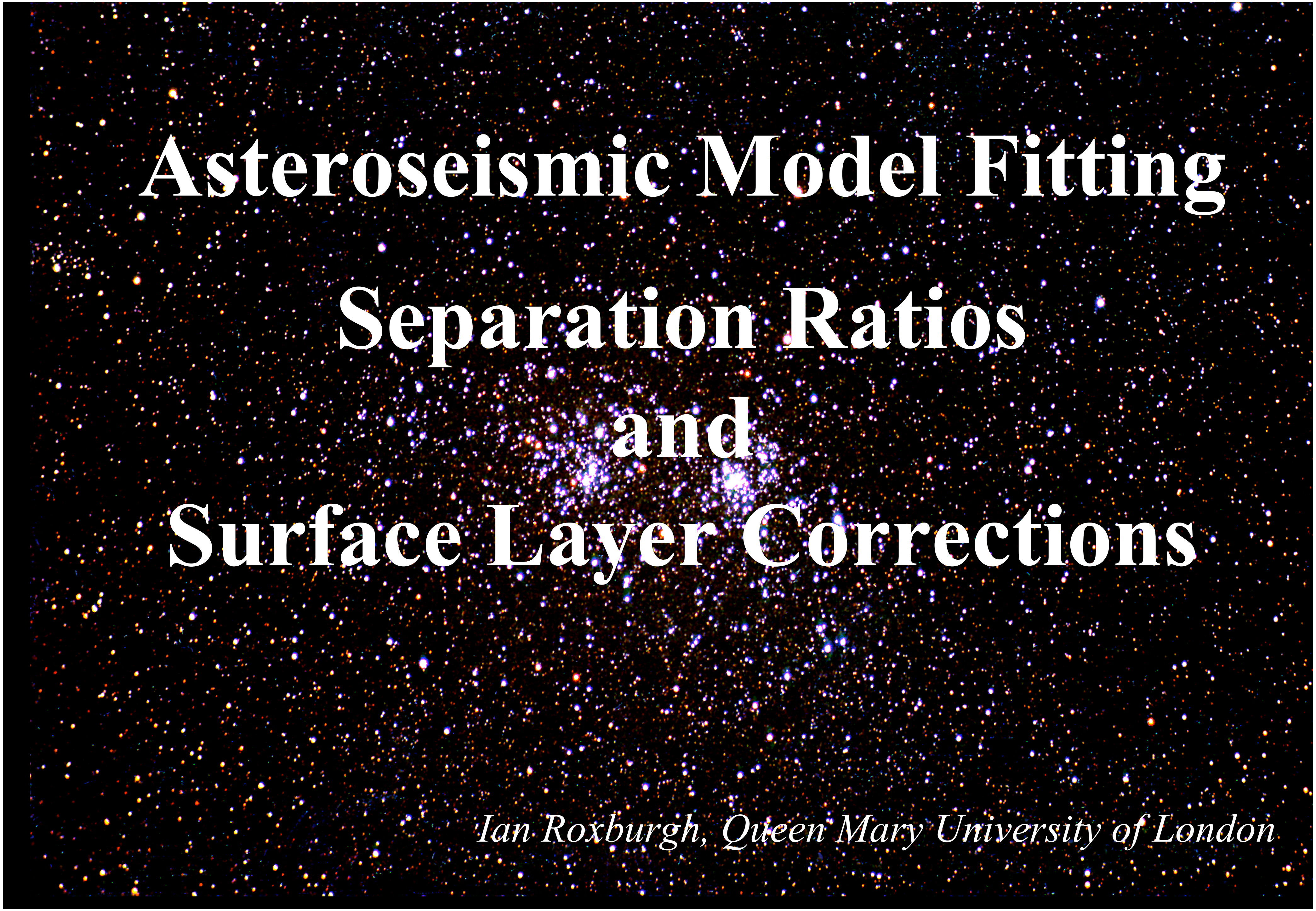


Asteroseismic Model Fitting

Separation Ratios and Surface Layer Corrections

Ian Roxburgh, Queen Mary University of London

Conclusions

1) *Separation ratios to be compared at **same** frequency (interpolate model values)*
2) *Surface offset changes properties of model - if a "corrected" model fits observations interior structure not same. Better not to use at all.*
3) *Large separations does not accurately scale with $(M/R^3)^{1/2}$ - considerable error*
4) *Take care when fitting observations to models!*

Separation ratios as a diagnostic of stellar interiors
Ratios subtract off main effect of outer layers

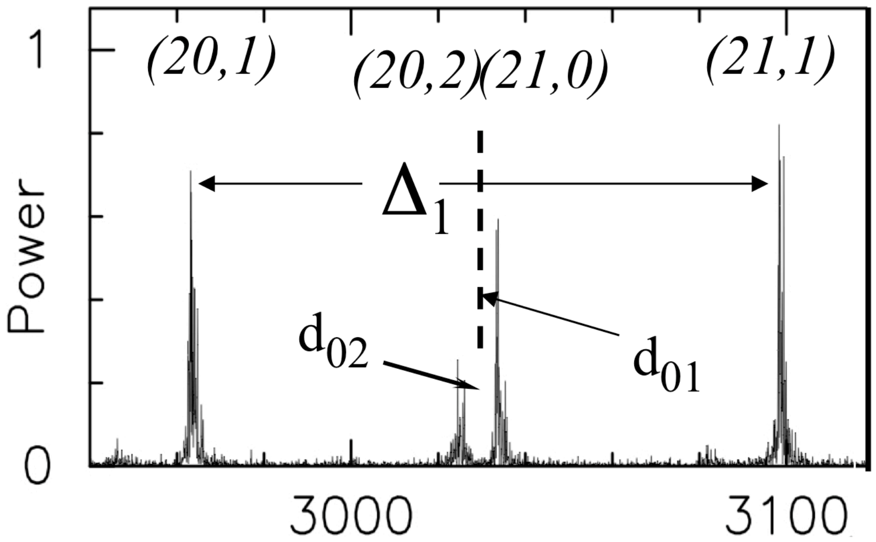
Large separations

$$\Delta_\ell(n) = \nu_{n,\ell} - \nu_{n-1,\ell}$$

Small separations

$$d_{02}(n) = \nu_{n,0} - \nu_{n-1,2},$$

$$d_{01}(n) = \nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2$$



Separation ratios

$$r_{02}(n) = \frac{\nu_{n,0} - \nu_{n-1,2}}{\nu_{n,1} - \nu_{n-1,1}}, \qquad r_{01}(n) = \frac{\nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2}{\nu_{n,1} - \nu_{n-1,1}}$$

$$\text{Best fit :} \quad \chi^2 = \sum_1^N \left(\frac{r_{0\ell}^{obs}(n) - r_{0\ell}^{mod}(n)}{\sigma_{0\ell}^r(n)} \right)^2$$

Separation ratios as a diagnostic of stellar interiors
Ratios subtract off main effect of outer layers

