Voronoi & Delaunay Tessellations; and the multiscale cosmic web

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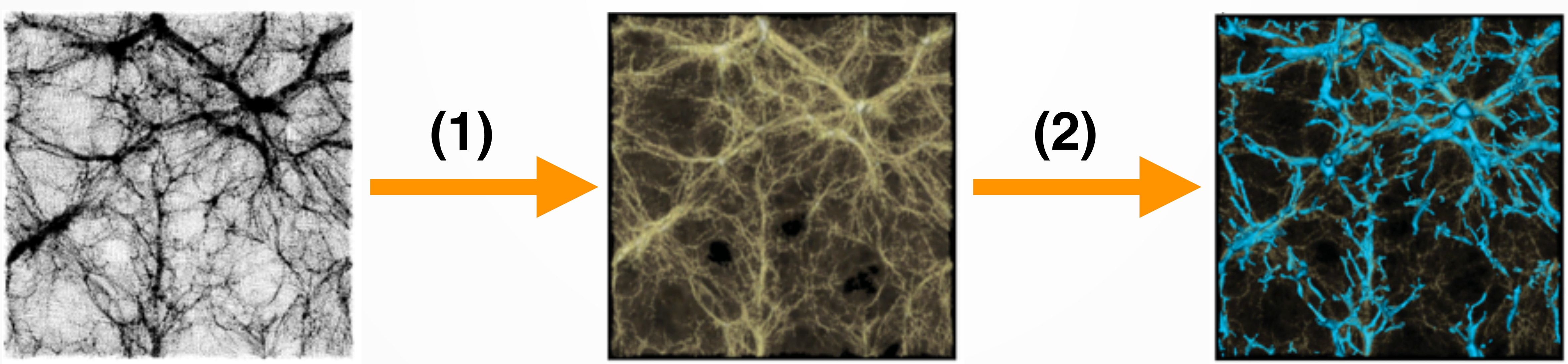
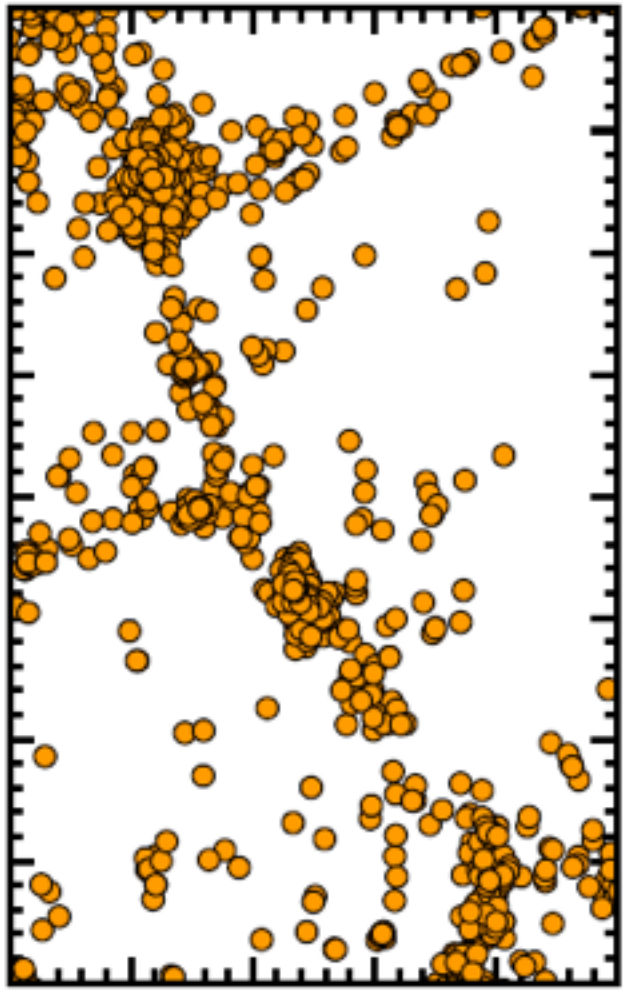
Overview

1. **Density estimation**
2. **Multiscale identification of the cosmic web**



1. Density estimation with: Voronoi and Delaunay Tessellations

Density estimation



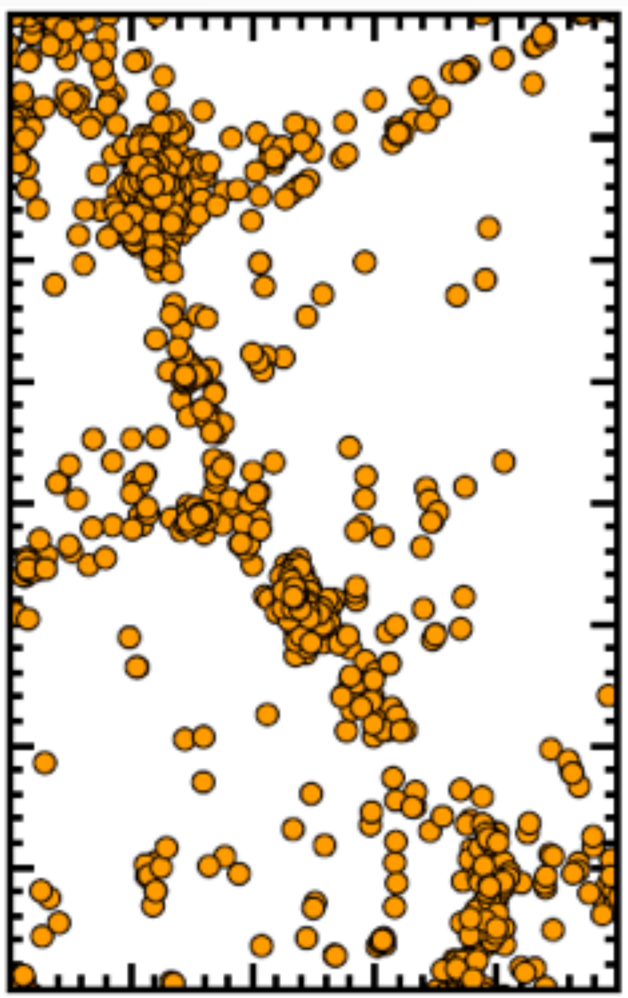
A. Non-adaptive:

- Nearest grid point
- Cloud in cell
- Triangular shape cloud

B. Adaptive:

- Smoothed particle hydrodynamics (SPH)
- **Voronoi Tessellations**
- **Delaunay Tessellations**

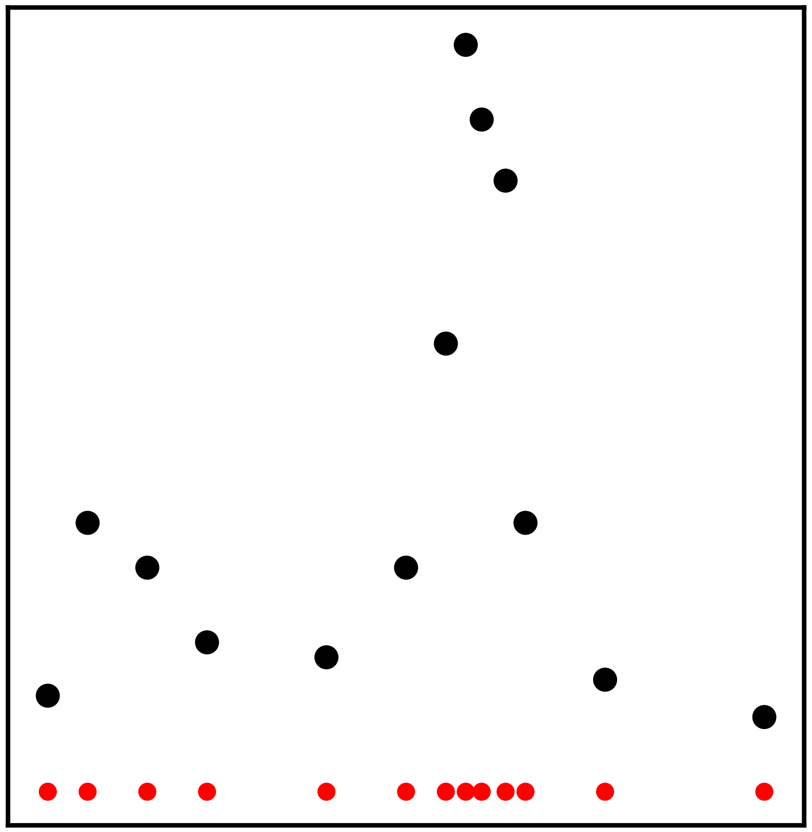
Voronoi & Delaunay density estimation methods



