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TRIANGULATING THE UNIVERSE'S LAWS AND CONTENTS WITH THE 3-POINT CORRELATION FUNCTION

FUERTEVENTURA COSMOLOGY SUMMER SCHOOL
CANARY ISLANDS
19 SEPTEMBER 2017

OUTLINE

Why?

Data landscape

Four areas 3PCF can help with

How?

How do we measure it, model it, and match model to measurement?

Synthetic approach

History + cutting edge

MAJOR QUESTIONS IN COSMOLOGY

How do galaxies form?

What is the full theory of gravity?

What are the initial conditions?

What is the nature of dark energy?

WHAT IS THE DATA LANDSCAPE?

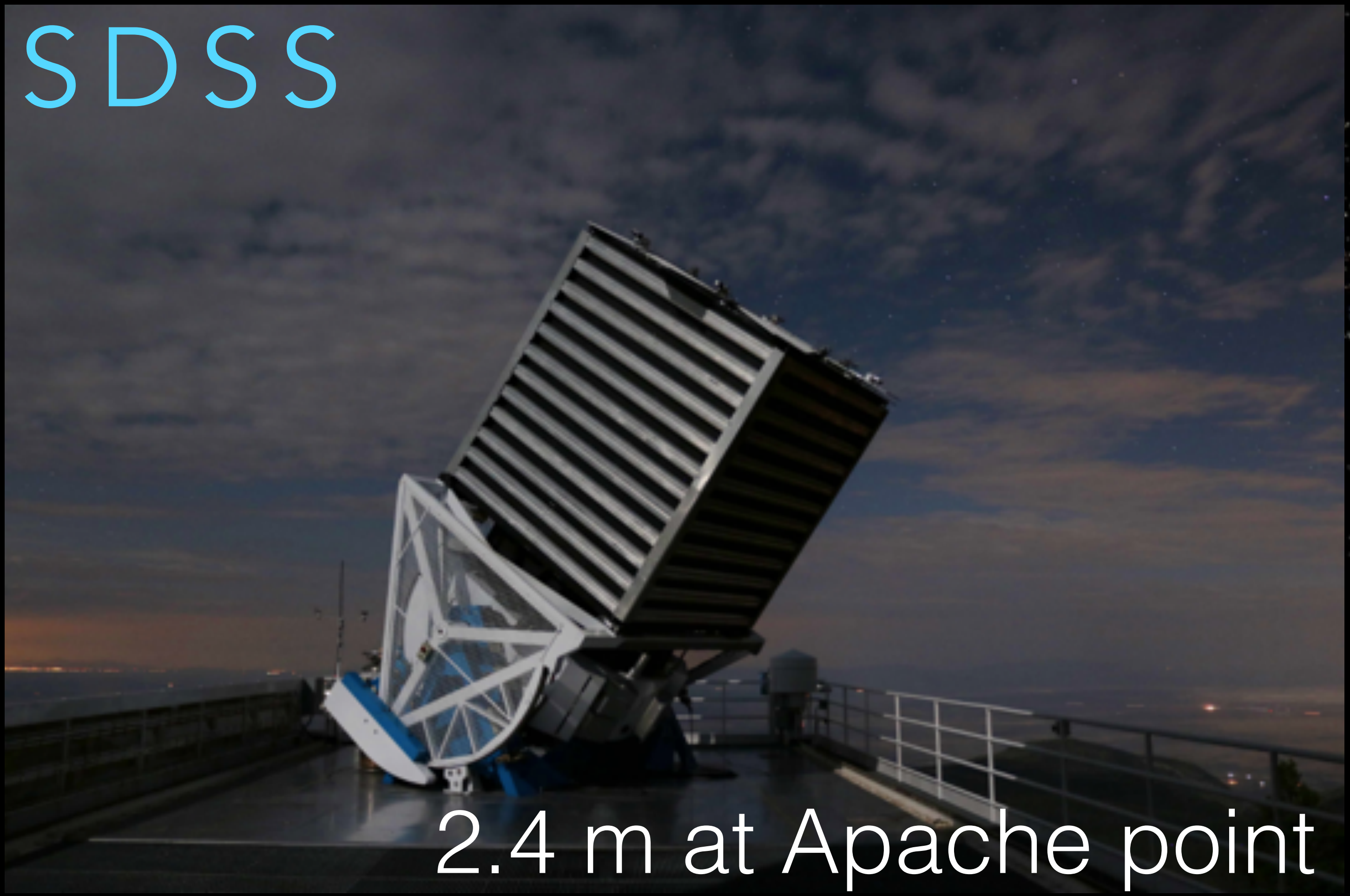
Current: BOSS, eBOSS, DES (photo)

Future: Dark Energy Spectroscopic Instrument (DESI)

