Puzzling features of data from asteroseismology space missions

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Motivation

- Old problems like the search for the solar g-modes still unsolved.

- New questions to be answered like the huge number of frequencies detected in some intermediate mass stars.

- Data Analysis in Asteroseismology: Fourier Transform, Wavelet Transform, etc. all based in square integrable functions.

- No test of the assumptions are done.

- In this work we try to answer this question: Does the time series comes from an analytic function? Surprisingly, **the most plausible answer is negative**

Outline

- Objective: **Analyticity test**.
- An extension of the continuity concept to discrete time series: the **conectivities**.
- Two approximations: **spline polynomials** and autoregressive methods (**ARMA**).
- A case of study: **HD174936 (ID7613). Simulation**
- Light curves from **CoRoT, Kepler and SOHO**.
- Conclusions.

Objective

The discrete version of the Parseval's Theorem states that:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

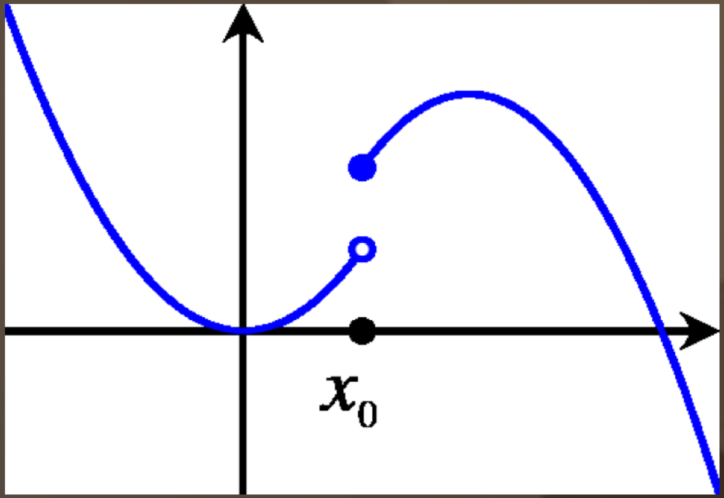
Assuming that the series x[n] is a square summable series, that is, the series x[n] satisfies:

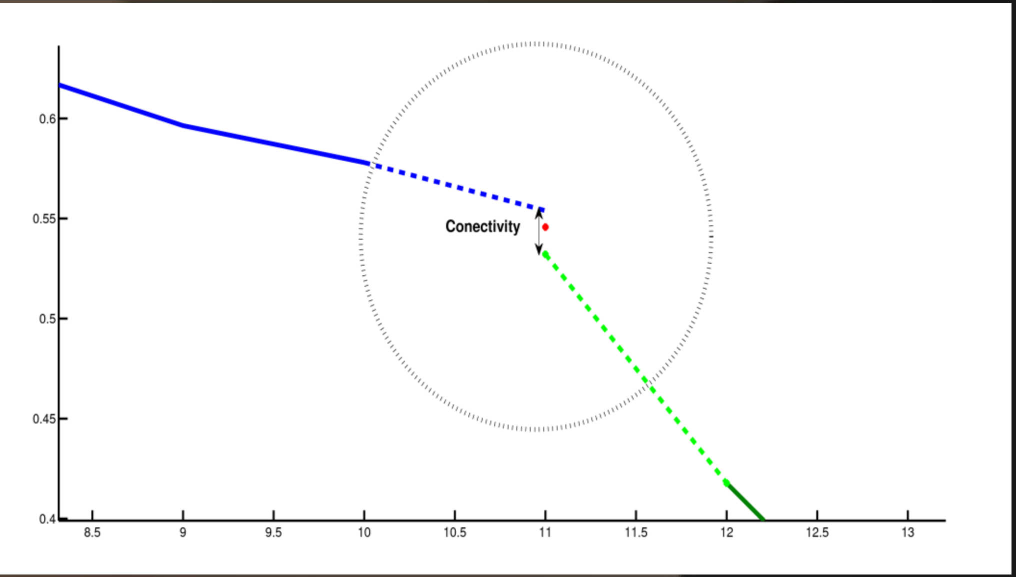
$$\sum_{n=-\infty}^{\infty} |x_n| < \infty$$

Then x[n] can be expressed as: $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k].e^{i2\pi kn/N}$

This is true only under the condition of uniform continuity or analyticity of the data series. If this condition is not met Parseval's theorem is no longer valid and neither Fourier nor any others methods based on that assumption should be applied. Before applying any analysis technique we need to test for the analyticity of the data series.

Conectivities





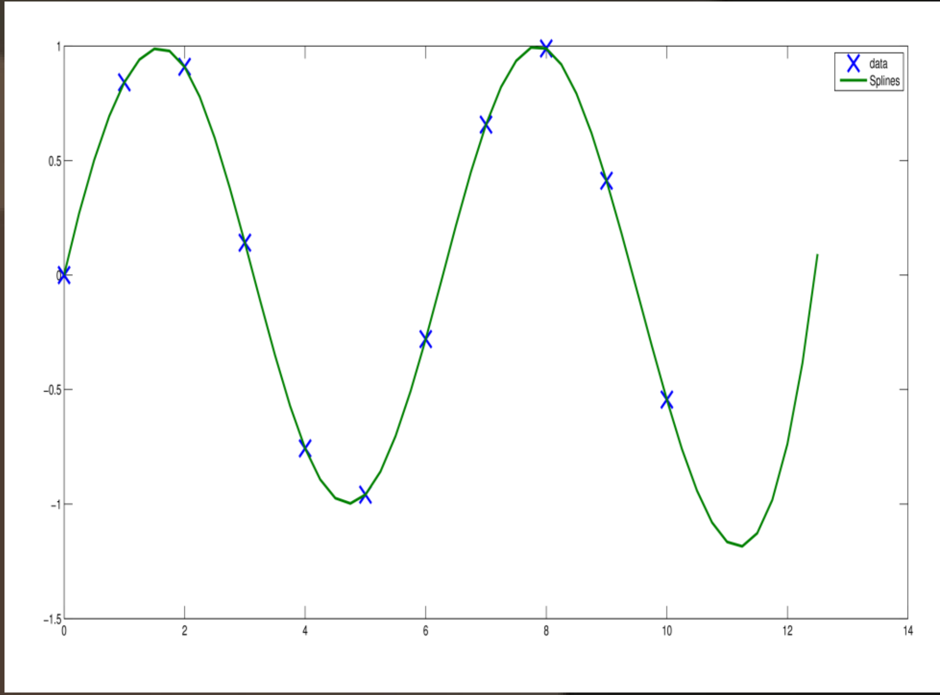
In functional analysis, it is a condition for continuity at a point x0 that both one-sided limits must exist at x0 and be equal to f(x0)

- Forward and backward extrapolations at a point from the data bracketing the point as an approximation for one-sided limits.

- Random not-correlated (white noise) differences as approximation of continuity.

Two approximations: Splines

- A fitting model which can approximate arbitrarily well continuous functions.
- Stone-Weierstrass theorem: a function that is uniformly continuous in a closed interval [a,b] can be approximated arbitrarily well by a polynomial of degree n.
- In our case, we add an additional constraint to the polynomials: analyticity.



- The problem is posed in a general way and consequences may apply also for a harmonic basis (Fourier) and a smooth basis, in general.
- We use splines as approximation, a smooth piecewise polynomial function. These function have their first and second derivatives continuous, therefore, fullfilling our requirements for the test of analyticity.
- Splines also avoid Runge's phenomenon.

Two approximations: ARMA

- Second approximation: Autoregressive methods
- Autoregressive methods do not make use of any form of analytic expression.

$$x_n + a_{n-1}x_{n-1} + a_{n-2}x_{n-2} + \ldots + a_{n-p}x_{n-p} = \varepsilon_n \longrightarrow x_n + \sum_{i=1}^{p} a_i x_{n-i} = \varepsilon_n$$

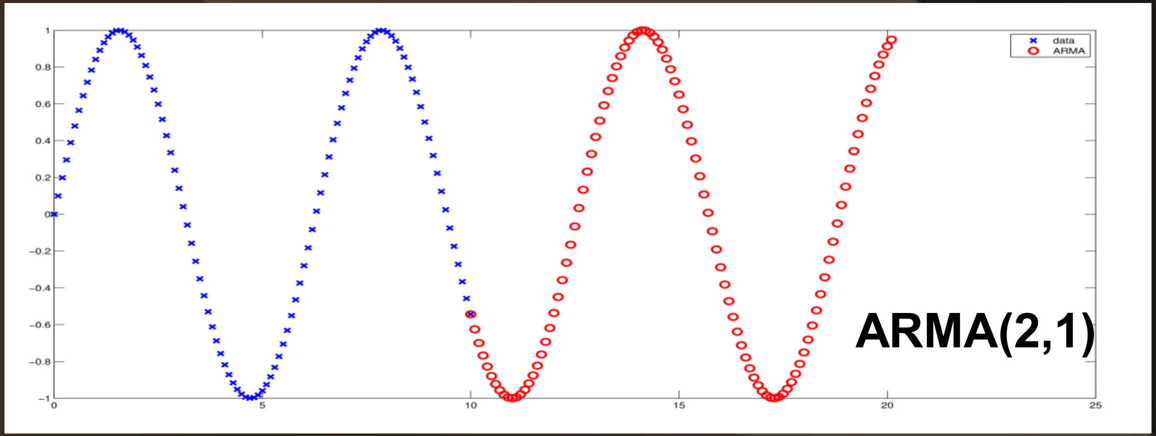
AR(p)

- We fit autoregressive moving average models (ARMA) which are defined through a recursive formula like this:

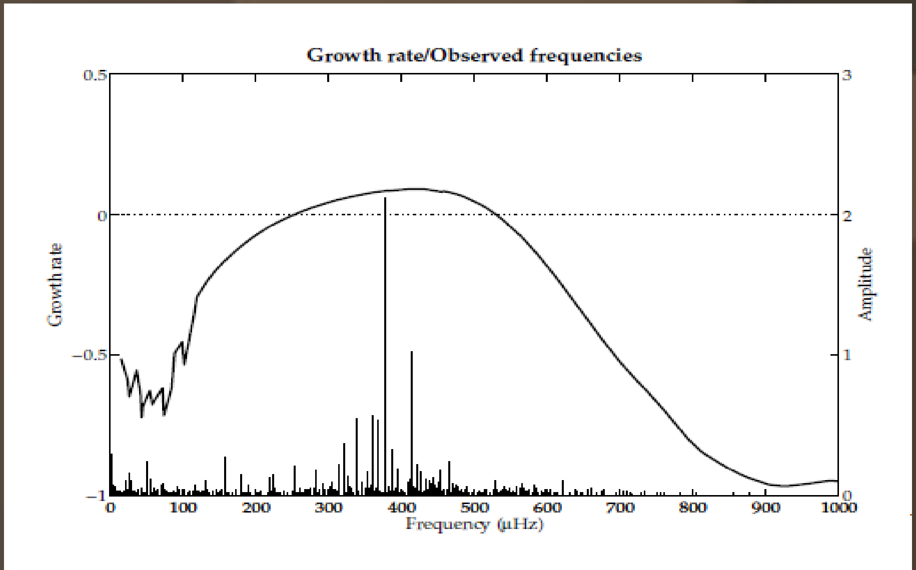
$$x_n + \sum_{i=1}^{p} a_i x_{n-i} = \varepsilon_n + \sum_{j=1}^{q} b_j \varepsilon_{n-j}$$

ARMA(p,q)

An ARMA model can represent a sine wave with only three terms. The number of terms increase according to the frequency content of the signals.

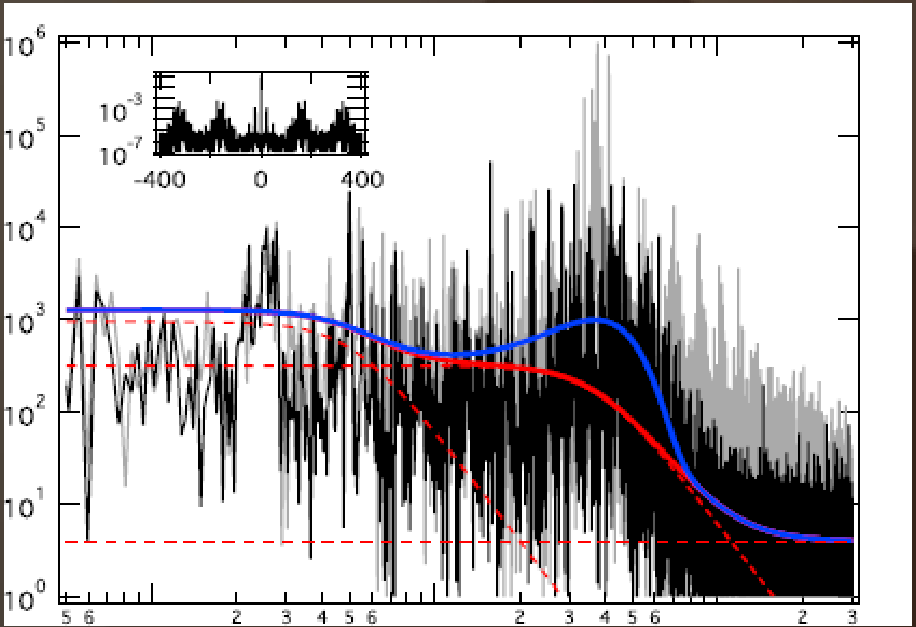


Case of study: HD 174936



García-Hernandez et al. 2009, A&A 509, 79

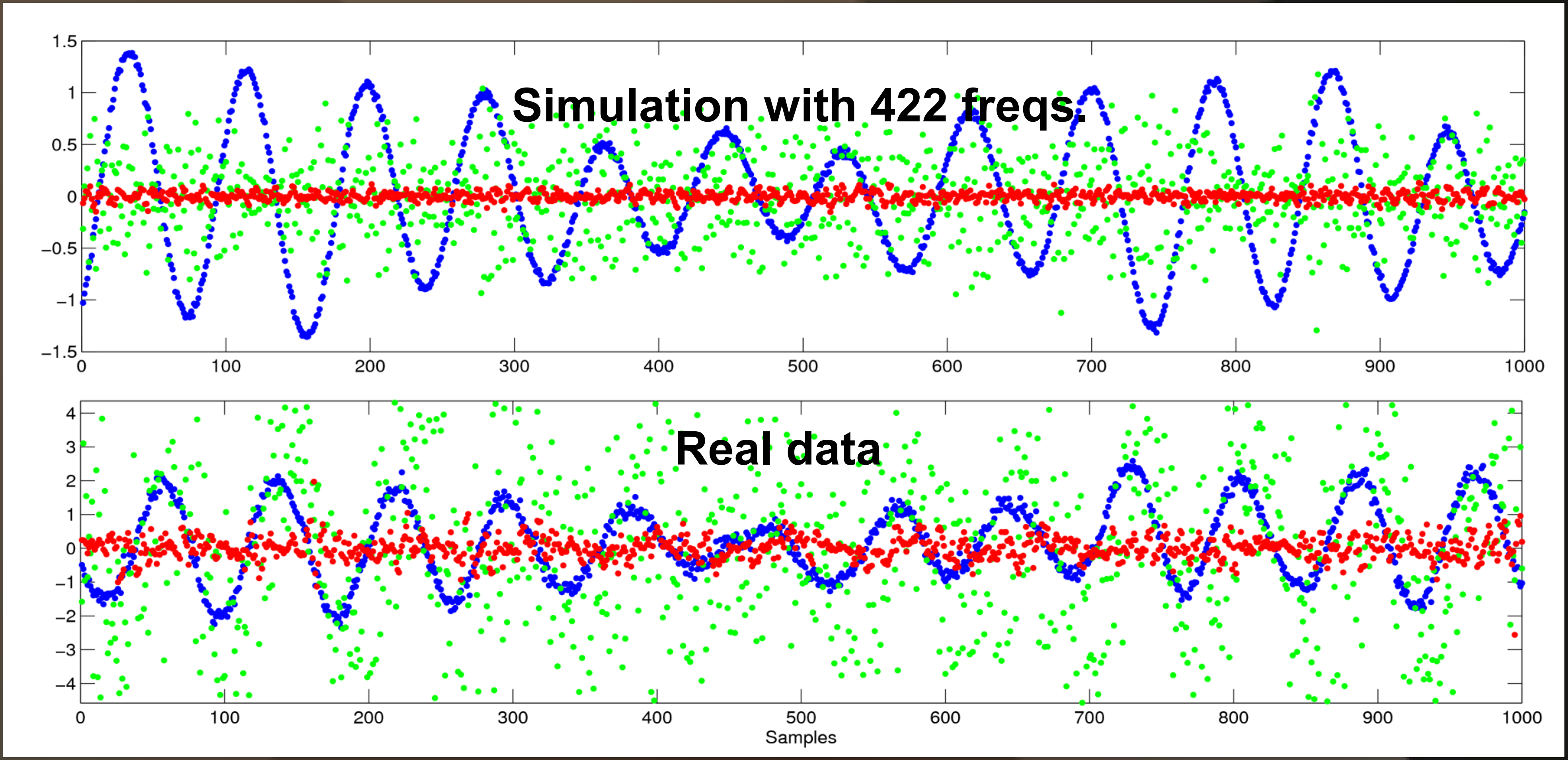
- 422 frequencies detected
- Not one of the models studied give the necessary instability range to represent the full range in the observations.



Kallinger & Matthews, 2010 ApJ 711 L35

- They propose that most of the peaks in Fourier spectra are the signature of a non-white granulation background noise and less than a 100 freqs are actual p-modes.

Case of study: HD 174936
Real data VS simulations



Splines do not reduce residuals to a white noise process.

