Redshift Space Distortion Introduction

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Outline

- What's RSD? — Anisotropic properties on observed galaxy clustering statistics in redshift space induced by peculiar velocity

- Why RSD? — Degeneracy breaking between DE and MG

- Why modeling RSD? — More cosmological information on non-linear scales, tighter cosmological constrains. Statistical or systematic uncertainty dominated?

- How to model RSD? — (1) real space non-linearity (2) non-linear mapping formula from real to redshift space (Taylor expansion) (3) galaxy density and velocity bias

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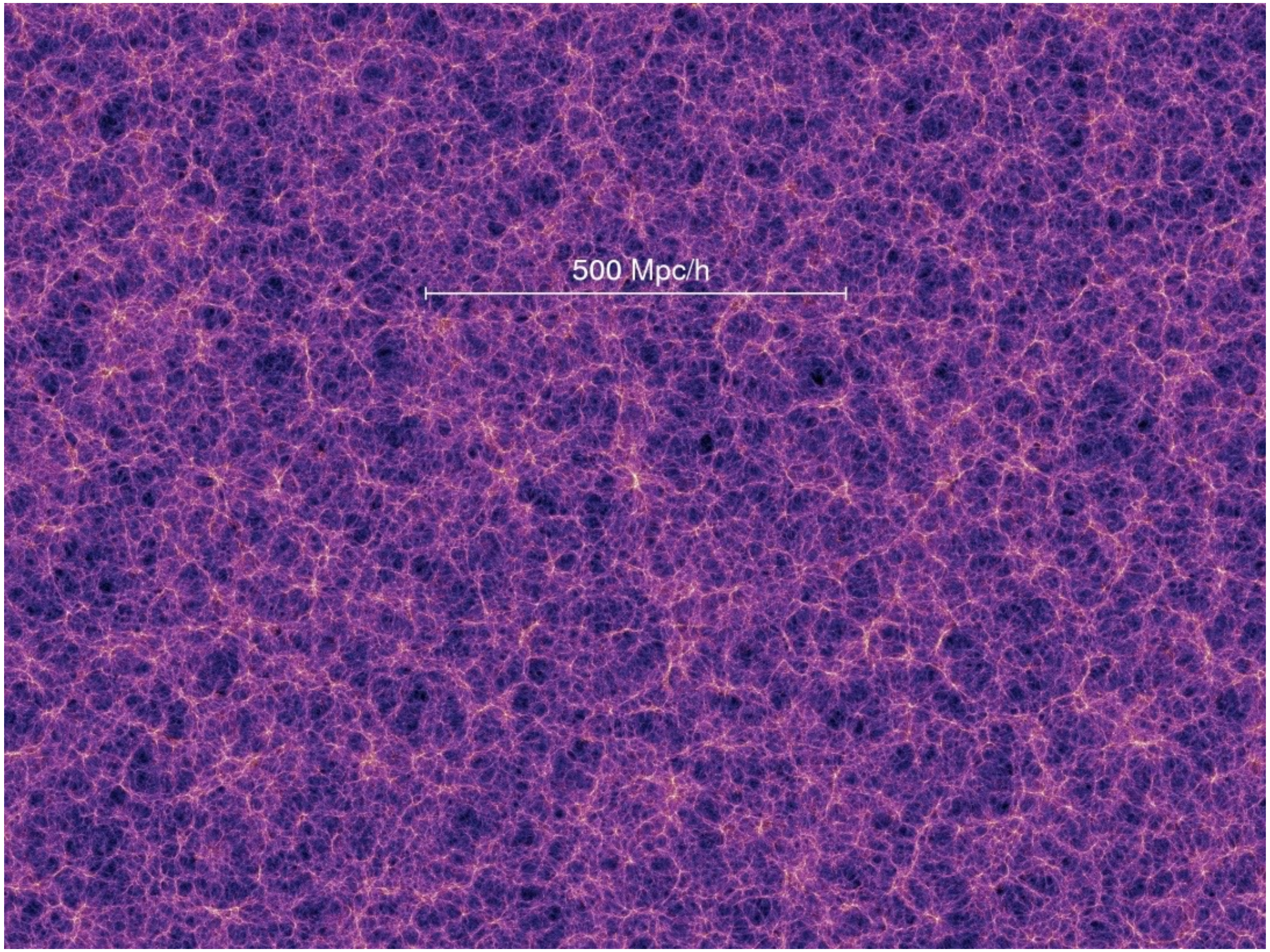
What's "distortion"

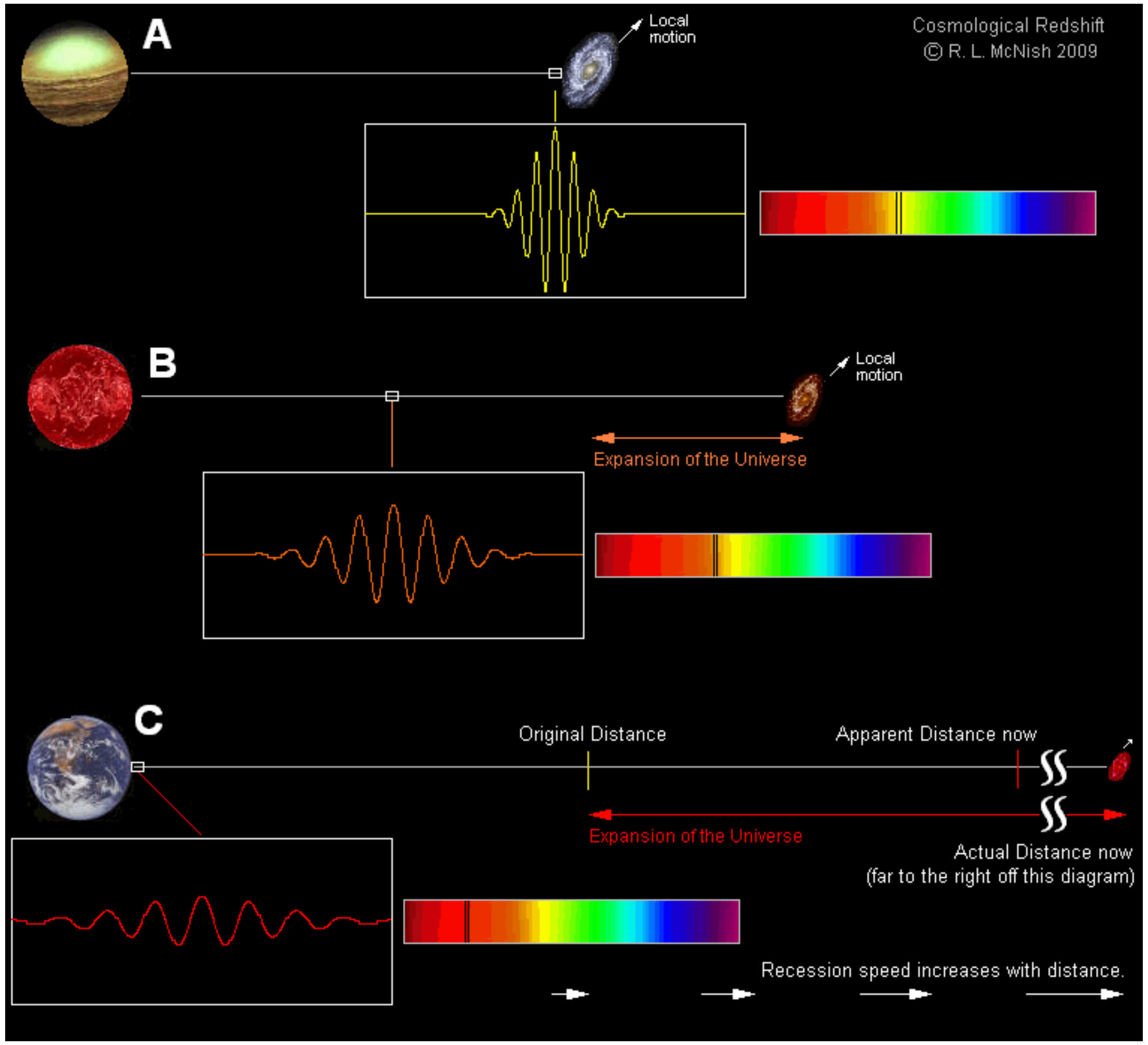
- In real space
- Cosmological principle: 1. Isotropy   2. homogeneity

