On the relation between observed and predicted global seismic quantities in red-giant stars

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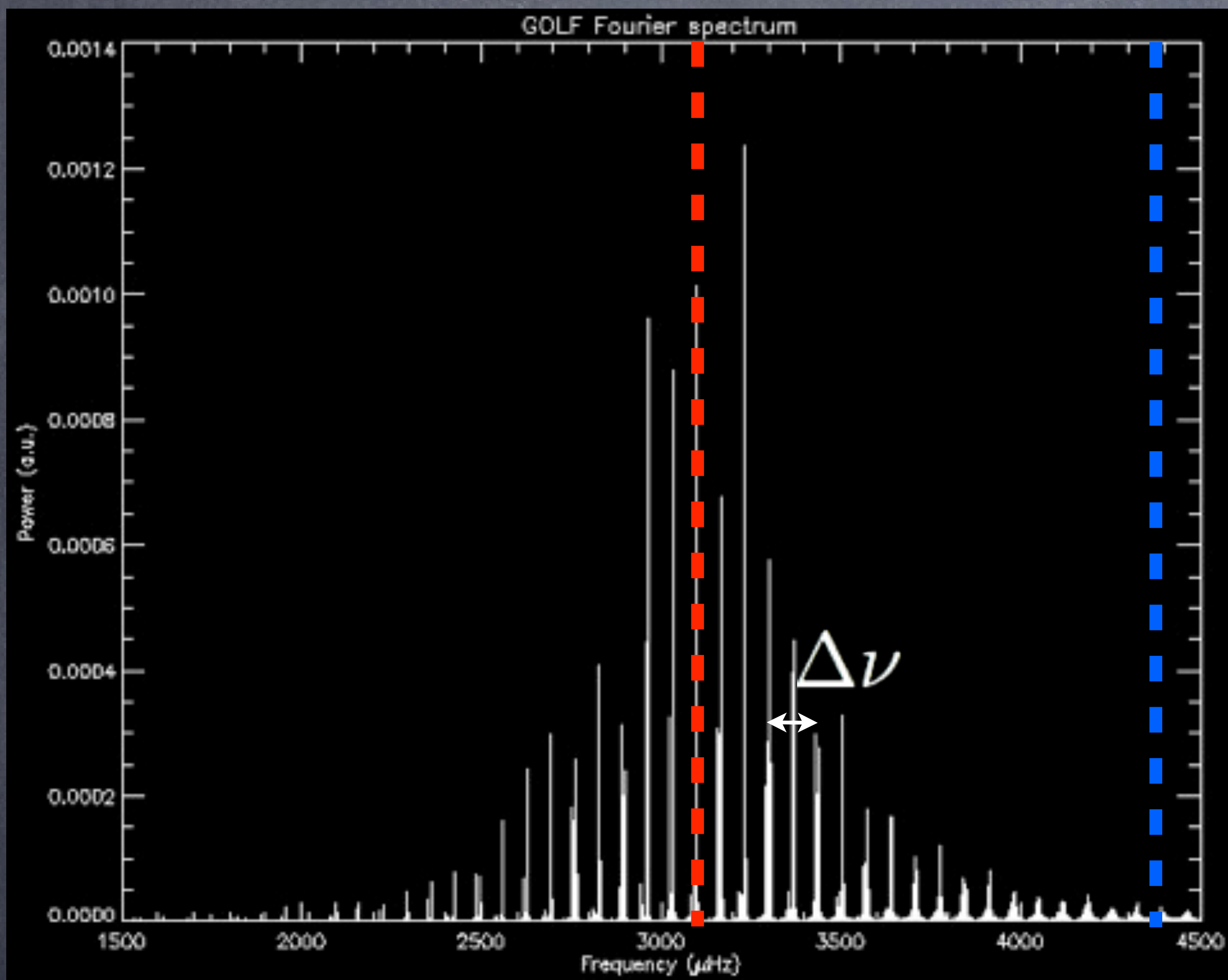
Preliminary definitions & problematic

- Global seismic parameters



$\nu_{\rm max}$ is the frequency of the maximum height in the power spectrum

$\nu_{\rm c}$ is the cut-off frequency

$\Delta\nu$ large separation

For gravity modes, we also measure period spacing $\Delta\Pi$

One can also use the maximum amplitude (i.e. at the ν_{max}) $A_{\rm max}$

Preliminary definitions & problematic

- Global seismic parameters

Theory (scaling relations): asymptotic developments, etc... →

$$\Delta\nu \propto \langle\rho\rangle^{1/2}$$

$$\nu_{\mathrm{max}} \propto \frac{g}{\sqrt{T_{\mathrm{eff}}}}$$

$$\Delta\Pi \propto \langle\rho\rangle_{\mathrm{core}}^{-1/2}$$

$$A_{\mathrm{max}} \propto \left(\frac{L}{M}\right)^{\alpha}$$

From all those information you can:

✓ obtain «first-order» contraints for modelling

✓ perform ensemble asteroseismology through the use of scaling relations

Preliminary definitions & problematic

- Global seismic parameters

Theory (scaling relations): asymptotic developments, etc... →

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- Of course it is not so simple...

✓ In fact, there is always quite a leap between observables and theoretical predictions

✓ This is particular crucial for red-giants

Outline

- Period spacing of red-giant stars

  ✓ Asymptotic relation for deriving the period spacings

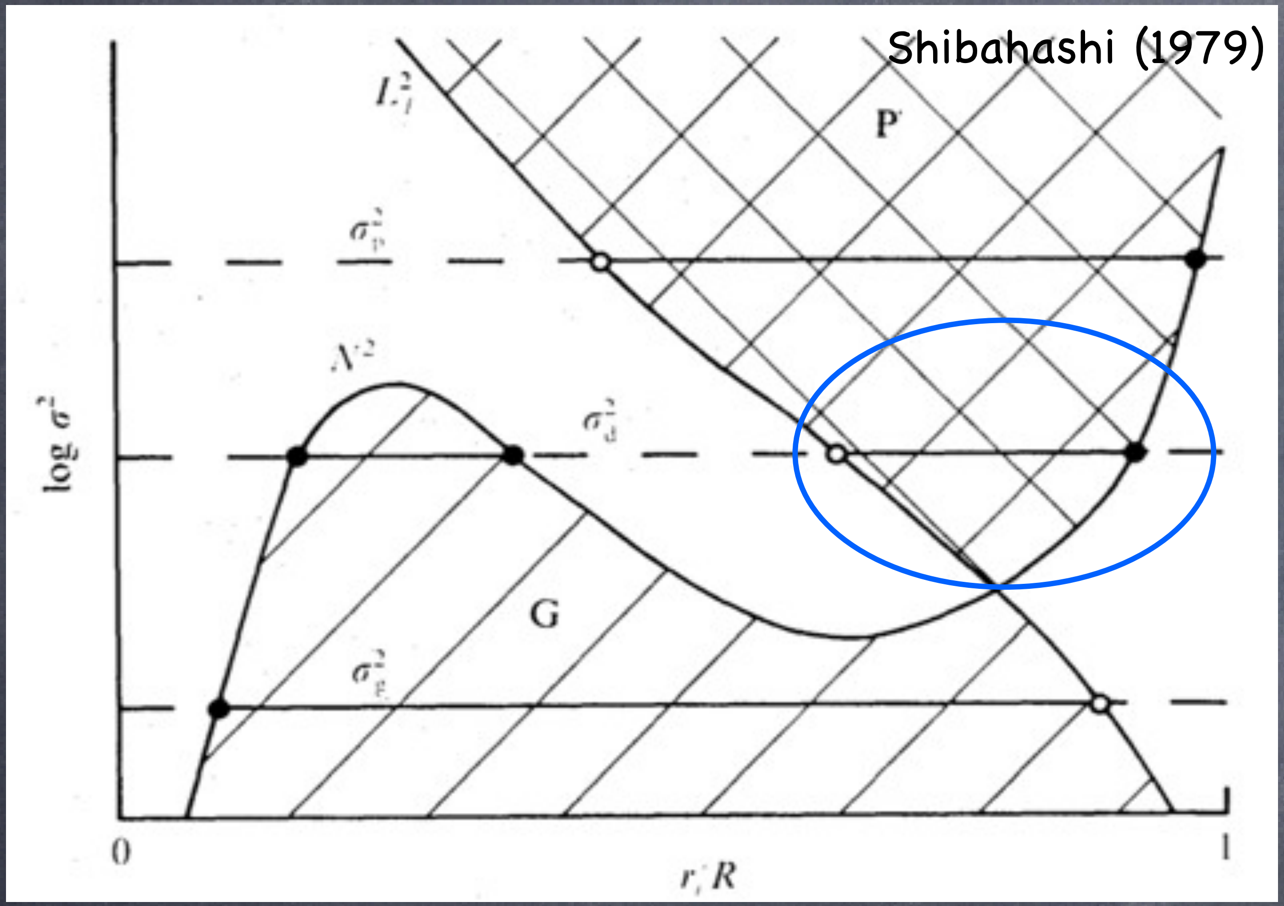
- Amplitudes of red-giant stars

  ✓ Evidence of non-adiabatic effects

  ✓ The need for both radial velocities and photometric observations

✓ Why red-giant stars are so special? Mixed modes

The dispersion relation reads $k_r^2 = \frac{1}{\omega^2 c_s^2}\left(\omega^2 - N^2\right)\left(\omega^2 - S_\ell^2\right)$



There are two propagative cavities ($k_r^2 > 0$):

- upper cavity: acoustic modes can exist

$$\omega >> S_\ell \quad and \quad \omega >> N$$

✓ Why red-giant stars are so special? Mixed modes

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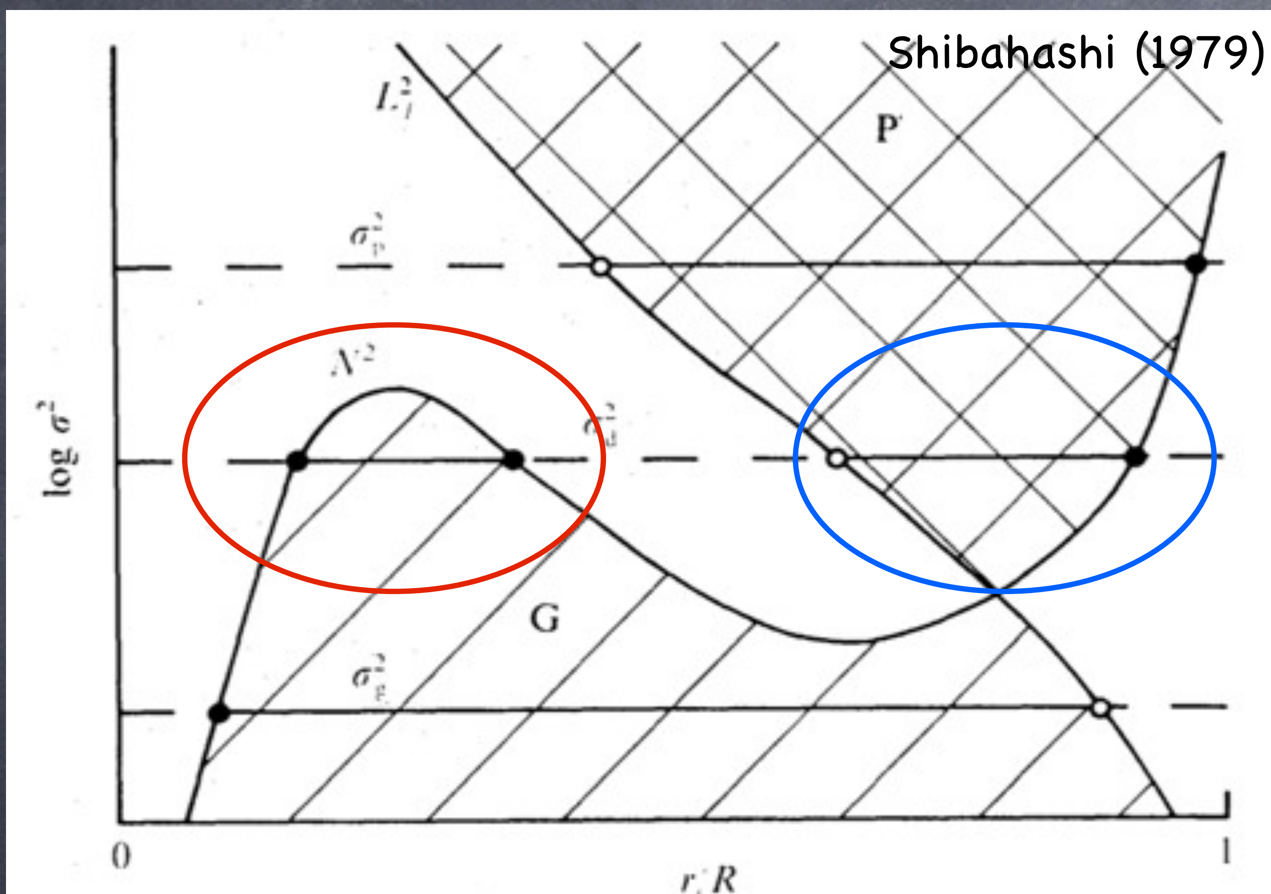

