New insight on the large separation: observational approach to the asymptotic value

B. Mosser

and

E. Michel, K. Belkacem, M. J. Goupil,
A. Baglin, C. Barban, J. Provost, R. Samadi, M. Auvergne, and C. Catala

Laboratoire d'Études Spatiales et d'Instrumentation en Astrophysique

Forewords

✓ I will present what I understood from Benoit's work

Forewords

- ✓ I will present what I understood from Benoit's work
- ✓ This is not necessarily the same understanding as for Benoit!

Forewords

- ✓ I will present what I understood from Benoit's work
- ✓ This is not necessarily the same understanding as for Benoit!
- ✓ So for questions

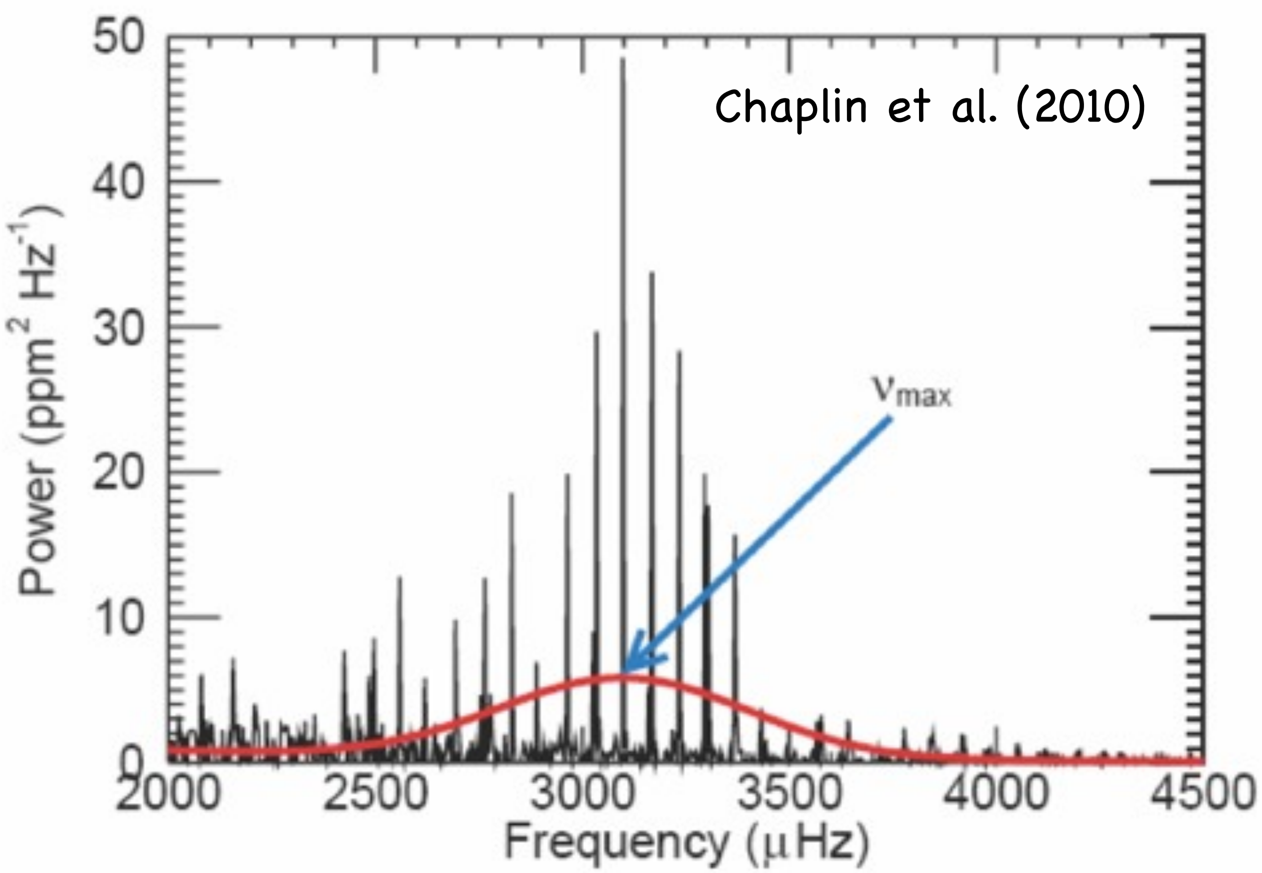
Benoit Mosser <benoit.mosser@obspm.fr>

Context: ensemble asteroseismology

With the CoRoT and Kepler space-borne missions:

✓ High-quality, long-term observations → precise mode parameters

✓ A large number of stars showing solar-like oscillations discovered

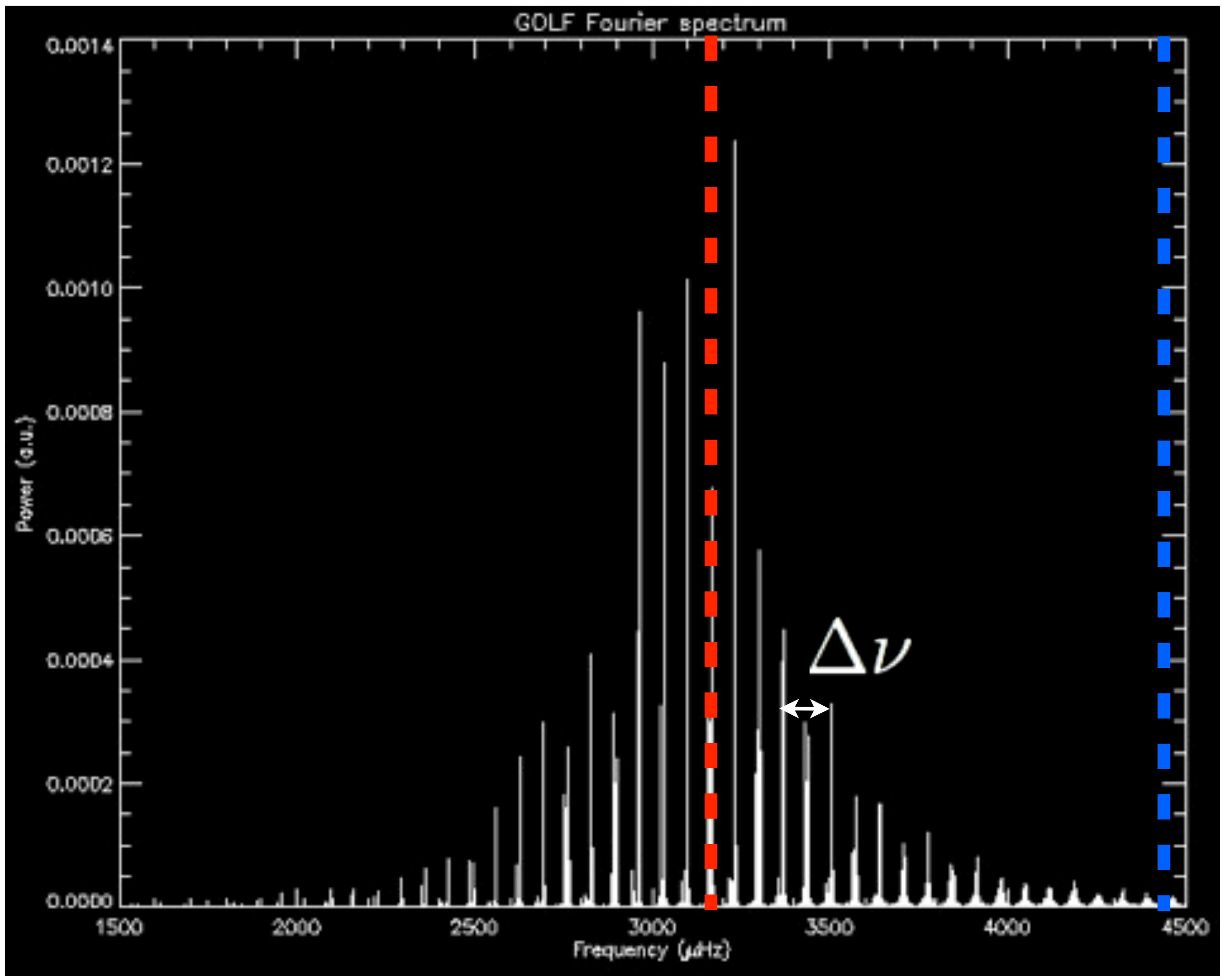


→ Individual mode fitting: but time and labor consuming for a large sample

→ Automatic and large-scale extraction of global seismic indices ($\Delta\nu$, ν_{max})

Context: ensemble asteroseismology

Seismic global parameters:



$\nu_{\max}$ ν_c

$\nu_{\max}$ is the frequency of the maximum height in the power spectrum

ν_c is the cut-off frequency

$\Delta\nu$ large separation

Context: ensemble asteroseismology

✓ large separation versus mean density

$$\Delta\nu \propto \langle\rho\rangle^{1/2} \propto \left(\frac{M}{R^3}\right)^{1/2}$$

✓ frequency of the maximum height versus cut-off frequency

$$\nu_{\max} \propto \nu_{\mathrm{c}} \propto \frac{c_s}{2H_{\mathrm{p}}} \propto \frac{g}{\sqrt{T_{\mathrm{eff}}}} \propto \frac{M}{R^2\sqrt{T_{\mathrm{eff}}}}$$

Context: ensemble asteroseismology

✓ large separation versus mean density

$$\Delta\nu \propto \langle\rho\rangle^{1/2} \propto \left(\frac{M}{R^3}\right)^{1/2}$$

✓ frequency of the maximum height versus cut-off frequency

$$\nu_{\max} \propto \nu_c \propto \frac{c_s}{2H_p} \propto \frac{g}{\sqrt{T_{\text{eff}}}} \propto \frac{M}{R^2\sqrt{T_{\text{eff}}}}$$

$$\rightarrow \quad R \propto \nu_{\max}\,\Delta\nu^{-2}\,T_{\text{eff}}^{1/2}$$

$$M \propto \nu_{\max}^3\,\Delta\nu^{-4}\,T_{\text{eff}}^{3/2}$$

For a given effective temperature one can deduce an estimate of mass and radius

R and M (log g) are often named the seismic mass and radius (seismic gravity)

Context: ensemble asteroseismology

✓ large separation versus mean density

$$\Delta\nu \propto \langle\rho\rangle^{1/2} \propto \left(\frac{M}{R^3}\right)^{1/2}$$