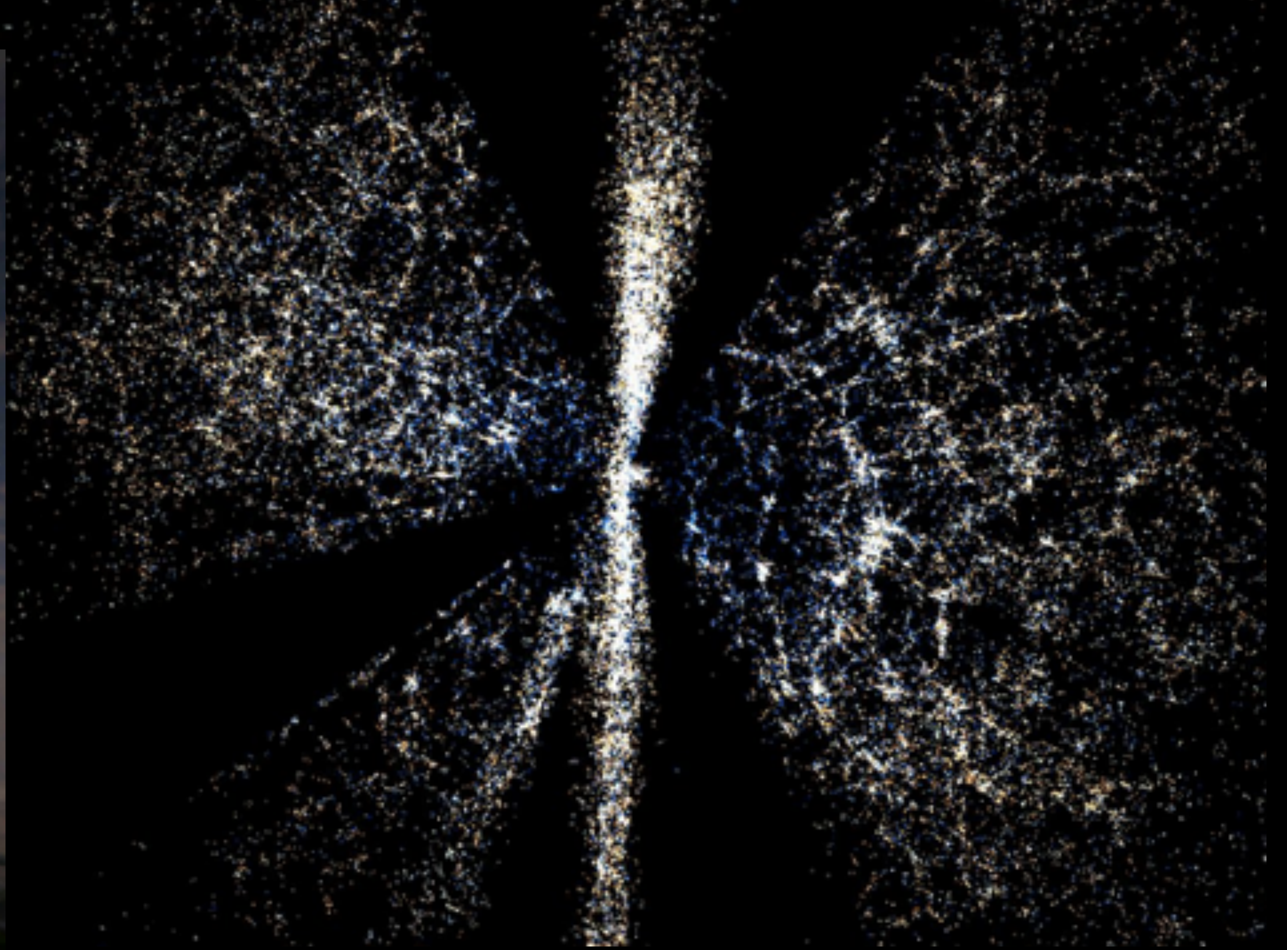
+other spectroscopic: Euclid, WFIRST

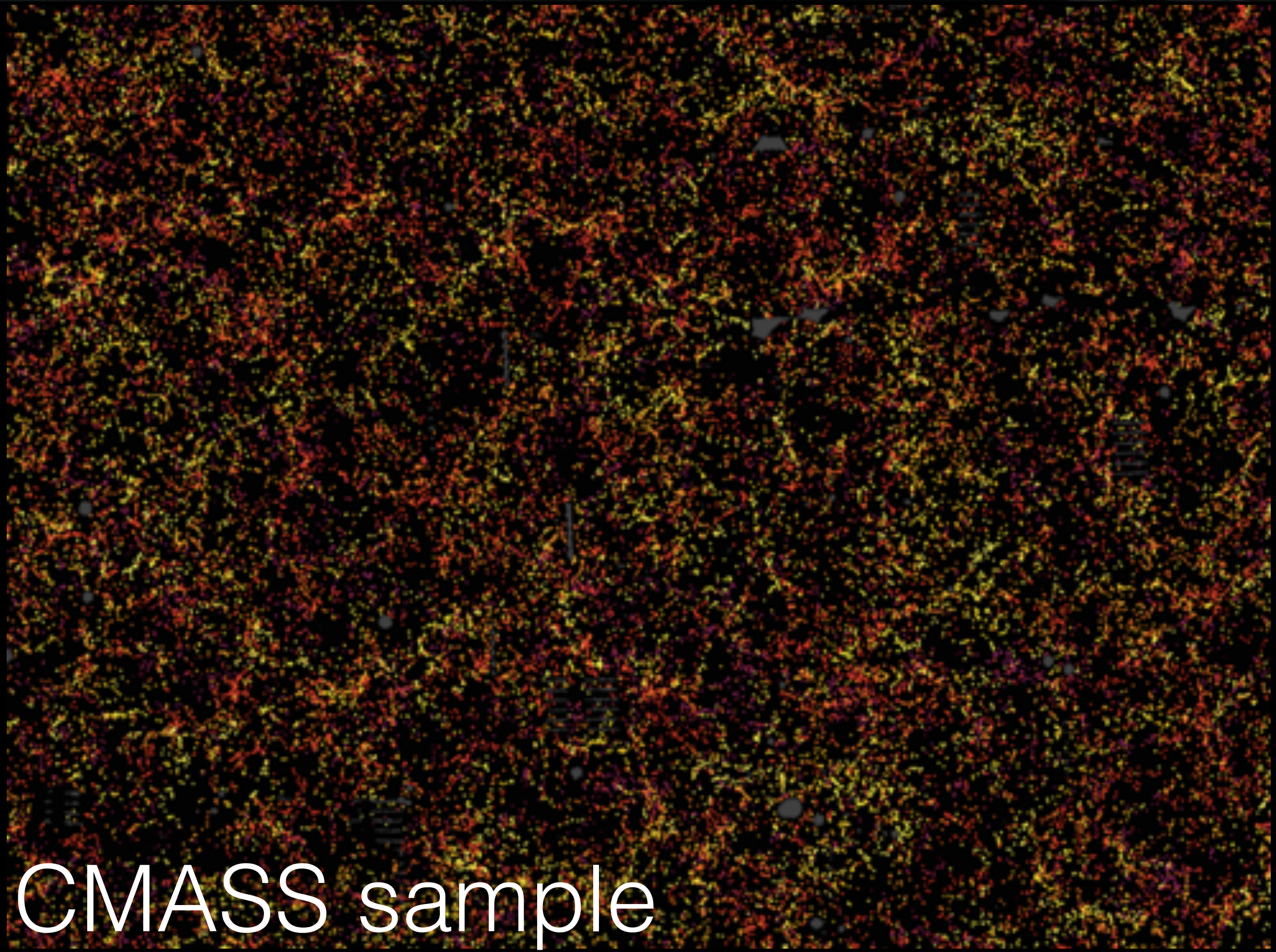
+other photometric: LSST

SDSS

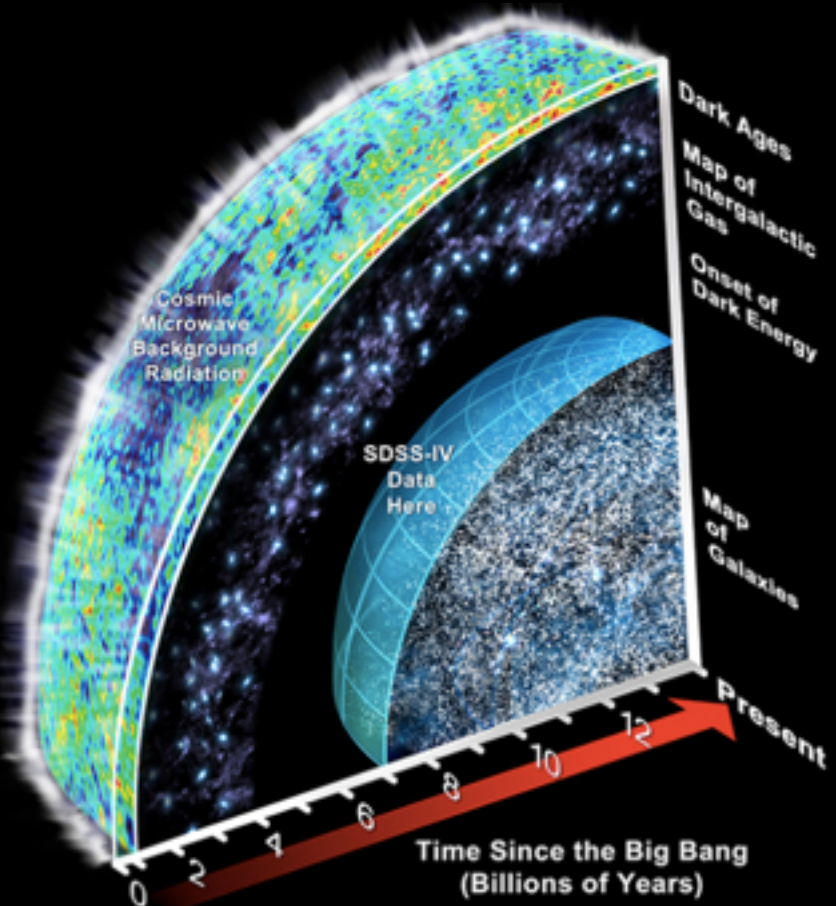


2.4 m at Apache point





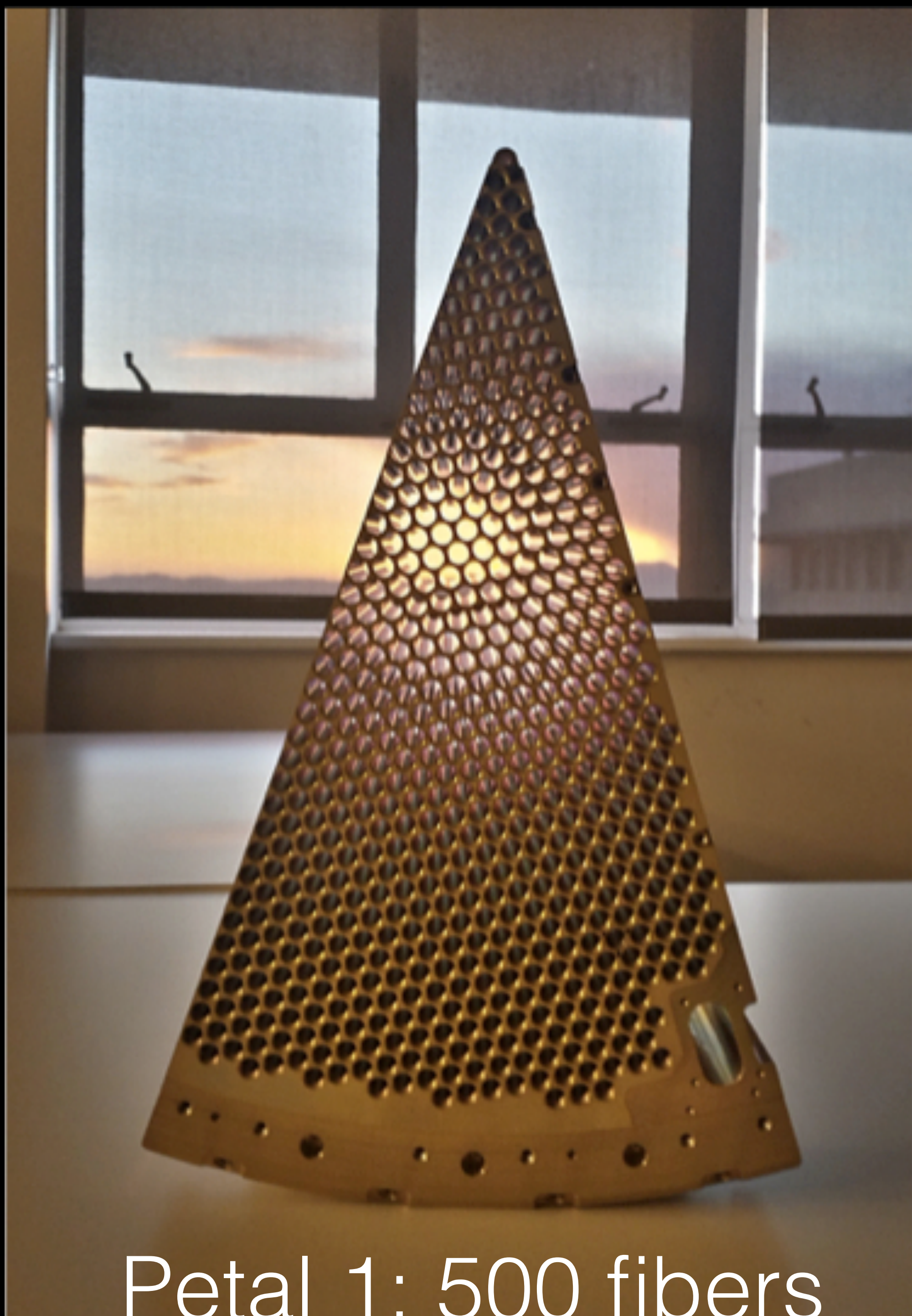
CMASS sample

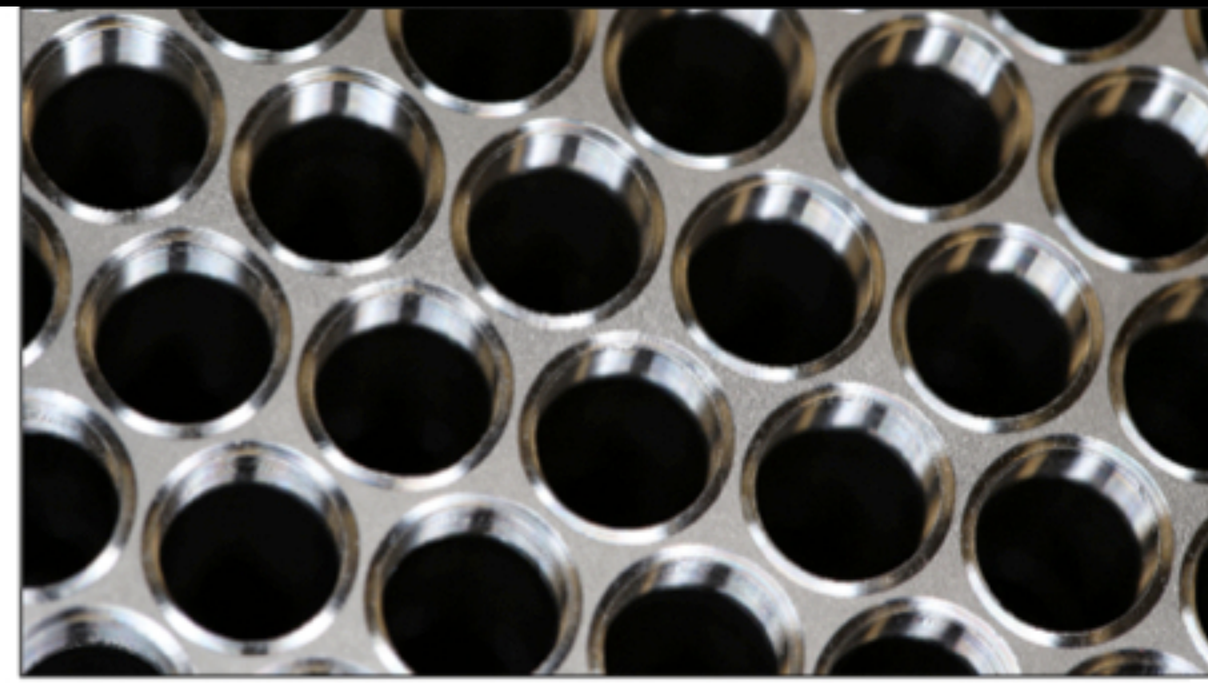


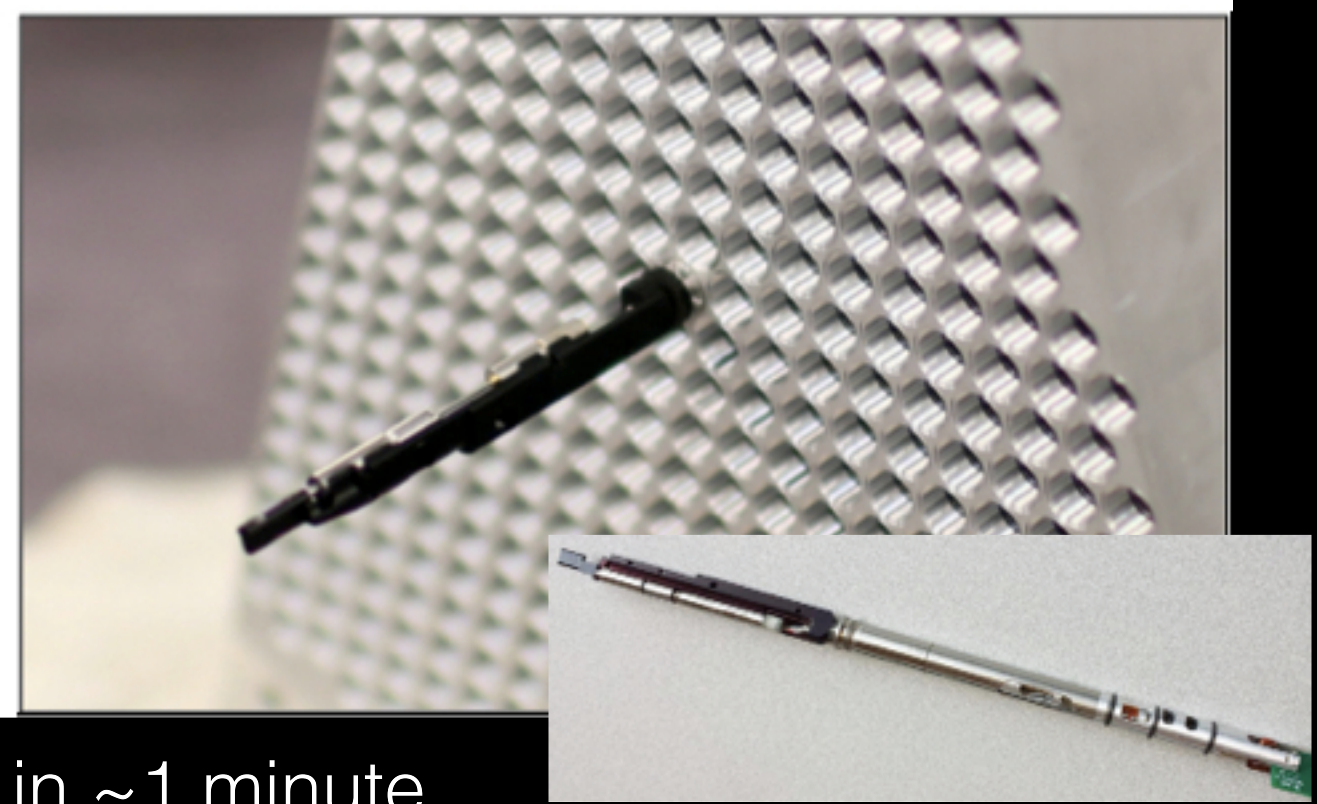
SDSS

http://astro.uchicago.edu/cosmus/projects/sloangalaxies/ http://www.sdss.org/press-releases/astronomers-map-a-record-breaking-1-2-million-galaxies-to-study-the-properties-of-dark-energy/ http://www.sdss.org/wp-content/uploads/2016/07/sdss_gaulme1.jpg

DARK ENERGY SPECTROSCOPIC INSTRUMENT


Petal 1: 500 fibers





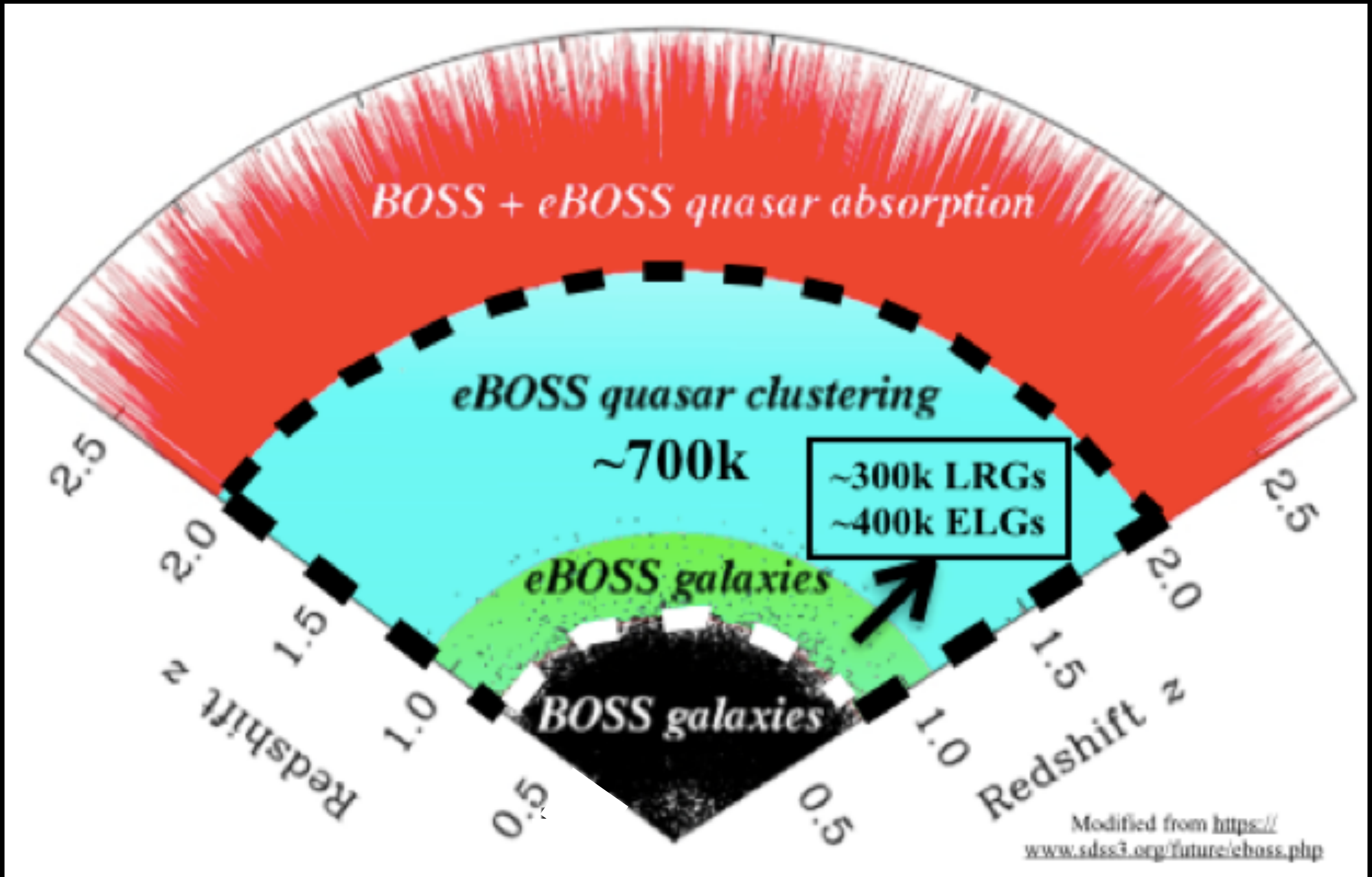
5k robotic positioners: reconfigure in ~1 minute

http://newscenter.lbl.gov/wp-content/uploads/sites/2/2016/08/first-petal-2.jpg

http://www.symmetrymagazine.org/sites/default/files/images/standard/proto10mm_pos.jpg

BOSS and eBOSS ~1 million galaxies each

DESI will get 30 million spectra: ELGs, LRGs, Quasars



How will we use this data to answer the key questions?