There are two propagative cavities ($k_r^2 > 0$):

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- inner cavity: gravity modes

$$\omega << S_\ell \quad and \quad \omega << N$$

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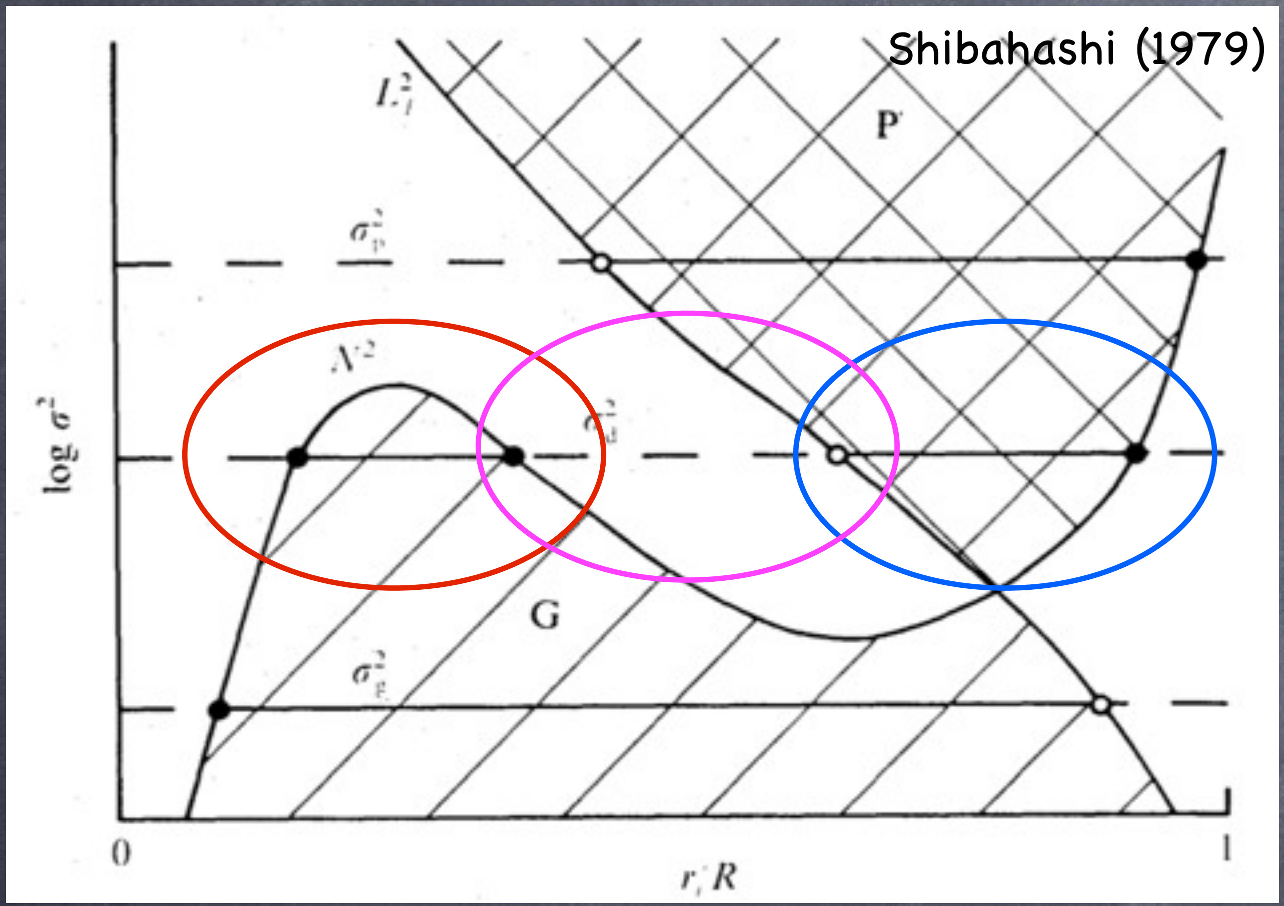

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- upper cavity: acoustic modes can exist

$$\omega >> S_\ell \quad and \quad \omega >> N$$

- inner cavity: gravity modes

$$\omega << S_\ell \quad and \quad \omega << N$$

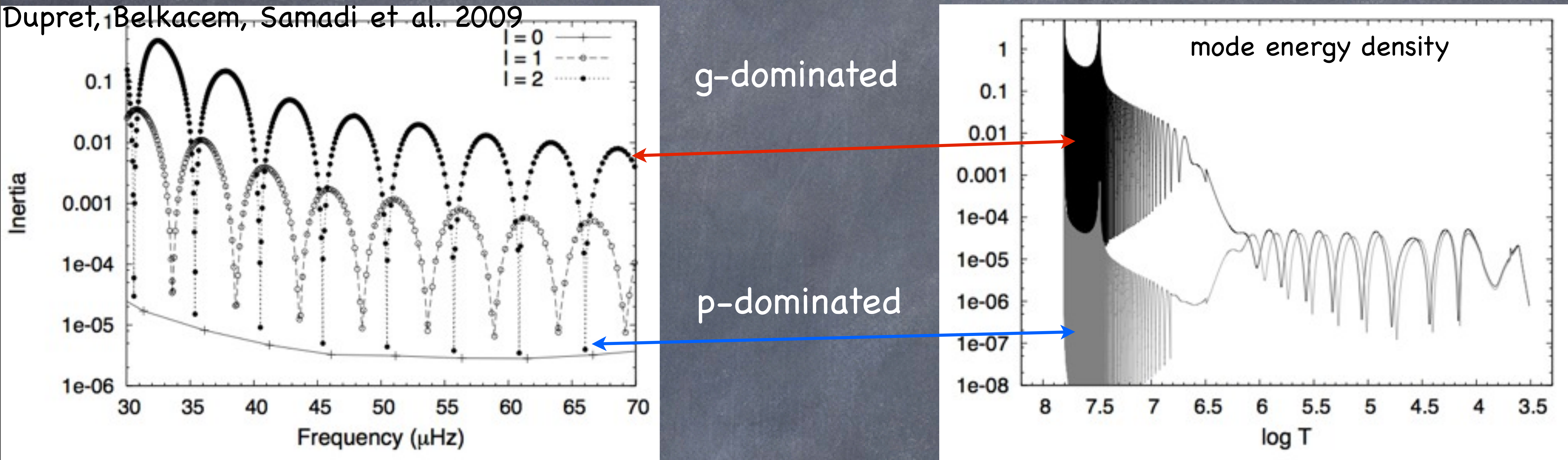
- intermediate region: modes are evanescent, it couples the cavities

→ modes behave as p-mode in the upper-layers and as a g-mode in the inner layers: they are mixed

✓ Why red-giant stars are so special? Mixed modes

Mode inertia is a good indicator of the degree of mixing $I = \int_0^M \vec{\xi}^2 \mathrm{d}m$

Dupret, Belkacem, Samadi et al. 2009



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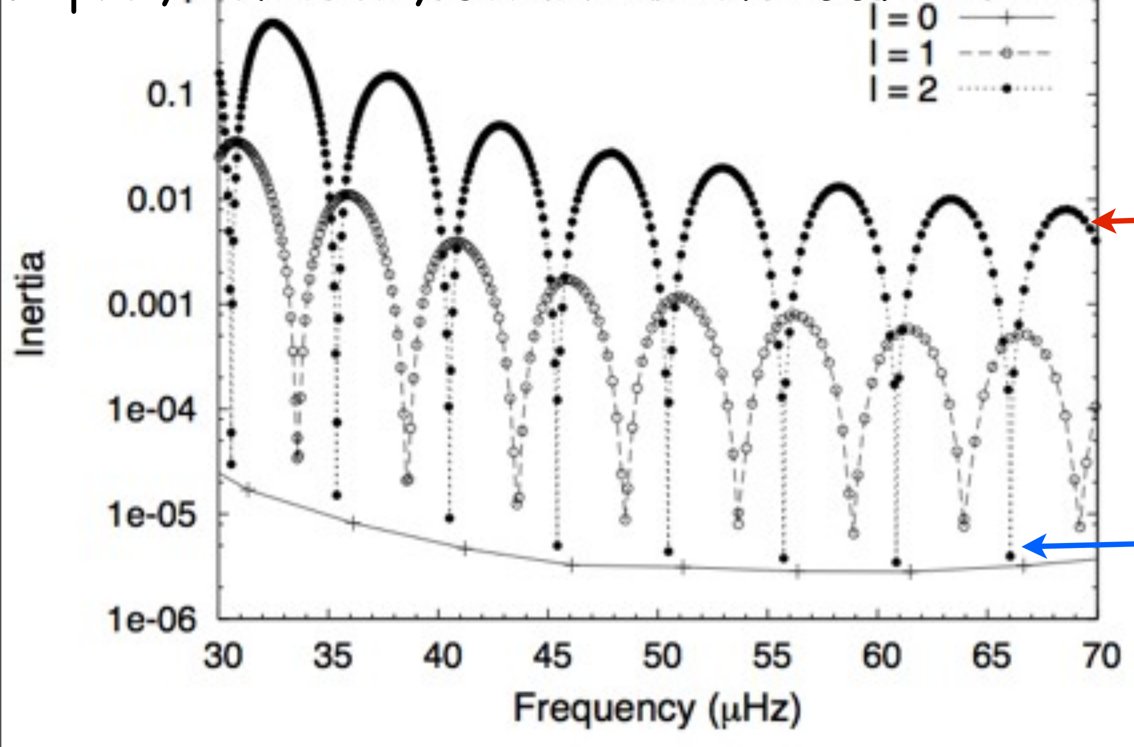
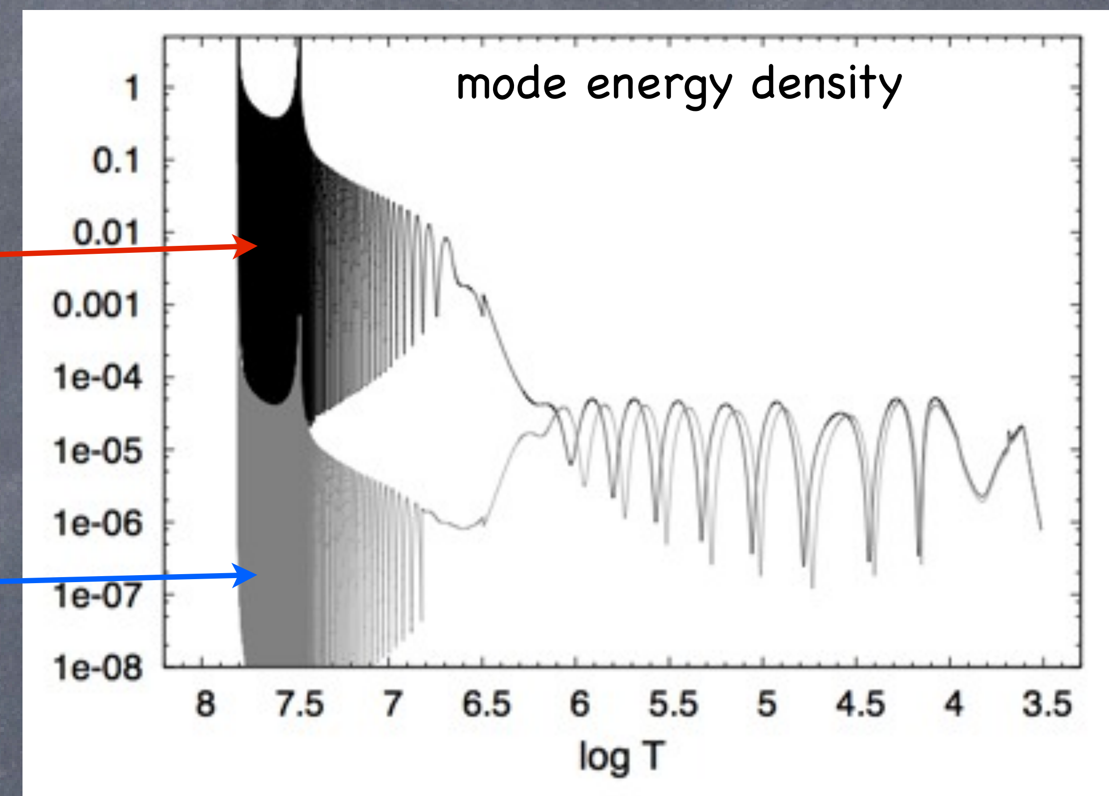
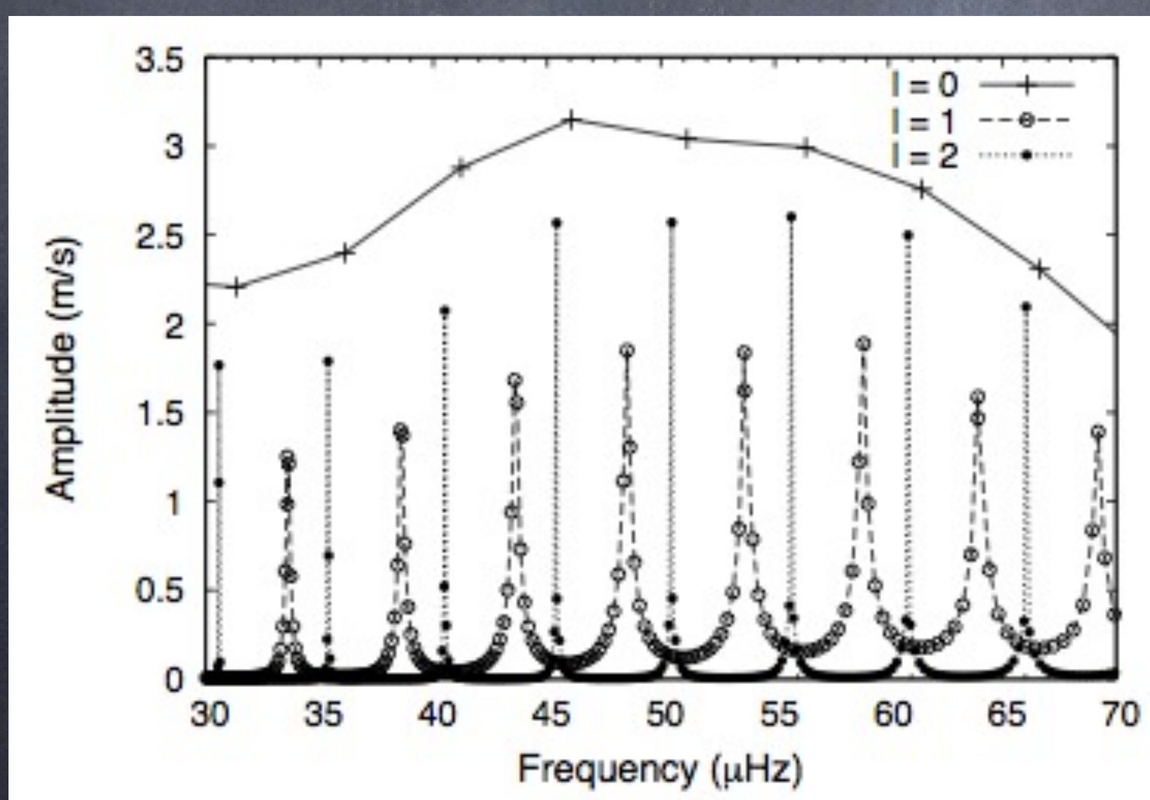


