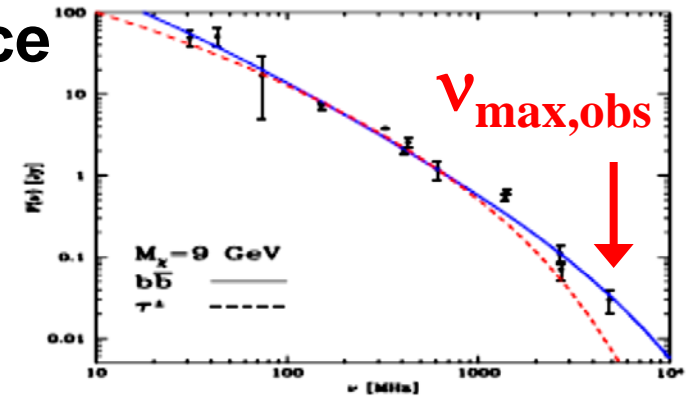
Sensitivity to DM composition

b-b model preferred by RH spectra with neutralino mass $M_\chi \sim 40-60$ GeV (□ CRESST-II results)

Sensitivity to DM particle mass

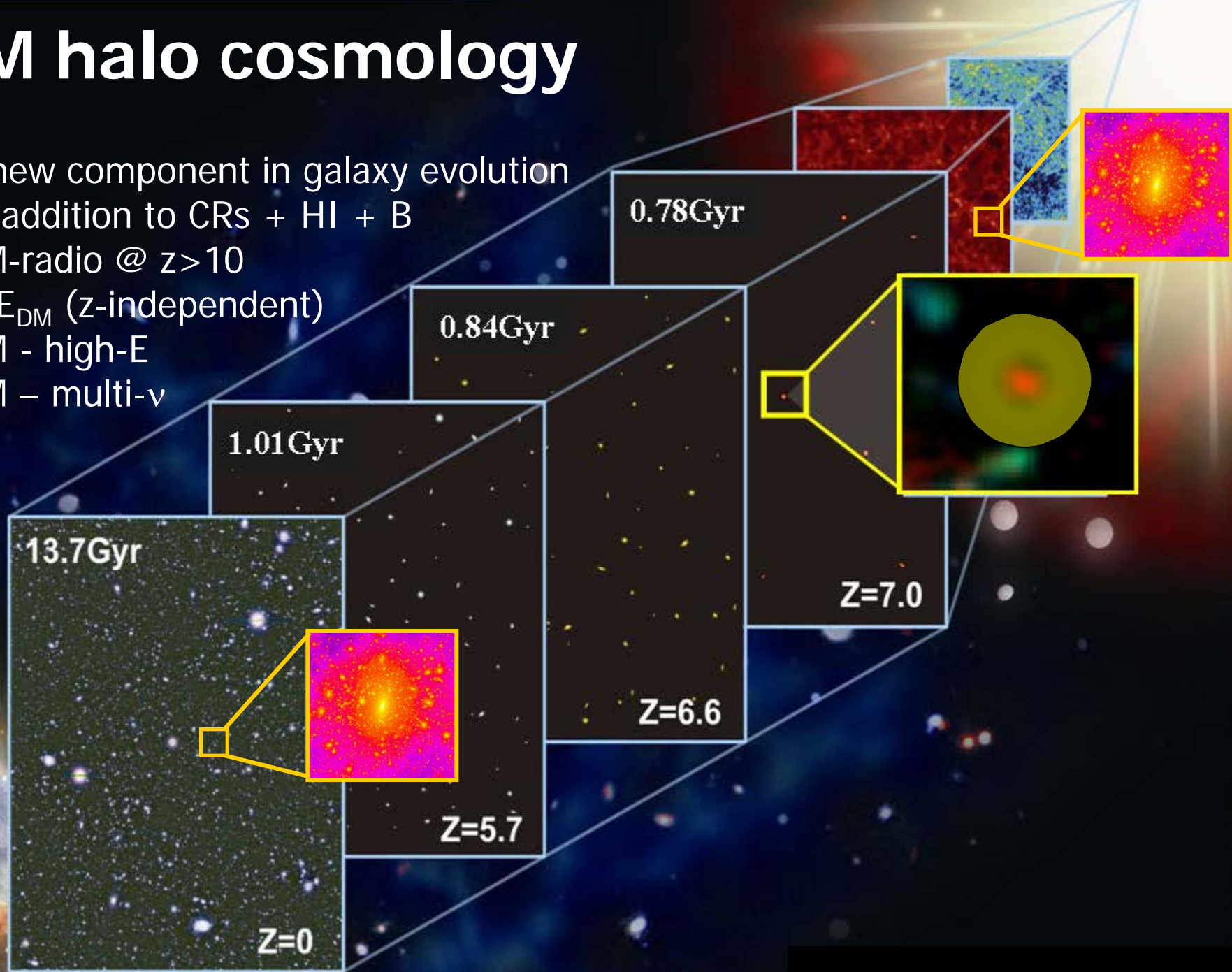
Lower l.
$$M_\chi \geq \frac{16.44}{k} \text{ GeV} \left(\frac{\nu_{\text{THALX,obs}}}{\text{GHz}} \right)^{1/2} B_\mu^{-1/2}$$

Upper l.
$$M_\chi \leq 74.3 \text{ GeV} \left(\frac{\nu_{\text{THALX}}}{10 \text{ GHz}} \right)^{1/2} B_\mu^{-1/2}$$



DM halo cosmology

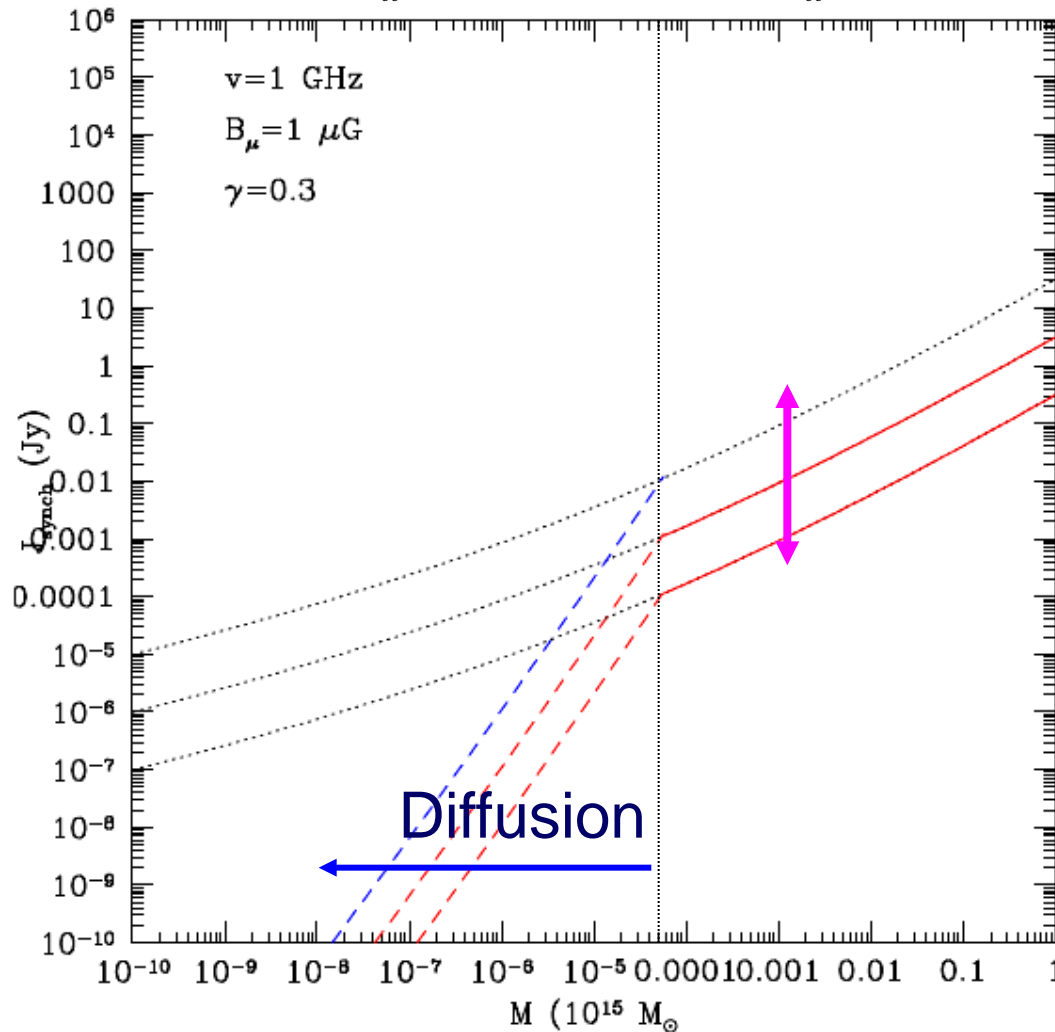
- A new component in galaxy evolution in addition to CRs + HI + B
- DM-radio @ $z > 10$
- SZE_{DM} (z-independent)
- DM - high-E
- DM - multi- ν



Radio emission from DM halos

$$\frac{R_{halo}}{L_{Diff}} \lll 1$$

$$\frac{R_{halo}}{L_{Diff}} \ggg 1$$



$$\langle \sigma V \rangle \rho_{DM}^2$$

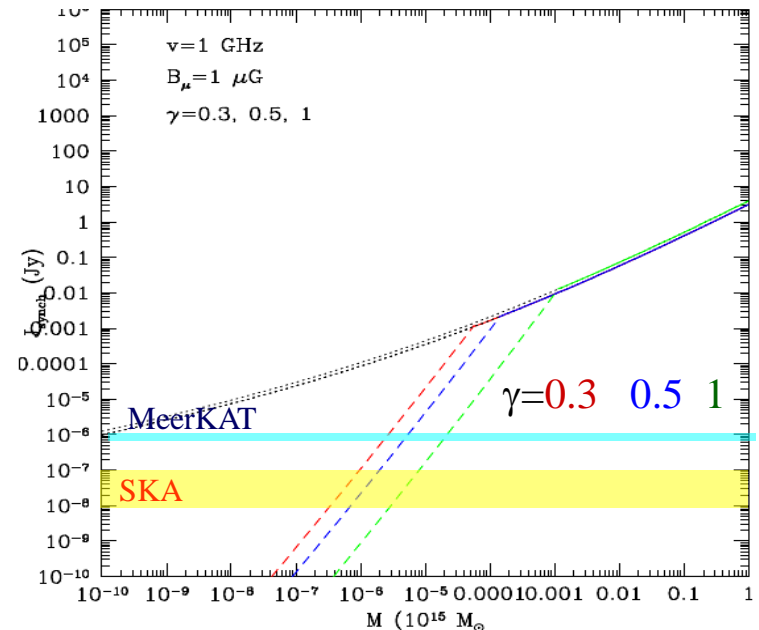
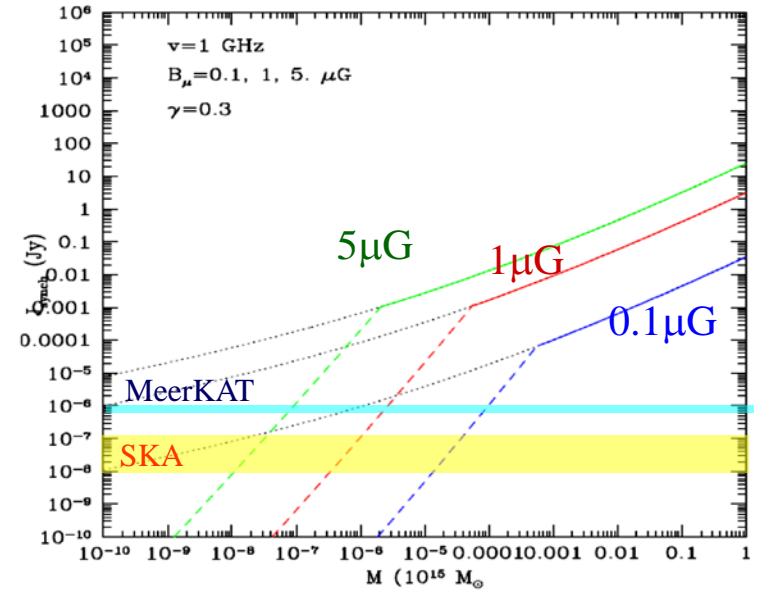
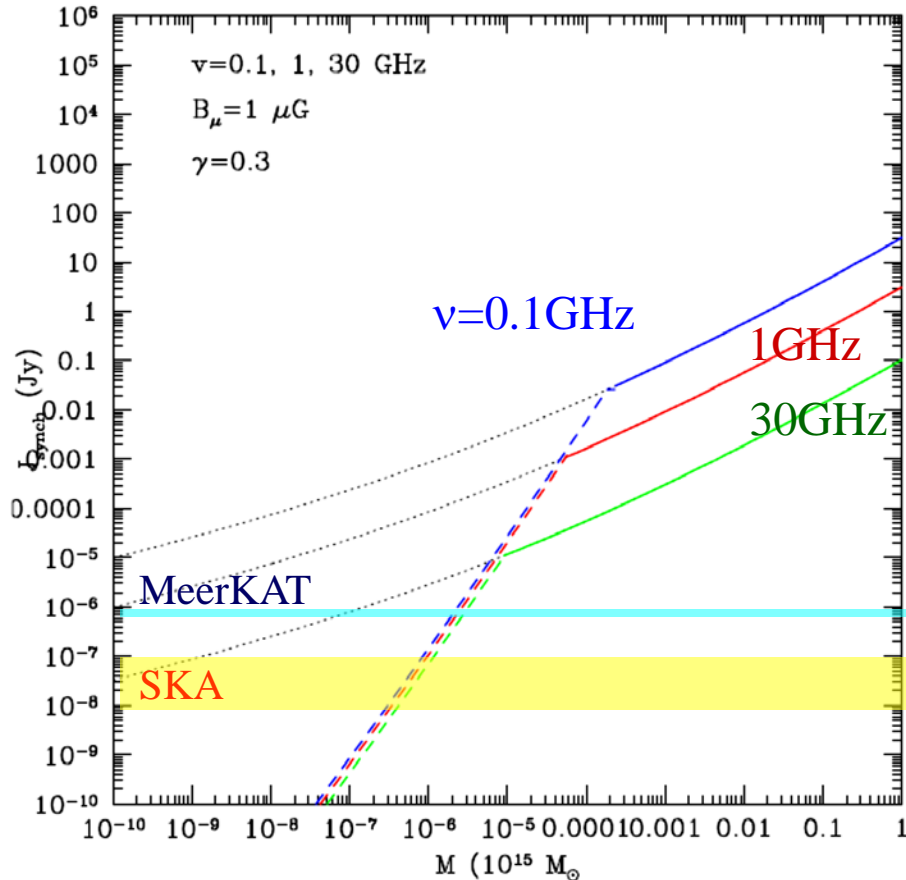
$$\propto \langle \sigma V \rangle M_{\chi}^{-2}$$

$$L_{Diff} = [D(E, B) \cdot \tau_{loss}]^{1/2}$$

Radio emission from DM halos

SKA can probe $\sim 10^7 - 10^8 M_\odot$ DM halos with standard $\langle \sigma V \rangle$ and M_χ

- Dwarf galaxies (\rightarrow DM halos)
- Proto-galaxies (\rightarrow DM halos)