Case of study: HD 174936
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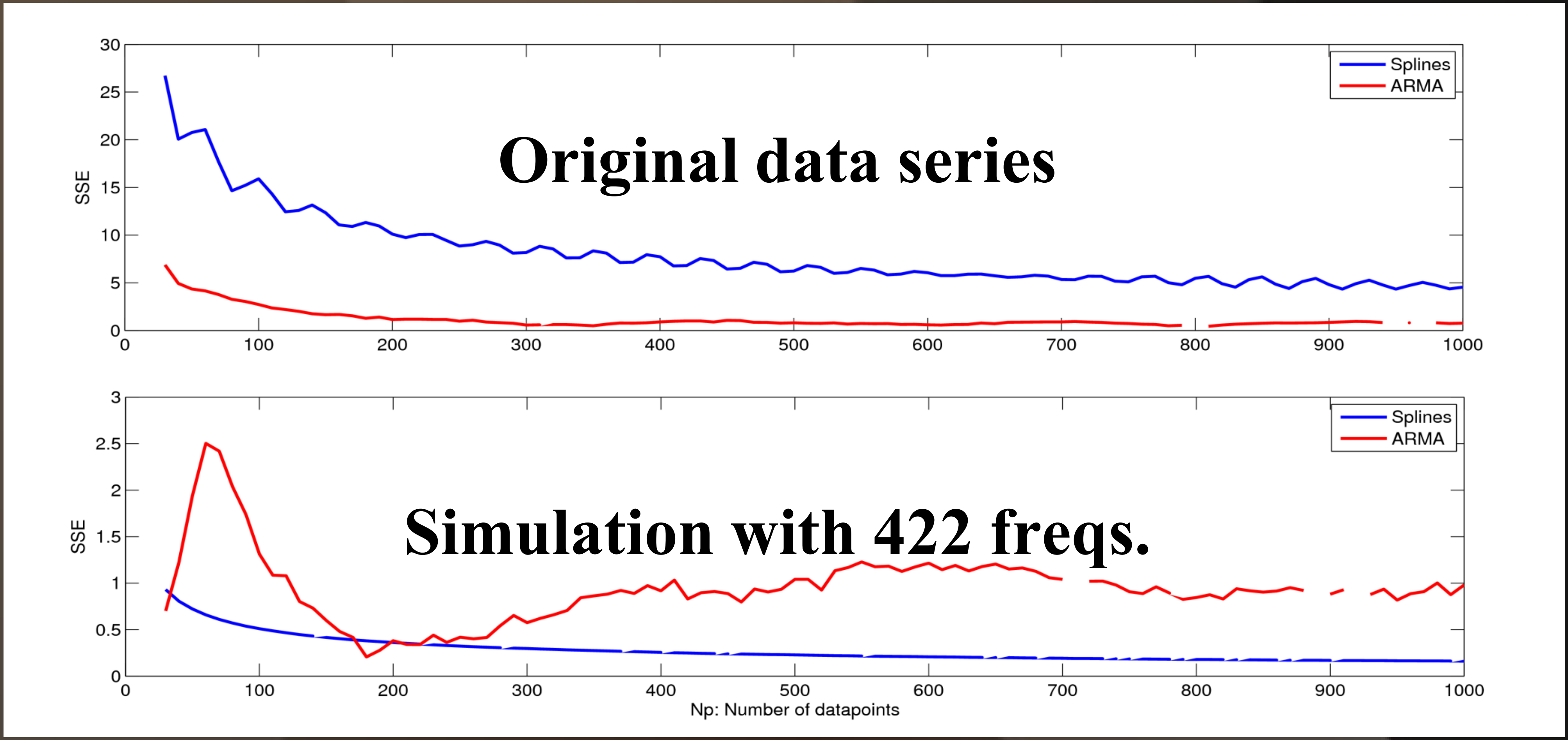


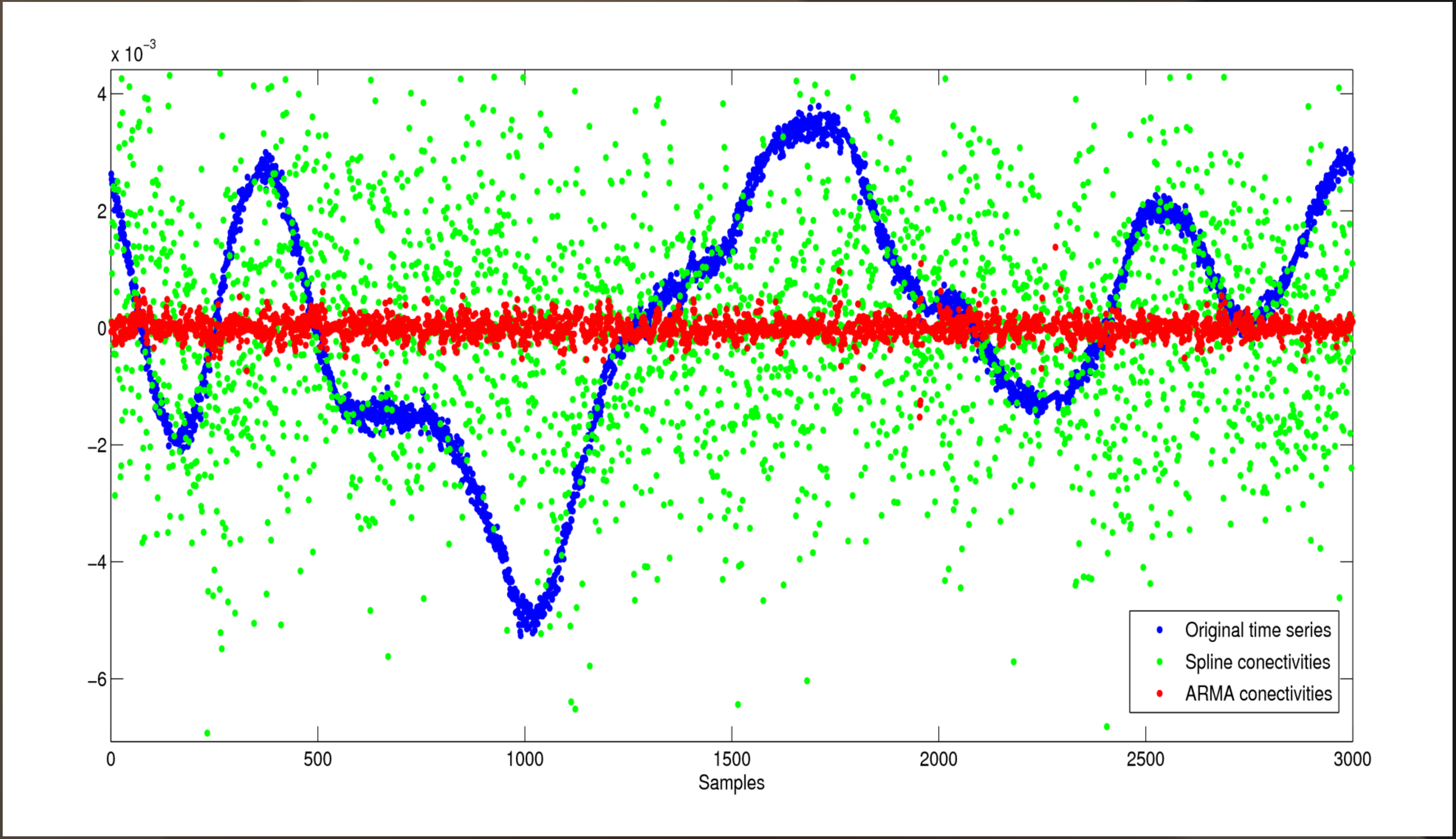
ARMA(20,1)

Case of study: HD 174936

The huge number of frequencies detected in the light curve analysis of HD 174936 cannot reproduce the original light curve but a few terms of an autoregressive model does it.
In short, what is the meaning of the term frequency?

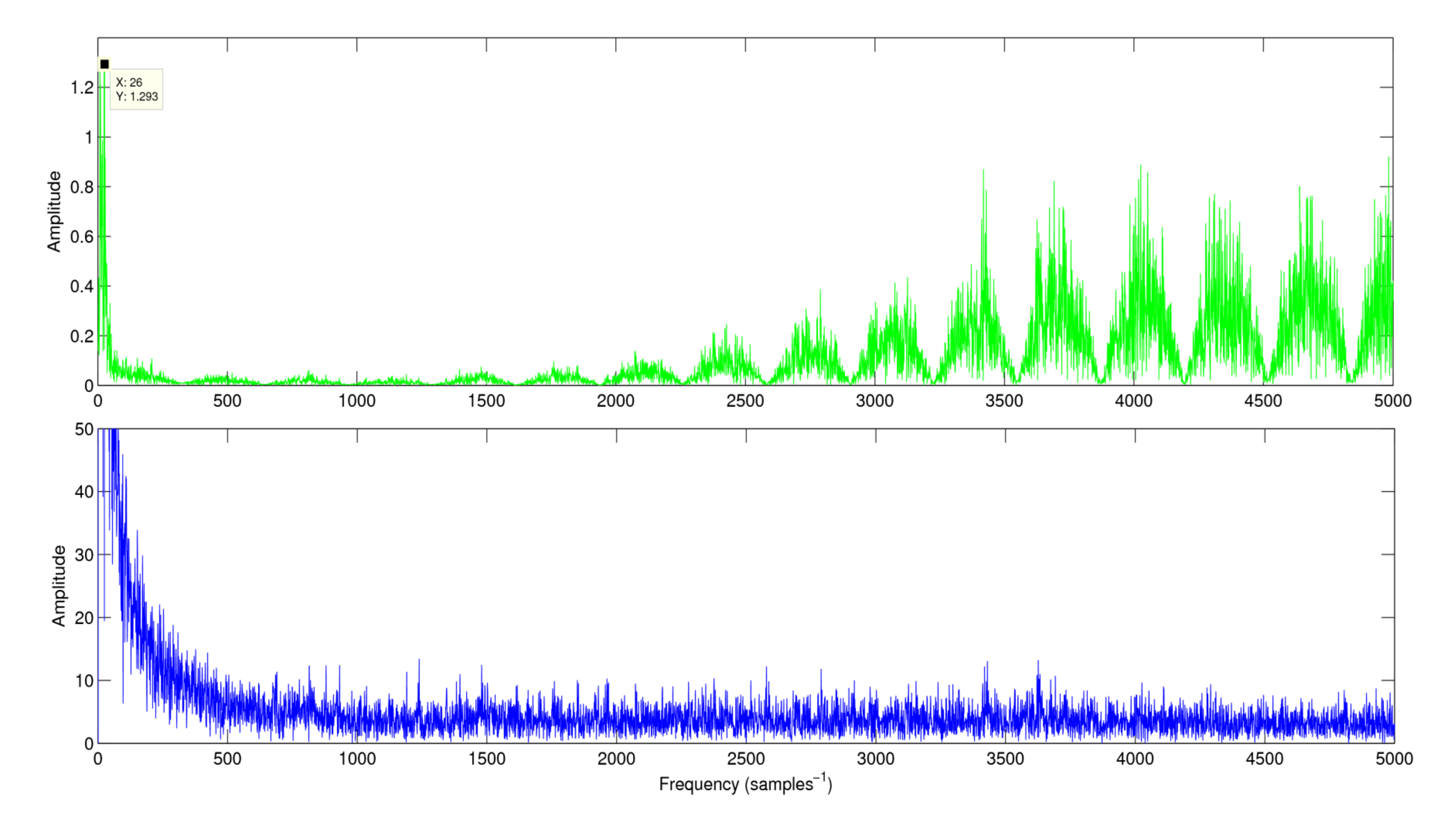
Light curves from CoRoT, Kepler and SOHO

