The Spline-AoV Periodogram

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Abstract

Periodograms are a family of analytic methods for finding periodic patterns in light curves and other time series. The choice for one periodogram over another generally hinges on its sensitivity (its ability to detect low SNR patterns) and the accuracy of its measured periods. As a general rule, the performance of periodograms can be improved by making assumptions about the periodic patterns one is searching for. However, this comes at a cost of performing poorly in cases where the assumptions do not hold. In this poster I introduce a new class of non-specialized spline-based periodograms that can operate on unevenly sampled observations. These periodograms are a generalization of the Schwarzenberg-Czerny (1989) Analysis of Variance (AoV) periodogram, which in this new framework can be called: 0,0-spline-AoV. When comparing this original AoV periodogram with a 1,1-spline-AoV periodogram, one finds that the new algorithm is slightly more sensitive and about twice as accurate in determining the period of eclipsing binaries in CoRoT-like light curves.

Method

Our method is based on the Schwarzenberg-Czerny Analysis of Variance (AoV) periodogram[(a)], which operates as follows: The periodogram iterates through a large series of candidate periods, folding the given light curve (LC) by each period, and examining the phased curve. The phased curve is divided into a group of phase bins and is effectively fitted by a step function, where the average of each bin determined a step (Figure 1). The periodogram score of each candidate period is then calculated as being proportional to a weighted variance of these step values, divided by the variance of the residuals. The spline-AoV periodogram generalizes this procedure by fitting the phased LC with a spline. Each bin is fitted with an N^{th} degree polynomial, where the polynomials are constrained so that their joints are cyclic[(b)] and can be differentiated M times before becoming discontinuous. This piecewise function is called an *M,N-spline*.

One can construct M,N-splines with any pair of integer values such that $N \geq M \geq 0$. One must fit (N-M+1) free parameters per bin, so that at least this number of data point are needed in each bin to avoid degeneracies[(c)]. The simplest spline is a 0,0-spline, which exactly describes the step function of the original AoV periodogram. However, for splines with $M \geq 1$, the entire spline must be fit at once, which can be done by simultaneously solving (N·B) linear equations, where B is the number of bins.

Results

To compare the effectiveness of the original AoV (or 0,0-spline-AoV) periodogram with the new spline-AoV (1,1-spline-AoV) periodogram, I ran both algorithms on sets of 1000 simulated eclipsing binary (EB) LCs. The LC sampling was constructed to be similar to CoRoT long-run LCs, with a 512 second sampling cadence over 100 days. In each set the eclipse duration and depth were held fixed, while the Gaussian noise realization and the epoch of the eclipse were randomized. After computing the both periodograms for each LC, the most-likely period of each method were compared with the simulation's true period. If a period error was large (frequency error >0.005 day^{-1} or about $\Delta P/P > 0.006$), the solution was considered unsuccessful and was rejected. The probability of a random period passing this criterion by chance (i.e. a false positive) is about 1%. The "success rate" of each method is the fraction of successful solution it produced, and the "accuracy" of each method is the median fractional absolute period error ($|\Delta P|/P$) among the successful solutions.

Both methods were found to have very similar success rates (Figure 2), with the spline-AoV being very slightly better, though both methods perform excellently when the eclipse depth is >0.5σ. However, the accuracy of the spline-AoV solutions were about twice as good as the original AoV solutions, while the errors of both methods were typically more than an order of magnitude better than the success criterion described above (Figure 3). To understand why the spline-AoV produces significantly smaller errors, I compared the periodogram peaks produced by both methods (Figure 4) and found that the spline-AoV peak is much smoother, so that determining its maximum is much more reliable. Both peaks may be offset from the true period by a small amount due to the LC noise, however the "roughness" of the AoV peak often doubles the size of this error.

Finally, I compare the behavior of the two periodogram methods, when applied to LCs with varying depth (Figure 5), duration (Figures 6), and number of bins (Figure 7). In all cases the accuracy of the spline-AoV periodogram is significantly superior, with exceptions only for very low-SNR eclipses (depth < 0.25σ). Furthermore, both methods have a sharp falloff in their success rate when the eclipse duration is shorter than 0.03P. This is a consequence of the fact that such narrow eclipses will usually be completely containes within a single bin and so will not be fitted correctly by either method. Using a larger number of bins can alleviate this problem, however it will bring about a side-effect of increasing the number of fitted free parameters, which may have the periodogram over-fit the phased LC and so degrade its success rate and accuracy. This limitation is more pronounced with the spline-AoV than with the AoV periodogram, even when they have an equal number of free parameters.