Make a 3-D map of the distribution of galaxies

https://www.pinterest.com/pin/79235274667115008/

Search for patterns

https://www.pinterest.com/pin/462322717978863523/

http://www.freakingnews.com/Angel-Butterflies-Design-by-Escher-Pictures-26905.asp

Search for patterns

Homogeneous

Isotropic

https://www.pinterest.com/pin/462322717978863523/

http://www.freakingnews.com/Angel-Butterflies-Design-by-Escher-Pictures-26905.asp

Laws of physics: same everywhere and in all directions
This is what we *mean* by laws

Galaxy clustering will be so too*

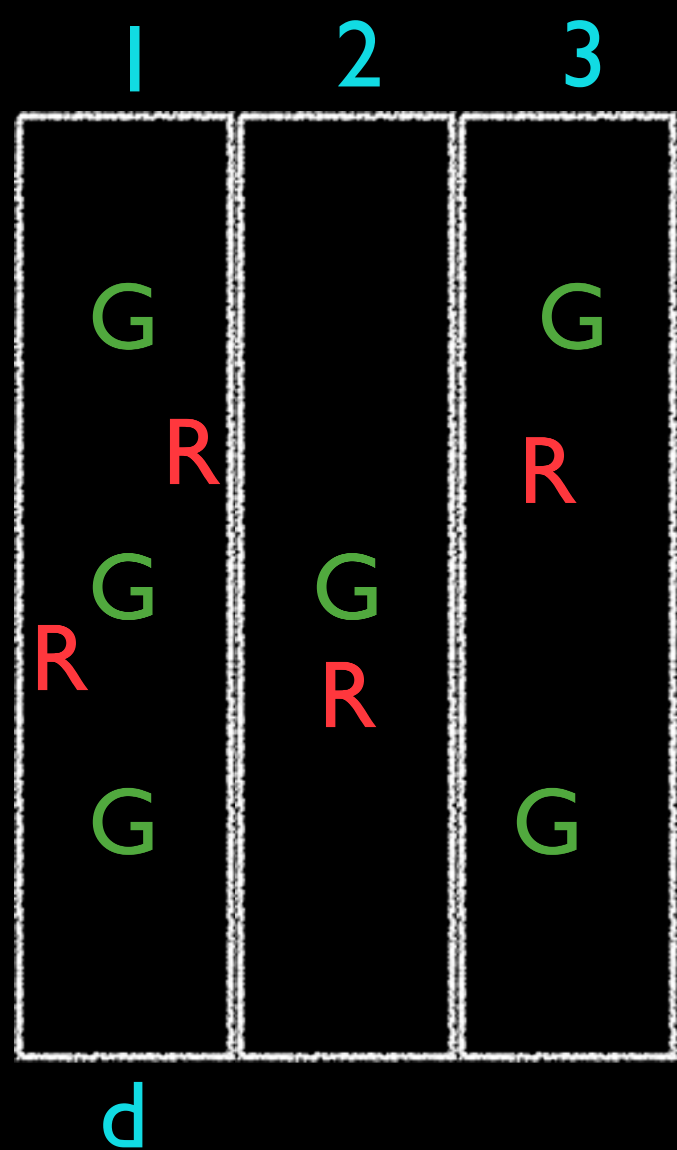
Need a way to *mod out* translations and rotations

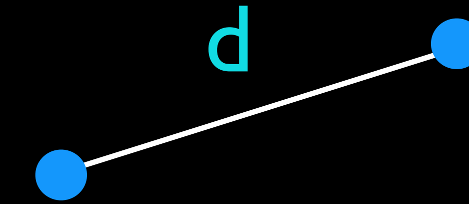
CORRELATION FUNCTIONS

2-point correlation function (2PCF): count excess pairs of galaxies over random

2PCF = (GG - RR)/RR

(in detail, we use Landy-Szalay, which actually is $(G-R)^2$/RR)





2PCF	2/3		2	
total	3	5	2	6
13			2	6
23	1	2		
12	2	3		
	d		2d	

For a Gaussian Random Field density, the 2PCF, or its Fourier-space analog the power spectrum, contains all the information

Intuitively: **Gaussian has a mean and variance—2 parameters**

We know cosmic mean density, and 2PCF is a generalization of the variance (zero-lag ξ is the variance)

But I am here to talk about the 3PCF . . .
excess triangles over random

because Universe is not just a GRF!

1) non-linear structure formation

and

2) possible primordial non-Gaussianity

RECENT LARGE-SCALE 3PCF

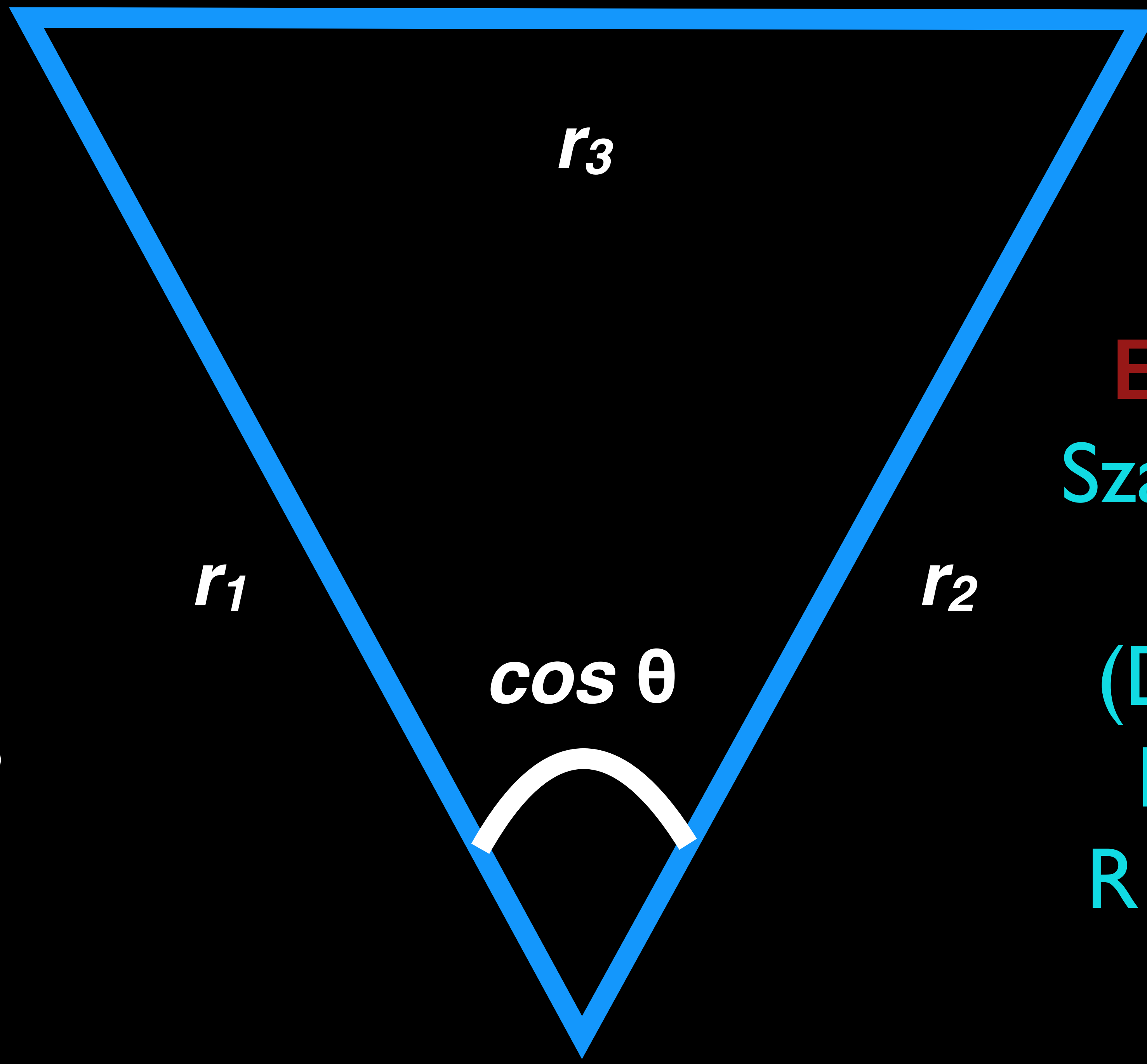
SDSS: Nichol+2006: 37k LRGs 2, 3 and 4:1, 40 Mpc/h,
Kulrkarni+2007, 51k SDSS DR3 LRGs, 1, 2, 3:1, 30 Mpc/h
Gaztanaga+2009: 40k DR7 LRGs, 2-3sigma BAO
Marin 2011: 106k DR7 LRGs, 2:1, 90 Mpc/h
McBride 2011a, b: 220k galaxies SDSS DR6, 2:1, 27 Mpc/h
Guo+2014: DR7 LRGs 1, 2, 3:1, 40 Mpc/h
Guo+2015: DR11 CMASS LRGs, 2:1, 40 Mpc/h
Slepian+ 2016a,b, 2017: 800k DR12 CMASS LRGs, all configs. up to 180 Mpc/h

WiggleZ: Marin+2013: 1, 2, and 3:1, 90 Mpc/h.

2dFGRS: Jing & Borner 2004: 60k, 40 Mpc/h
Wang+2004: 250k, 20 Mpc/h.

PARAMETRIZATION & ESTIMATOR

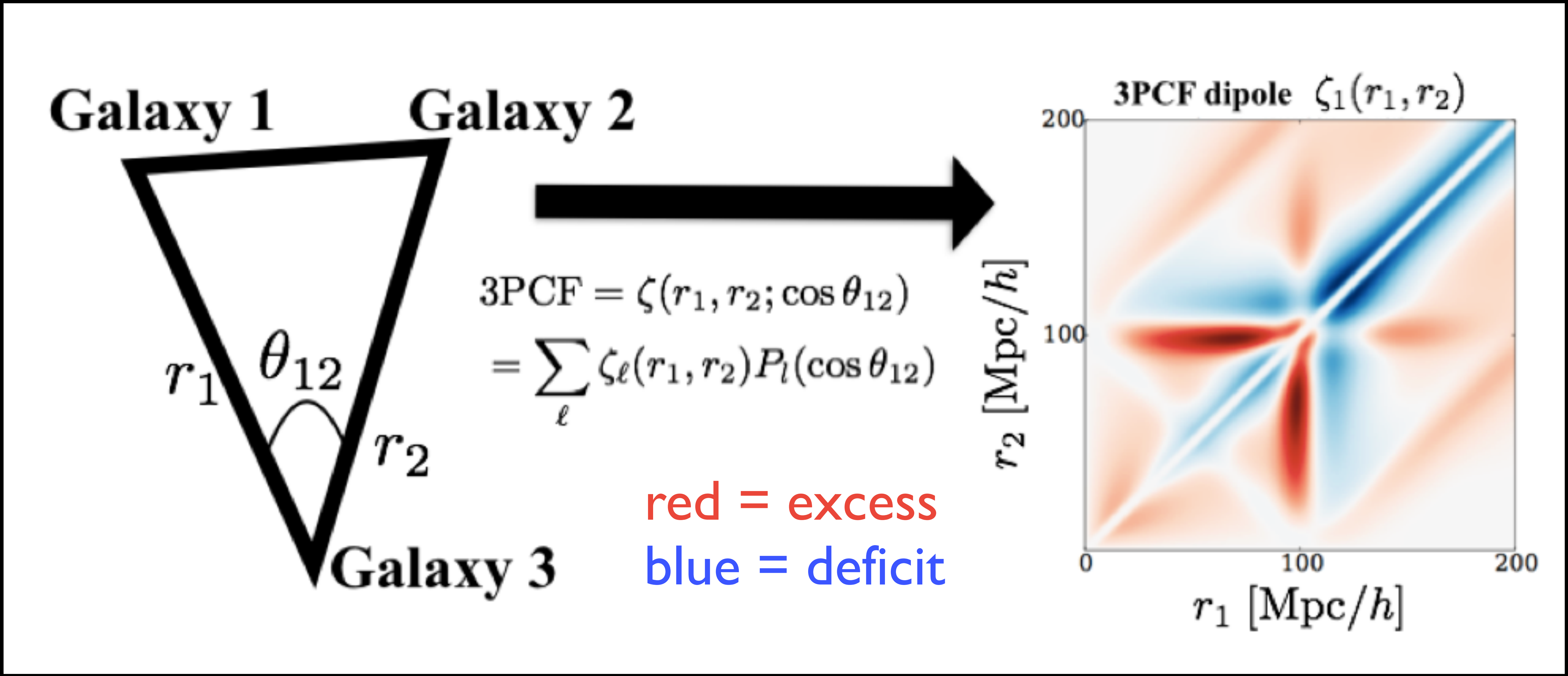
r_1, r_2, θ:
McBride
+2011a,b
(SDSS)



r_1, r_2 / r_1, θ:
Marin+2013
(WiggleZ)
Guo+2015
(SDSS)

Estimator:
Szapudi-Szalay
(1998)
$(D-R)^3/RRR$,
D = Data,
R = Random

MULTIPOLE BASIS



Color shows # of triangles with given side lengths; angle dependence is projected onto Legendre polynomial P_l

Basis proposed by Szapudi 2004, developed in several papers by Slepian & Eisenstein (2015, 16, 17)

The 3PCF is computationally expensive:

if you have 10 friends, choosing two to take to lunch is 10 X 9 = 90, but choosing 3 is
10 X 9 X 8 = 720—you are double-booked!

For large N, N^3 vs. 2PCF's N^2

WHY USE THE 3PCF?

1) **Galaxy formation: biasing**

2) **Gravity: RSD**

3) **Initial Conditions: Primordial Non-Gaussianity**

4) **Dark Energy: BAO**

BIASING

$$\delta_g = b_1 \delta_m + b_2[\delta_m^2 - <\delta_m^2>] + b_t S_{ij} + b_v[v_{bc}^2 - <v_b c>^2]$$

$$\delta_m = \delta_{lin} + \delta^{(2)} + \delta^{(3)} + \cdots$$

Linear bias is like the contrast on a photocopier, or the volume on a speaker

One can also see biases as derivatives with respect to different parameters

Historical: Dekel & Rees 1987
PT review: Bernardeau+2002

Biasing review: Desjacques+2016

WHY CAN'T WE JUST USE THE 2PCF?

2PCF: $\xi_{gg} \propto \sigma_8^2 b_1^2$

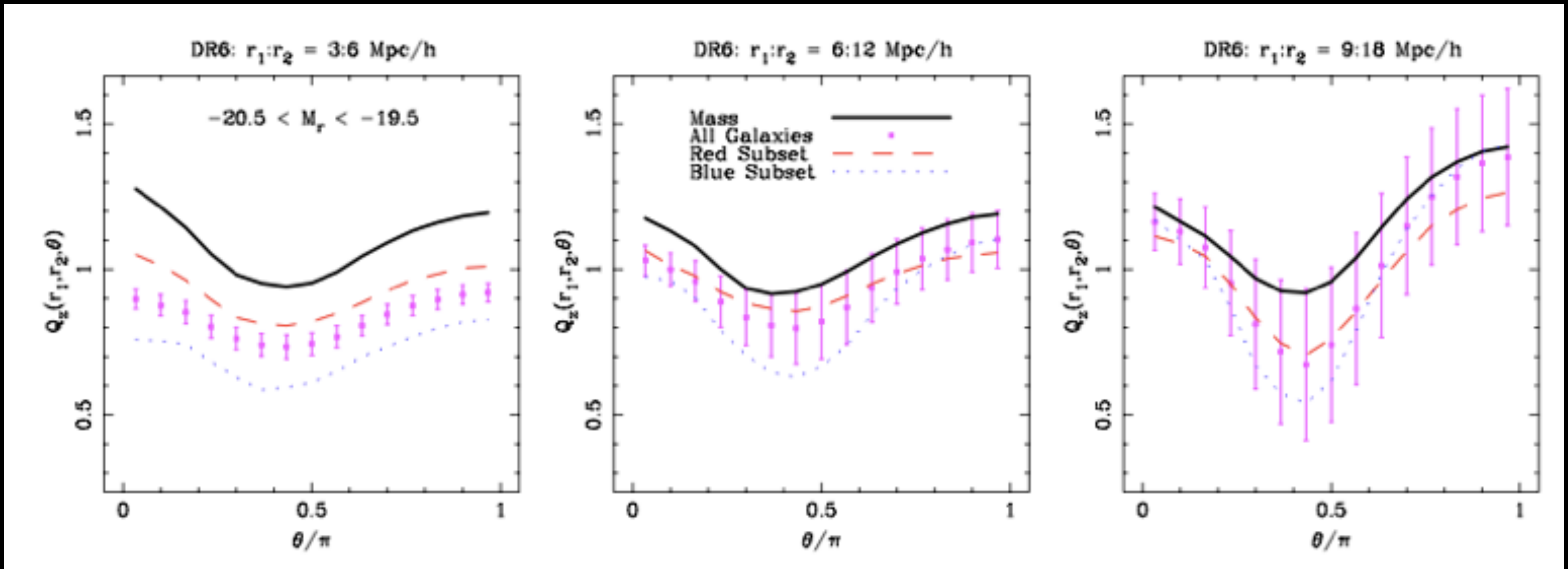
3PCF: $\zeta_{ggg} \propto \sigma_8^4 b_1^3$

Could just fix σ₈ from CMB, but if we want it and *b₁* from LSS, need to break the degeneracy

$$\frac{\xi_{gg}^2}{\zeta_{ggg}} = b_1$$

RSD AFFECT ON BIAS?

