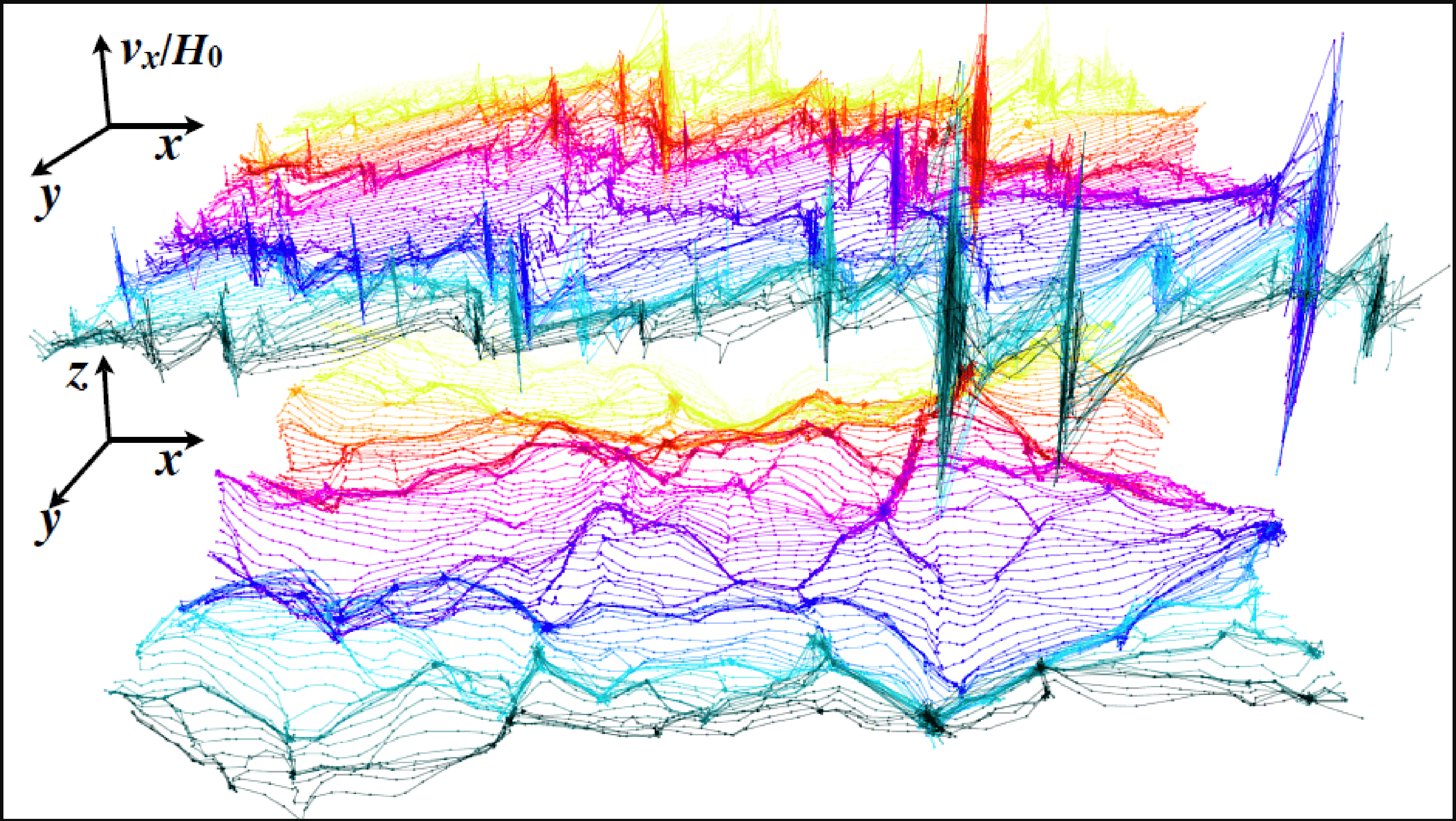
# The Cosmic Web in Phase-Space





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# Outline

- Collisionless fluids: the Vlasov-Poisson equation
- The Zel'dovich approximation
- Shell-crossing and caustics
- The dark matter phase-space sheet

# Collisionless Fluids

Self-gravitating, collisionless system (e.g. distribution of dark matter) evolves via *Vlasov-Poisson* equations.

$f$ = phase-space density distribution function

$\phi$ = gravitational potential

$\rho$ = projected density

**Vlasov-Poisson in 1-D in some units:**

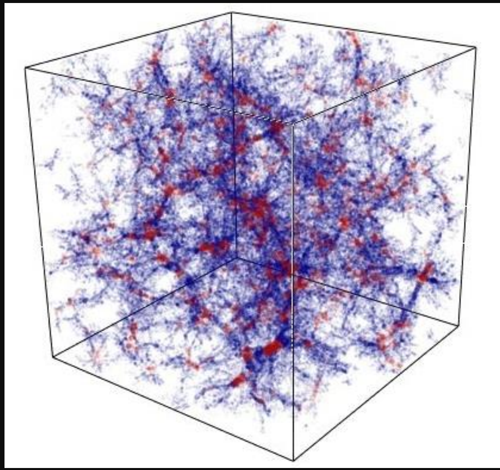
$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial v} - \frac{\partial \phi}{\partial x}\frac{\partial f}{\partial v} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} = 2\rho(x,t)$$

$$\rho(x,t) \equiv \int f(x,v',t)\mathrm{d}v'$$

*N*-body simulations solve (in 3-D) by representing continuous dark matter fluid by a set of massive particles, discretizing the collisionless fluid.

# Collisionless Fluids



- $N$-body discretization: $f \sim$ set of Dirac delta functions
  - Reduce impact by *softening gravitational forces* (SPH) or using *adaptive mesh refinement* (AMR)
- Can solve Vlasov-Poisson numerically in 1-D without discretization *(Colombi & Touma 2008, 2014)*:
  - Define small patches in Lagrangian space (initial conditions), where by Liouville theorem, $f[x(t), v(t), t] = constant$
  - Area of each patch conserved because collisionless fluid is incompressible in phase-space → **"waterbags"**

