

HECTOR GIL-MARIN (ILP-LPNHE)



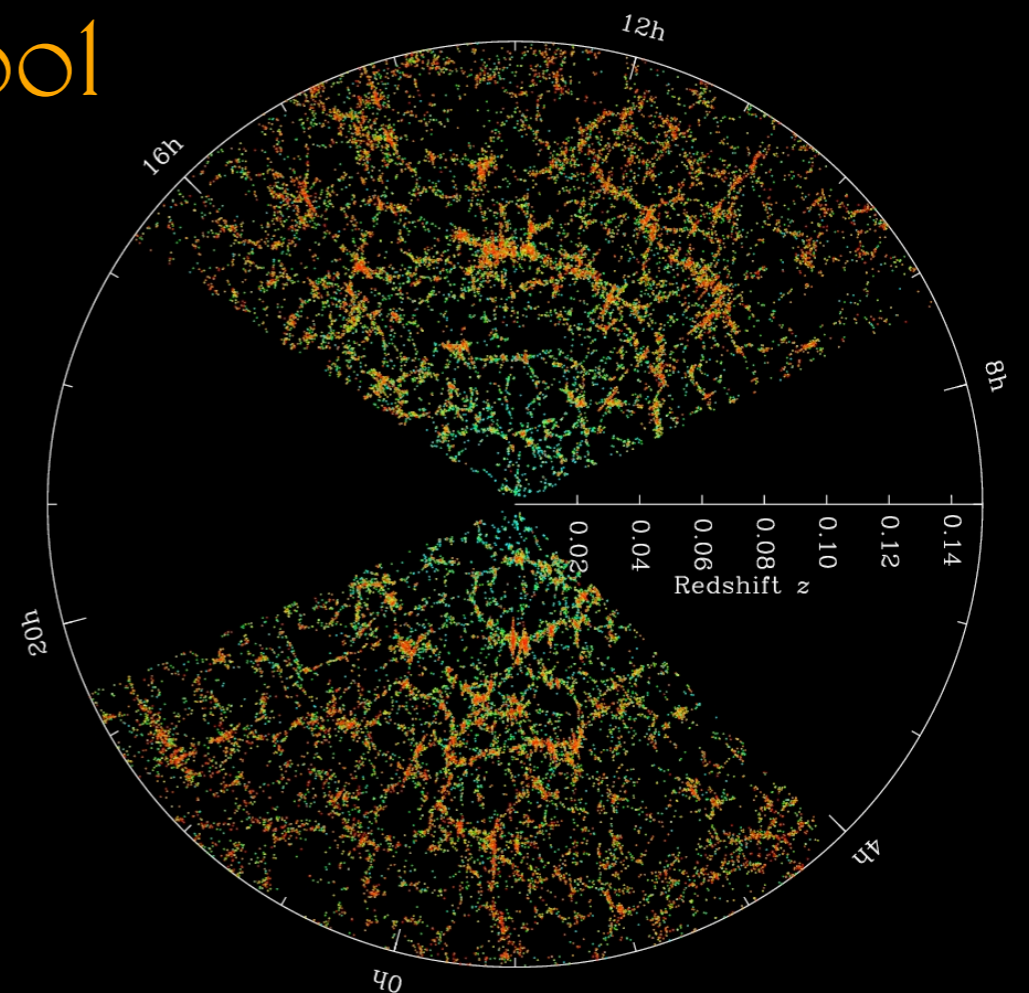
COSMOLOGY WITH GALAXY SURVEYS

Fuerteventura Cosmology School

REDSHIFT SPACE DISTORTIONS

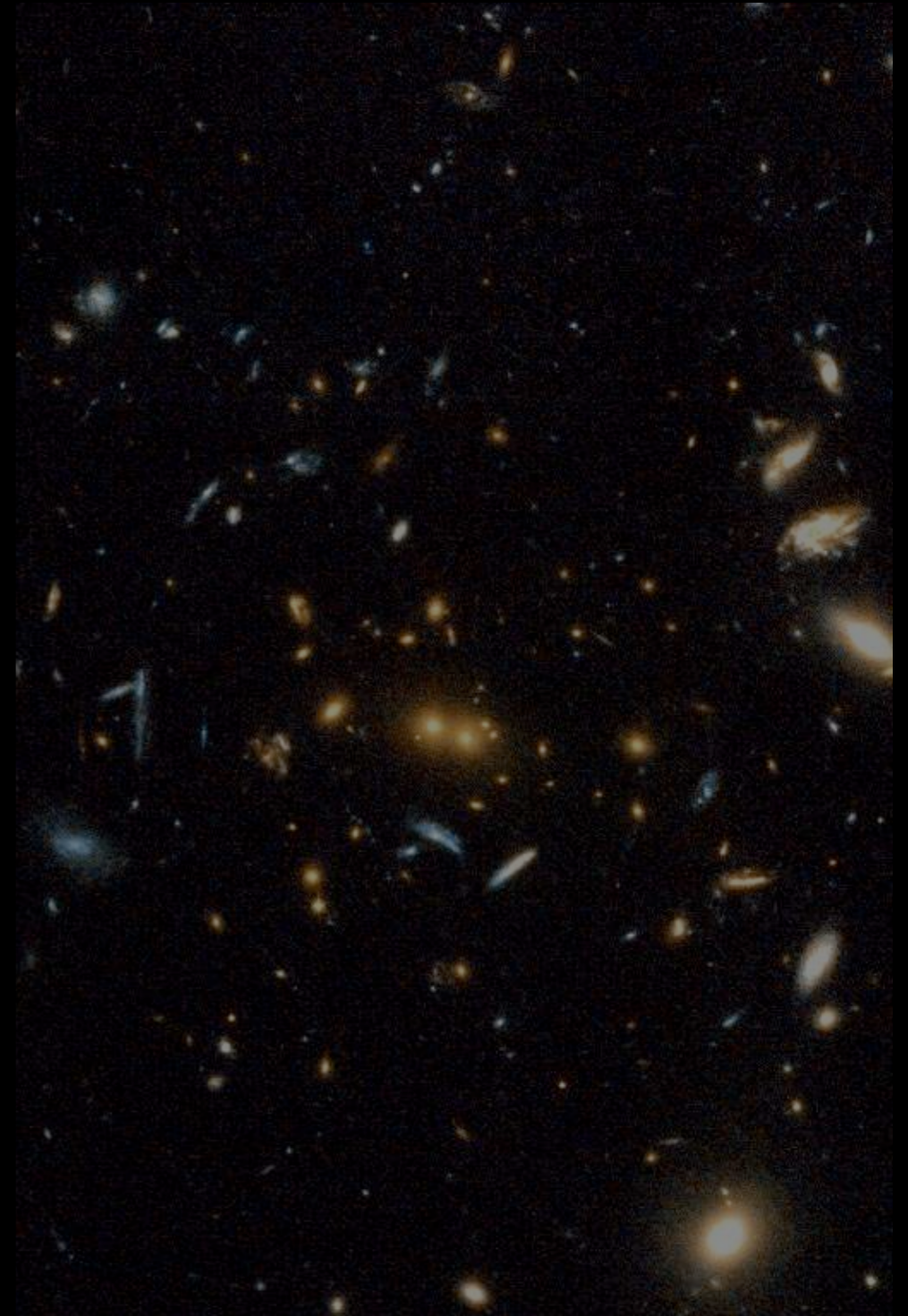
BARYON ACOUSTIC OSCILLATIONS

HIGHER ORDER STATISTICS



Outline

- **$P(k)$ Estimator & Selection Function**
- **RSD & BAO in actual surveys**
- **Bispectrum signal**



Introduction

Dark matter over-density, $\delta(r) \equiv \frac{\rho(r)}{\bar{\rho}} - 1$ $\xrightarrow{\text{FT}}$ $\delta(k) = \int \delta(x) e^{ikx} dx$

2-point correlation function (2pCF), $\langle \delta(r) \delta(r+R) \rangle = \xi(R)$

Power Spectrum (PS) $\langle \delta(k) \delta^*(k') \rangle = (2\pi)^3 P(k) \delta^D(k+k')$

The PS and 2-pCF are the *observables* for RSD and BSO

Power Spectrum

Key ingredients for cooking a $f\sigma_8$ / BAO measurement



1. Measure $P^{(0)}(k)$, $P^{(2)}(k)$ from data
2. Survey selection function
3. Model dark matter power spectra
4. Galaxy bias model
5. Covariance (error)

Mix it with a MCMC sampler and serve it cold!



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M. Crocce
D. Blas

W. Percival

A. Cuesta

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Measuring $P(k)$ from surveys

Real life problems

- Non-uniform distribution of galaxies
- Varying photometric conditions
- Fibre limitations



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FKP function (Feldman et al. 1994)

$$F_2(\vec{r}) \Rightarrow \frac{w(\vec{r})}{\sqrt{I_2}} [n(\vec{r}) - \alpha n_s(\vec{r})]$$

weight

normalisation

random catalogue

same features than data catalogue but with no clustering structure

$$\alpha = \frac{N_g}{N_s}$$

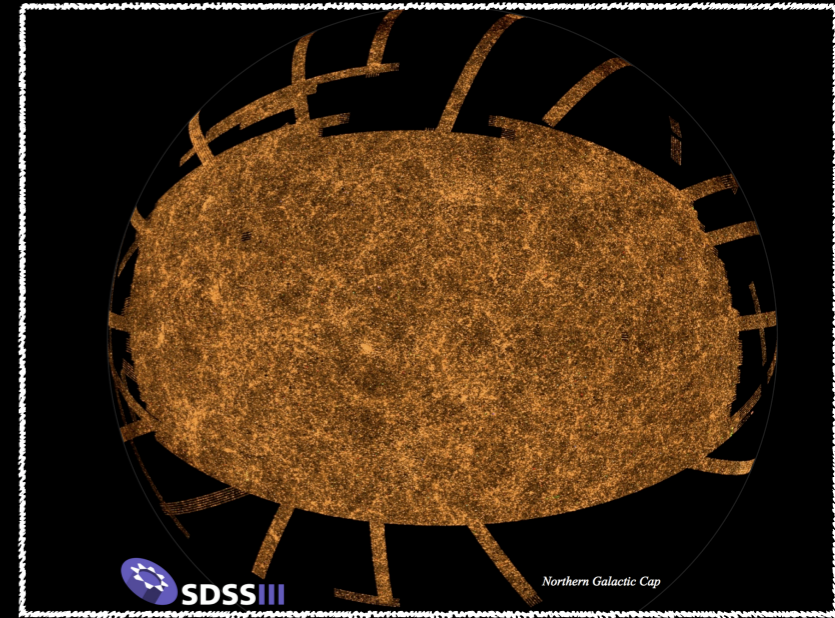
actual galaxy catalogue

$$I_2 = \int [w(\vec{r}) \bar{n}(\vec{r})]^2 d^3r$$

Measuring $P(k)$ from surveys

Real life problems

- Non-uniform distribution of galaxies
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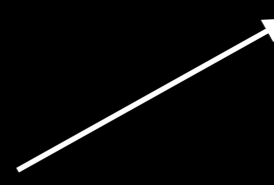
FKP function (Feldman et al. 1994)

$$F_2(\vec{r}) = \frac{w(\vec{r})}{\sqrt{I_2}} [n(\vec{r}) - \alpha n_s(\vec{r})]$$

Yamamoto et al. 2006

$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \mathcal{P}_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

line of sight dependence



$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

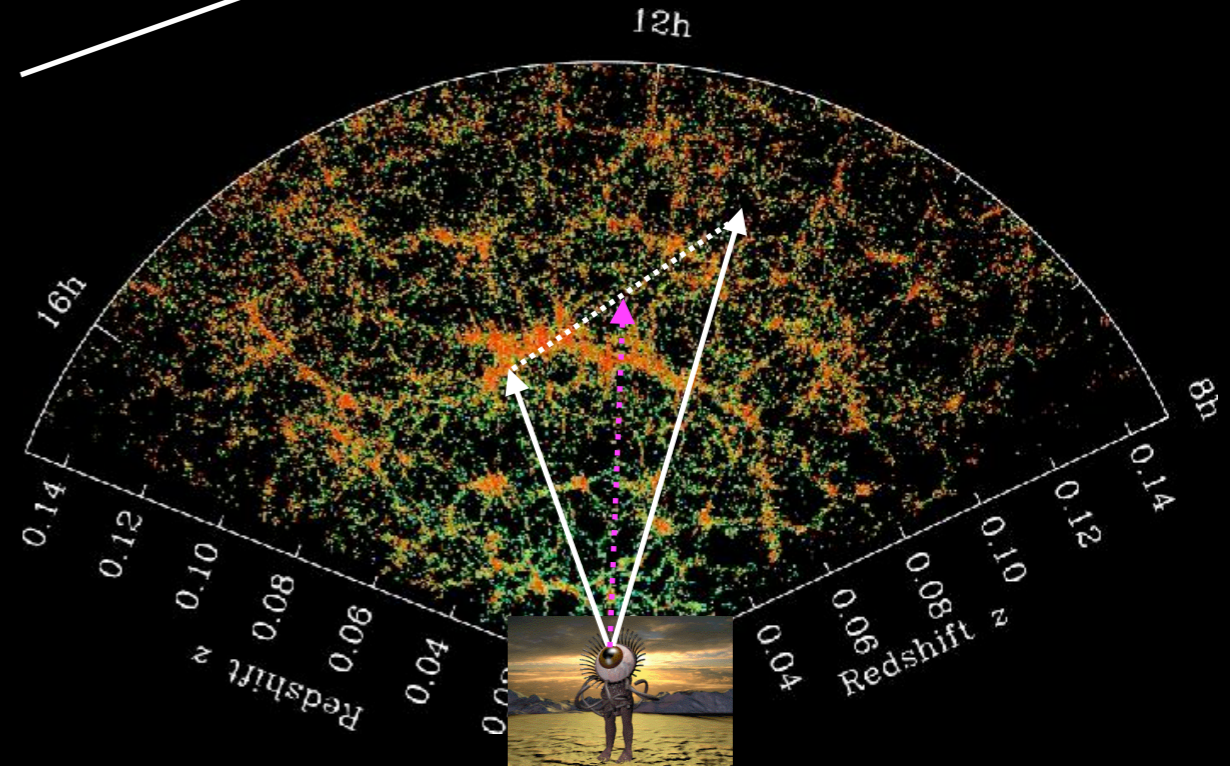
Line of sight dependence through

$$\vec{r}_h = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

Legendre Polynomials

$$\wp_0(x) = 1 \quad \text{Monopole}$$

$$\wp_2(x) = \frac{1}{2}(3x^2 - 1) \quad \text{Quadrupole}$$



For the **monopole** no-LOS dependence,

$$\tilde{P}_g^{(0)}(k) = \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} - P_0^{noise}(k) \right]$$

Integrals are separable

$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

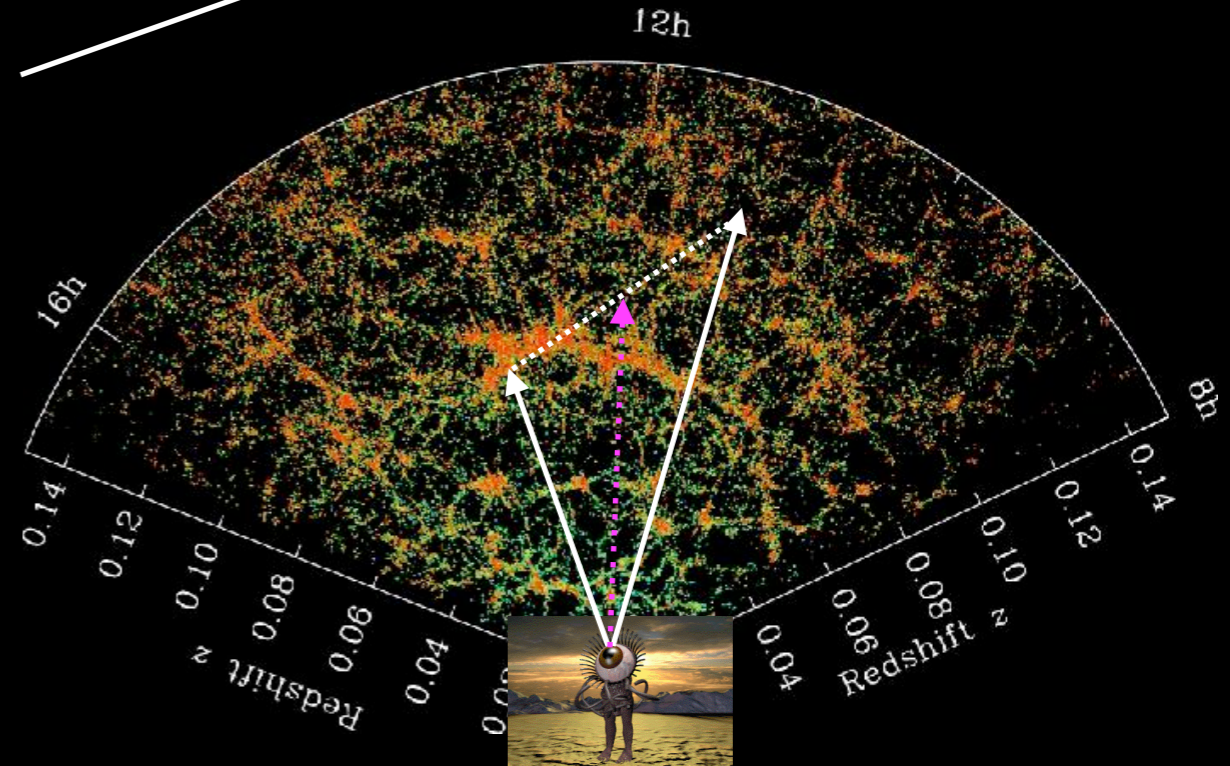
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$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