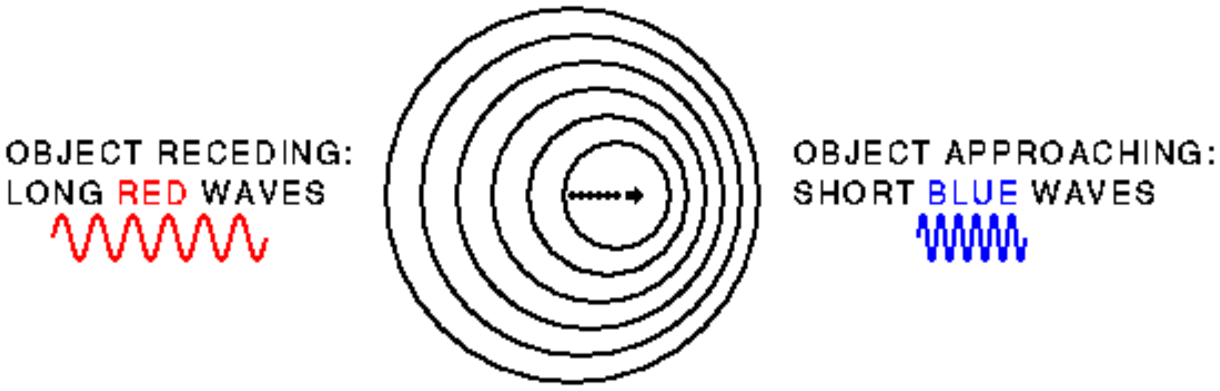


Millennium simulation

What's "distortion"

- In redshift space: latitude, longitude, redshift
- Redshift = cosmological redshift + doppler redshift



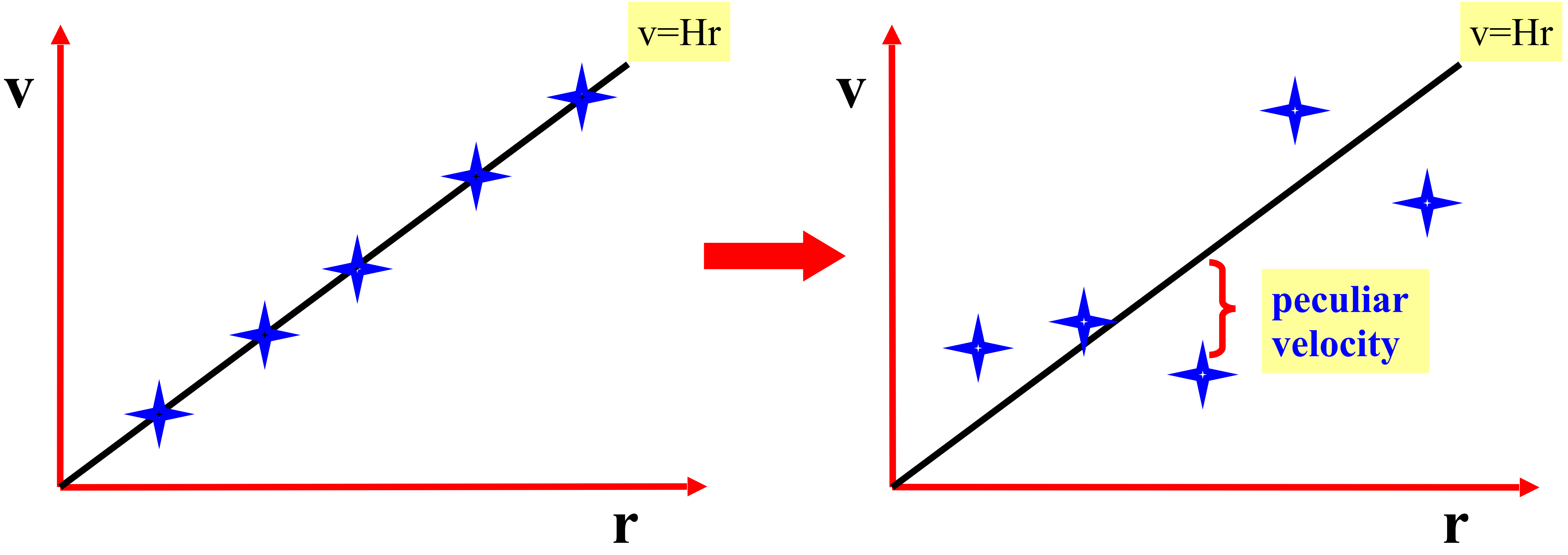


$$1 + z = \frac{\lambda_{obs}}{\lambda_{rest}} = \left(\frac{1+\frac{V}{c}}{1-\frac{V}{c}}\right)^{\frac{1}{2}} \approx 1 + \frac{V}{c}, \qquad for\ c \gg V$$

Millennium simulation

Peculiar velocity: a window to the dark universe

- **Matter distribution in our universe is inhomogeneous**
- **Gravitational attraction arising from inhomogeneity perturbs galaxies and causes deviation from the Hubble flow**