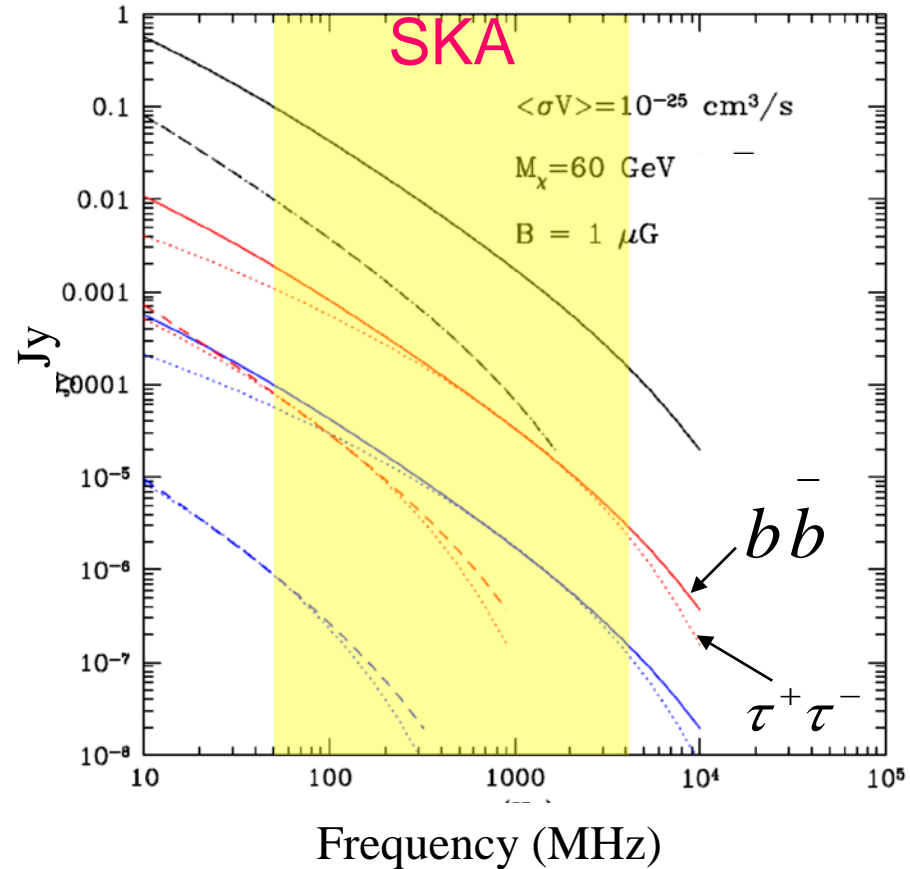
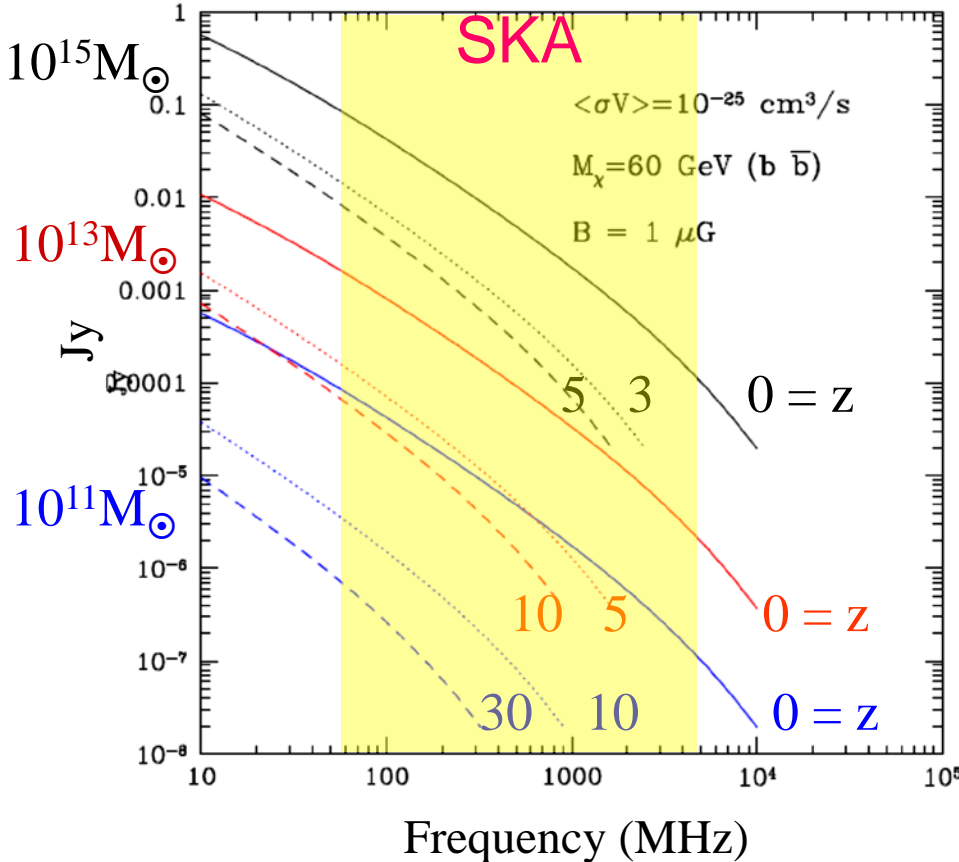
Probing early DM halos @ radio

High-z DM halos

Early proto galaxies

High-z DM halos

χ mass & composition



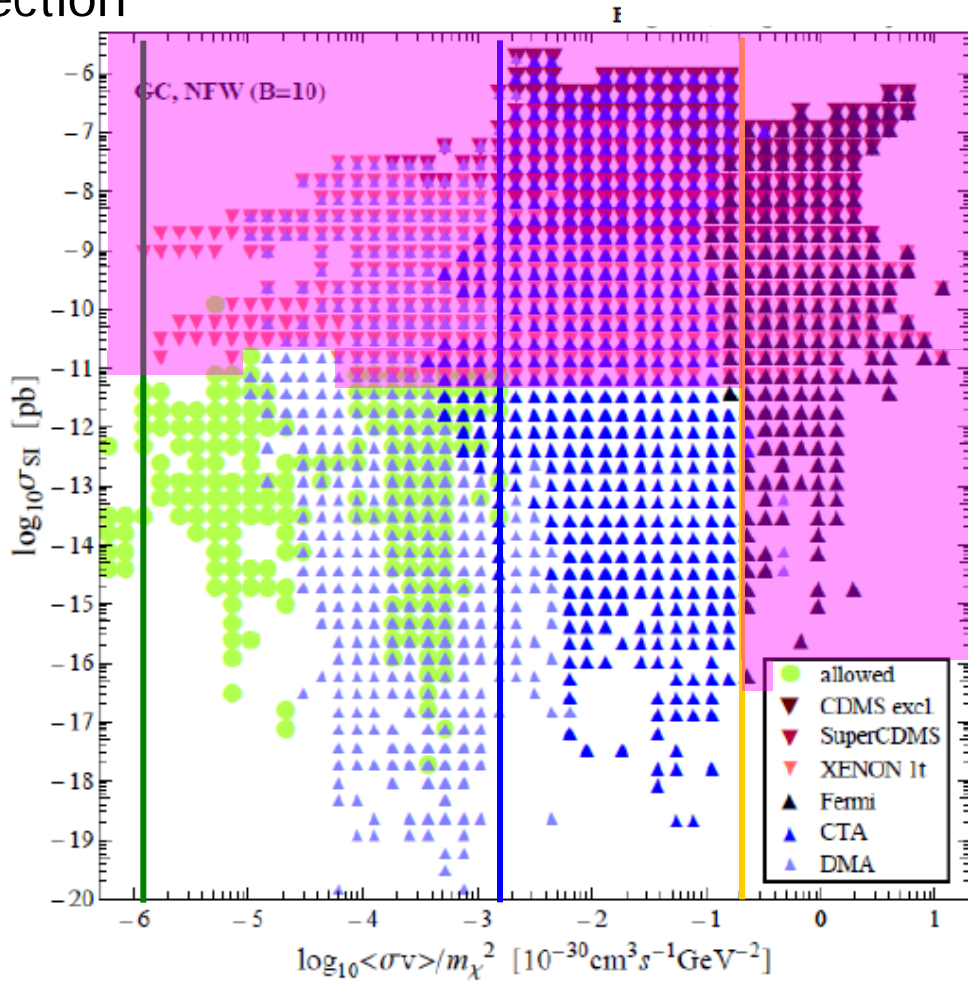
M_χ from radio spectrum cutoff

$$\left\{ \begin{array}{l} v \approx (3.7 \text{ MHz}) B_\mu \left(\frac{E}{\text{GeV}} \right)^2 \\ v_{\text{max}} \sim (3.7 \text{ MHz}) B_\mu \left(\frac{k M_\chi}{\text{GeV}} \right)^2 \end{array} \right\} M_\chi \geq \frac{16.44}{k} \text{ GeV} \left(\frac{v_{\text{max,obs}}}{\text{GHz}} \right)^{1/2} B_\mu^{-1/2}$$

Exploring DM universes

Direct
Detection

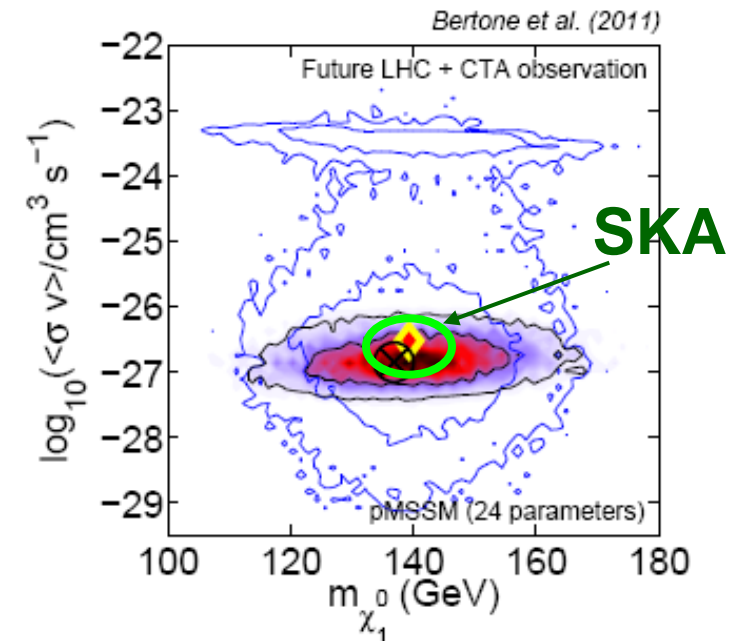
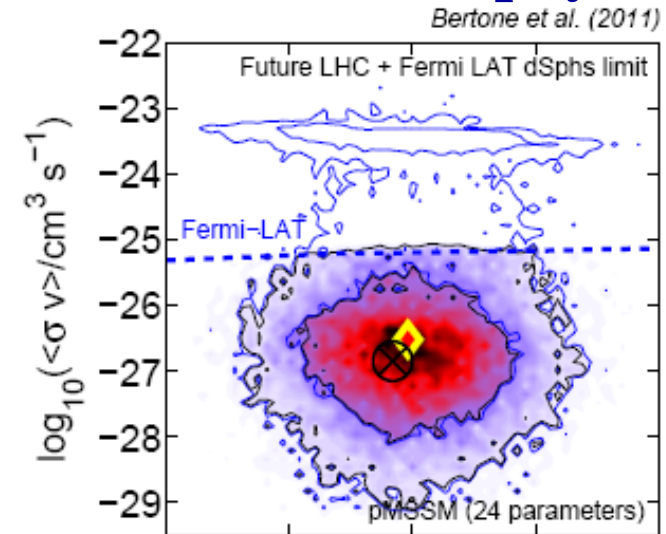
DM detectors + Astrophysics



← SKA ← CTA ← Fermi

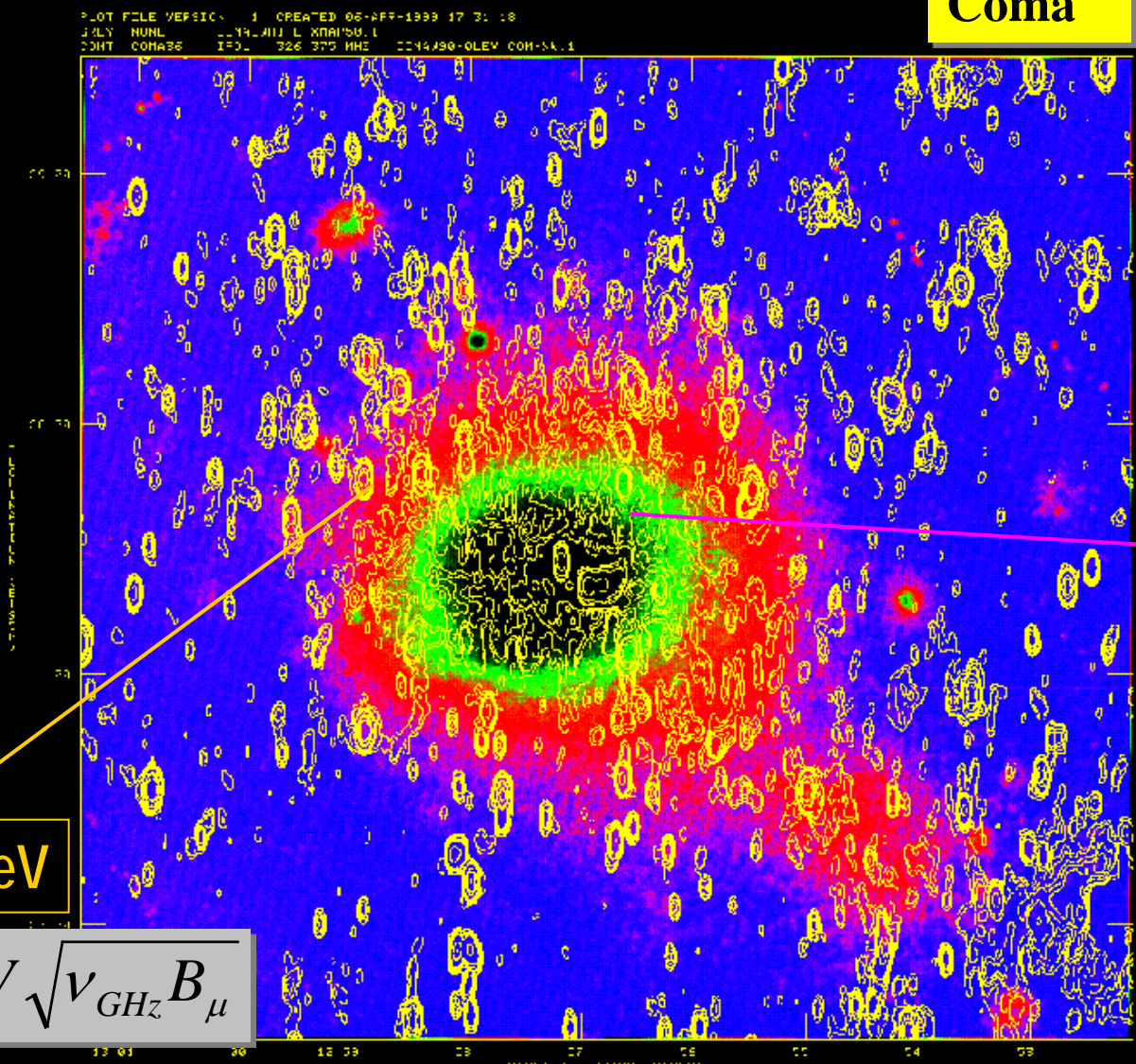
← Indirect Detection

LHC + Astrophysics



CRs in clusters: radio emission

Coma



$E_e \sim \text{a few GeV}$

$E_e \geq \text{keV}$

$$E \approx 16.6 \text{ GeV} \sqrt{v_{\text{GHz}} B_{\mu}}$$

GREY SCALE FLUX RANGE: 0.3 0 237.4
CONT PLAR TELL: 0.00001 0.00001

CRs in clusters: Hard X-Rays

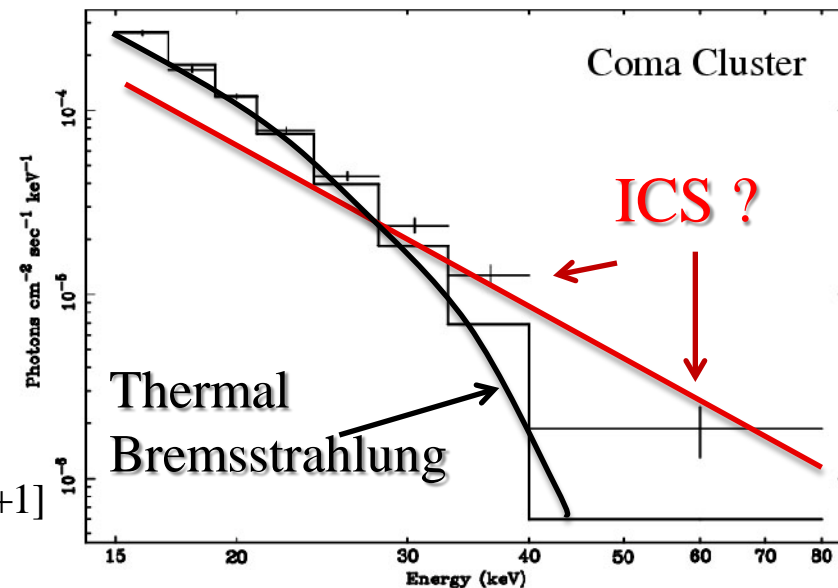
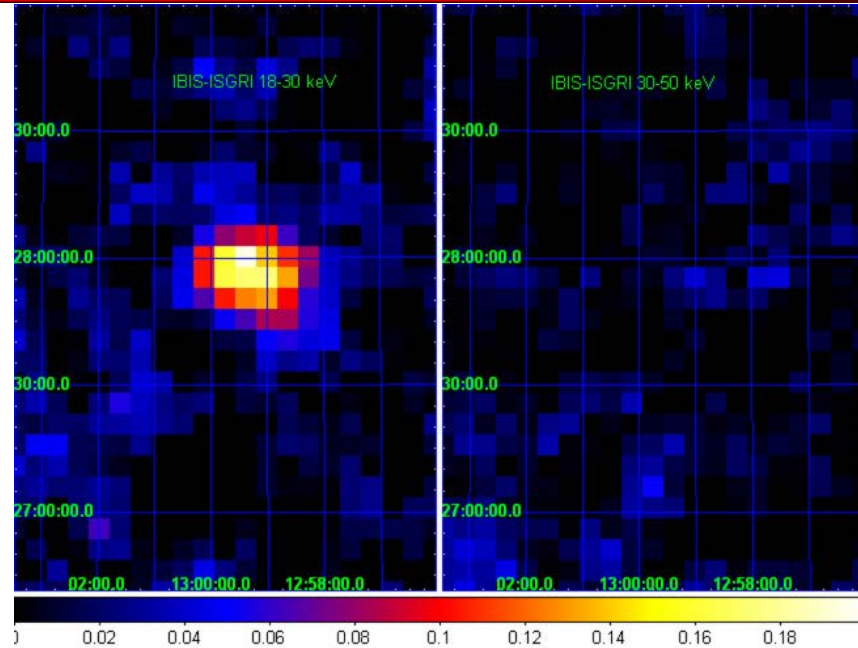
Beppo-SAX
INTEGRAL