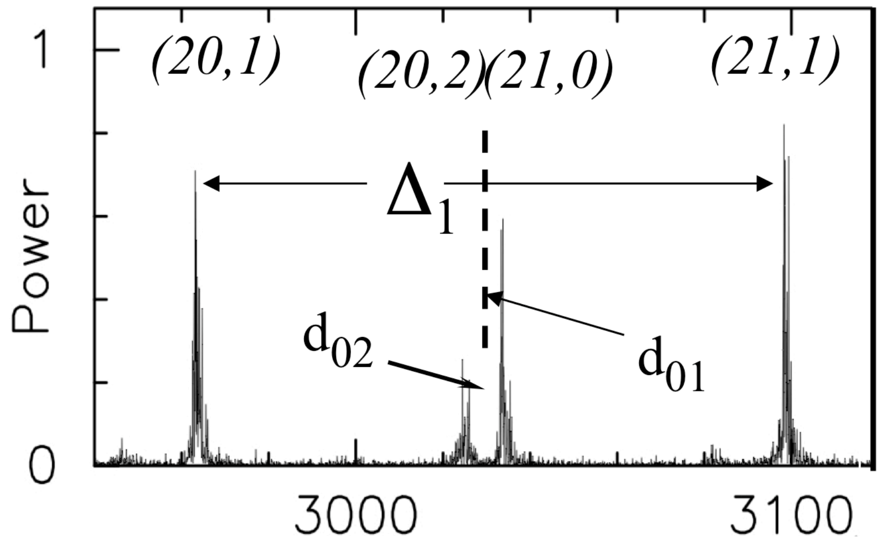
Large separations

$$\Delta_\ell(n) = \nu_{n,\ell} - \nu_{n-1,\ell}$$

Small separations

$$d_{02}(n) = \nu_{n,0} - \nu_{n-1,2},$$

$$d_{01}(n) = \nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2$$



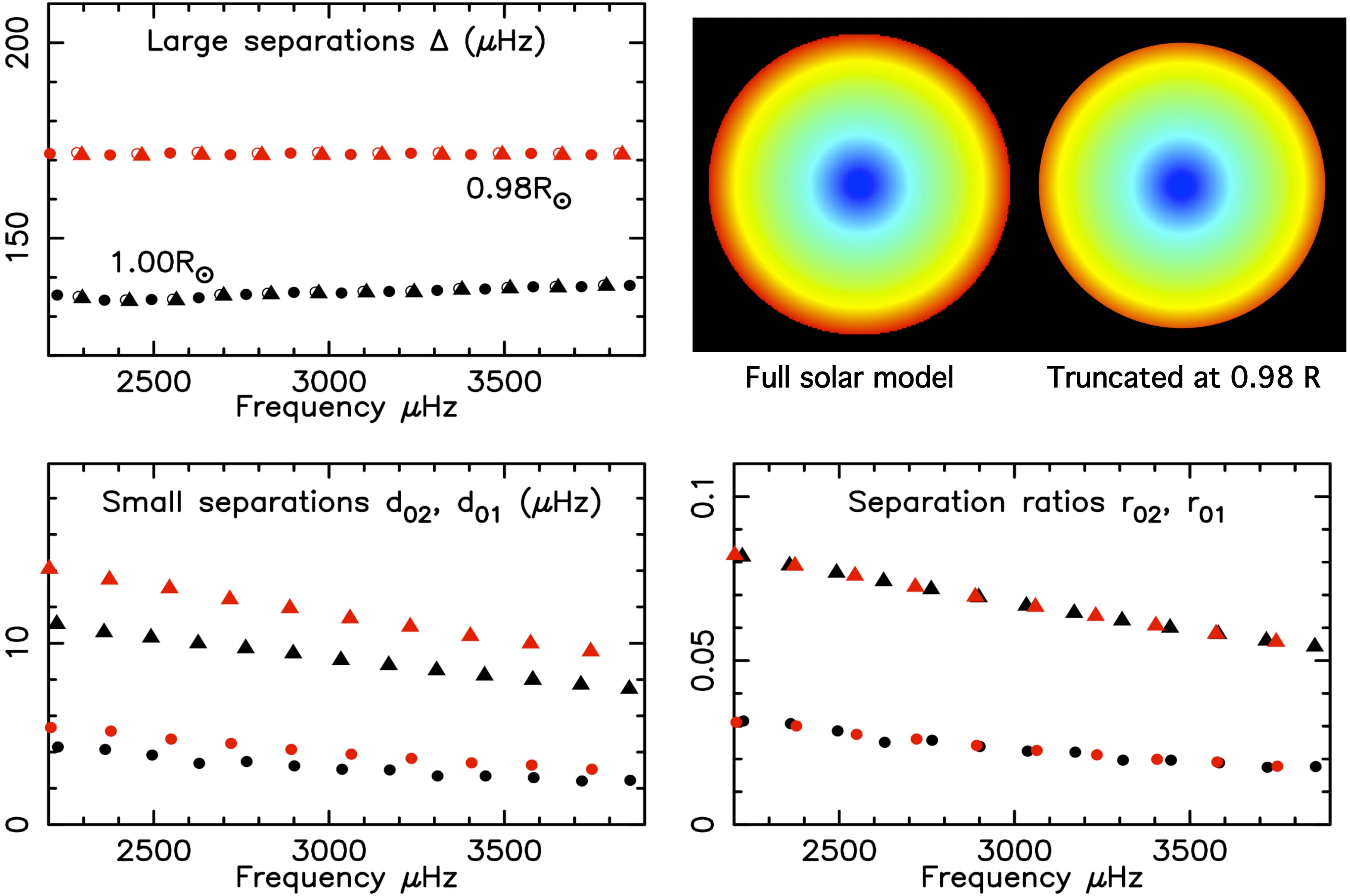
Separation ratios

$$r_{02}(n) = \frac{\nu_{n,0} - \nu_{n-1,2}}{\nu_{n,1} - \nu_{n-1,1}}, \qquad r_{01}(n) = \frac{\nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2}{\nu_{n,1} - \nu_{n-1,1}}$$

Best fit : $$\chi^2 = \sum_1^N \left(\frac{r_{0\ell}^{obs}(n) - r_{0\ell}^{mod}(n)}{\sigma_{0\ell}^r(n)} \right)^2$$

WRONG !

Separation ratios independent of surface layers

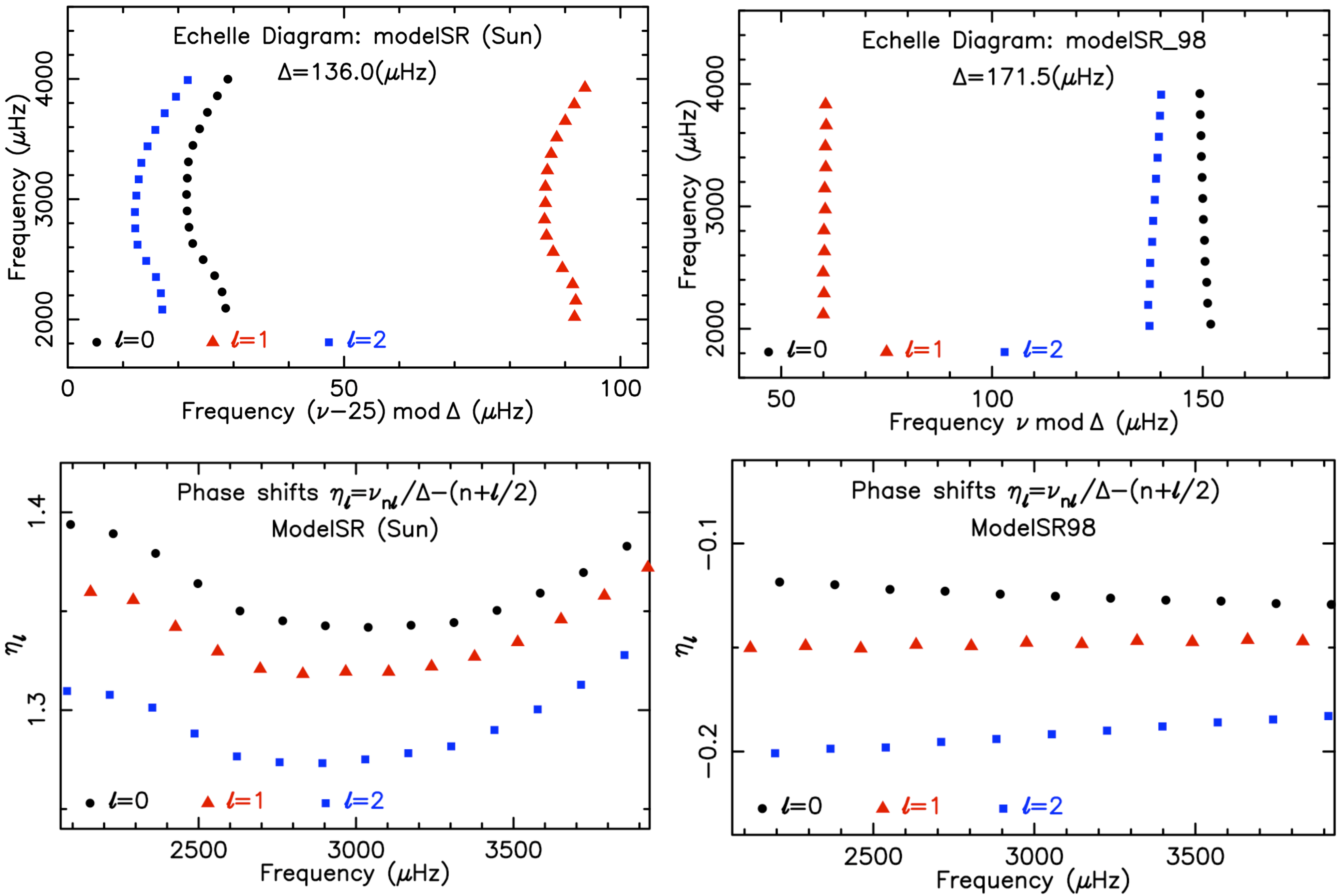


Separation ratios lie on same curve $r_{01}(\nu)$, $r_{02}(\nu)$ but not at the same frequencies and not at the same n values