# Gaussian initial conditions: $f(x,v) = 4e^{-(x^2+v^2)/0.08}$

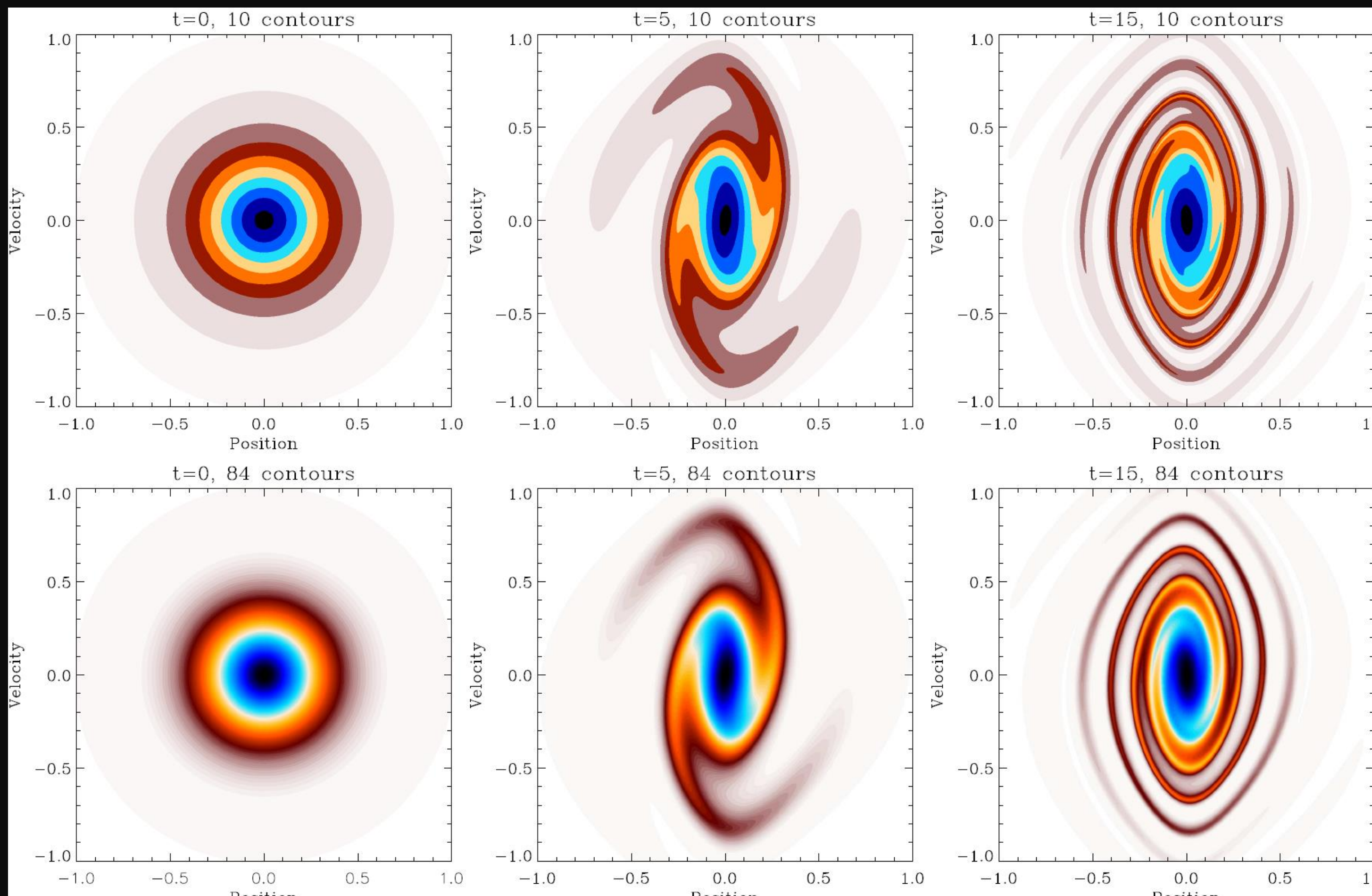
*time evolution →*

10 waterbags

$v$

84 waterbags

$v$




*(Colombi & Touma 2014)*

$x$ $x$ $x$

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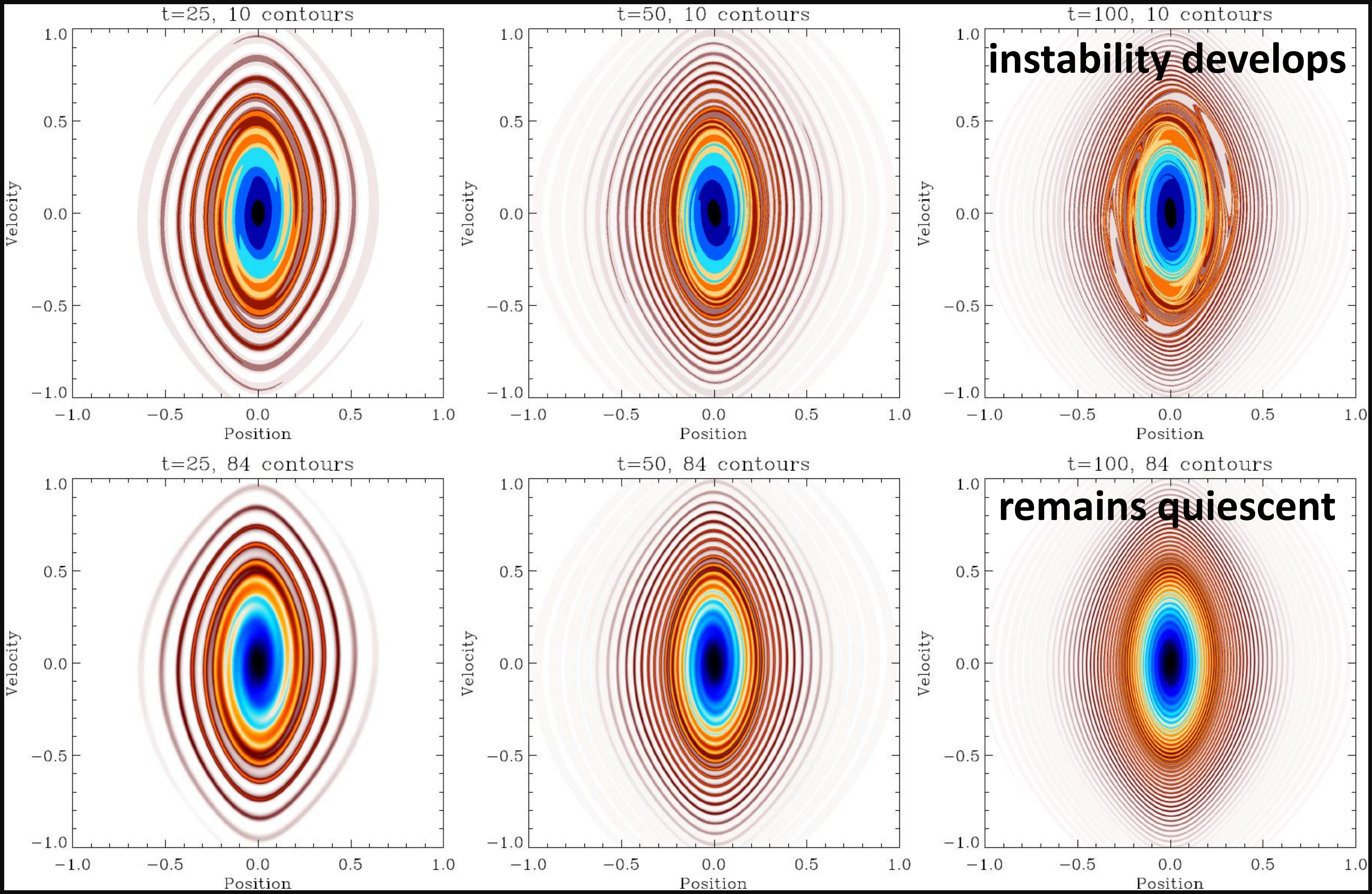