First detection of hard X-rays
in Coma at $E > 20\text{keV}$

HXR spectrum has a slope
consistent with the
synchrotron radio spectrum

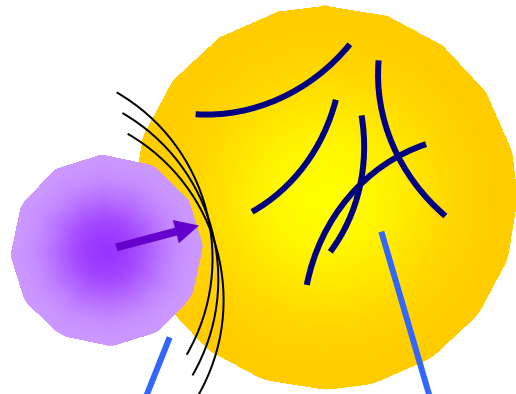
$$I_{ICS}(\nu) \propto E^{-(p-1)/2}$$

$$I_{Sync}(\nu) \propto E^{-(p-1)/2} \cdot B^{[(p-1)/2+1]}$$



Cosmic rays in clusters

Acceleration $t_{acc} \ll t_{loss}, t_{eq}$



Direct

Stochastic

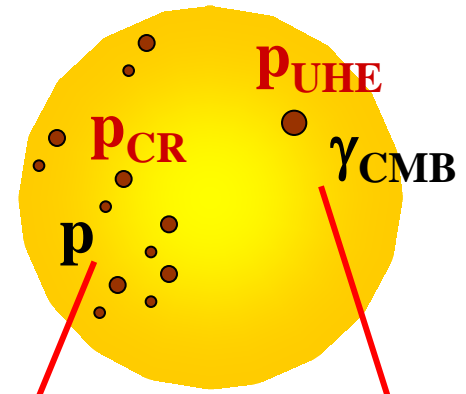


e^-, p



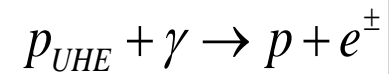
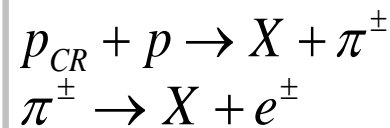
e^-, p

In-situ $\partial n_e / \partial t \approx 0$



$p_{CR} - p$

$p_{UHE} - \gamma_{CMB}$



$E_e \approx 10^{-5,-6} GeV$

Bremsstrahlung Bremsstrahlung

$E_e \approx GeV$

ICS on CMB

ICS on CMB

$E_e \approx 10^{5-6} GeV$

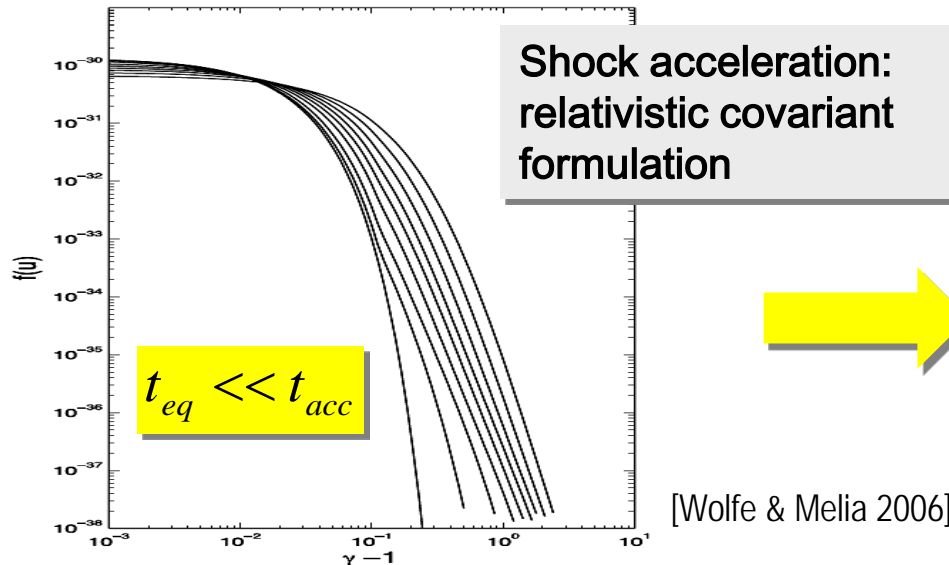
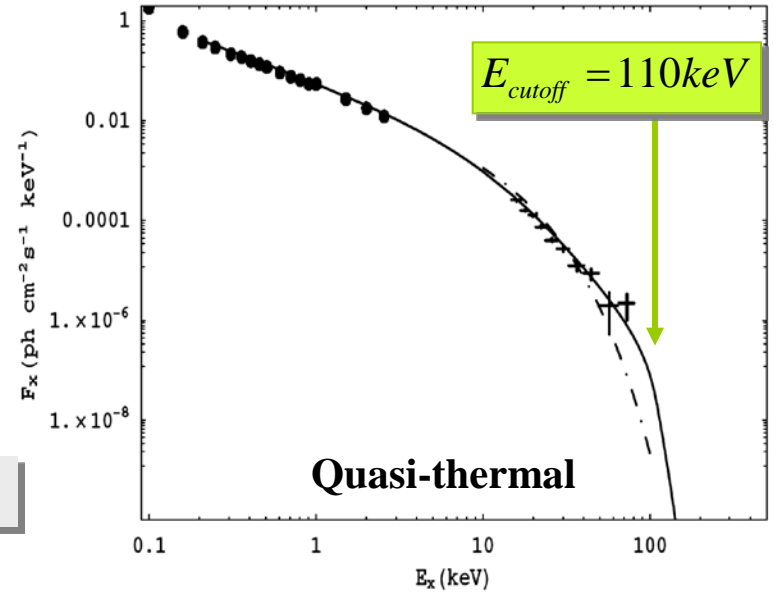
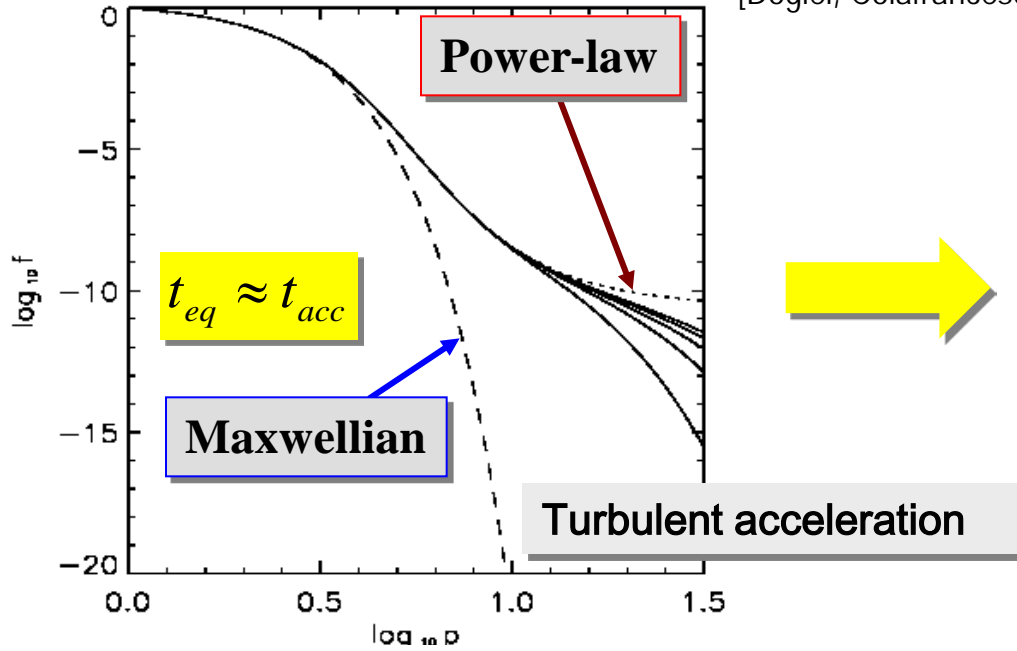
Bremsstrahlung

ICS on CMB

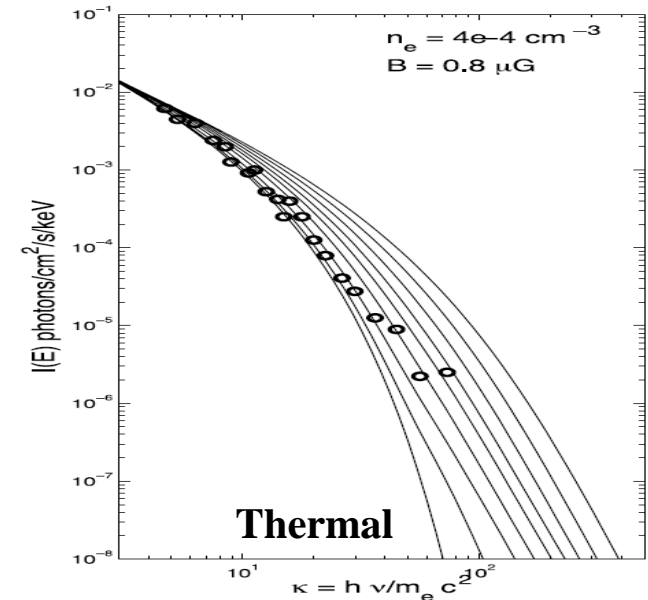
Synchrotron

CR direct acceleration efficiency

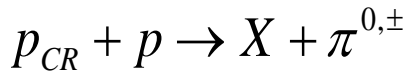
[Dogiel, Colafrancesco et al. 2007]



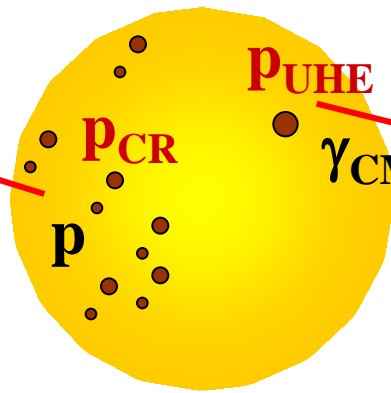
[Wolfe & Melia 2006]



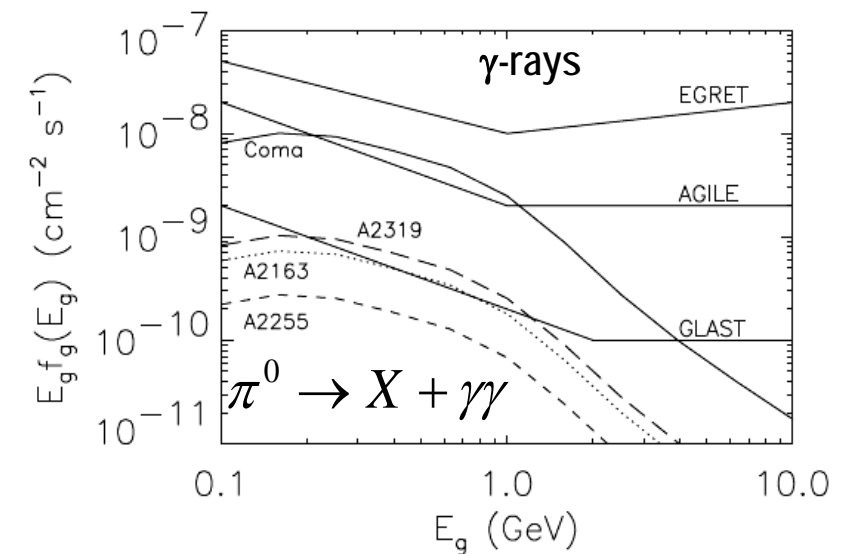
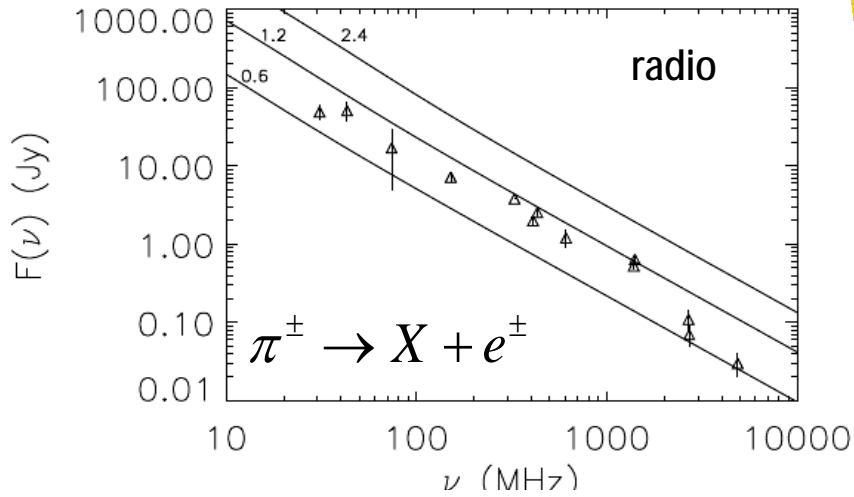
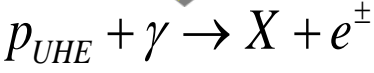
CRs of high-E \rightarrow *in-situ* production



$$p_{CR} - p$$



$$p_{UHE} - \gamma_{CMB}$$



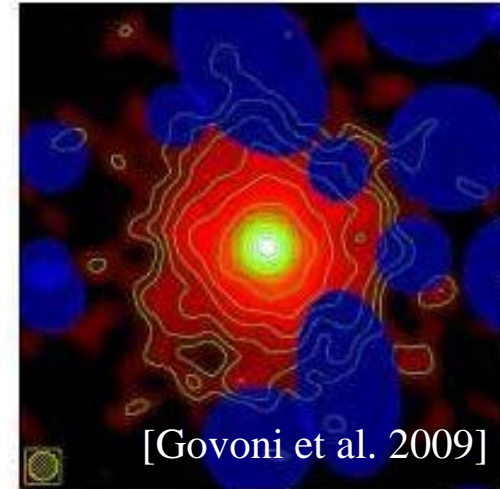
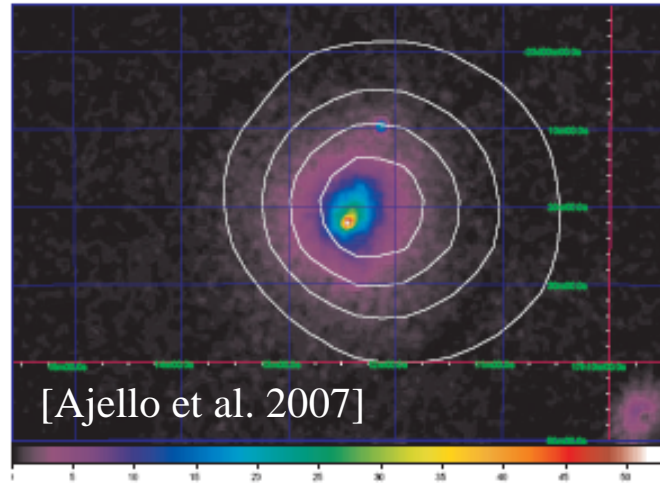
If we interpret the HXR emission in clusters as due to ICS of CRs, we have three consequences:

1. The number density of CRs is high
2. The cluster B field must be very low $B \sim 0.15 \mu\text{G}$
3. The ICS and $\pi^0 \rightarrow \gamma\gamma$ gamma-ray emission exceeds the observed upper limits

Examples: Ophiuchus and Perseus

Ophiuchus cluster

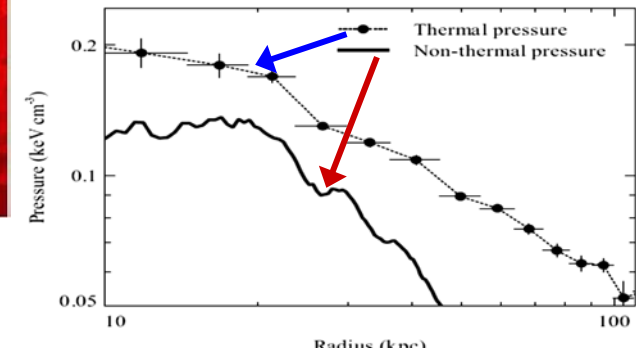
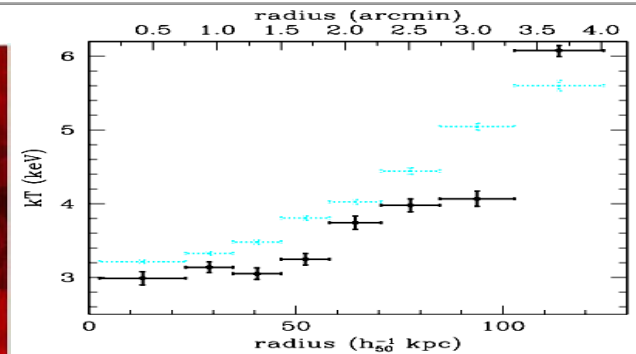
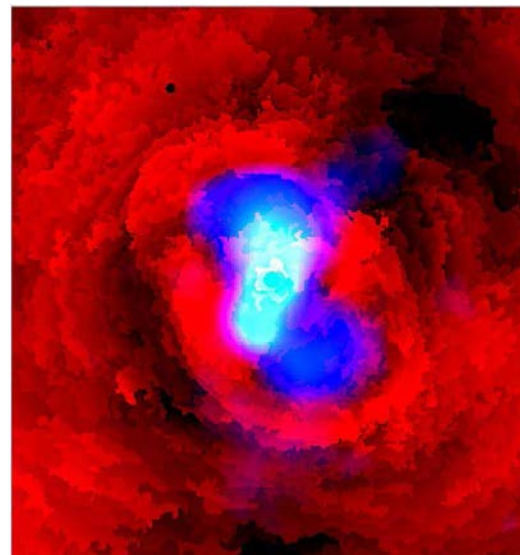
- Single-T ~ 9.5 keV
- no-Cool Core
- no AGN in the core
- Radio halo @1.4GHz



(DM, CRs, WRs,...)