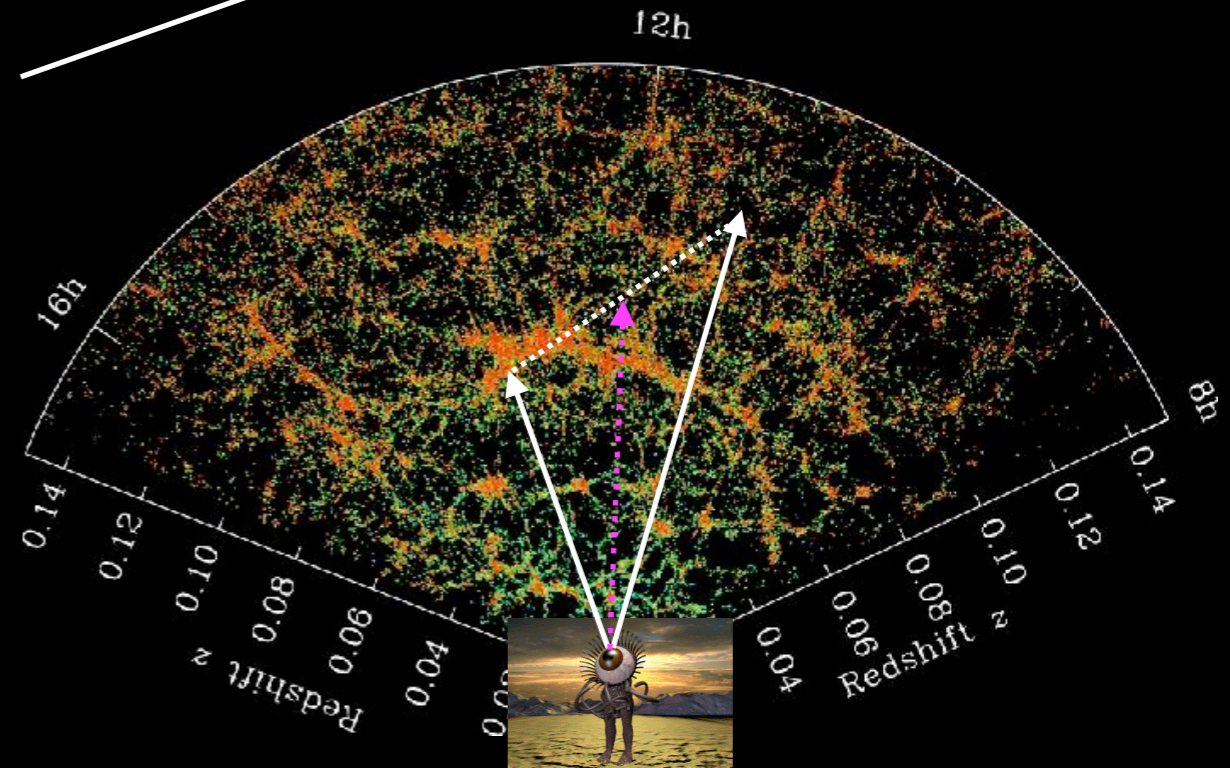
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$$\tilde{P}_g^{(0)}(k) = \int \frac{d\Omega_k}{4\pi} \left[F_2(\vec{k}) F_2^*(\vec{k}) - P_0^{noise}(k) \right]$$

$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

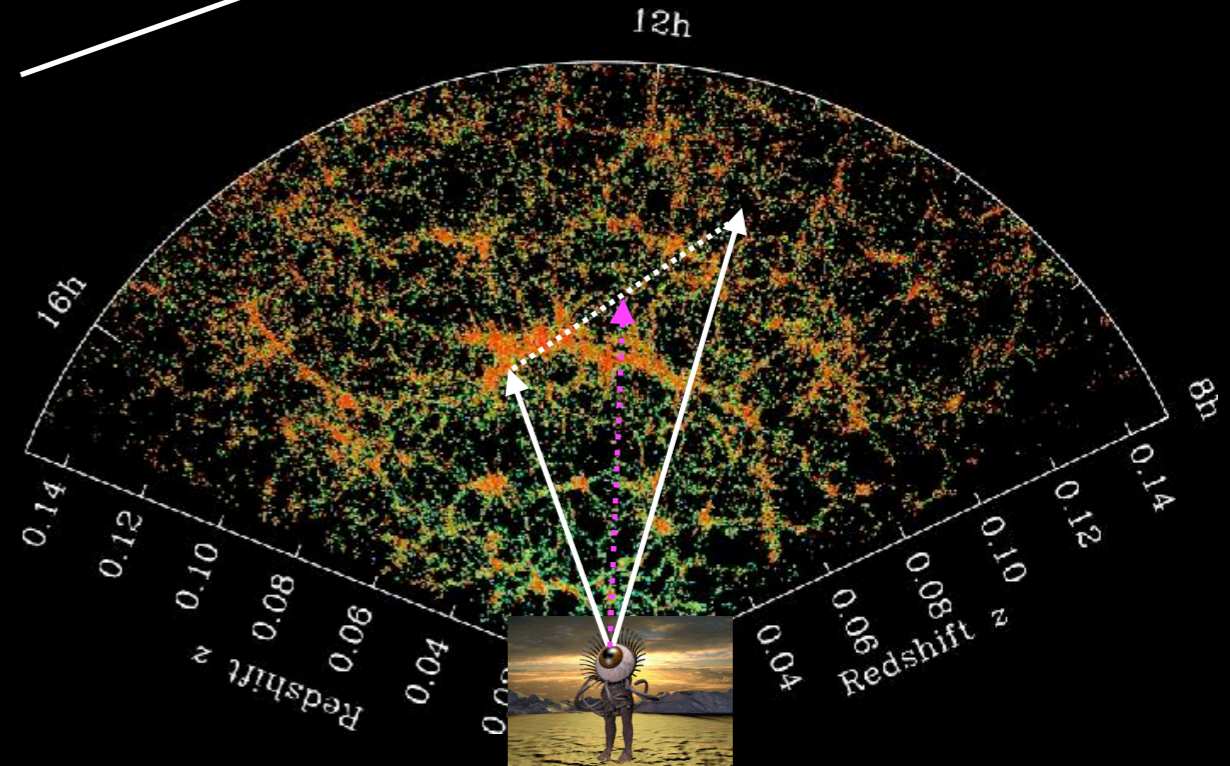
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For the **quadrupole**,

$$\tilde{P}_g^{(2)}(k) = 5 \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_2(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

Integrals are **not** separable

$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

Line of sight dependence through

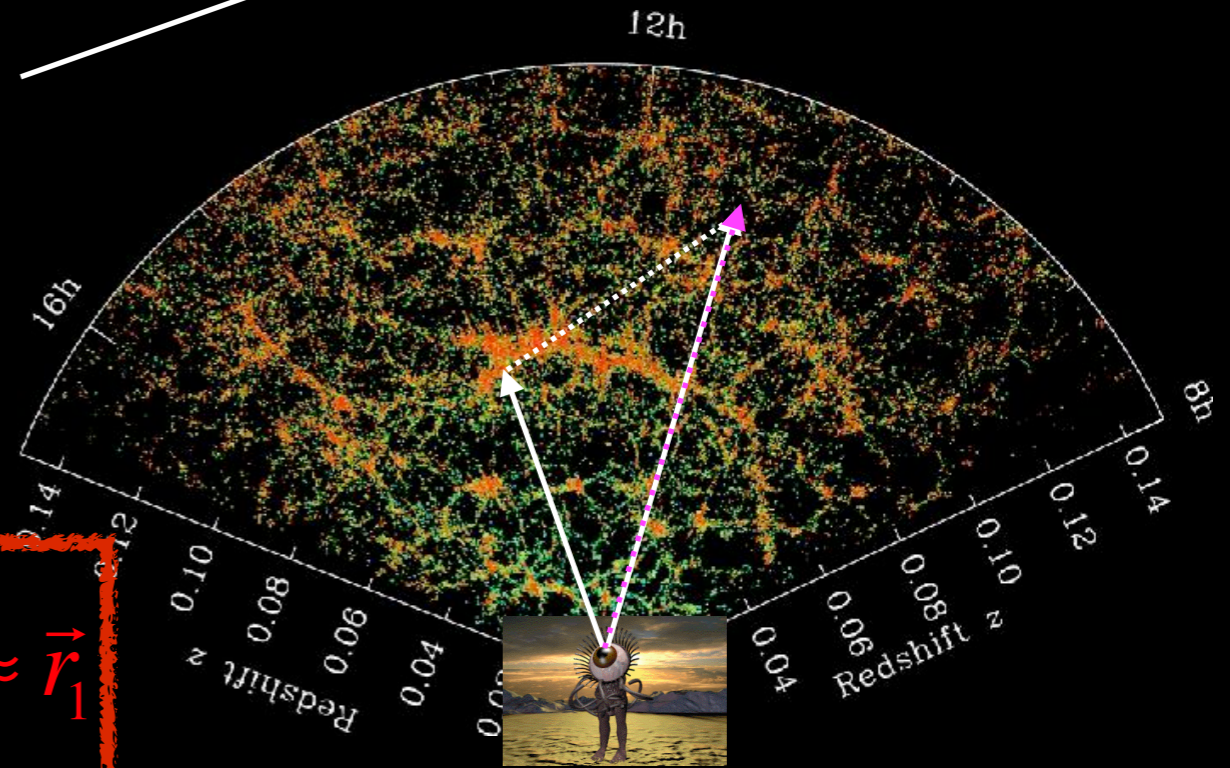
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For the **quadrupole**,

$$\tilde{P}_g^{(2)}(k) = 5 \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_2(\hat{k} \cdot \hat{r}_1) - P_\ell^{noise}(k) \right]$$

Integrals are separable

$$\tilde{P}_g^{(\ell)}(k) = (2\ell + 1) \int \frac{d\Omega_k}{4\pi} \left[\int d^3r_1 \int d^3r_2 F_2(\vec{r}_1) F_2(\vec{r}_2) e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \wp_\ell(\hat{k} \cdot \hat{r}_h) - P_\ell^{noise}(k) \right]$$

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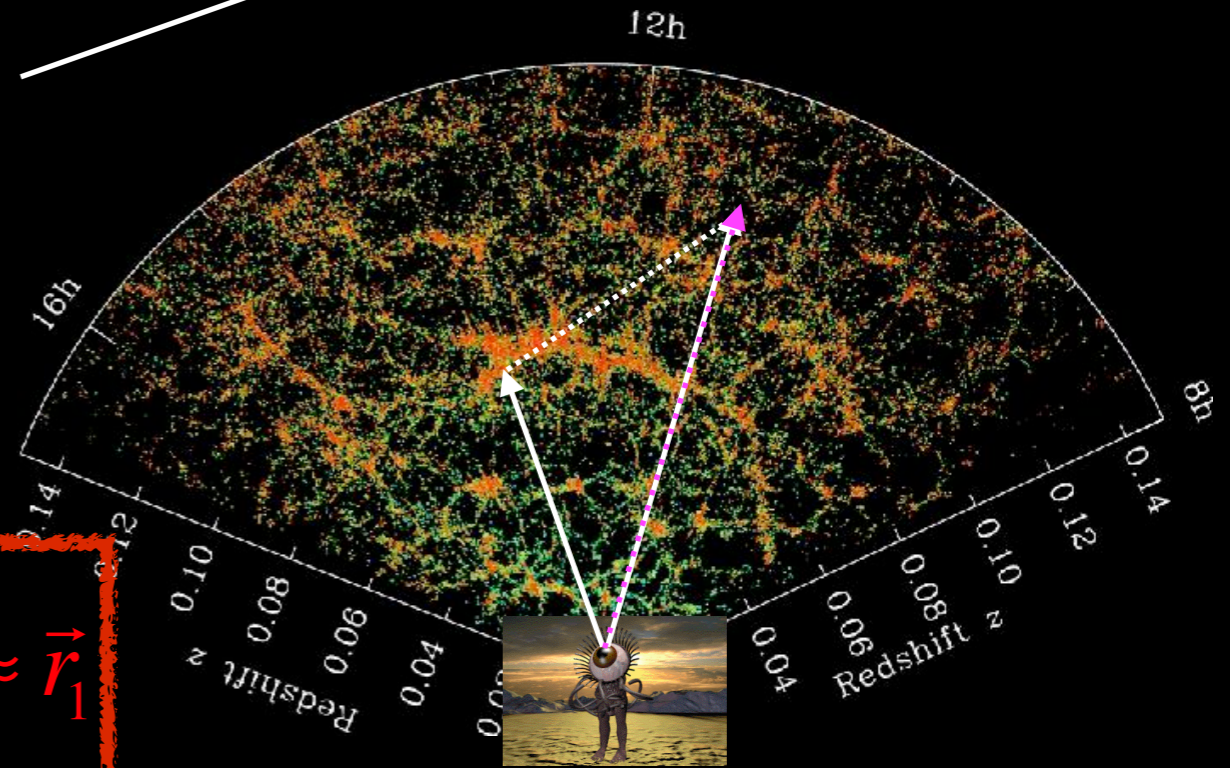
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For the **quadrupole**,

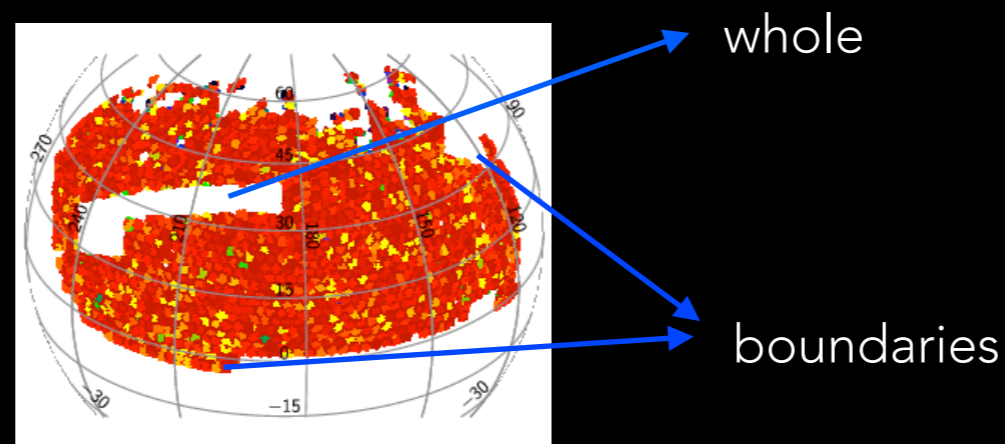
$$\tilde{P}_g^{(2)}(k) = 5 \int \frac{d\Omega_k}{4\pi} \left[\underbrace{\int d^3r_1 F_2(\vec{r}_1) e^{i\vec{k} \cdot \vec{r}_1}}_{\text{FT-like}} \wp_2(\hat{k} \cdot \hat{r}_1) \underbrace{\int d^3r_2 F_2(\vec{r}_2) e^{-i\vec{k} \cdot \vec{r}_2}}_{\text{FT}} - P_0^{noise}(k) \right]$$

Survey selection function

Issue: Estimators previously presented, $\tilde{P}_g^{(\ell)}(k)$ do not measure the actual underlying power spectrum multipoles.

Reason: Non-uniform distribution of galaxies get imprinted inevitably in $F(k)$ after the FT!

Consequence: The estimators $\tilde{P}_g^{(\ell)}(k)$ measure a convolution between the actual underlying power spectrum multipoles and the survey geometry function



Survey selection function

Generalization to higher order multipoles: Wilson et al. 2016

Hankel Transforms: $\hat{P}^{(\ell)}(k) = 4\pi(-i)^\ell \int r^2 \hat{\xi}^{(\ell)}(r) j_\ell(kr) dr$

spherical Bessel functions

Survey selection function

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spherical Bessel functions

Masked monopole and quadrupole

$$\hat{\xi}^{(0)}(r) = \xi^{(0)}(r)W_0^2(r) + \frac{1}{5}\xi^{(2)}(r)W_2^2(r) + \frac{1}{9}\xi^{(4)}(r)W_4^2(r) + \dots$$

$$\hat{\xi}^{(2)}(r) = \xi^{(0)}(r)W_2^2(r) + \xi^{(2)}(r)\left[W_0^2(r) + \frac{2}{7}W_2^2(r) + \frac{2}{7}W_4^2(r)\right] + \dots$$