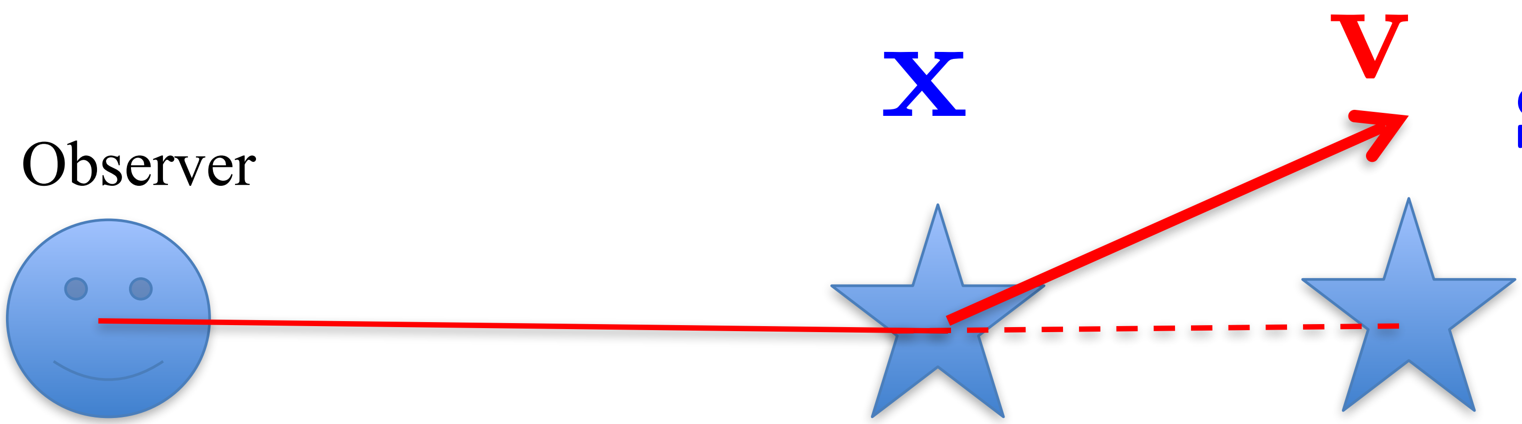


PJZ'PPT

Redshift space distortion

$$z^o = z + v_z$$

$$\mathbf{s} = \mathbf{x} + \frac{v_z}{H}\hat{z}$$



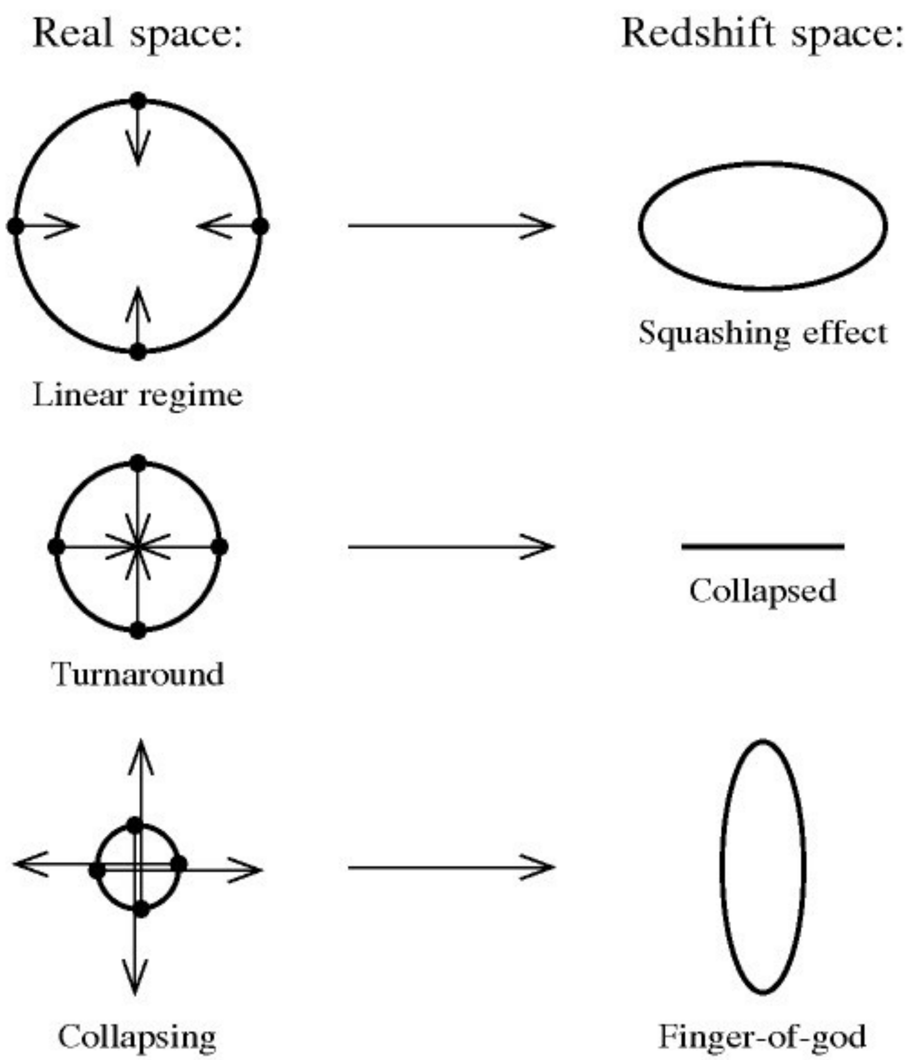
- Peculiar velocity changes the galaxy redshift and hence distorts the galaxy distribution in an anisotropic way
- Galaxy clustering along the line of sight is different to that perpendicular to the line of sight

PJZ'PPT

Redshift space distortion: Kaiser effect + Finger-of-god

$$P_0^s(k) = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right) P(k)$$
$$P_2^s(k) = \left(\frac{4}{3}\beta + \frac{4}{7}\beta^2\right) P(k)$$
$$P_4^s(k) = \frac{8}{35}\beta^2 P(k) \, .$$

=f/b



A. J. S. Hamilton, astro-ph/9708102

SDSS DR9, CMASS, 2D correlation function

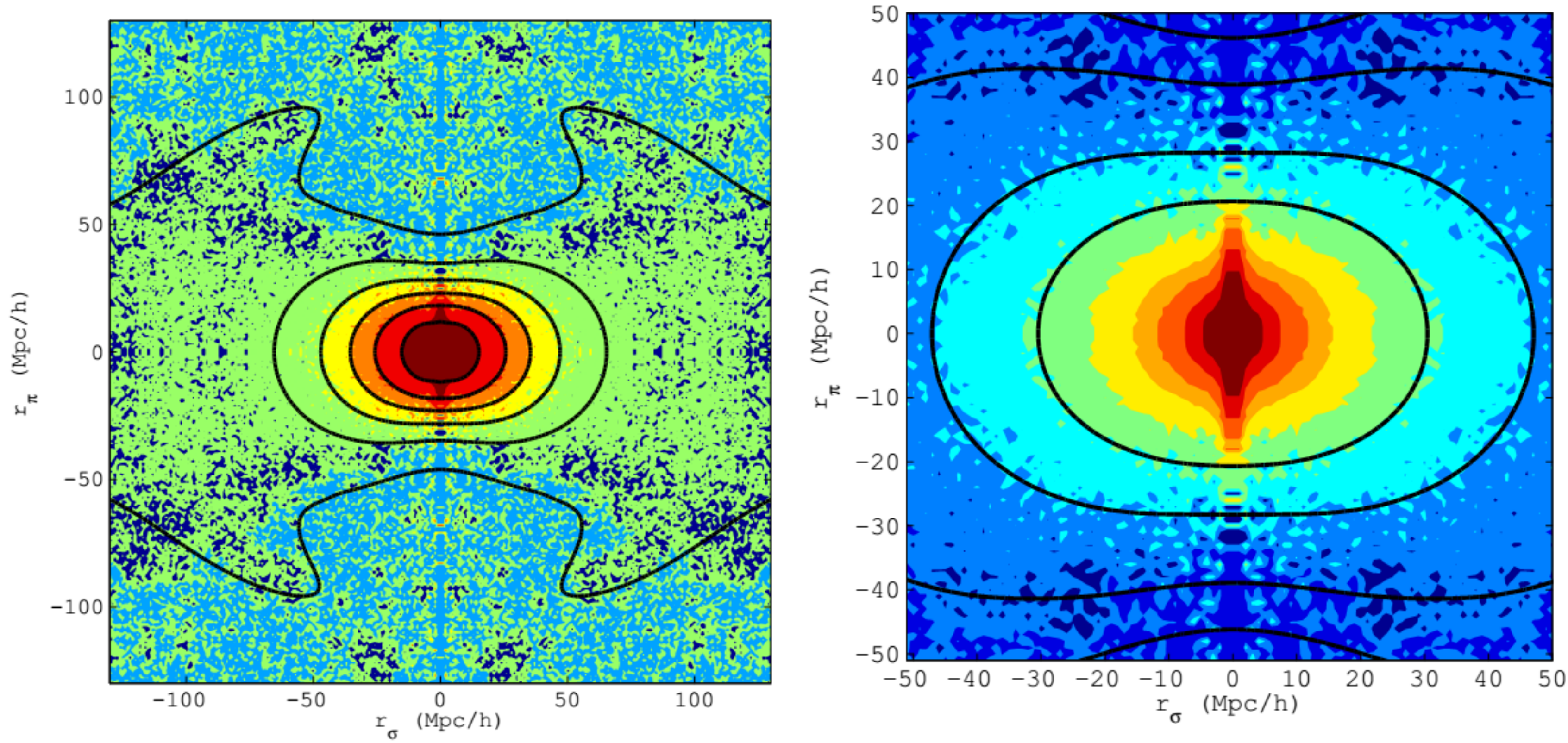


Figure 3. *Left panel*: Two-dimensional correlation function of CMASS galaxies (color) compared with the best fit model described in Section 6.1 (bla
Contours of equal ξ are shown at [0.6, 0.2, 0.1, 0.05, 0.02, 0]. *Right panel*: Smaller-scale two-dimensional clustering. We show model contours at [0.
0.01, 0]. The value of ξ_0 at the minimum separation bin in our analysis is shown as the innermost contour. The $\mu \approx 1$ "finger-of-god" effects are sma
scales we use in this analysis.

Reid et al. 1203.6641

Why RSD?