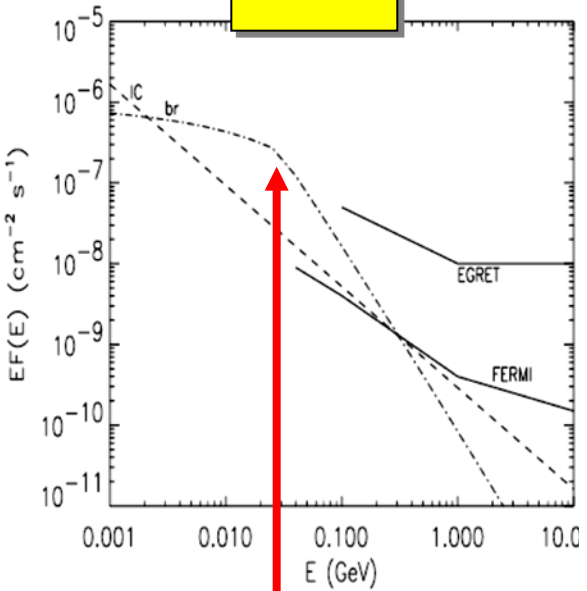
Perseus cluster

- Multi-T
- Cool Core
- AGN-dominated core
- Mini Radio halo
- Non-thermal plasma

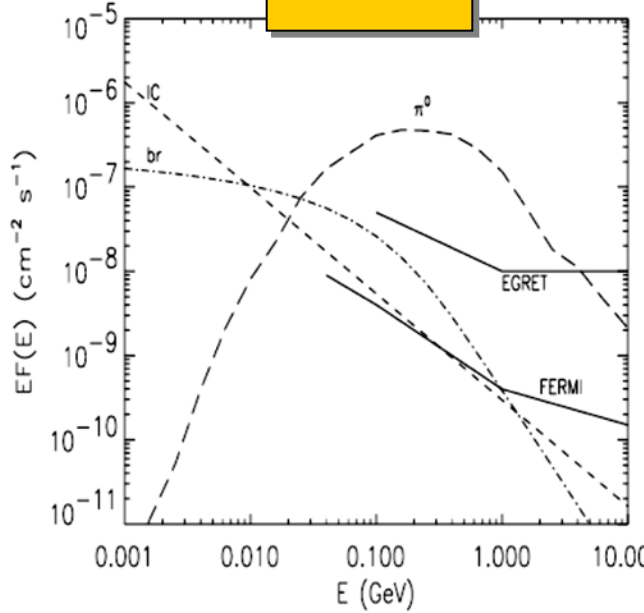


(DM, CRs, WRs, BH,..)

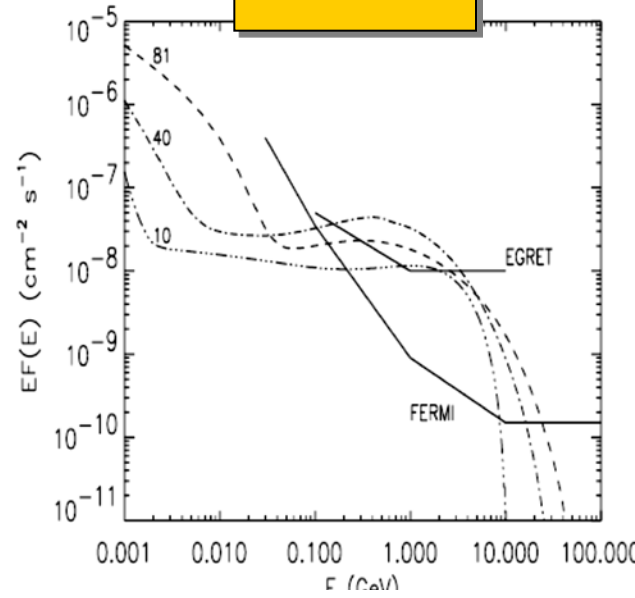
PEM



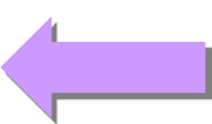
SEM-pp



SEM-DM



γ -ray emission normalized to the HXR emission

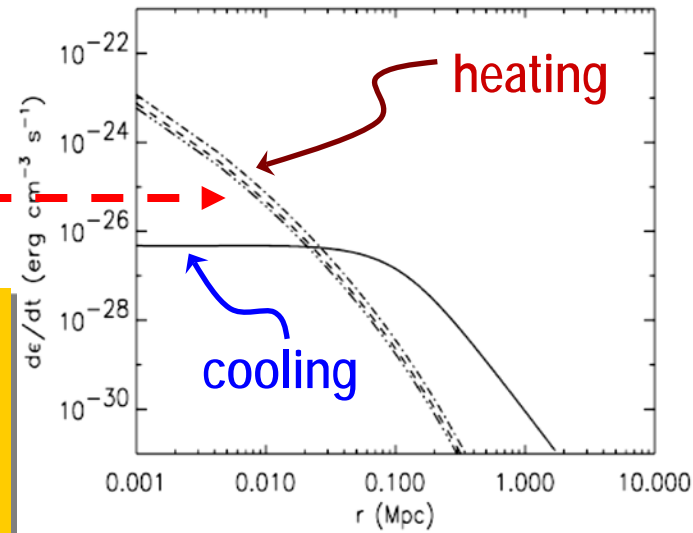


Excessive Heating



Need cut-off E_e spectrum at low $E_e \sim 30$ MeV

Untenable SEM models

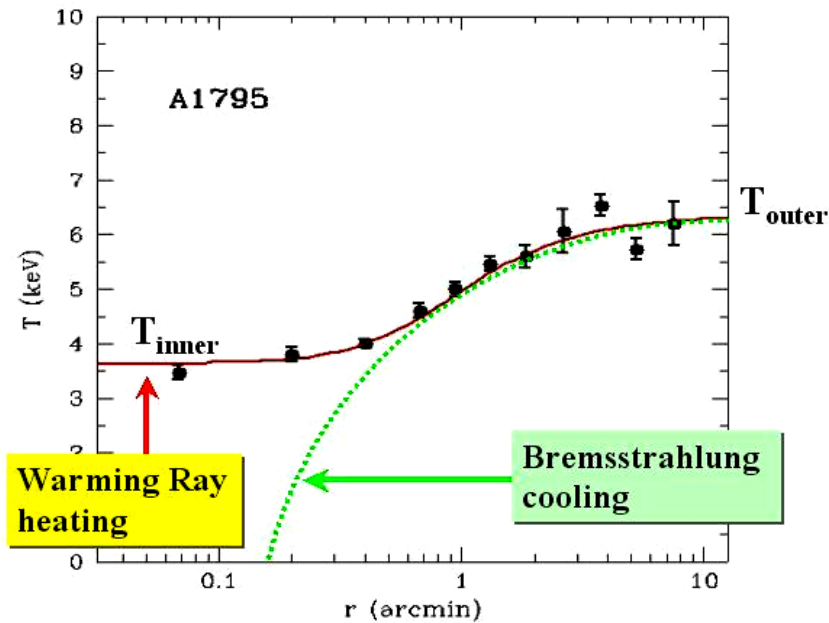


A consistent model: Warming Rays

$$3kn(r) \frac{dT(r,t)}{dt} = \left(\frac{dE}{dt} \right)_{WR} - \left(\frac{dE}{dt} \right)_X$$

$$\left(\frac{dE}{dt} \right)_{WR} = bn^2(r) \quad \text{Heating}$$

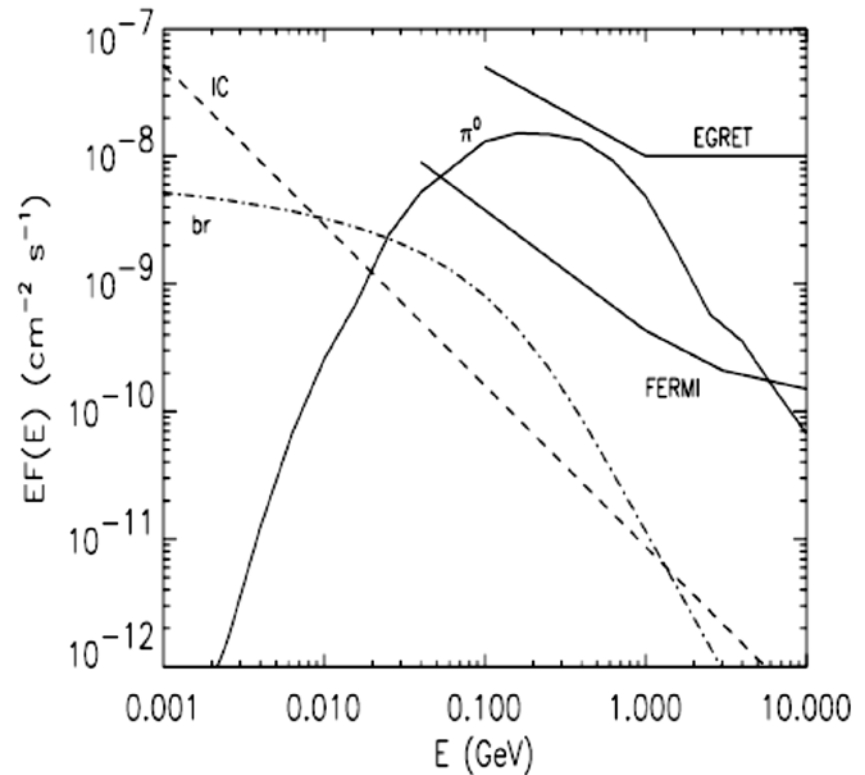
$$\left(\frac{dE}{dt} \right)_X = an^2(r)T^{1/2} \quad \text{Cooling}$$



Warming Ray Model

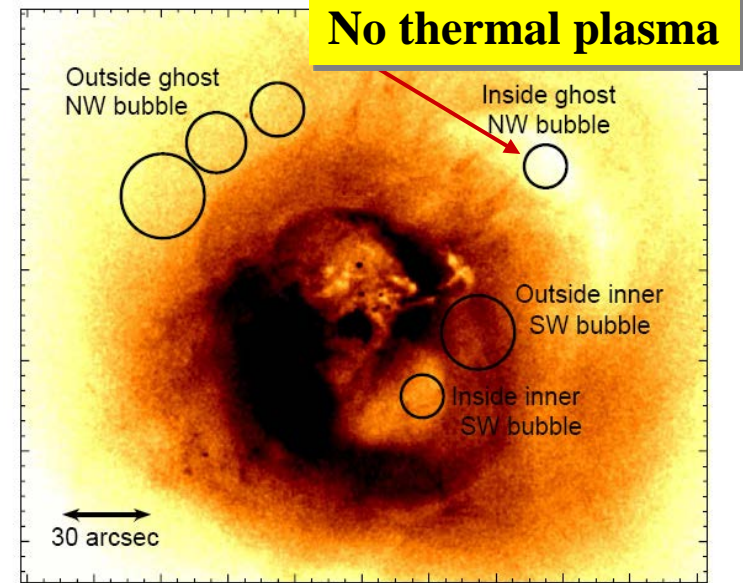
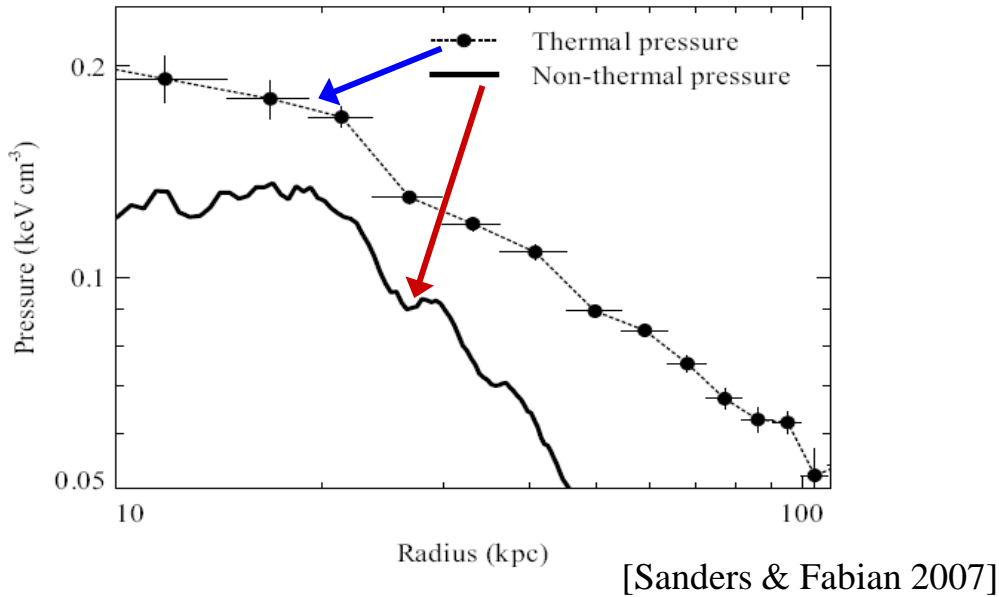
[Colafrancesco, Dar & deRujula 2004]

[Colafrancesco & Marchegiani 2008]



A self-consistent description of non-thermal phenomena in clusters based on the ability to recover the thermal structure of clusters.