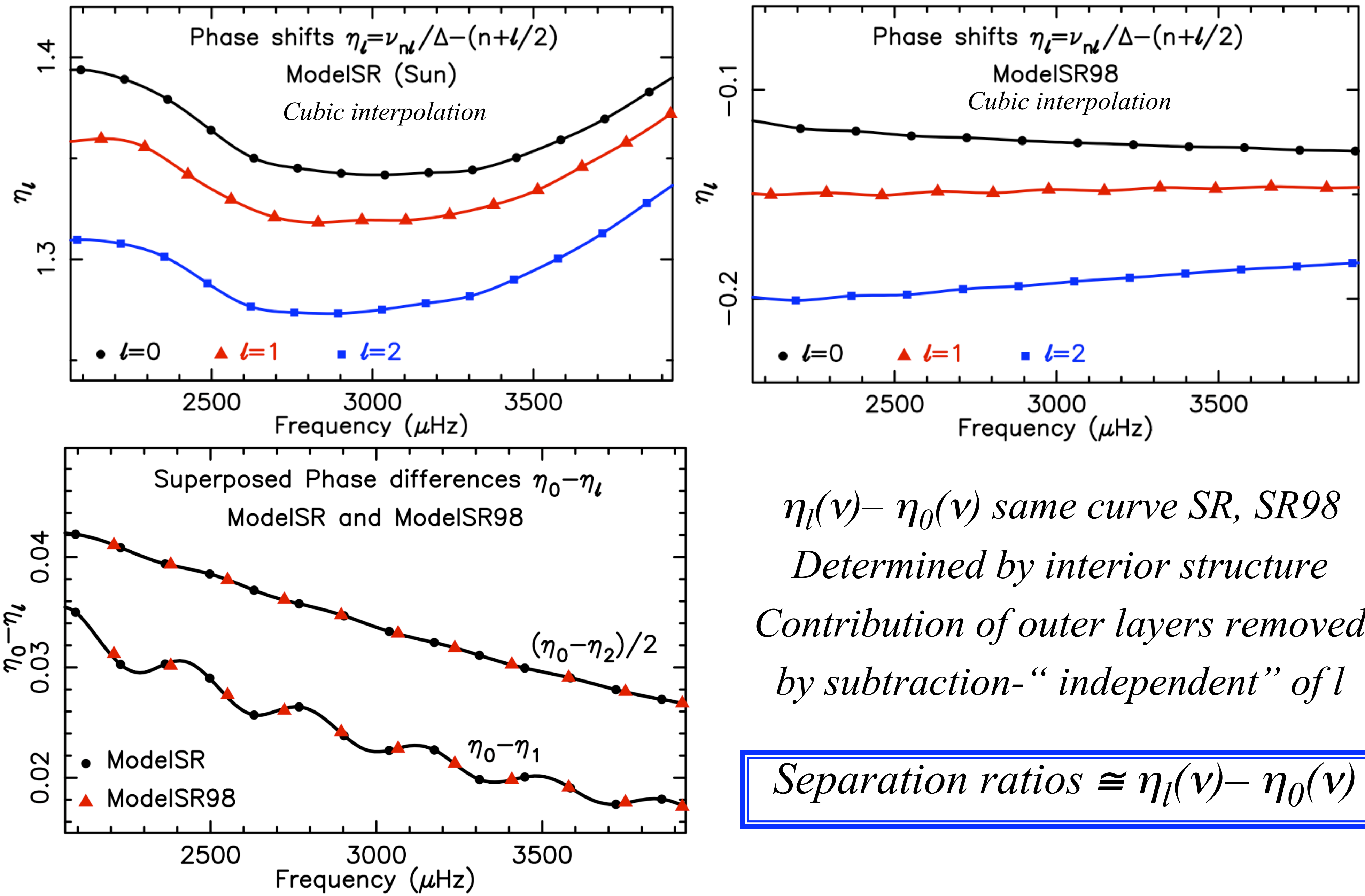
	$1.00R_\odot$			$0.98R_\odot$		
n	ν_{n0}	r_{01}	r_{02}	ν_{n0}	r_{01}	r_{02}
14	–	–	–	2294.81	0.0301	0.0789
16	2296.25	0.0308	0.0790	2637.24	0.0247	0.0759
18	2564.44	0.0251	0.0717	2979.84	0.0226	0.0664
20	2834.77	0.0238	0.0742	3322.51	0.0199	0.0607
22	3106.56	0.0221	0.0645	3665.25	0.0178	0.0557
24	3379.25	0.0196	0.0600	–	–	–
26	3653.55	0.0175	0.0561	–	–	–

It is the functional form of $r_{01}(\nu)$, $r_{02}(\nu)$ that is signature of the interior structure; sample this at different frequencies for 2 models

Need some theoretical understanding



Echelle Diagram: modelSR (Sun)
Δ=136.0(μHz)
Frequency (μHz)
Frequency (ν−25) mod Δ (μHz)
ℓ=0 ℓ=1 ℓ=2
Echelle Diagram: modelSR_98
Δ=171.5(μHz)
Frequency (μHz)
Frequency ν mod Δ (μHz)
ℓ=0 ℓ=1 ℓ=2
Phase shifts ηℓ=νnℓ/Δ−(n+ℓ/2)
ModelSR (Sun)
ηℓ
Frequency (μHz)
ℓ=0 ℓ=1 ℓ=2
Phase shifts ηℓ=νnℓ/Δ−(n+ℓ/2)
ModelSR98
ηℓ
Frequency (μHz)
ℓ=0 ℓ=1 ℓ=2



$\eta_l(\nu) - \eta_0(\nu)$ same curve SR, SR98

Determined by interior structure

Contribution of outer layers removed by subtraction-" independent" of l

Separation ratios ≅ $\eta_l(\nu) - \eta_0(\nu)$

Why does it work - theory

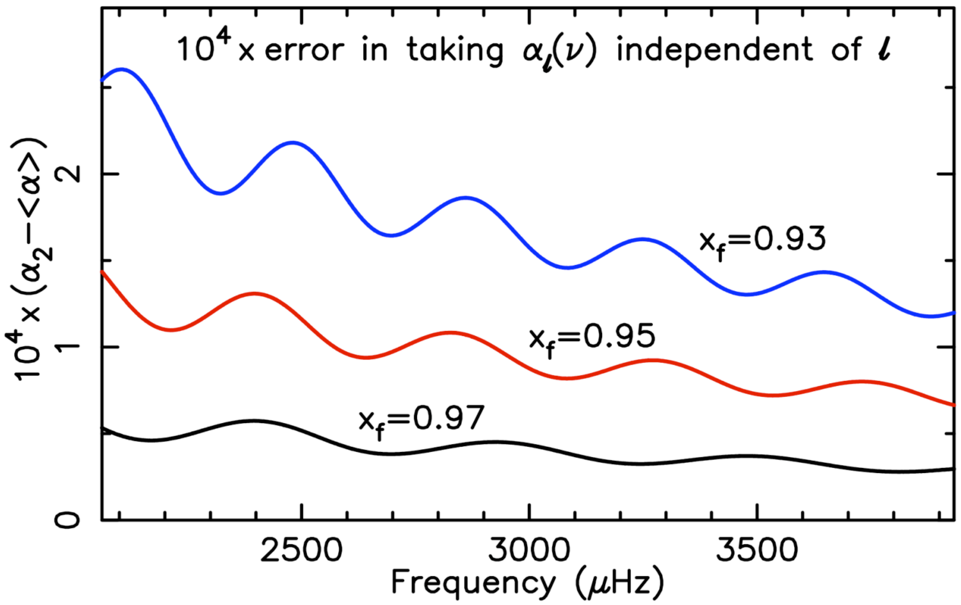
$\eta_\ell(\nu)$ determined by structure;

ψ the scaled oscillating pressure

$$\frac{2\pi\nu\psi_\ell}{d\psi_\ell/dt} = \tan[2\pi\nu t - \ell\pi/2 - \pi\,\eta_\ell(\nu, t)]$$

$$\eta_\ell(\nu, t_f) = -\int_0^{t_f} \mathcal{F}_\ell(\nu, t, \eta_\ell, \rho, P, \Gamma_1)\, dt$$

$$t_f = T, \psi_\ell = 0 : \frac{\nu_{n,\ell}}{\Delta_T} = n + \ell/2 + \eta_\ell(\nu_{n,\ell})$$



Split $\eta_\ell = \alpha_\ell - \delta_\ell$: $\alpha_\ell(\nu, t_f) = \eta_\ell(\nu, T) - \eta_\ell(\nu, t_f)$, $\delta_\ell(\nu, t_f) = -\eta_\ell(\nu, t_f)$