✓ frequency of the maximum height versus cut-off frequency

$$\nu_{\max} \propto \nu_c \propto \frac{c_s}{2H_p} \propto \frac{g}{\sqrt{T_{\text{eff}}}} \propto \frac{M}{R^2\sqrt{T_{\text{eff}}}}$$

$$R \propto \nu_{\max}\,\Delta\nu^{-2}\,T_{\text{eff}}^{1/2}$$
$$M \propto \nu_{\max}^3\,\Delta\nu^{-4}\,T_{\text{eff}}^{3/2}$$

For a given effective temperature one can deduce an estimate of mass and radius

R and M (log g) are often named the seismic mass and radius (seismic gravity)

A wealth of information are thereafter derived;

✓ "Model-independent" determination of stellar parameters (e.g., Mosser et al. 2010, Chaplin et al. 2011, ...)"

✓ Constraints on stellar evolution: e.g. evidence of mass loss (e.g. Mosser et al. 2012)

✓ Improved determination of log g and T_{eff} (e.g., Bruntt et al. (2010), Batalha et al. 2011, Creevey et al. 2012, Morel & Miglio (2012))

✓ and much more...

Problematic

But before using the scaling relation ($\Delta\nu$-$<\rho>$), it can be useful to understand it

Problematic

But before using the scaling relation ($\Delta\nu$-$\langle\rho\rangle$), it can be useful to understand it

→ The underlying physical hypothesis of the $\Delta\nu$-$\langle\rho\rangle$ relation: homology

Problematic

But before using the scaling relation (Δν-<ρ>), it can be useful to understand it

→ The underlying physical hypothesis of the Δν-<ρ> relation: homology

Two stars are homologous if $\frac{r}{R} = \frac{r'}{R'}$ → $\frac{m}{M} = \frac{m'}{M'}$

For two homologous stars
(e.g., Cox & Giuli 1968; Kippenhahn & Weigert 1990) $\frac{c_s}{c'_s} = \left(\frac{M}{M'}\right)^{1/2} \left(\frac{R}{R'}\right)^{-1/2}$

Therefore: $\mathcal{R} = \frac{\Delta\nu}{\Delta\nu'} = \left[\int_0^{R'} \frac{\mathrm{d}r'}{c'_s}\right] \left[\int_0^{R} \frac{\mathrm{d}r}{c_s}\right]^{-1} = \left(\frac{R}{R'}\right)^{-3/2} \left(\frac{M}{M'}\right)^{1/2} = \left(\frac{\langle\rho\rangle}{\langle\rho'\rangle}\right)^{1/2}$

→ For homologous stars the Δν-<ρ> relation is exact

Problematic

A comment on the $\Delta\nu$-$\langle\rho\rangle$ relation: this is not a novelty in asteroseismology!

It is very similar to the case of classical pulsators:

$$\Pi_0 \propto \int_0^R \frac{\mathrm{d}r}{c_s} \quad \Longrightarrow \quad \Pi_0 \propto \langle\rho\rangle^{-1/2}$$

where Π_0 is the period of the fundamental radial mode.

- It is the basis of the period-luminosity relation for Cepheids

- This relation is known for a long time (Eddington 1917) and has been extensively investigated (e.g., Ledoux & Walraven 1958, Cox 1980, etc...)

The $\Delta\nu$-$\langle\rho\rangle$ is the equivalent of the Π_0 - $\langle\rho\rangle$ for solar-like pulsators

Problematic

- It is therefore crucial to get a good proxy of the «asymptotic» value of the large separation

$$\Delta\nu = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

Problematic

- It is therefore crucial to get a good proxy of the «asymptotic» value of the large separation

$$\Delta\nu = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

- Is the measured large separation a good proxy of the theoretical expected value?

- Are we able to approach the asymptotic large separation?

Problematic

- It is therefore crucial to get a good proxy of the «asymptotic» value of the large separation

$$\Delta\nu = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

- Is the measured large separation a good proxy of the theoretical expected value?

- Are we able to approach the asymptotic large separation?

- Some sources of departure from the asymptotic expansion

  ✓ surface effects

  ✓ glitches (discontinuities in the sound speed profile)

  ✓ departure from the asymptotic domain of validity

Problematic

- It is therefore crucial to get a good proxy of the «asymptotic» value of the large separation

$$\Delta\nu = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

- Is the measured large separation a good proxy of the theoretical expected value?

- Are we able to approach the asymptotic large separation?