Survey selection function

Generalization to higher order multipoles: Wilson et al. 2016

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HT of the masked PS

HT of the true PS

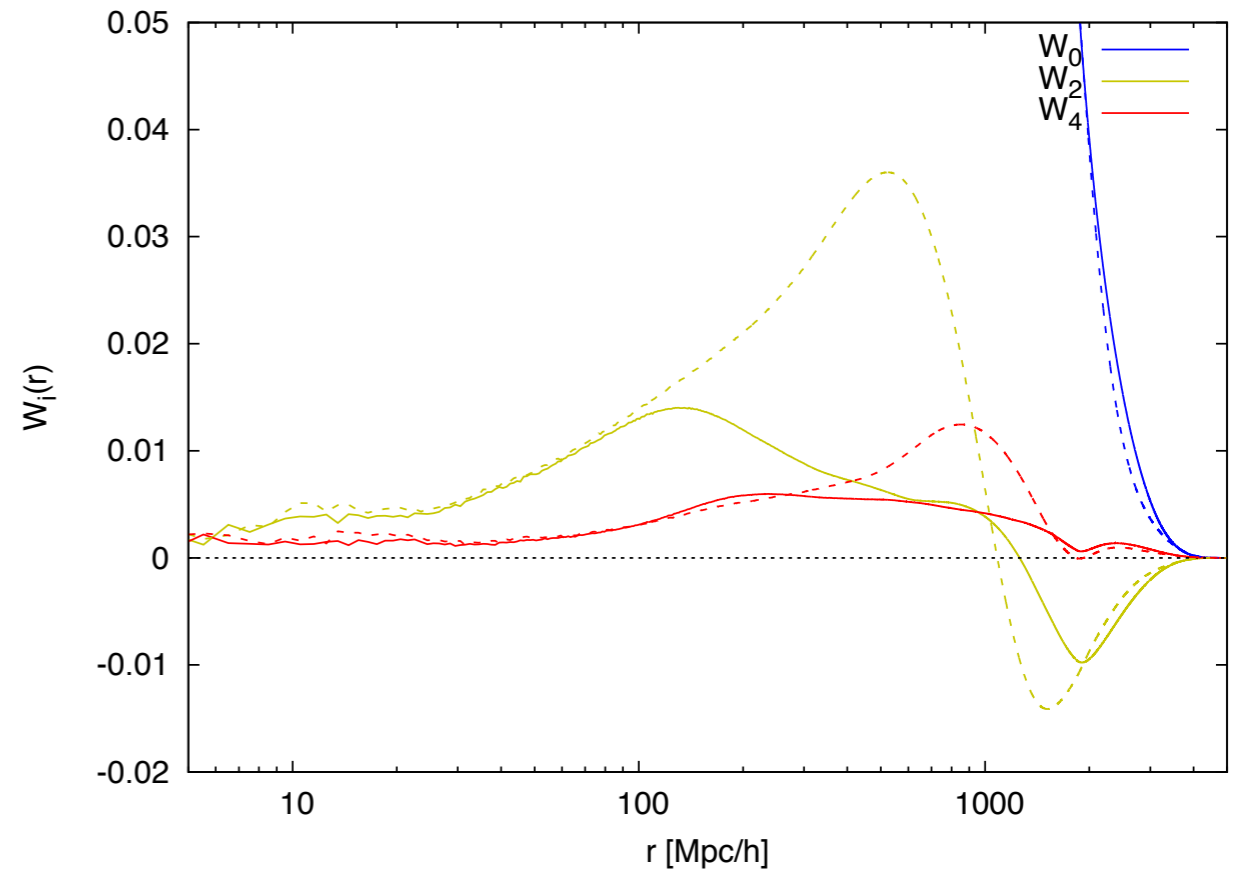
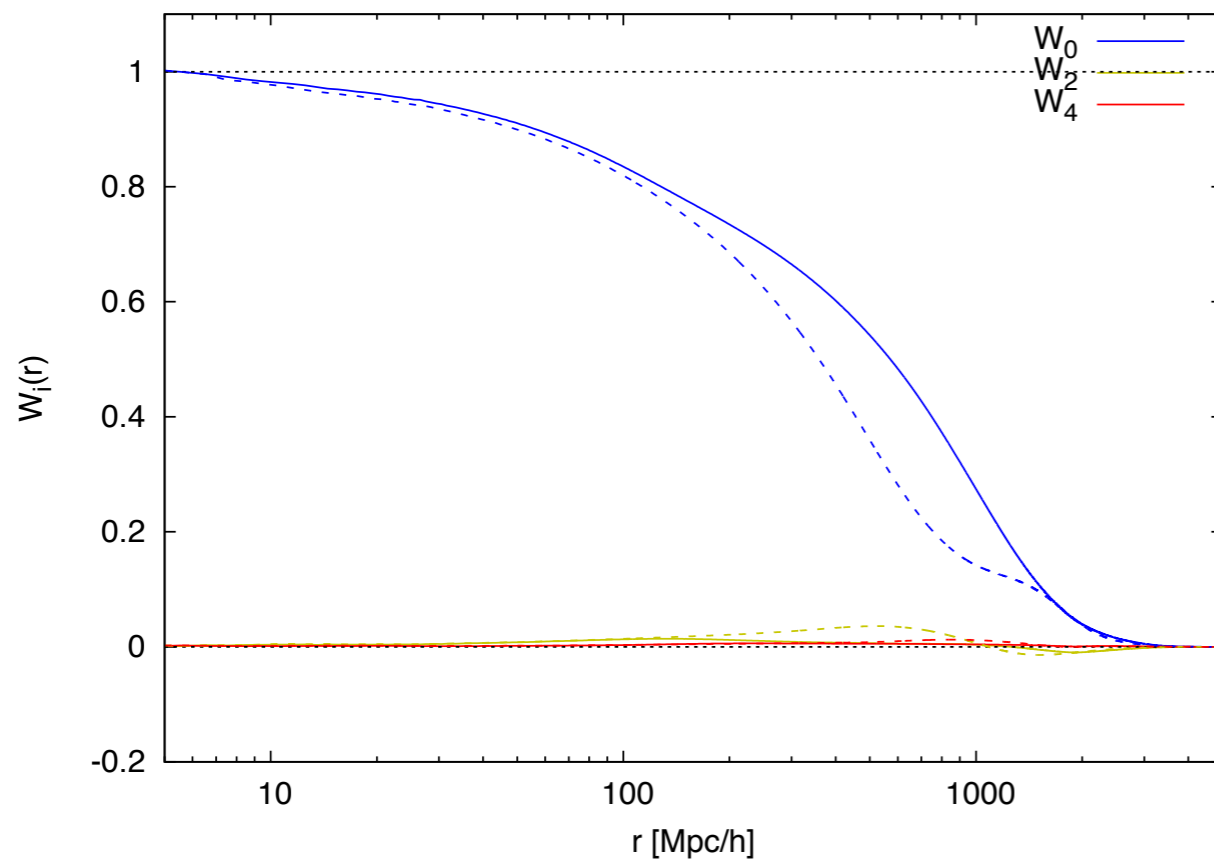
$$\hat{\xi}^{(0)}(r) = \xi^{(0)}(r) W_0^2(r) + \frac{1}{5} \xi^{(2)}(r) W_2^2(r) + \frac{1}{9} \xi^{(4)}(r) W_4^2(r) + \dots$$

$$\hat{\xi}^{(2)}(r) = \xi^{(0)}(r) W_2^2(r) + \xi^{(2)}(r) \left[W_0^2(r) + \frac{2}{7} W_2^2(r) + \frac{2}{7} W_4^2(r) \right] + \dots$$

Window Function $\begin{cases} W_0(r) \rightarrow 1 \\ W_2(r) \rightarrow 0 \\ W_4(r) \rightarrow 0 \end{cases}$ when no selection effect

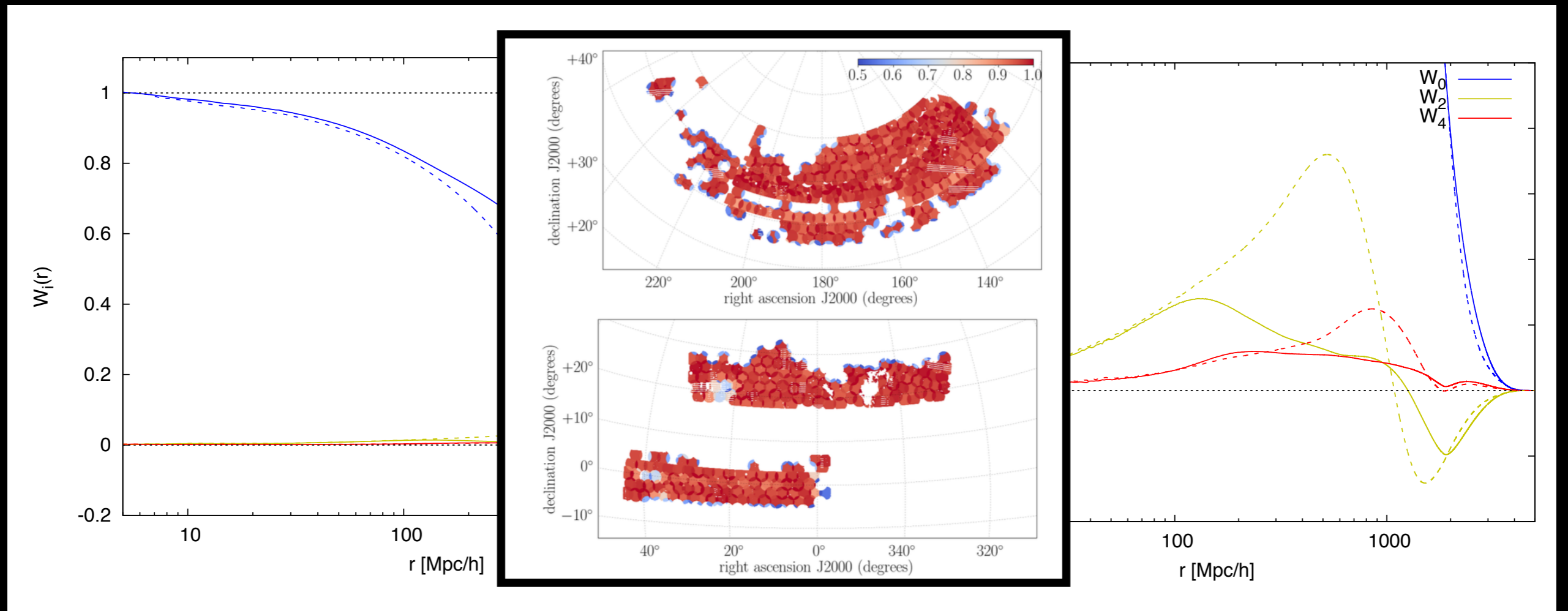
Survey selection function

Generalization to higher order multipoles: Wilson et al. 2016



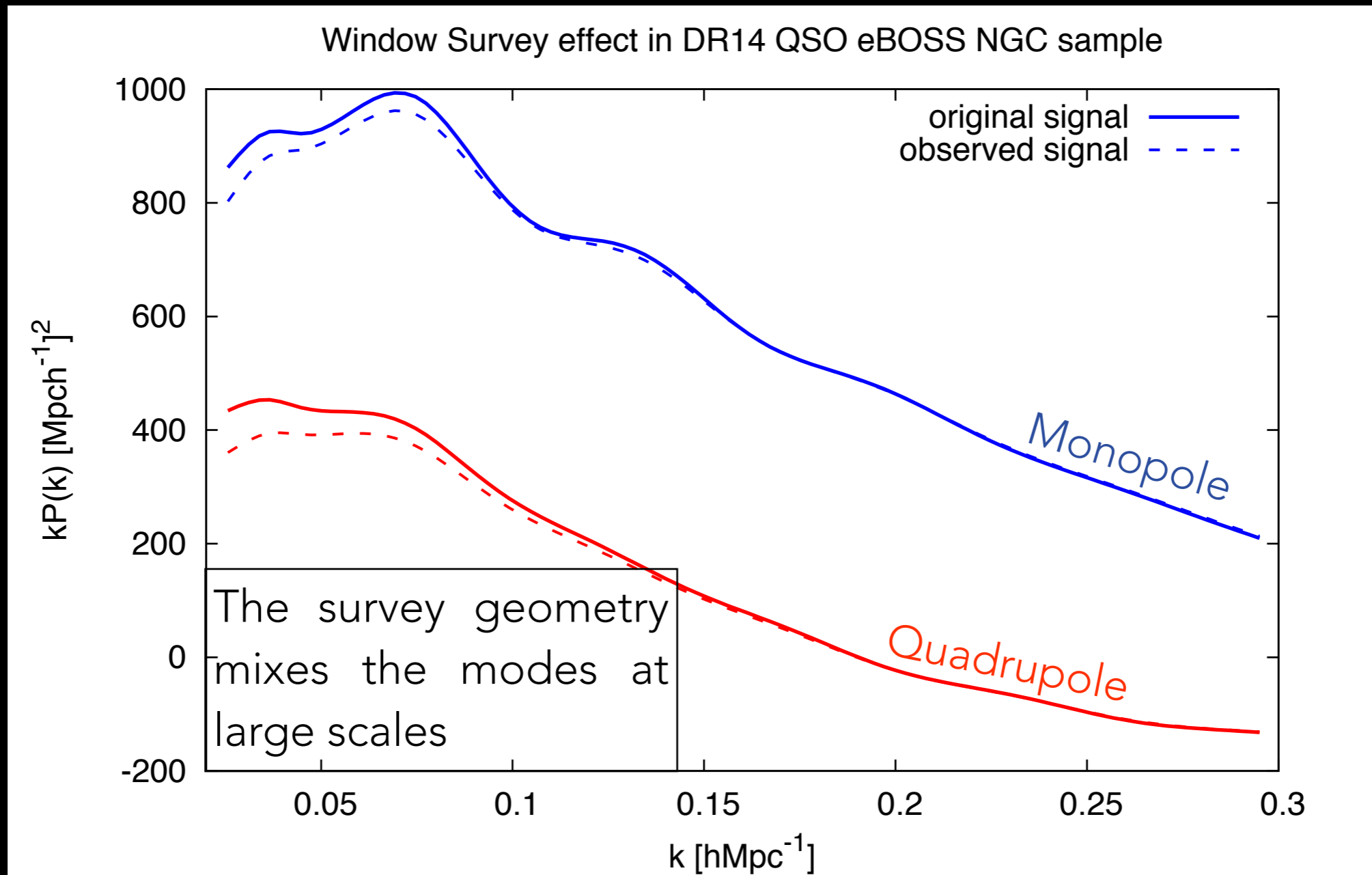
Survey selection function

Generalization to higher order multipoles: Wilson et al. 2016



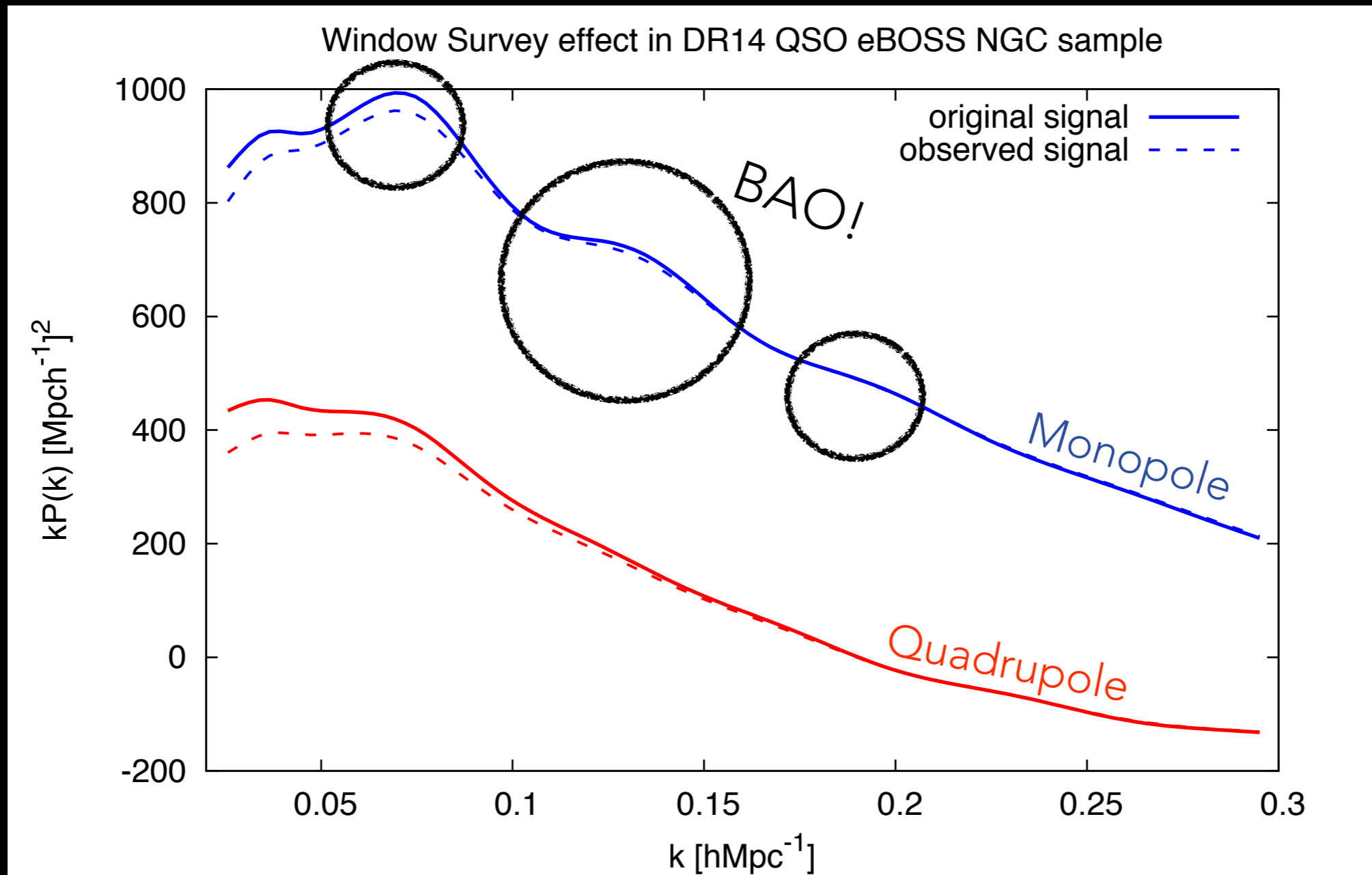
Survey selection function

Importance of the window function for BAO



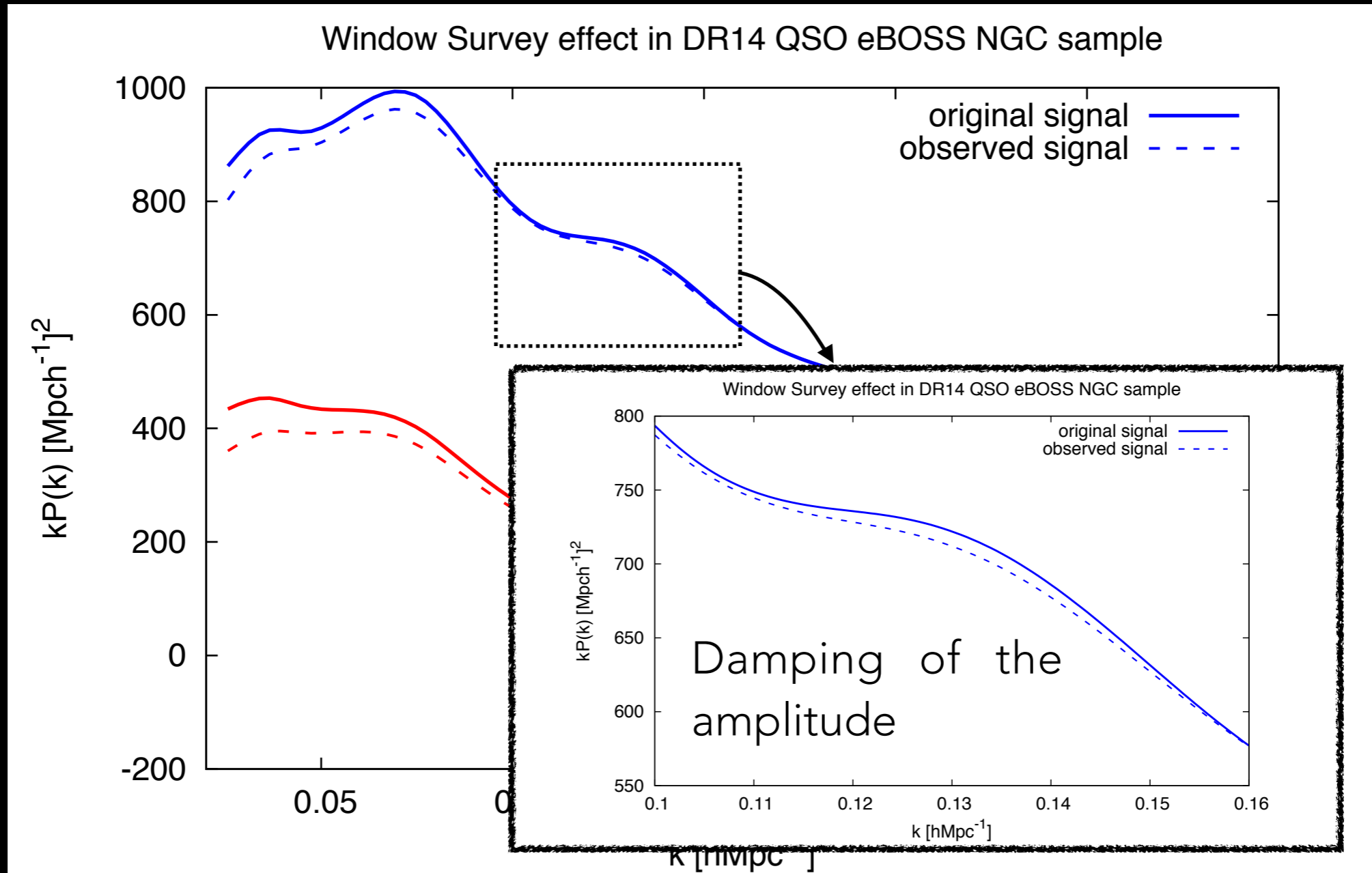
Survey selection function

Importance of the window function for BAO



Survey selection function

Importance of the window function for BAO



Survey selection function

Importance of the window function for RSD

$$P_g(k, \mu; z) = [b_1(z) + f(z)\mu^2]^2 P_{lin}(k, z_0) [\sigma_8^2(z) / \sigma_8^2(z_0)] + \dots$$

$$P_g^{(0)}(k; z) = \frac{P_{lin}(k; z_0)}{\sigma_8^2(z_0)} \sigma_8^2(z) \left[b_1(z)^2 + \frac{2}{3} f(z)b_1(z) + \frac{1}{5} f^2(z) \right] + \dots$$

$$P_g^{(2)}(k; z) = \frac{P_{lin}(k; z_0)}{\sigma_8^2(z_0)} \sigma_8^2(z) \left[\frac{4}{3} f(z)b_1(z) + \frac{4}{7} f^2(z) \right] + \dots$$