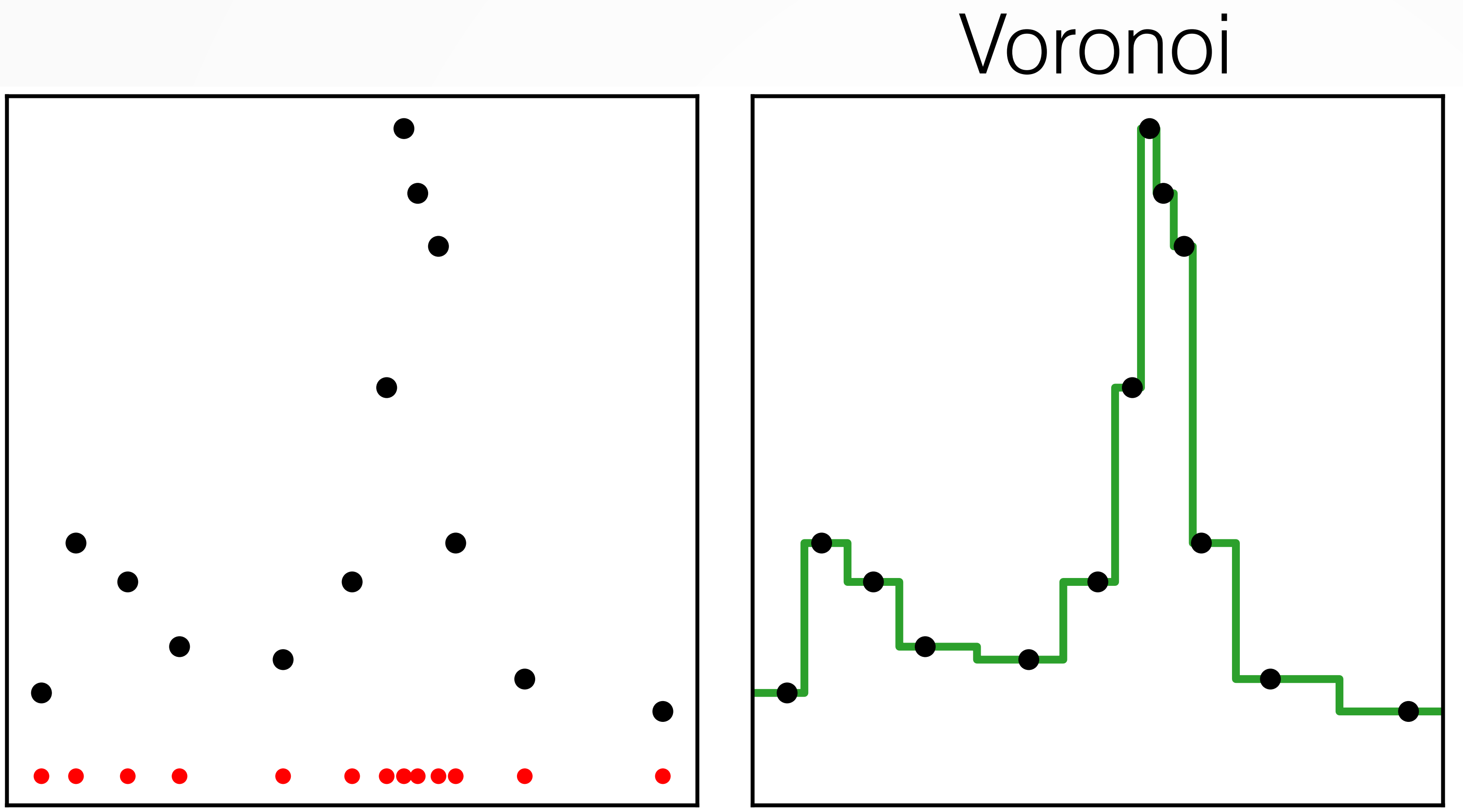
- Self-adaptive to the local distribution of tracers
- Preserves the hierarchical character of the matter distribution
- Preserves the anisotropies of the matter distribution
- Parameter free
- Volume weighted quantities (most methods give mass-weighted quantities)

Schaap & van de Weygaert (2000); van de Weygaert & Schaap (2009)

Voronoi & Delaunay: 1D

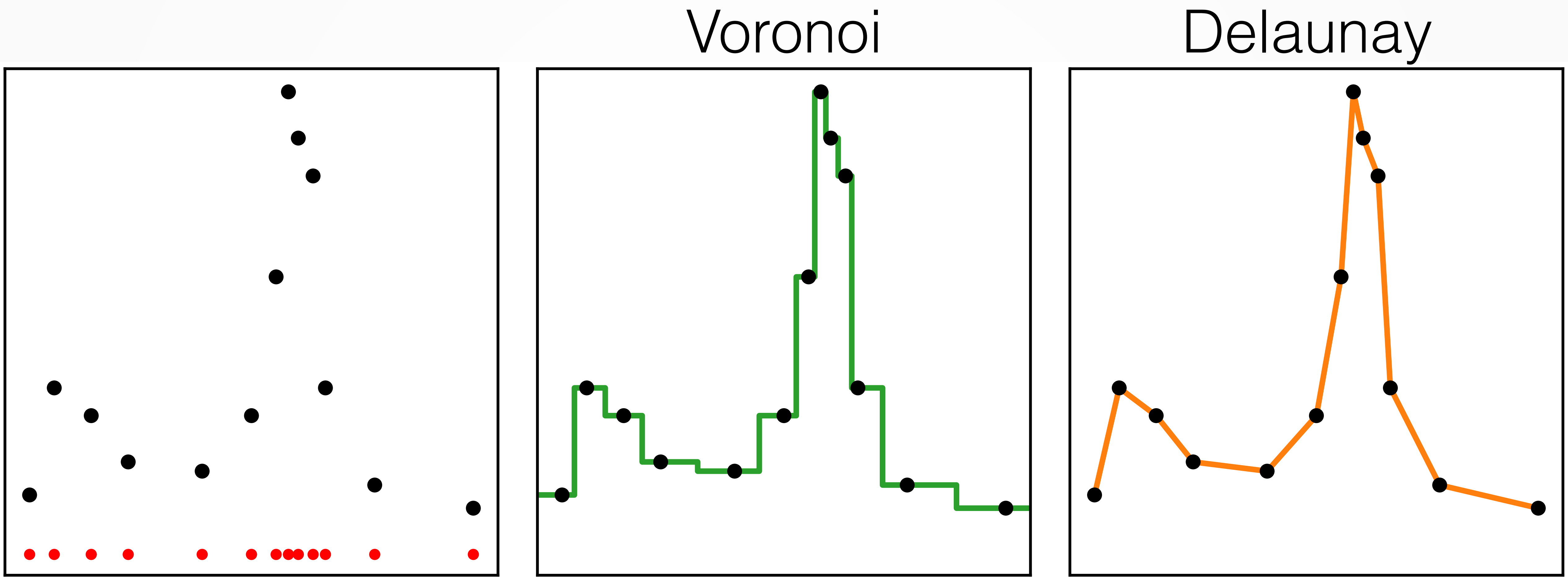


Voronoi & Delaunay: 1D



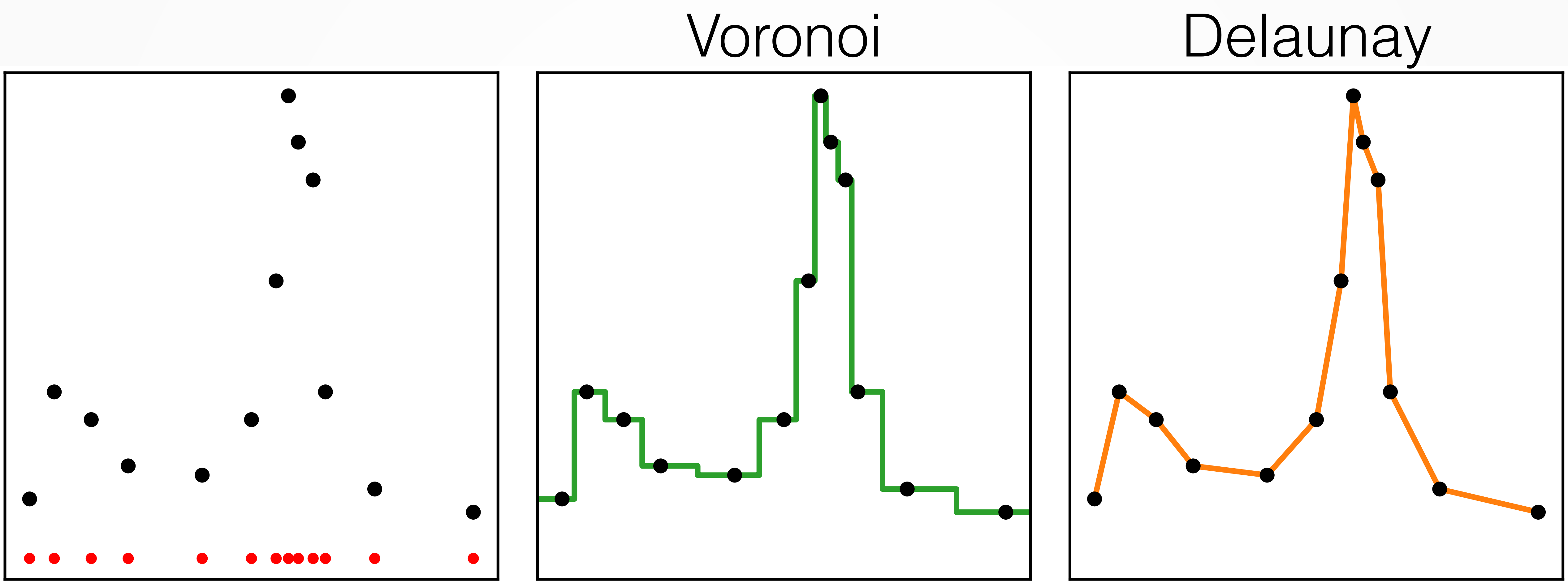
- **Voronoi:** take the value of the nearest point