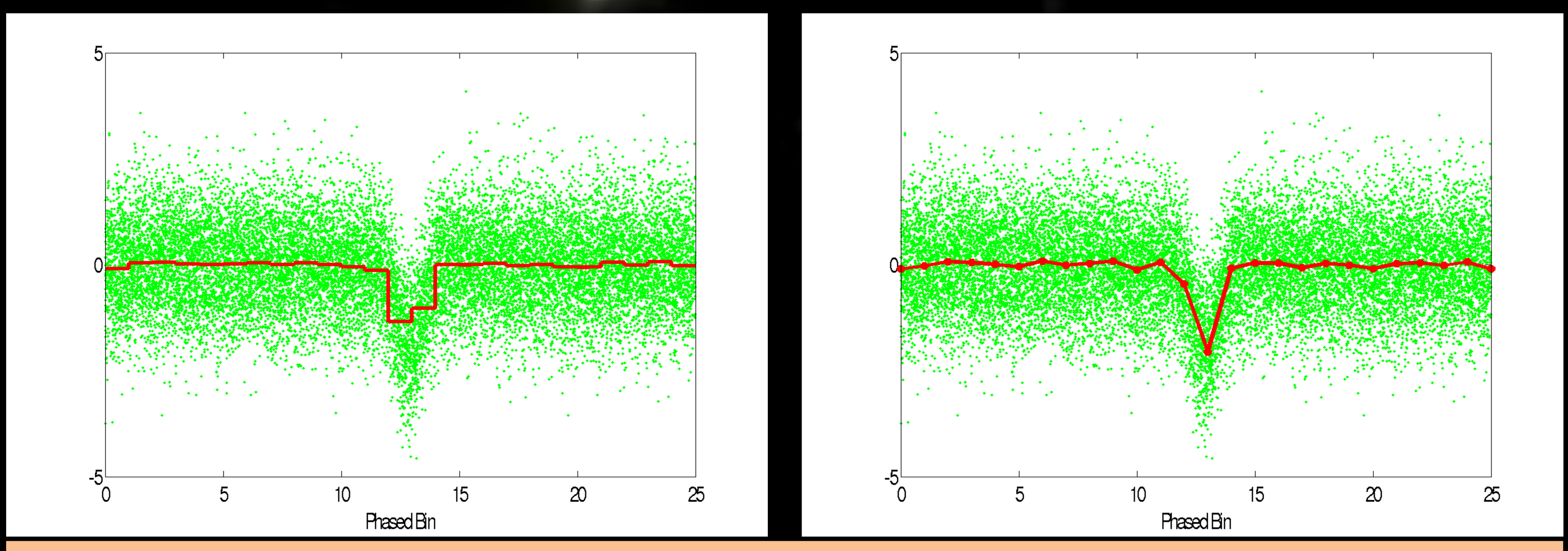

Fig 1. A simulated phased EB light curve fitted with a 25-bin AoV (or 0,0-spline-AoV) step-function (left), compared with the identical data fitted with a 25-bin 1,1-spline-AoV (right). The Y-axis is scaled to the noise standard deviation. (eclipse duration = 0.1P ; eclipse depth = 2σ)