2 Eq. & 2 free param.

$$b_1(z)\sigma_8(z)$$

$$f(z)\sigma_8(z)$$

Survey selection function

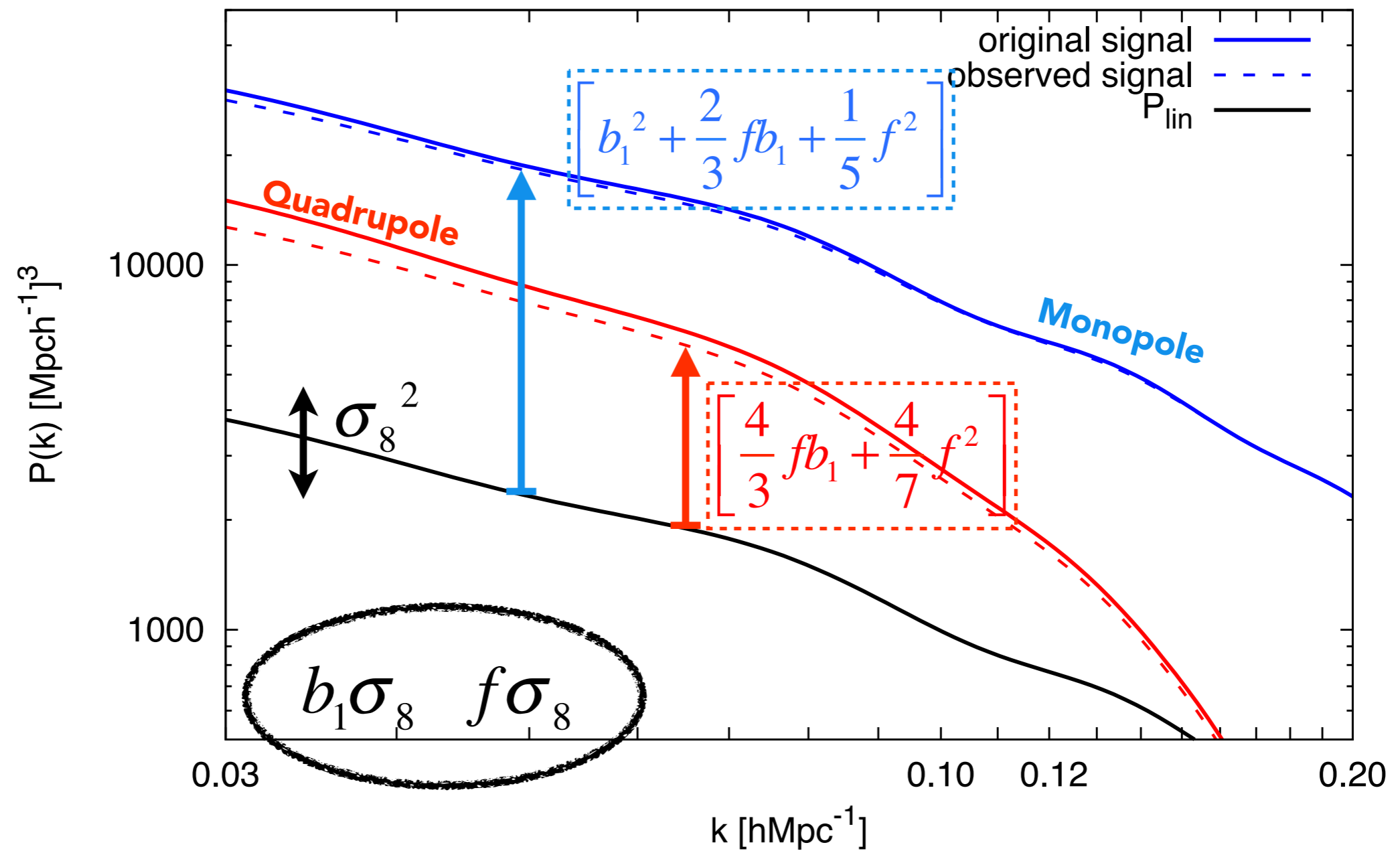
Import

$$P_g(k, \mu)$$

$$P_g^{(0)}(k; \dots)$$

$$P_g^{(2)}(k; \dots)$$

Window Survey effect in DR14 QSO eBOSS NGC sample



Bispectrum

- Quantity which is essentially non-linear

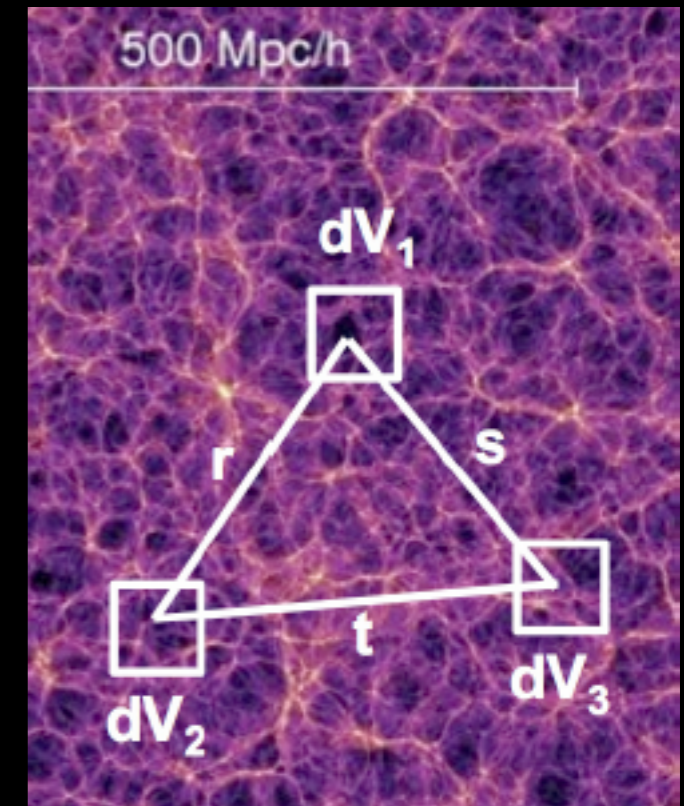
If all the $\delta(\mathbf{k})$ modes evolve linearly (and the initial conditions are Gaussian) the bispectrum is 0 and all the information on the system of objects/galaxies is described by the power spectrum.

Probability of finding 3 galaxies separated by r , s and t : $P_3(r, s, t) = [1 + \xi_2(r) + \xi_2(s) + \xi_2(t) + \zeta(r, s, t)] dV_1 dV_2 dV_3$
 The bispectrum is defined as the FT of ζ ,

$$B(\mathbf{k}_1, \mathbf{k}_2) \equiv \int dr ds \zeta(\mathbf{r}, \mathbf{s}) e^{-i\mathbf{r} \cdot \mathbf{k}_1} e^{-i\mathbf{s} \cdot \mathbf{k}_2}$$

Since, $\zeta(r, s, t) \equiv \langle \delta(\mathbf{x} + \mathbf{r}) \delta(\mathbf{x} + \mathbf{t}) \delta(\mathbf{x}) \rangle_x$

$$B(k_1, k_2, k_3) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle \delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$



Bispectrum

- Quantity which is essentially non-linear

If all the $\delta(\mathbf{k})$ modes evolve linearly (and the initial conditions are Gaussian) the bispectrum is 0 and all the information on the system of objects/galaxies is described by the power spectrum.

$$\delta(k) = \delta^{(1)}(k) + \delta^{(2)}(k) + \dots$$

Perturbative expansion
(see. M. Crocce Talk)

$$\delta^{(n)}(\vec{k}) = \int \tilde{F}_n(\vec{q}_1, \dots, \vec{q}_n) \delta^{(1)}(\vec{q}_1) \dots \delta^{(1)}(\vec{q}_n) d^3 q_1 \dots d^3 q_n$$

$$\vec{k} = \vec{q}_1 + \dots + \vec{q}_n$$

$\tilde{F}_n(\vec{q}_1, \dots, \vec{q}_n)$ non-symmetrised kernel of n-order

To be computed from recursive relations

Bispectrum

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$$\langle \delta(k)\delta(k') \rangle = \langle \delta^{(1)}(k)\delta^{(1)}(k') \rangle + 2\langle \delta^{(1)}(k)\delta^{(2)}(k') \rangle + \langle \delta^{(2)}(k)\delta^{(2)}(k') \rangle + 2\langle \delta^{(1)}(k)\delta^{(3)}(k') \rangle + \dots$$

Bispectrum

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linear term

$\langle \delta(k)^2 \rangle$

prim. nG term

$\langle \delta(k)^3 \rangle$

1-loop terms

$\langle \delta(k)^4 \rangle$

Bispectrum

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$$\langle \delta(k)\delta(k')\delta(k'') \rangle = \langle \delta^{(1)}(k)\delta^{(1)}(k')\delta^{(1)}(k'') \rangle + 2\langle \delta^{(1)}(k)\delta^{(1)}(k')\delta^{(2)}(k'') \rangle + \dots$$

Bispectrum

- Quantity which is essentially non-linear

If all the $\delta(\mathbf{k})$ modes evolve linearly (and the initial conditions are Gaussian) the bispectrum is 0 and all the information on the system of objects/galaxies is described by the power spectrum.

$$\delta(k) = \delta^{(1)}(k) + \delta^{(2)}(k) + \dots$$

Perturbative expansion
(see. M. Crocce Talk)

$$\langle \delta(k)\delta(k')\delta(k'') \rangle = \langle \delta^{(1)}(k)\delta^{(1)}(k')\delta^{(1)}(k'') \rangle + 2\langle \delta^{(1)}(k)\delta^{(1)}(k')\delta^{(2)}(k'') \rangle + \dots$$

primordial nG term
linear term

$\langle \delta(k)^3 \rangle$

tree-level terms

$\langle \delta(k)^4 \rangle$

1-loop terms

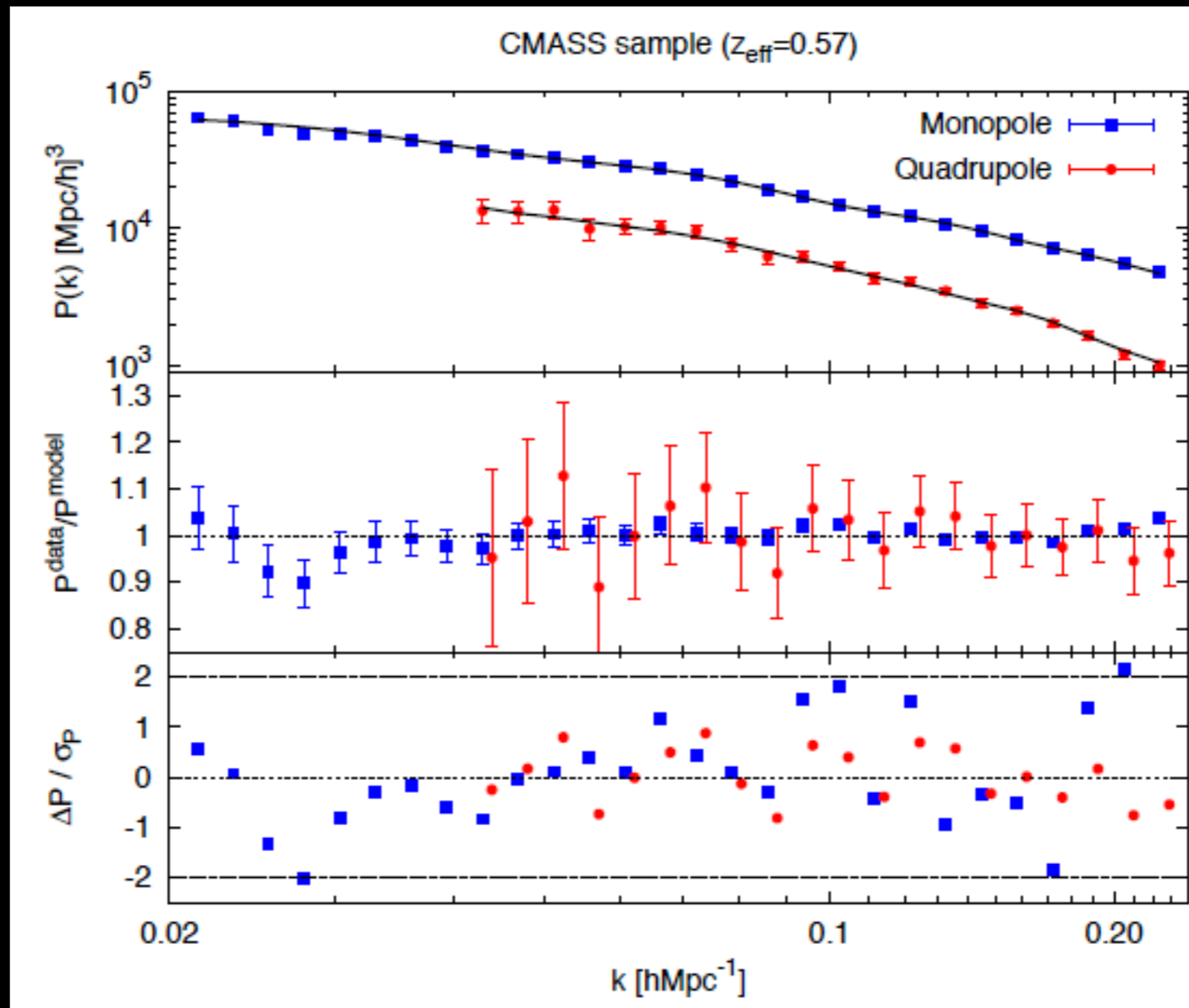
$\langle \delta(k)^6 \rangle$

$\langle 222 \rangle$
 $\langle 114 \rangle$
 $\langle 123 \rangle$

Sefusatti et al. 2010

Bispectrum

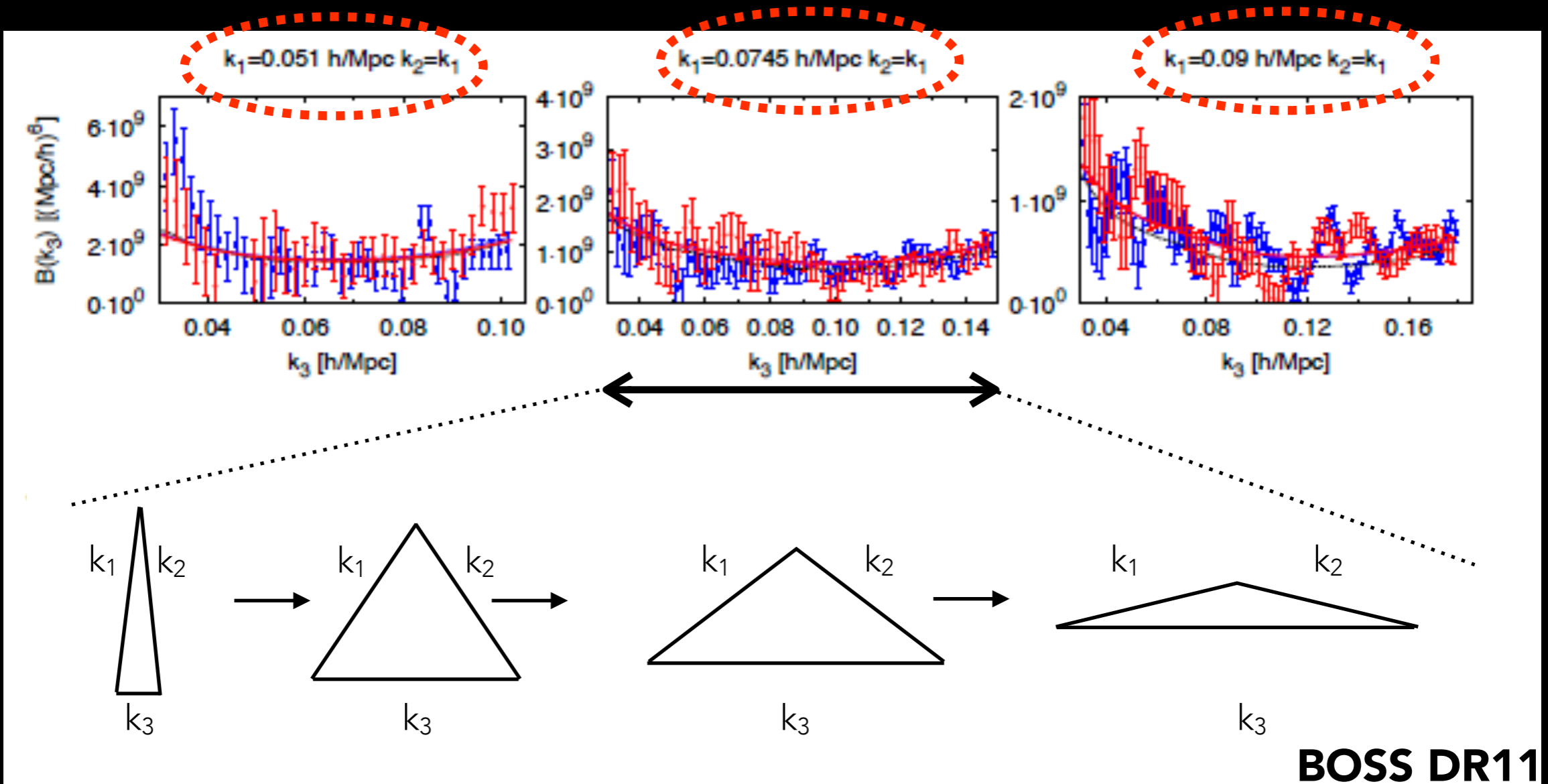
Plotting the Power Spectrum $P(k, \mu) \rightarrow P^{(0)}(k), P^{(2)}(k), \dots$