Voronoi & Delaunay: 1D



- **Voronoi:** take the value of the nearest point
- **Delaunay:** interpolate linearly between nearby points

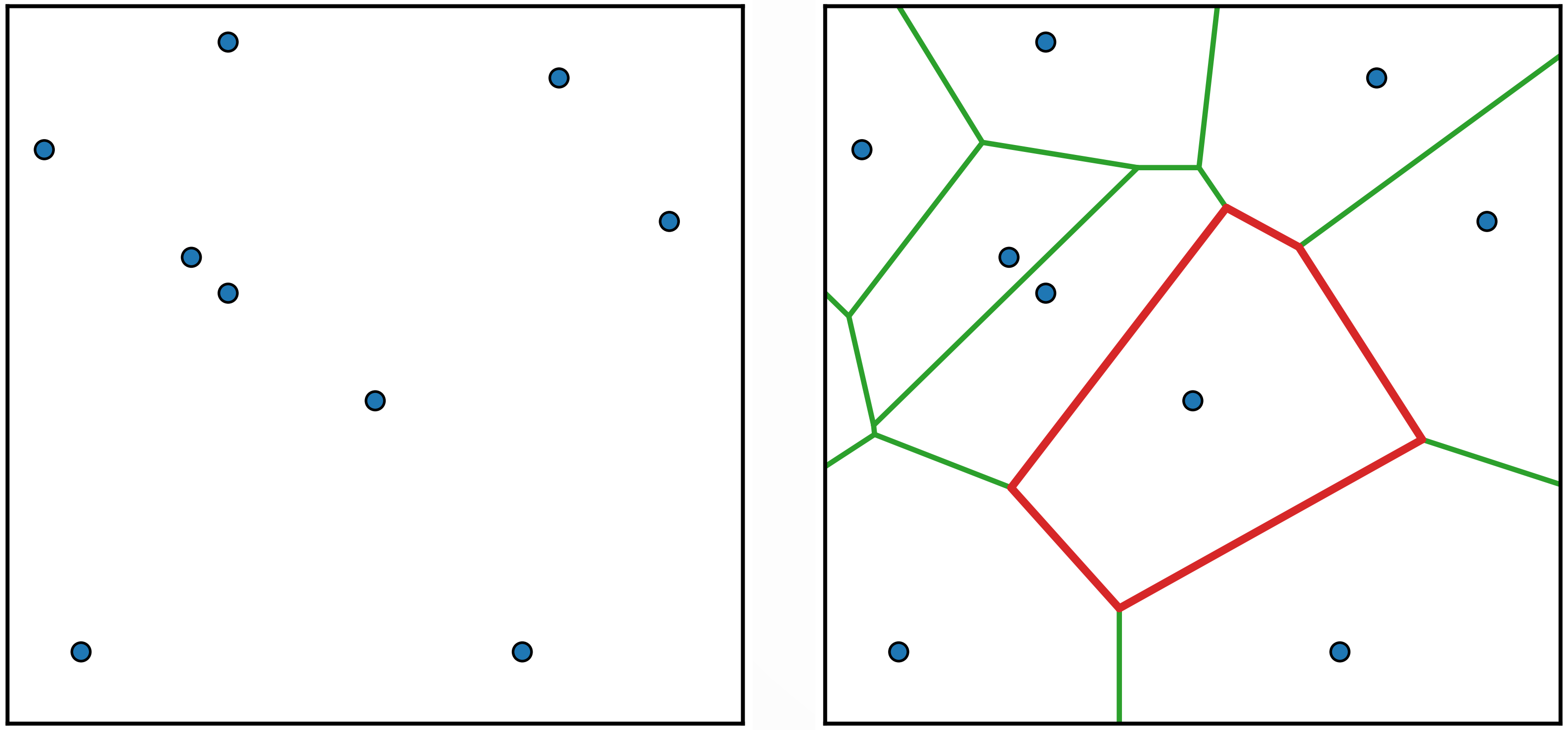
Voronoi & Delaunay: 1D



Taylor expansion: $\widehat{f}(\mathbf{x}) = f(\mathbf{x}_0) + \widehat{\nabla f}\Big|_j \cdot (\mathbf{x} - \mathbf{x}_0)$ + higher order

Voronoi tessellations: 2D

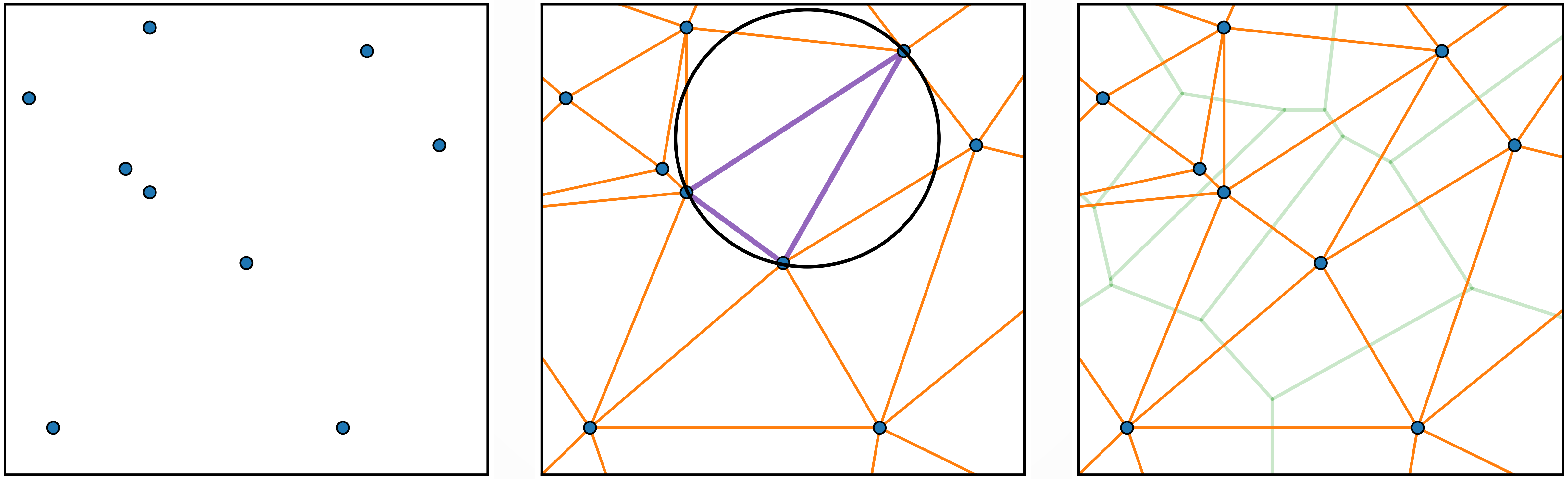
- **Voronoi:** take the value of the nearest point