RSD also enter so as to be degenerate with the linear bias



Take ratio of observed 3PCF to DM simulations that have been RSD-ed using their velocities (McBride+2011a,b)

But builds sim-dependence into reported results, and not a simple ratio if biases other than b_l

REMARKABLY . . .

2PCF: $\xi_{gg} \propto \sigma_8^4 b_1^4 (1 + \frac{4}{3}\beta) + \mathcal{O}(\beta^2)$

3PCF: $\zeta_{ggg} \propto \sigma_8^4 b_1^3 (1 + \frac{4}{3}\beta) + \mathcal{O}(\beta^2)$

$\beta = f/b_1$ $\qquad f = d\ln D/d\ln a \approx \Omega_{m,eff}^{0.55}(z)$

So ratio argument still works (but again, assuming only linear biasing)

BUT LET'S BE MORE REALISTIC

We'd like to study bias terms beyond linear: quadratic, tidal tensor, relative velocity, etc.

This is where the 3PCF has a key advantage over the 2PCF

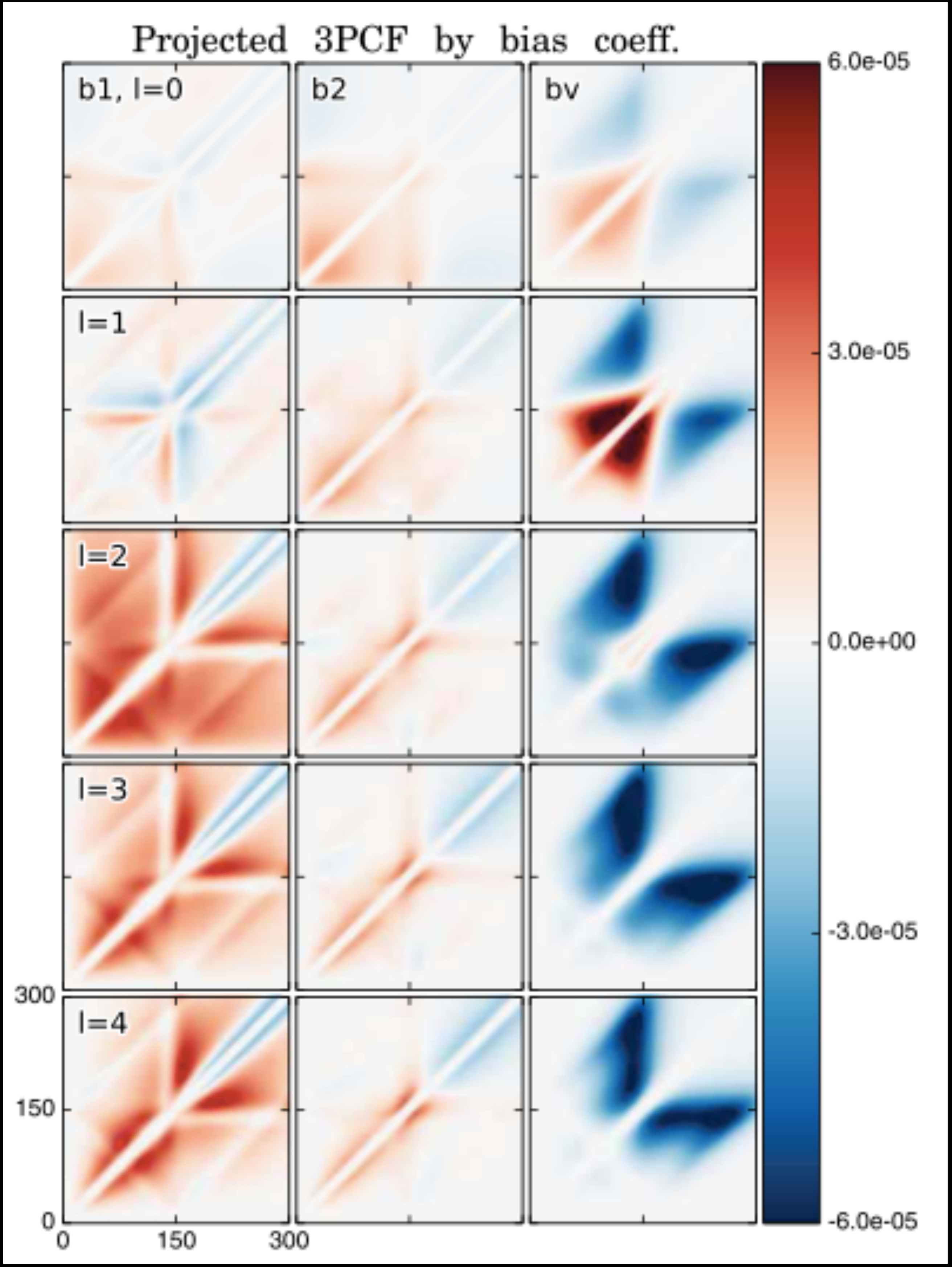
These bias terms enter the 3PCF at leading (4th) order in the linear density field but are sub-leading in the 2PCF

A BIAS MACHINE

So in the 2PCF, you are using *one* curve to look for many parameters each of which is producing a *small* fractional change to the total signal

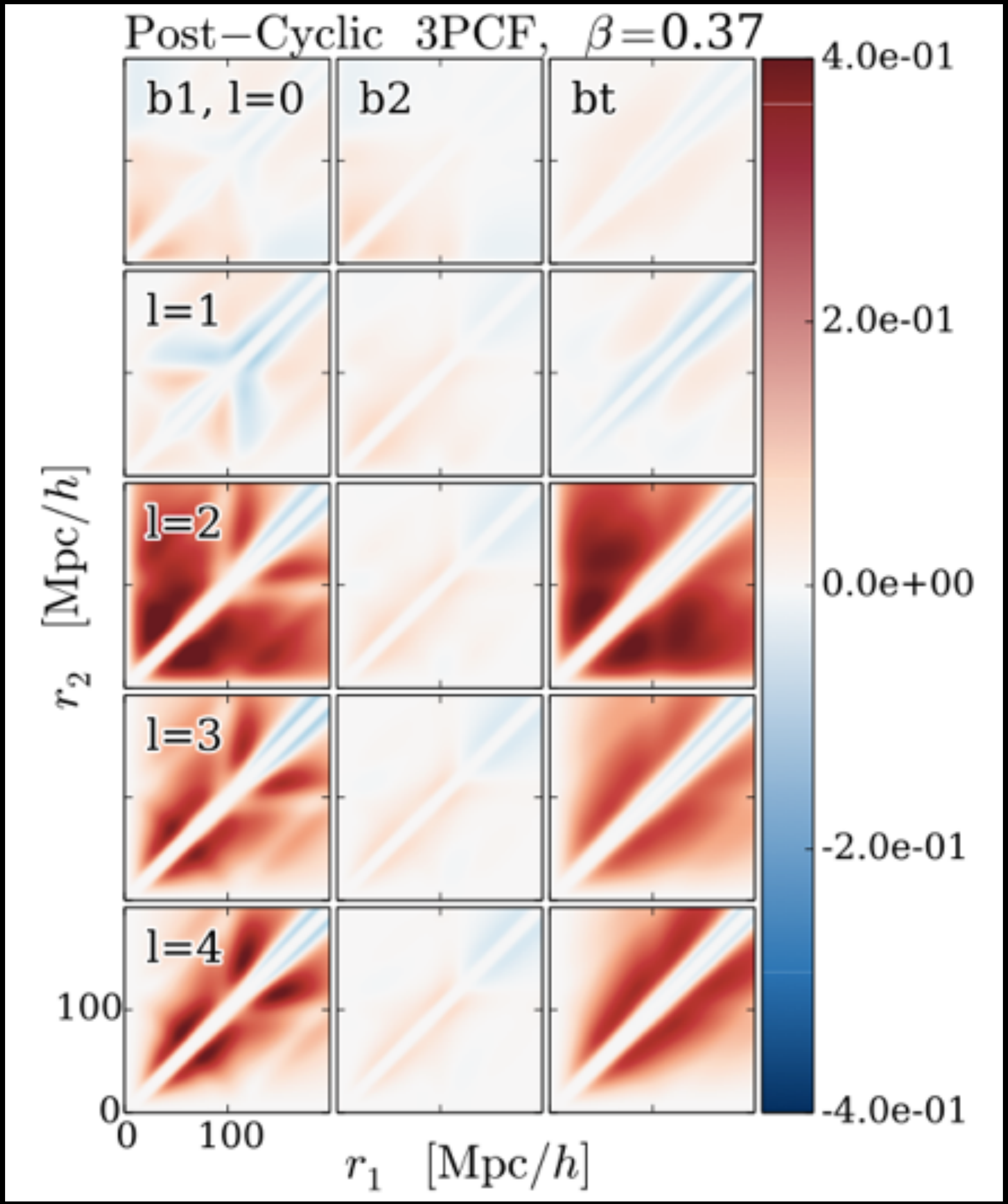
In the 3PCF, you are using many triangle configurations to look for these parameters and each can produce an order unity change to the signal

IN REAL SPACE

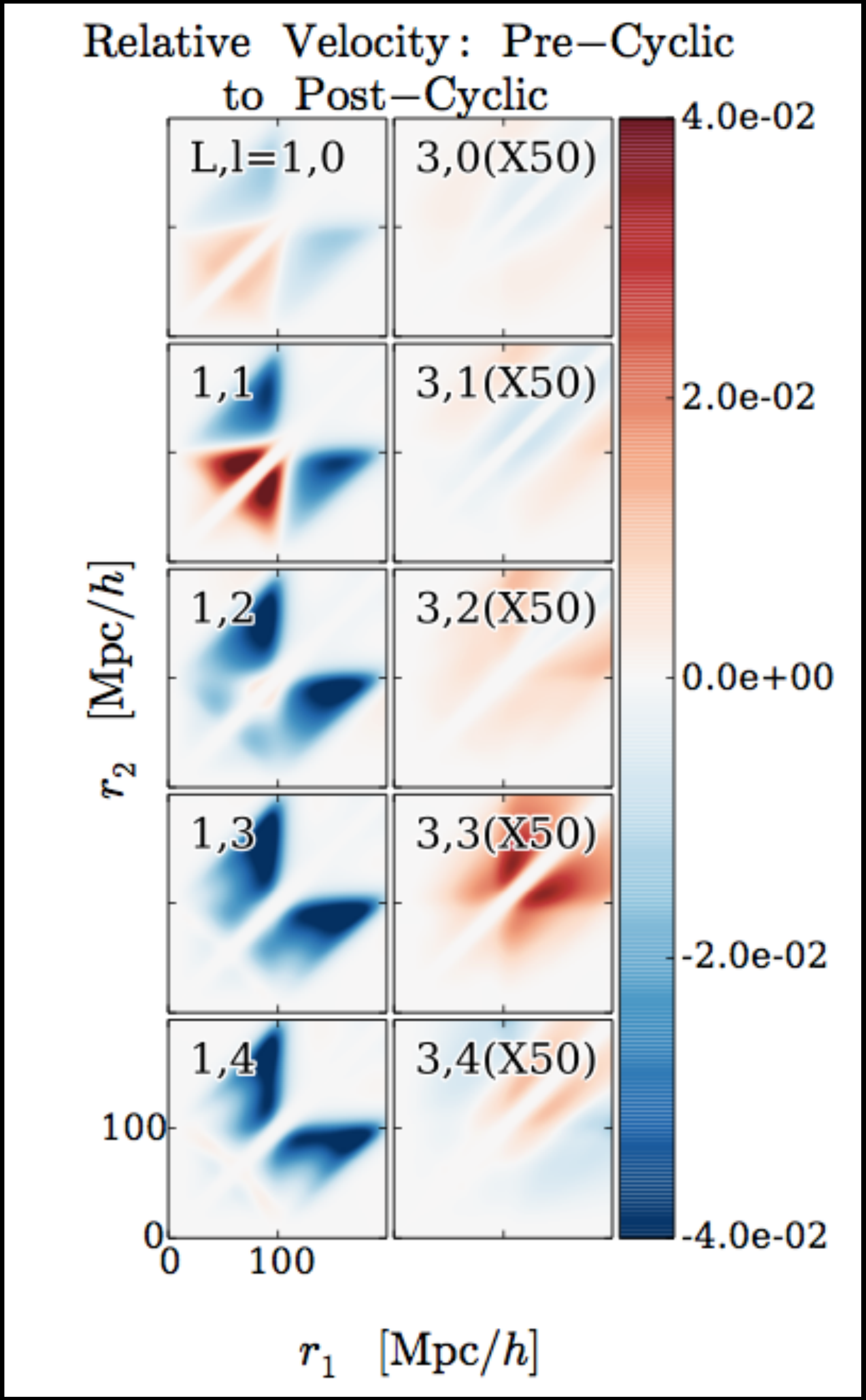


3PCF split out by bias and multipole moments, as function of 2 triangle sides

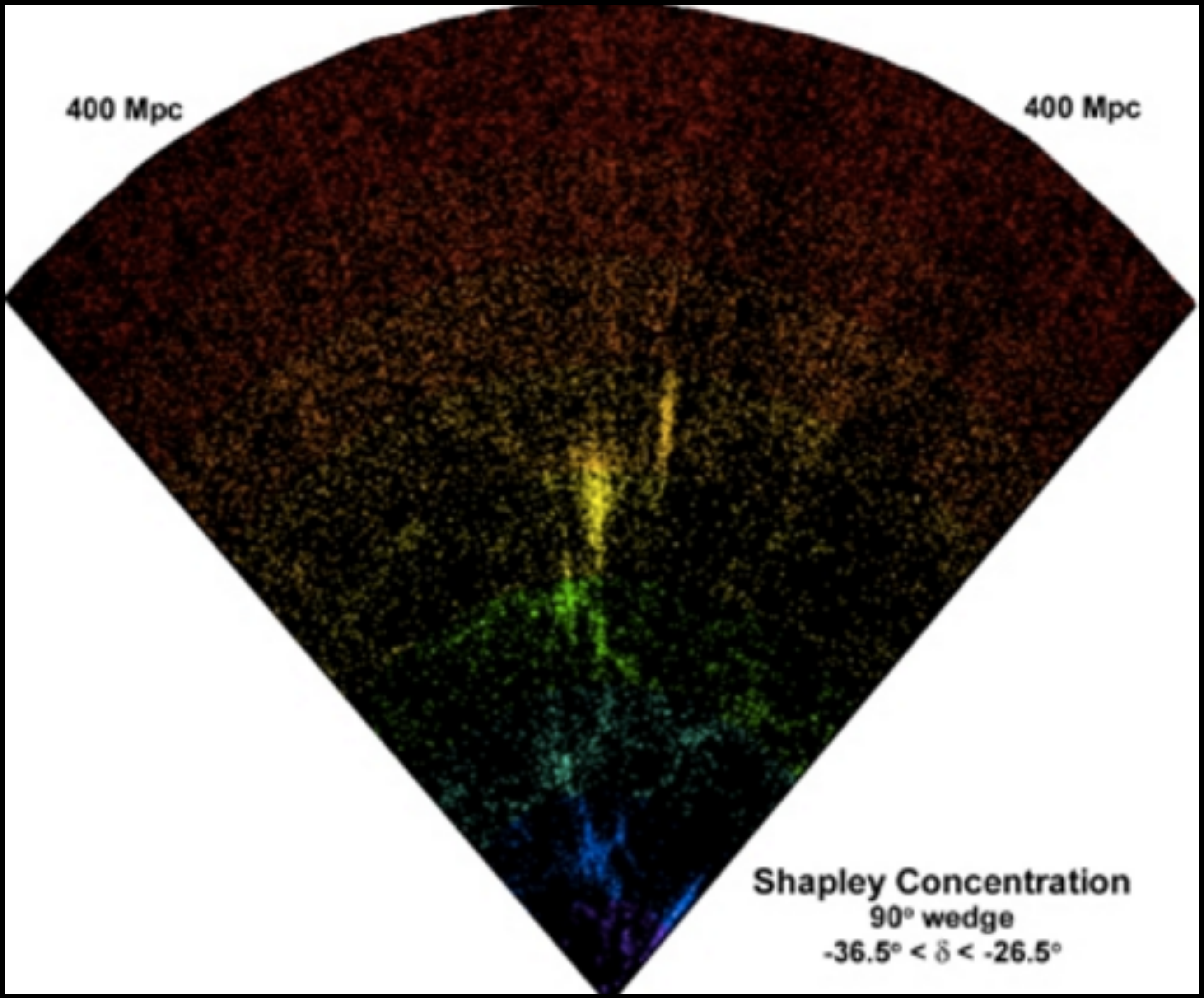
IN REDSHIFT SPACE



Recent work: high-precision constraints on relative velocity biasing, linear bias, hints at tidal tensor bias w/BOSS (Slepian+16a,b,17)