g-dominated

p-dominated



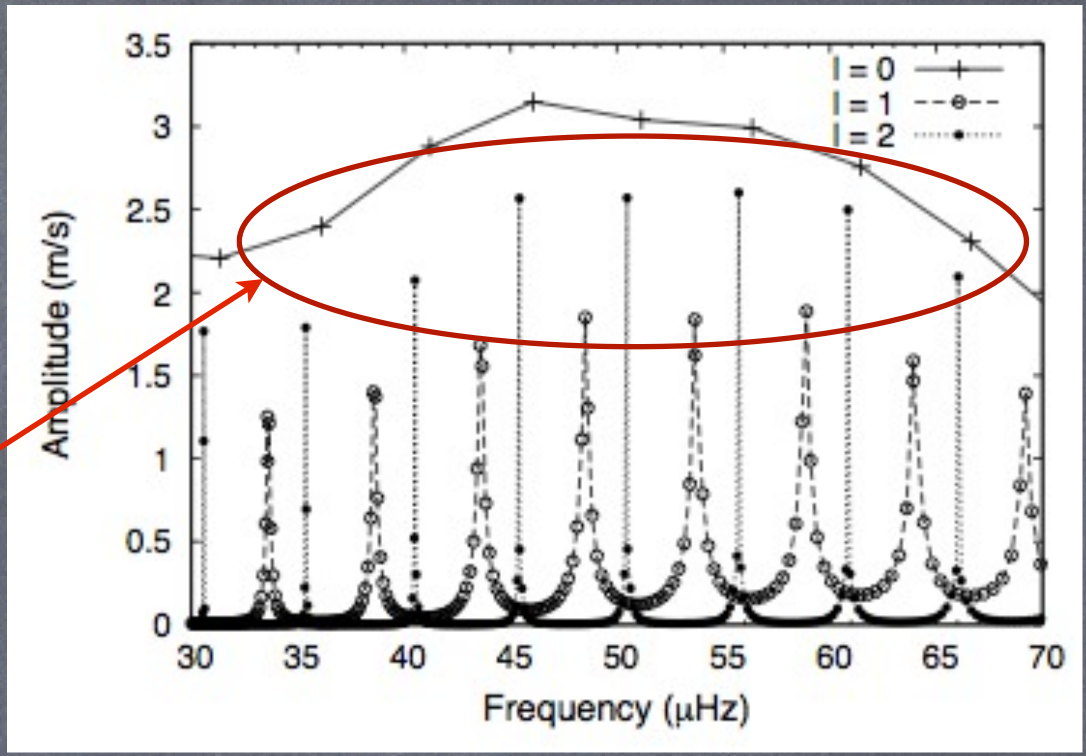


- we mainly observe p-dominated mixed modes
- As time goes on, the detected mixed modes become g-dominated

From an observational point of view:

- So the quest for mixed modes began

- Mixed modes (p-dominated) have been detected by both CoRoT and Kepler



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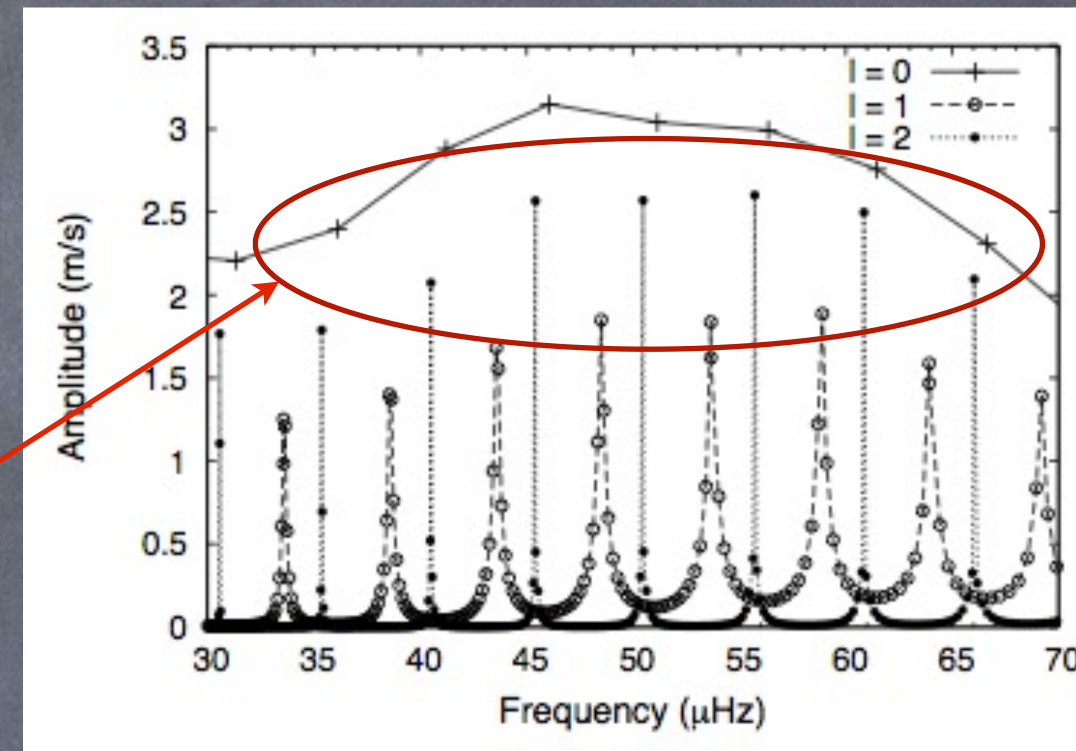

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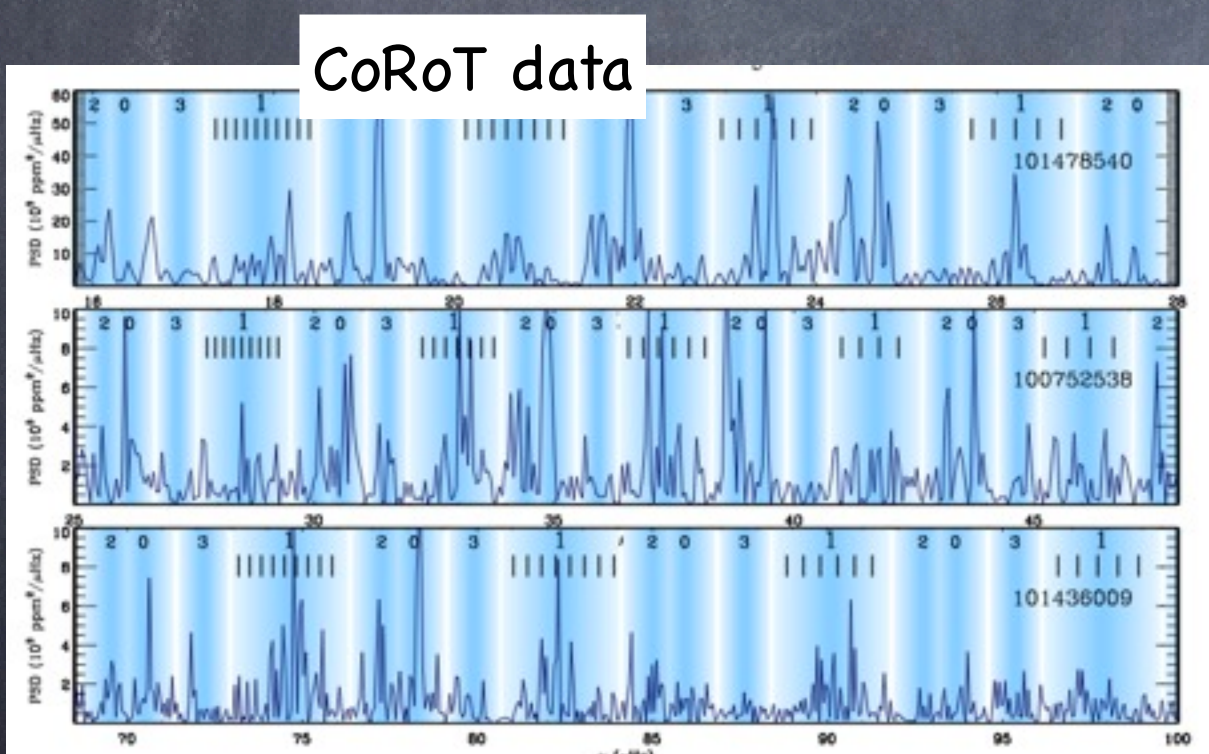
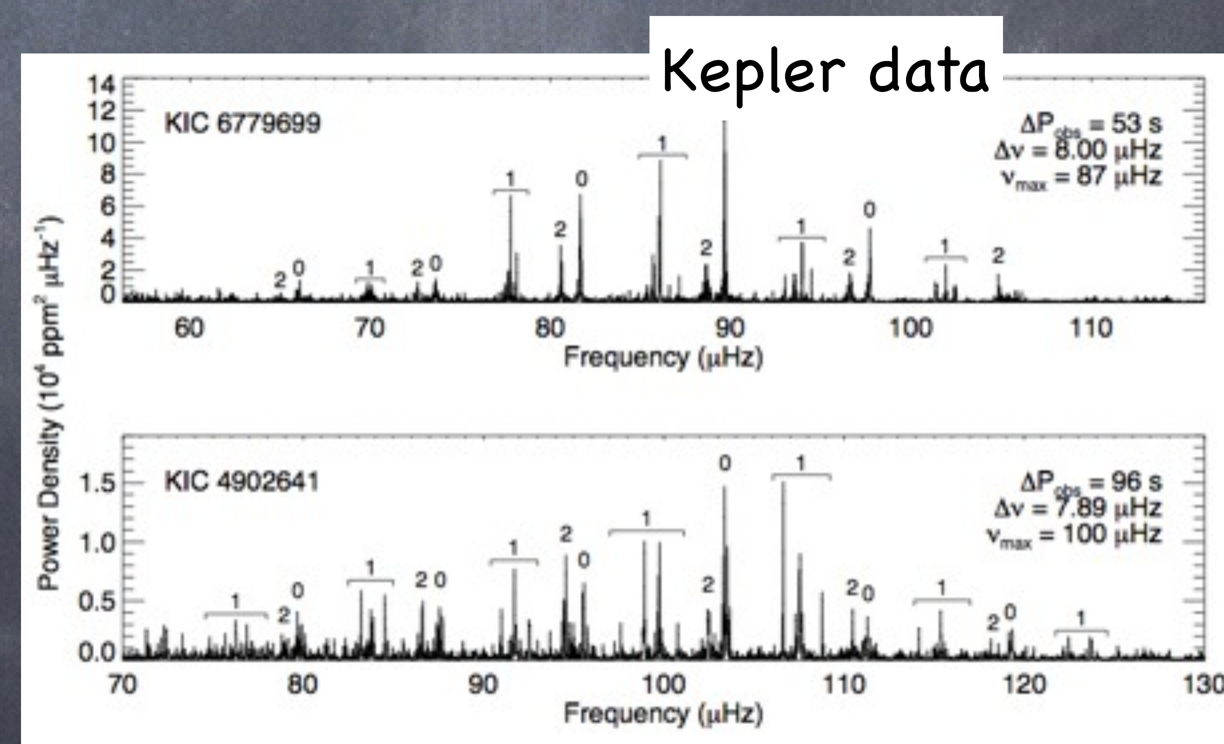