Density of each tracer: $$\widehat{\rho}_i = \frac{m_i}{V(\mathcal{V}_i)}$$

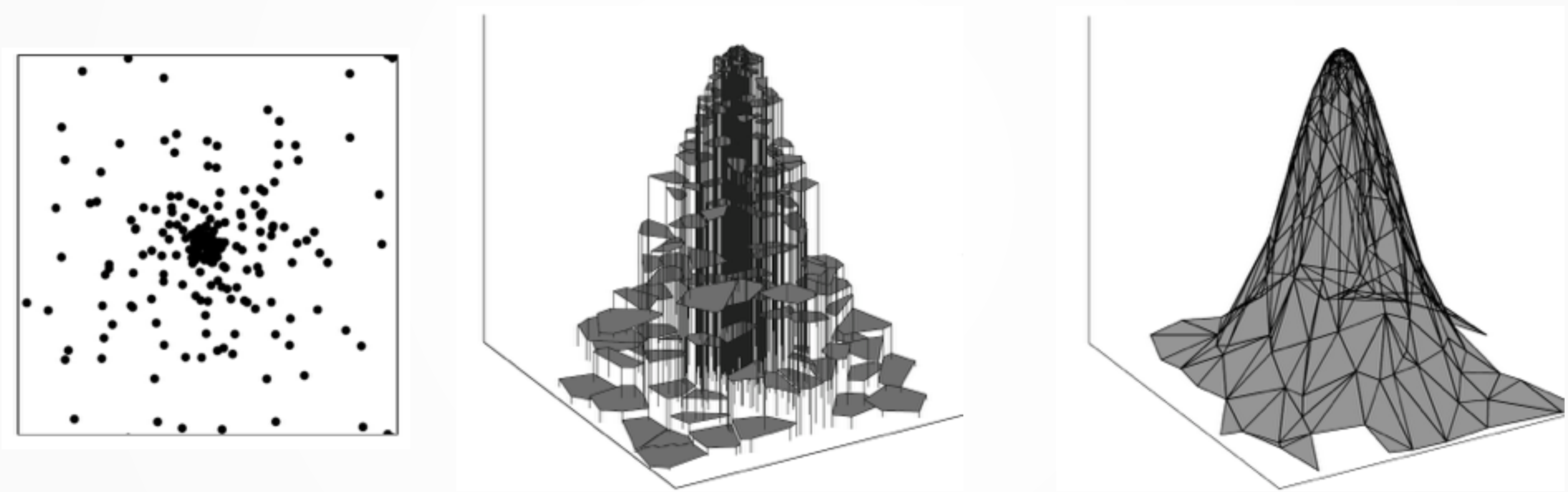
Delaunay tessellations: 2D

- **Delaunay:** interpolate linearly between nearby points



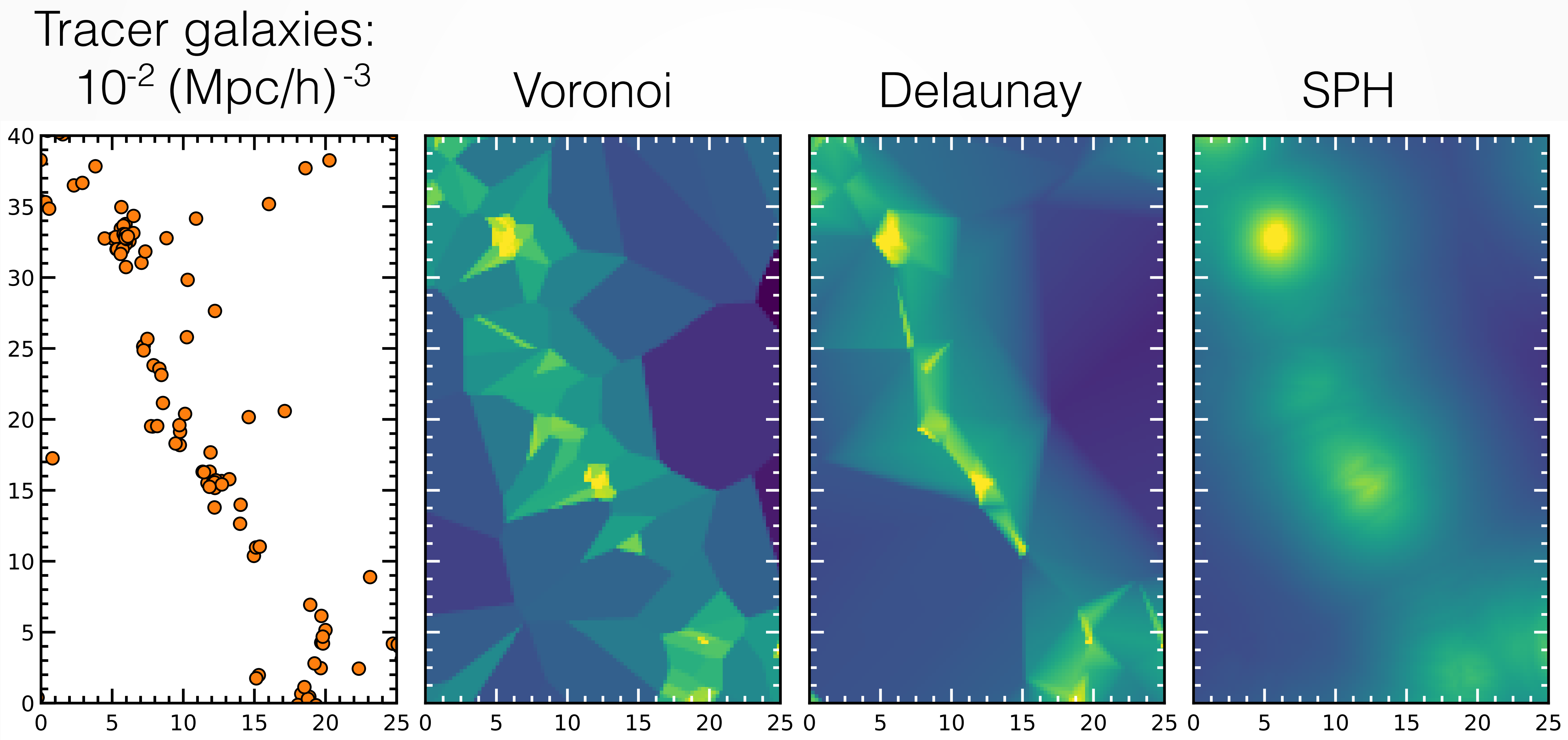
Density of each tracer: $$\widehat{\rho}(\mathbf{x}_i) = \frac{(D+1)m_i}{V(\mathcal{W}_i)}$$ $$V(\mathcal{W}_i) = \sum_{j=1}^{N_{\mathcal{T},i}} V(\mathcal{T}_{j,i})$$

Voronoi and Delaunay densities: 2D



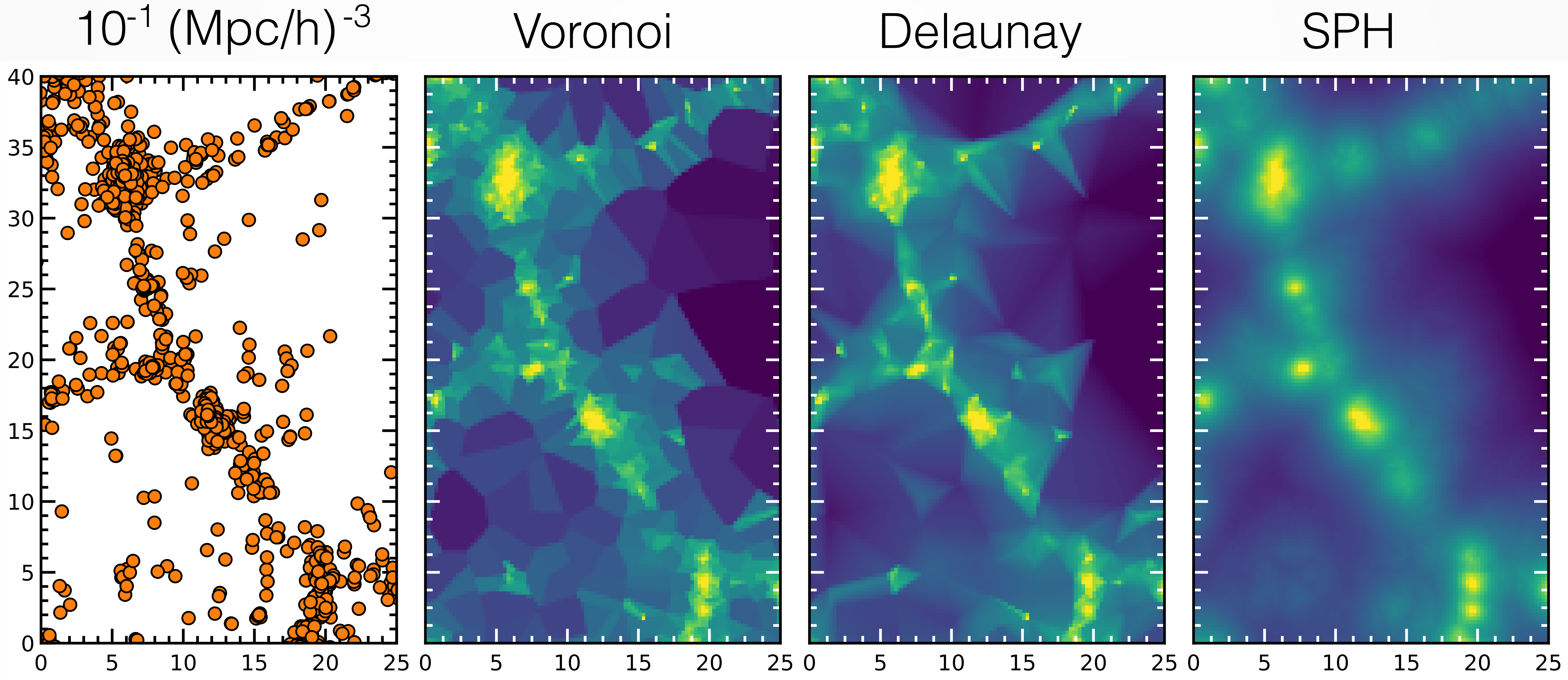
reproduced from Schaap (2007), PhD thesis