$\alpha_\ell(\nu, t_f) = \alpha(\nu, t_f)$ very weakly dependent on ℓ in outer layers

$\eta_0(\nu) - \eta_\ell(\nu) = \delta_\ell(\nu, t_f) - \delta_0(\nu, t_f)$ independent of α from outer layers

and very weakly varying with t_f in intermediate outer layers

Comparing observed and model separation ratios

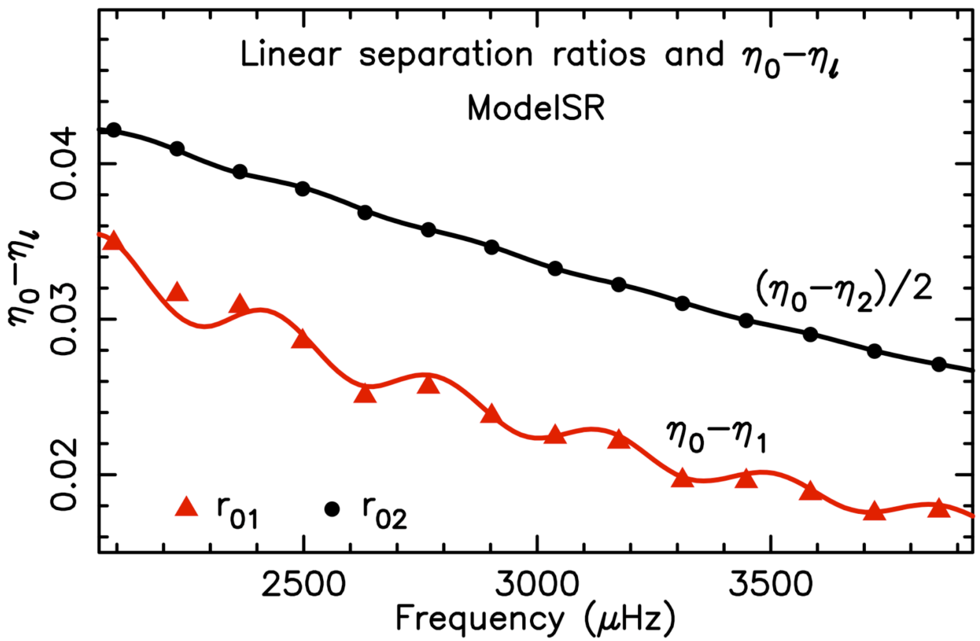
$$\eta_\ell(\nu_{n\ell}) = \frac{\nu_{n\ell}}{\Delta_T} - (n + \ell/2), \quad \eta_0(\nu_{n,0}) = \frac{\nu_{n,0}}{\Delta_T} - n : \quad \text{linear interpolation}$$

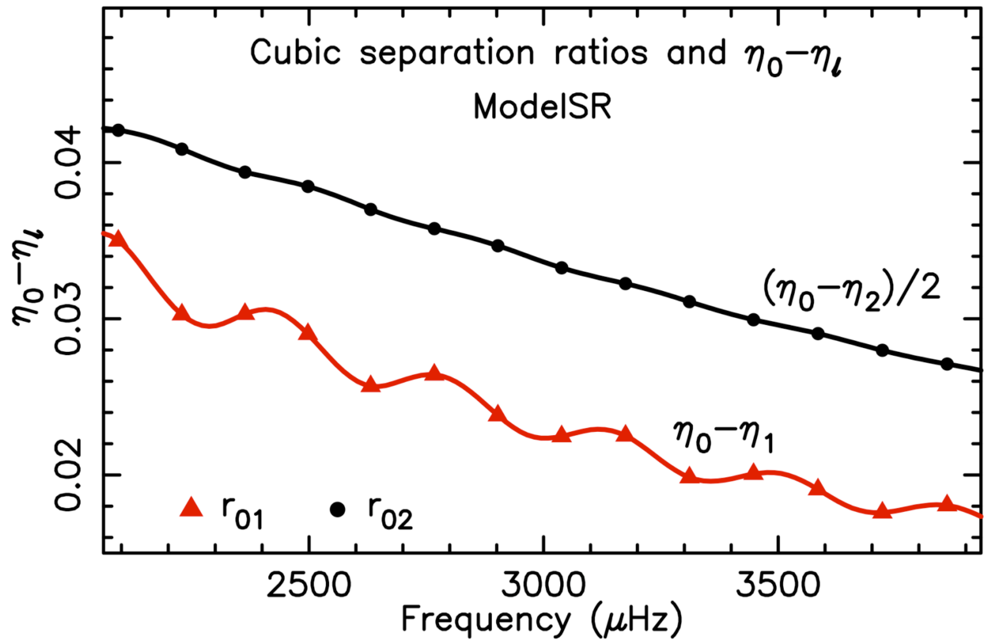
$$\eta_\ell(\nu_{n,0}) \approx \eta_\ell(\nu_{n-1,\ell}) + \frac{\eta_\ell(\nu_{n,\ell}) - \eta_\ell(\nu_{n-1,\ell})}{\nu_{n,\ell} - \nu_{n-1,\ell}}(\nu_{n,0} - \nu_{n-1,\ell})$$

$$\eta_\ell(\nu_{n,0}) - \eta_0(\nu_{n,0}) \approx r_{0,\ell} \quad \text{cubic interpolation } \eta_\ell(\nu_{n,0}) \text{ more accurate}$$

$$\chi^2 = \sum_1^N \left(\frac{r_{0\ell}^{obs}(\nu^{obs}) - r_{0\ell}^{mod}(\nu^{obs})}{\sigma_{0\ell}^r(\nu^{obs})} \right)^2$$

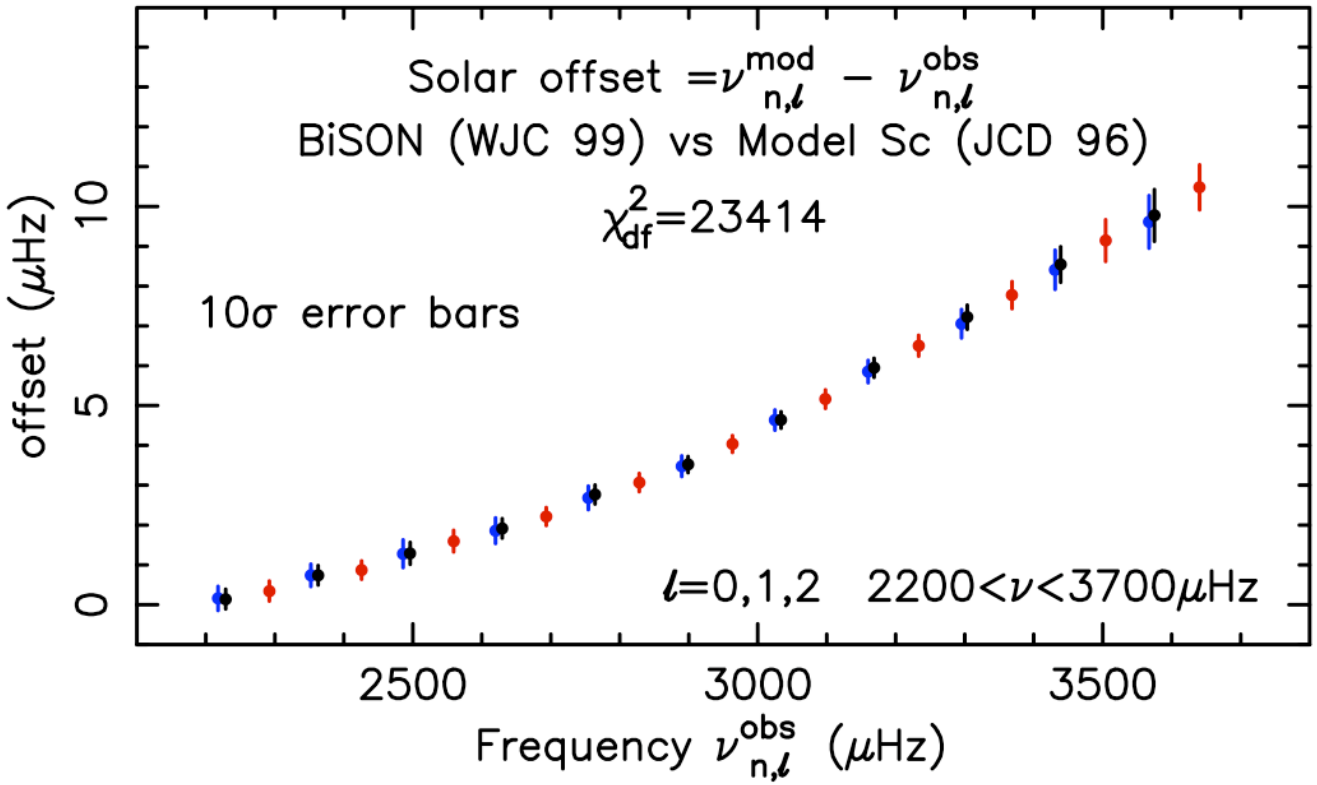
CORRECT !