Mosser et al. (2011)



Bedding, Mosser, Huber, et al. Nature, (2011)

The first issue was to discriminate between Clump stars and RGB stars

RGB (red giant branch): hydrogen shell burning

Red clump stars: helium central burning

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RGB (red giant branch): hydrogen shell burning

Red clump stars: helium central burning

- From mixed modes frequencies one can deduce an «effective» period spacing
- it depends mainly on the core of the giants

→ only mixed modes can lifted this «degeneracy» of the HR diagram

The first issue was to discriminate between Clump stars and RGB stars

RGB (red giant branch): hydrogen shell burning

Red clump stars: helium central burning

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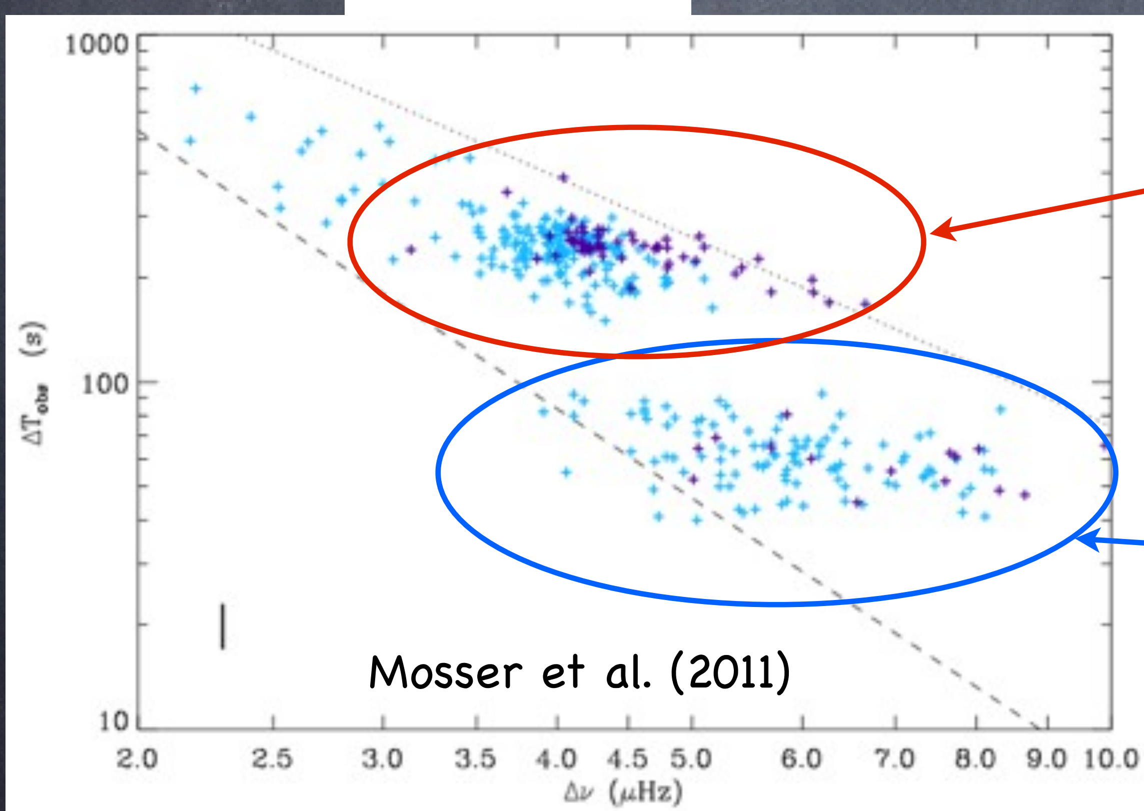
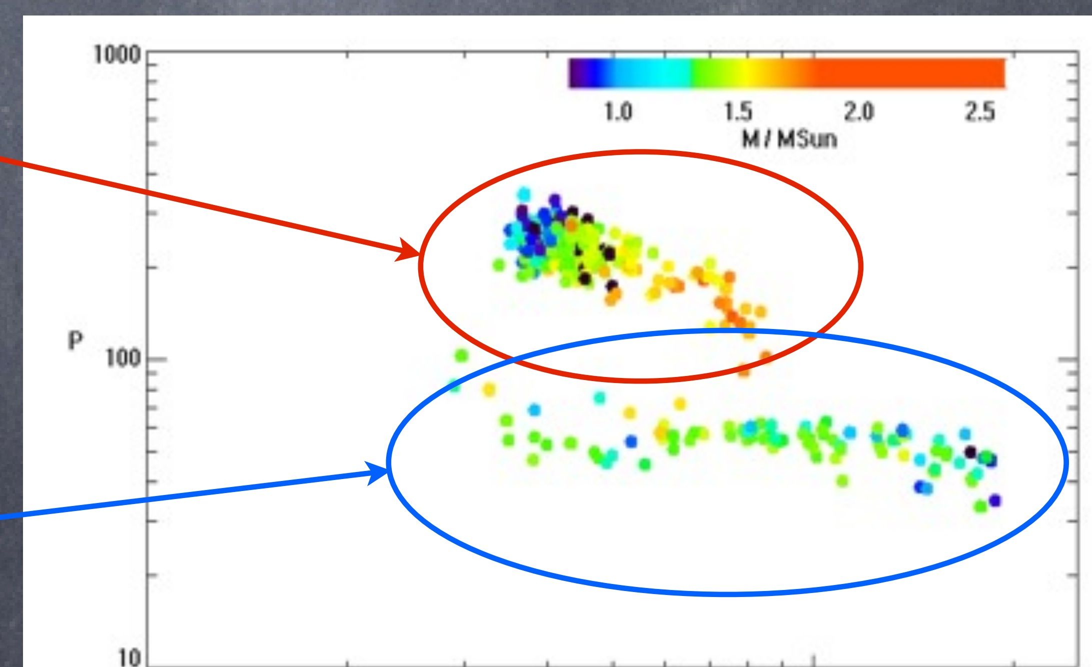
CoRoT data



Clump stars

Red Giant Branch stars

Kepler data

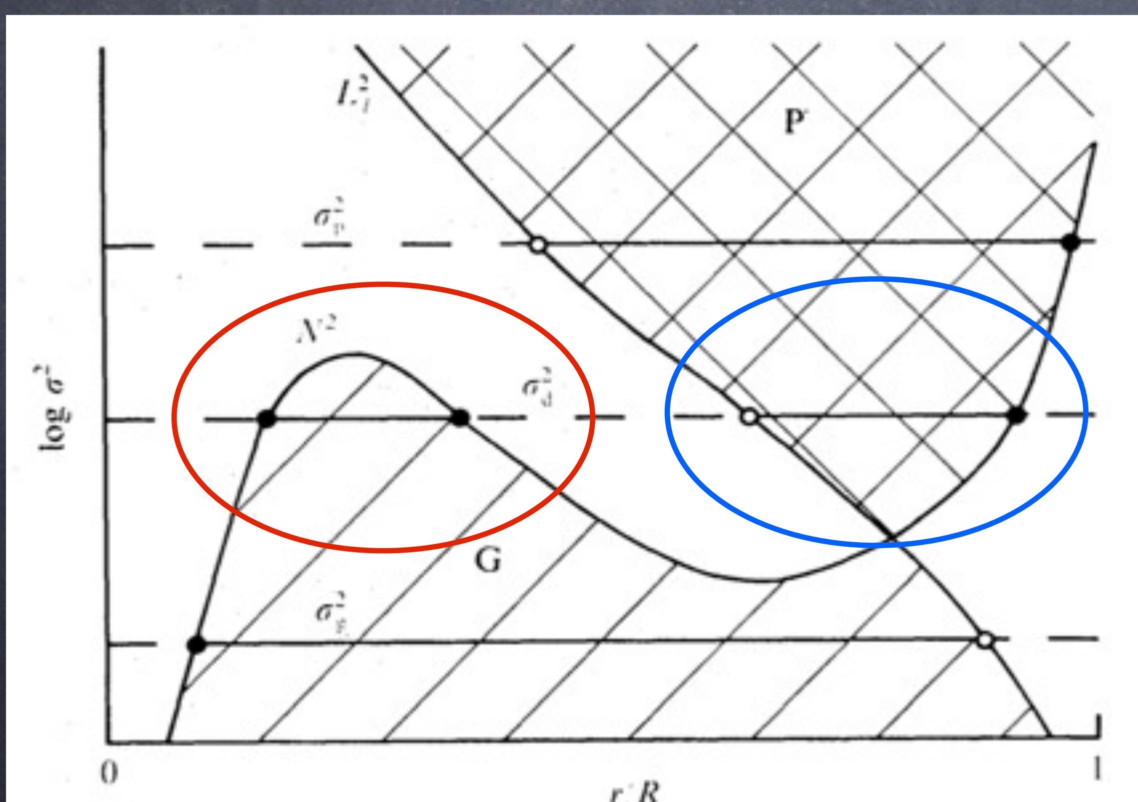


Bedding, Mosser, Huber, et al. (2011)

- Can we use the observed period spacings and compare them to the theory?