BHs, WRs & Cooling Flows

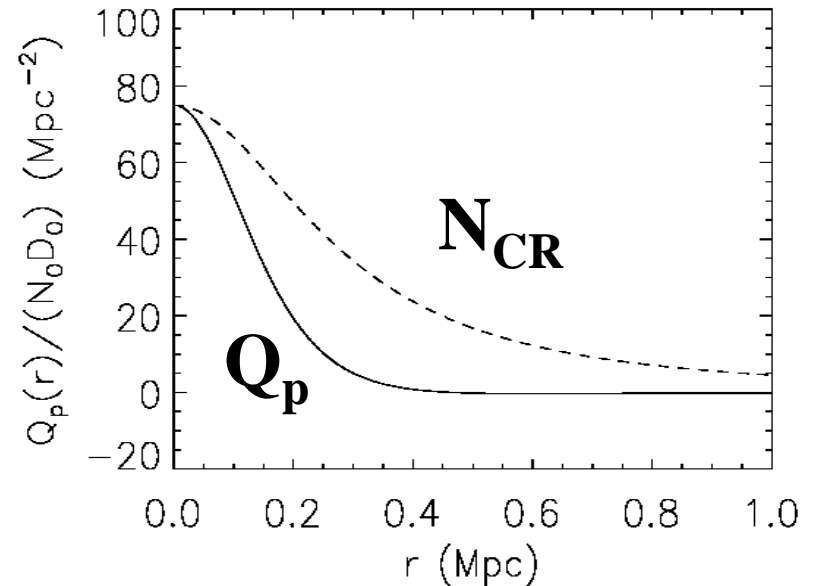


$$\frac{\partial N}{\partial t} - \nabla(D\nabla N) - \frac{\partial(b_p N)}{\partial E} = Q_p$$



$$N_{CR}(r) \sim [n_{th}(r)]^\alpha$$

[Colafrancesco & Marchegiani 2008]

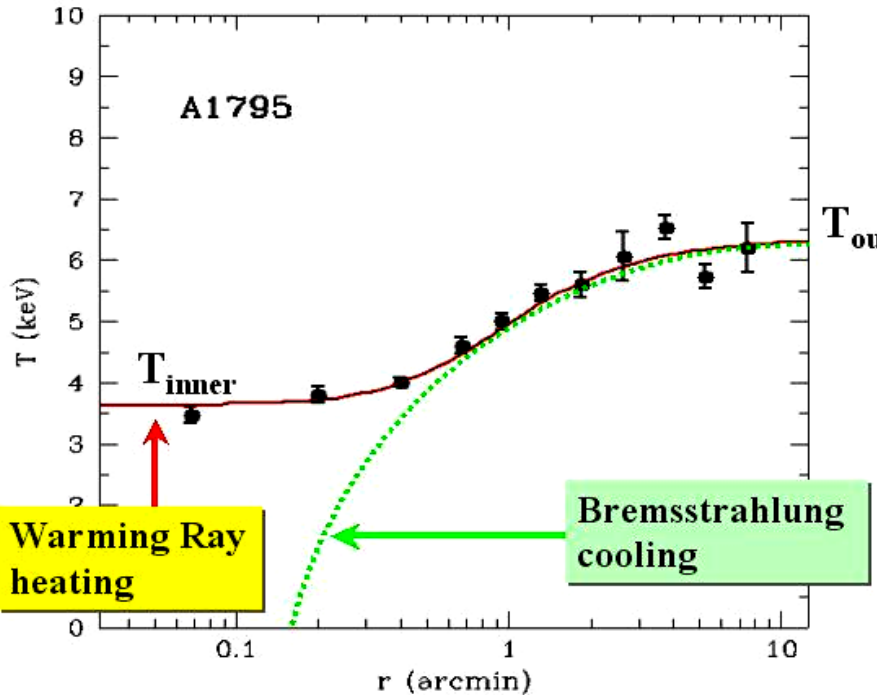


Warming Rays in cool cores

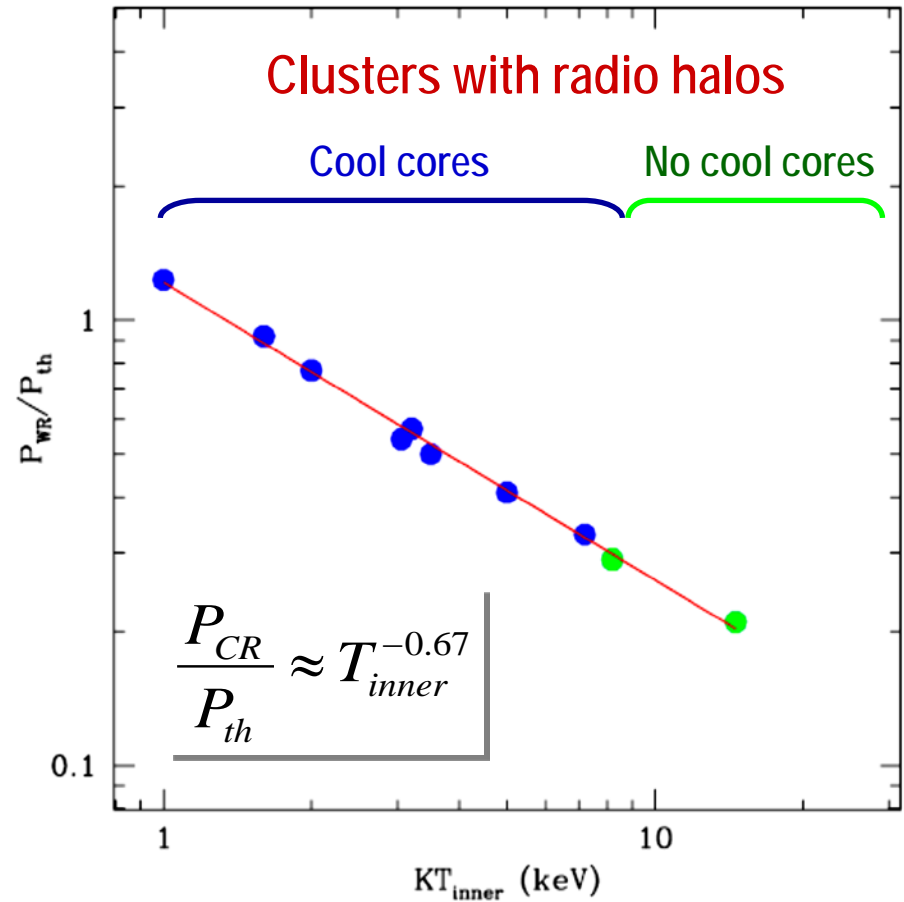
$$3kn(r) \frac{dT(r,t)}{dt} = \left(\frac{dE}{dt} \right)_{WR} - \left(\frac{dE}{dt} \right)_X$$

$$\left(\frac{dE}{dt} \right)_{WR} = bn^2(r) \quad \text{Heating}$$

$$\left(\frac{dE}{dt} \right)_X = an^2(r)T^{1/2} \quad \text{Cooling}$$



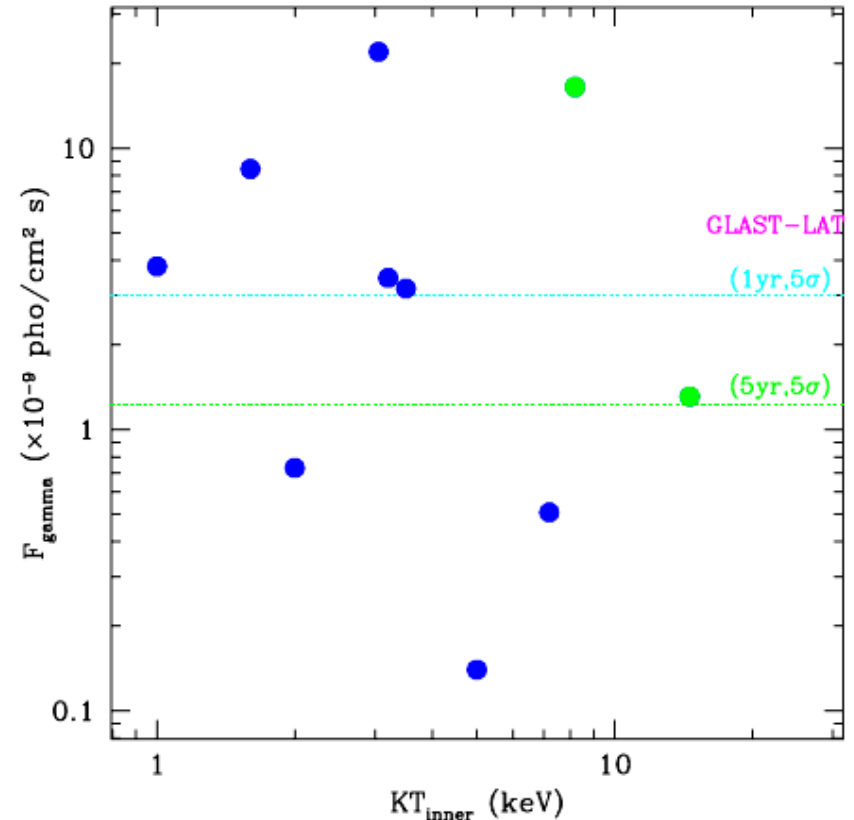
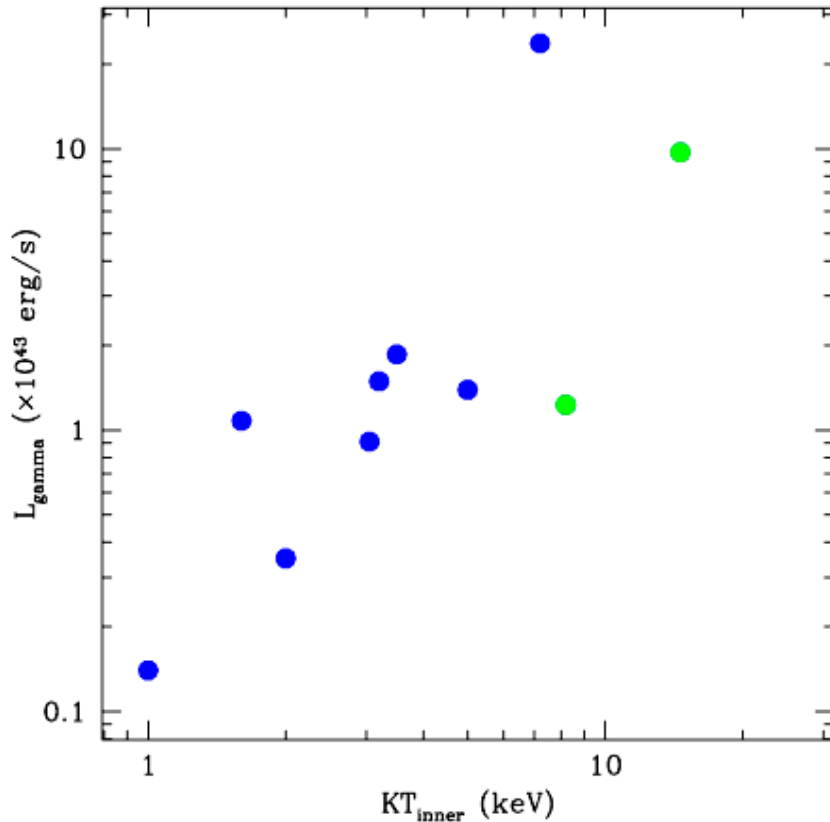
[Colafrancesco, Dar & deRujula 2004]



[Colafrancesco & Marchegiani 2007]

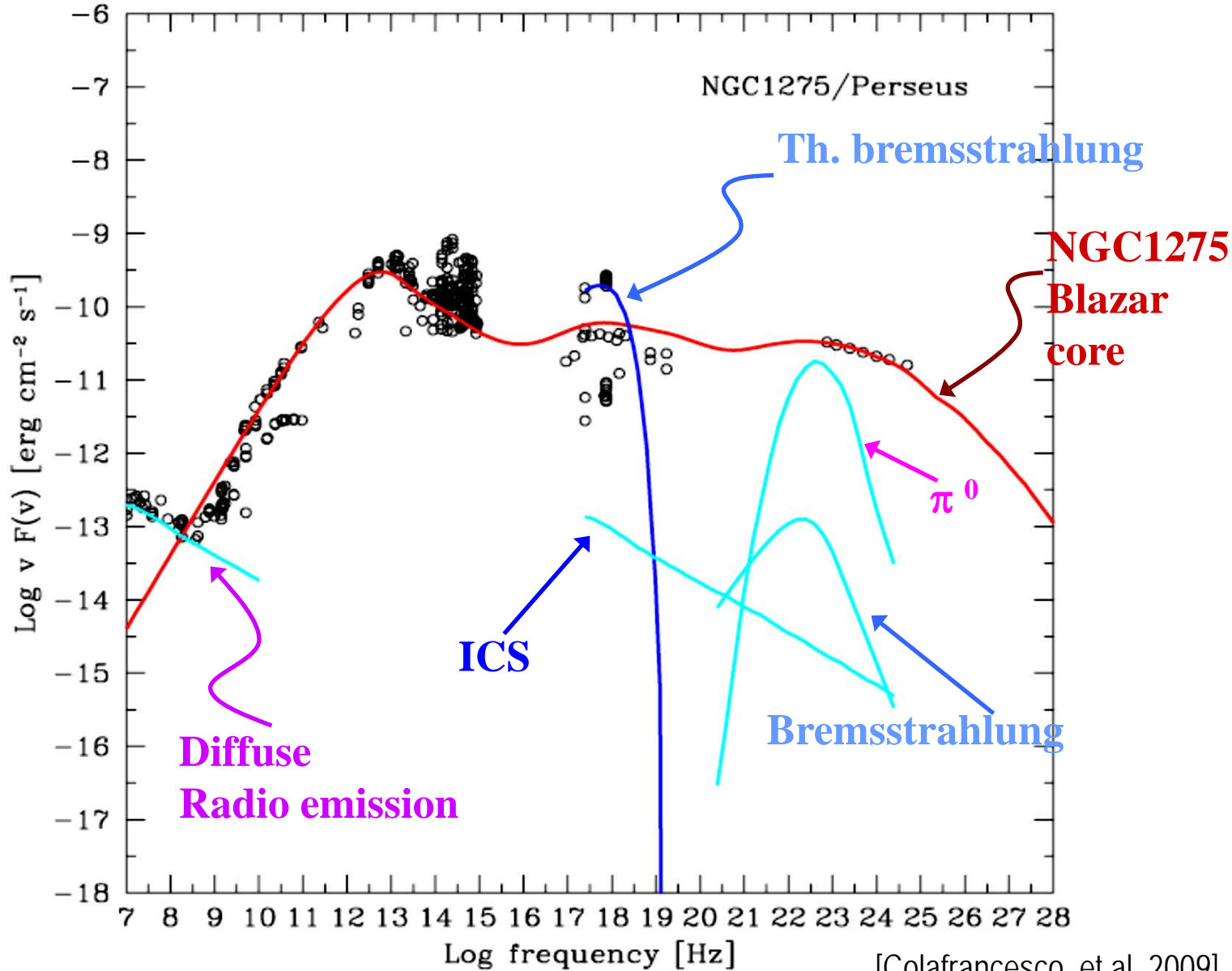
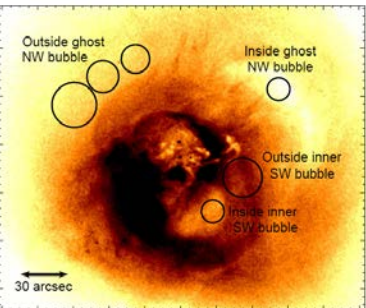
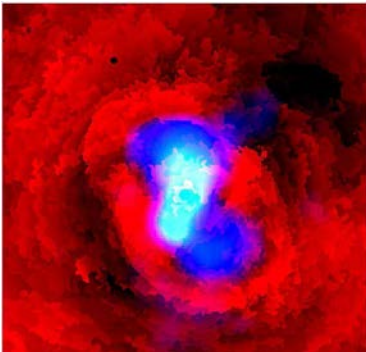
WRs, HXR and γ -rays

Cluster	α	$n_{WR,0}$ cm^{-3}	P_{WR}/P_{th}	F_{γ} $\text{cm}^{-2} \text{s}^{-1}$	L_{γ} erg s^{-1}	F_{HXR} $\text{erg cm}^{-2} \text{s}^{-1}$
A262	0.83	2.20×10^{-3}	1.23	3.89×10^{-9}	1.43×10^{42}	3.87×10^{-14}
A2199	0.83	2.31×10^{-3}	0.92	8.43×10^{-9}	1.08×10^{43}	3.06×10^{-13}
A133	0.84	4.56×10^{-4}	0.77	7.30×10^{-10}	3.53×10^{42}	6.10×10^{-15}
Perseus	0.91	4.98×10^{-4}	0.54	2.20×10^{-8}	9.91×10^{42}	1.59×10^{-13}
Hydra	0.97	6.24×10^{-4}	0.57	3.46×10^{-9}	1.49×10^{43}	2.57×10^{-14}
A1795	0.96	5.55×10^{-4}	0.50	3.17×10^{-9}	1.86×10^{43}	2.41×10^{-14}
A2390	0.94	2.21×10^{-4}	0.41	1.41×10^{-10}	1.39×10^{43}	6.17×10^{-16}