- Some sources of departure from the asymptotic expansion

    ✓ surface effects

    ✓ glitches (discontinuities in the sound speed profile)

    ✓ departure from the asymptotic domain of validity

Asymptotic relation

- The asymptotic relation for pressure modes, at first order, reads (e.g. Tassoul 1980)

$$\nu_{n,\ell} = \Big(\underbrace{n}_{\text{radial order}} + \frac{\underbrace{\ell}_{\text{angular degree}}}{2} + \underbrace{\epsilon}_{\text{phase shift = 1/4 in Tassoul (1980)}}\Big)\,\Delta\nu \qquad \text{and} \qquad \Delta\nu = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

Asymptotic relation

- The asymptotic relation for pressure modes, at first order, reads (e.g. Tassoul 1980)

$$\nu_{n,\ell} = \left(n + \frac{\ell}{2} + \epsilon\right) \Delta\nu \qquad \text{and} \qquad \Delta\nu = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

radial order

angular degree

phase shift = 1/4 in
Tassoul (1980)

- The main condition of validity for this equation is

$$n \gg 1$$

Asymptotic relation

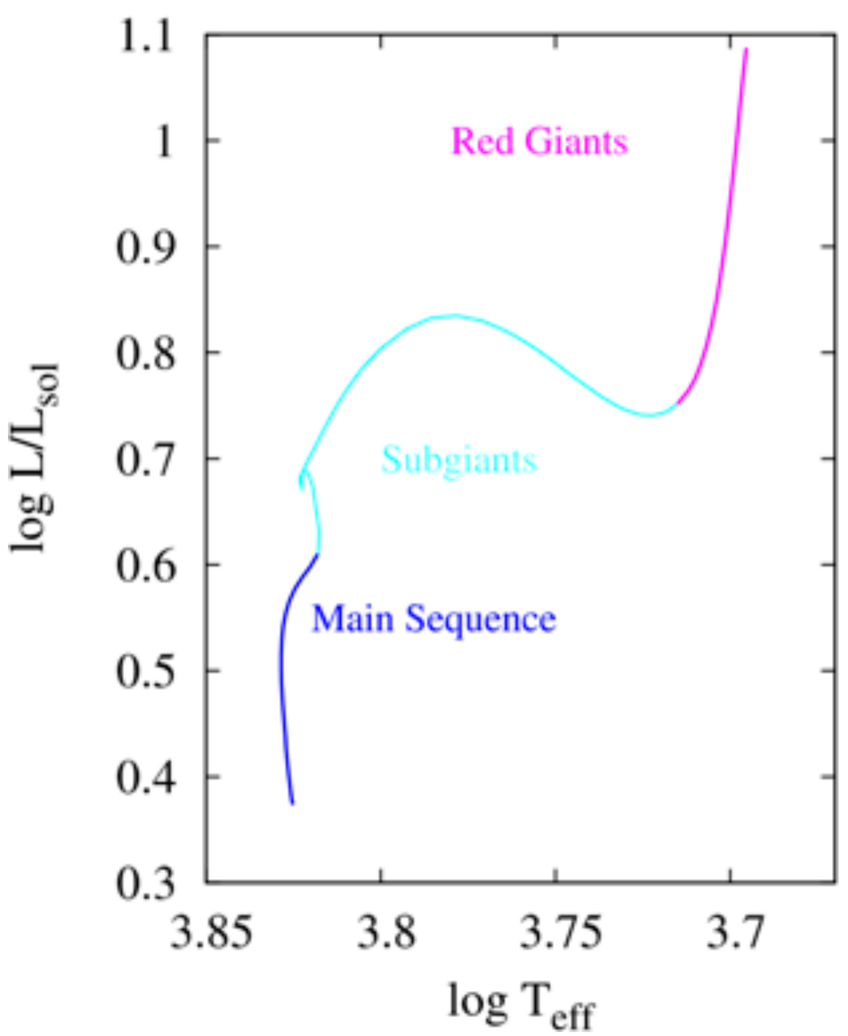
- The asymptotic relation for pressure modes, at first order, reads (e.g. Tassoul 1980)

$$\nu_{n,\ell} = \left(n + \frac{\ell}{2} + \epsilon \right) \Delta\nu \qquad \text{and} \qquad \Delta\nu = \left(2 \int_0^R \frac{\mathrm{d}r}{c_s} \right)^{-1}$$

radial order

angular degree

phase shift = 1/4 in Tassoul (1980)



- The main condition of validity for this equation is

$$n \gg 1$$

main-sequence stars $n_{max} \sim 25$ (i.e. n at ν_{max})

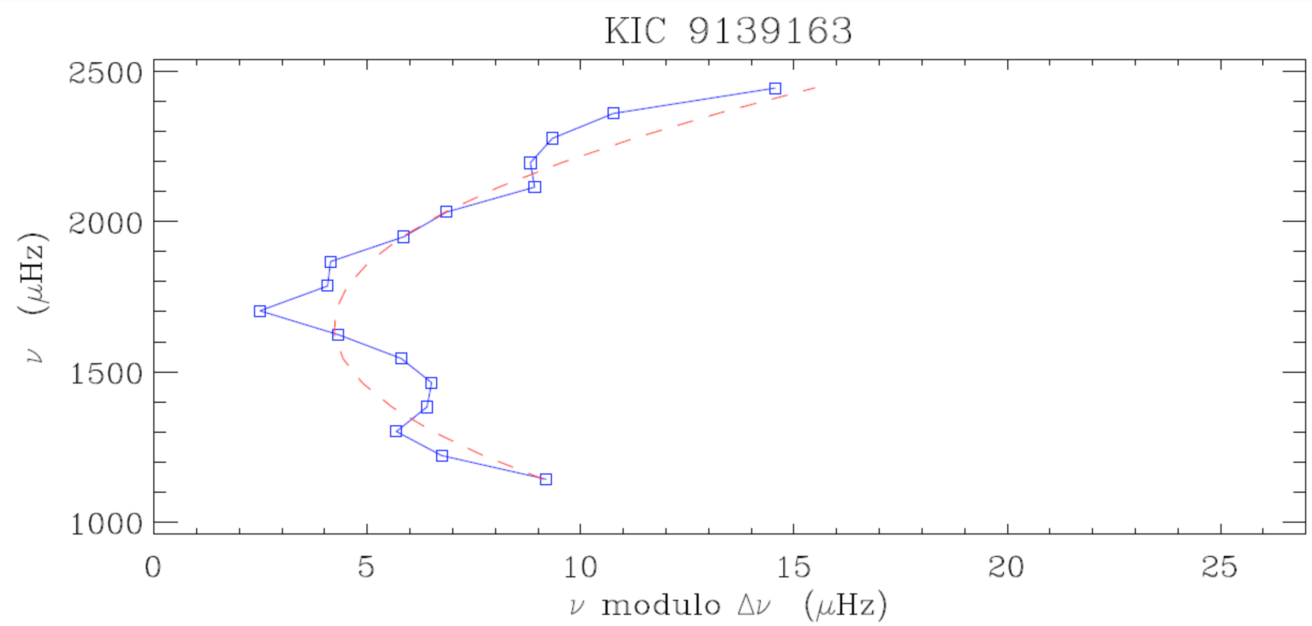
Botton of RG $n_{max} \sim 15$ (i.e. n at ν_{max})