Bispectrum

Plotting the Bispectrum

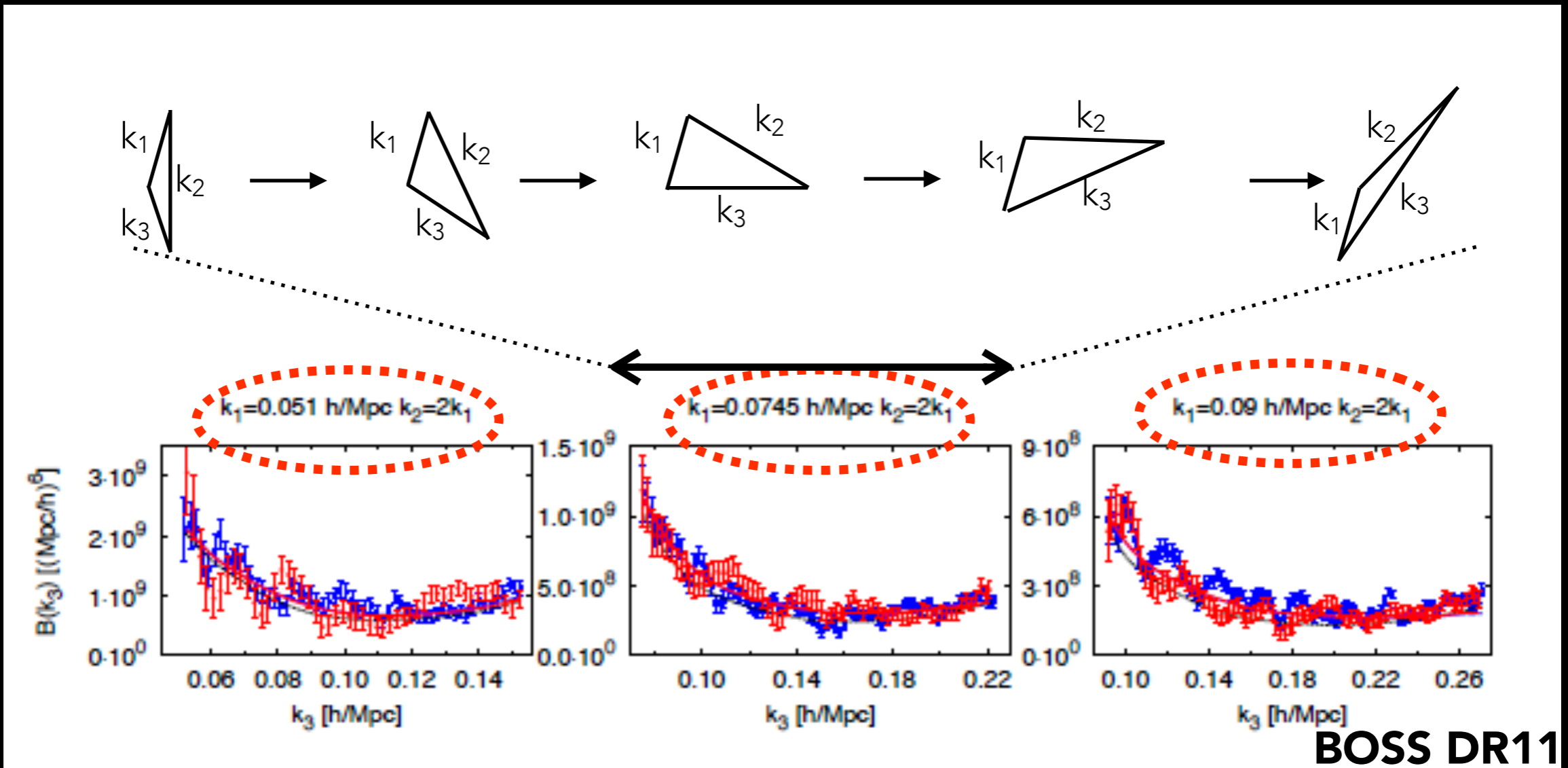
$$B(k_1, k_2, k_3) \rightarrow B(k_3; k_1, k_2)$$



Bispectrum

Plotting the Bispectrum

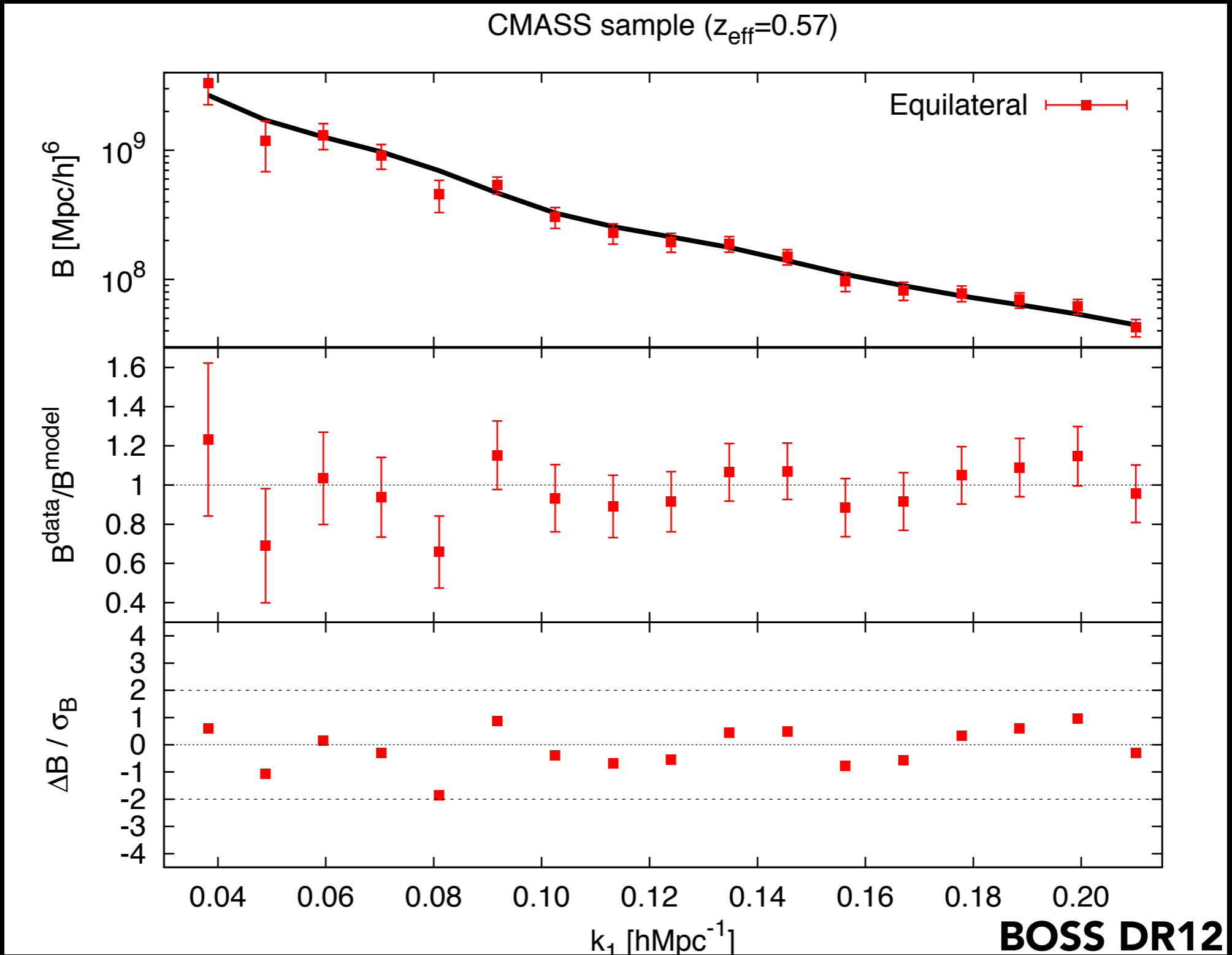
$$B(k_1, k_2, k_3) \rightarrow B(k_3; k_1, k_2)$$



BOSS DR11

Bispectrum

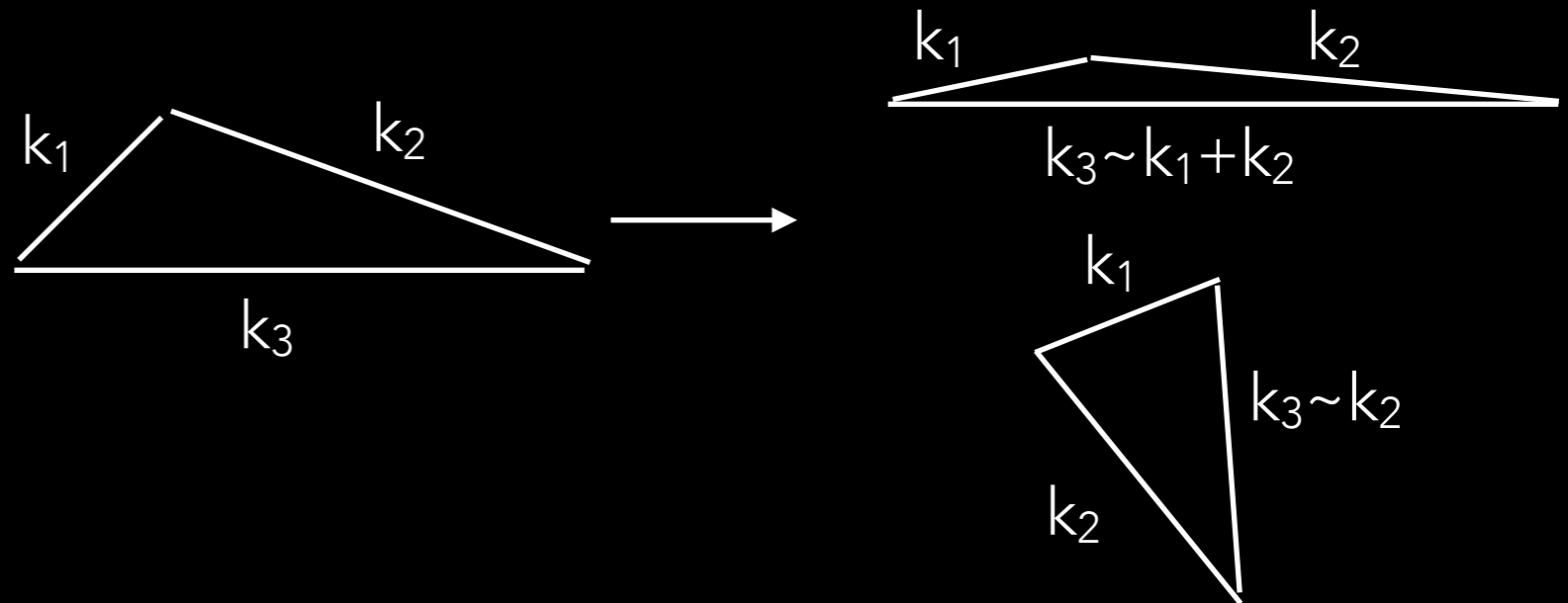
$$B(k_1, k_2, k_3) \rightarrow B^{equi}(k_1)$$



Bispectrum

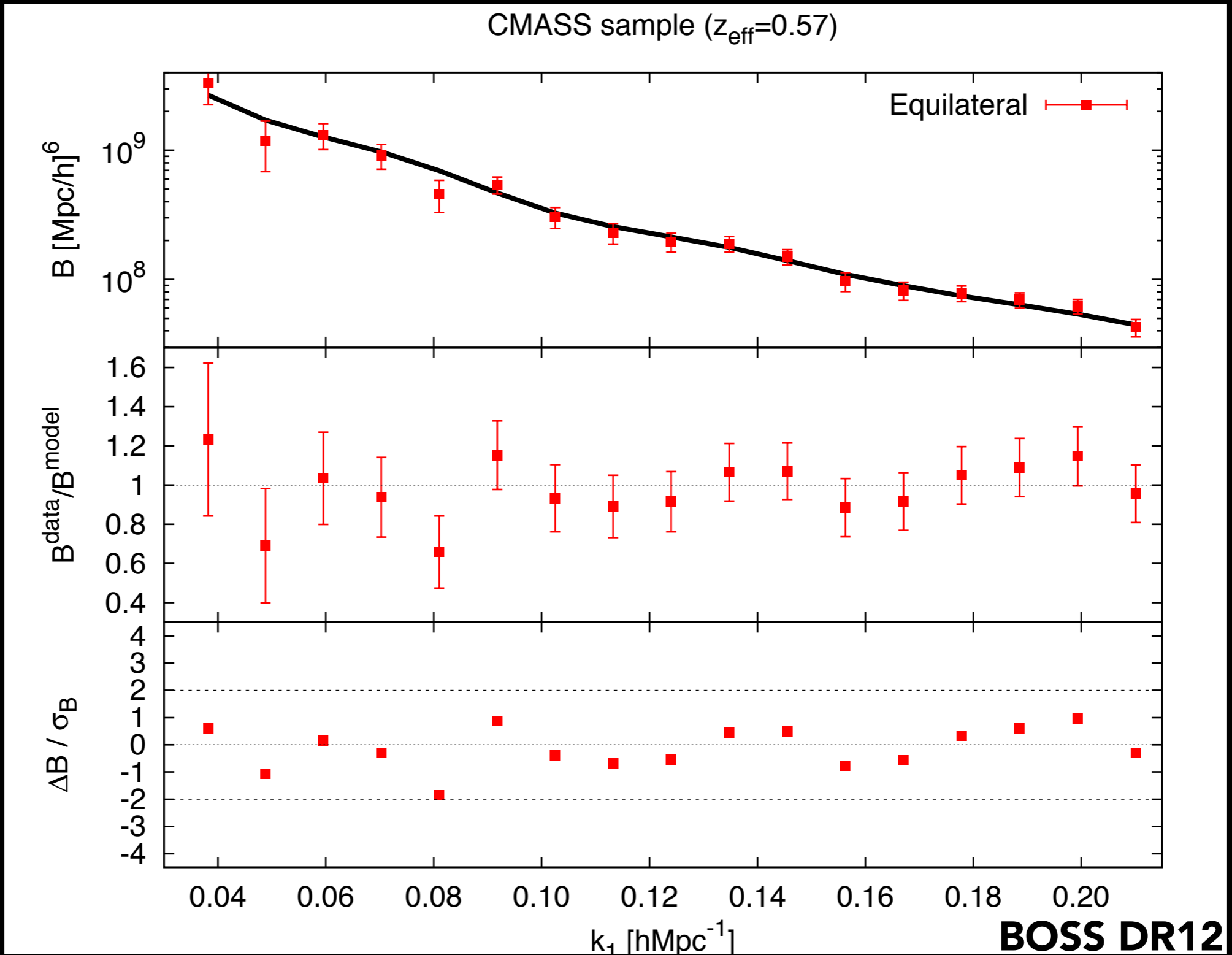
Without loss of generality $k_1 \leq k_2 \leq k_3 \longrightarrow k_1 + k_2 \geq k_3 \geq k_2$

index	k_1	k_2	k_3
1	0.01	0.01	0.01
2	0.01	0.01	0.02
3	0.01	0.02	0.02
4	0.01	0.02	0.03
5	0.01	0.03	0.03
6	0.01	0.03	0.04
...
i	0.02	0.02	0.02
$i+1$	0.02	0.02	0.03
$i+2$	0.02	0.02	0.04
$i+3$	0.02	0.03	0.03
...
n	k_{\max}	k_{\max}	k_{\max}