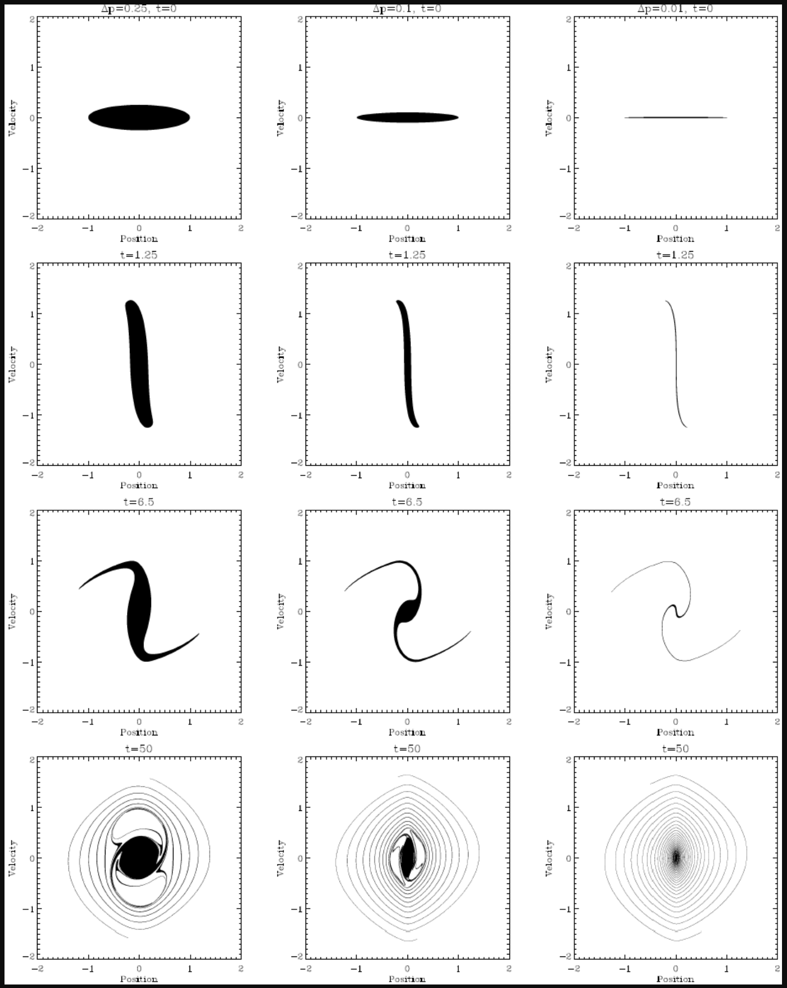

$x$

*(Colombi & Touma 2014)*

# Approaching the cold case

"Cold" initial conditions have very small velocity dispersion

*initial velocity dispersion decreases →*




*time evolution ↓*

*(Colombi & Touma 2014)*

# Approaching the cold case

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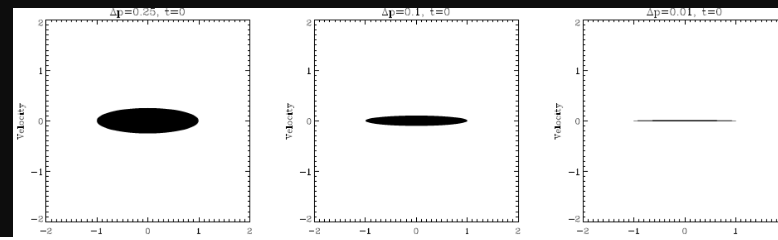
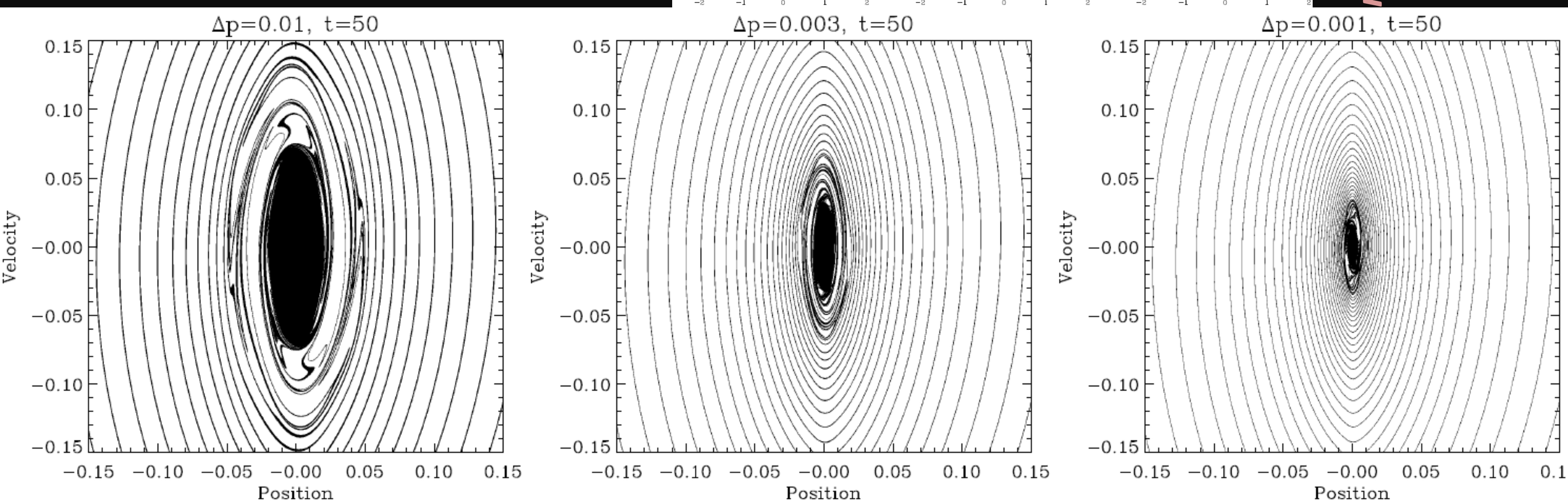
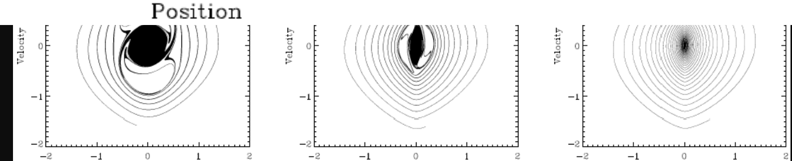
*time evolu*




*(Colombi & Touma 2014)*

# Analytic Approximation: Zel'dovich

- The Zel'dovich approximation maps initial Lagrangian coordinates, $q$, to co-moving Eulerian coordinates, $x$ *(Zel'dovich 1970; Shandarin & Zel'dovich 1989; Bouchet et al. 1995):*

$$x(q,t) = q - D(t)\nabla_q \phi(q)$$

where

$D(t)$ = linear growth function

$\phi(q)$ = initial displacement potential, proportional to the gravitational potential.

- Poisson's equation is then

$$D(t)\nabla_q^2 \phi(q) = \delta_0(q,t)$$

Note that the mapping from $q$ to $x$ is only valid up to shell-crossing!

# Analytic Approximation: Zel'dovich

- Before shell-crossing, we can write the density in terms of the Jacobian of the Lagrangian to Eulerian transformation:

$$\frac{\rho(x,t)}{\bar{\rho}} = \left|\frac{\partial x_i}{\partial q_j}\right|^{-1} = \frac{1}{J(q,t)} = 1 + \delta(q(x))$$

($J$ is the determinant of the deformation tensor.)

- The Jacobian can be written in terms of eigenvalues of symmetric matrix $d_{ij}(q)$:

$$d_{ij}(q) = \frac{\partial^2 \phi(q)}{\partial q_i \partial q_j}$$

$$J(q,t) = (1 - D\lambda_1)(1 - D\lambda_2)(1 - D\lambda_3) = \frac{1}{1 + \delta_q}$$