Tip of RG $n_{max} \sim 1$ (i.e. n at ν_{max})

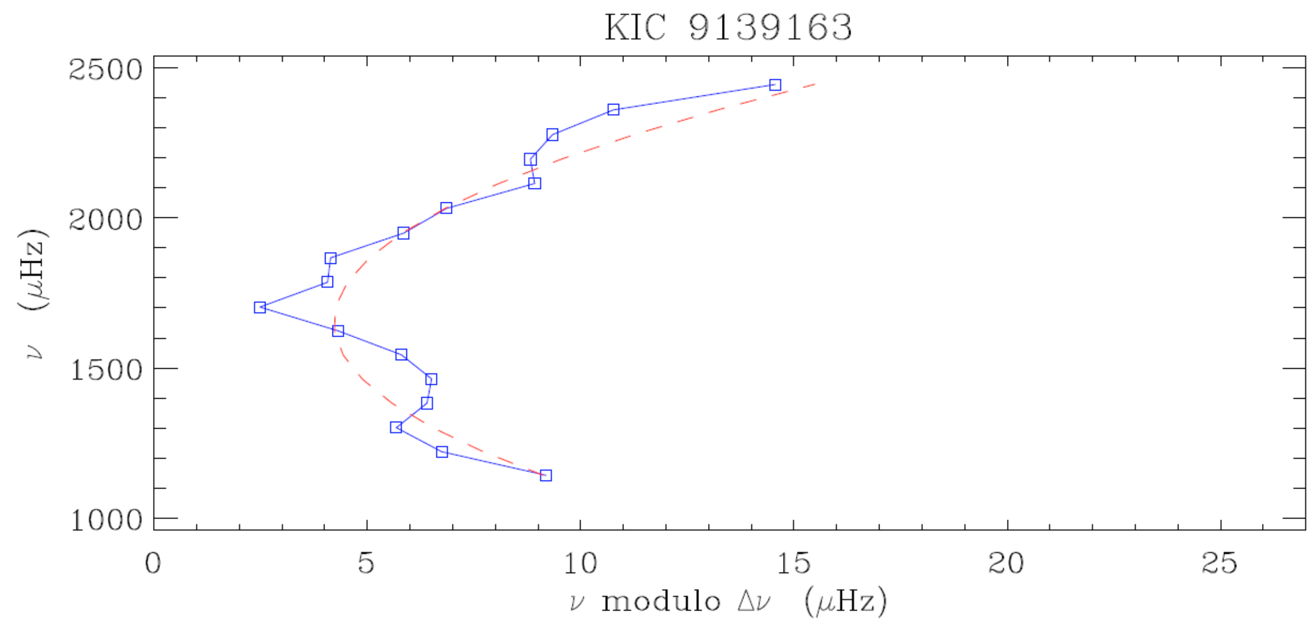
→ it is likely that for most of the observed stars, the (first-order) asymptotic relation hardly applies

The modelling

- In an Echelle diagram, the departure from the asymptotic relation can be measured by the curvature



The modelling



- In an Echelle diagram, the departure from the asymptotic relation can be measured by the curvature

- Mosser et al. (2010) and Mosser et al. (2013) proposed an empirical way to take into account the curvature

$$\nu_{n,\ell=0} = \left(n + \epsilon_{\rm obs} + \frac{\alpha_{\rm obs}}{2}\left[n - n_{\rm max}\right]^2\right) \Delta\nu_{\rm obs}$$

the third term mimics a linear gradient in large separation

$$\frac{\nu_{n+1,\ell=0} - \nu_{n-1,\ell=0}}{2} = \left(1 + \alpha_{\rm obs}\left[n - n_{\rm max}\right]\right) \Delta\nu_{\rm obs}$$

The modelling

- Now assuming that the asymptotic relation applies with the second-order correction for radial modes

$$\nu_{n,0} = \left(n + \epsilon_{\rm as} + \underline{\frac{A_{\rm as}}{n}}\right)\Delta\nu_{\rm as}$$