RSD



Fingers of God along l.o.s. due to infall (Jackson 1972) and on larger scales, transverse Kaiser pancakes (Kaiser 1987); review: Hamilton (1998)

PROBING GRAVITY

Large-scale RSD are generated by coherent growth of structure and can be used to probe gravity

$$\beta = f/b_1 \qquad\qquad f = d\ln D/d\ln a \approx \Omega^{0.55}_{m,eff}(z)$$

Exponent of 0.55 is assuming GR: it has a different value for other theories (Linder 2005; Raccanelli +2013)

WHAT ARE OUR OPTIONS?

Use multipoles of 2PCF/power spectrum, or wedges in angle to line of sight

Try the grueling push to higher *k* ?

Hand+2017 shows this is very hard: 19 free parameters roughly get you to k = *0.4 h/Mpc* but this only buys 15-30% improvement on $f\sigma_8$ relative to k = *0.2 h/Mpc*

OR . . .

Open up the use of the 3PCF. There are lots of triangles, and lots of ways for each side to project onto the line of sight

But . . . as we've seen, the spherically-averaged (isotropic) 3PCF has β degenerate with linear bias!

Why not use anisotropic 3PCF?

PNG

Generically, inflation predicts some deviation from GRF initial conditions

$$\Phi(\vec{x}) = \Phi_G(\vec{x}) + f_{NL}^{loc}[\Phi_G^2(\vec{x}) - \langle \Phi_G^2(\vec{x}) \rangle]$$

Φ **is the Bardeen's curvature perturbation, G for Gaussian**

This is a bit analogous to non-linear biasing, or a Taylor series!

There can also be equilateral contributions (Fourier space), motivated by higher derivative and DBI inflationary models

Sefusatti & Komatsu 2007
also: Baumann notes

PNG

Creminelli & Zaldarriaga: squeezed limit PNG in single-field inflation is suppressed by (1 - n_s) ~ 0 for scale invariant perturbations

PNG will be a major goal of post-DESI LSS

Why? The CMB has led so far, but is more or less saturated at f_{nl} ~ 5 modulo a slight improvement from polarization

But f_{nl} is very hard with the 2PCF, in particular getting to order 1, where theory indicates is interesting

DESI-2 PROJECTION

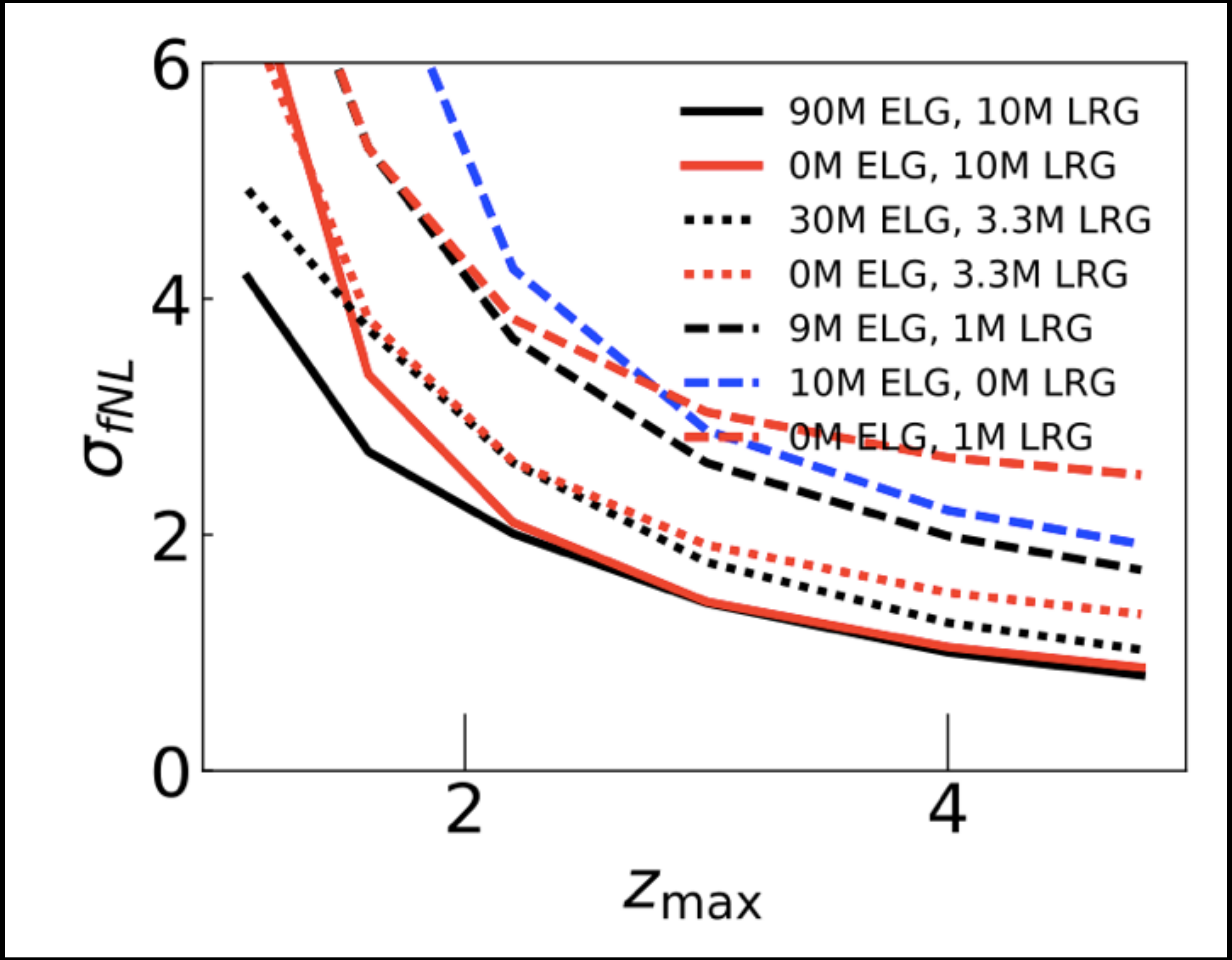


FIG. 16. Local non-Gaussianity constraints, for 14000 sq. deg. with uniform comoving density out to $z_{\max}$, for different numbers of "ELGs" (objects with bias $0.84D(0)/D(z)$) and "LRGs" (objects with bias $1.7D(0)/D(z)$). Bias is always capped to be no greater than the bias expected if the objects lived in the most massive halos with this number density.

Forecast: Pat McDonald for DESI-2 WG

3PCF AND PNG

The 3PCF is the leading-order statistic sensitive to PNG

A complication is that Dalal (2008) showed scale-dependent bias in the power spectrum can also reveal PNG

But signature is on large scales, and again . . . one curve, many parameters, many possible systematics on large scales (survey boundary, dust), also cosmic variance

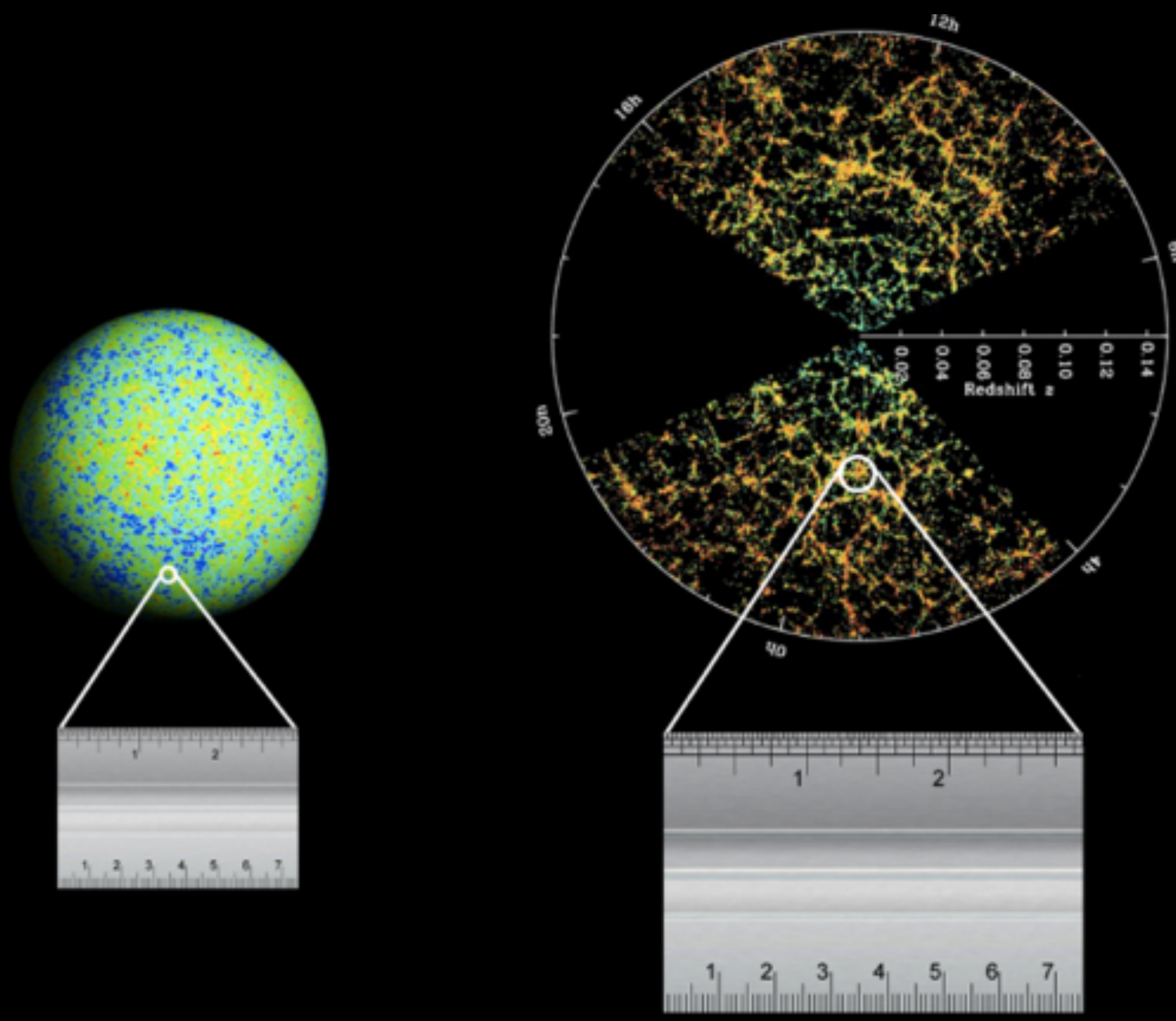
So 3PCF will still be vital for a robust detection

BARYON ACOUSTIC OSCILLATIONS: A STANDARD RULER

z ~ 1100

~14 billion years ago

(380,000 years after Big Bang)

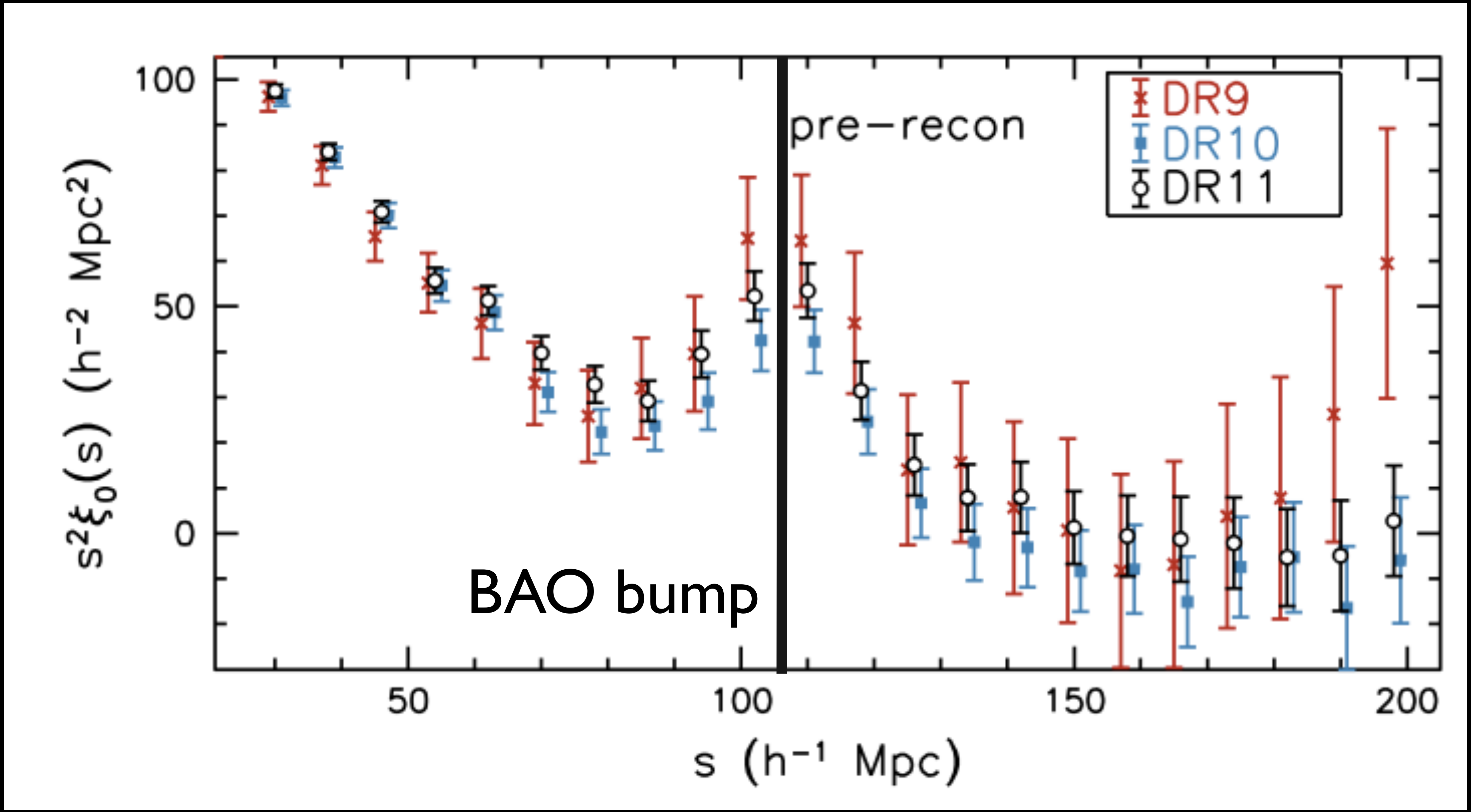


z ~ 1-0

(roughly 7 billion years ago to today)