Dark Energy? Modified Gravity?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu},$$

Modified gravity

Dark Energy

1. Background level:

$$ds^2 = c^2\, dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right]$$

$$H^2 = \frac{8\pi G}{3} \left(\sum_i \rho_i \right) - \frac{c^2 k}{R^2}$$

2. Perturbation level:

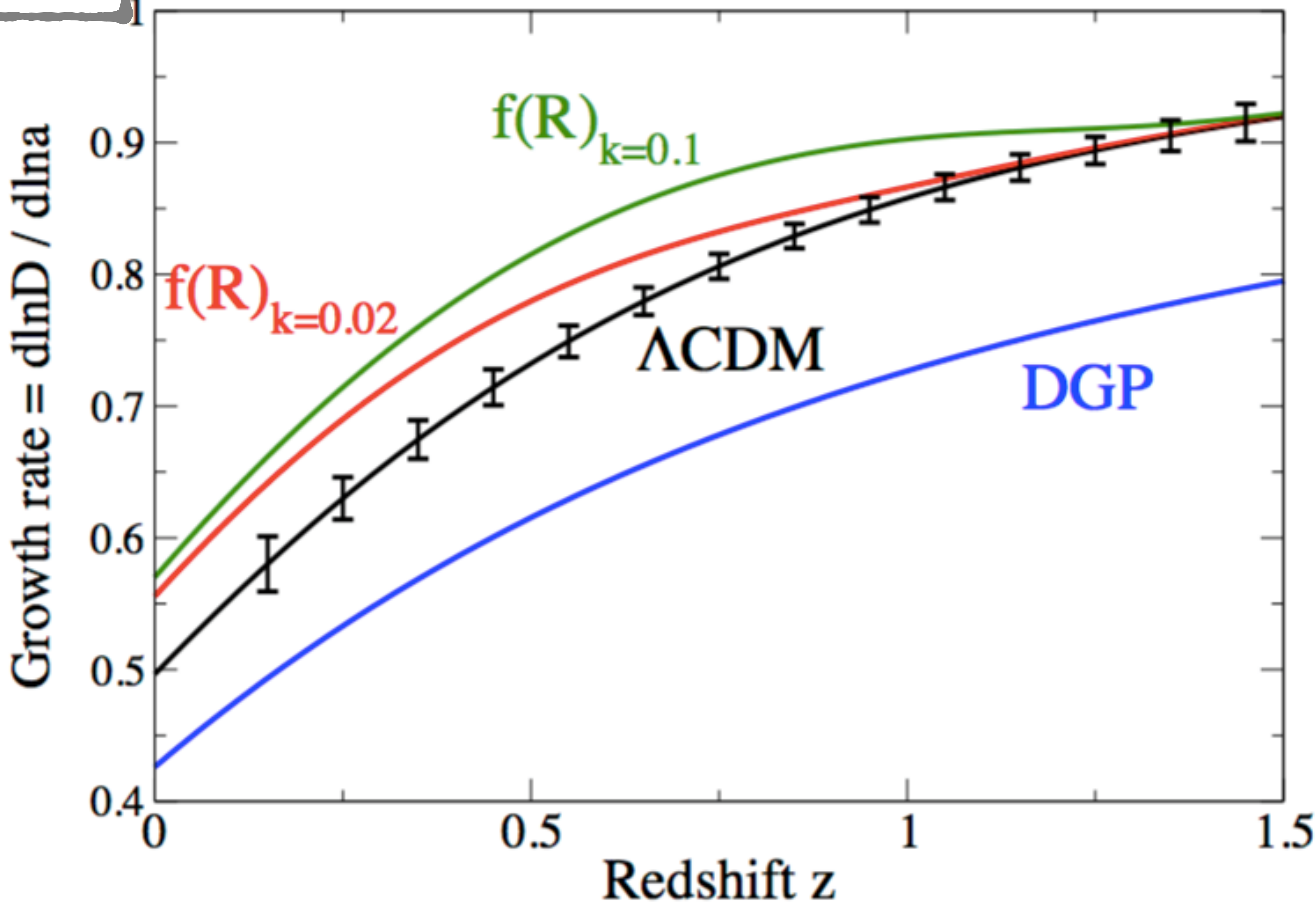
$$ds^2 = -(1 + 2\psi)dt^2 + (1 - 2\phi)a^2(t)d\vec{x}^2,$$

$$k^2\phi = -4\pi G a^2 \bar{\rho}_{\rm GR} \left[\delta_{\rm GR} + 3(1 + w)Ha \frac{\theta_{\rm GR}}{k^2} \right],$$
$$\simeq -4\pi G a^2 \bar{\rho}_{\rm GR} \delta_{\rm GR},$$

$$k^2(\phi - \psi) = 12\pi G a^2 (1 + w)\bar{\rho}\sigma$$

Jain, Zhang 0709.2375

For DESI



Huterer et al. 1309.5385

Redshift space distortion:
small scale modelling and systematic errors

$$N_k \propto k^3$$

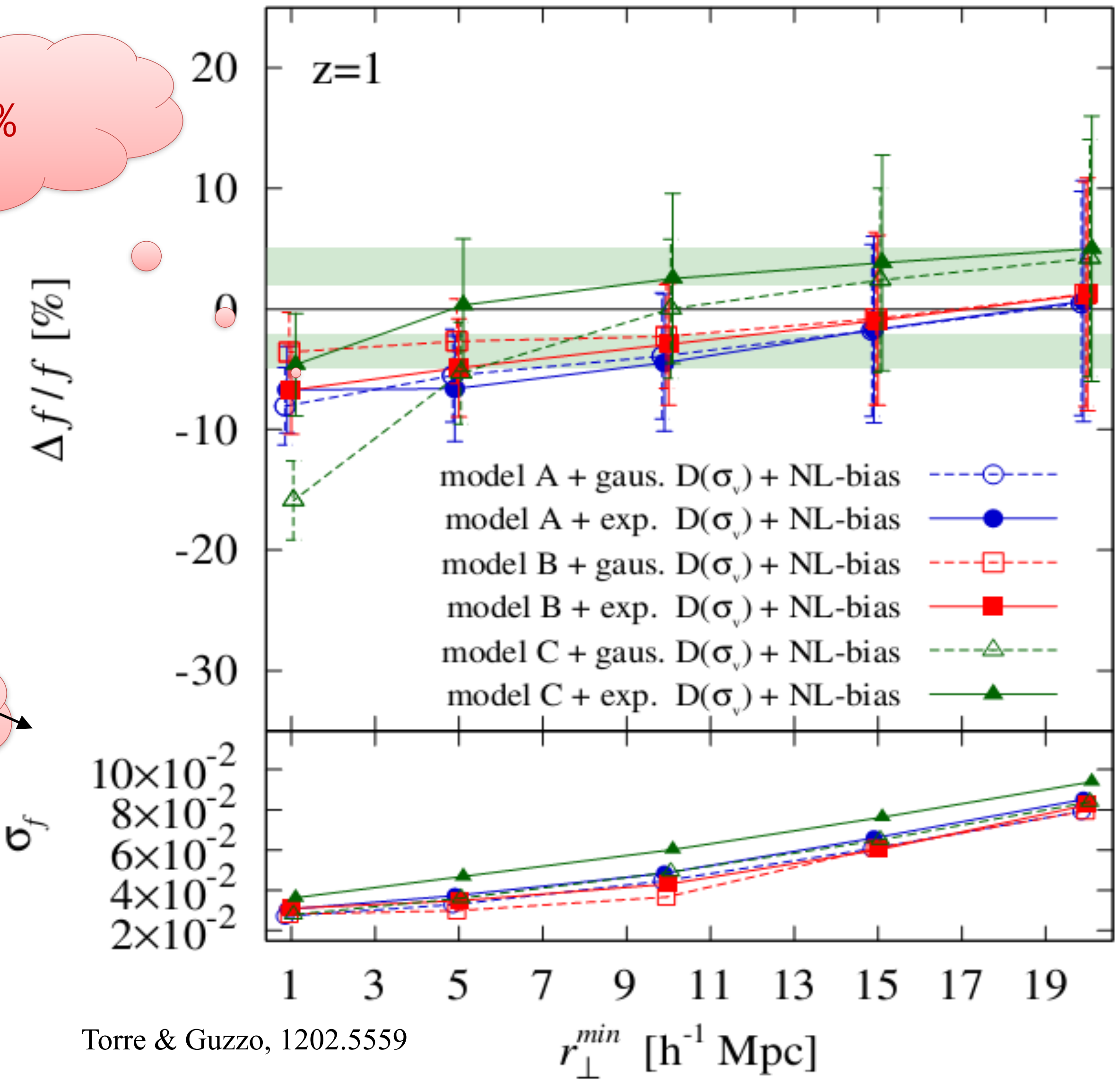
Theoretically:
1. Nonlinear mapping
2. Nonlinear evolution
3. Bias modelling