- CoRoT 100 – HD 49434 np = 30 ARMA(20,1)



Light curves from CoRoT, Kepler and SOHO

- CoRoT 100 – HD 49434 Frequency domain.
- Spline conectivities (green), Original data (blue)



Light curves from CoRoT, Kepler and SOHO

- CoRoT 100 – HD 49434 Frequency domain.
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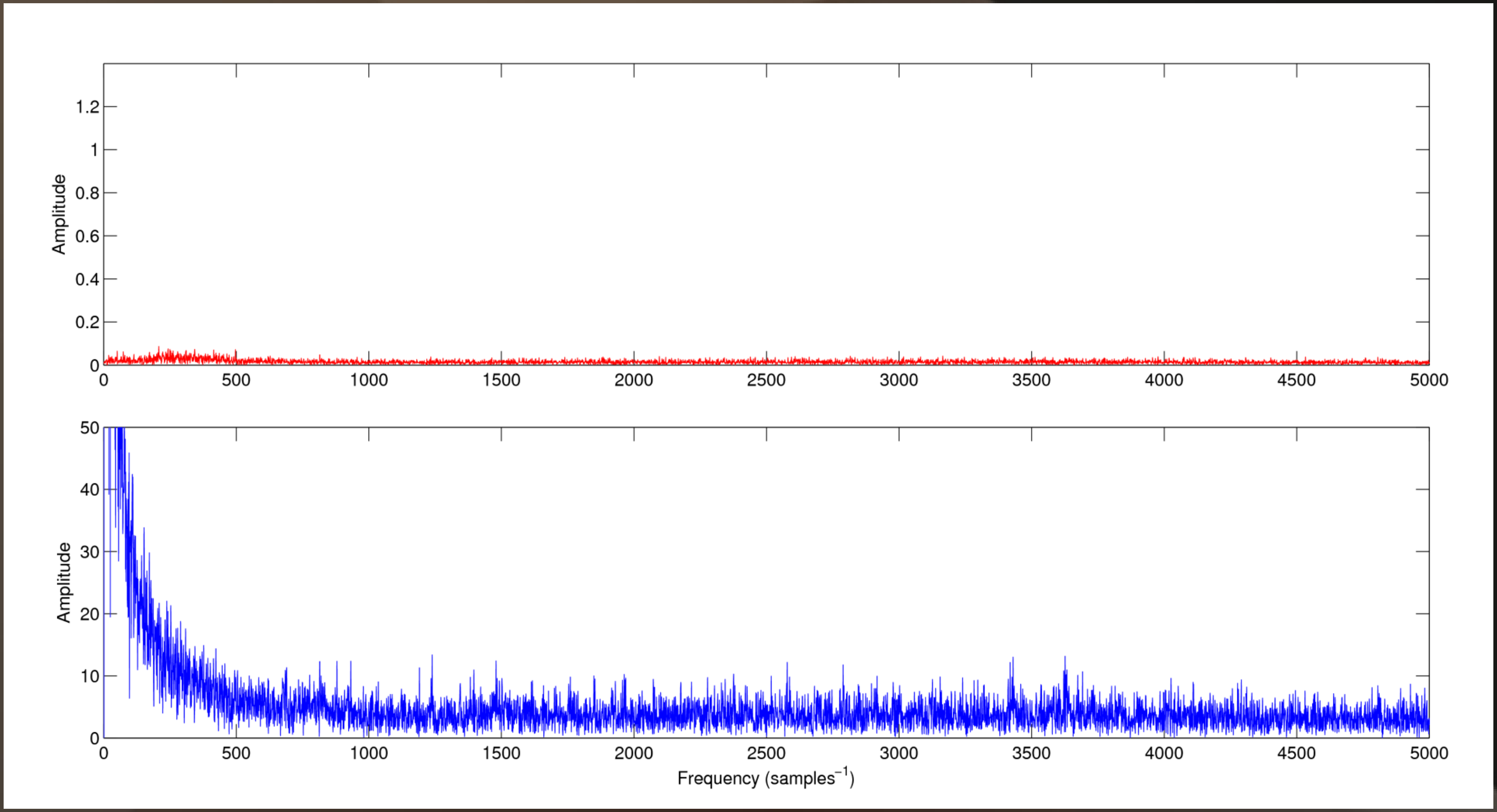
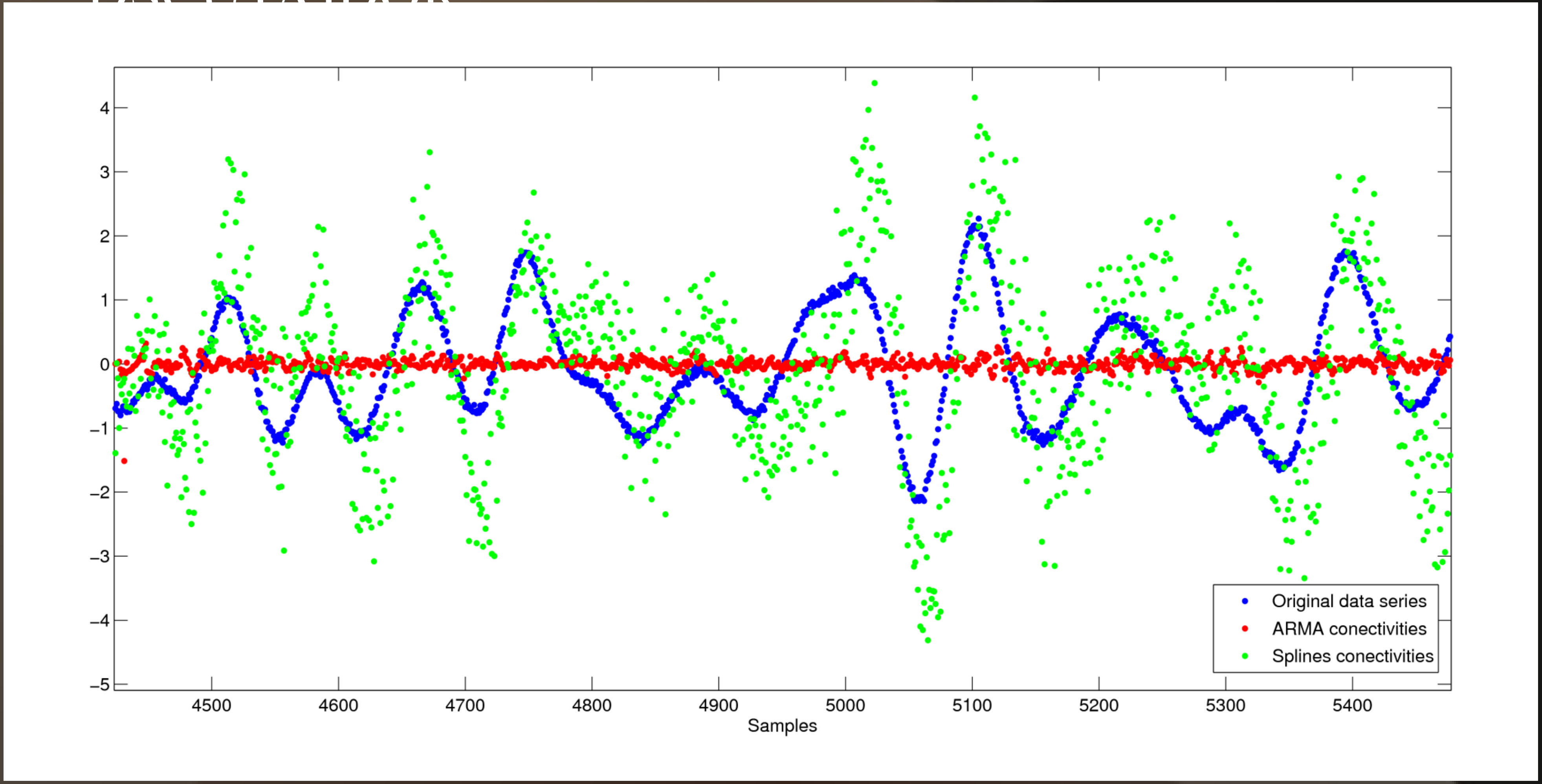