image: http://newscenter.lbl.gov/wp-content/uploads/ruler-cmb-today.jpg

BAO METHOD WITH THE 2PCF

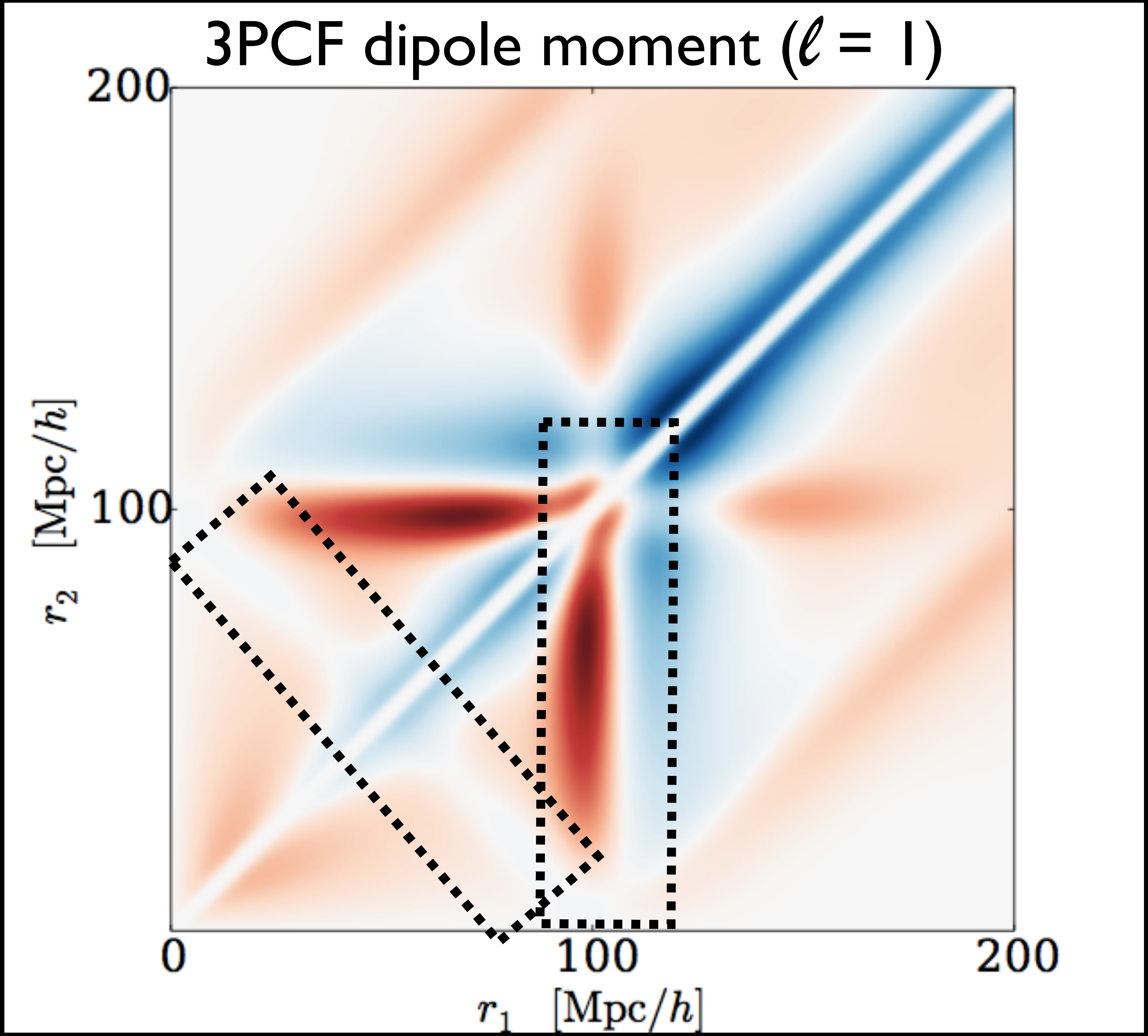


Cole et al. 2005

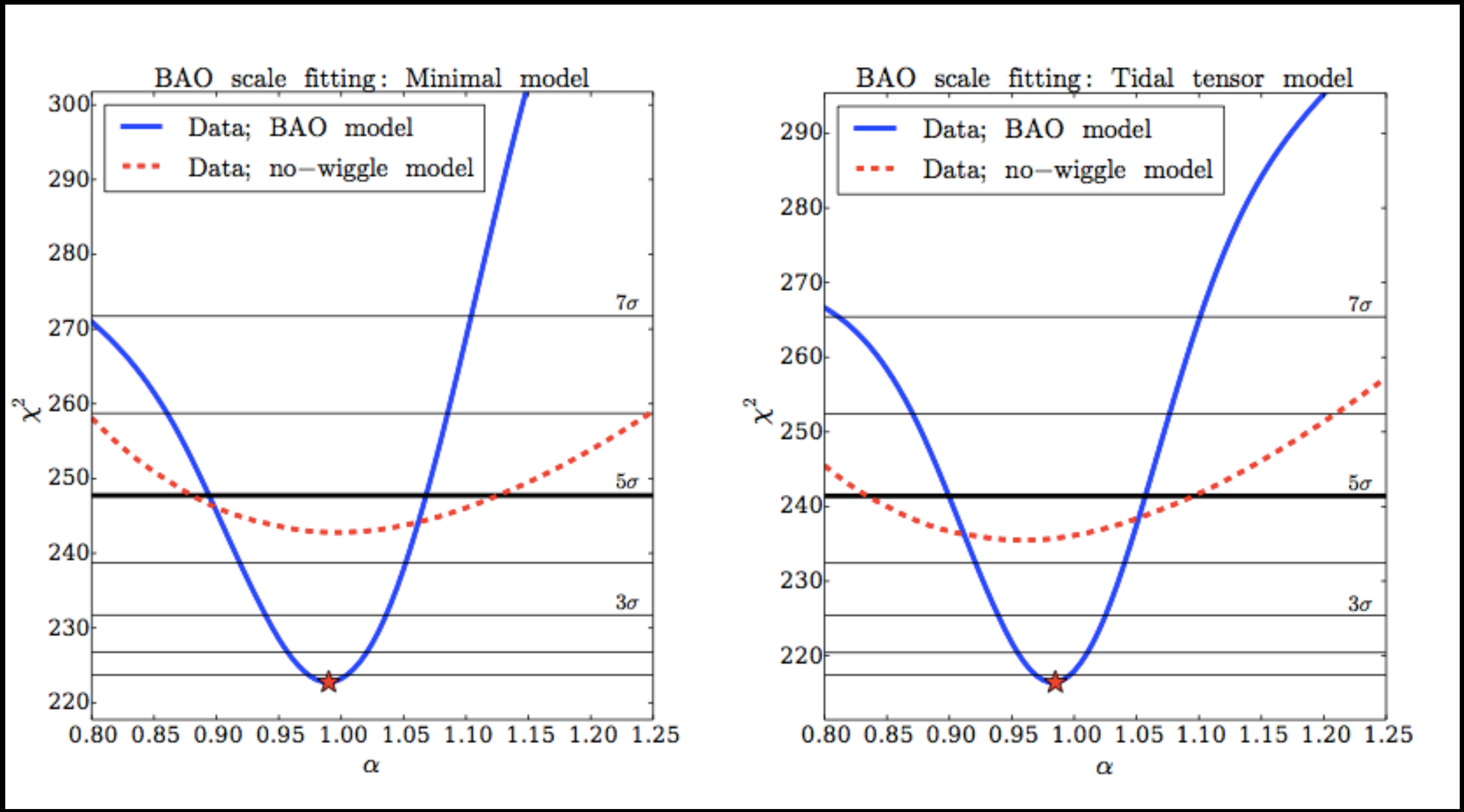
Eisenstein et al. 2005

Figure: Anderson et al. 2014

BAO IN THE 3PCF



THE FIRST HIGH-SIGNIFICANCE BAO DETECTION IN THE 3PCF



Measure distance to $z = 0.57$ (6 billion years in the past) with 1.7% precision: first use of BAO method in 3PCF

Data: Slepian+1607.06097, 1512.02231,1607.06098

Model: SE1607.03109

BUT A COMPLICATION . . .

Comparing unreconstructed 2PCF with 3PCF, 3PCF is like making BOSS 10% longer—seems like a win, for zero extra dollars and not much computer time!

But Schmittfull+2015 shows density field reconstruction brings this info nearly fully into 2PCF

Slepian+2017 confirmed this empirically using mocks pre and post-recon

But mocks don’t have systematics; data does
Valuable to have an independent method of getting this information

AND IN SOME CASES . . .

Sampling is so sparse that you don't have enough info on the density to do reconstruction: e.g. quasars

In these cases, 3PCF is the only way to access this additional BAO information

-eBOSS quasar BAO search underway right now!
(Peter Choi, Graziano Rossi, ZS, David Schlegel, Shirley Ho, Daniel Eisenstein)
-z~1.5—not many points on the Hubble diagram there, and will be a pathfinder for DESI

WE'VE DISCUSSED WHY

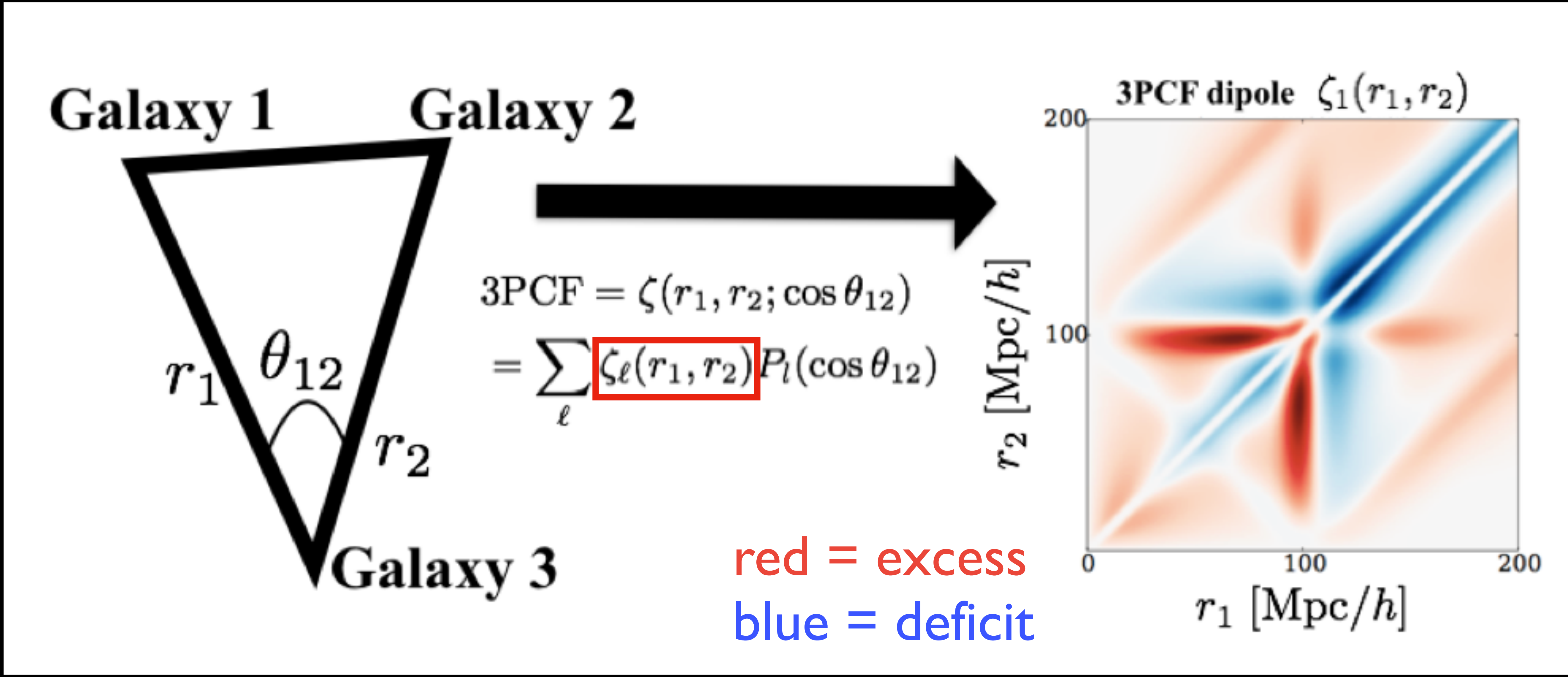
Now: how? Three challenges

1) Measurement (N^3)
2) Modeling
3) Matching measurement to model (covariance)

Previous algorithms: *kd*-tree (Moore+2001, Gray+2001, used in McBride, Marin, others; Zhang & Pen 2005 also uses a tree. Not ideal for galaxy surveys which are sparse.