Problem: Solar models don't fit solar data

$$f(\nu_{n,\ell}^{mod}) = \nu_{n\ell}^{obs} - \nu_{n\ell}^{mod} \qquad \chi_{df}^2 = \frac{1}{N_\nu}\sum^{N}\frac{(\nu_{n\ell}^{obs} - \nu_{n\ell}^{mod})^2}{\sigma_{n\ell}^2}$$



Poor modelling of outer layers - non-adiabatic convection (MLT),
Non-adiabatic oscillations – atmosphere - micro/macro physics
Need to correct for unknown effects of outer layers

Surface Layer corrections Kjeldsen et al (2008)

Fit solar offset by simple power law $f(\nu) = a\nu^b$ using average ν and Δ

Fit average $<\nu_{n,0}>$ *and* $<\Delta>$ [*least squares fit to* $\nu_{n,0} - <\nu_{n,0}>$]

For other stars and models:

Assume b is <u>universal constant</u> the same (4.9) for all stars/models

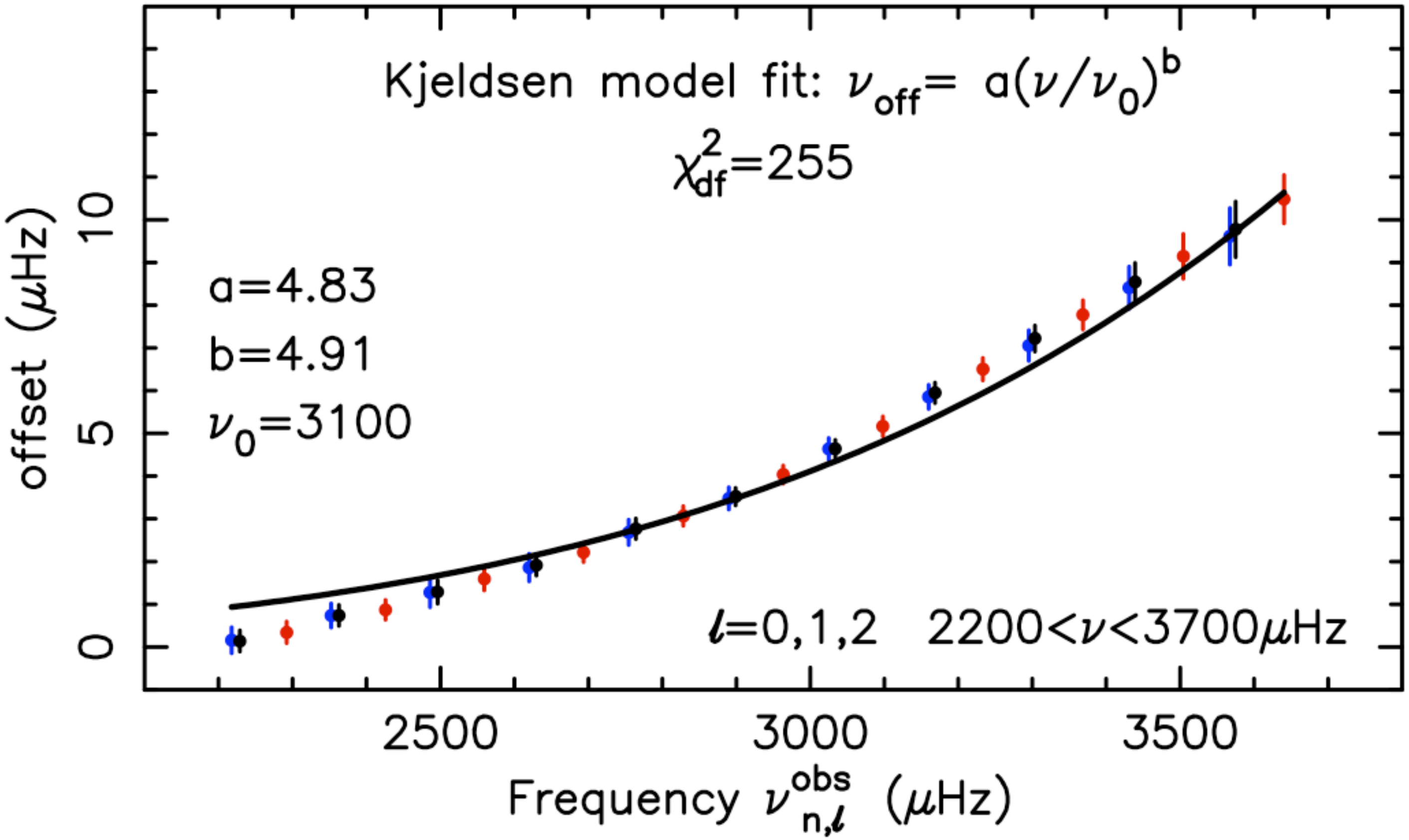
Determine a by scaling average ν and Δ

Add this to model ν then test goodness of fit of "corrected" ν to data ν

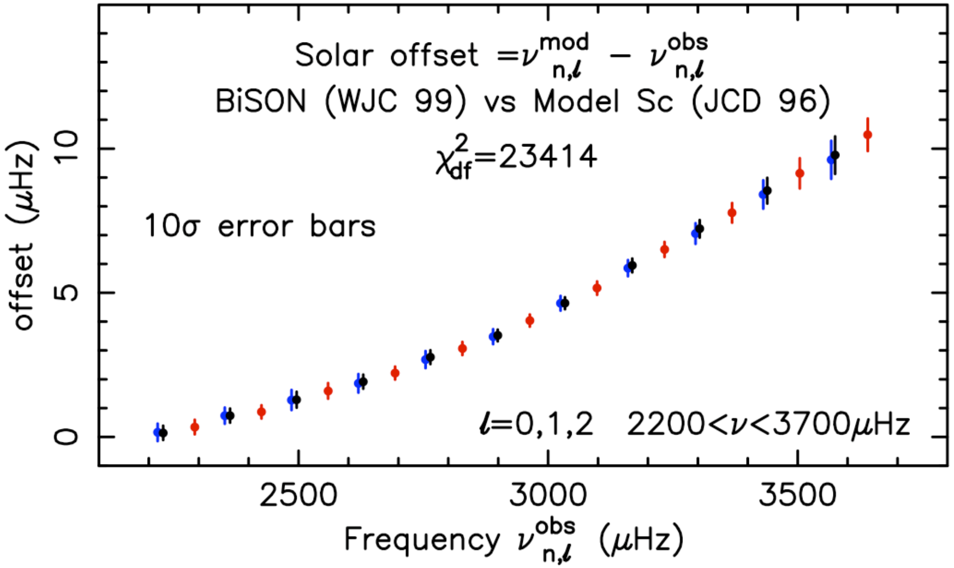
This is now included in Asteroseismic Portal (Metcalfe e al 2009) !!