- No: asymptotic period spacings concerns gravity modes (not p-dominated mixed modes)

- We need some theoretical framework for relating «effective» and asymptotic value of the period spacing



Following the Unno et al. (1989) approach to get the phase relation for asymptotic mixed modes

↓

Goupil et al. derived a asymptotic relation for mixed modes

Asymptotic relation for mixed modes

$$\nu = \nu_p + \frac{\Delta\nu}{\pi} \arctan\left[q \tan\pi\left[\frac{1}{\Delta P \nu} - \epsilon_g\right]\right]$$

Goupil, Belkacem, Marques, Mosser et al. in prep

frequency of 'pure' p modes

Asymptotic relation for mixed modes

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large separation

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coupling factor

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REAL period spacing

Asymptotic relation for mixed modes

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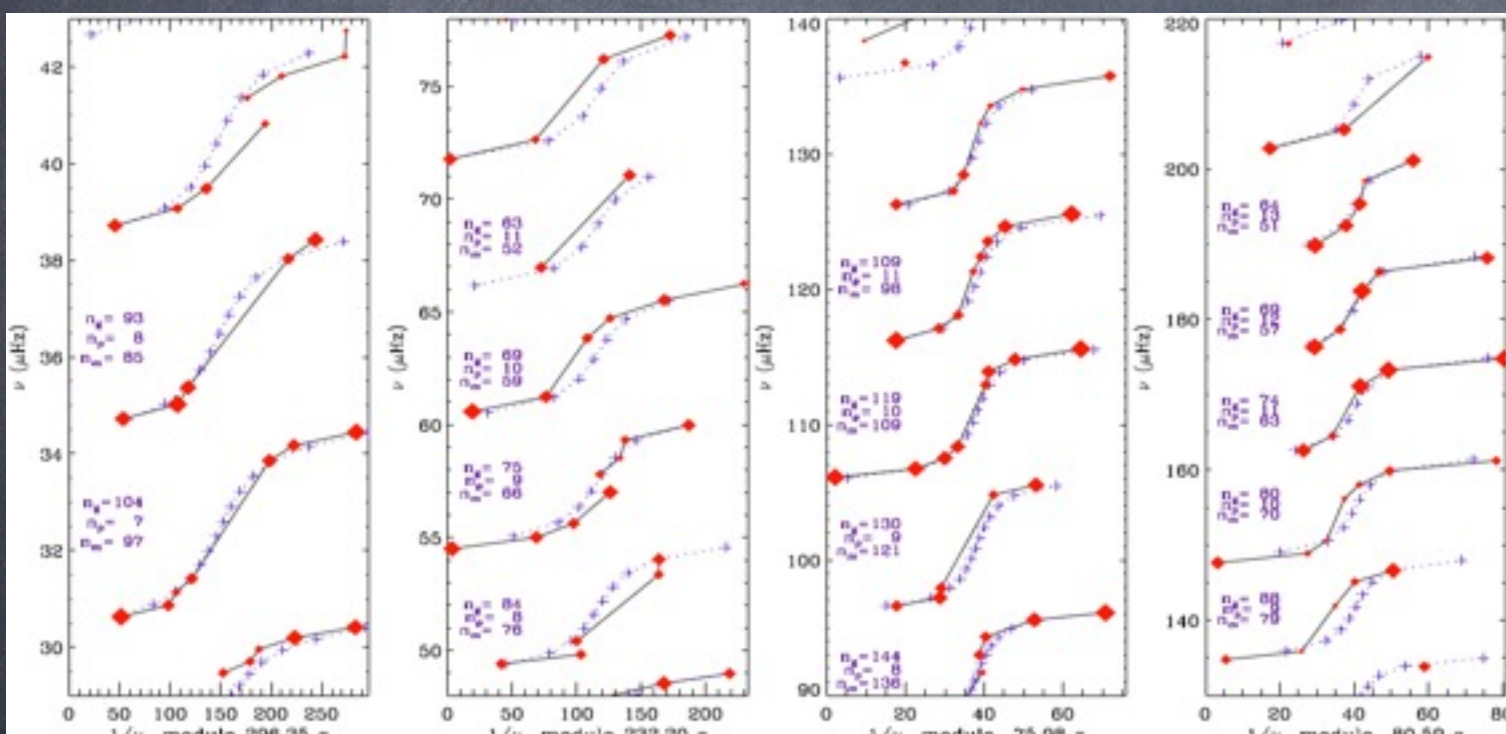
B. Mosser, M.J. Goupil, K. Belkacem et al. (2012)



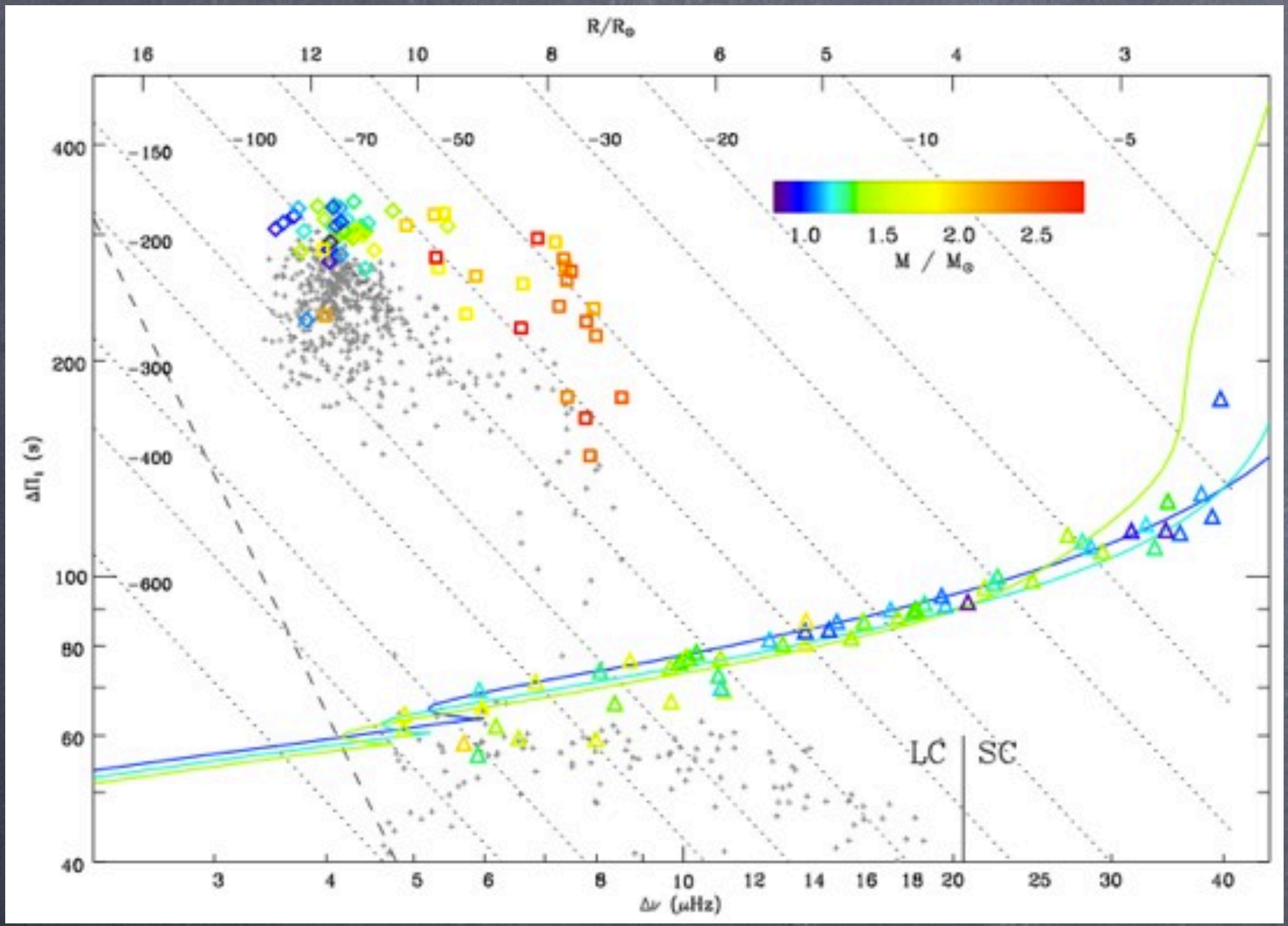
frequency versus mode period modulo the period spacing

Through a fit of the Period échelle diagram

Period spacing

Asymptotic relation for mixed modes

Period spacing vs large separation



Mosser et al. (2012)

period spacing from models computed with the asymptotic relation

good agreement with the observations

We have direct constraints on the core of stars

For more insight into the period spacings of red-giant stars see the recent paper of J. Montalban (Montalban et al. 2013, arXiv:1302.3173)

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- Period spacing of red-giant stars