Light curves from CoRoT, Kepler and SOHO

- KIC 006187665 HYB DSCUT/GDOR



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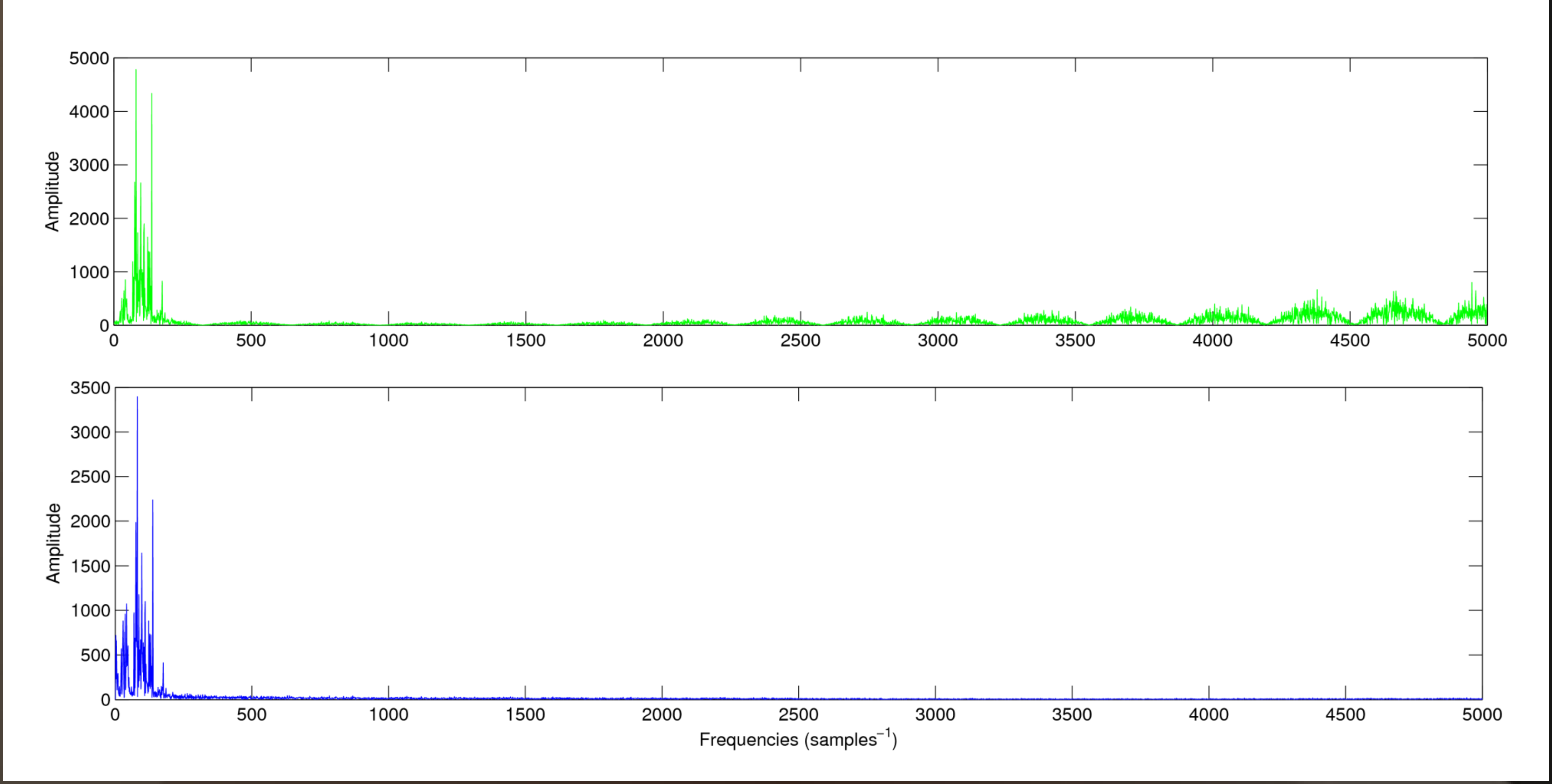

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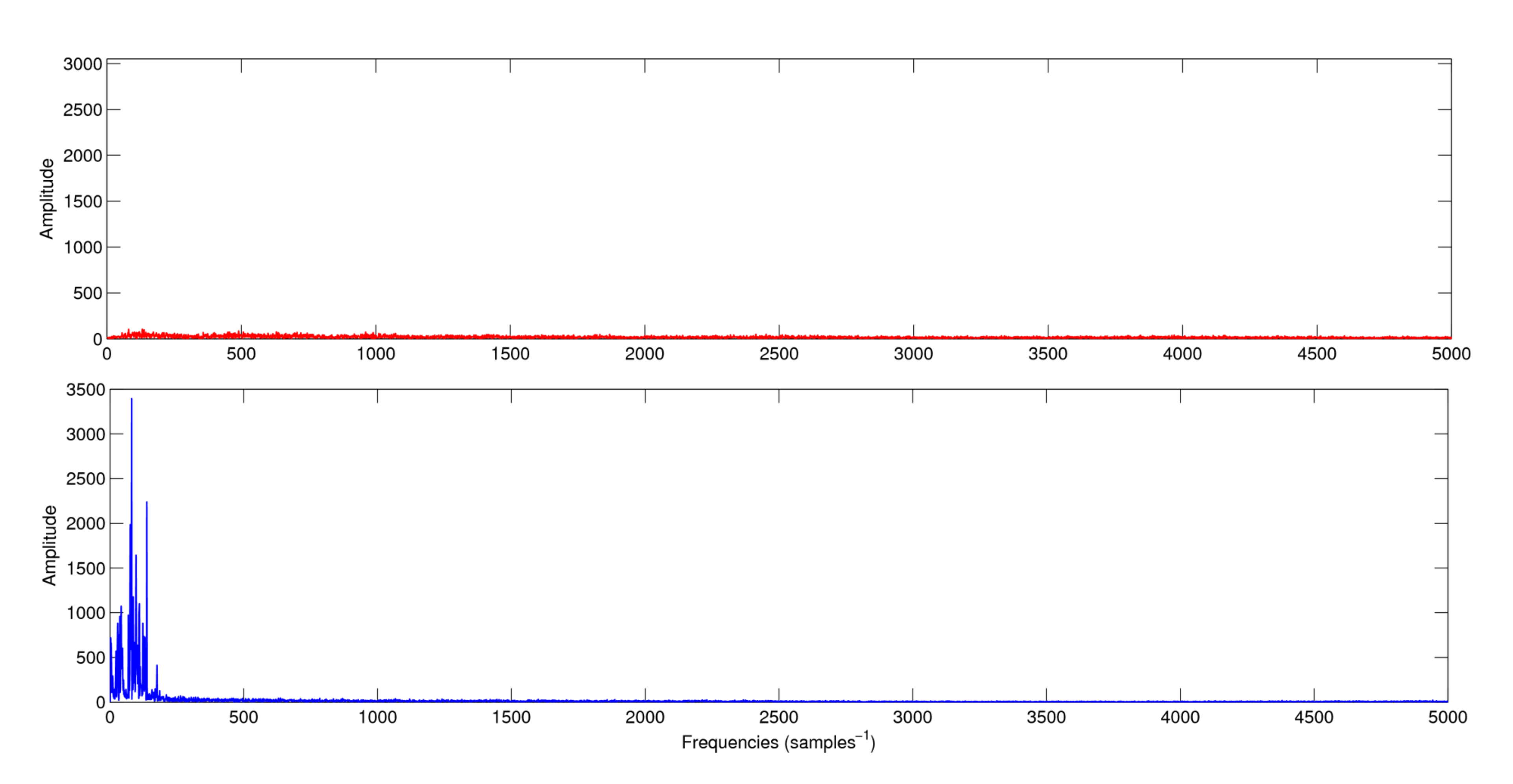
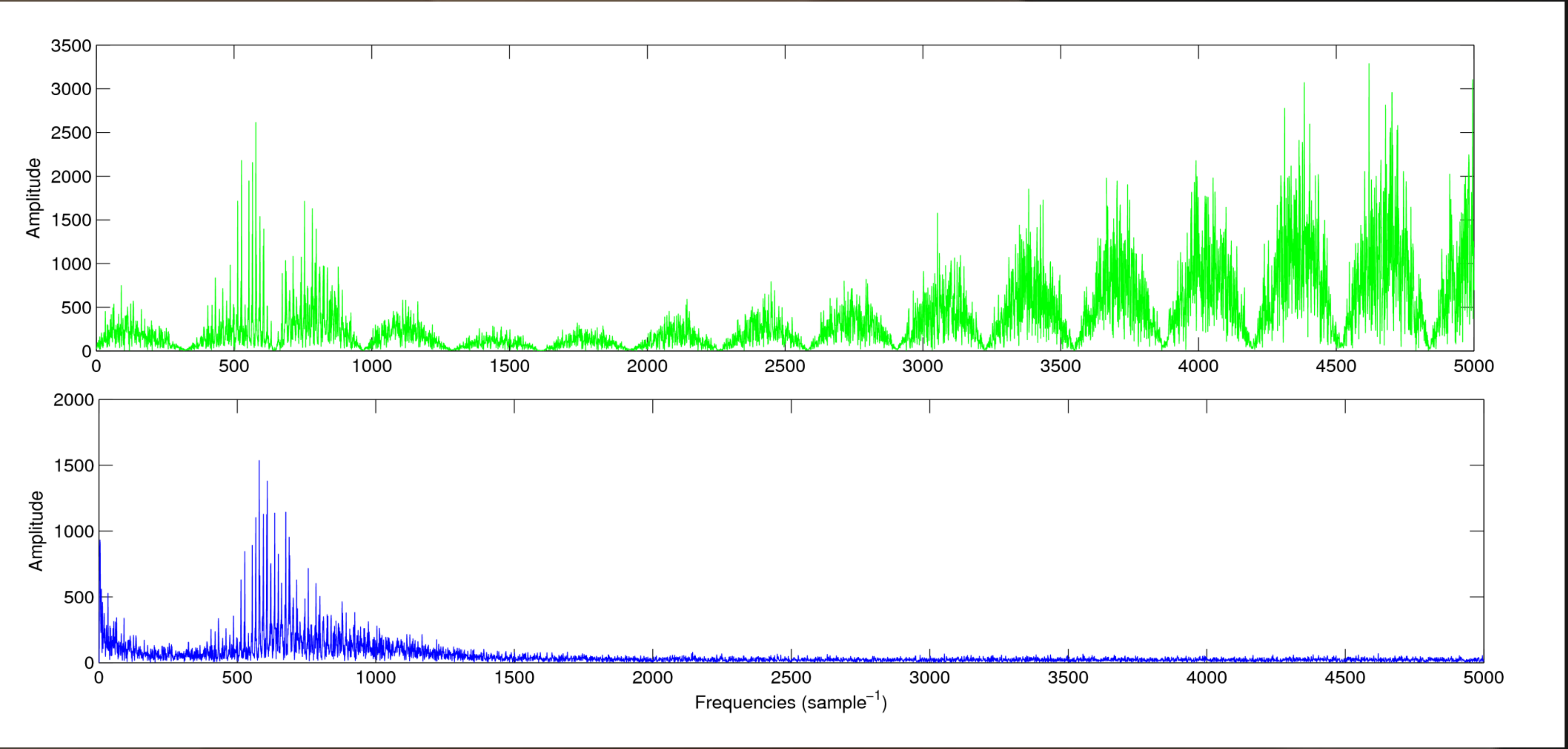