A is a function of the star structure

The modelling

- Now assuming that the asymptotic relation applies with the second-order correction for radial modes

$$\nu_{n,0} = \left(n + \epsilon_{\mathrm{as}} + \frac{A_{\mathrm{as}}}{n}\right) \Delta\nu_{\mathrm{as}}$$

A is a function of the star structure

- Relating the second-order expansion and the empirical expansion give us

$$\Delta\nu_{\mathrm{as}} = \Delta\nu_{\mathrm{obs}} \left(1 + \frac{n_{\mathrm{max}}\alpha_{\mathrm{obs}}}{2}\right)$$

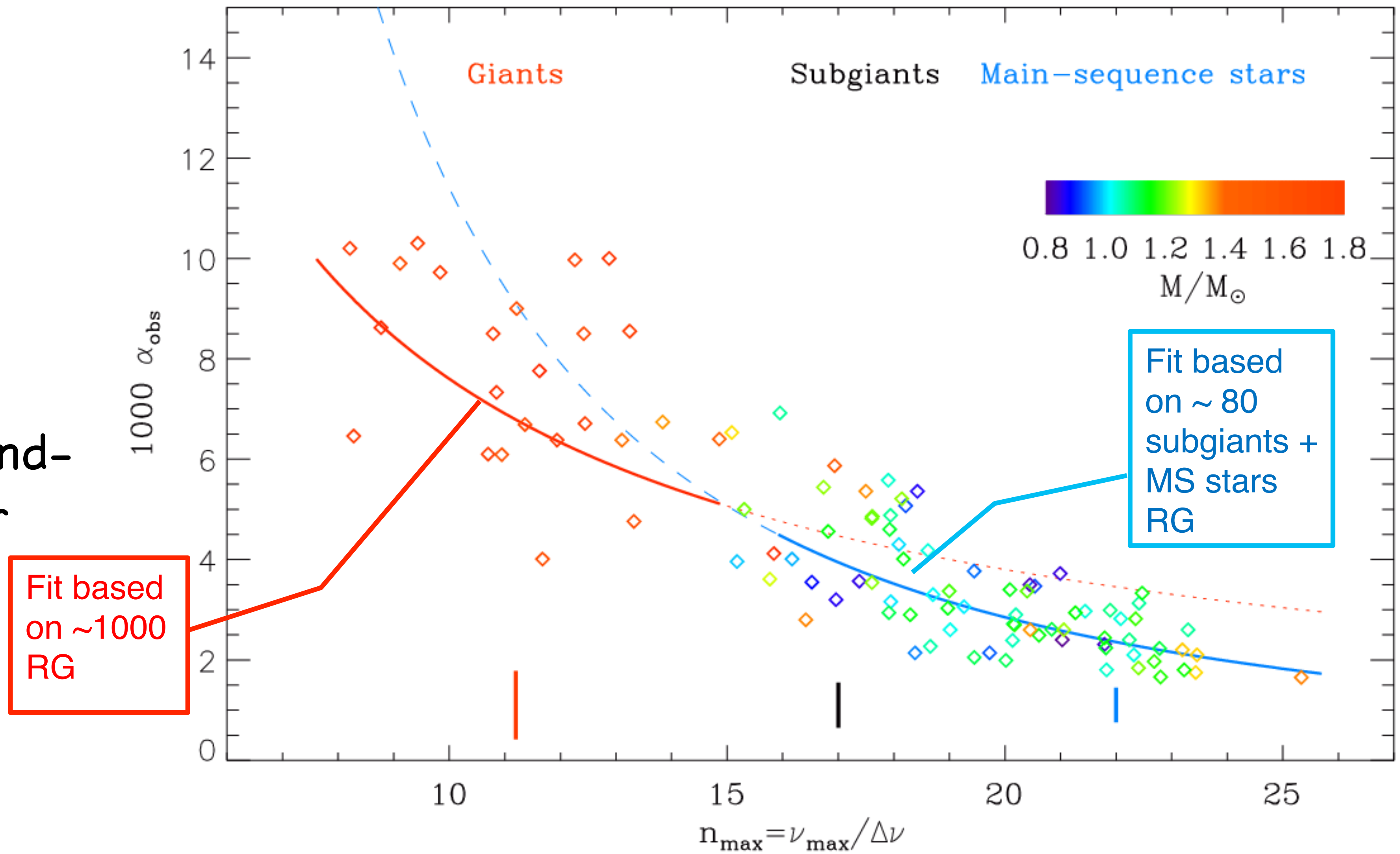
$$\epsilon_{\mathrm{as}} = \frac{\epsilon_{\mathrm{obs}} - n_{\mathrm{max}}^2\alpha_{\mathrm{obs}}}{1 + n_{\mathrm{max}}\frac{\alpha_{\mathrm{obs}}}{2}}$$

$\longrightarrow$ so you can derive a proxy of the asymptotic large separation with observable only

Results

- Observations of subgiants and main-sequence stars observed by CoRoT or by Kepler.