5-10%

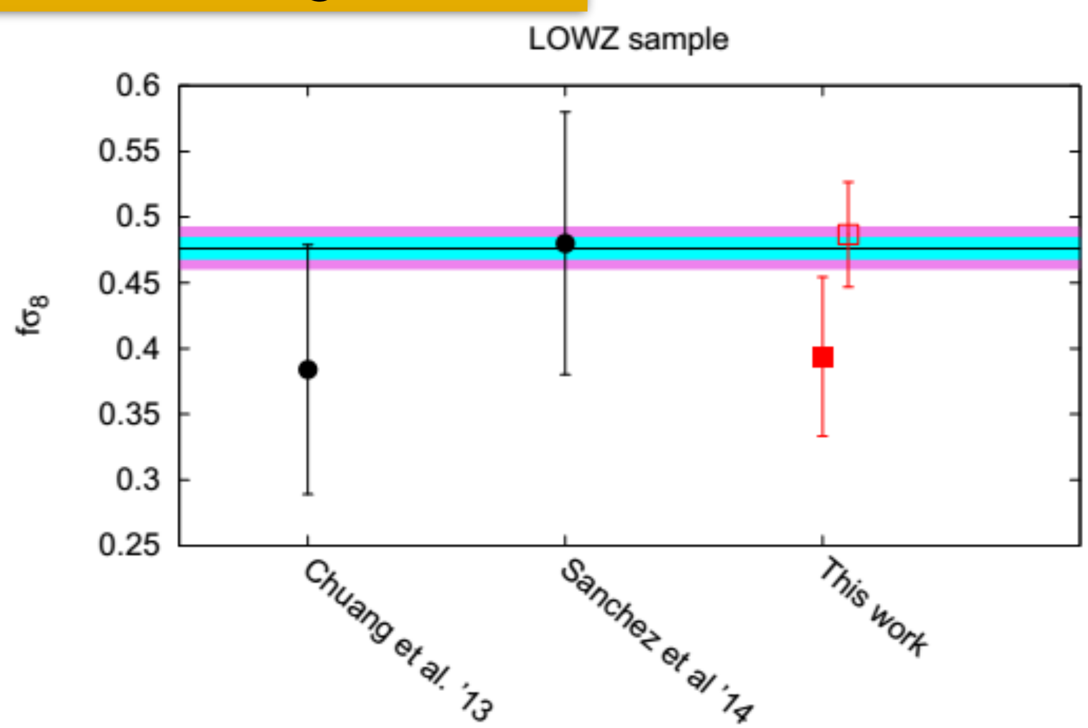
Observationally:

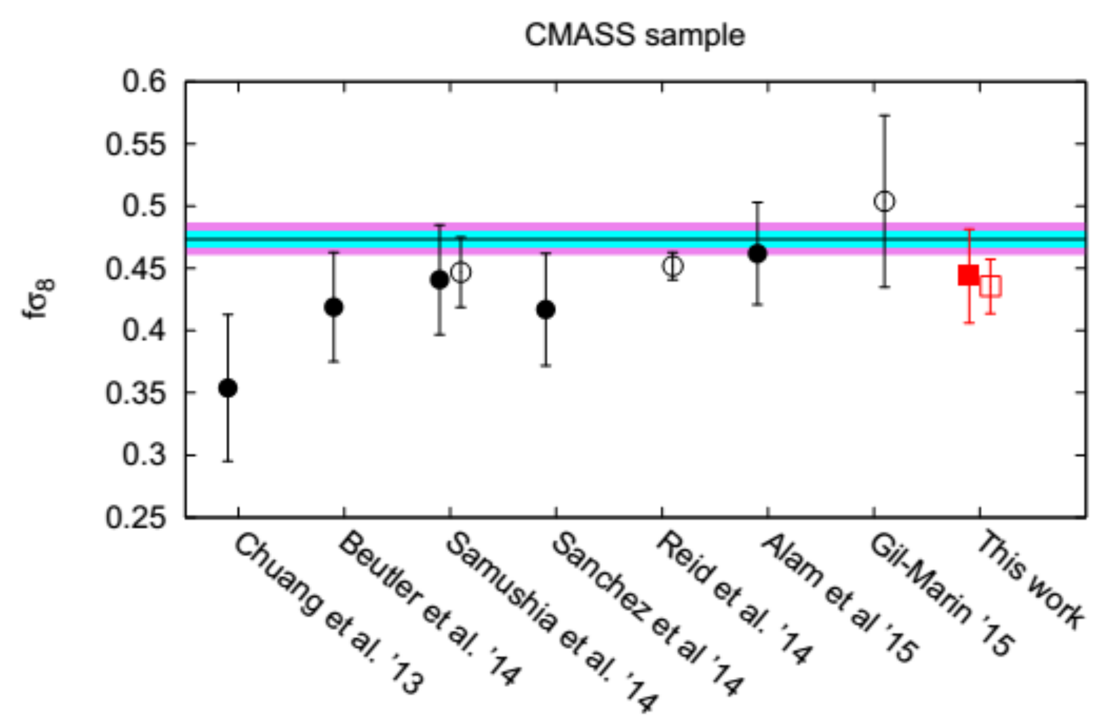
O(1%)

DESI, Euclid et all:



Torre & Guzzo, 1202.5559





Gil-Marin et al. 1509.06386

Redshift space distortion modelling
— power spectrum: in general three steps

1. Non-linear mapping of dark matter/halo/galaxy clustering from real space to redshift space

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle$$

TNS, 1006.0699

2.Non-linear evolution of density and velocity fields
——— Perturbation theory & High-resolution simulations

$$\delta_n(\mathbf{k}) = \int d^3\mathbf{q}_1 \cdots \int d^3\mathbf{q}_n\, \delta_\mathrm{D}(\mathbf{k}-\mathbf{q}_{1\cdots n}) F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_1(\mathbf{q}_1)\cdots\delta_1(\mathbf{q}_n)\,,$$

$$\theta_n(\mathbf{k}) = \int d^3\mathbf{q}_1 \cdots \int d^3\mathbf{q}_n\, \delta_\mathrm{D}(\mathbf{k}-\mathbf{q}_{1\cdots n}) G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_1(\mathbf{q}_1)\cdots\delta_1(\mathbf{q}_n)\,,$$

F. Bernardeau et al, 2002

3. Galaxy/halo density and velocity bias modelling

$$\delta_g(\boldsymbol{x}) = b_1\delta(\boldsymbol{x}) + \frac{1}{2}b_2[\delta(\boldsymbol{x})^2 - \sigma_2] + \frac{1}{2}b_{s^2}[s(\boldsymbol{x})^2 - \langle s^2\rangle] + \text{higher order terms},$$

McDonald&Roy, 0902.0991

Follow the derivation of TNS paper, Taruya et al. 1006.0699

From mass conservation:

$$\{1+\delta^{(S)}(\mathbf{s})\}d^3\mathbf{s} = \{1+\delta(\mathbf{r})\}d^3\mathbf{r}$$

$$\mathbf{s} = \mathbf{r} + \frac{v_z(\mathbf{r})}{aH(z)}\hat{z}$$

$$\delta^{(S)}(\mathbf{s}) = \left|\frac{\partial \mathbf{s}}{\partial \mathbf{r}}\right|^{-1}\{1+\delta(\mathbf{r})\} - 1$$

$$\delta^{(S)}(\mathbf{k}) = \int d^3\mathbf{r}\left\{\delta(\mathbf{r}) - \frac{\nabla_z v_z(\mathbf{r})}{aH(z)}\right\}e^{i(k\mu v_z/H + \mathbf{k}\cdot\mathbf{r})}$$

The redshift space power spectrum:

Intrinsically nonlinear; Taylor expansion and trunction

$$P^{(S)}(\mathbf{k}) = \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\langle e^{-ik\mu f\Delta u_z}\{\delta(\mathbf{r}) + f\nabla_z u_z(\mathbf{r})\} \times \{\delta(\mathbf{r}') + f\nabla_z u_z(\mathbf{r}')\}\rangle,$$

$$u_z(\mathbf{r}) = -v_z(\mathbf{r})/(aHf)$$

Redshift space distortion:
starting point for PS

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle$$

$$\delta_D(\boldsymbol{k}) + P_s(\boldsymbol{k}) = \int \frac{d^3 r}{(2\pi)^3} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \langle \mathrm{e}^{ifk_z\Delta u_z}[1+\delta(\boldsymbol{x})] \times [1+\delta(\boldsymbol{x}')]\rangle,$$

e.g. Scoccimarro astro-ph/0407214
Zhang 1207.2722

$$P^{(\mathrm{S})}(k,\mu) = \int \frac{d^3 r}{(2\pi)^3} e^{-i\,\boldsymbol{k}\cdot\boldsymbol{r}} \langle e^{ifk_z u_z}[1+\delta(\boldsymbol{x})] e^{-ifk_z u_z'}[1+\delta(\boldsymbol{x}')]\rangle$$