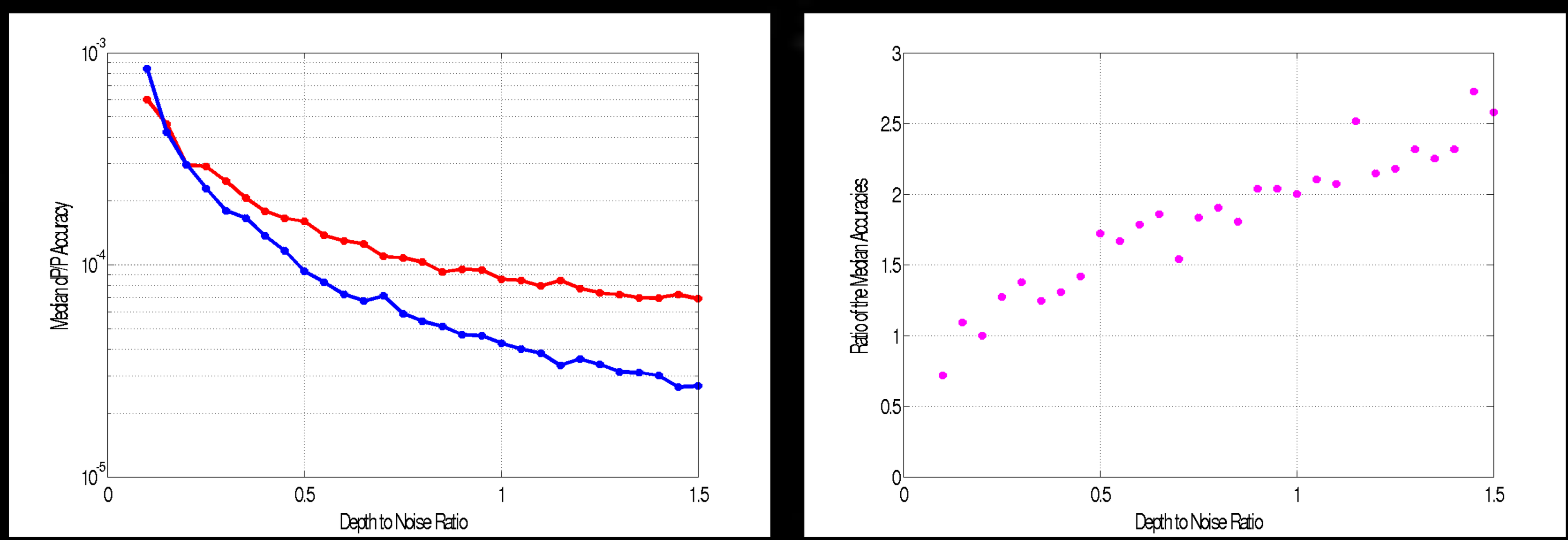

Fig 5. The accuracy (median $|\Delta P|/P$; smaller values are more accurate) of the AoV (red) and spline-AoV (blue) periodograms in determining the period of simulated EB light curves with varying eclipse depth. The accuracy of the spline-AoV improves faster than that of the AoV with higher SNR. (25 bins ; eclipse duration = 0.1P)

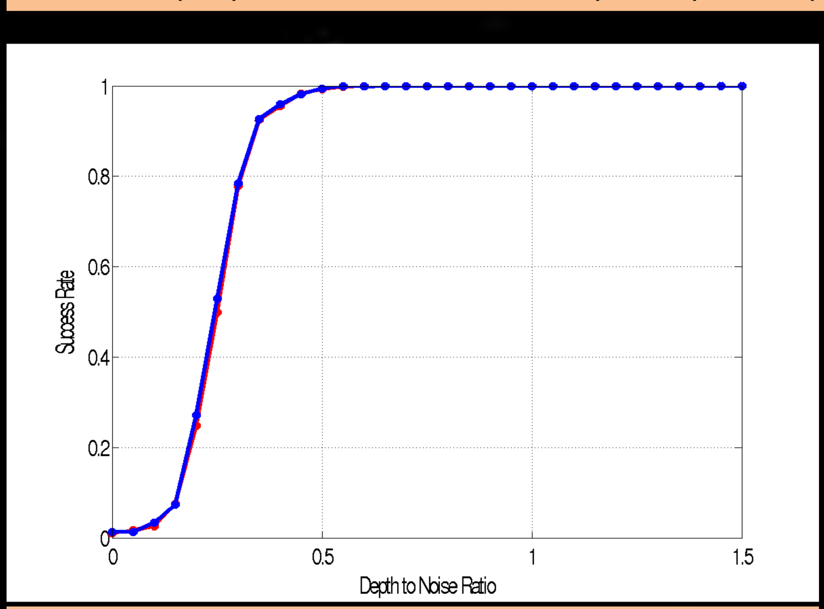

Fig 2. The success rate of the AoV (red) and the spline-AoV (blue) periodograms in finding the correct period of simulated EB light curves with varying eclipse depth. (25 bins ; eclipse duration = 0.1P)