- We also made use of ground-based observations and solar data



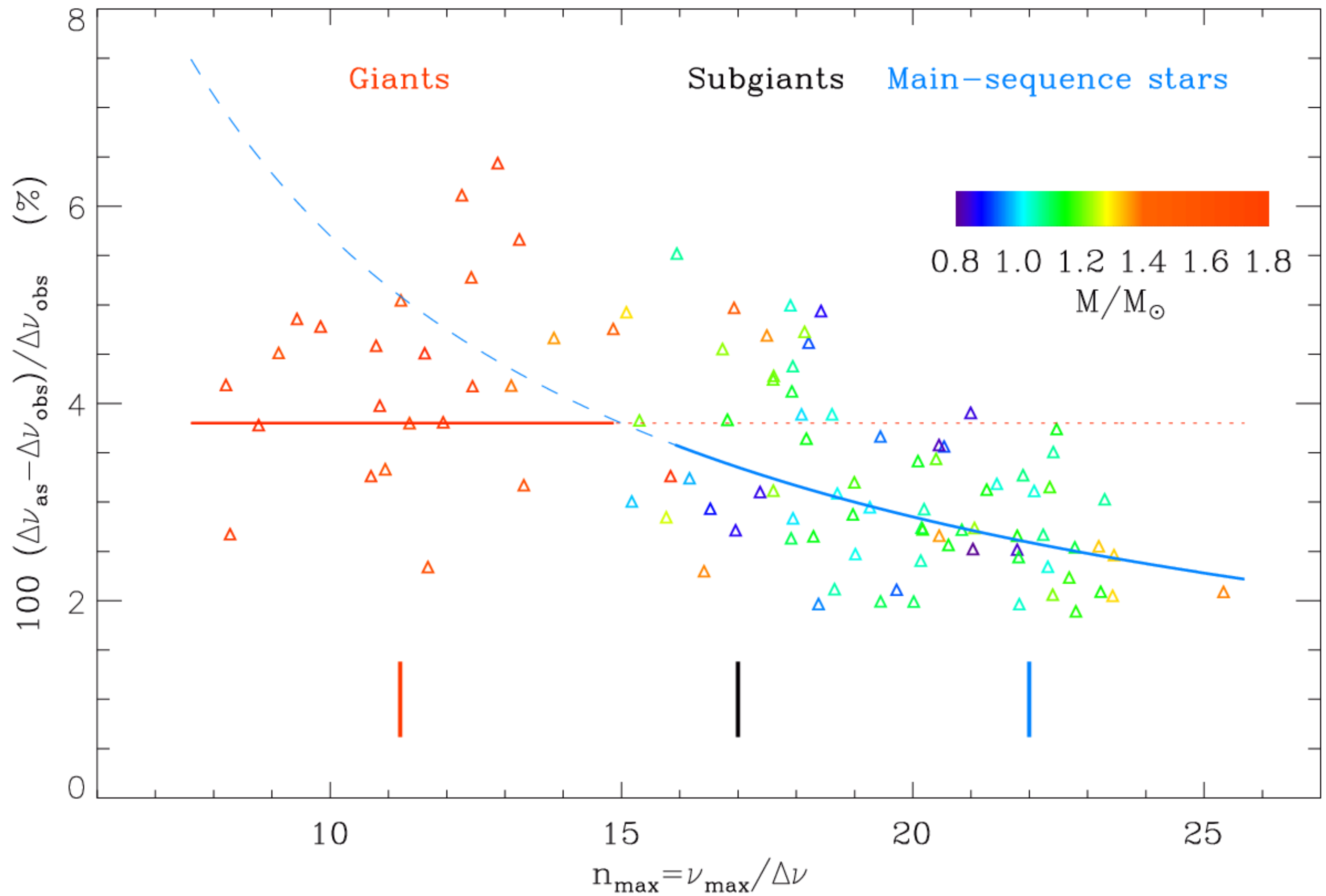
→ as expected the curvature increases as the star evolves on the red-giant branch

→ it seems that low-mass stars have systematically lower α_{obs} than high-mass stars

Results

- differences between the large separation measured at ν_{max} and the derived «asymptotic» large separation

$$\Delta\nu_{as} = \Delta\nu_{obs}\left(1 + \frac{n_{max}\alpha_{obs}}{2}\right)$$