Taylor expansion!

$$P^{ss}(\mathbf{k}) = \sum_{L=0}^{\infty} \frac{1}{L!^2}\left(\frac{k\mu}{H}\right)^{2L} P_{LL}(\mathbf{k}) + 2\sum_{L=0}^{\infty}\sum_{L'>L} \frac{(-1)^L}{L!\,L'!}\left(\frac{ik\mu}{H}\right)^{L+L'} P_{LL'}(\mathbf{k})$$

Seljak&McDonald 1109.1888

$$P^{(\mathrm{S})}(\boldsymbol{k}) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{-ik\mu f\Delta u_z}\{\delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r})\} \times \{\delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}')\}\rangle,$$

e.g. TNS 1006.0699

$$\langle e^{j_1A_1}A_2A_3\rangle = \exp\{\langle e^{j_1A_1}\rangle_c\}[\langle e^{j_1A_1}A_2A_3\rangle_c + \langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c].$$

Cumulant expansion

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \exp\{\langle e^{j_1A_1}\rangle_c\}[\langle e^{j_1A_1}A_2A_3\rangle_c + \langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c].$$

FoG

in terms of j_1

$$\langle e^{j_1A_1}A_2A_3\rangle_c + \langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c \simeq \langle A_2A_3\rangle + j_1\langle A_1A_2A_3\rangle_c + j_1^2\{\tfrac{1}{2}\langle A_1^2A_2A_3\rangle_c + \langle A_1A_2\rangle_c\langle A_1A_3\rangle_c\} + \mathcal{O}(j_1^3).$$

TNS 1006.0699
Zheng 1603.00101

The ‘extended’ TNS model

$$P^{(S)}(\boldsymbol{k}) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{-ik\mu f \Delta u_z} \{\delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r})\} \times \{\delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}')\}\rangle,$$

Definition:

$$j_1 = -ik\mu f, \qquad A_1 = u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'),$$
$$A_2 = \delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r}), \qquad A_3 = \delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}').$$

$$P^{(S)}(k, \mu) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle,$$

$$\langle e^{j_1 A_1} A_2 A_3 \rangle = \exp\{\langle e^{j_1 A_1}\rangle_c\}[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c].$$

Important for separation of FoG

$$P^{(S)}(k, \mu) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \exp\{\langle e^{j_1 A_1}\rangle_c\}[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c].$$

Scale dependent part+scale independent part

$$\begin{aligned} &D^{\rm FoG}_{\rm corr}(k\mu, \boldsymbol{x}) \left[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c\right] \\ &\simeq j_1^0 \langle A_2 A_3\rangle_c + j_1^1 \langle A_1 A_2 A_3\rangle_c \\ &+ j_1^2 \left\{\langle A_1 A_2\rangle_c \langle A_1 A_3\rangle_c + \frac{1}{2}\langle A_1^2 A_2 A_3\rangle_c - \langle u_z u_z'\rangle_c \langle A_2 A_3\rangle_c\right\} \\ &+ \mathcal{O}(j_1^3), \end{aligned} \qquad (13)$$

The ‘extended’ TNS model

1006.0699 & 1603.00101

Taylor expansion in terms of $j_1 = -ik\mu$

$$
\begin{aligned}
&D^{\rm FoG}_{\rm local}(k\mu,\boldsymbol{x})\left[\langle e^{j_1A_1}A_2A_3\rangle_c + \langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c\right] \\
&\simeq j_1^0\langle A_2A_3\rangle_c + j_1^1\langle A_1A_2A_3\rangle_c \\
&+j_1^2\left\{\langle A_1A_2\rangle_c\langle A_1A_3\rangle_c + \frac{1}{2}\langle A_1^2A_2A_3\rangle_c - \langle u_zu_z'\rangle_c\langle A_2A_3\rangle_c\right\} \\
&+\mathcal{O}(j_1^3)\,, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
P^{\rm (S)}(k,\mu) &= D^{\rm FoG}(k\mu\sigma_z)P_{\rm perturbed}(k,\mu) \\
&= D^{\rm FoG}(k\mu\sigma_z)[P_{\delta\delta} + 2\mu^2P_{\delta\Theta} + \mu^4P_{\Theta\Theta} \quad (15) \\
&\quad +A(k,\mu) + B(k,\mu) + T(k,\mu) + F(k,\mu)].
\end{aligned}
$$

parameter	physical meaning	value
Ω_m	present fractional matter density	0.3132
Ω_Λ	$1-\Omega_m$	0.6868
Ω_b	present fractional baryon density	0.049
h	$H_0/(100\ {\rm km\ s^{-1}Mpc^{-1}})$	0.6731
n_s	primordial power spectral index	0.9655
σ_8	r.m.s. linear density fluctuation	0.829
$L_{\rm box}$	simulation box size	$1890\ h^{-1}$Mpc
$N_{\rm p}$	simulation particle number	1024^3
$m_{\rm p}$	simulation particle mass	$5.46\times10^{10}h^{-1}M_\odot$

$$
\begin{aligned}
A(k,\mu) &= j_1\int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\;\langle A_1A_2A_3\rangle_c \\
&= j_1\int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\;\langle(u_z-u_z') \\
&\quad\times(\delta+\nabla_zu_z)(\delta'+\nabla_zu_z')\rangle_c
\end{aligned}
$$