  ✓ Asymptotic relation for deriving the period spacings

- Amplitudes of red-giant stars

  ✓ Evidence of non-adiabatic effects

  ✓ The need for both radial velocities and photometric observations

Amplitudes of red giants

Principle: driven and damped oscillator

- Mode amplitude is a balance between mode driving and damping

$$\frac{\mathrm{d}}{\mathrm{d}t}A^2 = \mathcal{P} - \eta A^2$$

- Amplitudes are of the order of

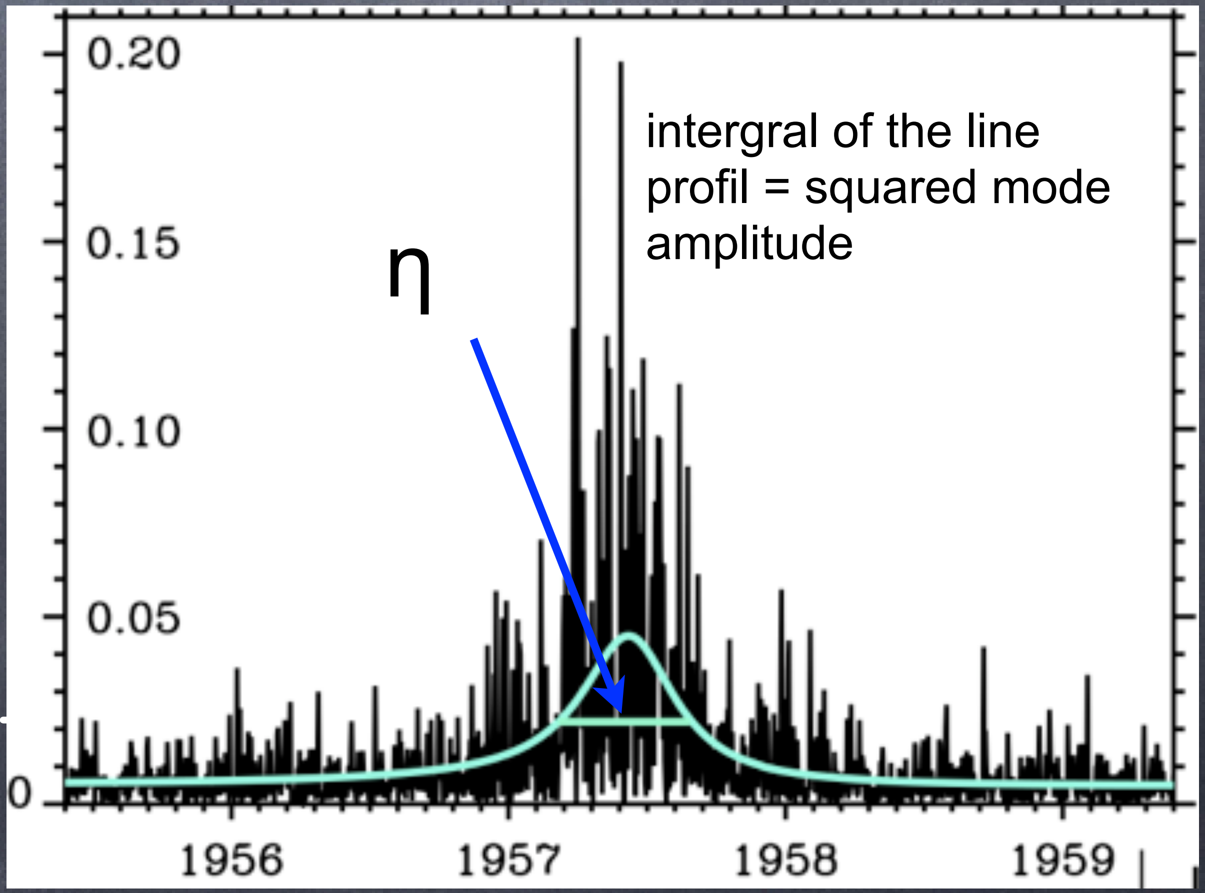
  ~3 ppm for the Sun

  ~100 ppm for red giants

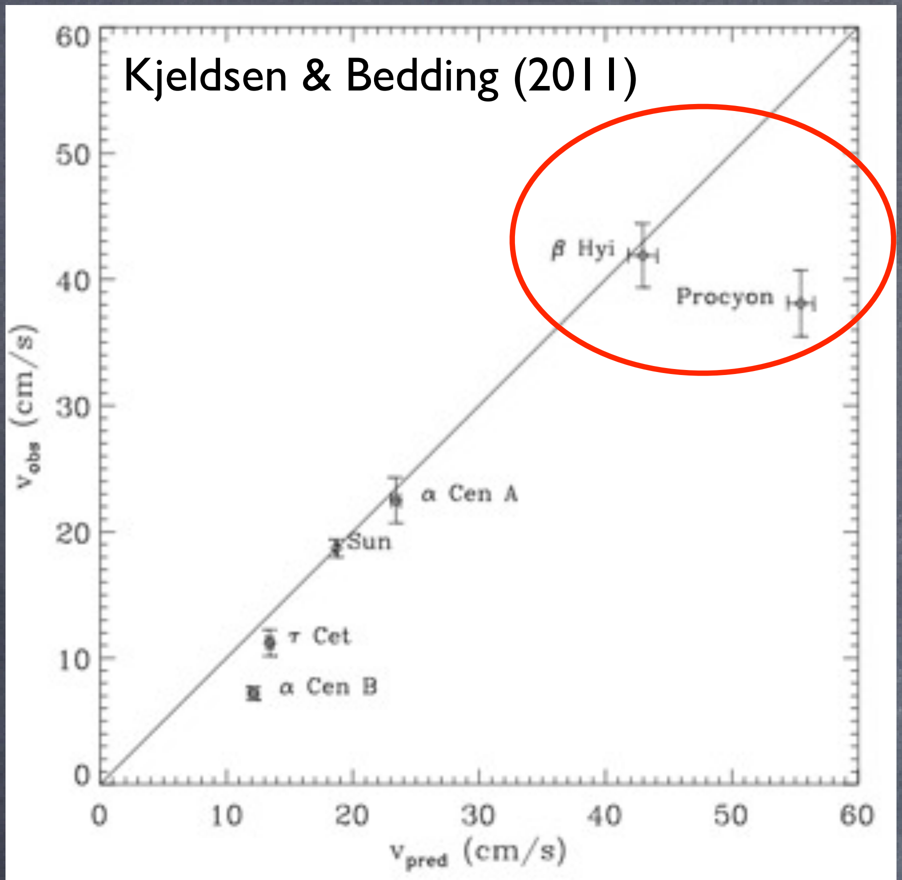
- Life-times are of the order of

  ~ days for the Sun

  ~ up to hundreds days for red gian



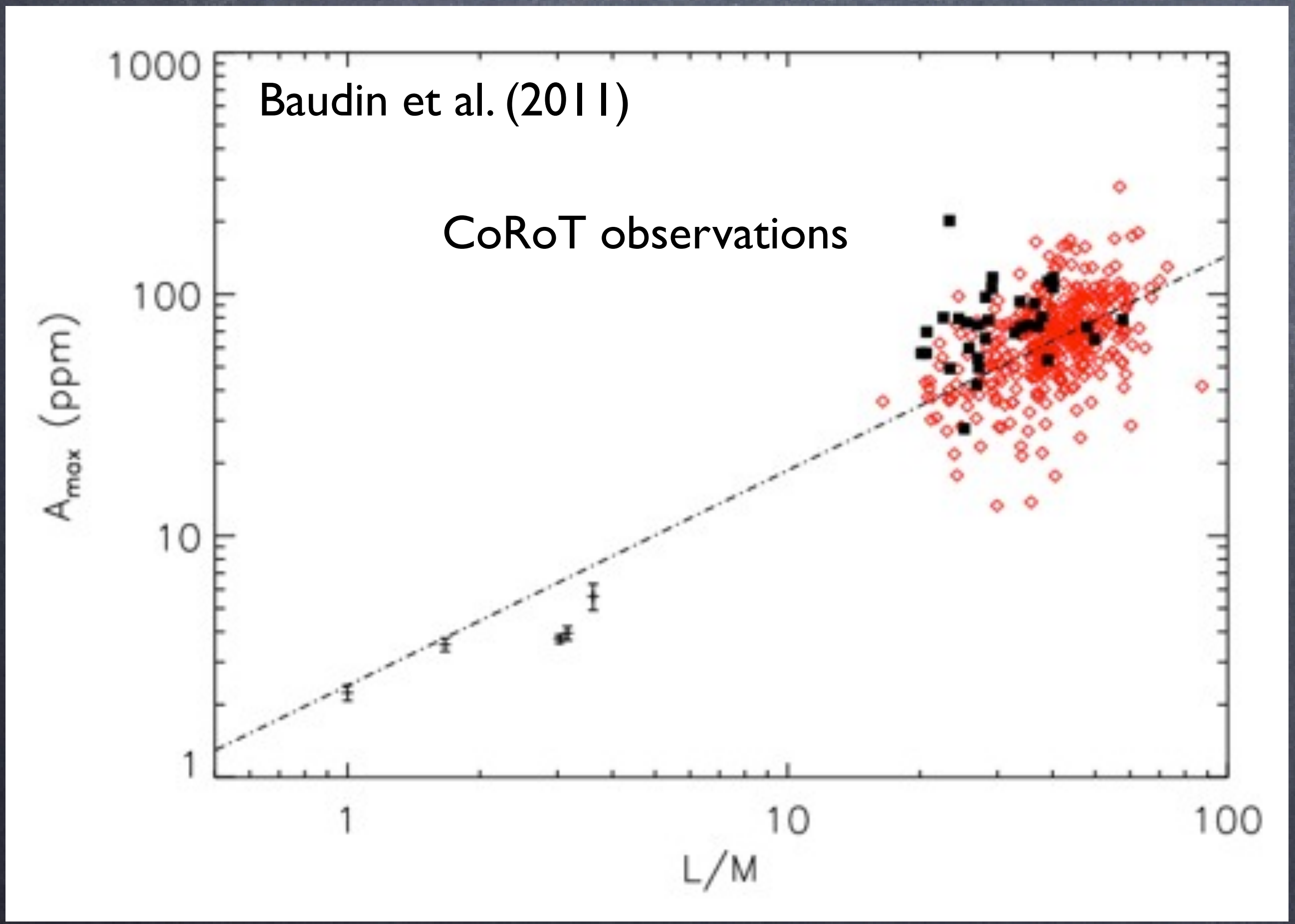
Amplitudes of red giants: observations

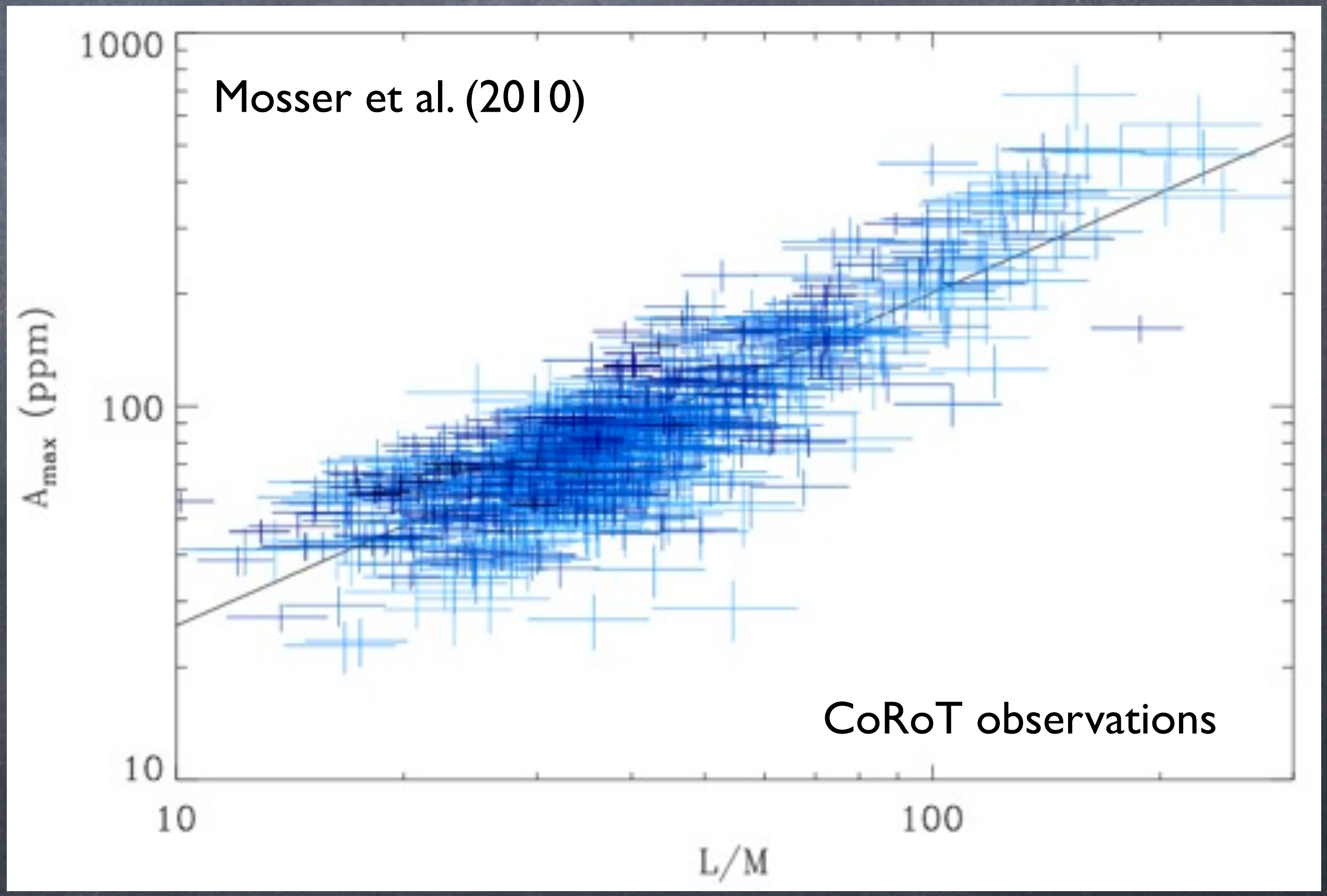


- From ground-based observations we have only a few stars

→ Observations vs Theory: not satisfactory for red giants

- From space-borne missions CoRoT and Kepler





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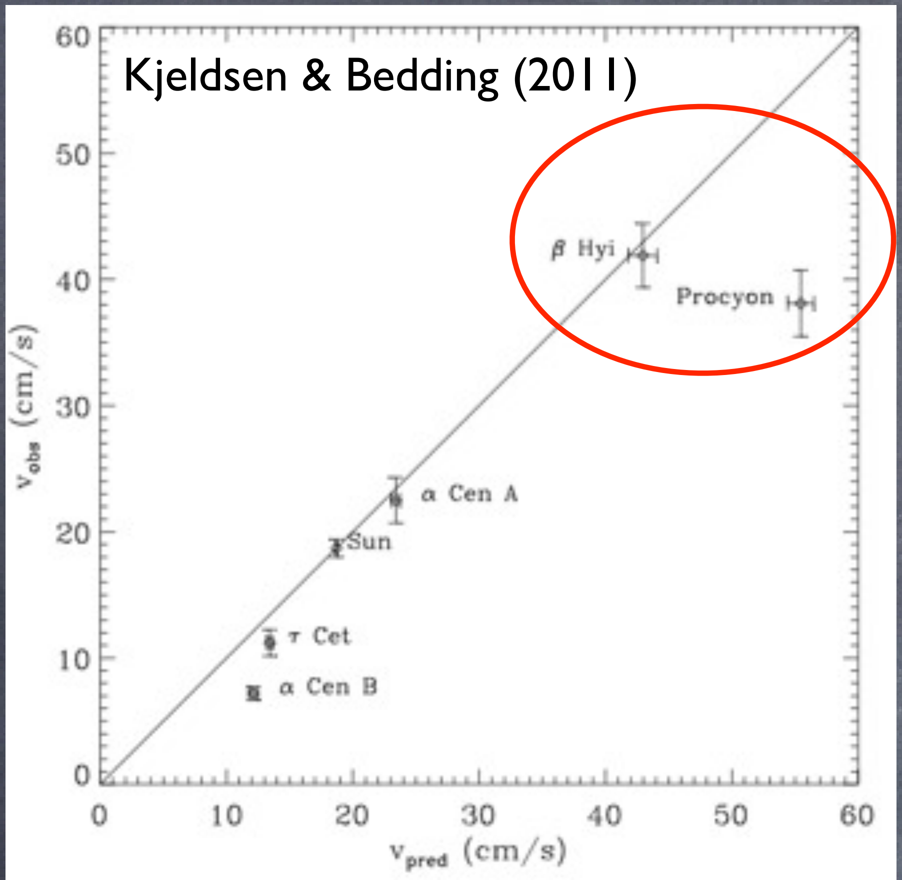