Voronoi and Delaunay densities

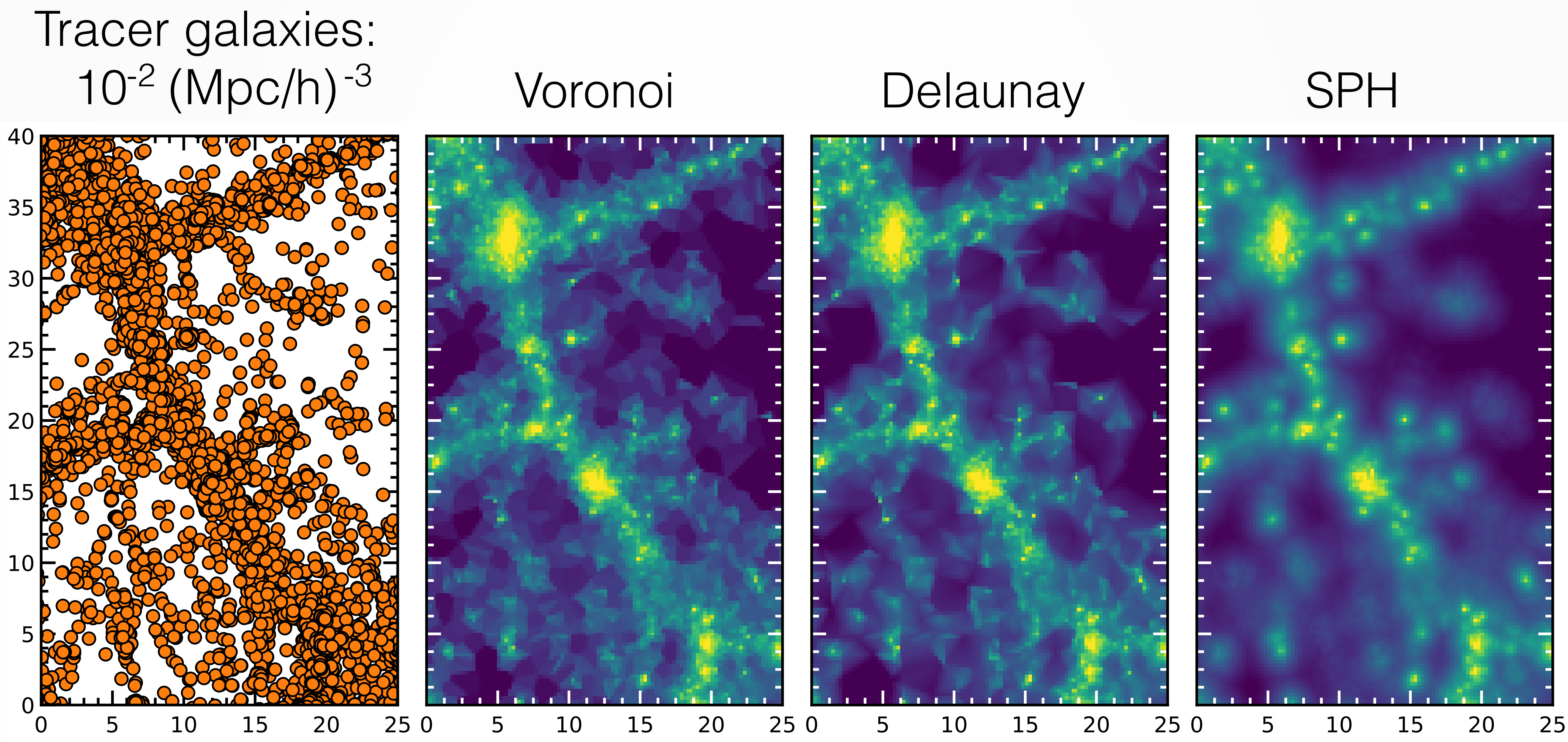


Voronoi and Delaunay densities

Tracer galaxies: 10^{-1} (Mpc/h)$^{-3}$



Voronoi and Delaunay densities



Voronoi and Delaunay tessellations: applications

- **DTFE: Delaunay Tessellation Field Estimator** for density, velocity fields and more (Schaap & van de Weygaert 2000, A&A, 363, L29) — *publicly available code* (MC & van de Weygaert 2011, arxiv:1105.0370; www.astro.rug.nl/~voronoi/DTFE/dtfe.html)
- **Void identification:** ZOBOV (Neyrinck 2008, MNRAS 386,2101), Watershed Void Finder (Platen+ 2007, MNRAS, 380, 551), Delaunay tetrahedra underdensities (Liang+ 2016, MNRAS, 459, L4020)
- **Halo/cluster identification:** VOBOZ (Neyrinck 2005, MNRAS 356, 1122), Voronoi Galaxy Cluster Finder (Ramella + 2001, A&A, 368, 776)

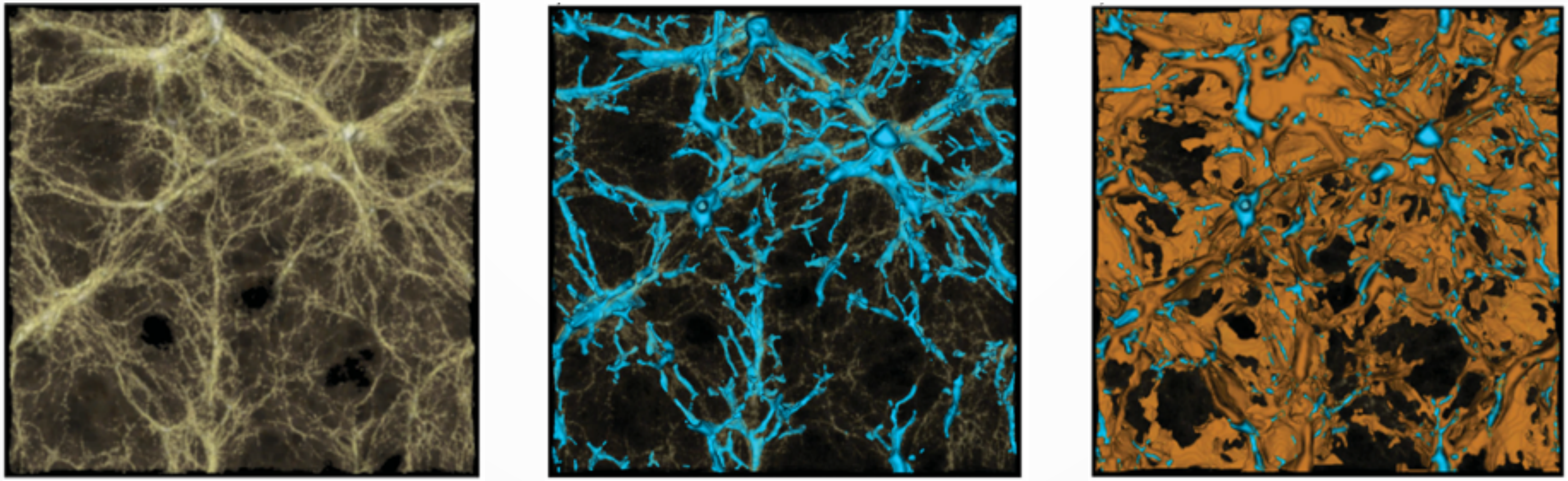
2. Multiscale identification of the cosmic web

Cosmic web identification: NEXUS

The cosmic web:

- Nodes / knots
- Filaments
- Walls / sheets
- Voids

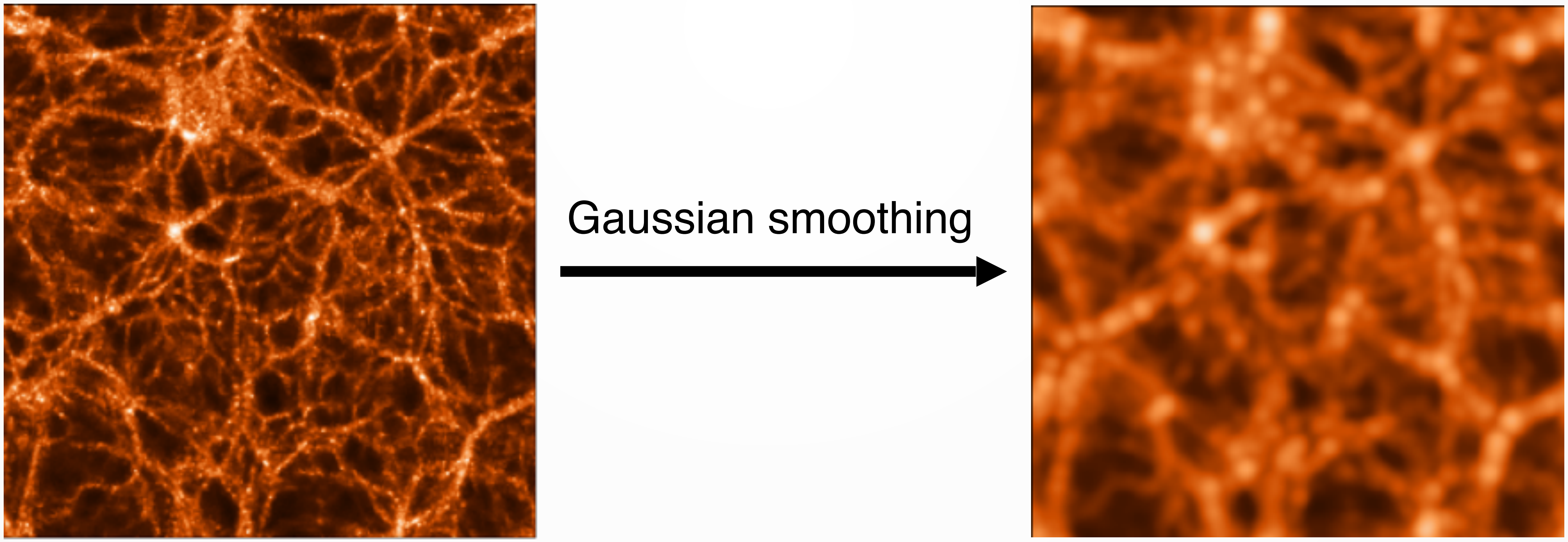
Density field — Filaments — Walls



MC + (2013)

Cosmic web identification: NEXUS

1. Smooth the input density field.



Cosmic web identification: NEXUS

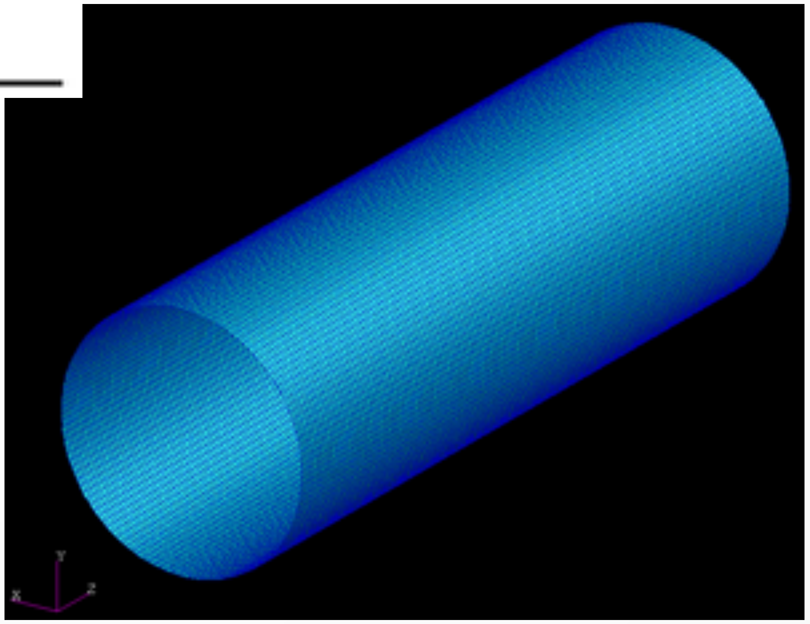
1. Smooth the input density field.
2. Compute the Hessian of the smoothed density field.

$$\mathbf{H}_{ij,R_n}(\boldsymbol{x}) = \frac{\partial^2 f_{R_n}(\boldsymbol{x})}{\partial x_i \partial x_j}$$

Cosmic web identification: NEXUS

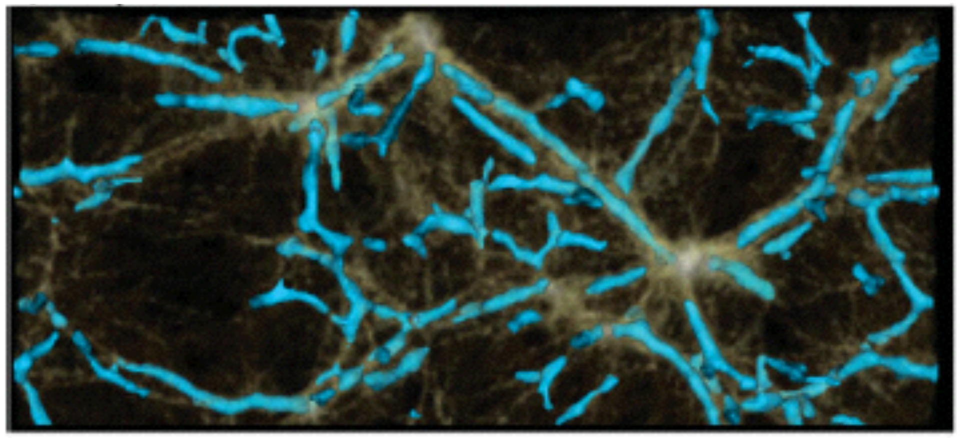
1. Smooth the input density field.
2. Compute the Hessian of the smoothed density field.
3. Use the Hessian eigenvalues to assign an environment signature to each point.

cluster	$\|\lambda_1\| \simeq \|\lambda_2\| \simeq \|\lambda_3\|$	$\lambda_1 < 0;\ \lambda_2 < 0;\ \lambda_3 < 0$
filament	$\|\lambda_1\| \simeq \|\lambda_2\| \gg \|\lambda_3\|$	$\lambda_1 < 0;\ \lambda_2 < 0$
wall	$\|\lambda_1\| \gg \|\lambda_2\|;\ \|\lambda_1\| \gg \|\lambda_3\|$	$\lambda_1 < 0$

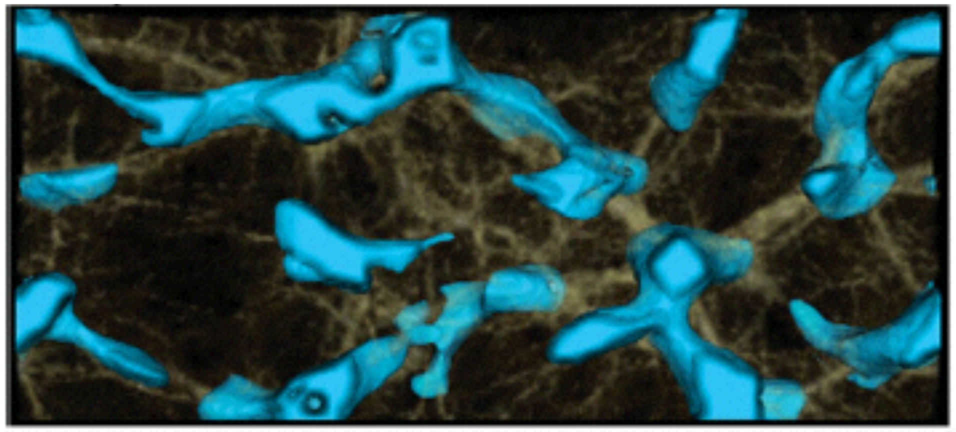


Cosmic web identification: NEXUS

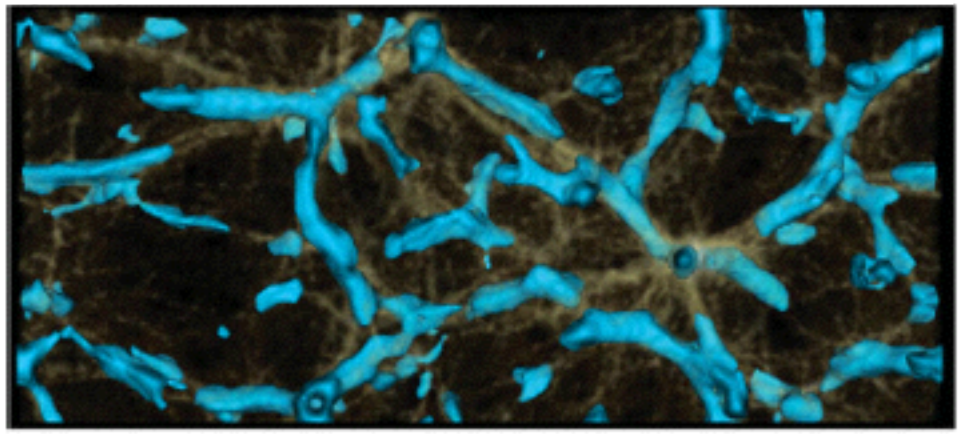
1 Mpc/h smoothing



4 Mpc/h smoothing



2 Mpc/h smoothing



Which one is the correct cosmic web?