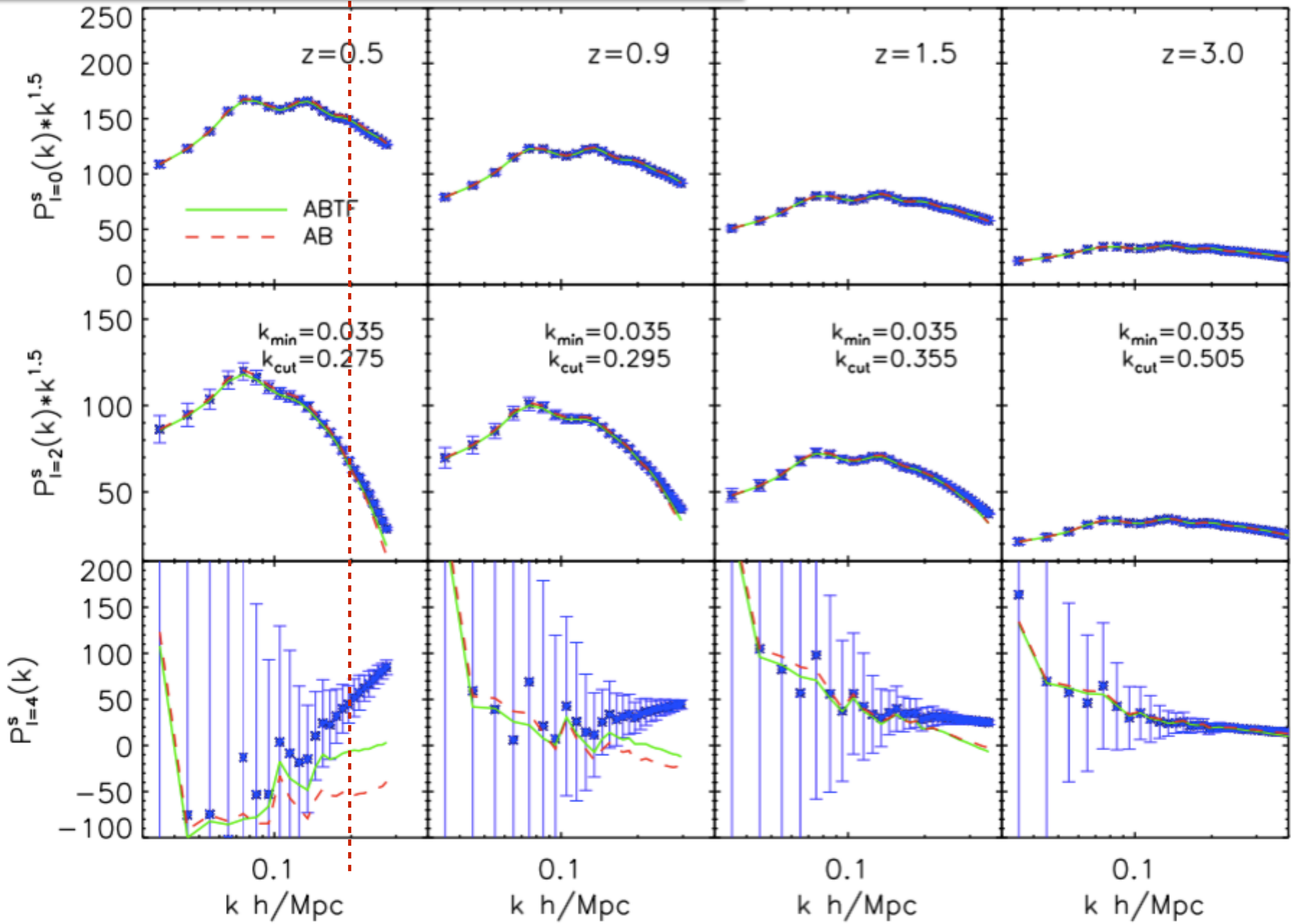
2-point

Multipole test

$$P_\ell^{(S)}(k) = \frac{2\ell+1}{2}\int_{-1}^{1} d\mu P^{(S)}(k,\mu)\mathcal{P}_\ell(\mu),$$

\sigma_v as free parameter, Gaussian FoG

1603.00101



RSD correlation function modelling—mapping formula: pairwise velocity PDF

$$1+\xi_S(s_\perp, s_\parallel) = \int dr_\parallel \, [1+\xi_R(r)] \, \mathcal{P}(r_\parallel - s_\parallel|\mathbf{r}) \, .$$

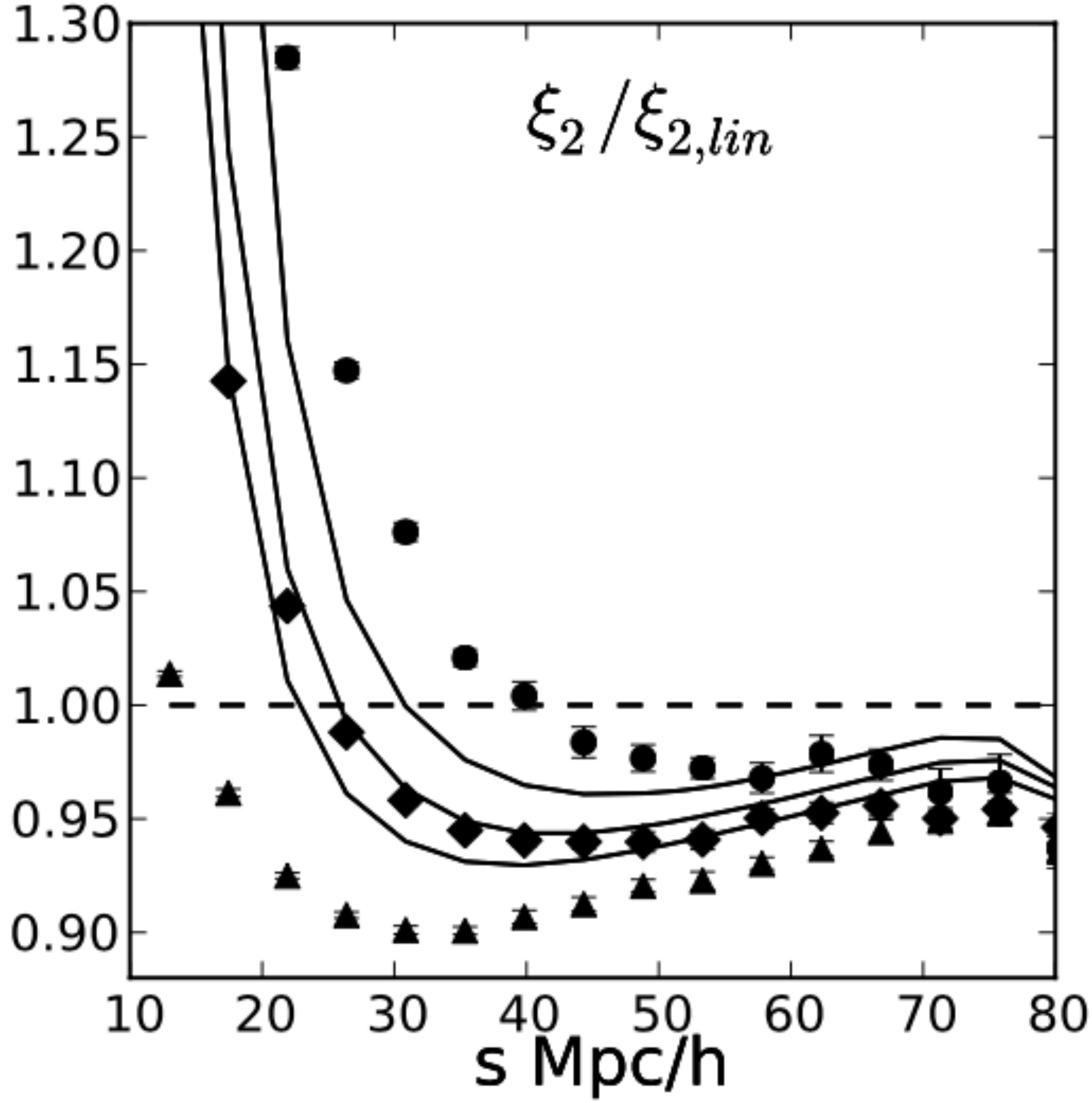
1407.4753

The scale-dependent Gaussian streaming model (GSM), Reid&White,2011

$$1+\xi_g^s(r_\sigma, r_\pi) = \int [1+\xi_g^r(r)] \, e^{-[r_\pi - y - \mu v_{12}(r)]^2/2\sigma_{12}^2(r,\mu)} \times \frac{dy}{\sqrt{2\pi\sigma_{12}^2(r,\mu)}}$$

$$\boldsymbol{v}_{12}(r) = v_{12}(r)\hat{\boldsymbol{r}} = -\hat{\boldsymbol{r}}\frac{fb}{\pi^2}\int dk \, k \, P_m^r(k) j_1(kr),$$

$$\sigma_{12}^2(r, \mu^2) = 2\left[\sigma_v^2 - \mu^2\Psi_\parallel(r) - (1-\mu^2)\Psi_\perp(r)\right]$$



$$1 + \xi_{\mathrm{g}}^{\mathrm{s}}(r_\sigma, r_\pi) = \int \left[1 + \xi_{\mathrm{g}}^{\mathrm{r}}(r)\right] \mathrm{e}^{-[r_\pi - y - \mu v_{12}(r)]^2/2\sigma_{12}^2(r,\mu)}$$