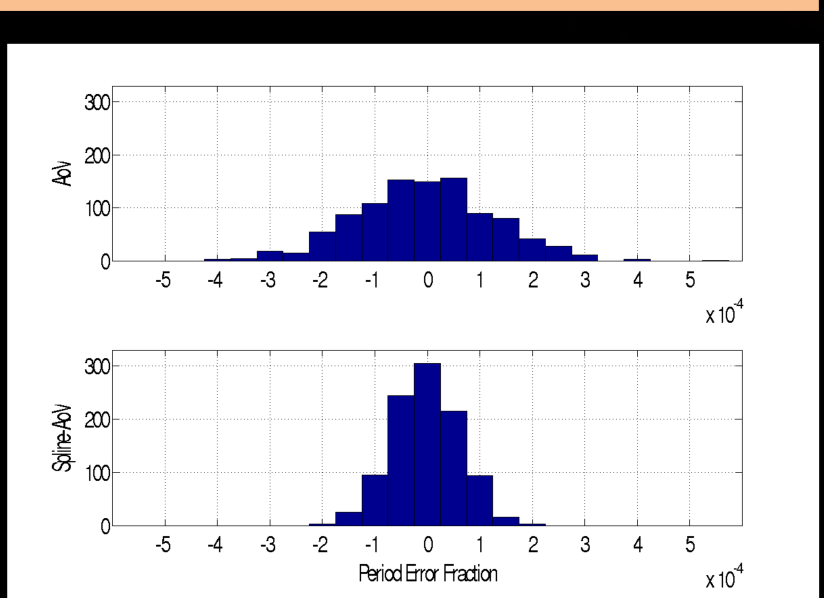

Fig 3. The fractional period error distributions ($\Delta P/P$) of the AoV (top) and spline-AoV (bottom) methods when applied to 1000 simulated EB light curves.
(25 bins ; eclipse depth = 1σ ; eclipse duration = 0.1P)

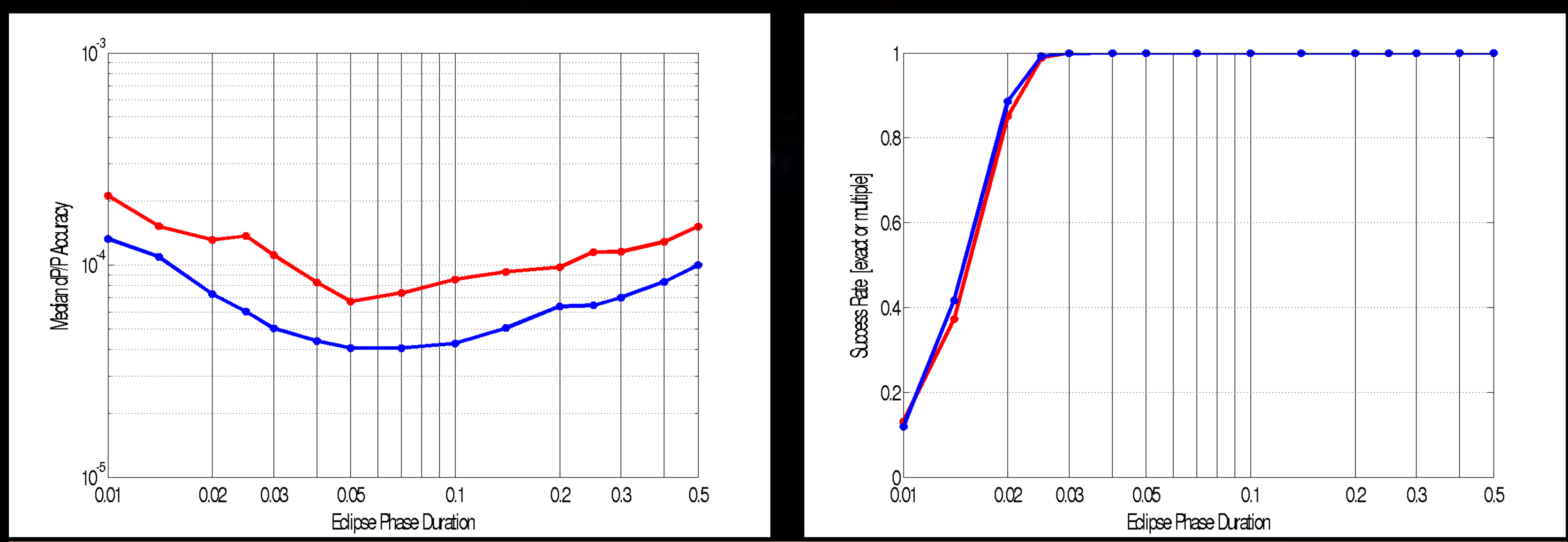

Fig 6. The accuracy (median $|\Delta P|/P$; left) and success rate (right) of the AoV (red) and spline-AoV (blue) periodograms for simulated EB light curves with varying eclipse durations. Note that harmonic multiples of the true period were also considered successful results, however such multiples only occur with very wide eclipses.
(25 bins ; eclipse depth =1σ)