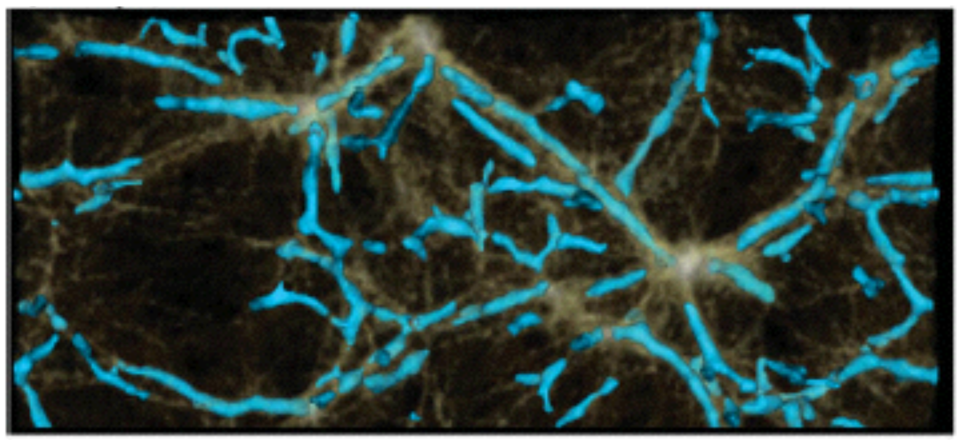
All of them. Different filtering scales are sensitive to structures of different sizes.

Cosmic web identification: NEXUS

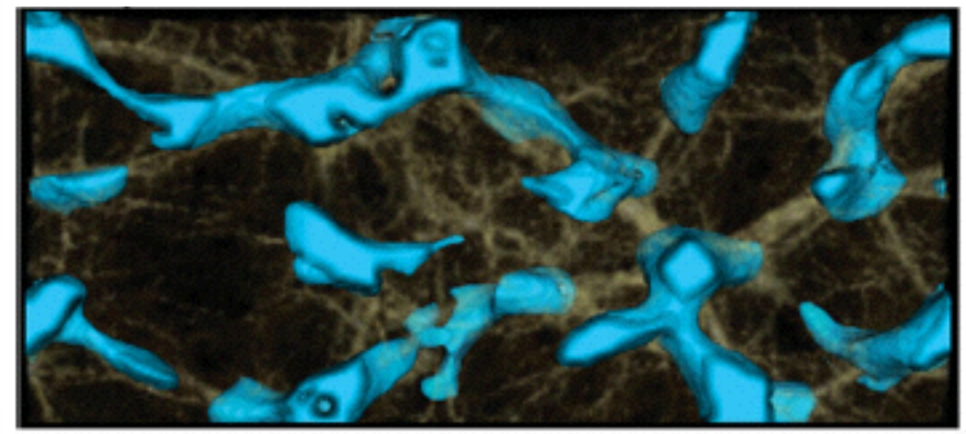
1. Smooth the input density field.
2. Compute the Hessian of the smoothed density field.
3. Use the Hessian eigenvalues to assign an environment signature to each point.
4. Combine information from a range of smoothing scales.

Cosmic web identification: NEXUS

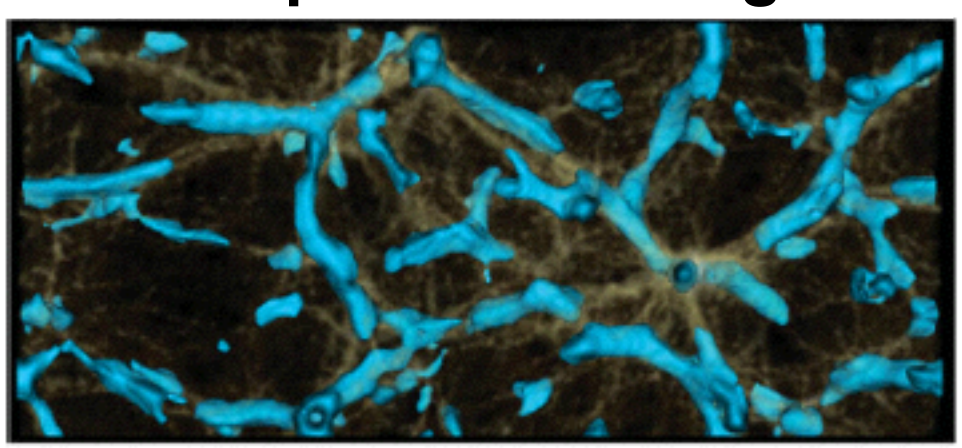
1 Mpc/h smoothing



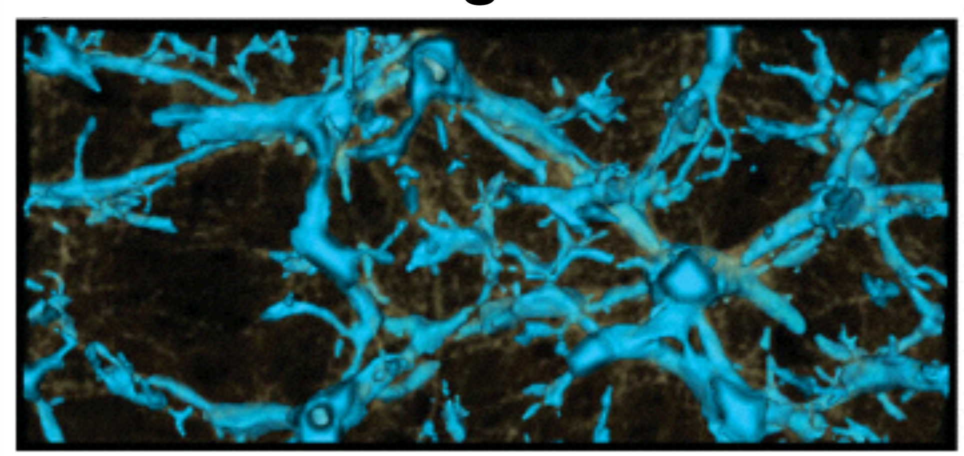
4 Mpc/h smoothing



2 Mpc/h smoothing

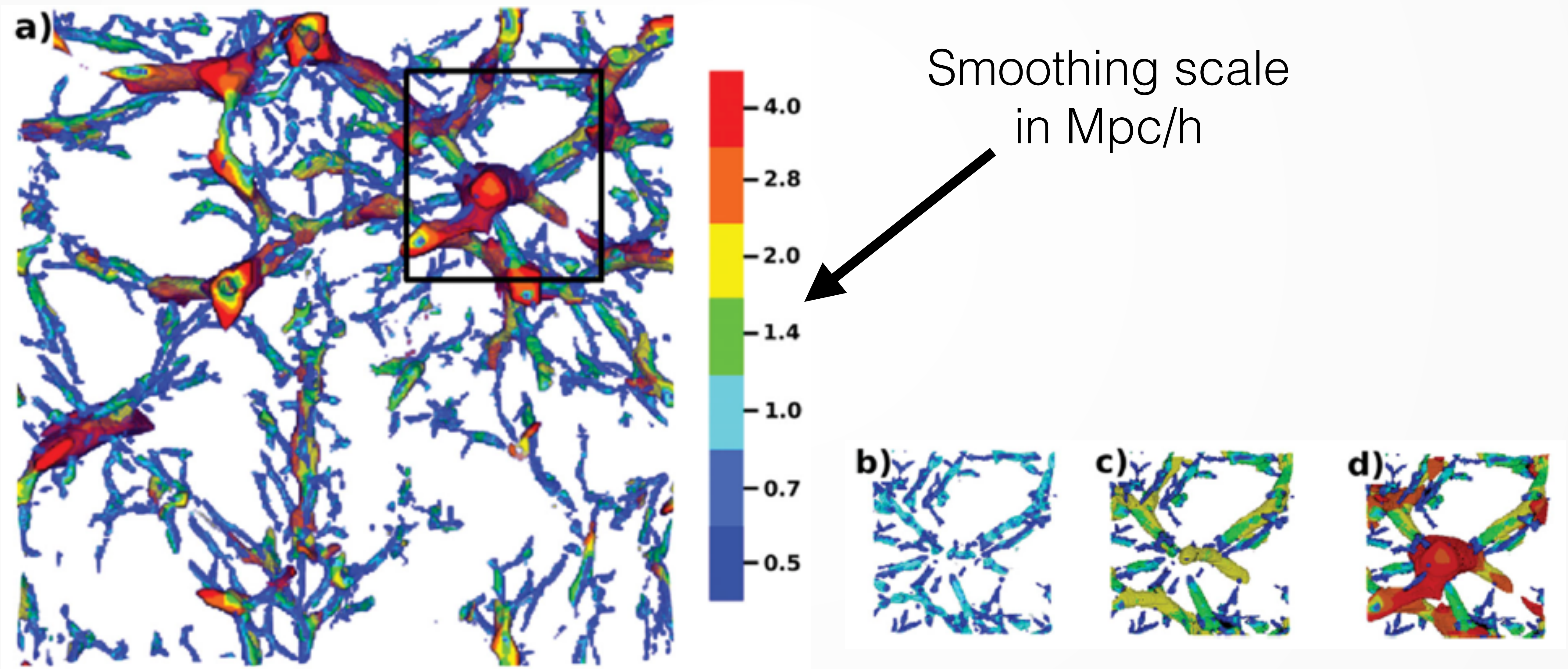


Combining all scales



MC + (2013)