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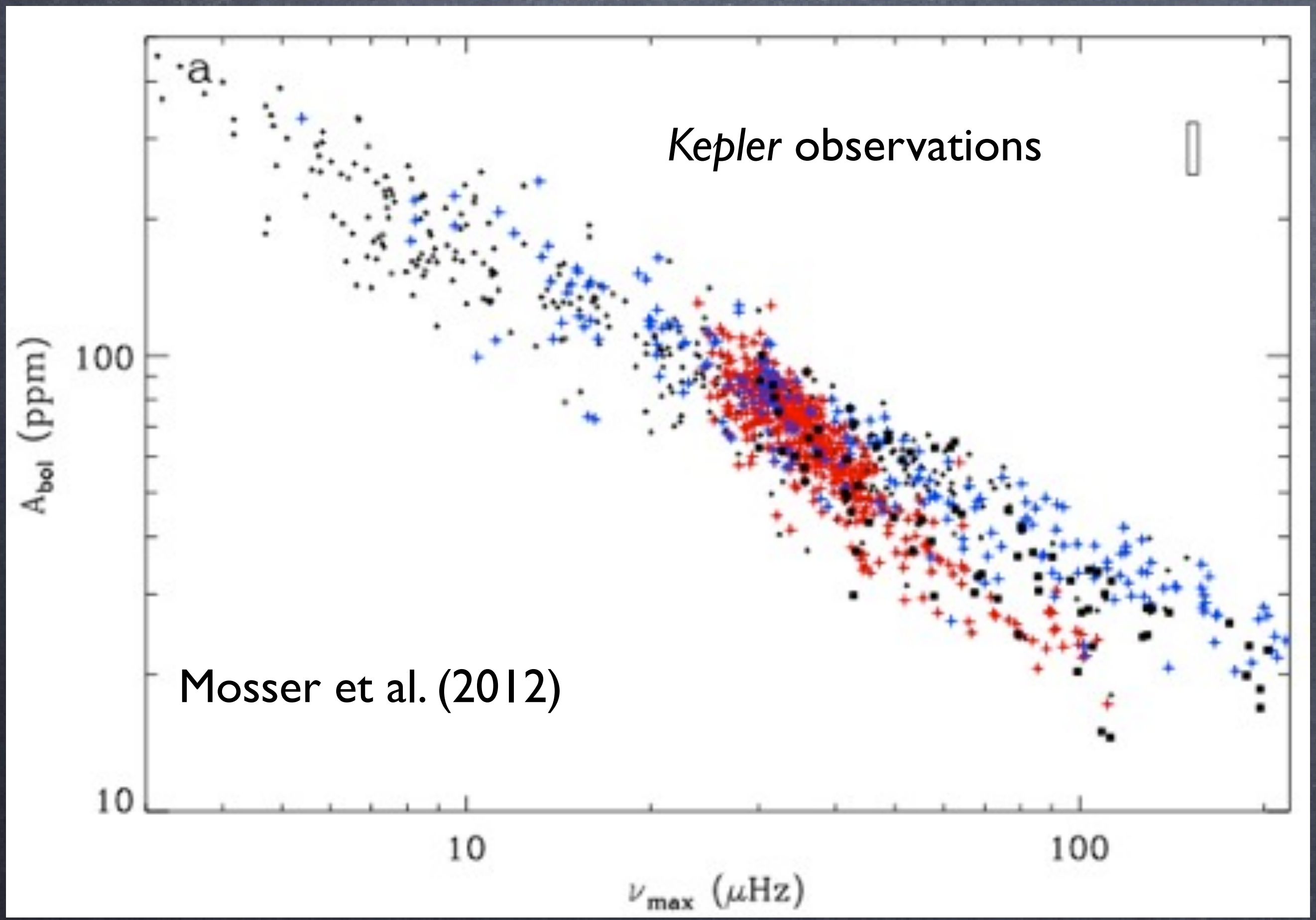

- RGB stars
- Red Clump stars
- stars with «depressed» dipolar modes

So what do we know about mode amplitudes in red giants and related scaling relations?

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Theory:

- Kjeldsen & Bedding (1995)

Derived from Christenssen-Dalsgaard & Frandsen (1983) calculations

Based on a equipartition principle (Goldreich & Keeky 1977)

$$A \propto \left(\frac{L}{M}\right)$$

- Houdek et al (1999)

Based on Balmforth (1992)'s formulation, closely derived from Goldreich & Keeky (1977) theory. Assumes implicitly a Gaussian eddy-time correlation, no thermal forcing

$$A \propto \left(\frac{L}{M}\right)^s$$

- Samadi et al (2007)

Based on Samadi & Goupil (2001 formalism), which is a generalization of Goldreich & Keeky (1977) theory. Thermal forcing; free choice for eddy-time correlation

Obsevations (empirical approaches):

- Kjeldsen & Beeding (2011)

Phenomenological approach. Assumes that the squared amplitude is proportional to the power in the granulation spectrum

$$A \propto \eta^{-0.5} \frac{L}{M^{1.5}\, T_{\rm eff}^{2.25}}$$

- Huber et al (2011)

Purely empirical. Coefficients fitted to match the observations

$$A \propto \frac{L^s}{M^t\, T_{\rm eff}^{r-1}}$$

So what do we know about mode amplitudes in red giants and related scaling relations?

In short: the overall picture is (at least) not clear for red-giant stars!

Scaling relation for mode amplitudes

Samadi et al. (2012)

$$\left(\frac{\delta L}{L}\right)_{\max} \propto \eta_{\max}^{-0.5} \left(\frac{L}{M}\right)^{1.55} \Delta\nu$$

Based on theoretical modelling and a set of 3D hydrodynamical simulations

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$$\eta_{\max}^{-1} \propto T_{\rm eff}^{-10.8}\, g^{0.3}$$

Based on full non-adiabatic calculations

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+ «classical» scalings →

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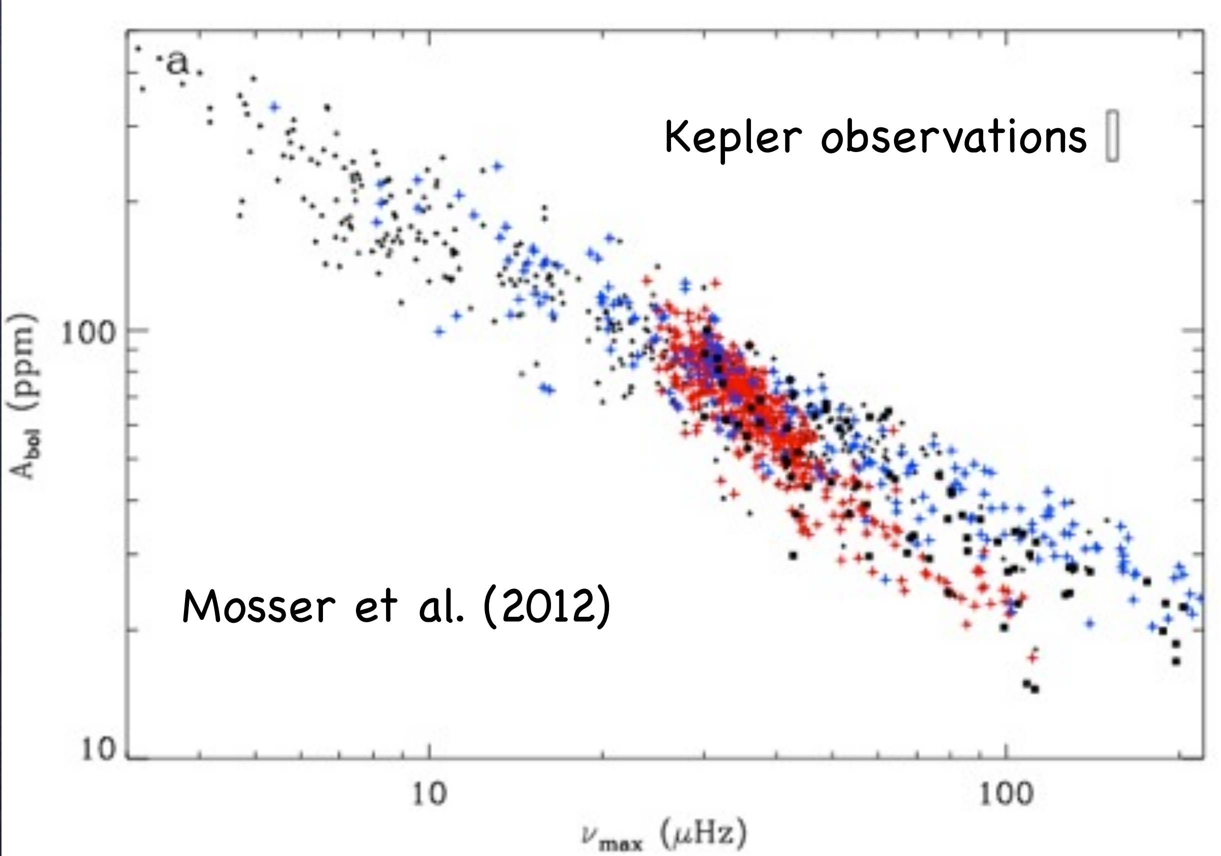

scaling with		coefficient	exponent
$\nu_{\max}$	all	860 ± 30	-0.71 ± 0.01
	clump	4700 ± 500	-1.18 ± 0.03
	RGB	650 ± 60	-0.63 ± 0.02

agreement between theory and observations, at least for RGB stars

What about the absolute values of amplitudes: evidence of non-adiabatic effects using CoRoT observations

It has been shown by Samadi et al (2007), for main-sequence stars

$$V_{\max} \propto \left(\frac{L}{M}\right)^{0.7}$$

confirmed by samadi et al. (2012) for red-giants

Now using an adiabatic conversion between mode velocity and luminosity

$$\left(\frac{\delta L}{L}\right)_{\max} \propto \left(\frac{L}{M}\right)^{0.7} \times \frac{1}{\sqrt{T_{\rm eff}}}$$

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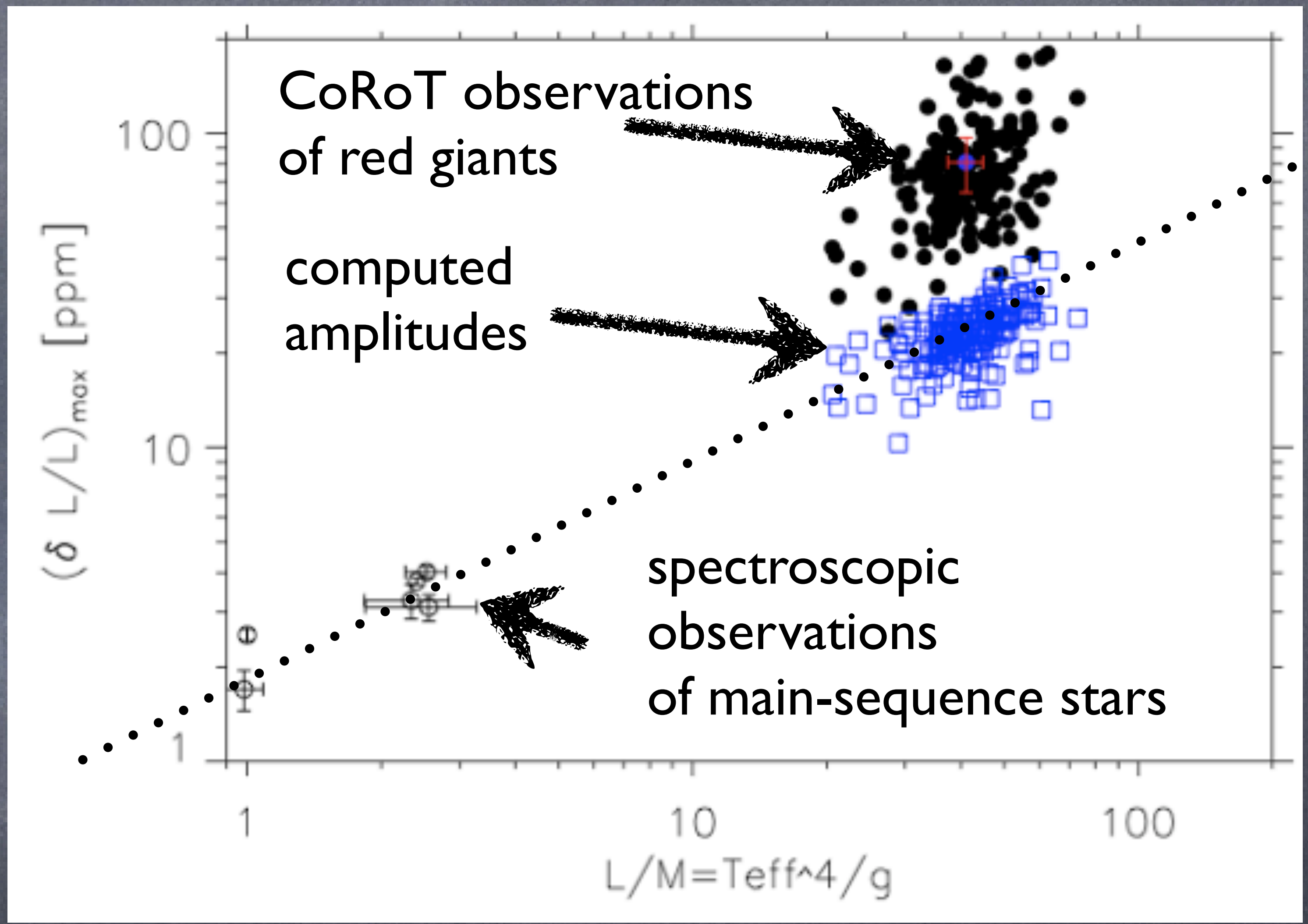