Light curves from CoRoT, Kepler and SOHO

- SOHO (GOLF). Frequency domain
- Spline conectivities (green), Original data (blue)



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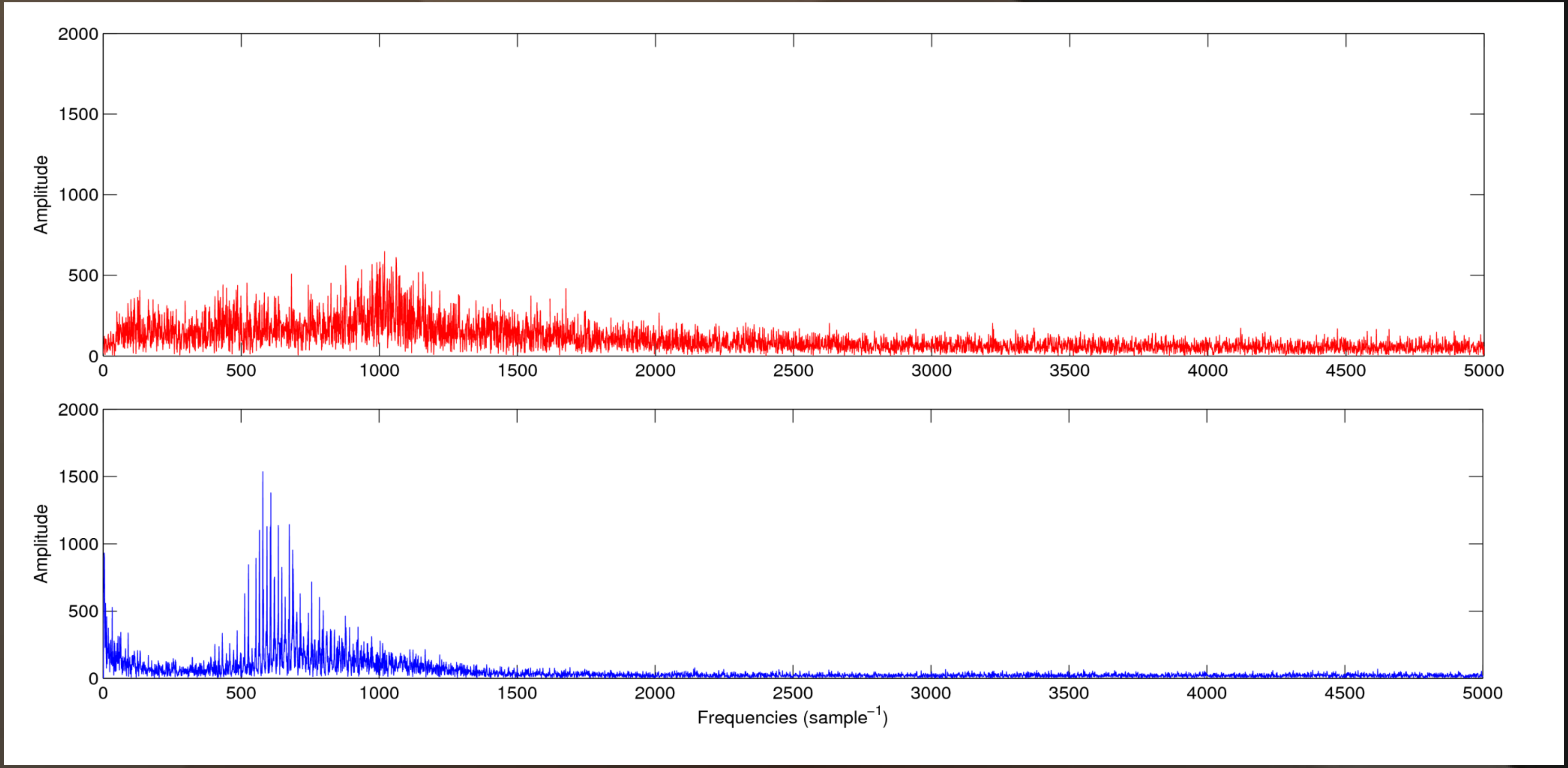
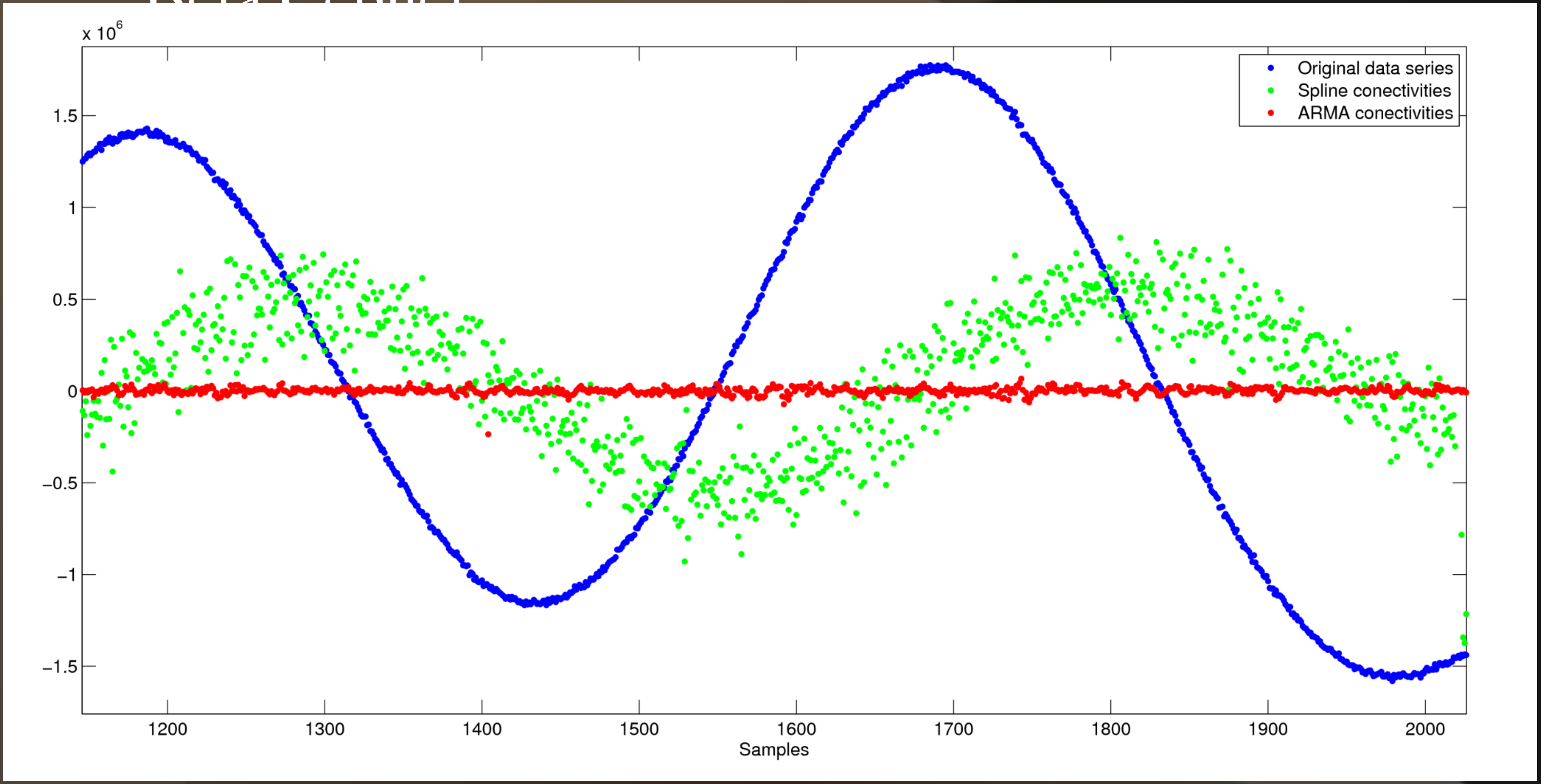