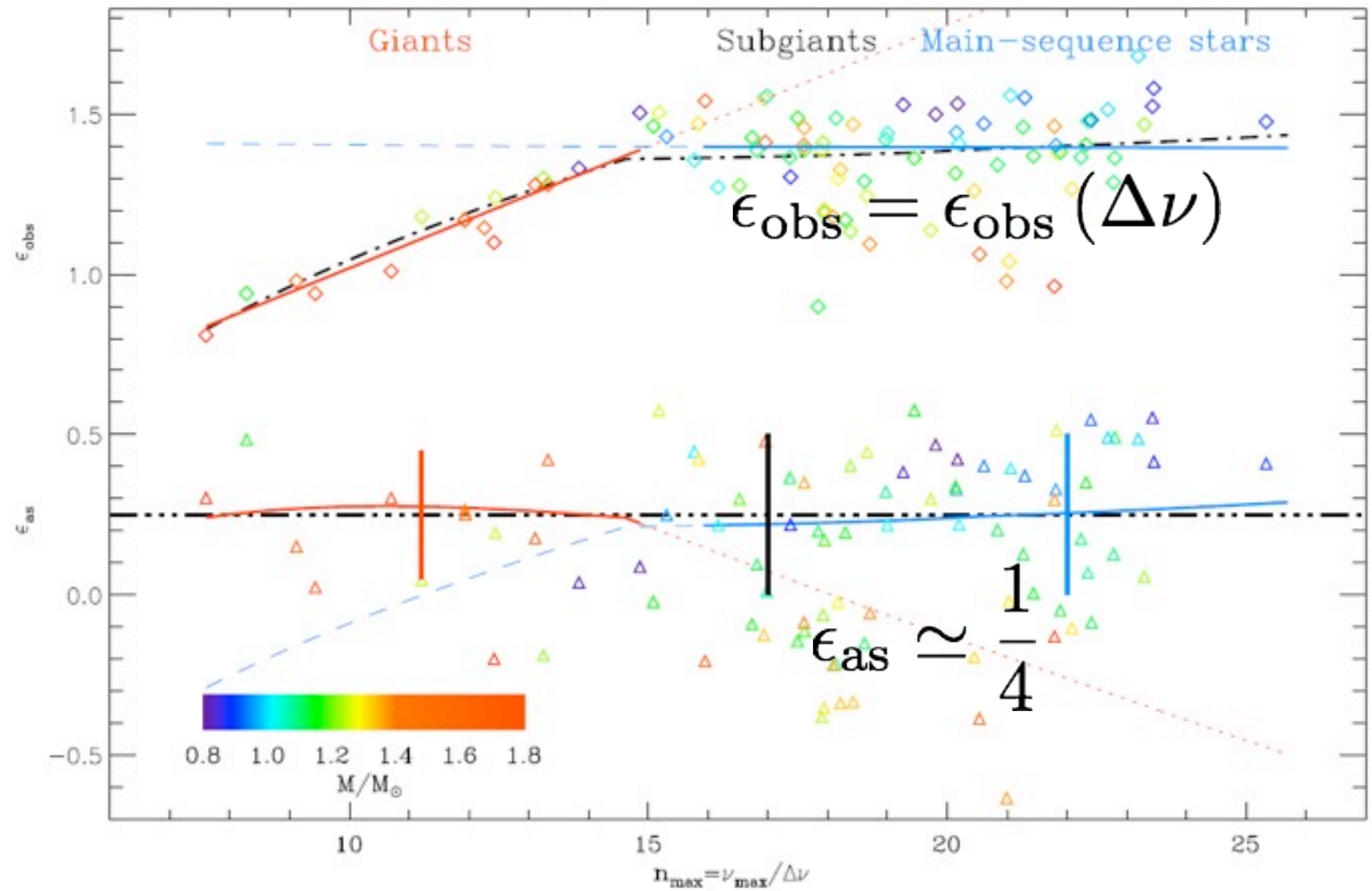
→ the difference between the large separation at ν_{max} and the «asymptotic» large separation is non-negligeable

→ again it is particularly important for red-giant stars

Results

- phase shift at ν_{max} and «asymptotic» phase shift

$$\epsilon_{as} = \frac{\epsilon_{obs} - n_{max}^2 \alpha_{obs}}{1 + n_{max} \frac{\alpha_{obs}}{2}}$$



→ ε_{obs} depends on the large separation what is clearly not expected from the asymptotic expansion

→ after correction, ε_{as} is rougly equals to 1/4 as expected by the asymptotic expansion by Tassoul (1980)

Consequences on seismic stellar parameters?

- Using the scaling relations with $\Delta\nu_{obs}$

$$R \propto \nu_{\max}\, \Delta\nu^{-2}\, T_{\rm eff}^{1/2}$$
$$M \propto \nu_{\max}^{3}\, \Delta\nu^{-4}\, T_{\rm eff}^{3/2}$$

→ biaised estimates of seismic masses and radius (and thus seismic gravity)

→ by using $\Delta\nu_{as}$ can lead to differences up to 6% for the stellar radius and 3% for the mass

Consequences on seismic stellar parameters?

- Using the scaling relations with $\Delta\nu_{obs}$

$$R \propto \nu_{\max} \, \Delta\nu^{-2} \, T_{\text{eff}}^{1/2}$$
$$M \propto \nu_{\max}^3 \, \Delta\nu^{-4} \, T_{\text{eff}}^{3/2}$$

→ biaised estimates of seismic masses and radius (and thus seismic gravity)

→ by using $\Delta\nu_{as}$ can lead to differences up to 6% for the stellar radius and 3% for the mass

- Mosser et al. (2013) then proposed

$$R_{\text{as}} \approx (1 - 2\zeta) R_{\text{obs}}$$
$$M_{\text{as}} \approx (1 - 4\zeta) M_{\text{obs}}$$

with

$$\zeta = \frac{0.57}{n_{\max}} \quad \text{for main sequence stars } (n_{\max} > 15)$$
$$\zeta = 0.038 \quad \text{for giant stars } (n_{\max} < 15)$$

Conclusion

✓ There is quite a leap between the expected and measured large separation

✓ Empirical corrections can be applied to account for the departure from the asymptotic regime

✓ Still work to be perform, for instance:

- including the effects of glitches

- investigate the validity of such an approach near the tip of the RGB that would be soon available through Kepler data