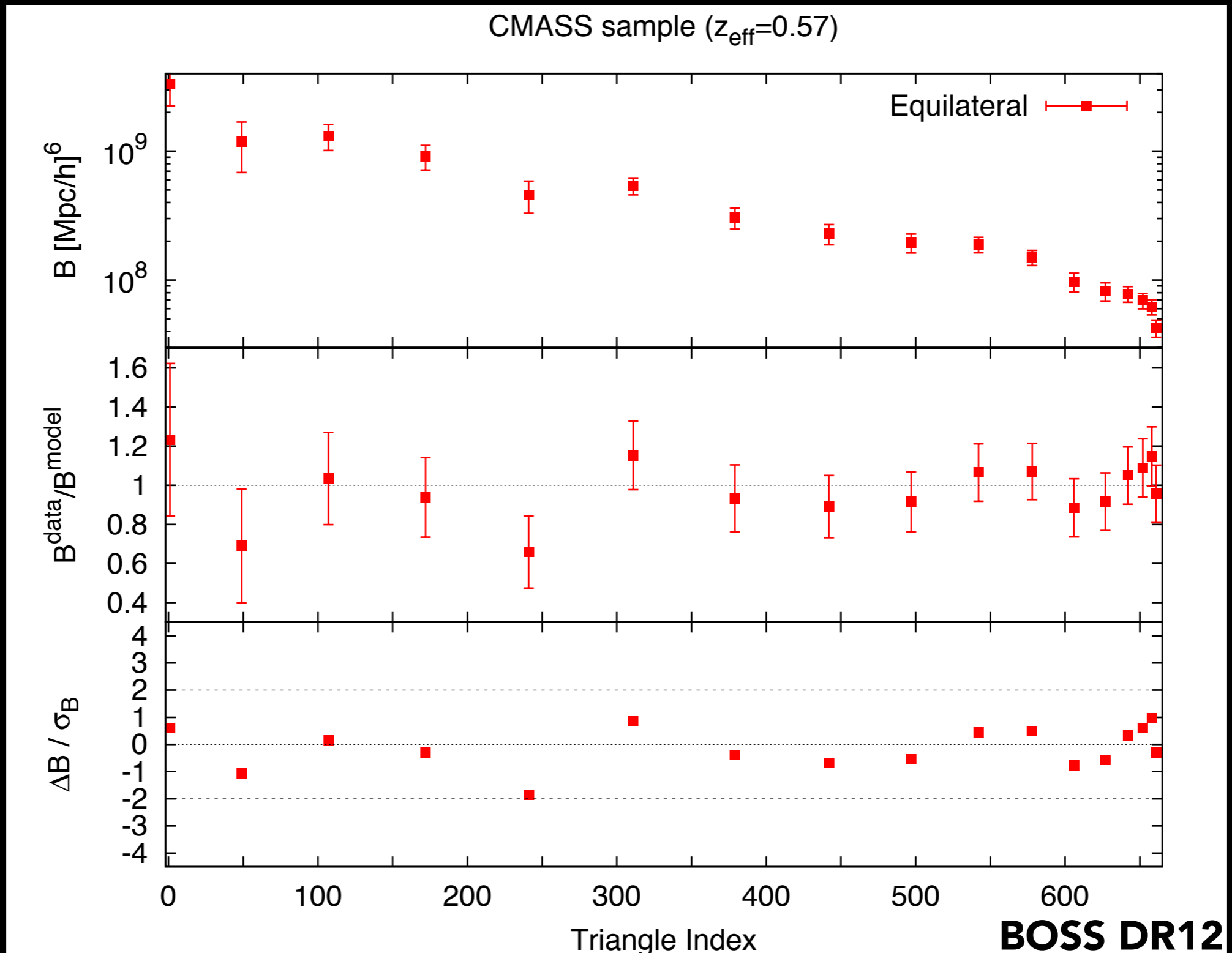
Bispectrum

$$B(k_1, k_2, k_3) \rightarrow B^{equi}(k_1)$$



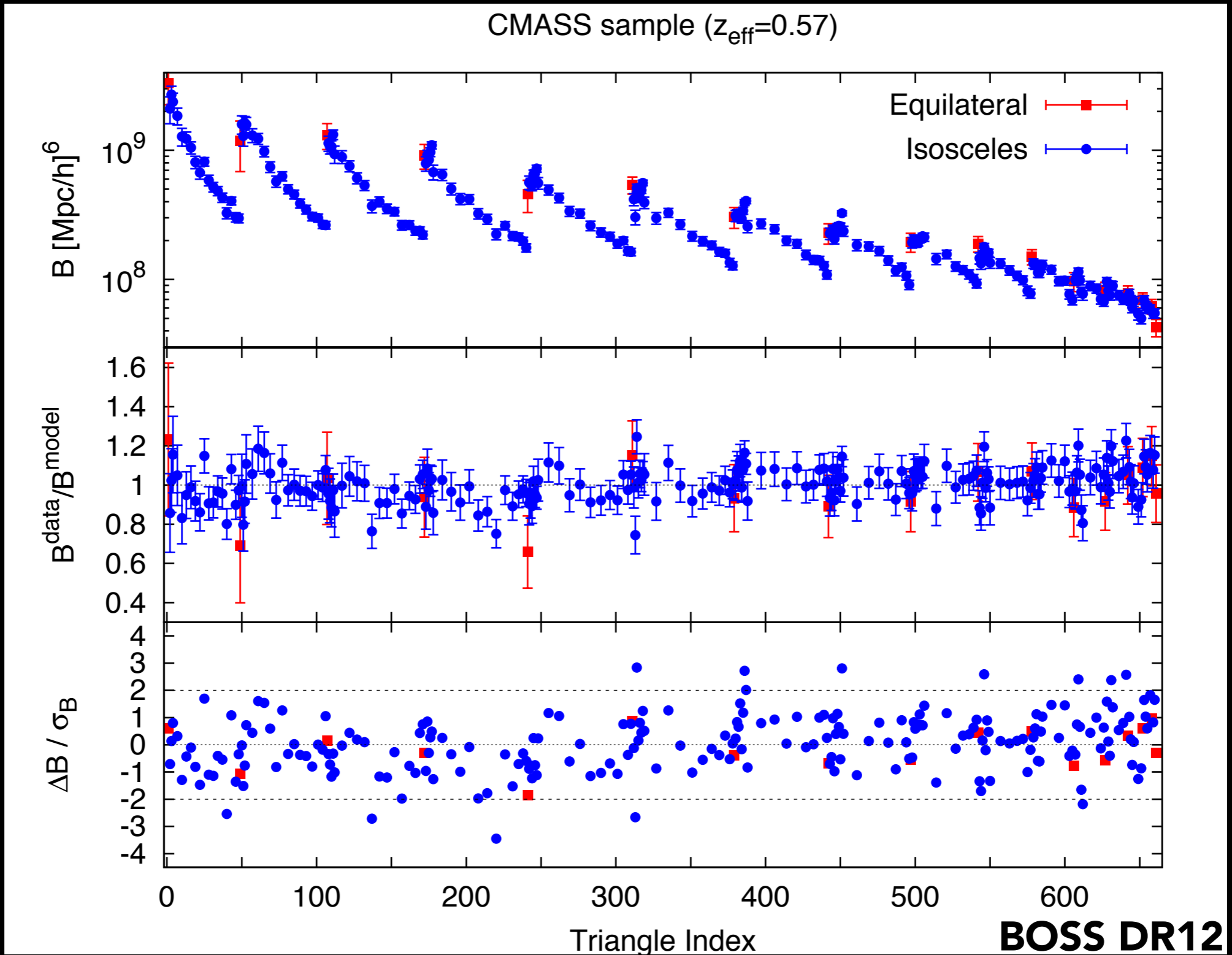
Bispectrum

$$B(k_1, k_2, k_3) \rightarrow B(I)$$



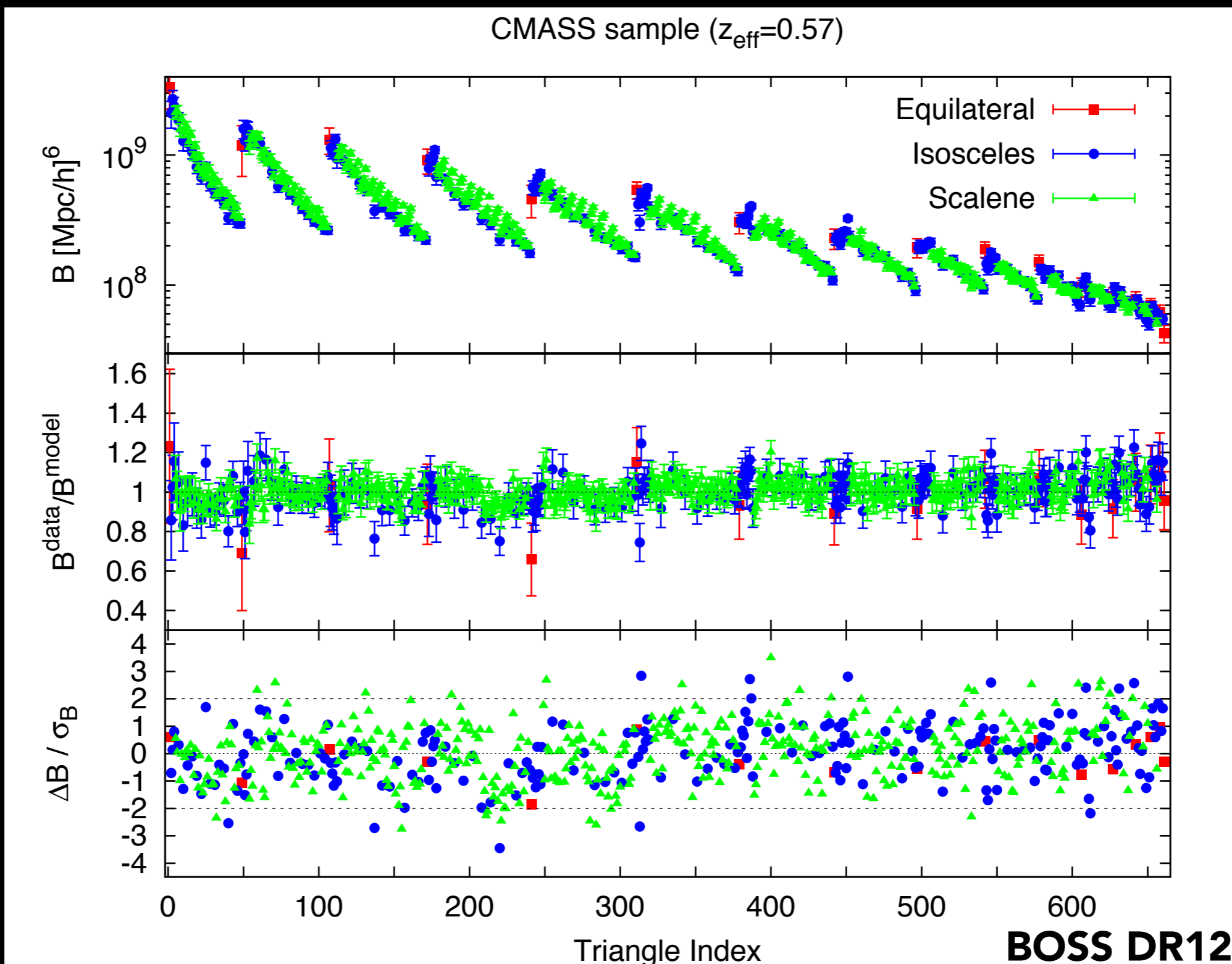
Bispectrum

$$B(k_1, k_2, k_3) \rightarrow B(I)$$



Bispectrum

$$B(k_1, k_2, k_3) \rightarrow B(I)$$



Bispectrum of dark matter

For a Gaussian initial conditions,

$$\langle \delta(k) \delta(k') \delta(k'') \rangle = 2 \langle \delta^{(1)}(k) \delta^{(1)}(k') \delta^{(2)}(k'') \rangle + \dots$$

$$B^{tree}(\vec{k}_1, \vec{k}_2) = 2P_{lin}(k_1)P_{lin}(k_2)\tilde{F}_2^{(s)}(\vec{k}_1, \vec{k}_2) + cyc.$$

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where...

$$F_2^{(s)}(\vec{k}_i, \vec{k}_j) = \frac{5}{7} + \frac{1}{2} \cos(\alpha_{ij}) \left[\frac{k_i}{k_j} + \frac{k_j}{k_i} \right] + \frac{2}{7} \cos^2(\alpha_{ij})$$

→ derived for a EdS

Weak dependence on Ω , but sensitive to modifications of GR

↓

Leading order term in **bispectrum** is sensitive to GR

Bispectrum of dark matter

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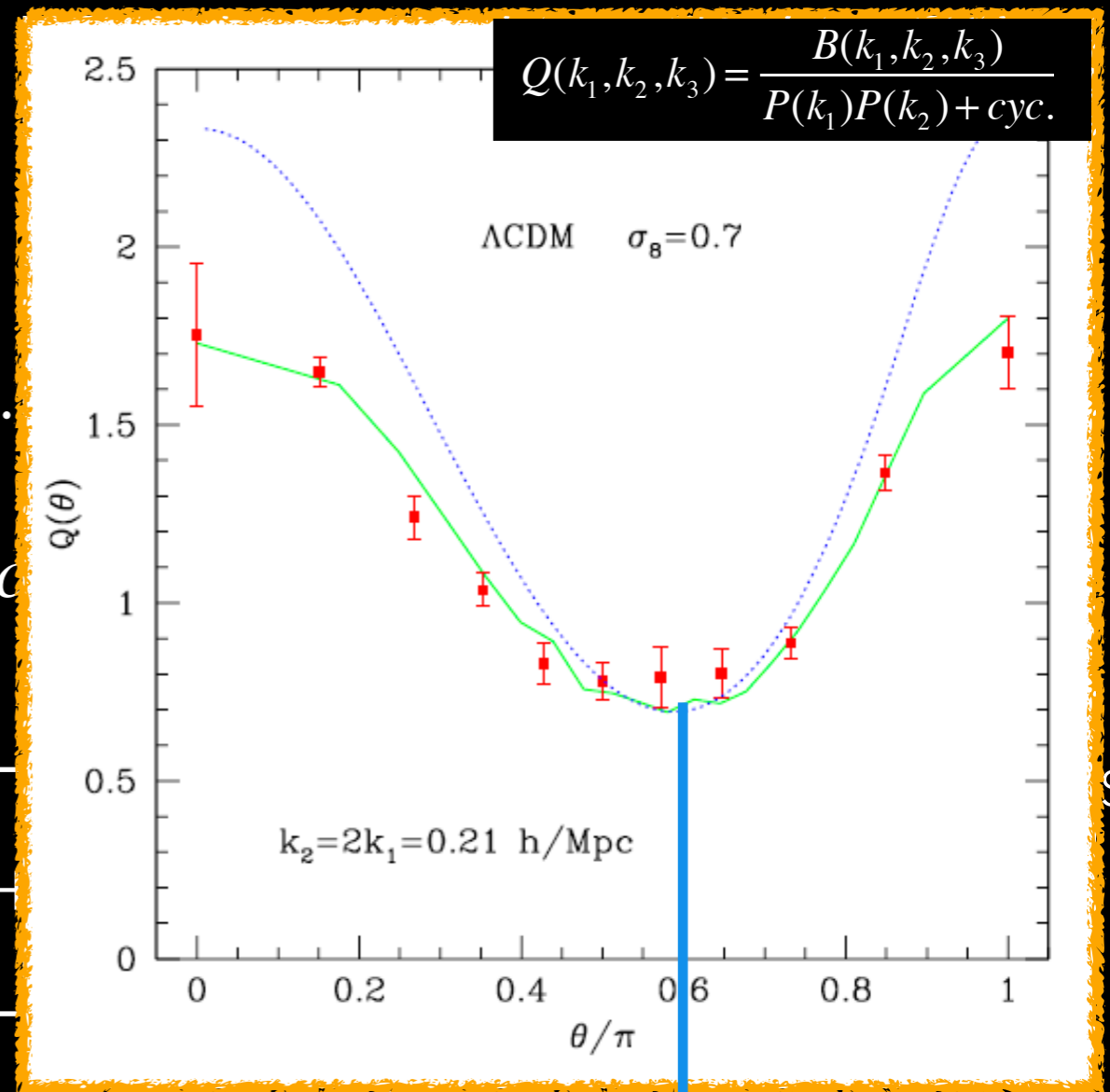
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Weak dependence on Ω , but sensitive to modifications of GR

Characteristic U-shape

Leading order term in **bispectrum** is sensitive to GR



Bispectrum of dark matter

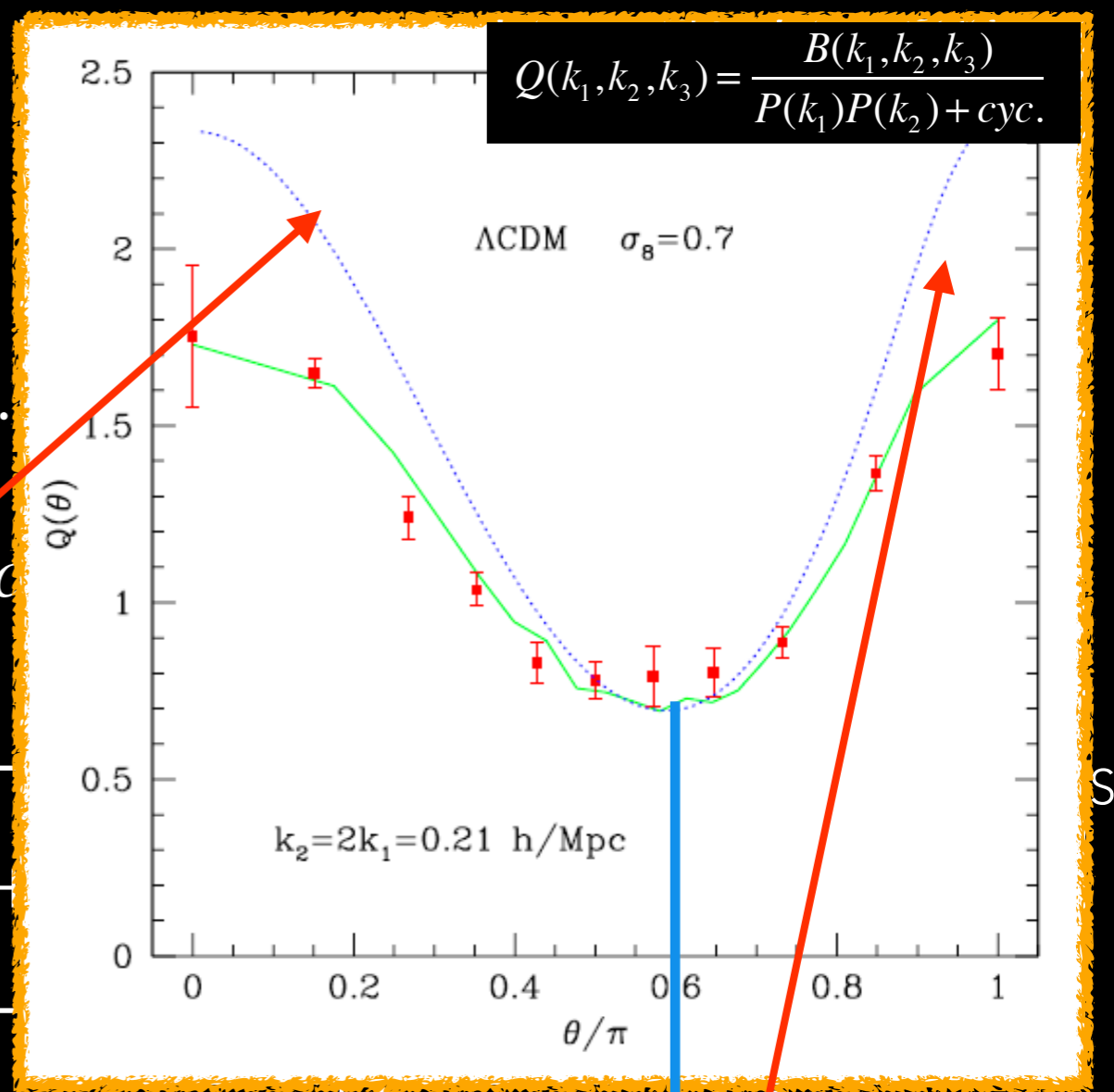
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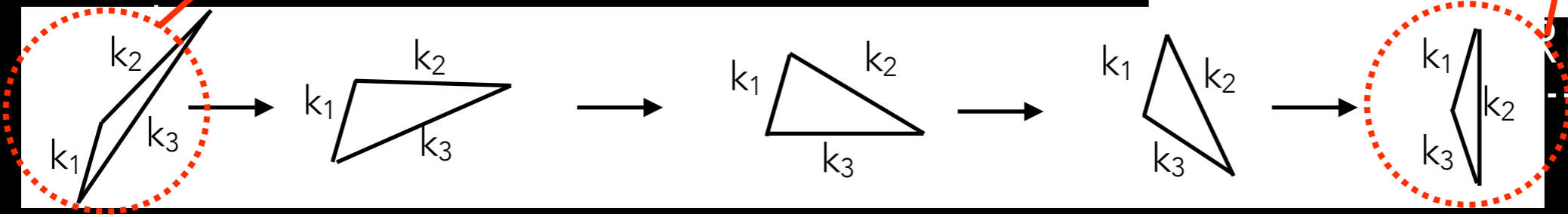
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Weak dependence on Ω , but sensitive to modifications of GR