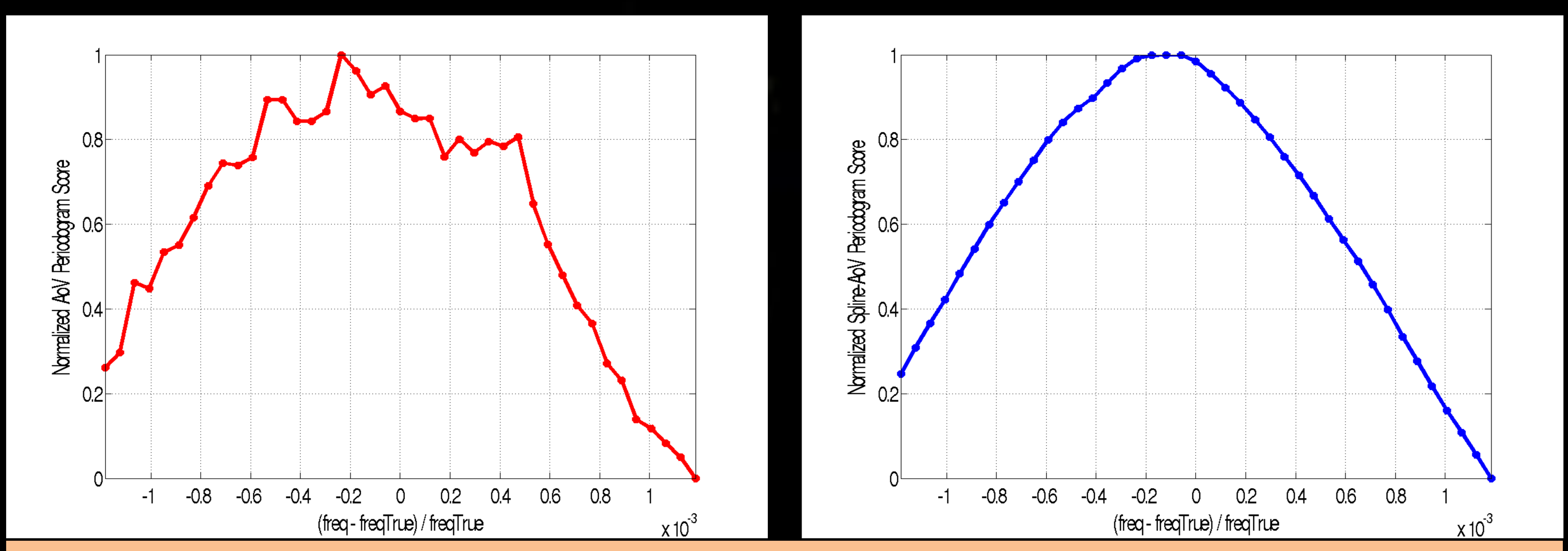

Fig 4. A highly magnified view of the AoV (left ; red) and spline-AoV (right ; blue) periodograms around the true period. Each method returns the location of its respective periodogram maximum. Noise in the LC can shift the maximum by a small amount, however the noise in the AoV peak adds significantly to the effect of this shift, and so increases the AoV period error. (25 bins ; eclipse duration = 0.1P ; eclipse depth = 0.5σ)

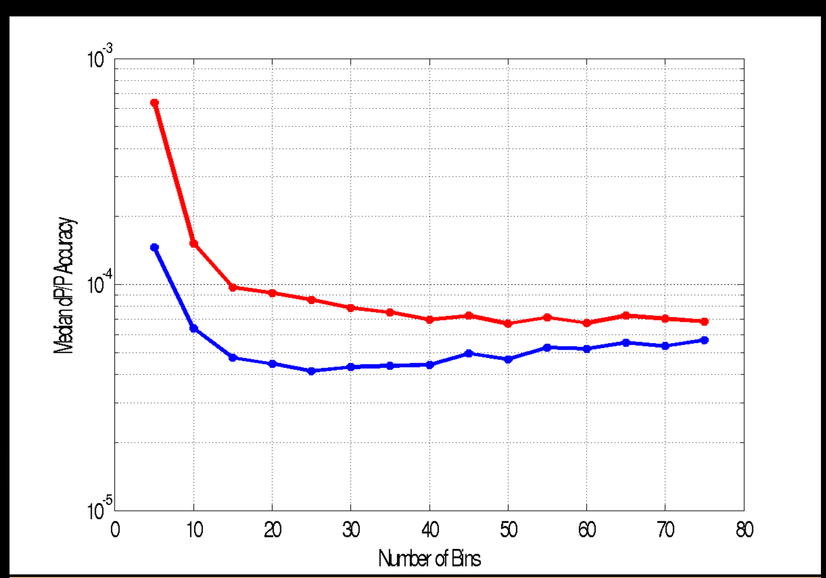

Fig 7. The accuracy (median $|\Delta P|/P$) of the AoV (red) and spline-AoV (blue) periodograms with various numbers of bins.
(eclipse duration = 0.1P ; eclipse depth =1σ)

Notes:

(a) Schwarzenberg-Czerny, A. 1989, MNRAS, 241, 153

(b) The end of the last polynomial is joined to the beginning of the first.

(c) There are some subtle exceptions to this statement.

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