→ Adiabatic computation and scaling relation under-estimate mode amplitudes for red giants

What about the absolute values of amplitudes: evidence of non-adiabatic effects using CoRoT observations

Most of the disagreement can be explained by non-adiabatic effects

$$i\sigma T \delta S = -\frac{\partial \delta L}{\partial m} \propto \frac{L}{M}$$

- For main-sequence stars L/M ~ 1-5
- For Red-Giants stars L/M ~ 30-100

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→

- It impacts the shape of the eigenfunction and therefore the driving

- It impacts the conversion between velocity and luminosity perturbations

Conversion between surface velocities and luminosities

Space-borne observations

$$\left(\frac{\delta L}{L}\right)_{\rm max}$$

⟷

Theoretical computations

$$V_{\rm max}$$

Conversion between surface velocities and luminosities

Space-borne observations $\left(\frac{\delta L}{L}\right)_{\max}$ ⟷ Theoretical computations $V_{\max}$

- adiabatic relation (e.g., Kjeldsen & Bedding 1995) $$\left(\frac{\delta L}{L}\right)_{\max} / V_{\max} \propto \sqrt{\frac{1}{T_{\rm eff}}}$$

commonly used with a solar calibration

→ ✓ physically not justified

✓ it does not work even for the Sun

Conversion between surface velocities and luminosities

Space-borne observations

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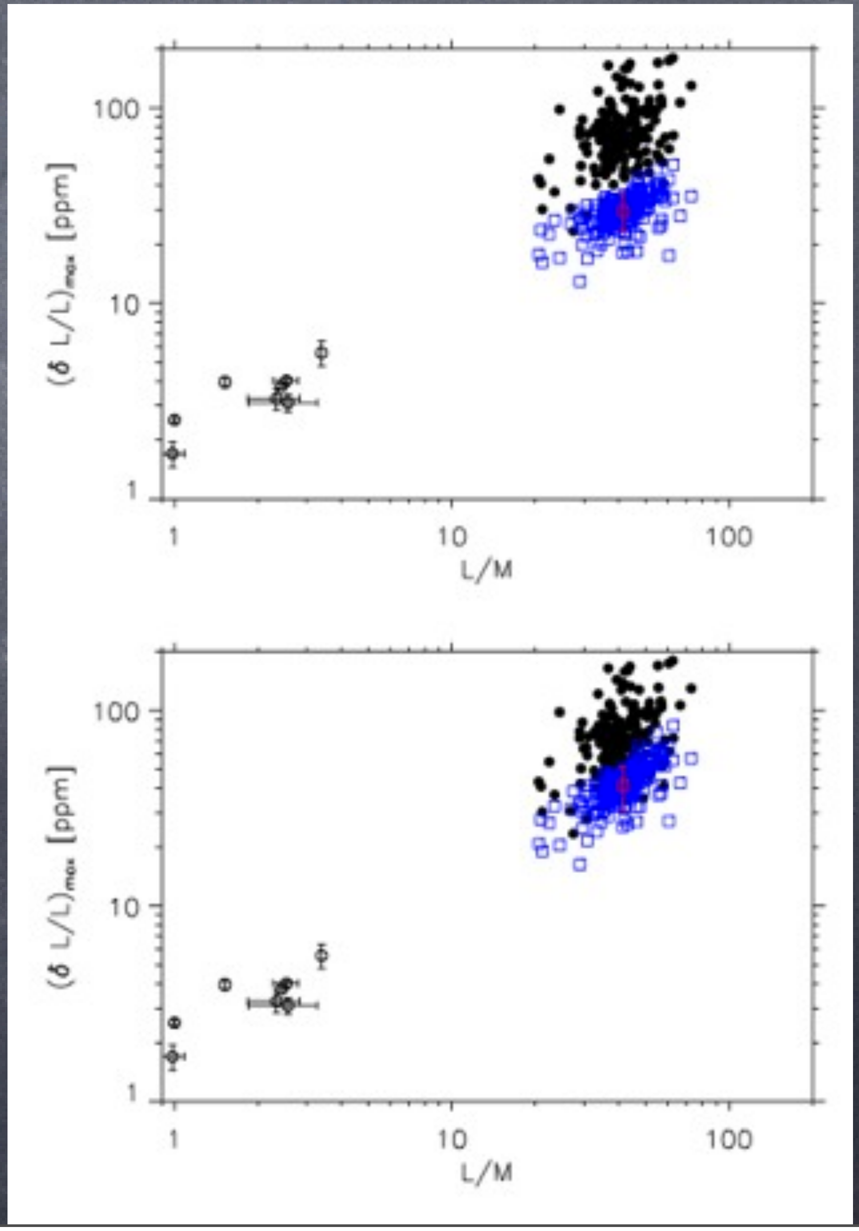
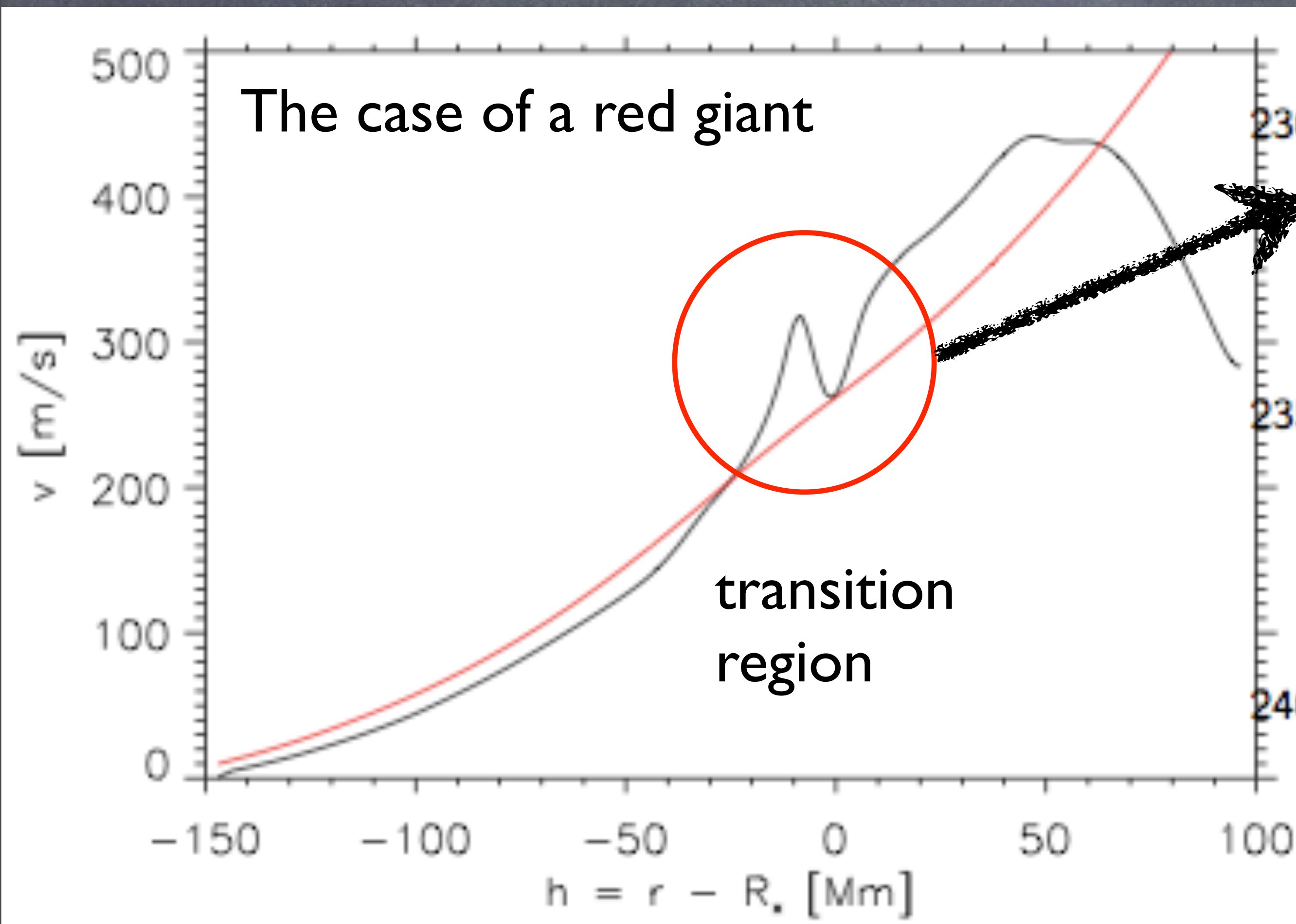
- full non-adiabatic calculations

Using the MAD code (Dupret et al. 2002) $\left(\frac{\delta L}{L}\right)_{\rm max} / V_{\rm max} \propto \left(\frac{L}{M}\right)^{0.25}$

→ Improves the agreement with the observations



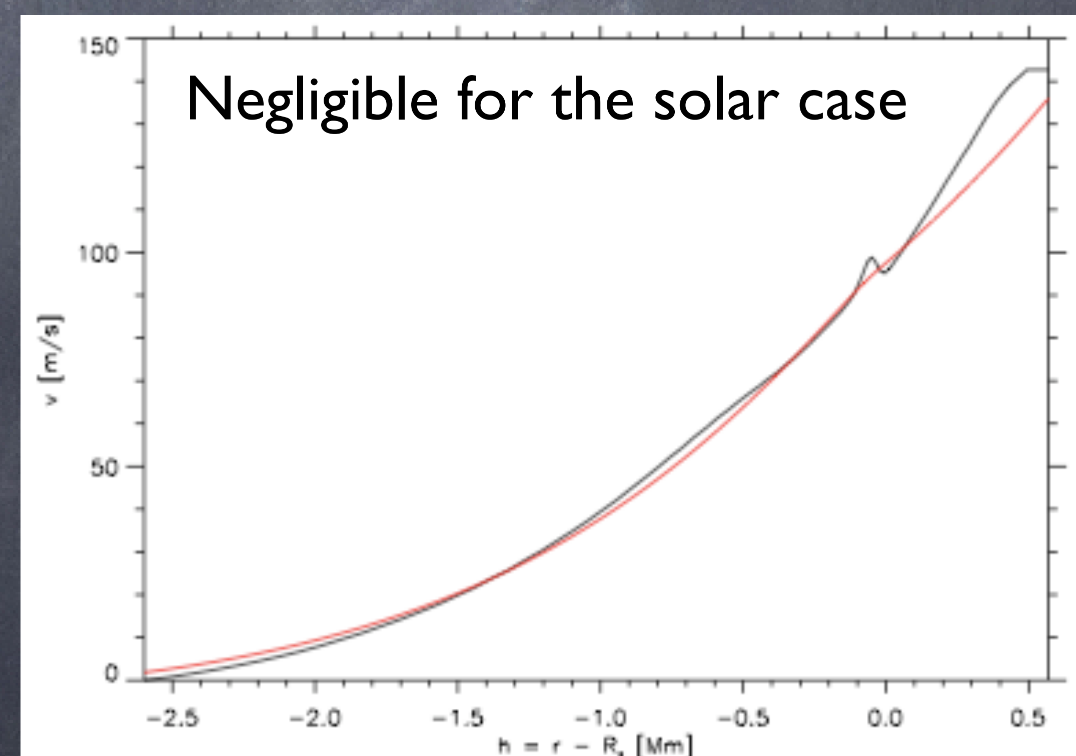
non-adiabatic effect on mode compressibility



mode displacement rapidly varies at the photosphere due to non-adiabatic effect

mode compressibility significantly increases, thus mode driving (~ factor 3)

$$\mathcal{P} \propto \frac{1}{I} \int dm \left| \frac{d\xi_r}{dr} \right|^2 E_{\rm eddy}\, \Lambda\, u_0$$



Hence, mode amplitude of red giants is potentially a way to get constraints on non-adiabaticity effects

Concluding remarks

- The wealth of high-quality observational data from CoRoT and Kepler opens up great opportunities for stellar physics, but...

- For period spacings (as well as for large separation!):
one has to be careful before any comparison between observations and models

- For mode amplitudes, the overall pictures is being more clear, but...

→ we crucially need constraints on non-adiabatic processes

ground-based observations, led by E. Poretti, are currently undertaken to this end, for several stars within the CoRoT red-giant working group