NGC 1275 / Perseus cluster

RG (3C84)
Mini RH
Sy 1.5
Blazar



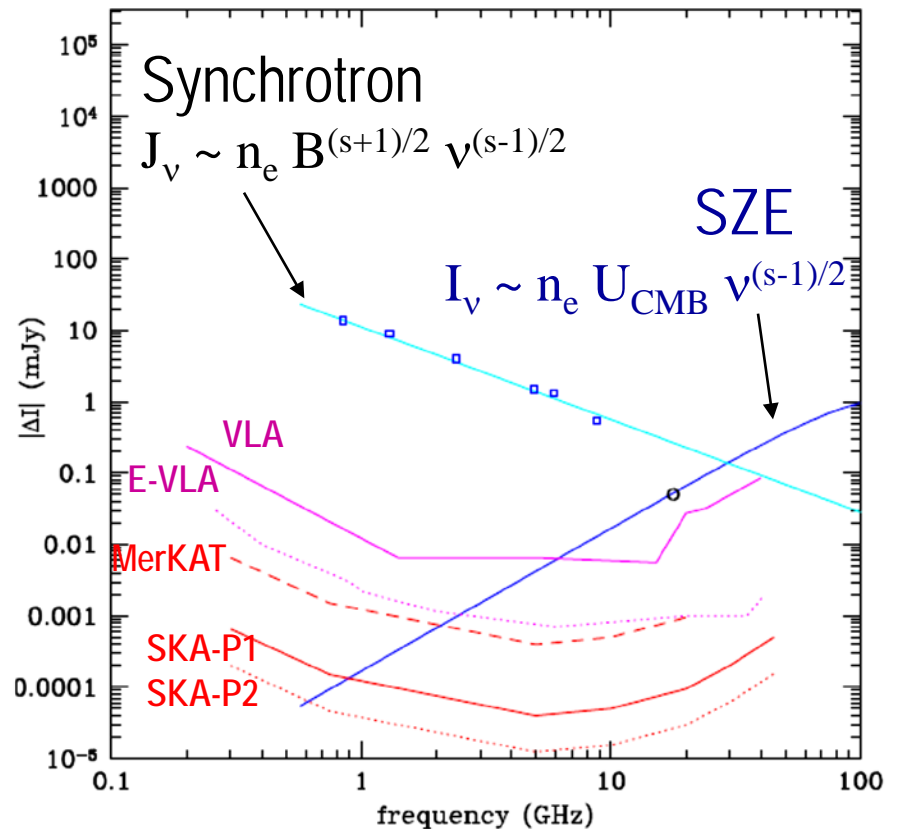
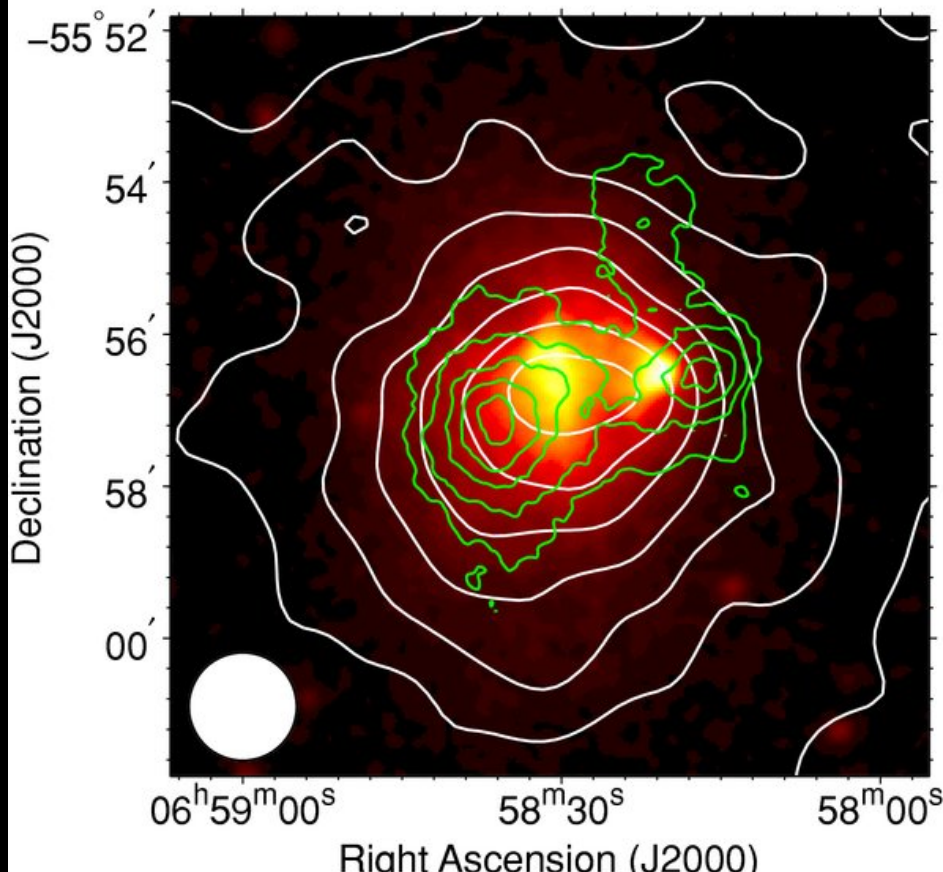
Radio Halos & Cosmic Rays

STRATEGY \Rightarrow SKA

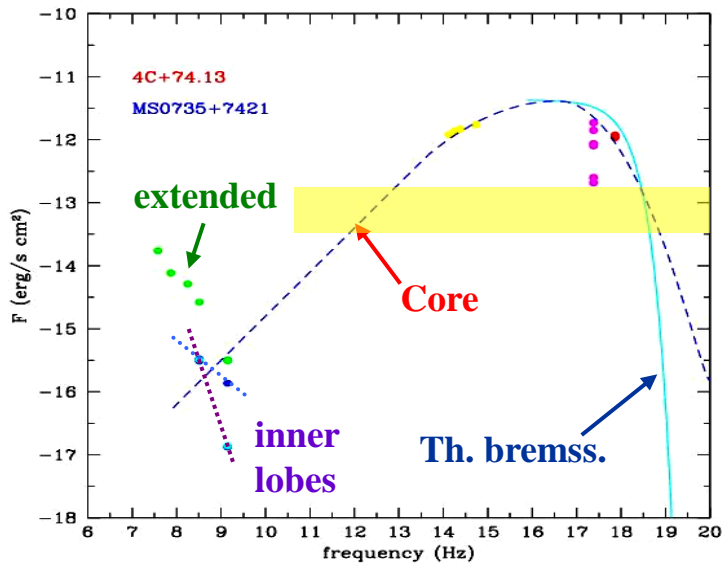
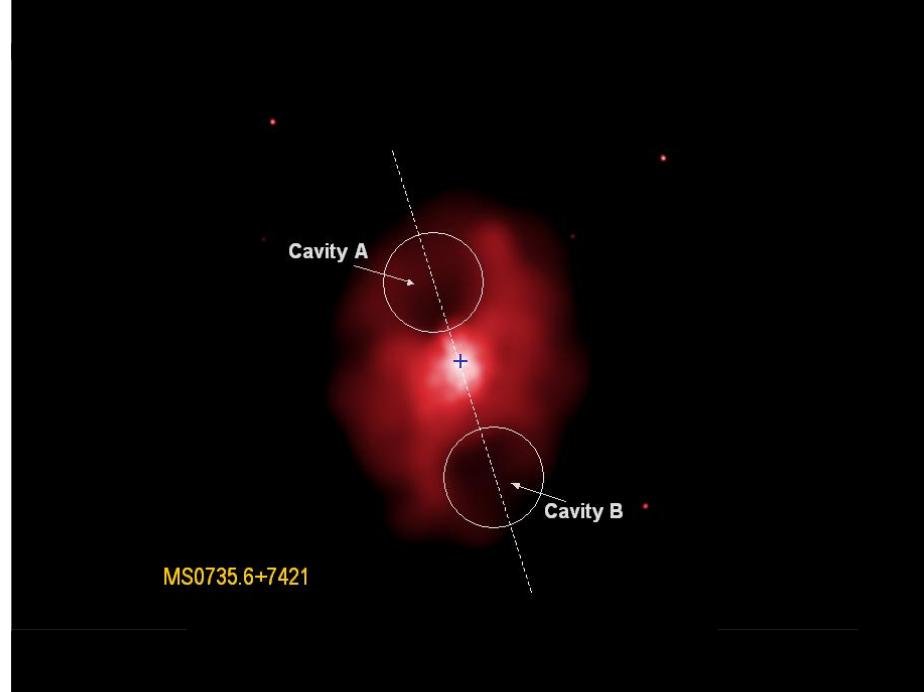
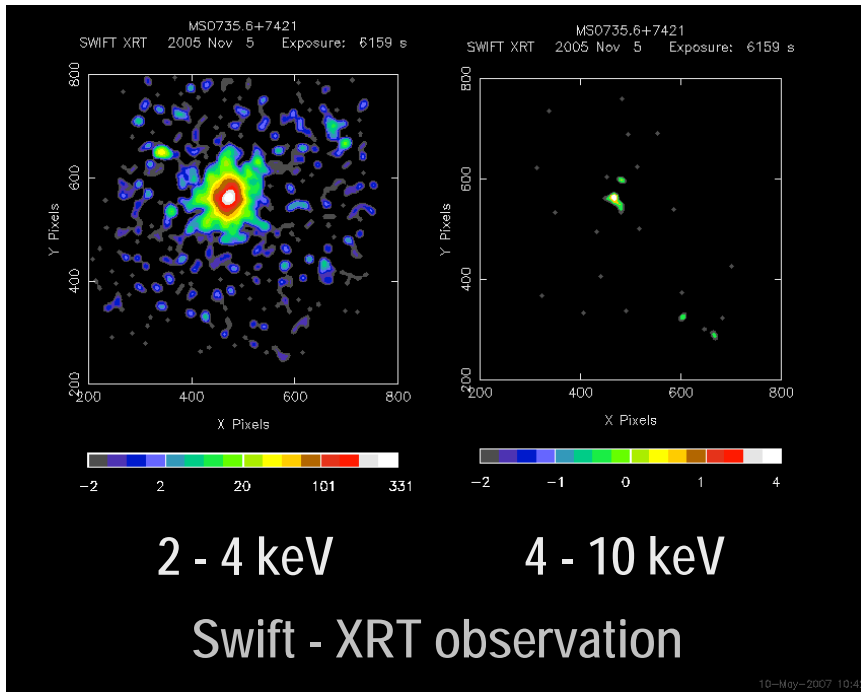
Derive both n_e and B from single SKA observations

Combine: radio + ICS

0.1-1 GHz + 30 GHz



Xrays from BHs & cavities in clusters



Multi- ν emission from cavities

Cavity E $\sim 10^{60-63}$ ergs

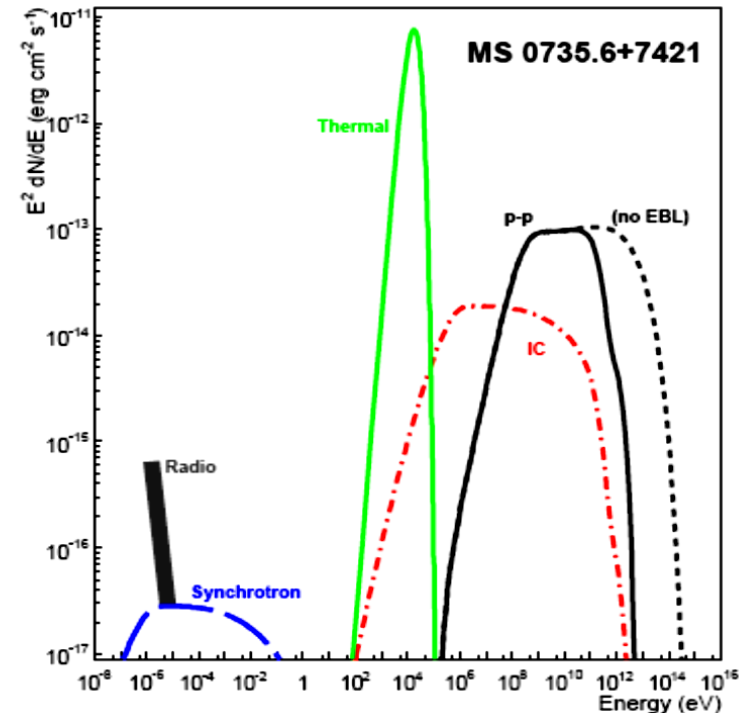
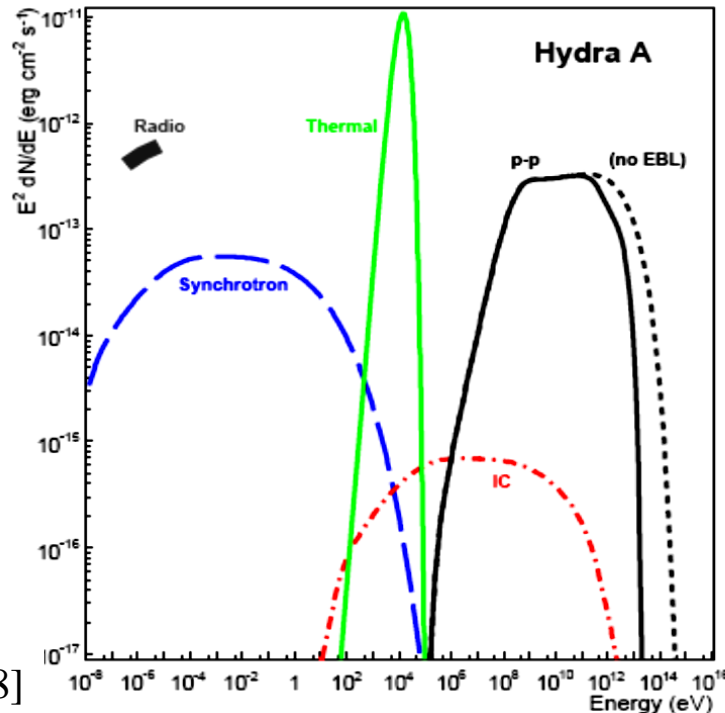
Cavity Age $\geq 10^8$ yrs

Diffusion D $\leq 10^{28}$ cm²/s



Cavities likely supported
by hadronic CRs

[Brighenti & Matthews 2007]



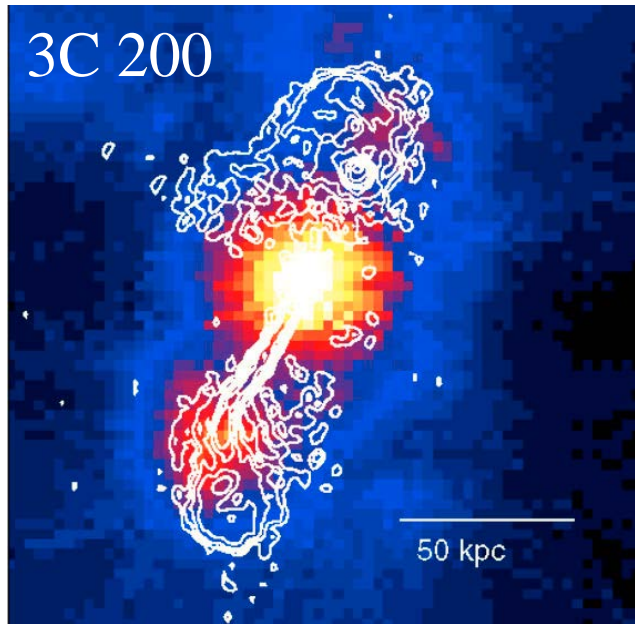
[Hinton et al. 2007]

[Domainko et al. 2008]

Radio galaxy jets



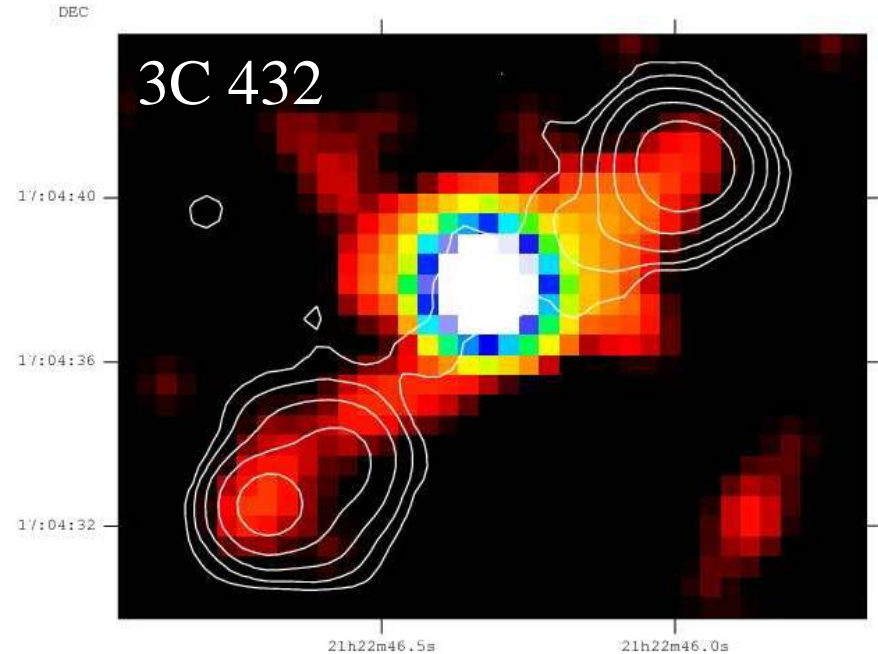
Radiogalaxy jets: emission



Chandra (color)+5GHz (contours)