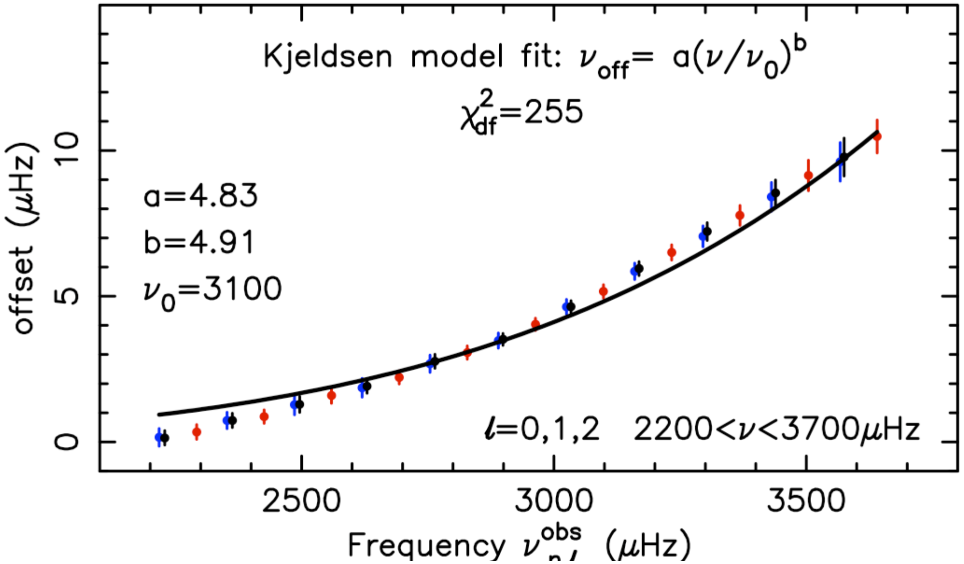
Kjeldsen et al (2008) model of Solar offset

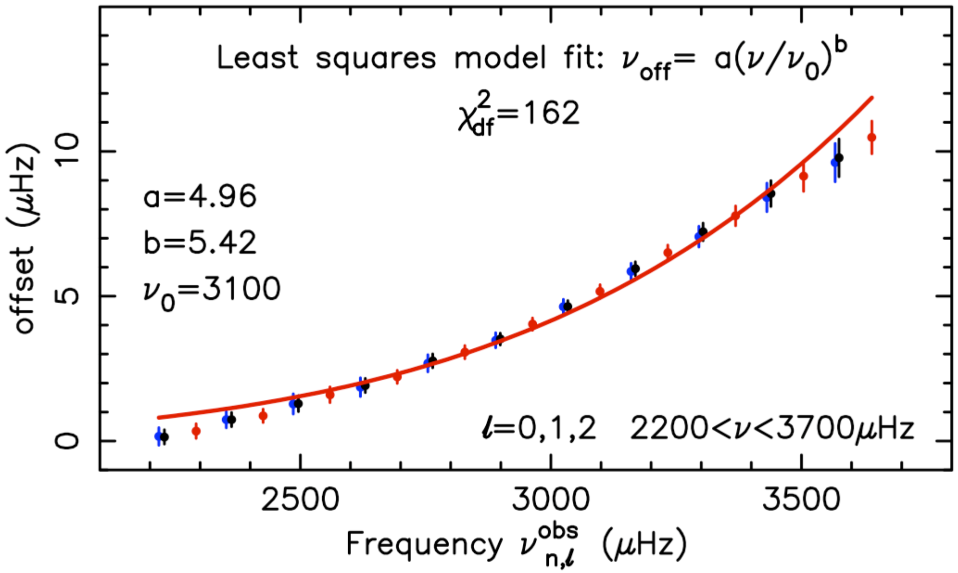
$$f(\nu_{n,\ell}) = a\left[\frac{\nu_{n\ell}^{obs}}{\nu_0}\right]^b \qquad \chi^2_{df} = \frac{1}{N_\nu - 2}\sum^{N_\nu - 2}\frac{(\nu_{n\ell}^{obs} - \nu_{n\ell}^{mod} - f(\nu_{n,\ell}^{mod}))^2}{\sigma^2_{n\ell}}$$

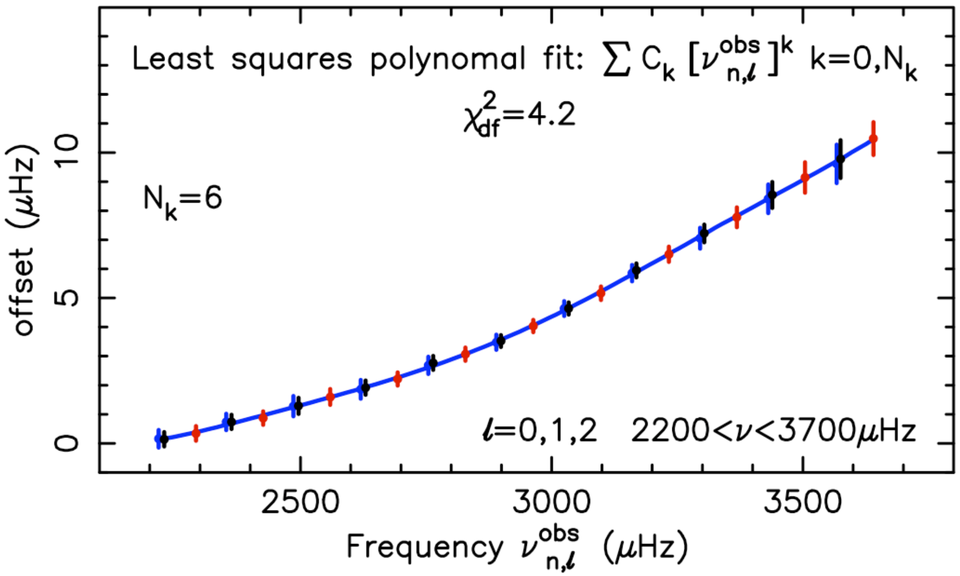


Fits to solar offset









Surface Layer corrections Kjeldsen et al

Fit solar offset by simple power law $f(\nu) = a\nu^b$ using average ν and Δ

Assume b is <u>universal constant</u> the same (4.9) for all stars/models

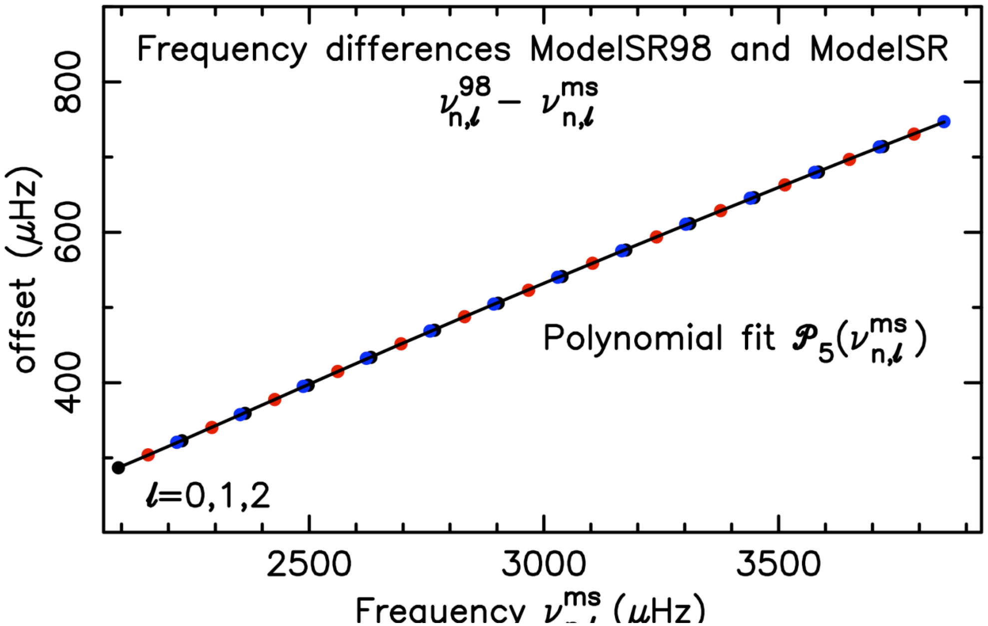
Determine a by scaling average ν and Δ

Add this to model ν then test goodness of fit of "corrected" ν to data ν

1. Simple power law fit bad even for Sun
2. There is neither theoretical nor observational justification for taking b as a <u>universal constant</u>! … expect $b=b\,(g,\ T_{eff},\ \alpha,\ X,\ Z_k,\ ...$

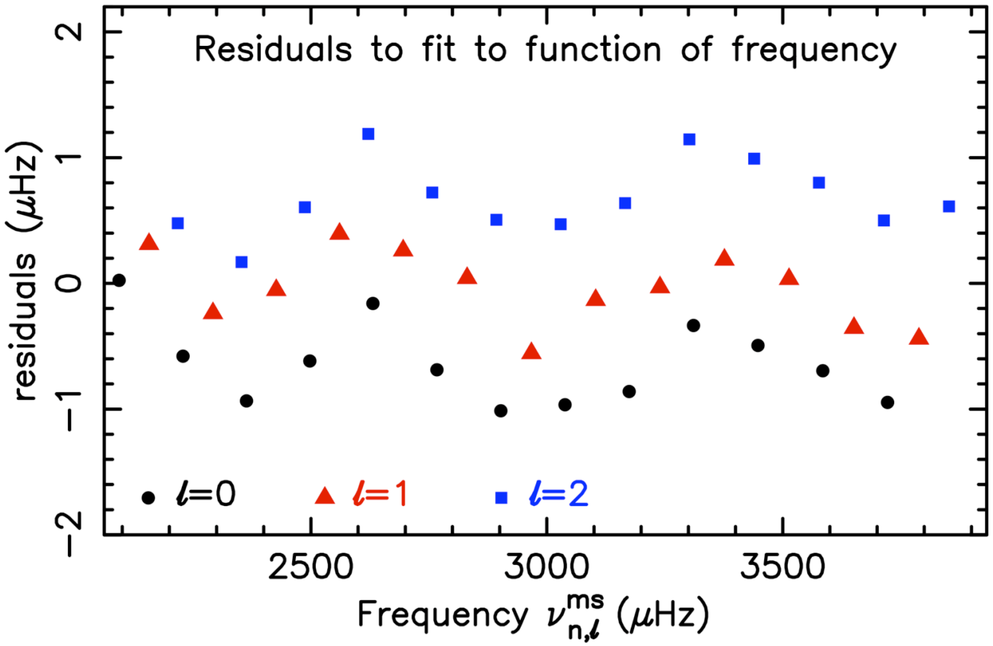
Adding a "correction" function f(ν) to model frequencies – best fit "corrected" model will not have internal structure of observed star