# Shell-crossing & Caustics

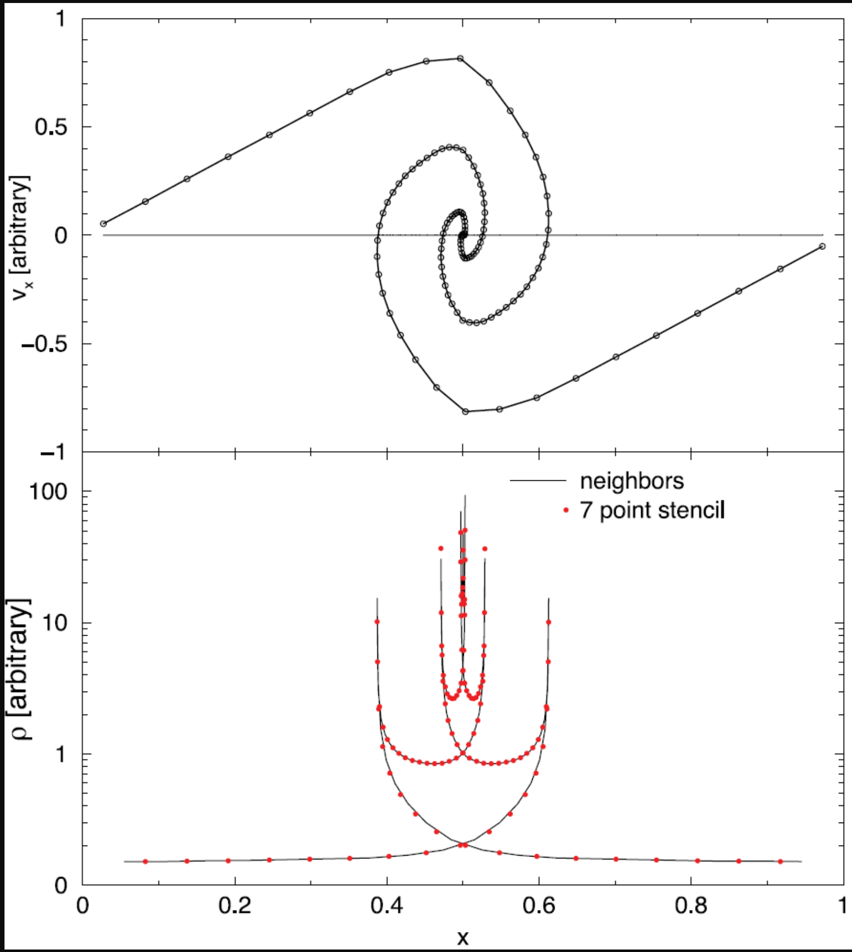
- A "pancake" forms perpendicular to the principal axis of the deformation tensor with largest negative eigenvalue (if any)
- Pancakes (or walls) are the first structures of the cosmic web
- Can also refer to them as "shell-crossing" or "caustics" but not as delicious

*"Pancake theory"*

# Shell-crossing & Caustics

- In 1-D, Zel'dovich plane-wave collapse of initially cold distribution (small initial velocity perturbation) forms spirals in phase-space and caustics in projected density

- Outside of caustics is *single-stream*, within is *multi-stream* region where velocity field is multi-valued.
  - (1, 3, 5, … streams)