ISOTROPIC 3PCF ALGORITHM

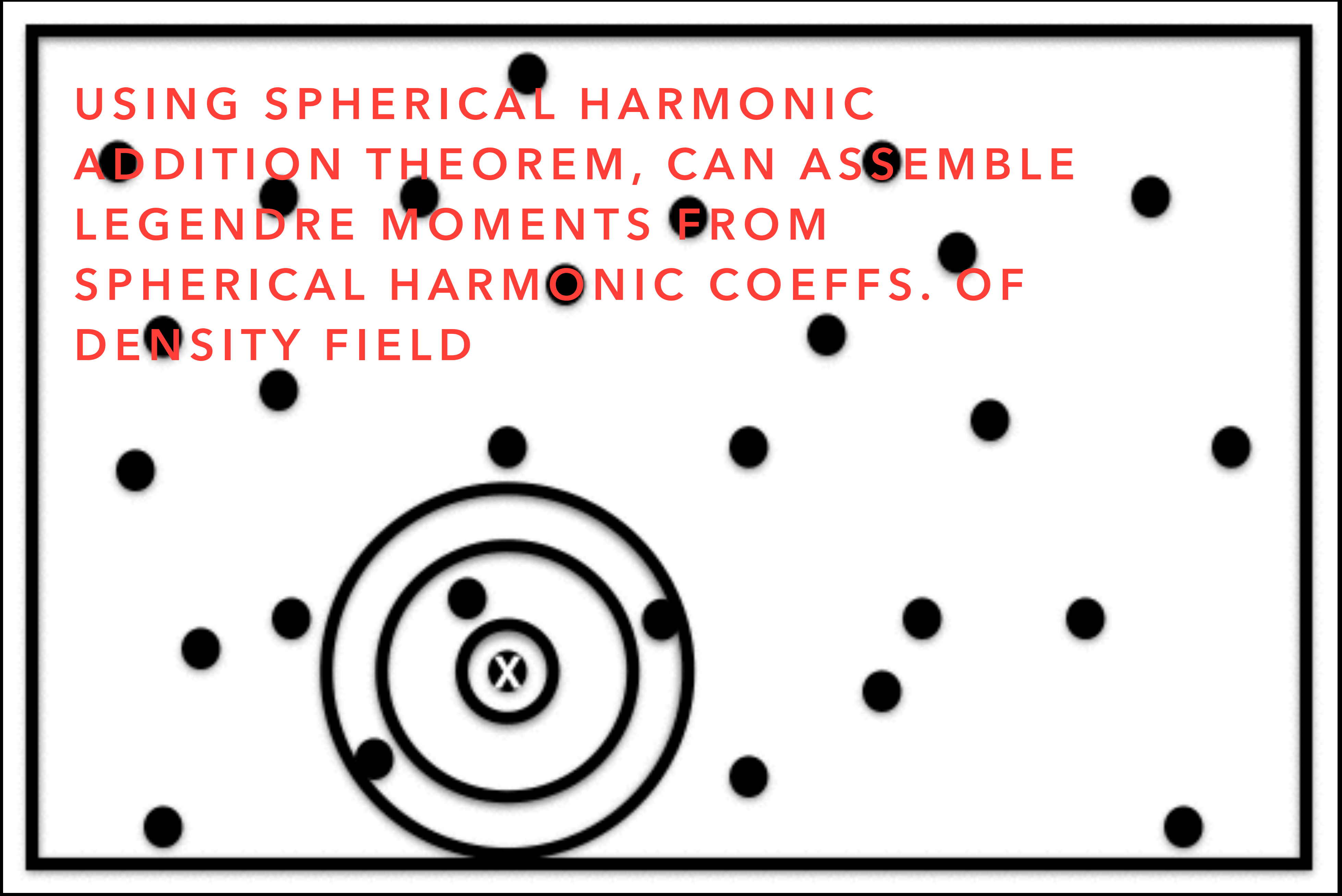
3-point correlation function (3PCF): excess triangles over random



Color represents # of triangles with given side lengths; in this panel, angle dependence is projected onto P_l

3PCF basis: Szapudi 2004, also Verde+2000
Algorithm: SE1506.02040, SE1506.04746

Around each galaxy, compute a_{lm} in spherical shells/radial bins



USING SPHERICAL HARMONIC ADDITION THEOREM, CAN ASSEMBLE LEGENDRE MOMENTS FROM SPHERICAL HARMONIC COEFFS. OF DENSITY FIELD

$$a_{lm}(r;\vec{s}) = \sum_{\text{gals } j \text{ in bin}} Y^*_{lm}(\hat{r}_j)$$

NOW ORDER N *ABOUT EACH* GALAXY, SO N^2 OVERALL

CAN BE EVEN FASTER WITH FOURIER TRANSFORMS

SE1506.02040, SE1506.04746

ADVANTAGES OF THE ALGORITHM

500X FASTER THAN A TRIPLET COUNT FOR 700K GALAXY TEST CASE

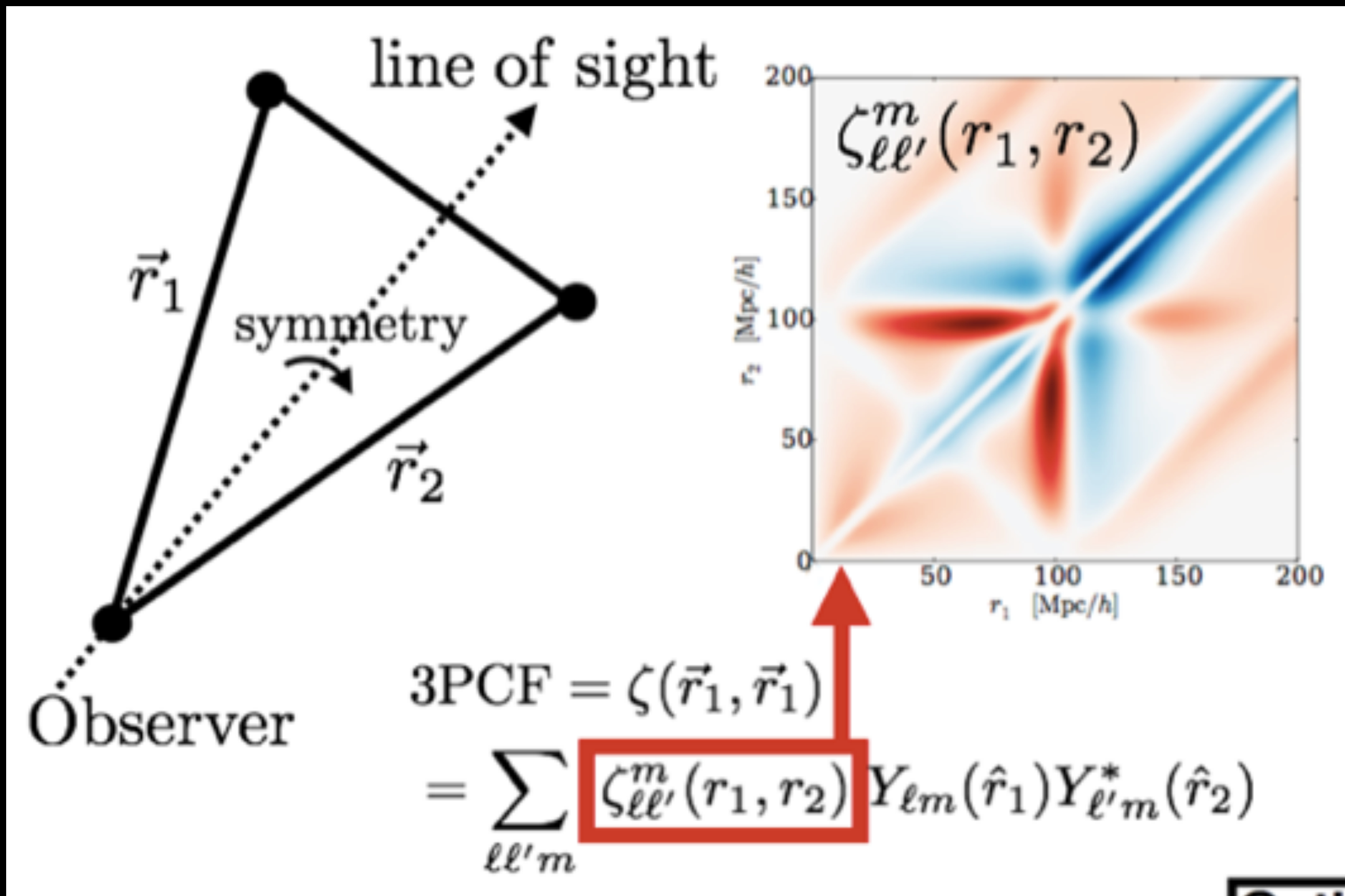
ONLY 6X SLOWER THAN COMPUTING A 2-POINT FUNCTION

HIGHLY ENABLING FOR USE OF 3PCF IN COSMOLOGY ALREADY

ANISOTROPIC 2PCF WITH ROTATING LOS FOR FREE BECAUSE GEOMETRY IS THE SAME

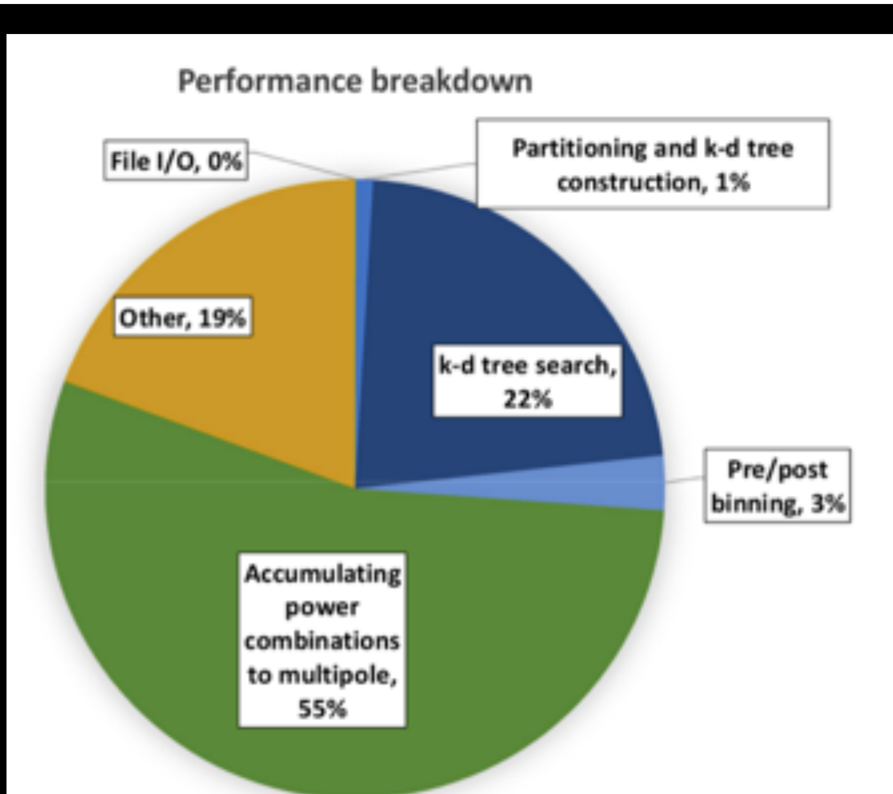
Details in SE15b, SE16a

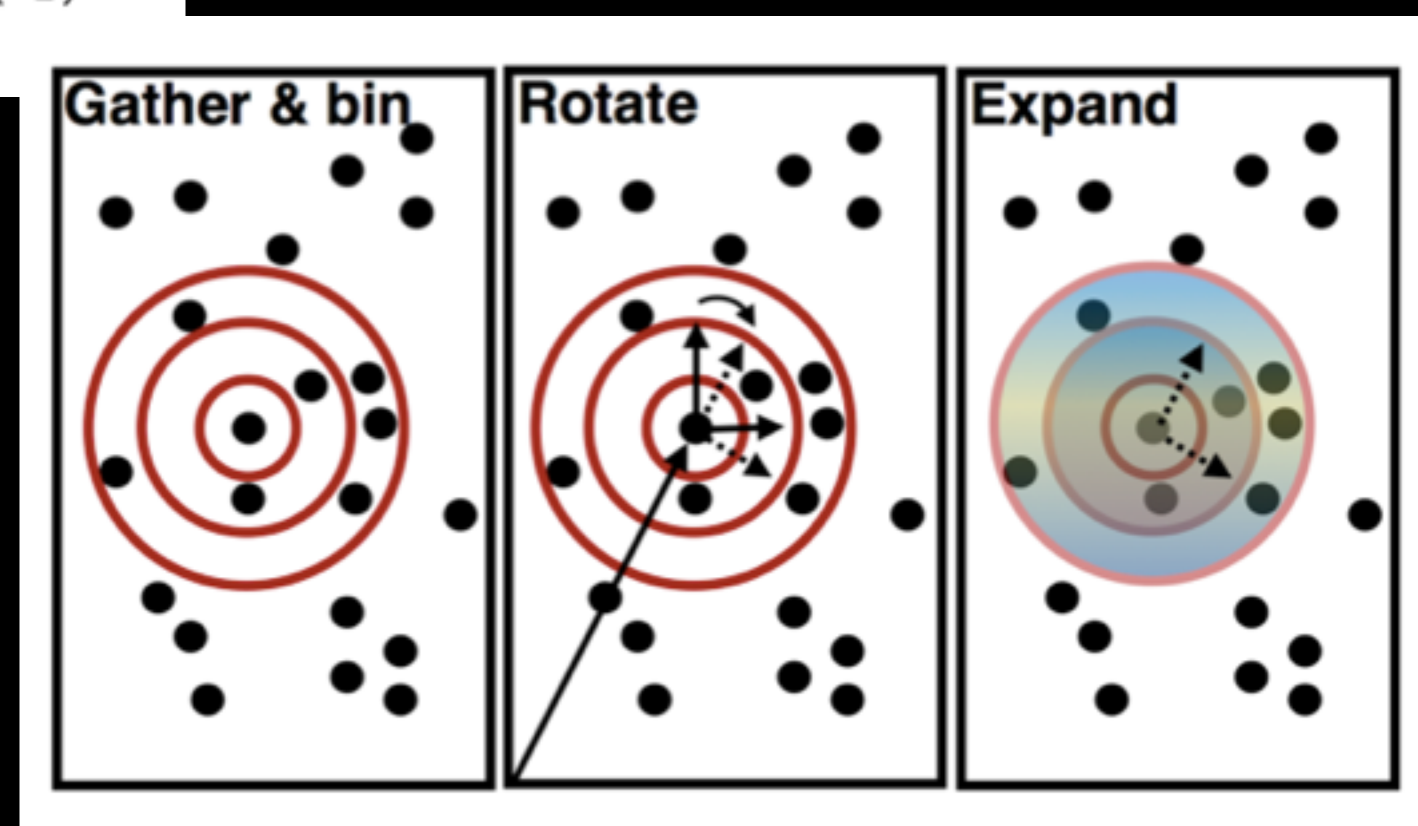
PROBING REDSHIFT-SPACE DISTORTIONS: ANISOTROPIC 3PCF



-Ran at scale on CORI
-9,600 nodes, 5.6 sustained PF
-80% peak for instruction mix
-Obtain harmonic coefficients with matrix algebra libraries

3PCF for 2 billion haloes in 20 minutes





Algorithm: SE17.; Implementation: Friesen+(incl. ZS) 17, accepted ACM: SC'17 conference; First use on data (BOSS): Nugent+17 in prep.