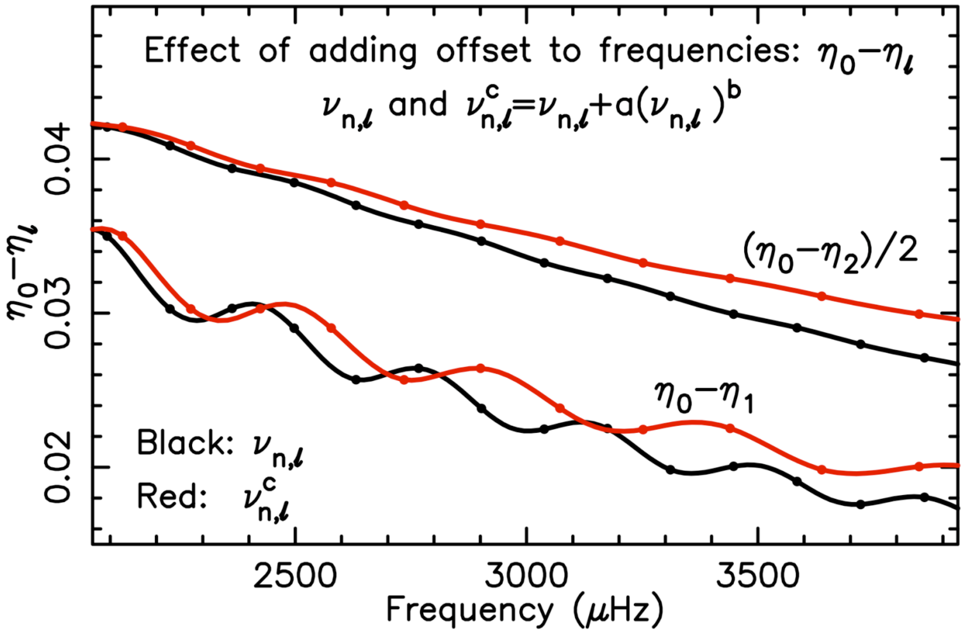
Effect of adding *f(ν)* to frequencies of model with same interior

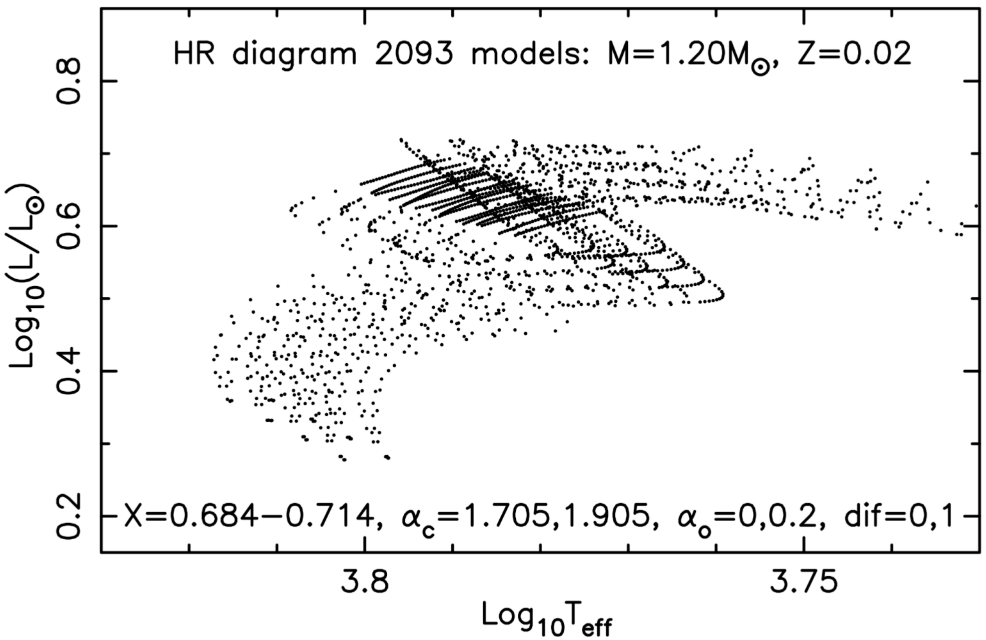


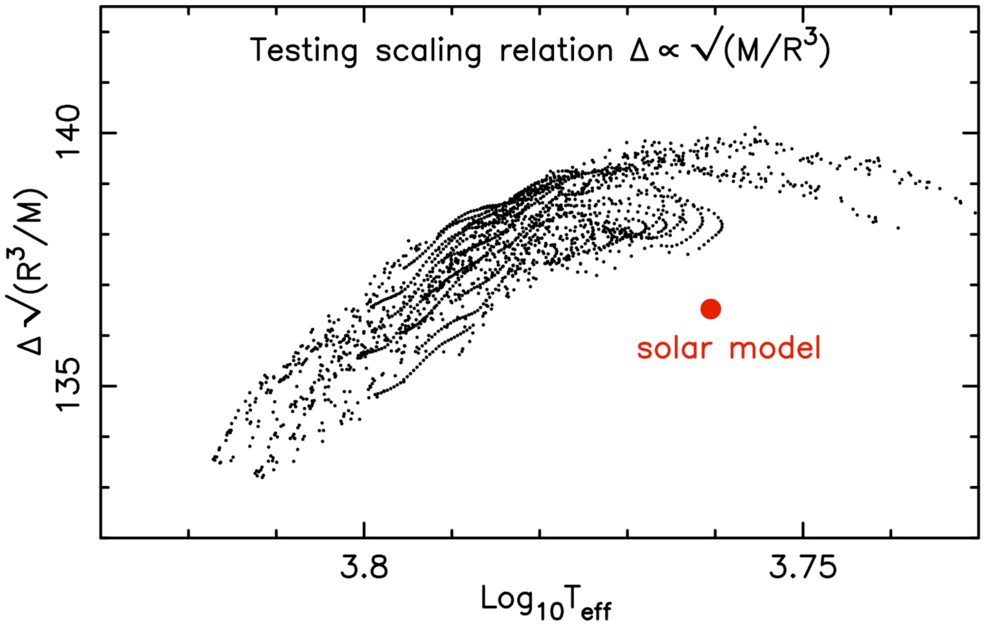
If same interior but added f(ν) doesn't fit frequencies

If add offset to model the phase differences $\eta_0 - \eta_l$ are not same









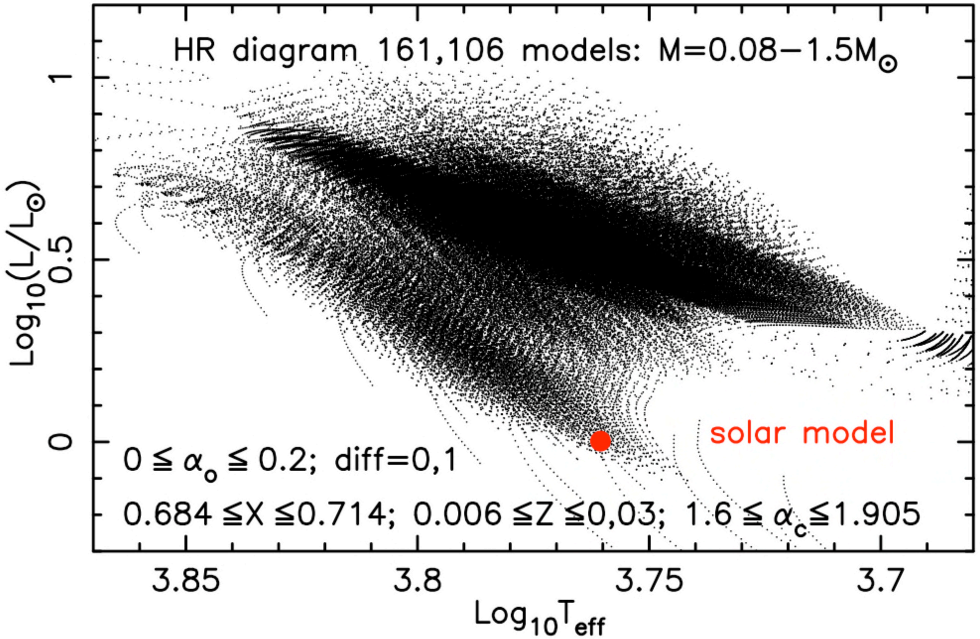
Testing the scaling relation on large separations