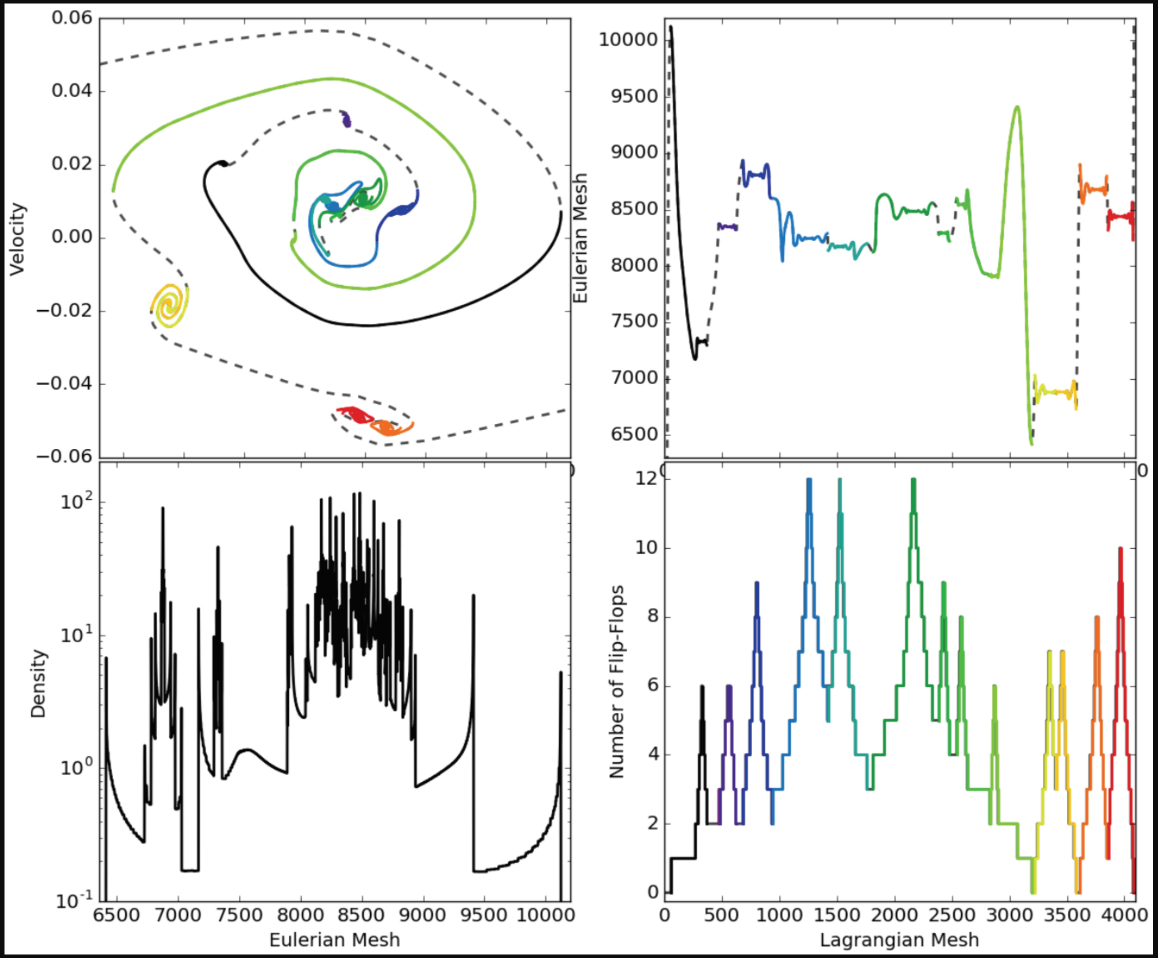

*(Abel, Hahn, & Kaehler 2012)*

# Collapse of a one-dimensional halo

substructure visible in phase-space

caustics in projected density

*(Shandarin & Medvedev 2016)*

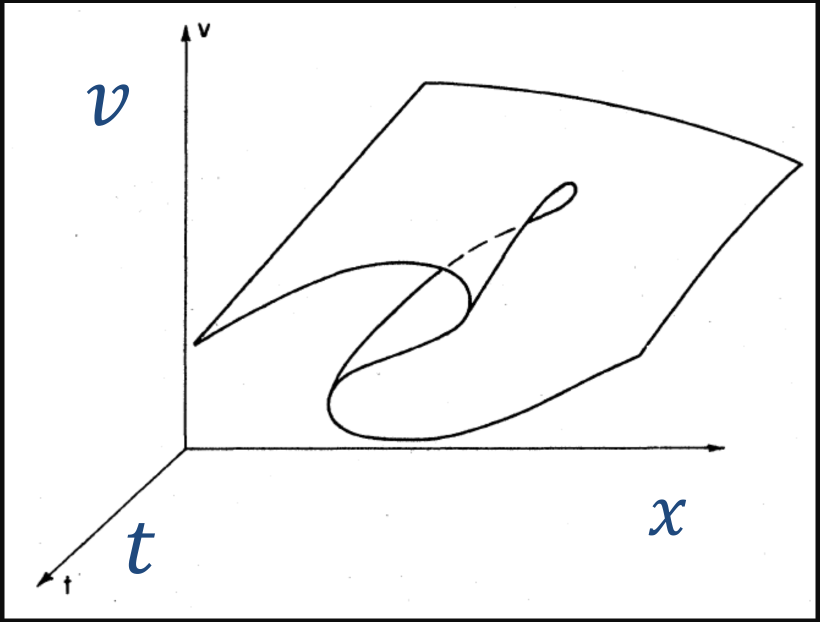



$x$ $q$

Lagrangian submanifold $x = x(q;t)$

number of crossings in time $t$

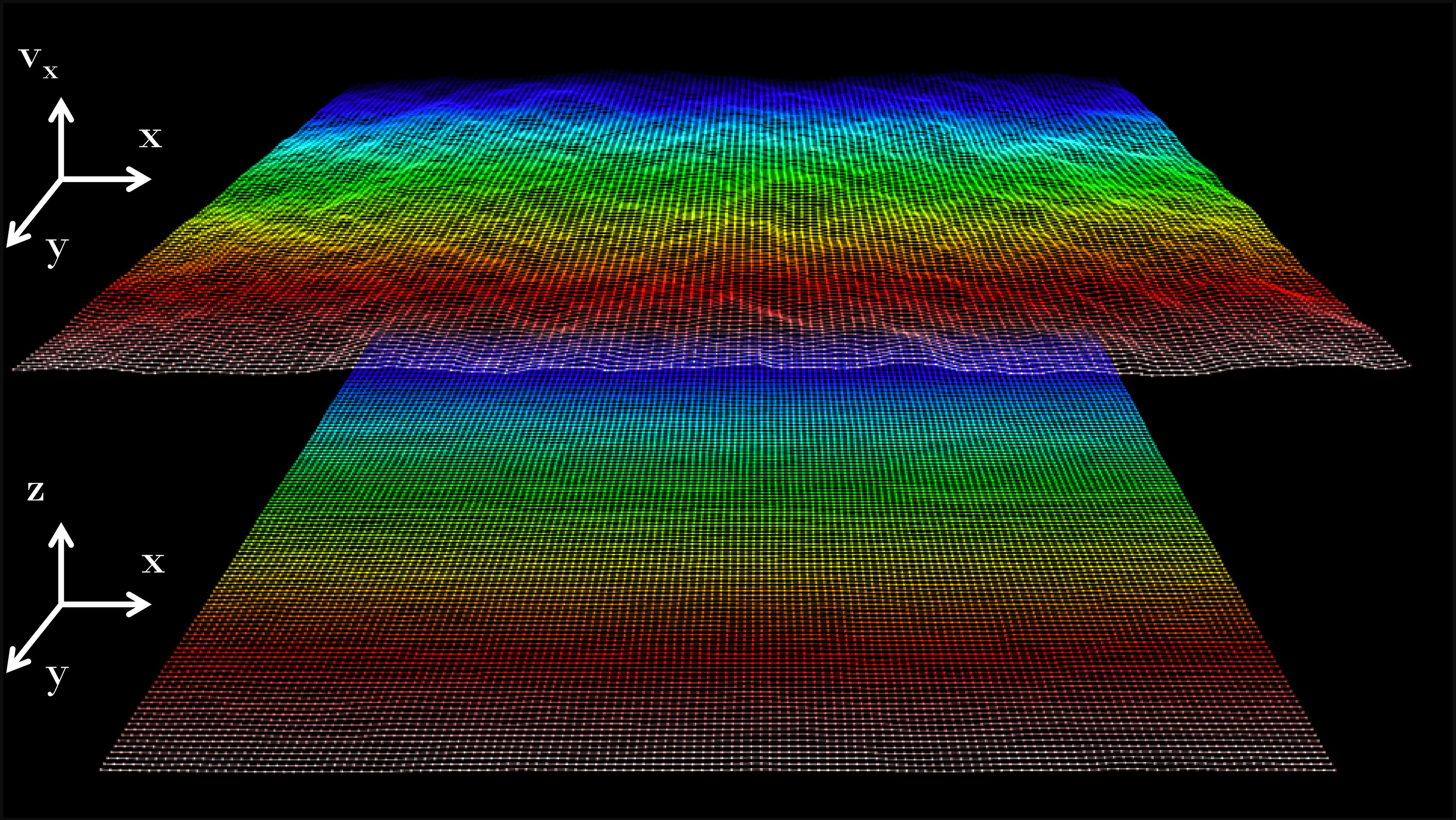
# The Phase-Space Sheet



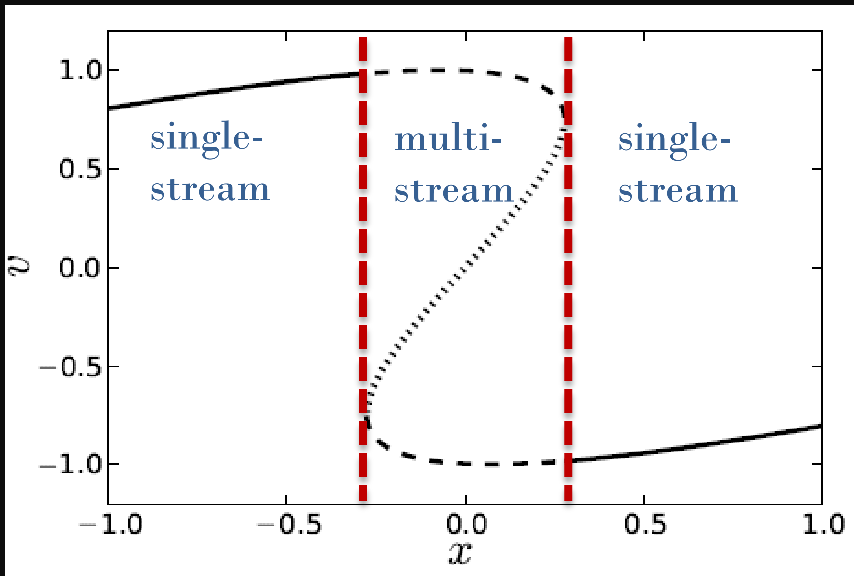

Time evolution of phase-space fold
*(Shandarin & Zel'dovich 1989)*

- 1-D collapse (left) creates folds in 2-D phase-space
- In the real world (i.e. simulations), cold dark matter undergoes 3-D collapse in 6-D phase space.
- This folding 3-D manifold is called the *dark matter sheet* or *phase-space sheet* when referring to 6-dimensional phase-space, or the *Lagrangian submanifold* when referring to the 6 dimensions of $(q, x)$ space.

# 2-D Lagrangian *N*-body slice



# The ORIGAMI Cosmic Web

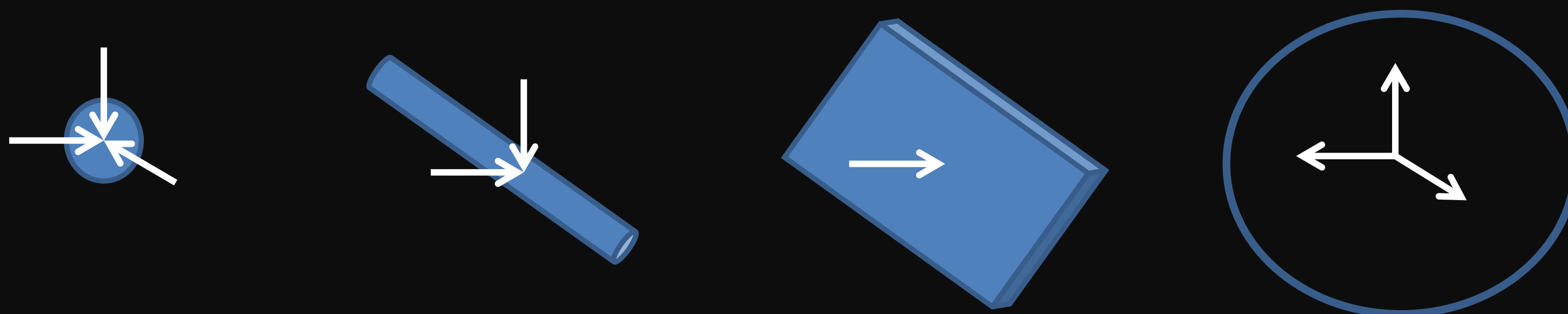



Find the phase-space folds by looking for simulation particles that are out of order along orthogonal axes