Characteristic U-shape



Bispectrum of galaxies

Do PT but on $\delta_g(\mathbf{k})$

PT expansion

$$\delta(k) = \delta^{(1)}(k) + \delta^{(2)}(k) + \dots$$

Bias model (McDonald & Roy 2009)

$$\delta_g(\mathbf{x}) = \underbrace{b_1 \delta(\mathbf{x})}_{\text{linear Eulerian}} + \underbrace{\frac{1}{2} b_2 [\delta(\mathbf{x})^2]}_{\text{non-linear Eulerian}} + \frac{1}{2} \left[\underbrace{b_{s2}}_{\frac{4}{7}(1-b_1) \text{ tidal tensor}} \right] \left[\underbrace{s(\mathbf{x})^2}_{\text{non-local Eulerian}} \right]$$

$$B_g(\vec{k}_1, \vec{k}_2) = b_1^4 \sigma_8^4 \left\{ 2P_{lin}(k_1)P_{lin}(k_2) \left[\frac{1}{b_1} F_2^{(s)}(\vec{k}_1, \vec{k}_2) + \frac{b_2}{2b_1^2} + \frac{2}{7b_1^2} (1-b_1) S_2(\vec{k}_1, \vec{k}_2) \right] + cyc. \right\}$$



Shape dependence sensitive to b_2 (non-linear term)
 Shape dependence sensitive to b_{s2} (tidal tensor term)

Bispectrum of galaxies

Pisone et al. (MNRAS, 2000)

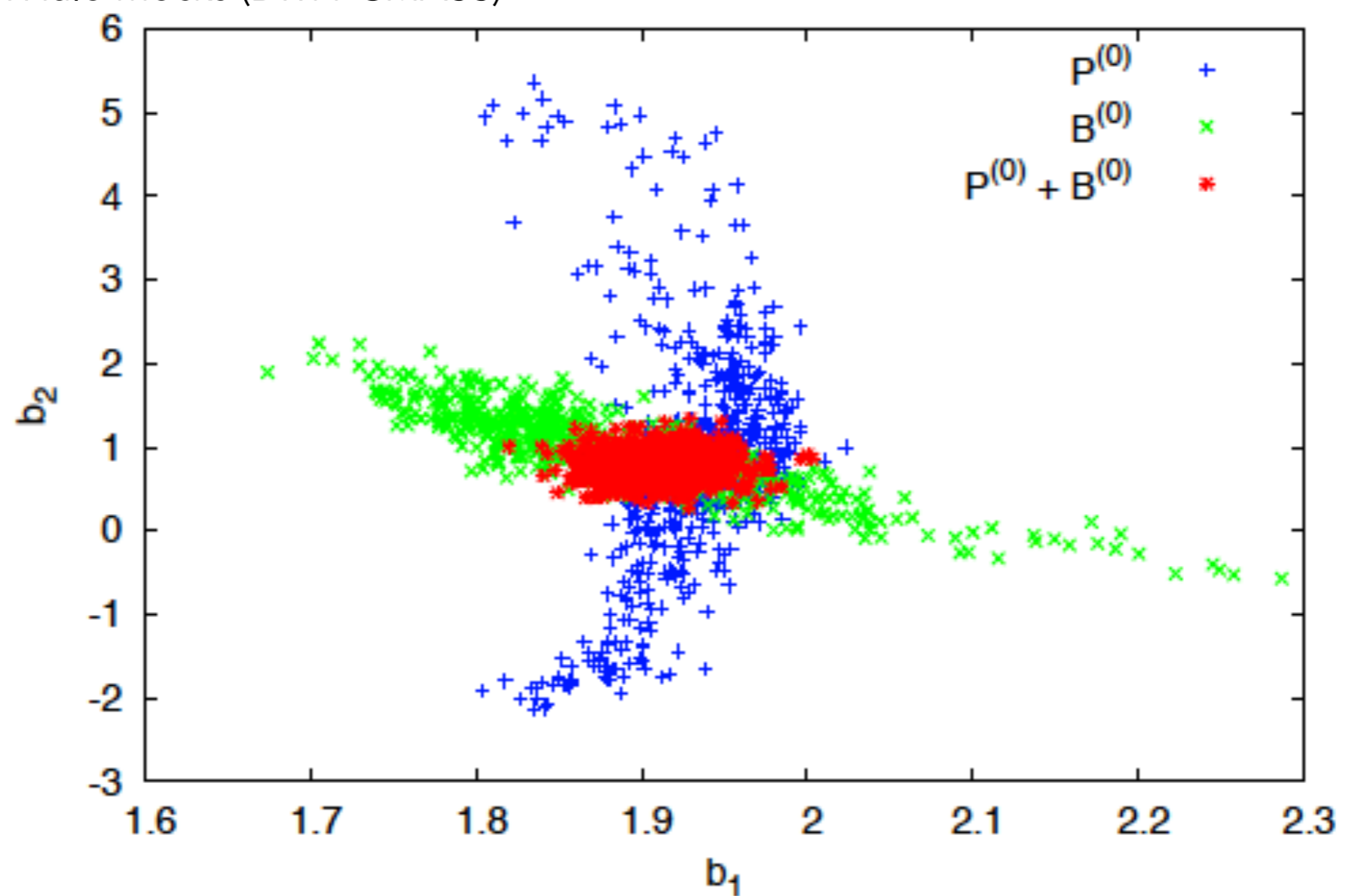
D_0

$\delta(l)$

B_g

Shap
Shap

600 PTHalo mocks (DR11 CMASS)



$\left. \begin{array}{l} \text{radial tensor} \\ s(x)^2 \end{array} \right\}$
 Eulerian

Gil-Marín et al. 2015

erm)
erm)

Bispectrum of galaxies in redshift space

Bispectrum of galaxies in configuration space

$$B_g(\vec{k}_1, \vec{k}_2) = b_1^4 \sigma_8^4 \left\{ 2P_{lin}(k_1)P_{lin}(k_2) \left[\frac{1}{b_1} F_2^{(s)}(\vec{k}_1, \vec{k}_2) + \frac{b_2}{2b_1^2} + \frac{2}{7b_1^2} (1-b_1) S_2(\vec{k}_1, \vec{k}_2) \right] + cyc. \right\}$$

Redshift space Kernels $\begin{cases} F_1 \rightarrow Z_1 \\ F_2 \rightarrow Z_2 \end{cases}$

$Z_1(\vec{k}) = (b_1 + f\mu^2)$ Kaiser like

$$Z_2(\vec{k}_i, \vec{k}_j) = b_1 \left[F_2(\vec{k}_1, \vec{k}_2) + \frac{f\mu k}{2} \left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2} \right) \right] + f\mu^2 G_2(\vec{k}_1, \vec{k}_2) + \frac{f^3 \mu k}{2} \mu_1 \mu_2 \left(\frac{\mu_2}{k_1} + \frac{\mu_1}{k_2} \right) + \frac{b_2}{2} + \frac{2}{7} (1-b_1) S_2(\vec{k}_1, \vec{k}_2)$$

velocity kernel

Bispectrum of galaxies in redshift space

$$B_g^{(s)}(\vec{k}_1, \vec{k}_2) = \sigma_8^4 \left[2P_{lin}(k_1)P_{lin}(k_2)Z_1(\vec{k}_1)Z_1(\vec{k}_2)Z_2^{(s)}(\vec{k}_1, \vec{k}_2) + cyc. \right]$$

Bispectrum of galaxies in redshift space

60 Nbody DM simulations

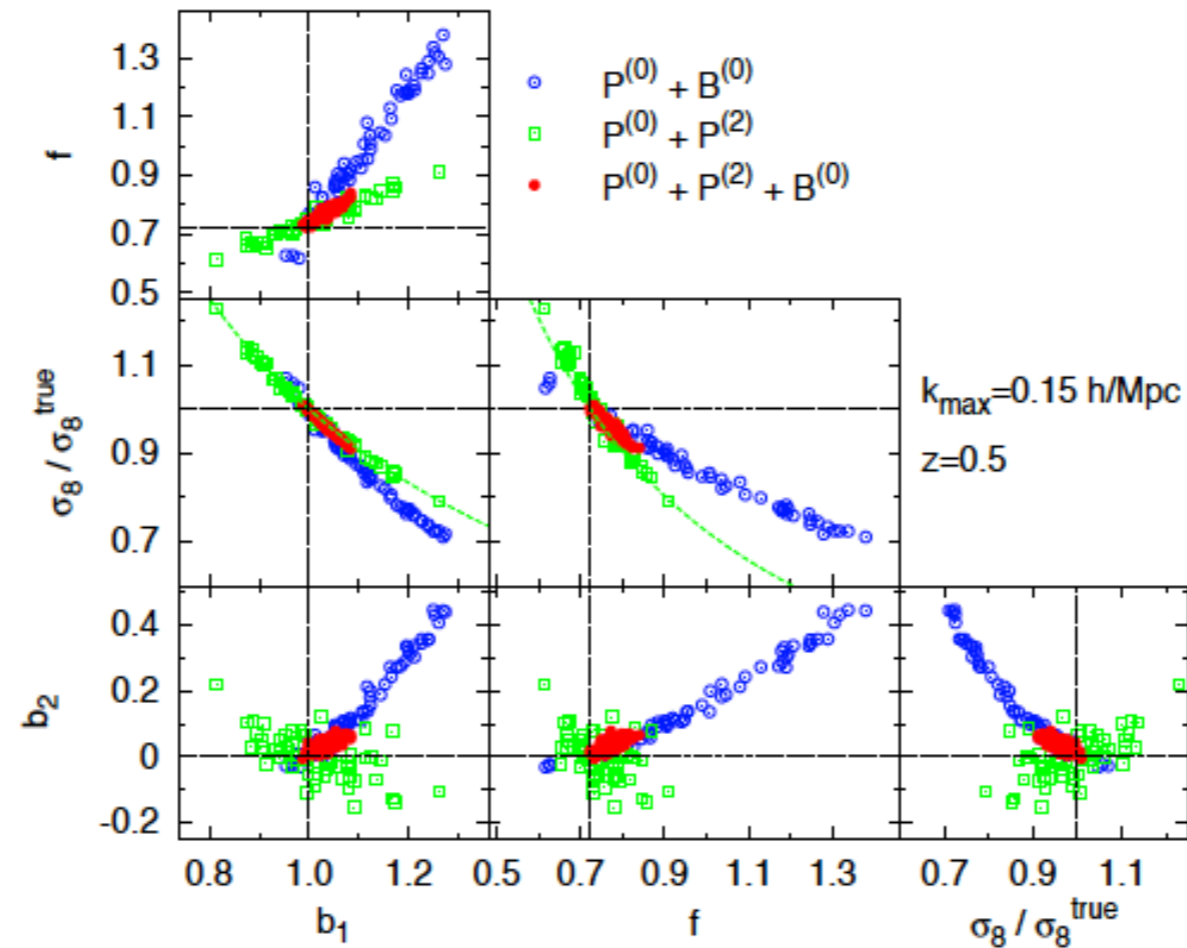


Figure 10. Best-fitting parameters for dark matter simulations in redshift space at $z = 0.5$ for $k_{\text{max}} = 0.15$ when different statistics are used: blue points correspond to $P^{(0)} + B^{(0)}$, green points to $P^{(0)} + P^{(2)}$ and red points to $P^{(0)} + P^{(2)} + B^{(0)}$ as indicated. The dashed black lines mark the true values. The green dashed lines mark the $b_1 \propto \sigma_8^{-1}$ and the $f \propto \sigma_8^{-1}$ relations. Note that $b_1, b_2, f, \sigma_8, \sigma_0^P, \sigma_0^B$ are varied as free parameters, although only b_1, b_2, f and σ_8 are shown for clarity.

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Bispectrum of galaxies in redshift space