Light curves from CoRoT, Kepler and SOHO

- KIC 010119517
  Beta Cephei



Light curves from CoRoT, Kepler and SOHO

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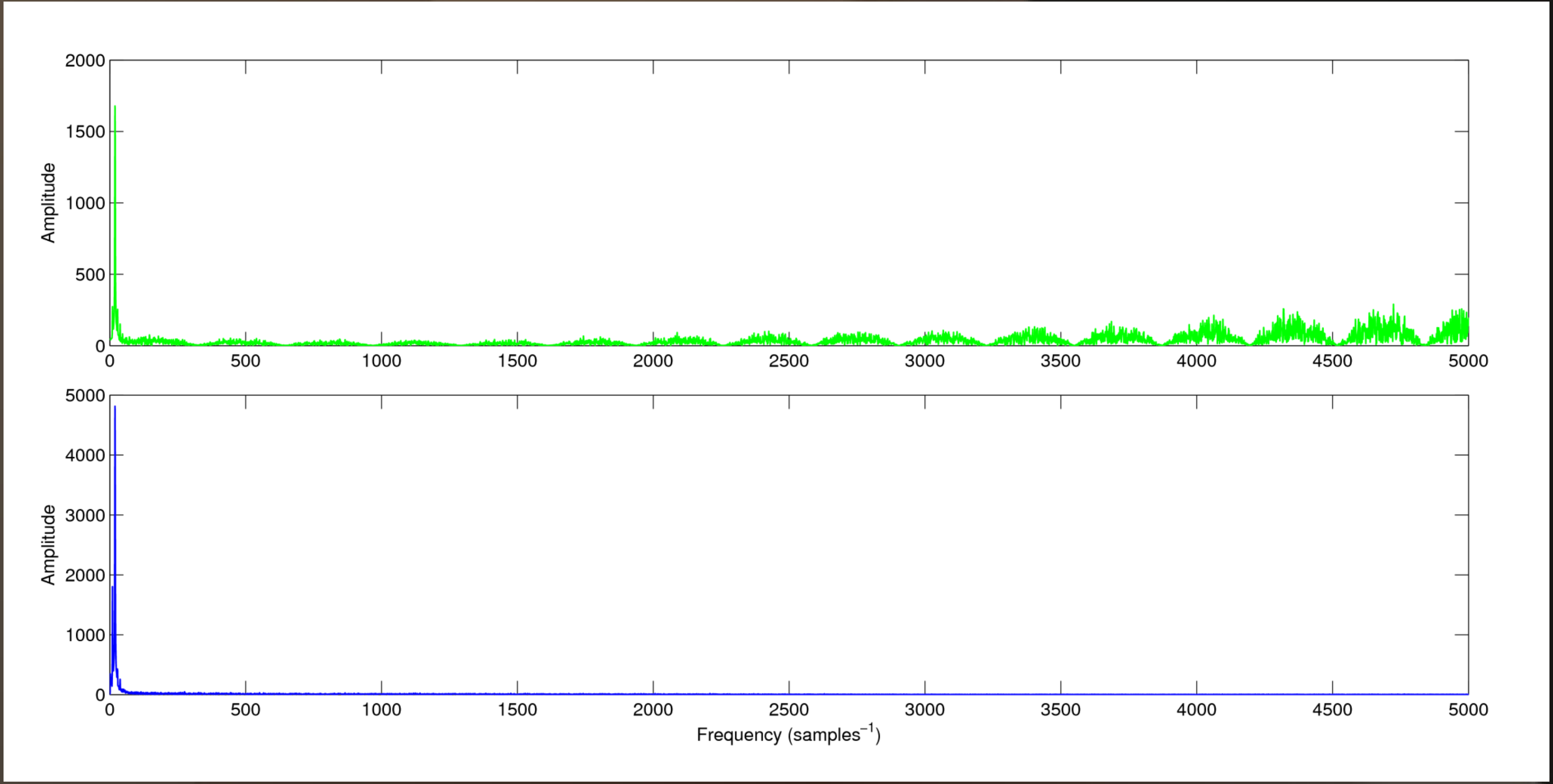

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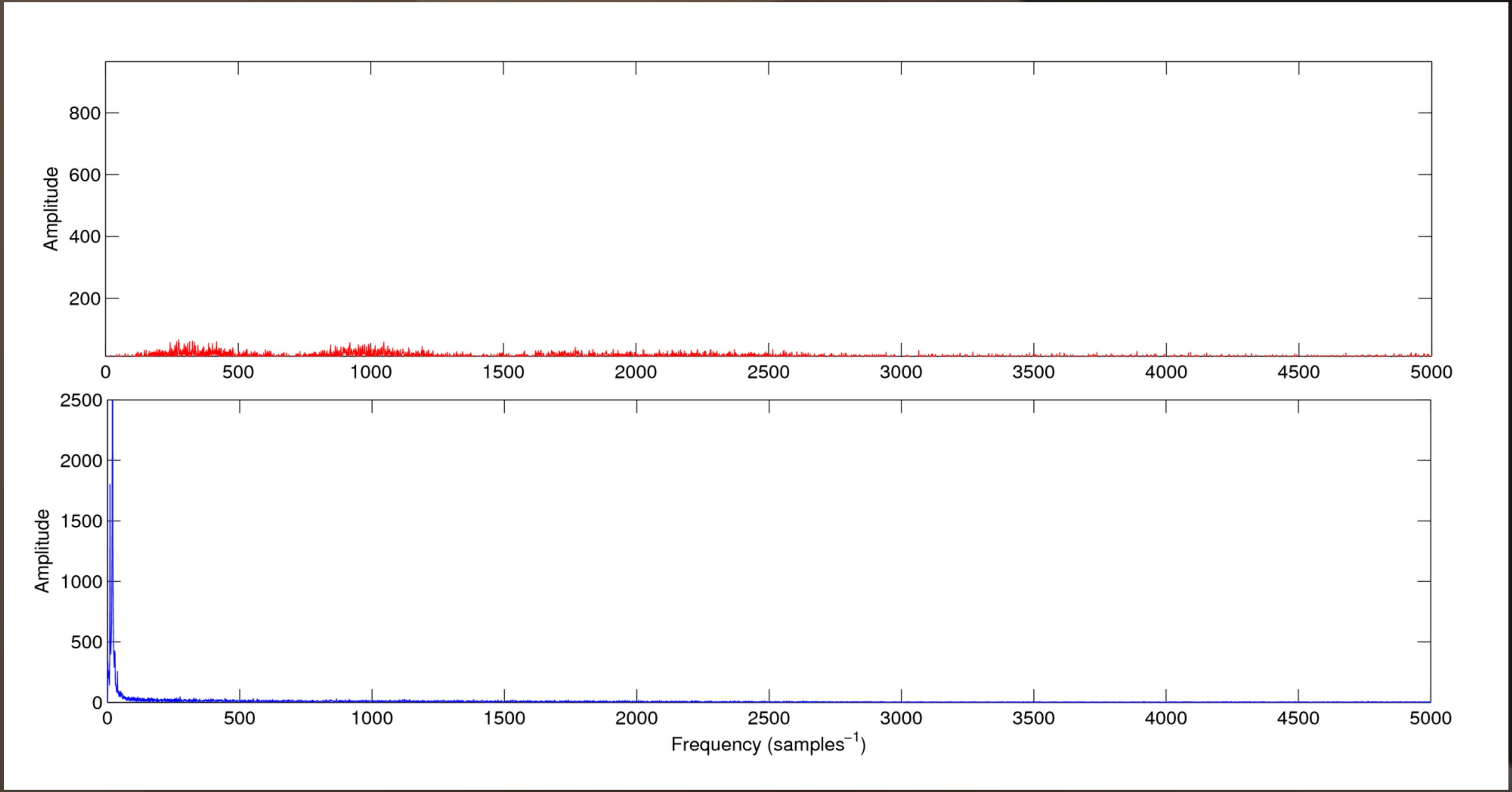
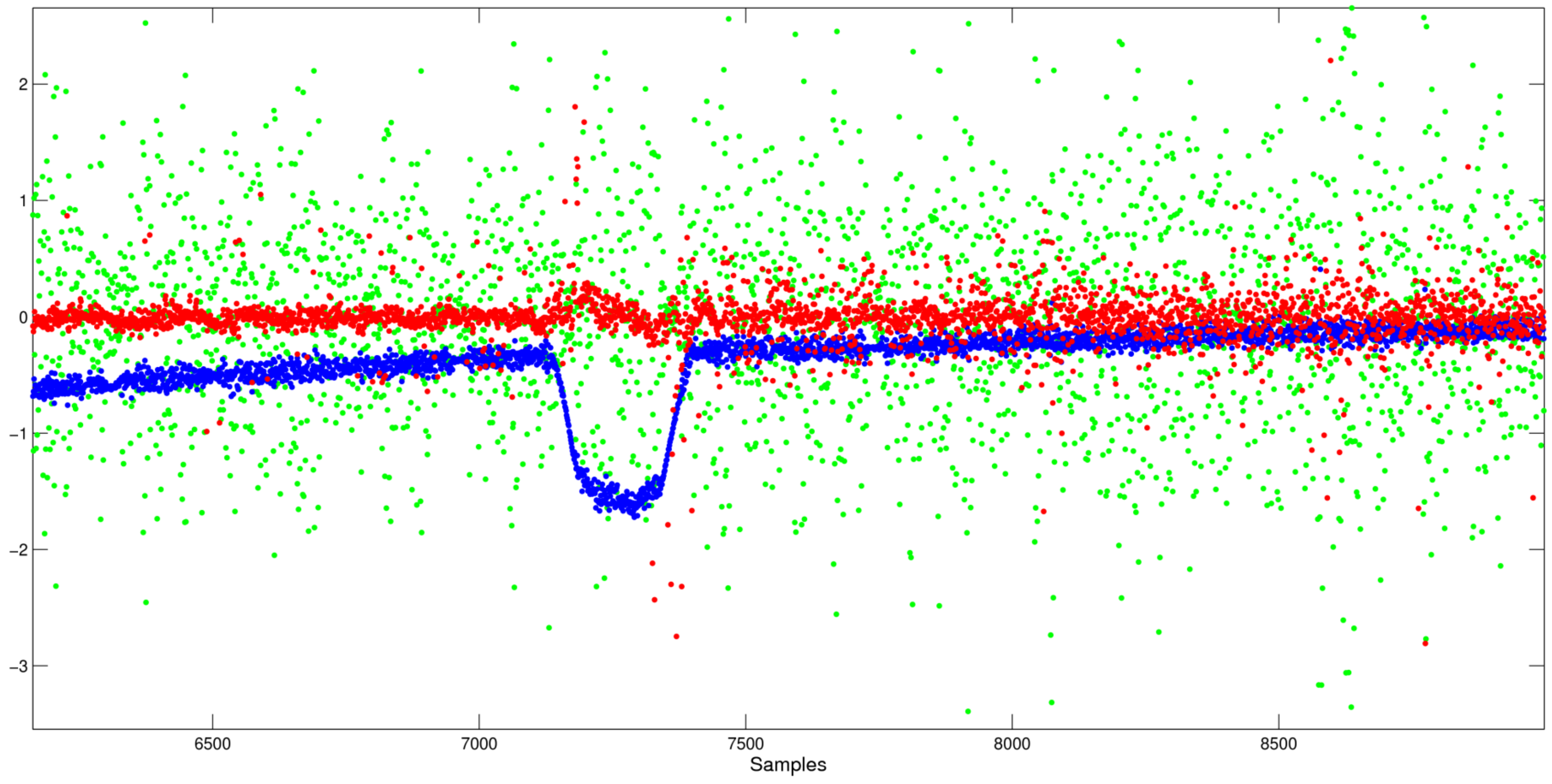