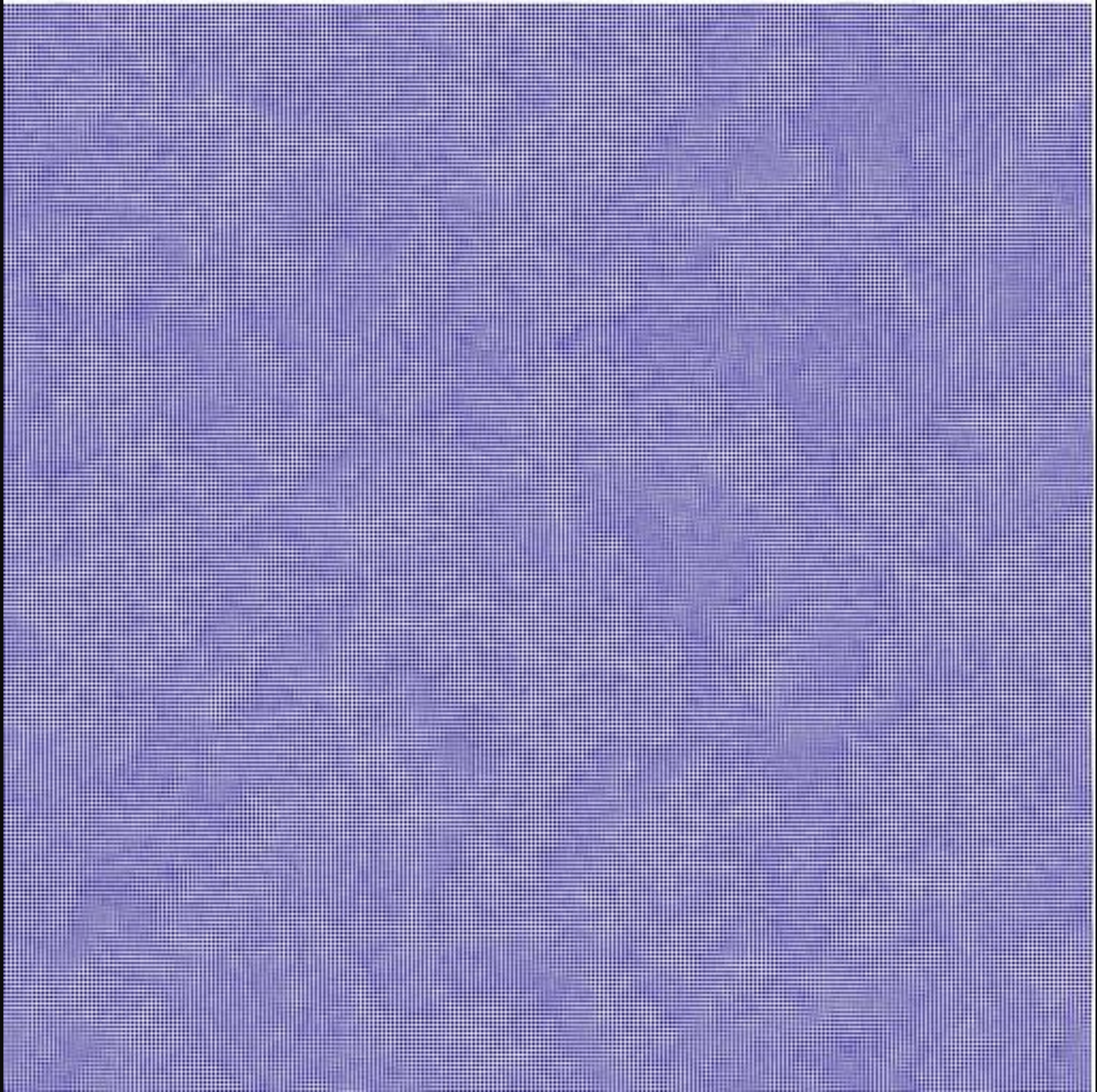
*(Falck, Neyrinck, & Szalay 2012)*

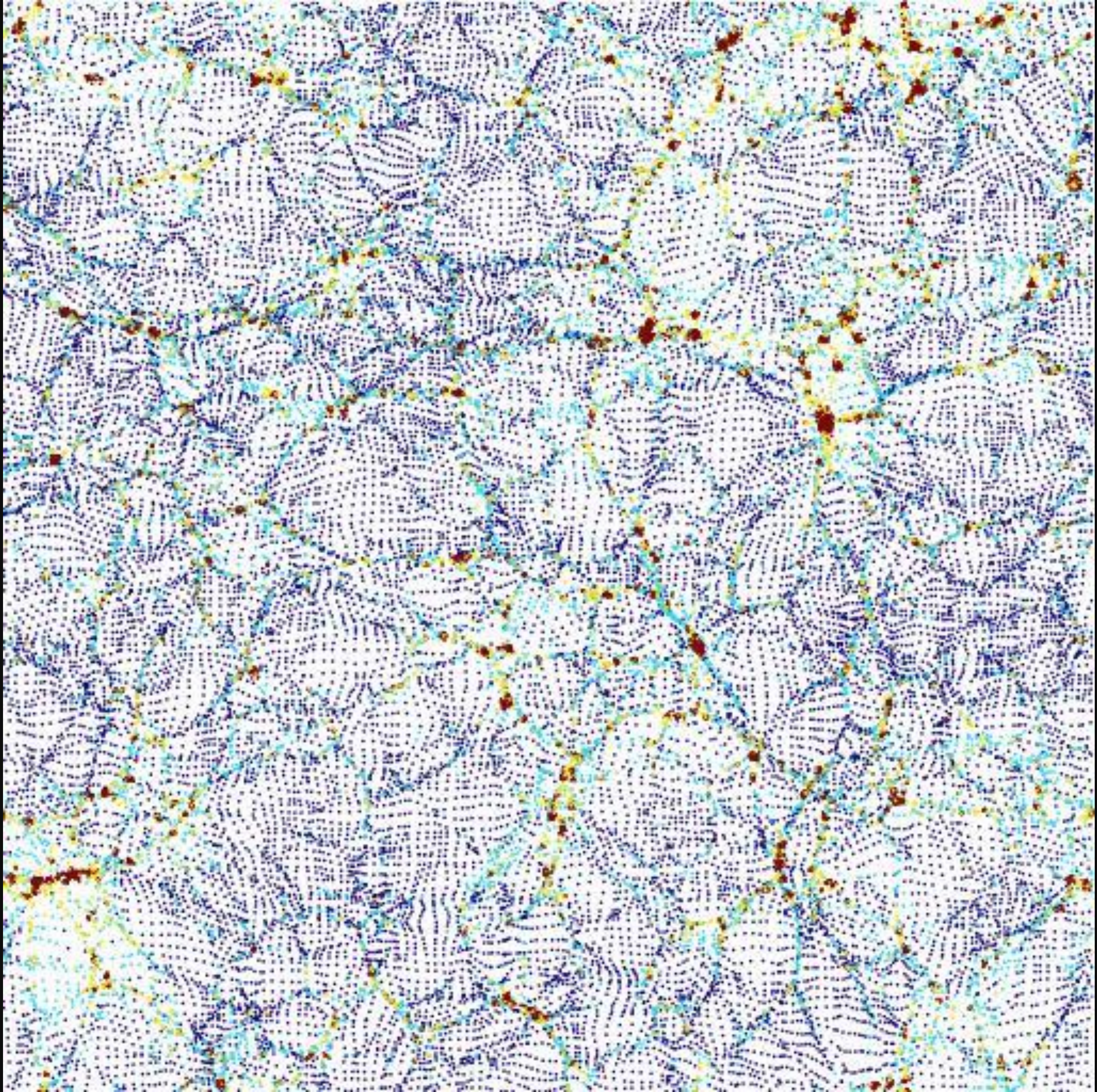
$$J(q,t) = (1 - D\lambda_1)(1 - D\lambda_2)(1 - D\lambda_3) = \frac{1}{1+\delta}$$