HANDLING RSD

This improved algorithm will enable us to extract information on RSD from the 3PCF

Also allows computing 3PCF for entire observable Universe (100 bn galaxies) in time it took me to fly here (~17 hours)

MODELING THE ISOTROPIC 3PCF

$$\ell = 0: \; b_1^3\Bigg\{\frac{34}{21}\left[1+\frac{4}{3}\beta+\frac{1154}{1275}\beta^2+\frac{936}{2975}\beta^3+\frac{21}{425}\beta^4\right] + \gamma\left[1+\frac{2}{3}\beta+\frac{1}{9}\beta^2\right]\Bigg\}\xi_1^{[0]}\xi_2^{[0]} + b_1^3(7\beta^2+3\beta^3)\kappa_0(r_1,r_2)$$

$$\ell = 1: \; -b_1^3\left[1+\frac{4}{3}\beta+\frac{82}{75}\beta^2+\frac{12}{25}\beta^3+\frac{3}{35}\beta^4\right] \times\left[\xi_1^{[1+]}\xi_2^{[1-]}+\xi_2^{[1+]}\xi_1^{[1-]}\right] + b_1^3(7\beta^2+3\beta^3)\kappa_1(r_1,r_2)$$

$$\ell = 2: \; b_1^3\Bigg\{\frac{8}{21}\left[1+\frac{4}{3}\beta+\frac{52}{21}\beta^2+\frac{81}{49}\beta^3+\frac{12}{35}\beta^4\right] + \frac{32\gamma}{945}\beta^2\Bigg\}\xi_1^{[2]}\xi_2^{[2]} + b_1^3(7\beta^2+3\beta^3)\kappa_2(r_1,r_2)$$

$$\ell = 3: \; -b_1^3\left[\frac{8}{75}\beta^2+\frac{16}{175}\beta^3+\frac{8}{315}\beta^4\right]\left[\xi_1^{[3+]}\xi_2^{[3-]}+\xi_2^{[3+]}\xi_1^{[3-]}\right] + b_1^3(7\beta^2+3\beta^3)\kappa_3(r_1,r_2)$$

$$\ell = 4: \; b_1^3\left[-\frac{32}{3675}\beta^2+\frac{32}{8575}\beta^3+\frac{128}{11025}\beta^4\right]\xi_1^{[4]}\xi_2^{[4]} + b_1^3(7\beta^2+3\beta^3)\kappa_4(r_1,r_2)$$

$$\ell \geqslant 5: \; b_1^3(7\beta^2+3\beta^3)\kappa_\ell(r_1,r_2)$$

Multipoles of the 3PCF; γ is b_2/b_1, tidal tensor can also be added easily

Key: write model in terms of 1-D and 2-D integrals of power spectrum

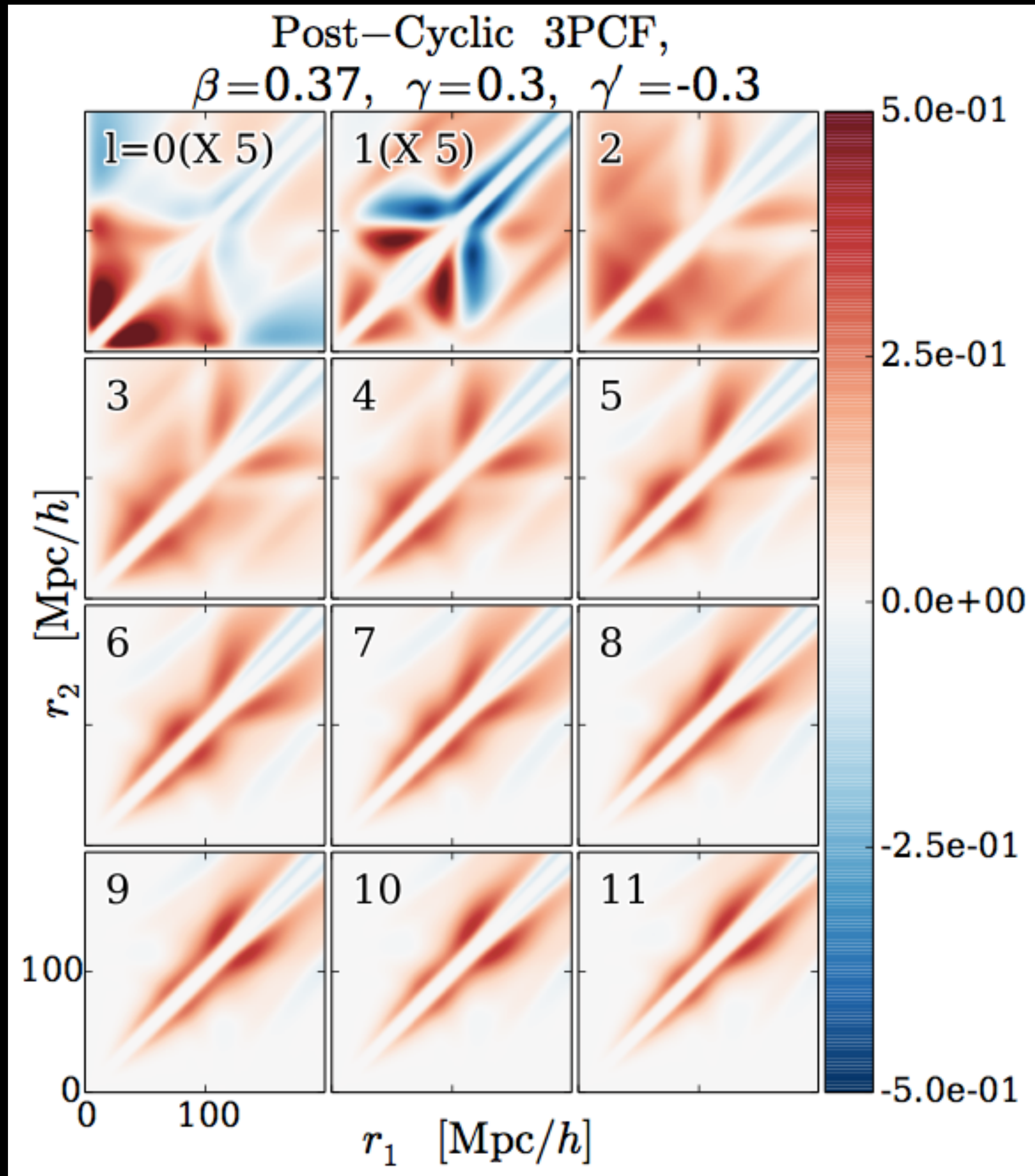
$$\xi_i^{[n]} = \int \frac{k^2 dk}{2\pi^2} P(k) j_n(kr_i)$$
$$\xi_i^{[n\pm]} = \int \frac{k^2 dk}{2\pi^2} k^{\pm 1} P(k) j_n(kr_i);$$

$$\kappa_\ell(r_1,r_2) = \frac{64}{77175}\Big[9I_{1\ell}(r_1,r_2) - 14I_{3\ell}(r_1,r_2) + 5I_{5\ell}(r_1,r_2)\Big],$$

$$I_{\mathcal{L}\ell}(r_1,r_2) = \sum_{l_1}(-1)^{l_1+\ell}(2l_1+1)(2\ell+1)\begin{pmatrix} l_1 & \ell & \mathcal{L} \\ 0 & 0 & 0\end{pmatrix}^2 \times \int r dr\, f_{\ell l_1}(r_1;r) f_{\ell l_1}(r_2;r),$$

Makes predictions quite fast to compute: eventually, embed in MCMC?

Scoccimarro Couchman Frieman 1999: base model
see also Rampf & Wong 2012
Slepian & Eisenstein 2016, 1607.03109



Slepian & Eisenstein 2016, 1607.03109

MATCHING MEASUREMENT TO MODEL

Requires the covariance. Very high-*d* matrix, and inverse is dominated by smallest eigenvalue, which is noisiest

Thus if estimating from mocks, need >>1 mock/dimension to render it invertible: demands an impossible number of mocks

Instead: analytically compute dominant, GRF covariance and use mocks to fit for 2 free parameters (shot noise and volume)

Analytic template is smoothly invertible

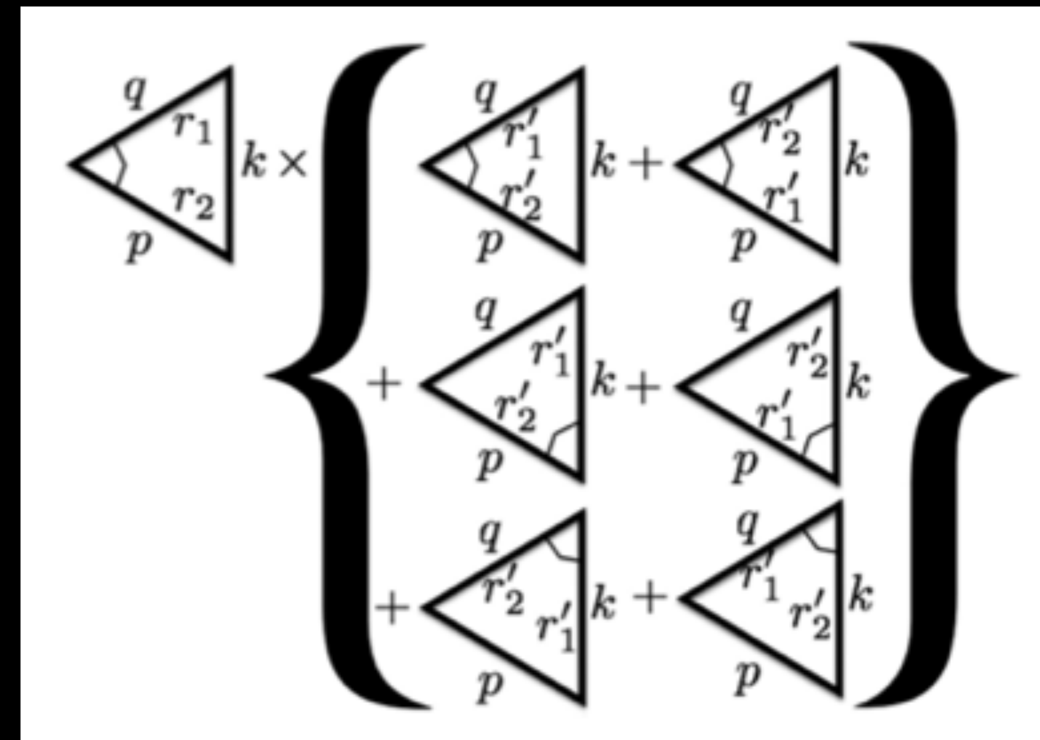
Isotropic covariance

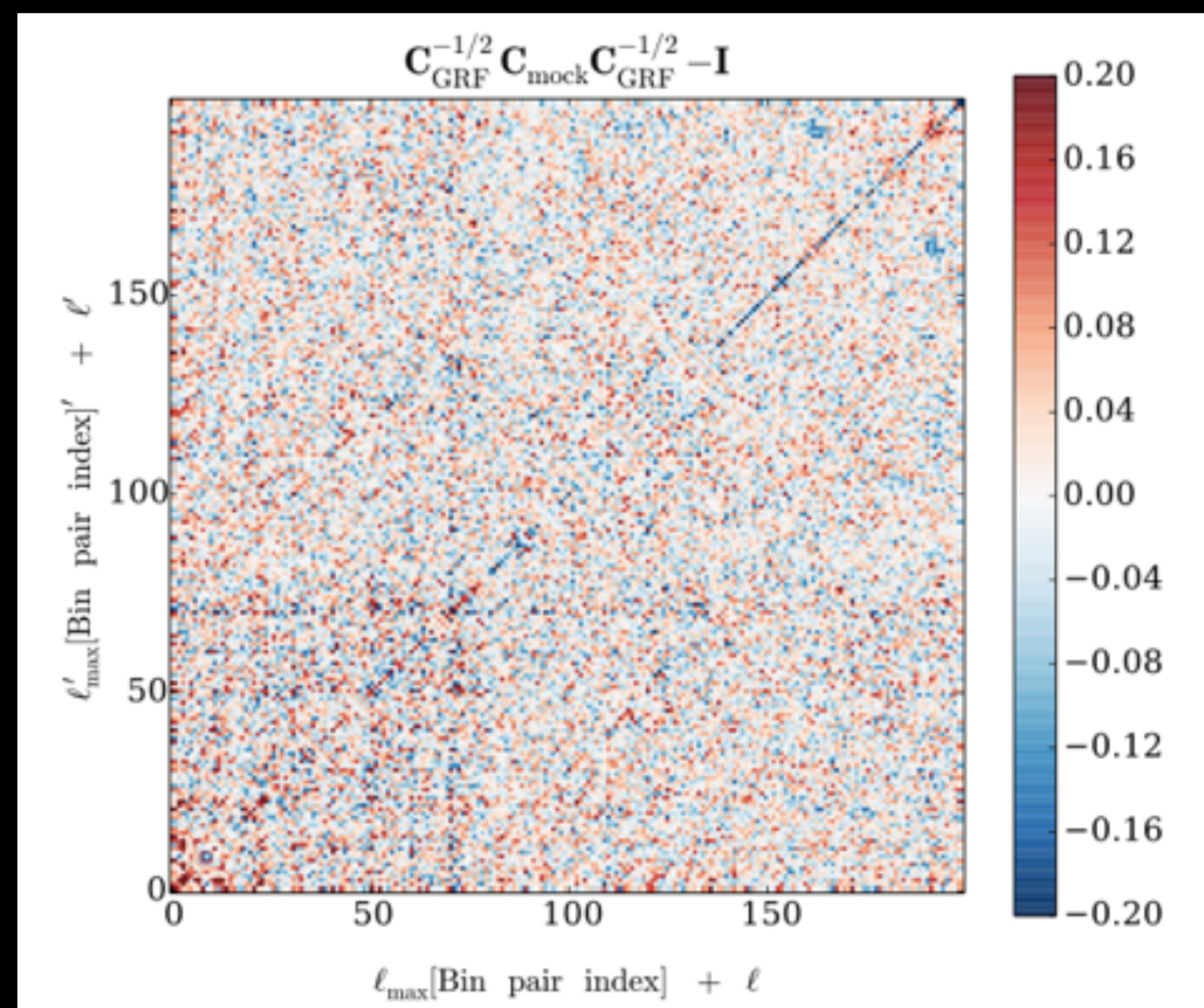
$$\mathrm{Cov}_{ll'}(r_1, r_2; r_1', r_2') = \frac{4\pi}{V}(2l+1)(2l'+1)(-1)^{l+l'}$$
$$\times \int r^2 dr \sum_{l_2}(2l_2+1)\begin{pmatrix} l & l' & l_2 \\ 0 & 0 & 0 \end{pmatrix}^2$$
$$\times \Big\{(-1)^{l_2}\xi_0(r)\Big[f_{l_2 ll'}(r;r_1,r_1')f_{l_2 ll'}(r;r_2,r_2')$$
$$+ f_{l_2 ll'}(r;r_2,r_1')f_{l_2 ll'}(r;r_1,r_2')\Big] + (-1)^{(l+l'+l_2)/2}$$
$$\times \Big[f_{ll}(r;r_1)f_{l'l'}(r;r_1')f_{l_2 ll'}(r;r_2,r_2')$$
$$+ f_{ll}(r;r_1)f_{l'l'}(r;r_2')f_{l_2 ll'}(r;r_2,r_1')$$
$$+ f_{ll}(r;r_2)f_{l'l'}(r;r_1')f_{l_2 ll'}(r;r_1,r_2')$$
$$+ f_{ll}(r;r_2)f_{l'l'}(r;r_2')f_{l_2 ll'}(r;r_1,r_1')\Big]\Big\}.$$

$$f_{ll}(r;r_1) = \int \frac{k^2 \mathrm{d}k}{2\pi^2} P(k) j_l(kr_1) j_l(kr)$$

$$f_{l_2 ll'}(r;r_1,r_1') = \int \frac{k^2 \mathrm{d}k}{2\pi^2} P(k) j_l(kr_1) j_{l'}(kr_1') j_{l_2}(kr).$$

In terms of 1 and 2-D integrals of the power spectrum





WHAT ABOUT CROSS COVARIANCE?

How do we combine fitted parameter values (e.g. linear bias) from the 3PCF with those from the 2PCF?

Use analytic auto-covariance to fit them separately, then use mocks to form the low-d ($2N_{params}$) cross-covariance matrix and combine parameters weighted by its inverse

Used this approach with BAO results from BOSS 3PCF

SUMMARY

The 3PCF is a uniquely powerful probe of galaxy biasing and primordial non-Gaussianity

Beyond-linear biasing enters at leading order. 3PCF is also the leading order statistic sensitive to PNG

It has discovery potential in both of these areas

3PCF also has potential to add to standard 2PCF analyses on RSD and BAO