Conclusions

- The tests carried out with CoRoT 7613 show that the description based in Fourier frequencies is incorrect.
- For all the stars analyzed we repeated the test of the variation of SSE with the number of points used for the modelling. The residual never reach zero but a finite asymptotic value.
- That implies that Parseval's theorem mathematical requirements are not fulfilled.

Conclusions

- We have demonstrated that this phenomenon is not an instrumental effect.
- Nor a physical effect related with the turbulence.
- Definetively, the signal is not reducible to a square summable function, hence it cannot be represented by a Fourier series.



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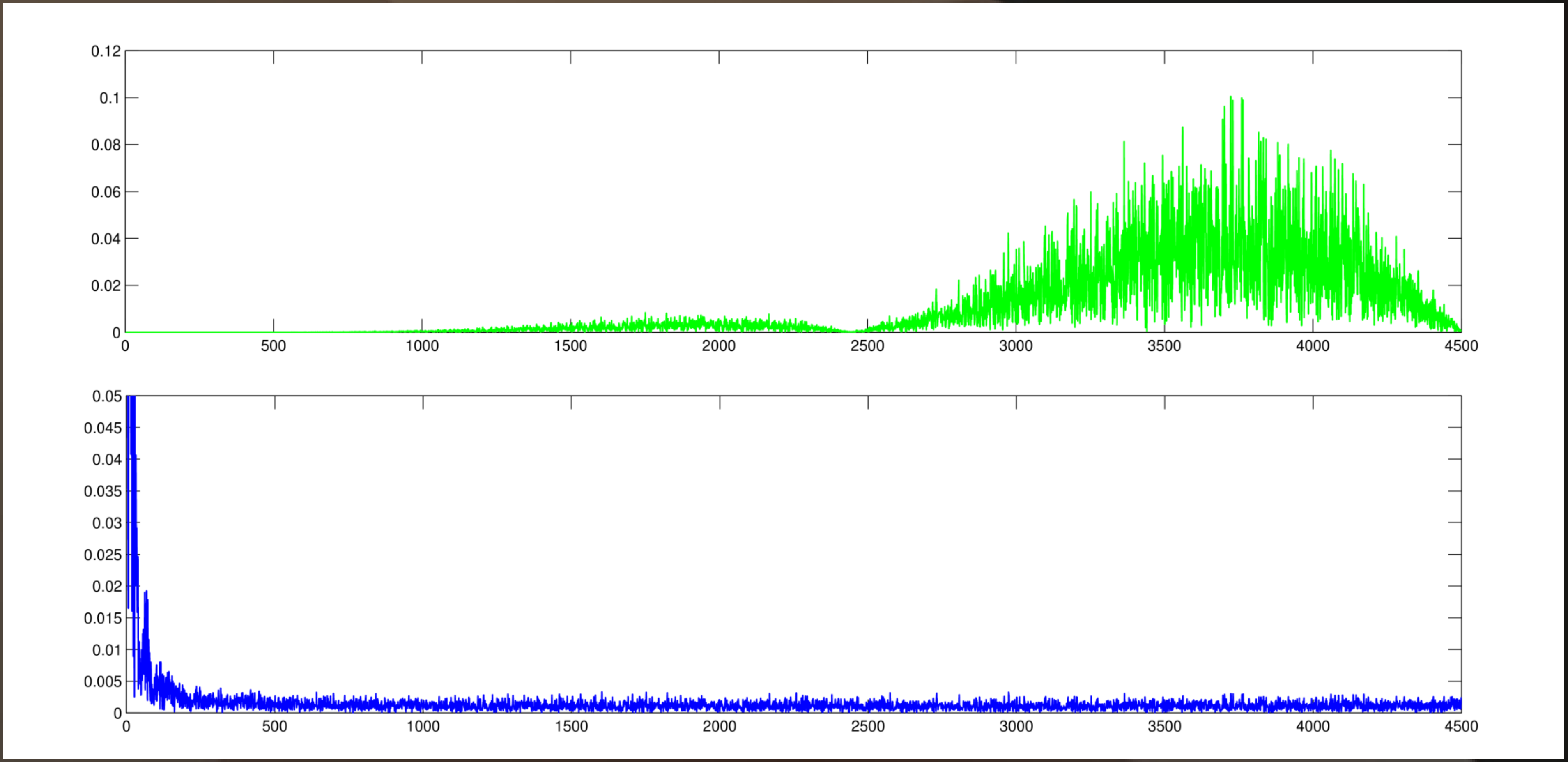
Samples



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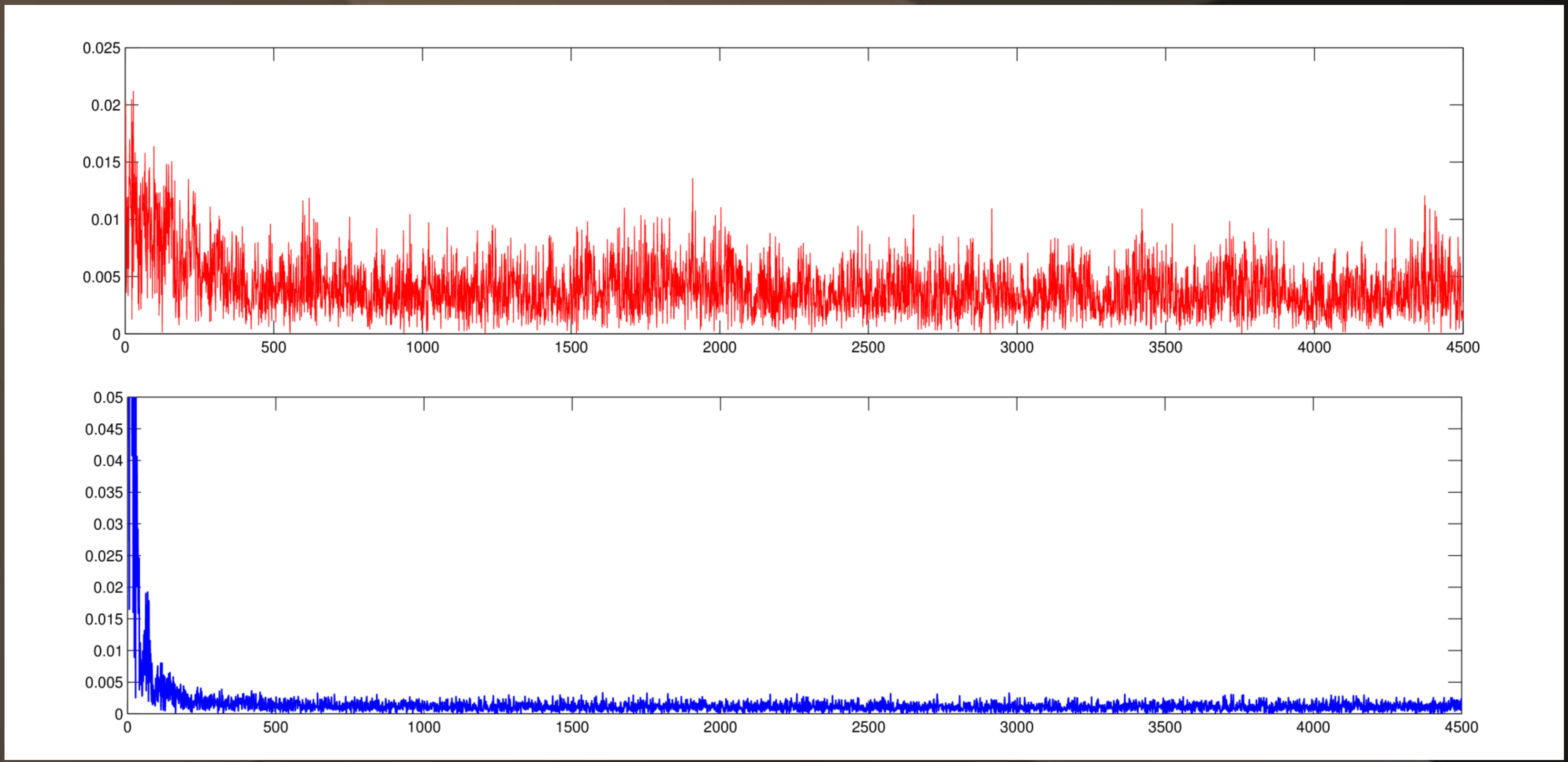
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