$$F_{radio} \approx \nu^{-\alpha} B^{2(\alpha+1)}$$

$$\alpha = (p - 1) / 2$$

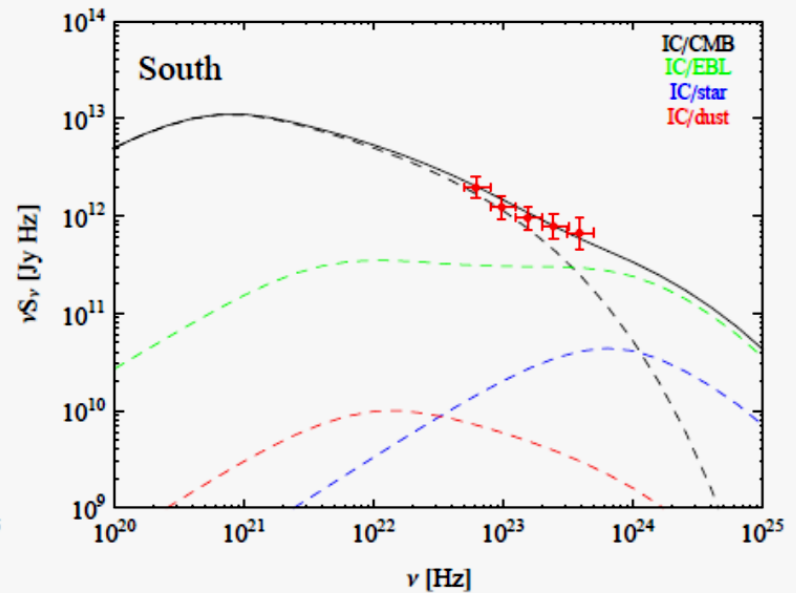
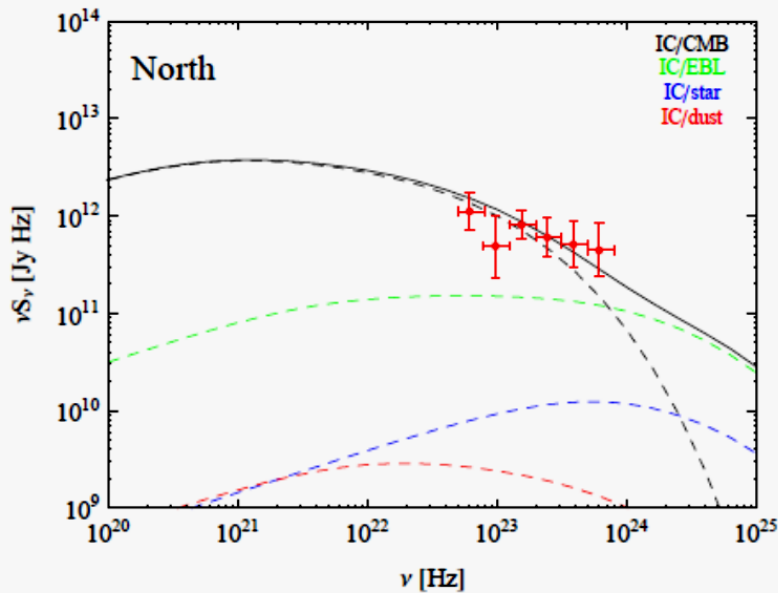
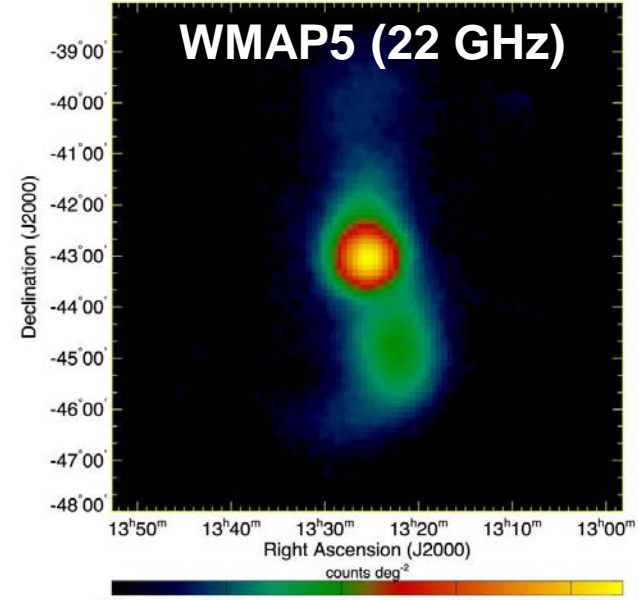
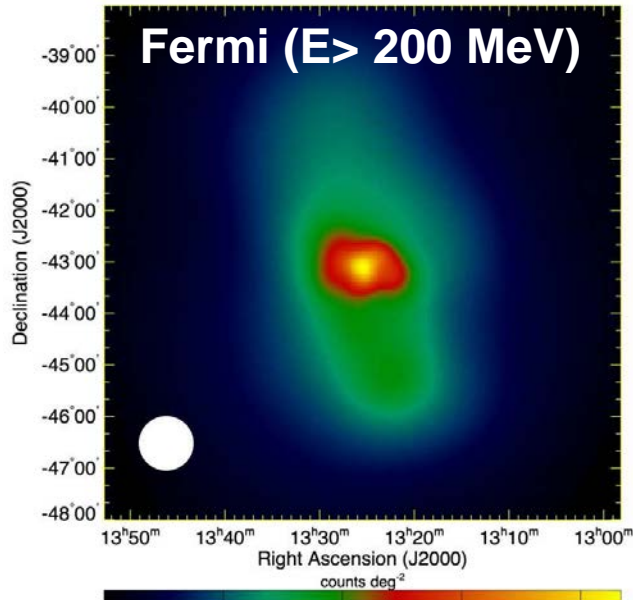


Chandra (color)+1.4GHz (contours)

$$F_{X-ray} \approx E^{-\alpha}$$

The co-spatial location and the similarity in the X-ray and radio spectra indicate a common parent population $\rightarrow N_e \sim E^{-p}$ for the electrons responsible for the jet/lobe emission

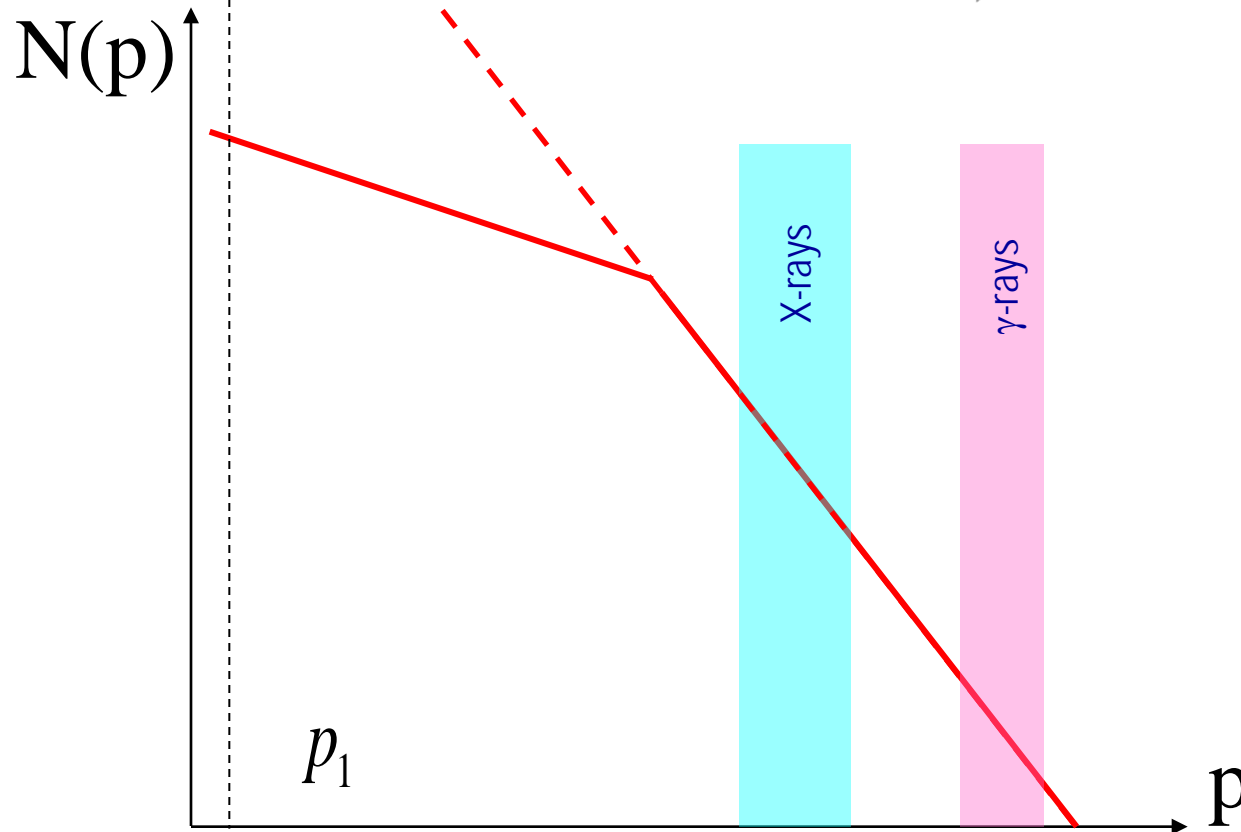
RGs: jet/lobe diffuse emission



Radiogalaxy jet energetics

$$U_e = \int_{p_1}^{\infty} dp N(p) (\sqrt{1 + p^2} - 1) m_e c^2$$

X-ray \rightarrow rough misleading measure of U_e
SZE \rightarrow reliable unbiased measure of U_e



$$h\nu \approx 0.35 \text{ keV} \left(\frac{E_e}{\text{GeV}} \right)^2$$

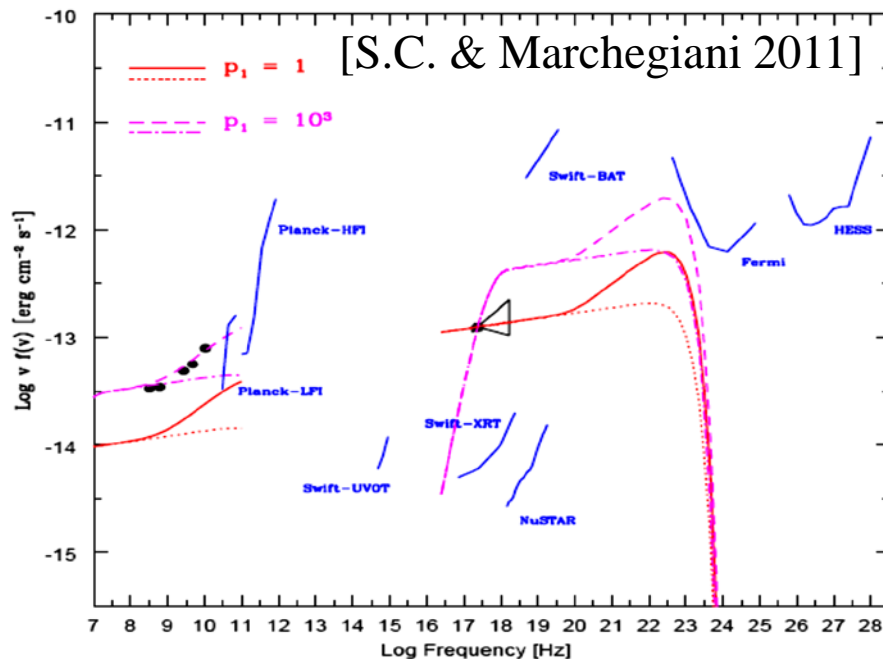
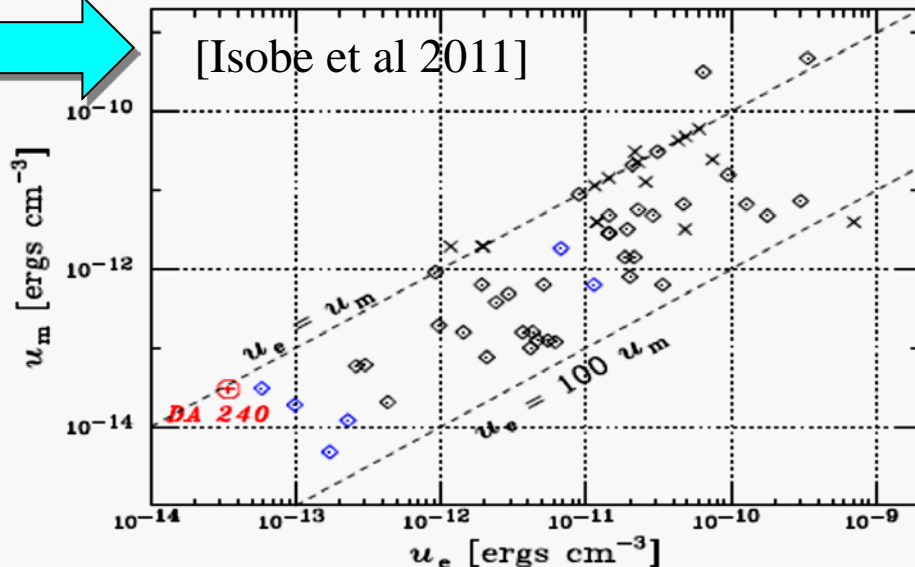
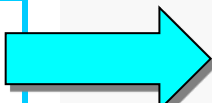
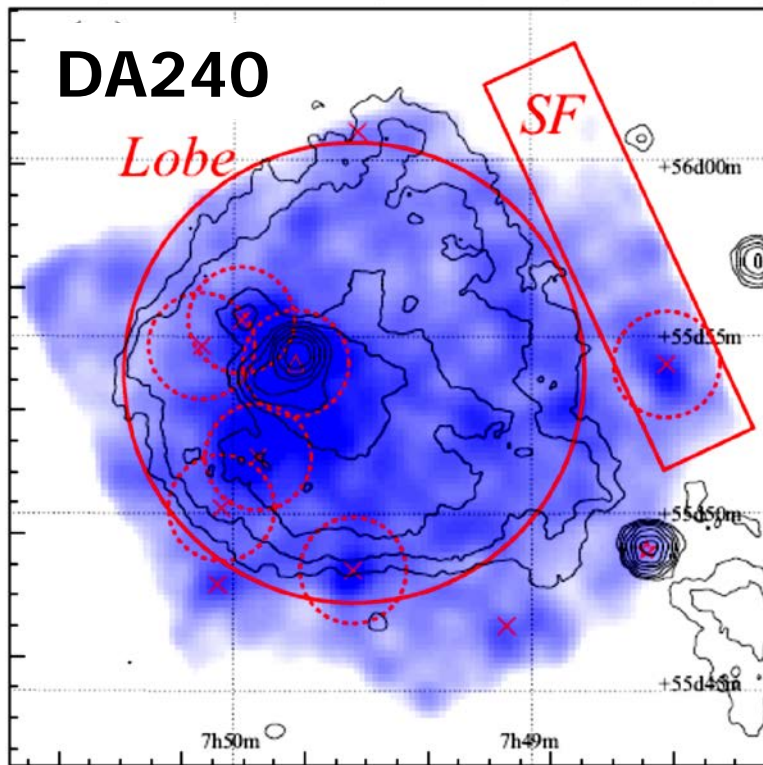
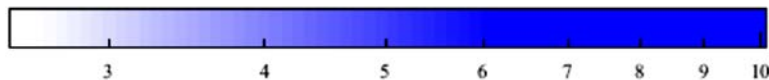
A tale of a giant radiogalaxy: DA 240

Suzaku obs. of U_e and U_B
 [Isobe et al. 2011]

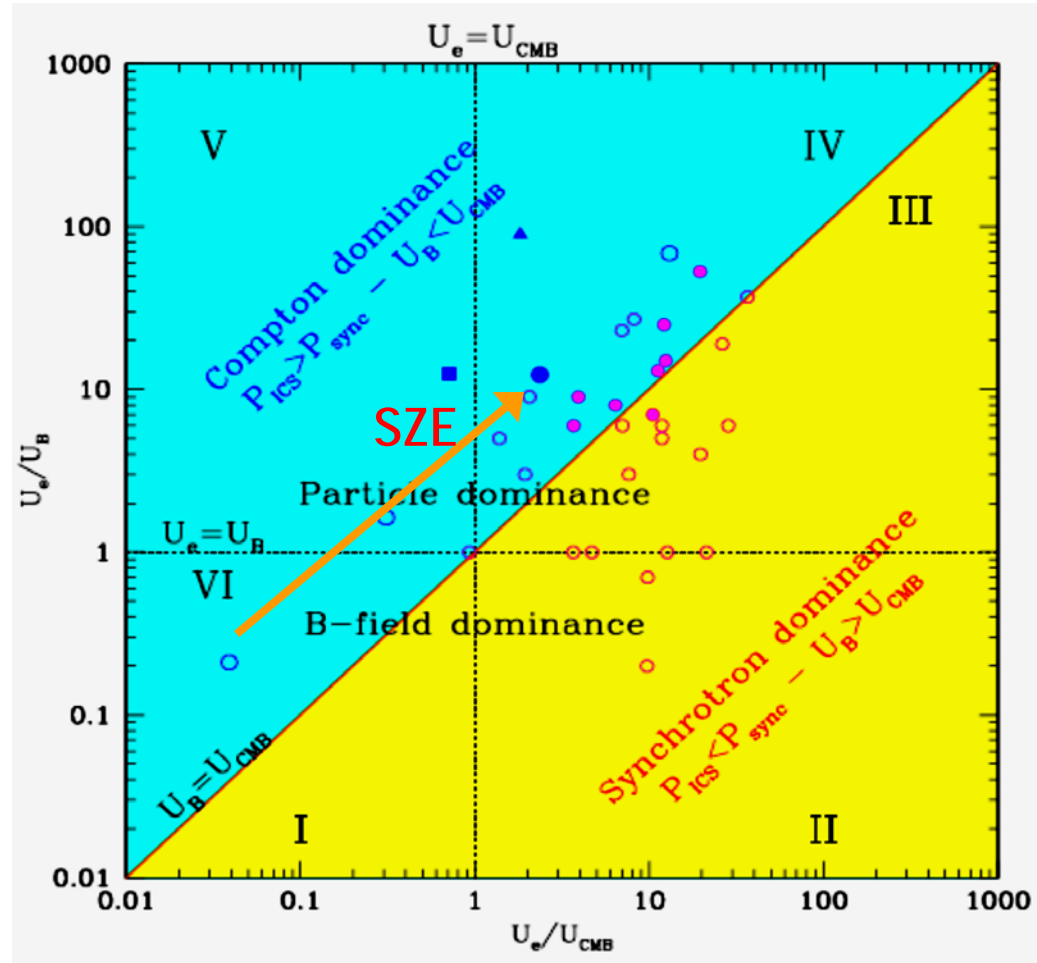
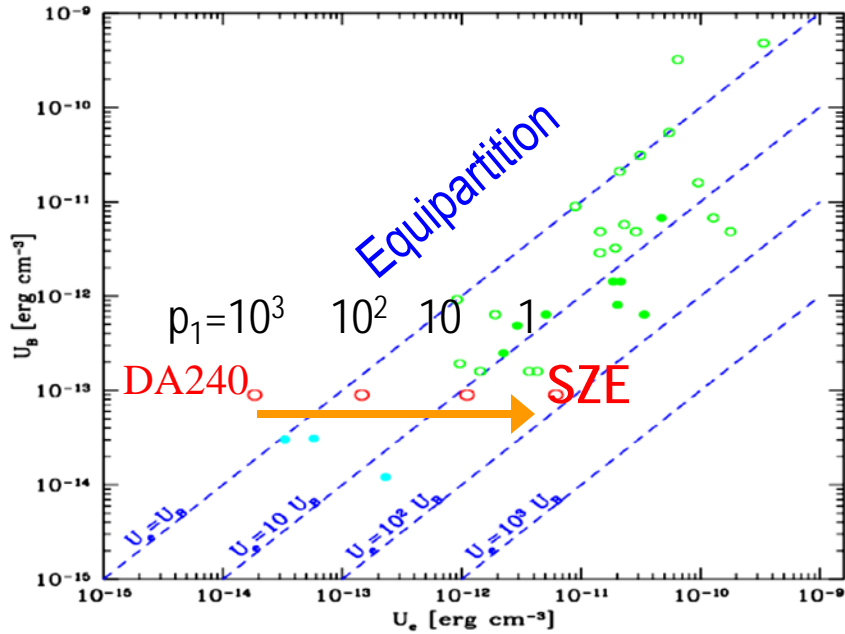
$$\Delta\gamma = 10^3 - 10^5$$