60 Nbody DM simulations

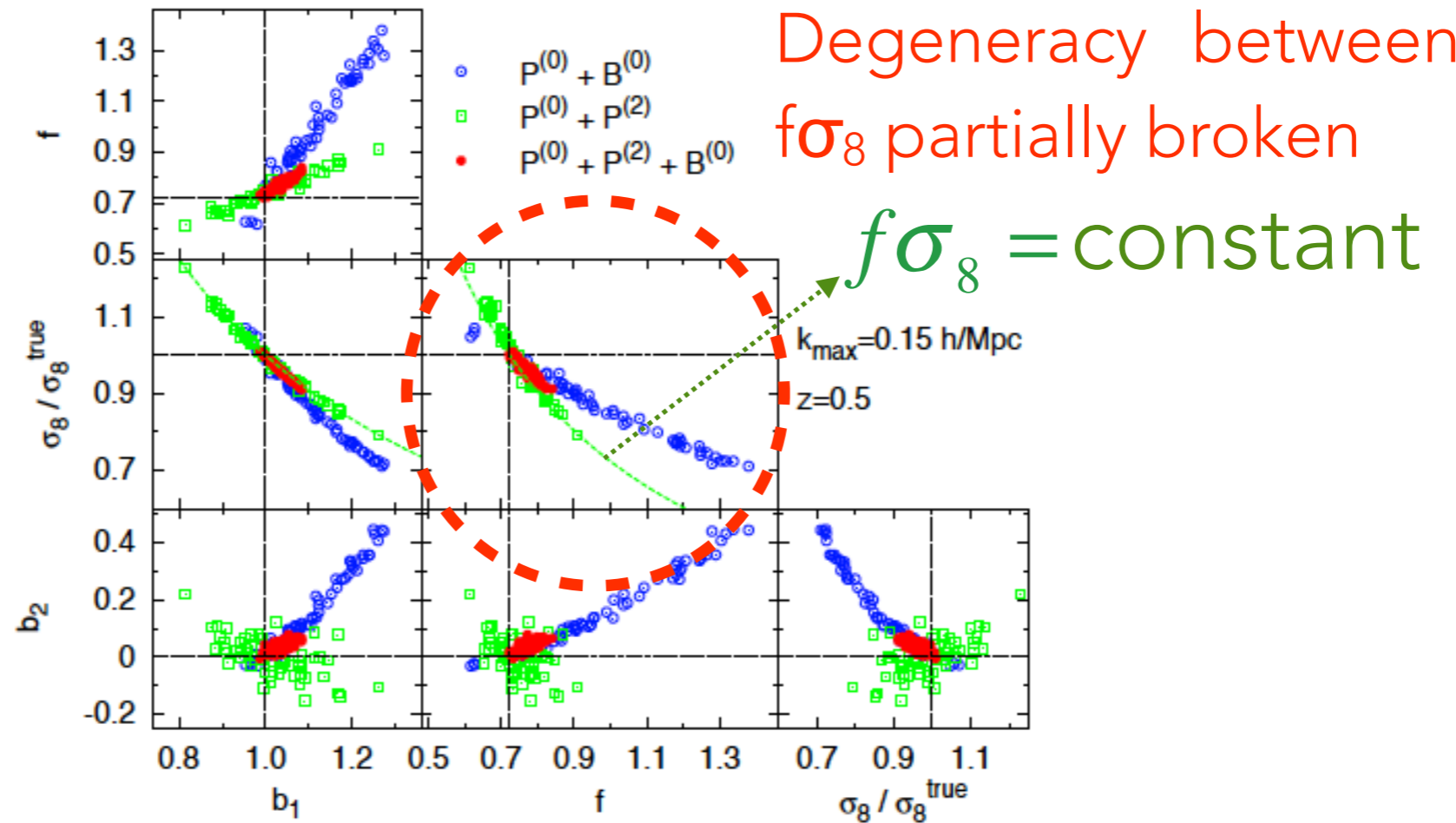
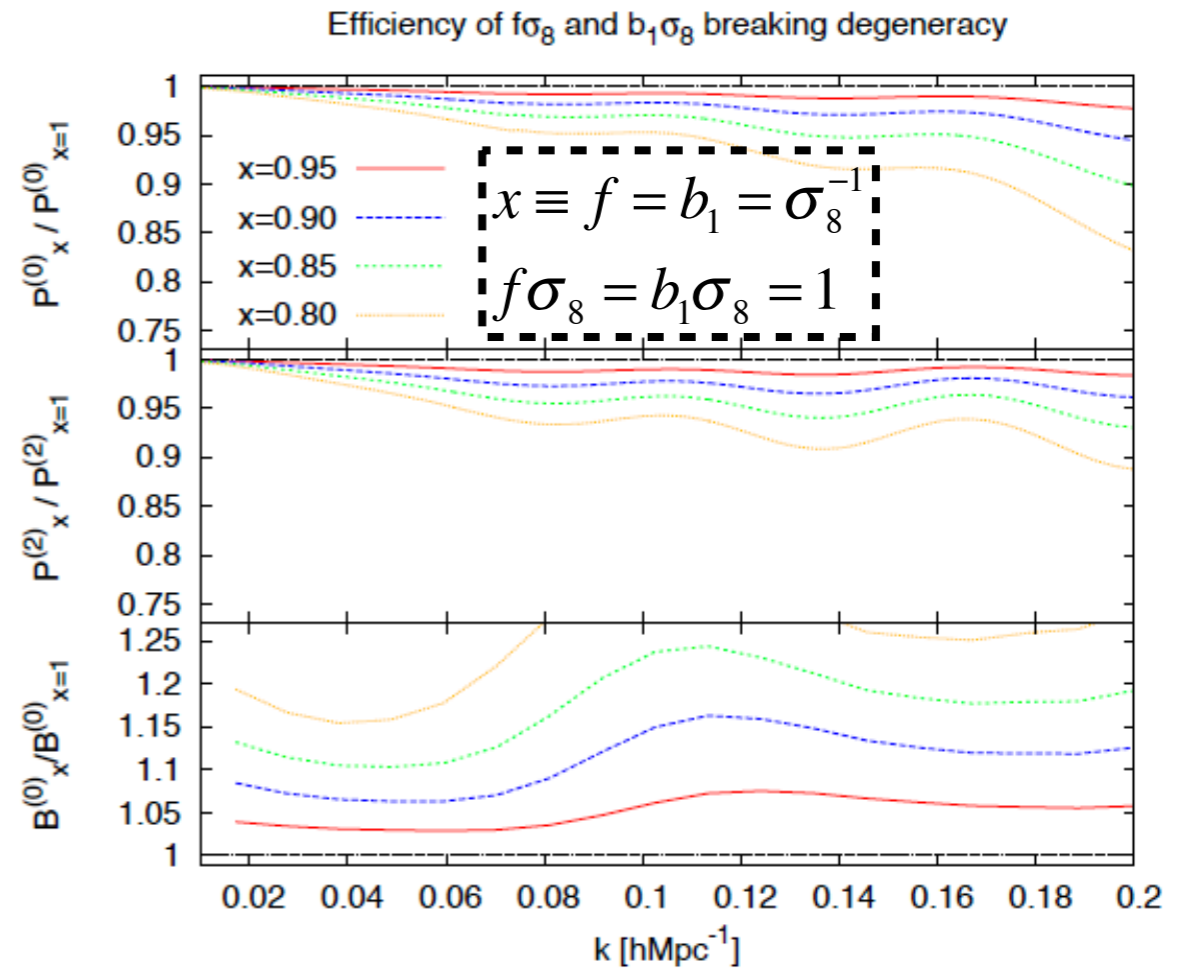
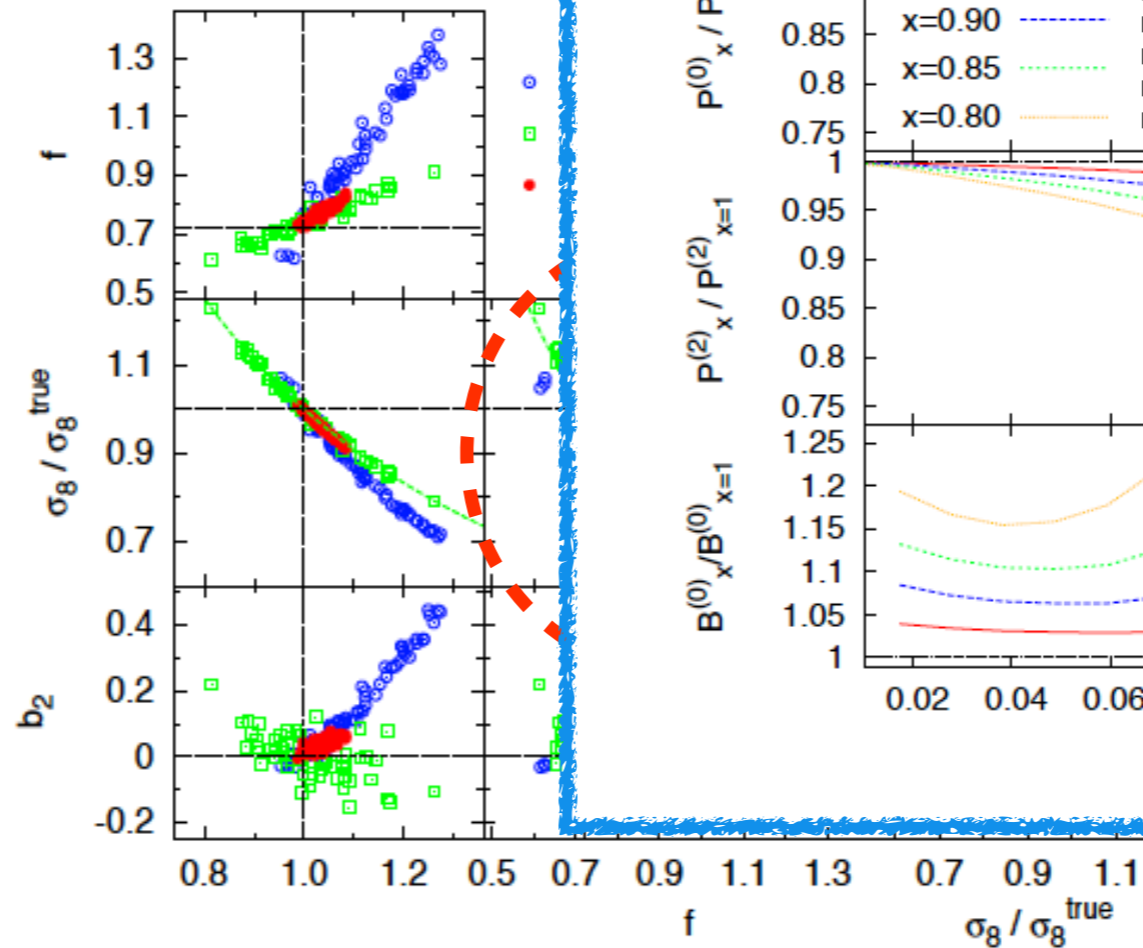


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Bispectrum of galaxies in redshift space

60 Nbody DM simulations



Remember...

$$P_g^{(0)}(k) = P_{lin}(k)\sigma_8^2 \left[b_1^2 + \frac{2}{3}fb_1 + \frac{1}{5}f^2 \right] + \dots$$

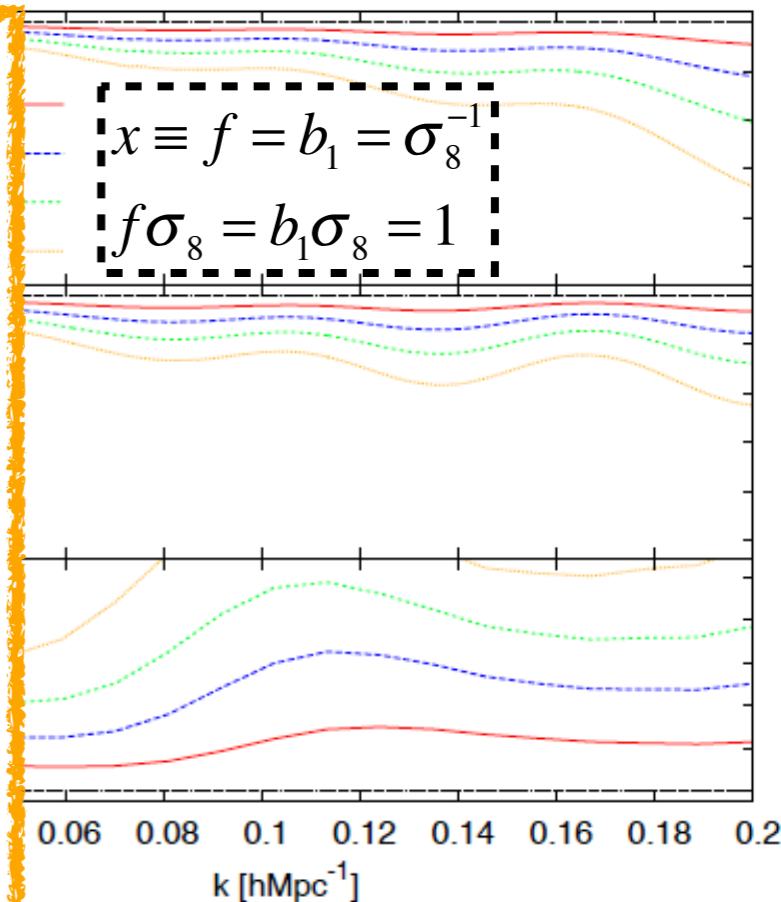
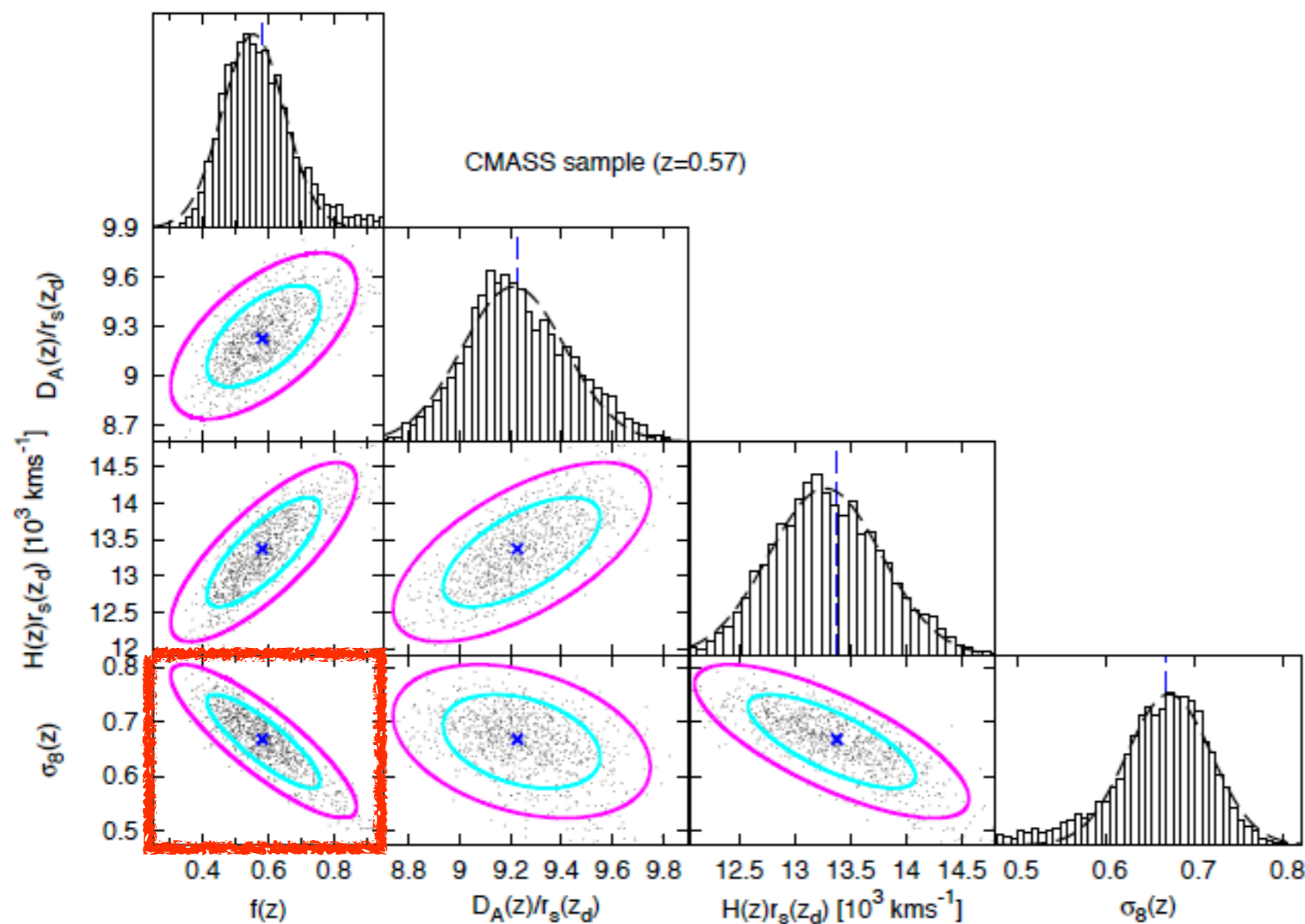
$$P_g^{(2)}(k) = P_{lin}(k)\sigma_8^2 \left[\frac{4}{3}fb_1 + \frac{4}{7}f^2 \right] + \dots$$

60 dark matter simulations in redshift space at $z = 0.5$ for $k_{max} = 0.15$ hMpc⁻¹. Blue points correspond to $P^{(0)} + B^{(0)}$, green points to $P^{(0)} + P^{(2)}$ and red points to $P^{(0)} + P^{(2)} + B^{(0)}$. The dashed black lines mark the true values. The green dashed lines mark the true values for $f\sigma_8$ and $b_1\sigma_8$. Note that $b_1, b_2, f, \sigma_8, \sigma_0^P, \sigma_0^B$ are varied as free parameters, and σ_0^P, σ_0^B are fixed for clarity.

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Bispectrum of galaxies in redshift space

Efficiency of $f\sigma_8$ and $b_1\sigma_8$ breaking degeneracy

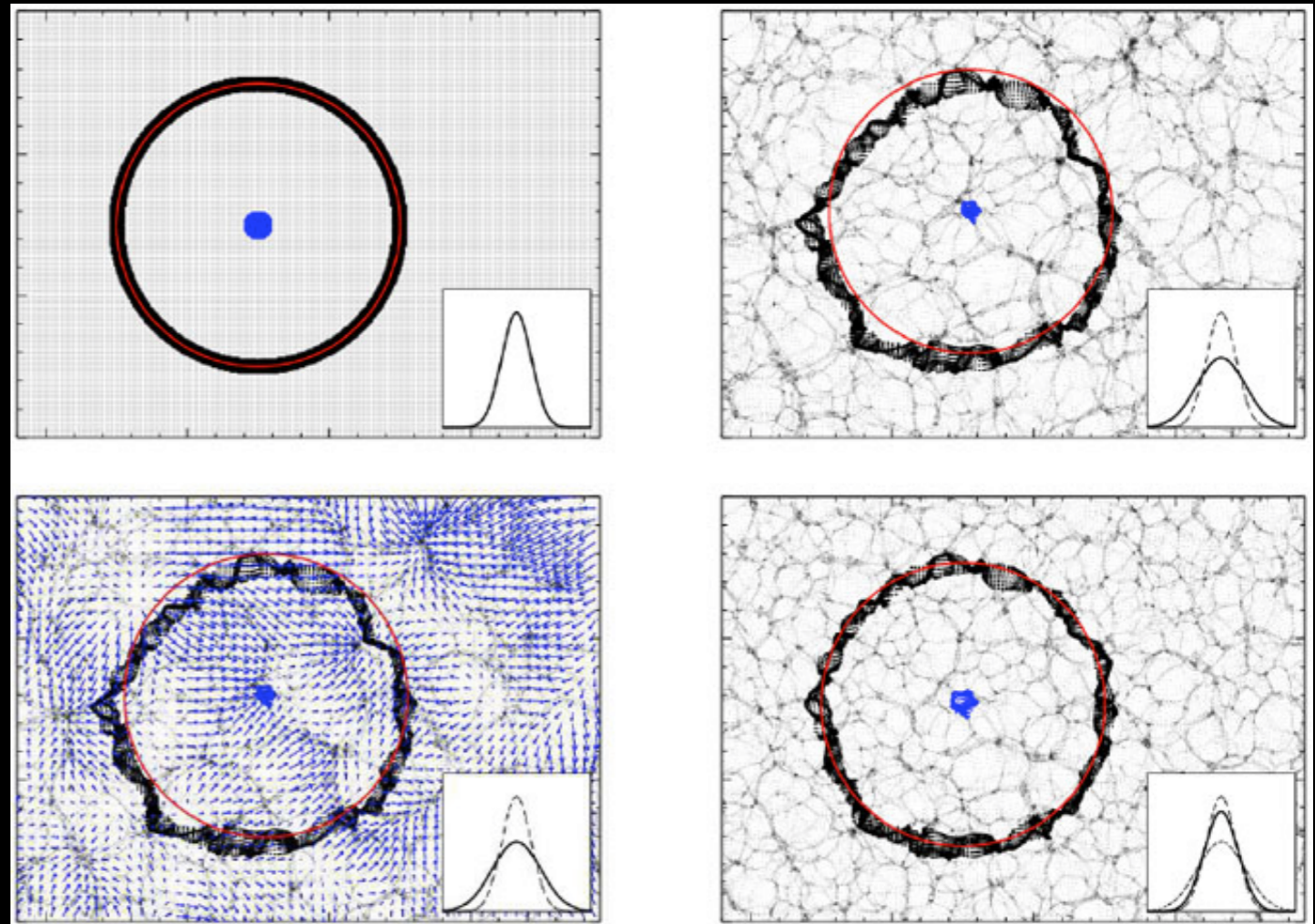


1.1
true
space at $z = 0.5$ for $k_{\max} = 0.15$
n points to $P^{(0)} + P^{(2)}$ and red
values. The green dashed lines
are varied as free parameters,

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Bispectrum and BAO

- Bispectrum as an alternative to reconstruction techniques.
- After reconstruction the bispectrum signal is significantly reduced to 0
- Reconstruction is pulling information from B back into P
- By measuring P and B in the pre-reconstructed field we can recover post-reconstruction BAO information without assuming GR nor Ω_m



(see W. Percival talk)

Bispectrum of galaxies in redshift space

Conclusions

- Bispectrum is a non-linear quantity (even at first order)
- Leading order term in **bispectrum** is sensitive to GR
- Shape dependence sensitive to **b_2** (non-linear term) and **b_{s2}** (tidal tensor term)
- f and σ_8 can be measured independently