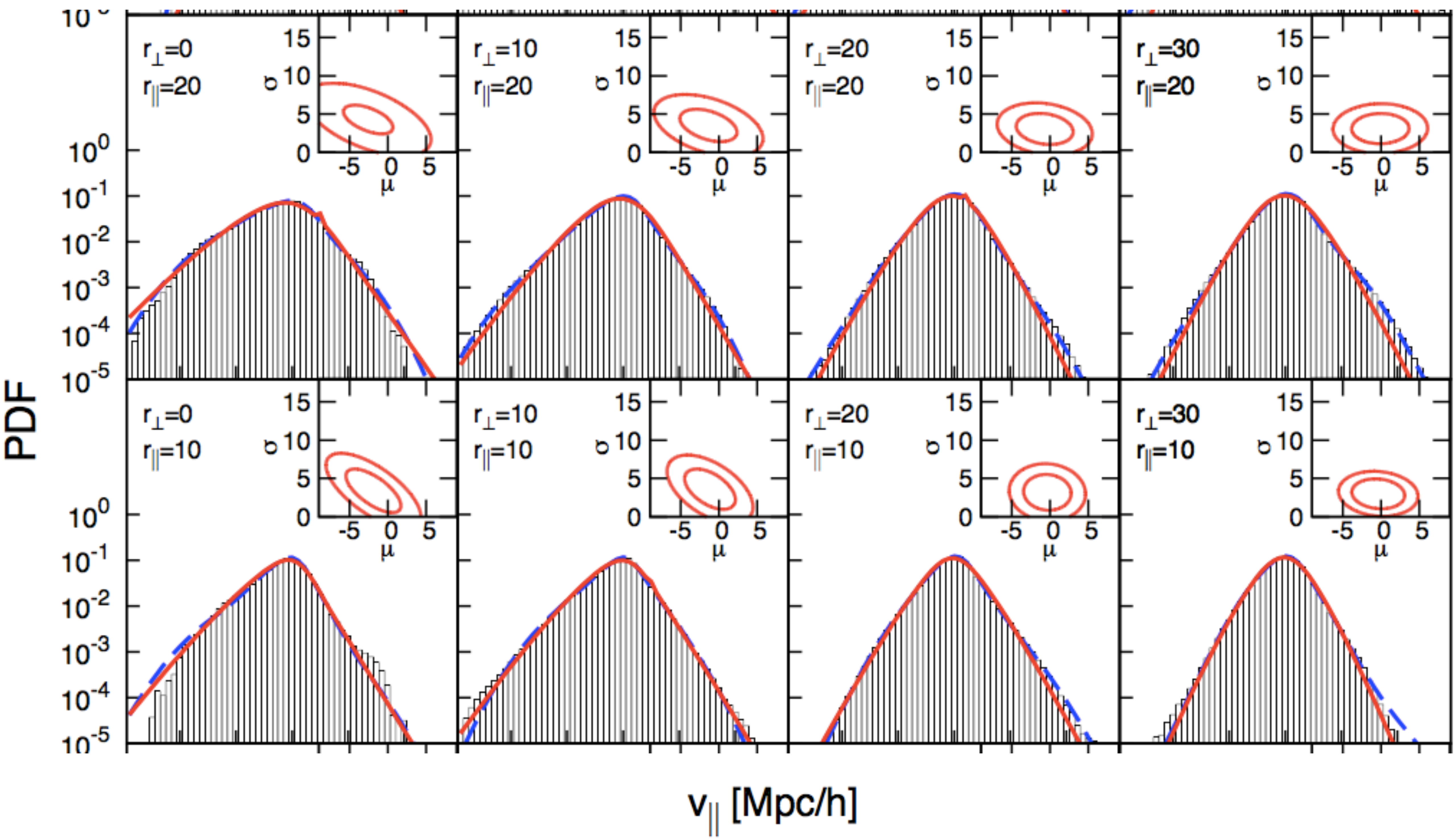
$$\boldsymbol{v}_{12}(r) = v_{12}(r)\hat{\boldsymbol{r}} = -\hat{\boldsymbol{r}}\frac{fb}{\pi^2}\int \mathrm{d}k\; k\; P_{\mathrm{m}}^{\mathrm{r}}(k) j_1(kr),$$

$$\sigma_{12}^2(r, \mu^2) = 2\left[\sigma_v^2 - \mu^2\Psi_{\|}(r) - (1-\mu^2)\Psi_{\perp}(r)\right]$$

Gaussian quasi-Gaussianity (GQG)

Improving the modelling of redshift-space distortions - II. A pairwise velocity model covering large and small scales

Davide Bianchi[1,4⋆], Will Percival[1] and Julien Bel[2,3,4]



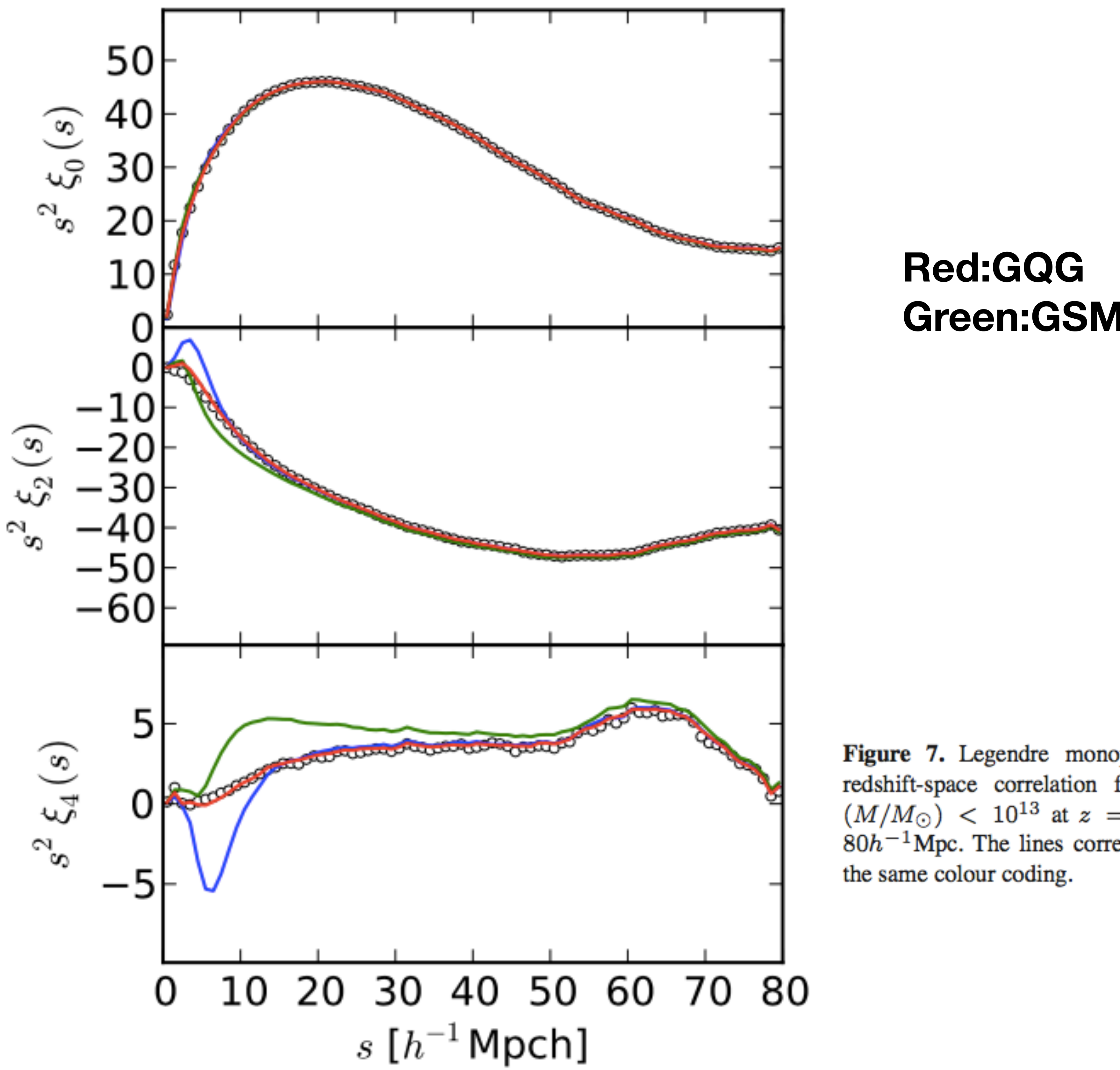


Figure 7. Legendre monopole, quadrupole and hexadecapole of the redshift-space correlation function, for the halo catalogue $10^{12} < (M/M_\odot) < 10^{13}$ at $z = 0$, on a large separation range, $0 < s < 80h^{-1}$Mpc. The lines correspond to the same models as in Fig. 4, with the same colour coding.

Future

- Real space nonlinearity: SPT, RegPT, EFT, Response function, Emulator… PT+simulation
    - For correlation function — theoretical calculation of first three moments of PDF
- Mapping formula: power spectrum — better Taylor expansion and truncation strategy
- Bias modelling: density and velocity bias

5. arXiv:1611.09787 [pdf, other]
Large-Scale Galaxy Bias
Vincent Desjacques, Donghui Jeong, Fabian Schmidt