Halos collapse along 3 axes, Filaments 2, Walls 1, and Voids 0

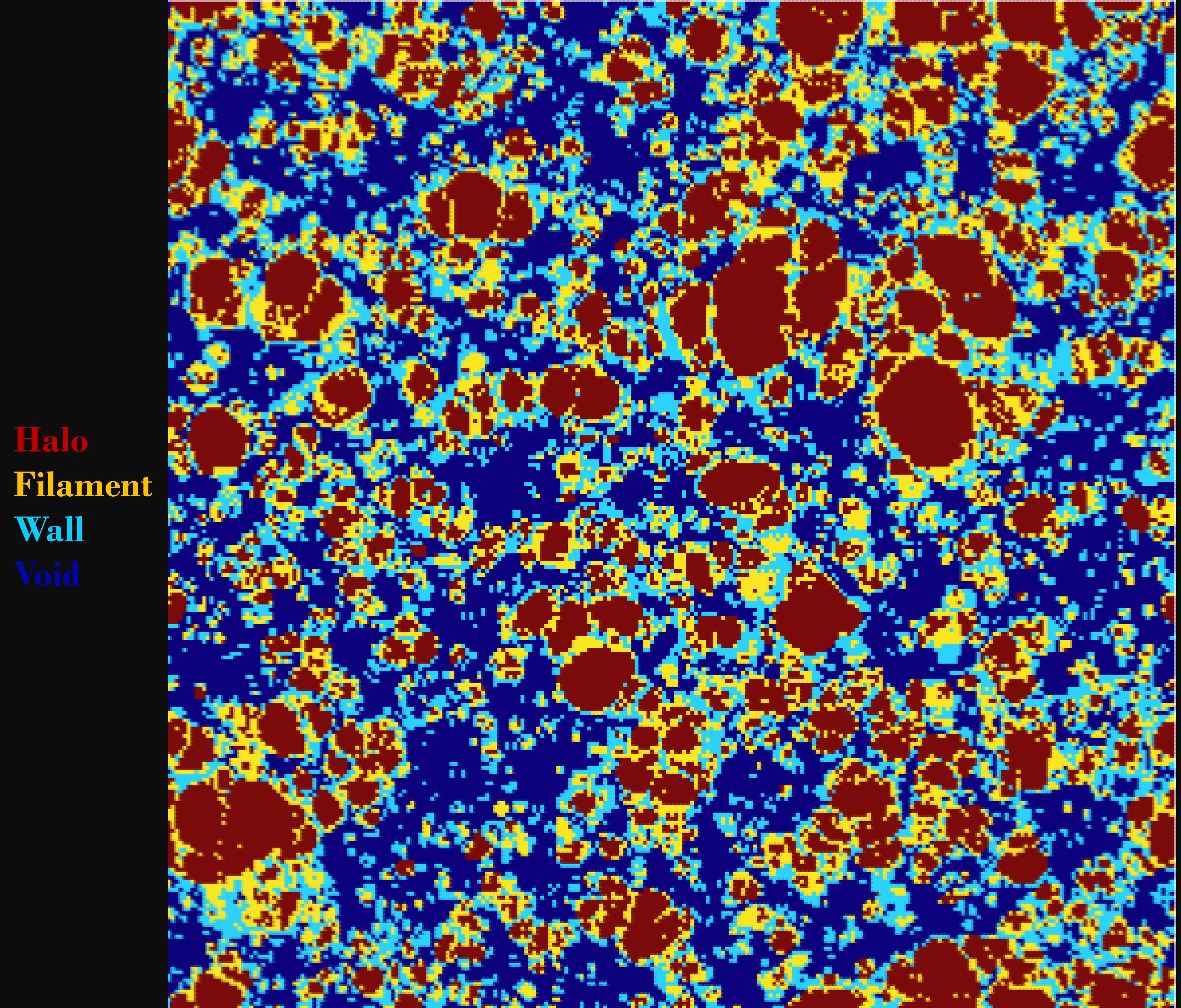


Halo
Filament
Wall
Void





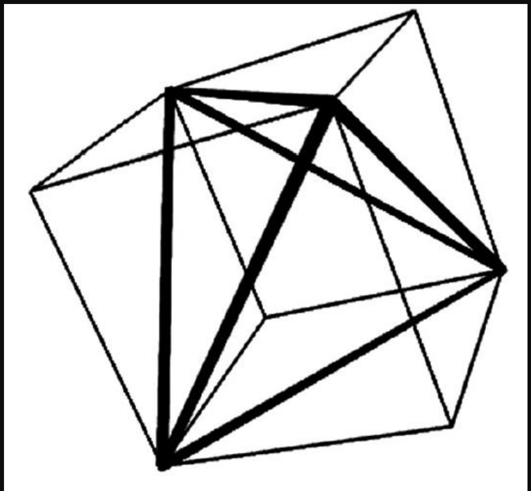
Halo
Filament
Wall
Void



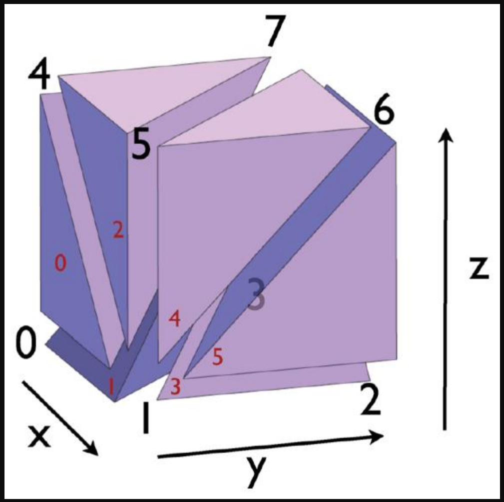
Halo
Filament
Wall
Void

# Tracing the evolution of the manifold

- Tessellate Lagrangian space, with *N*-body particles as vertices of tetrahedra.
- Shell-crossing occurs when tetrahedra invert.
- Instead of mass at particles, can spread mass over tetrahedra volumes.