$$\gamma_{\min} = 10^3 \Rightarrow (p_1 = 10^3)$$

$$U_e/U_B \sim 1.1 \quad (\sim \text{Equipartition})$$



SZE: RG lobe energetics revisited

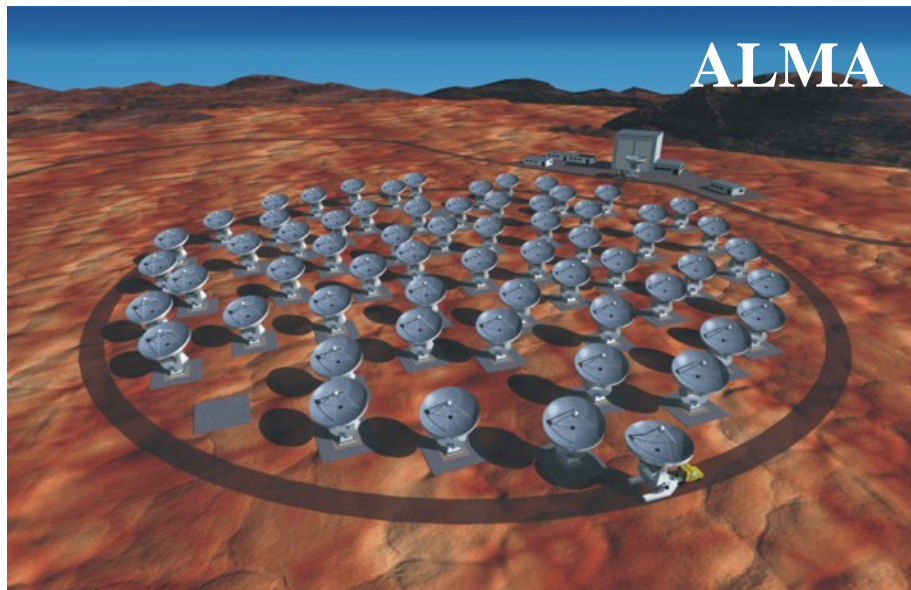


Synergy with ground-based exps.

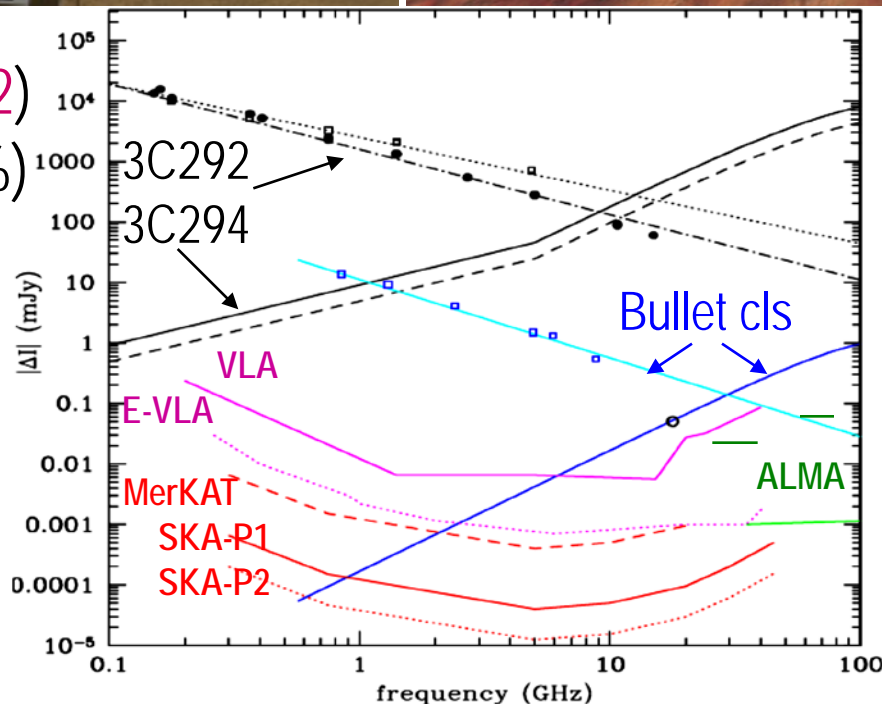
SKA



ALMA



- Approved (25/5/2012)
- SA (70%) & Au (30%)
SA: 0.7-15 GHz
Au: low- ν + high- ν
- Wide FOV
- Multi-beaming
- High survey speed
- Polarization

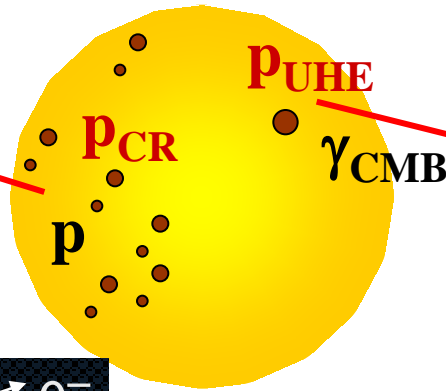


- Operating
- ESO
84 - 950 GHz
3.6" - 0.43"
- Small FOV
- Mosaicing mode
- Polarization

CRs of high-E → *in-situ* production

$$p_{CR} + p \rightarrow X + \pi^{0,\pm}$$

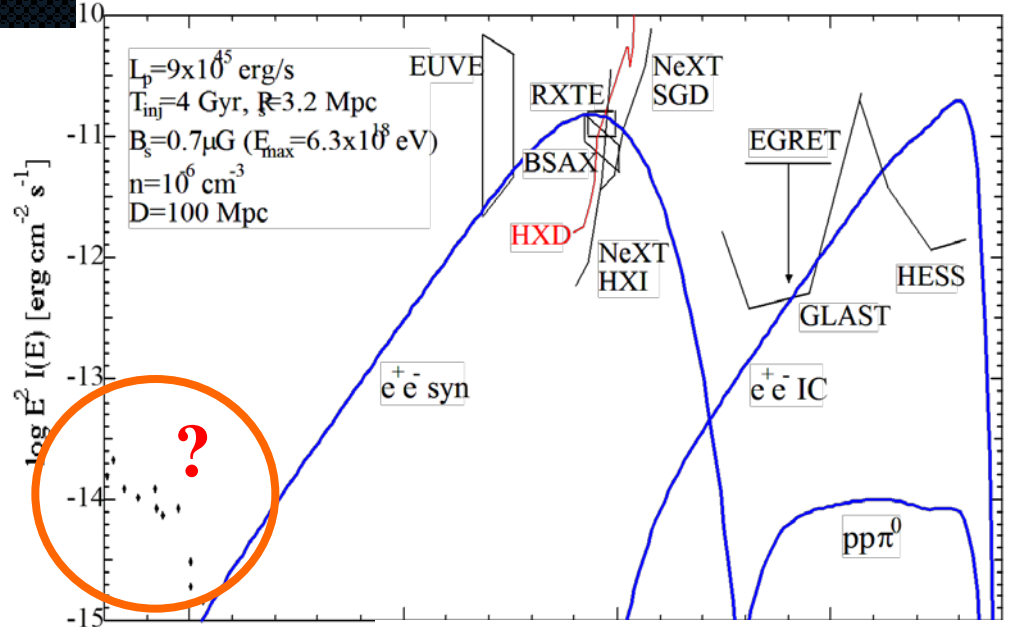
$p_{CR} - p$



$p_{UHE} - \gamma_{CMB}$

High-E jet source

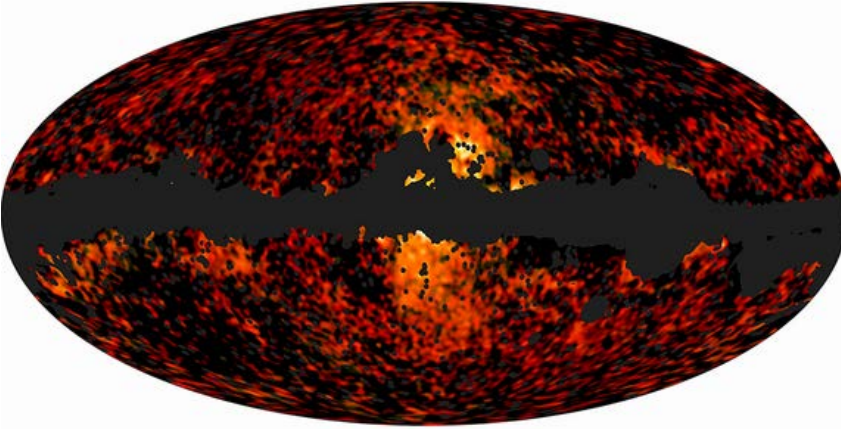
$$p_{UHE} + \gamma \rightarrow X + e^{\pm}$$



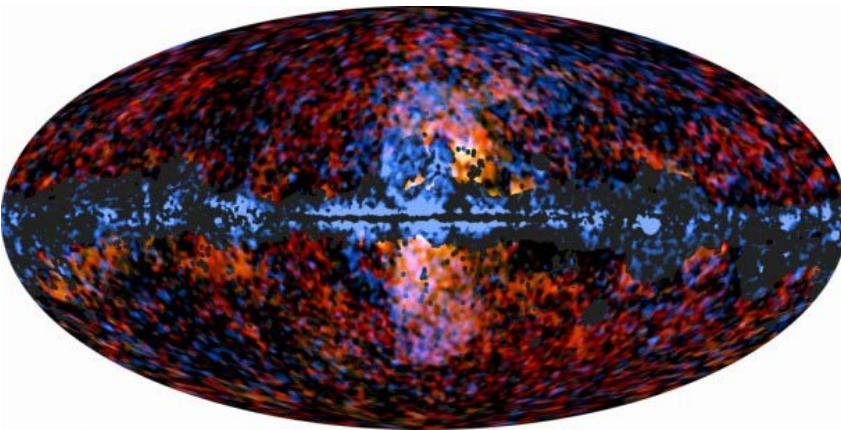
[Timokhin et al. 2002; Inoue et al. 2005]

Galaxy cores

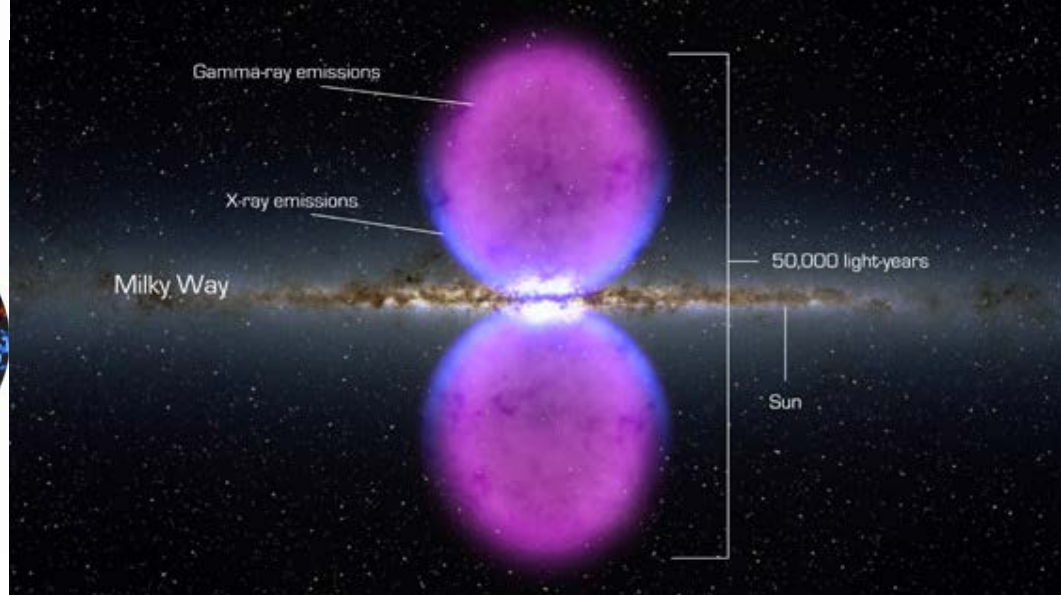
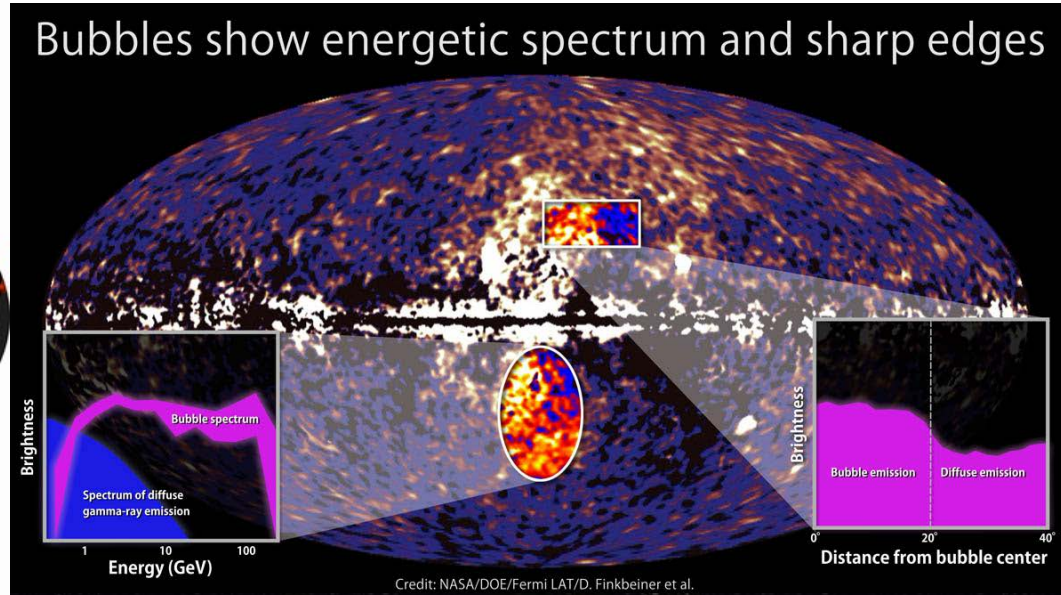
Galactic bubbles: Planck



Galactic bubbles:
Planck (red) + Fermi (violet)



Bubbles show energetic spectrum and sharp edges



Further readings

Colafrancesco: 2010MmSAI..81..104C
: 2008ChJAS...8...61C
: 2008MmSAI..79..213C
: 2010AIPC.1206....5C

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Rybicki and Lightman (1979): Radiative Processes in Astrophysics

Longair (1993): High Energy Astrophysics

Crocker, R.M.: arXiv:1112.6247, arXiv:1112.6249

Bergstrom, L.: arXiv:1202.1170

Fabian, A.C.: arXiv:1204.4114