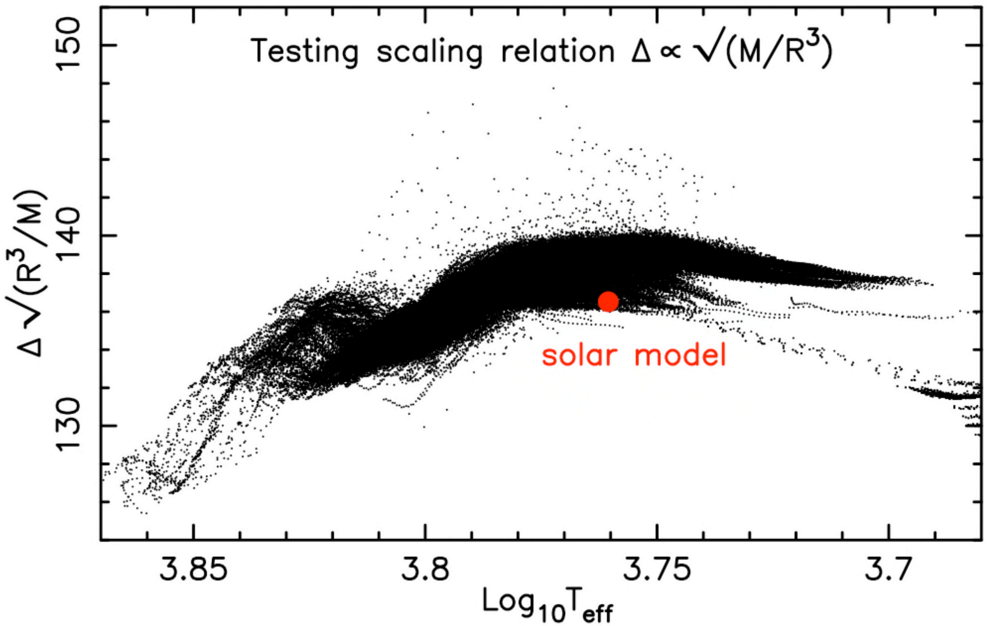
$$\Delta \propto \left(\frac{M}{R^3}\right)^{1/2}$$

$$\Delta_s = \Delta \left(\frac{R}{R_\odot}\right)^{3/2} \left(\frac{M_\odot}{M}\right)^{1/2} = \text{constant}$$

$$133 < \Delta_s < 140\ \mu\text{Hz}$$

Models: CLES (Miglio)+iwr





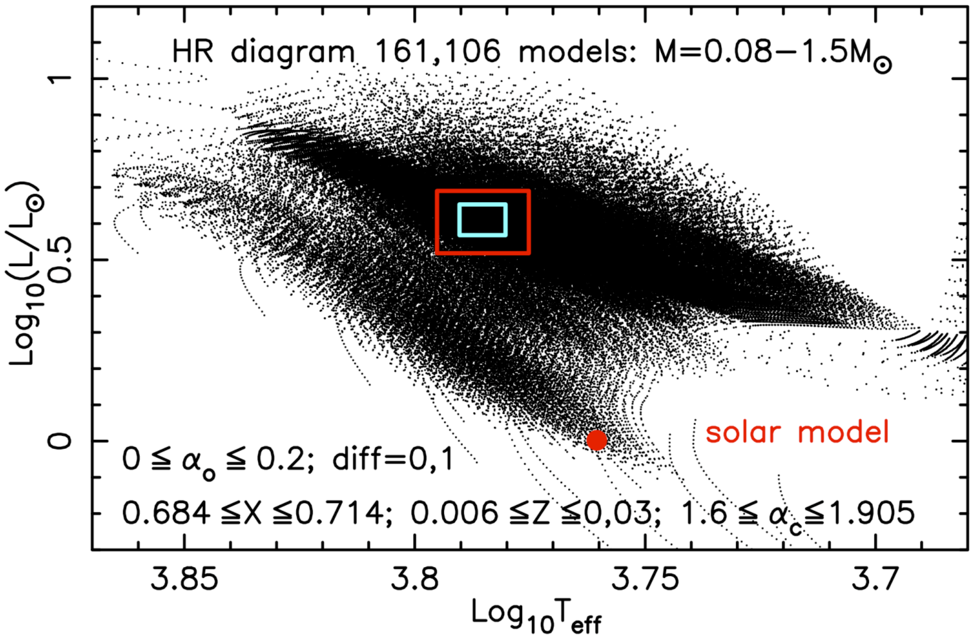
Testing the scaling relation on large separations

$$\Delta \propto \left(\frac{M}{R^3}\right)^{1/2}$$

$$\Delta_s = \Delta \left(\frac{R}{R_\odot}\right)^{3/2} \left(\frac{M_\odot}{M}\right)^{1/2} = \text{ constant}$$

$$120 < 126 < \Delta_s < 142 < 148 \ \mu\text{Hz}$$

Models: CLES (Montalban, Miglio)+iwr

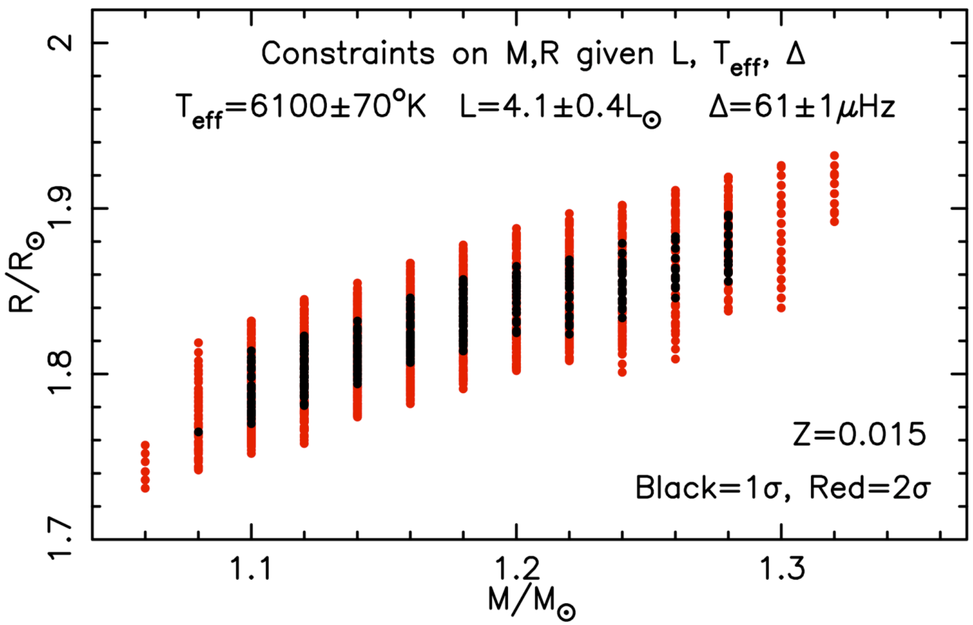


Constraining Mass using Large Separation

$L = 4.1 \pm 0.4\ L_{sun}$

$T_{eff} = 6100 \pm 70\ ^oK$

$\Delta = 61.0 \pm 1.0\ \mu Hz$



$1\sigma : 1.10 < M/M_{sun} < 1.28$
290 models 3.5-6.1 $x10^9y$

$2\sigma : 1.06 < M/M_{sun} < 1.32$
1153 models 3.2-6.6 $x10^9y$

Mass not well constrained !

Conclusions

1) *Separation ratios to be compared at **same** frequency (interpolate model values)*
2) *Surface offset changes properties of model - if a "corrected" model fits observations interior structure not same. Better not to use at all.*
3) *Large separations does not accurately scale with $(M/R^3)^{1/2}$ - considerable error*
4) *Take care when fitting observations to models!*