The multiscale web

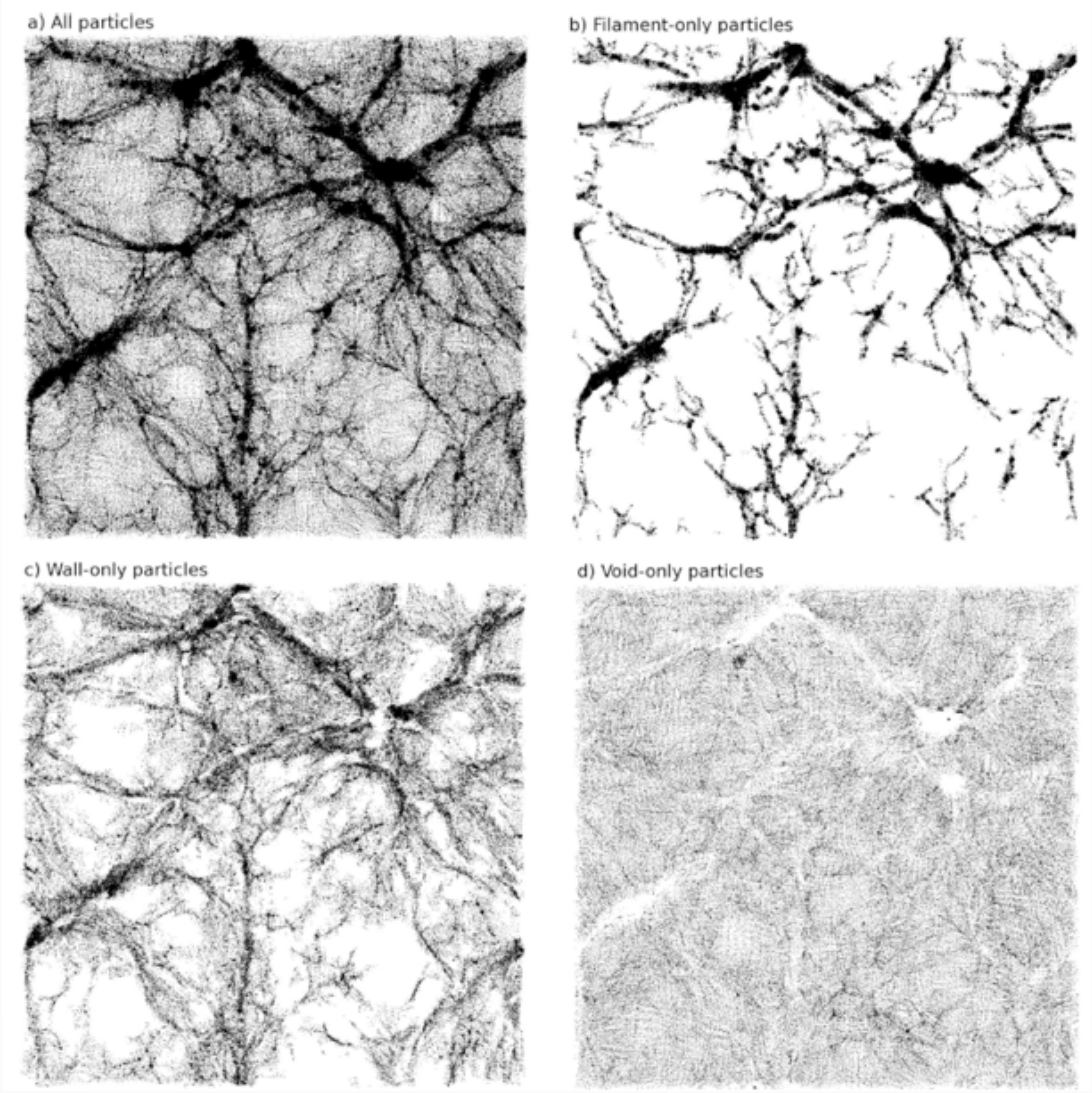


Smoothing scale in Mpc/h

MC + (2013)

Web & mass distribution

All particles

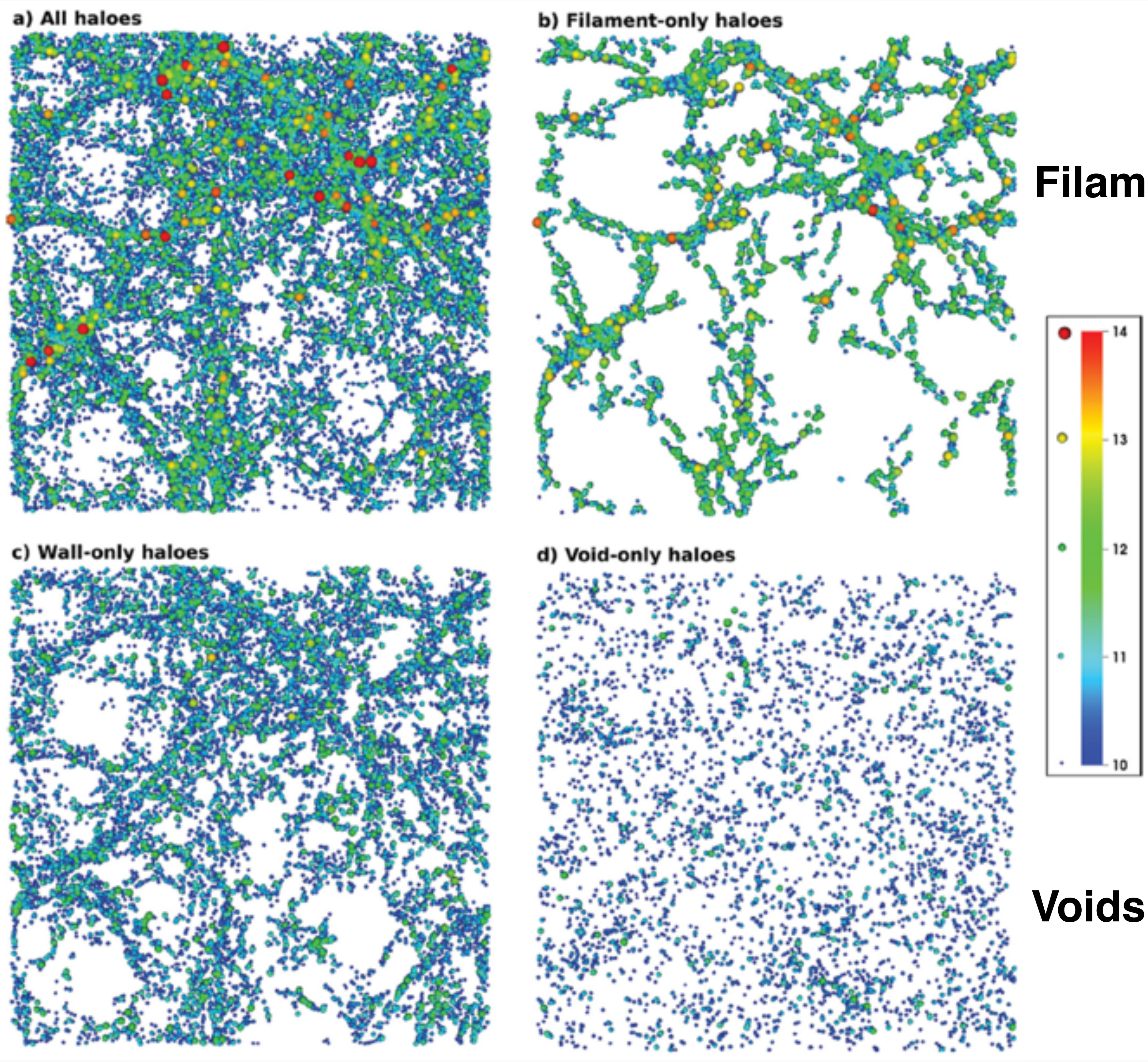


Filaments

Walls

Voids

Web & halo distribution



All haloes

Filaments

log halo mass

Walls

Voids

Summary

- Voronoi and Delaunay tessellations are self-adaptive and parameter free methods that preserve the hierarchical and anisotropic properties of the matter distribution.

- The cosmic web is highly hierarchical, with structures on a wide range of scales; we need a multiscale approach to identify structures over all scales.