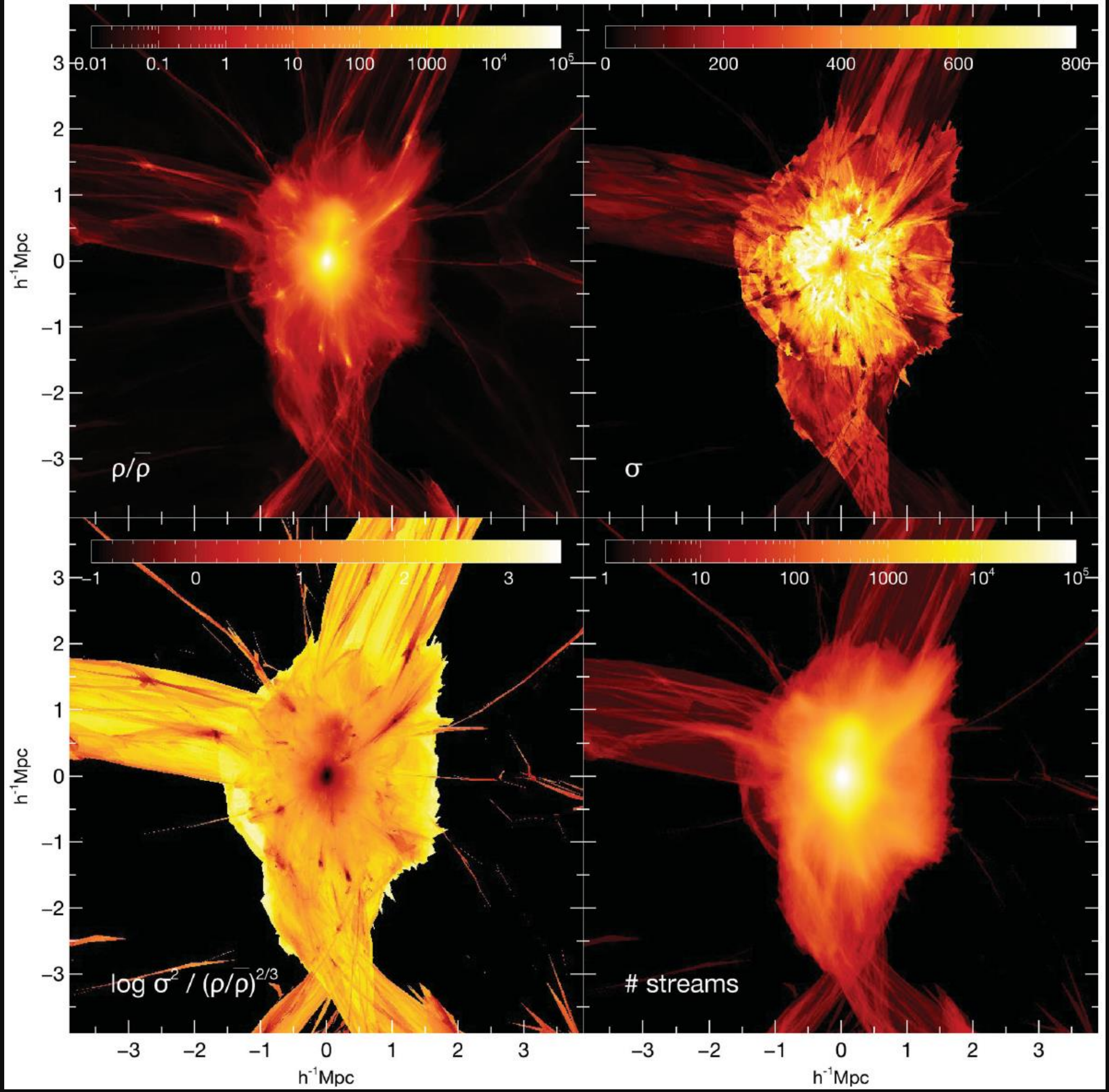
*(Shandarin, Habib, & Heitmann 2012)*




*(Abel, Hahn, & Kaehler 2012)*

density

velocity dispersion

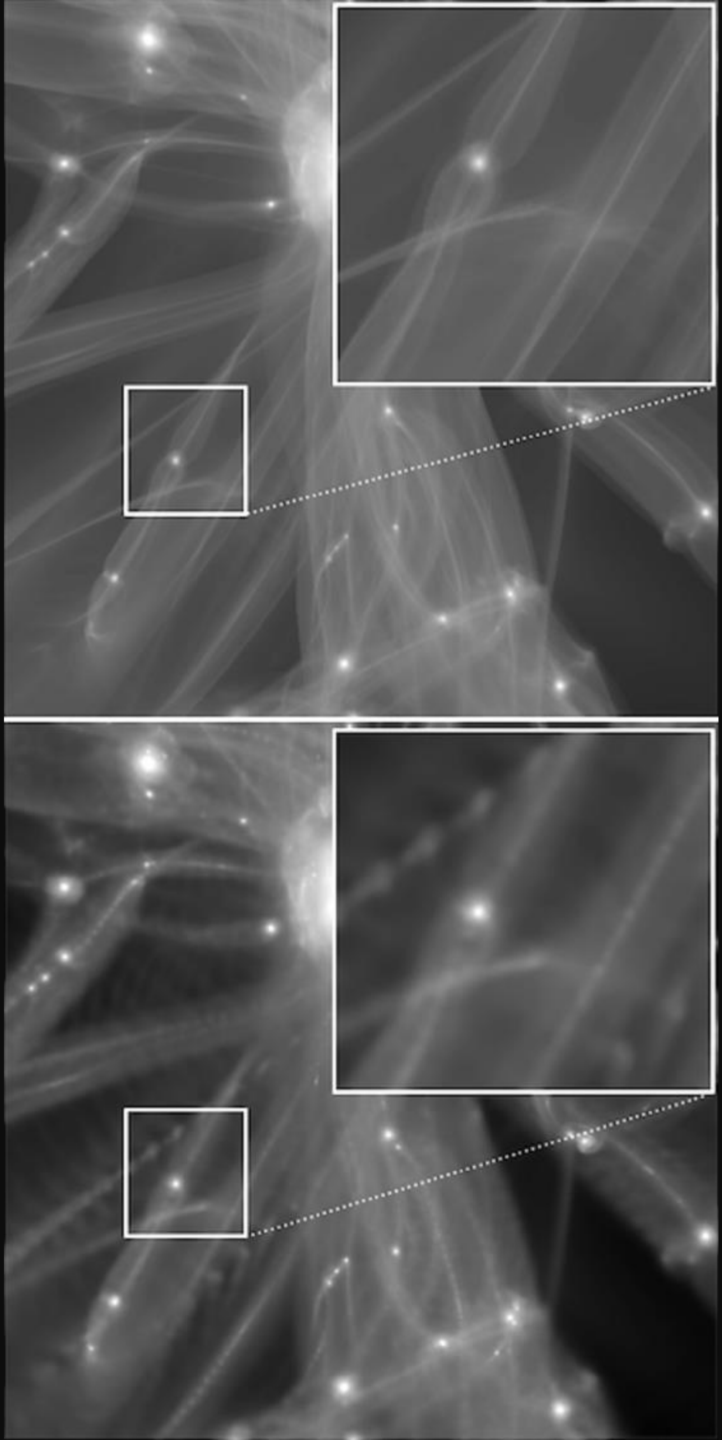



entropy

# of streams

*(Abel, et al. 2012)*

Using the phase-space sheet to compute density of a WDM simulation eliminates small granularity introduced with SPH kernel density



density from phase-space sheet

density from SPH kernel

*(Hahn, Abel, & Kaehler 2013)*

# Summary

- $N$-body simulations discretely solve the Vlasov-Poisson equations, but cold dark matter is a continuous collisionless fluid.
- The Zel'dovich approximation is a Lagrangian 1$^{st}$ order solution and gives insight on non-linear dynamics.
  - Spirals form in phase-space, and caustics in the 1-D projected density field at boundaries between single-stream and multi-stream regions
- Dark matter structure formation can be represented by collapse of a 3-D manifold in 6-D phase-space
  - Identify elements of cosmic web, count number of streams at any point, improve on $N$-body simulations, remove numerical fragmentation in WDM simulations, make cool visualizations, etc…