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Cosmological Perturbation Theory

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Why Large Scale Structure ?

- Number of modes in CMB (temperature) is saturated
- There is a large number of modes in 3 dimensions, if we could interpret them
- Information is highly complementary to CMB (low-z cosmic acceleration and growth of structure, breaking of degeneracies)

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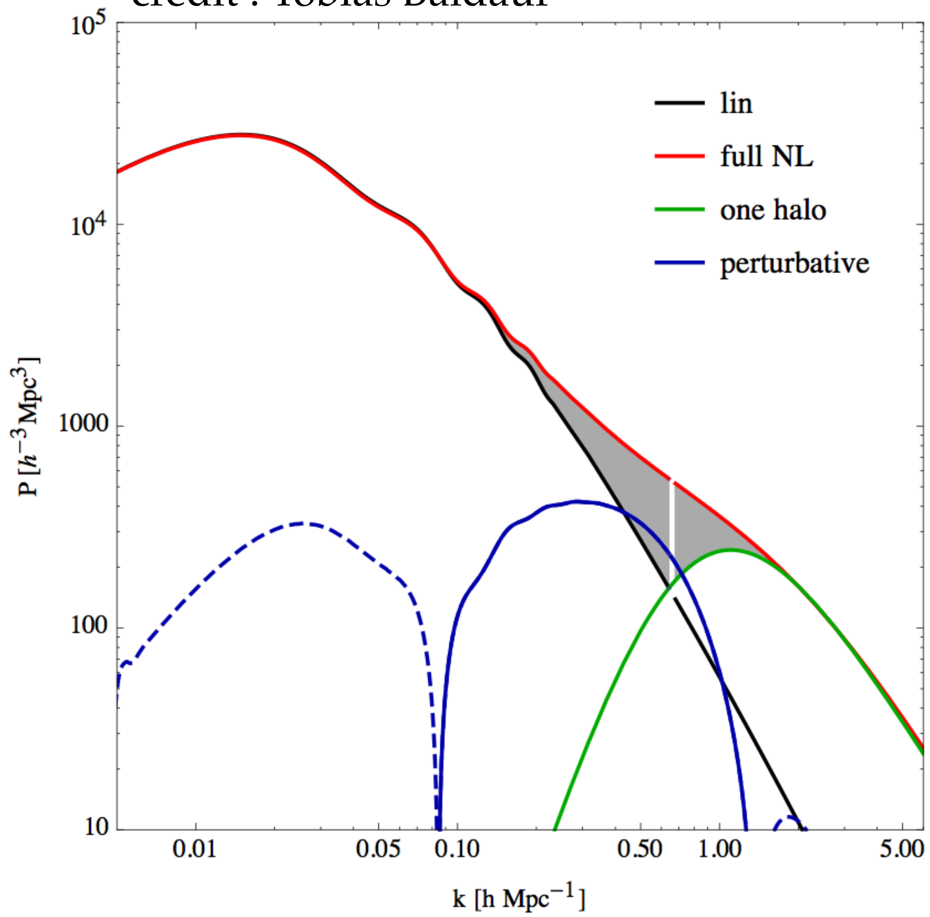
credit : Tobias Baldauf



Nonlinear Physics makes it complicated

- Linear theory (valid on large scales)
- Perturbative regime (weakly nonlinear)

Dark matter clustering
Galaxy bias
Redshift space distortions

- One-halo term (virialised, non PT)

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- Number of modes in CMB (temperature) is saturated
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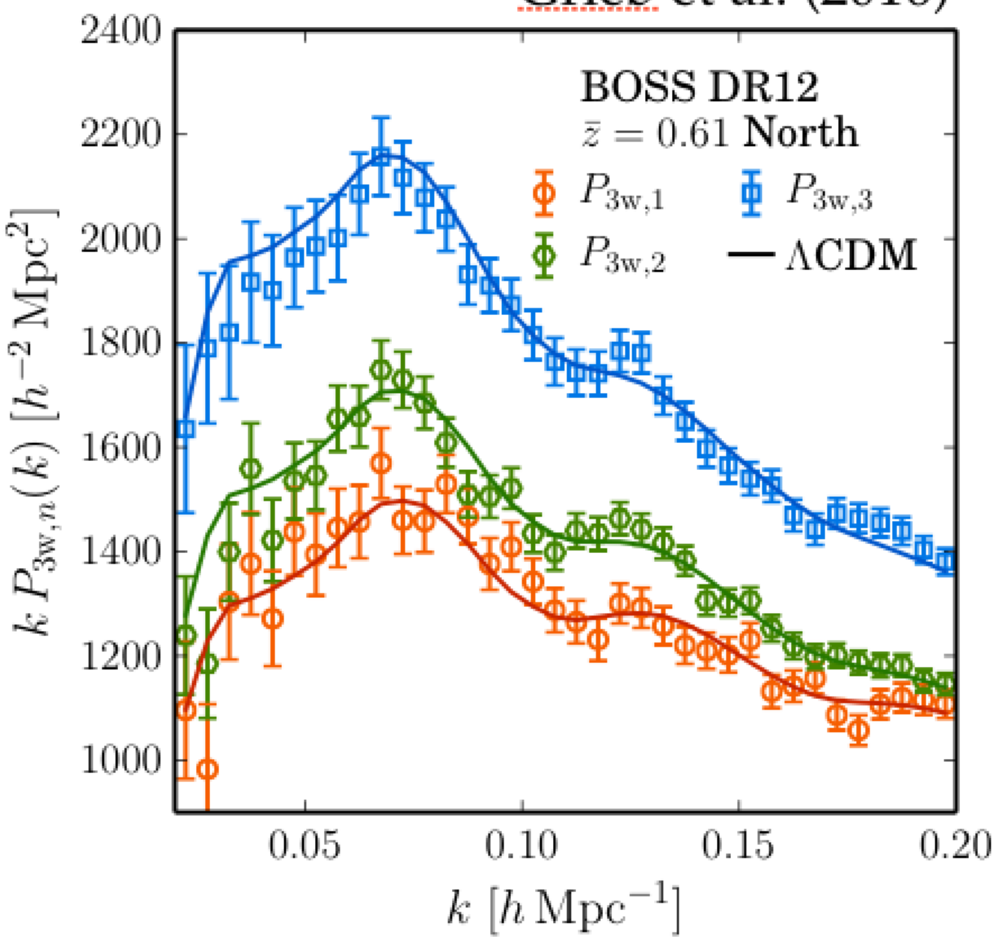
Grieb et al. (2016)



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On large scales evolution of perturbations is determined by Cold Dark Matter (CDM)

CDM can be taken as a collision-less fluid

Evolution of perturbations given by Vlasov Eq. (Collision-less limit of Boltzmann eq.)

↓

N-body codes

On sufficiently large scales where orbit crossing can be neglected Vlasov equation reduces to the dynamics of a pressure less perfect fluid (PPF)

↓

Standard Cosmological Perturbation Theory

Vlasov Equation (see PT review, Bernardeau et al arXiv 0112552 for full discussion on the next few slides)

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{a} \cdot \nabla f - a \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0,$$

$f(\bar{\mathbf{x}}, \mathbf{p}, \tau)$: Phase-space distribution function

: Momentum per unit mass

: Gravitational potential

To solve it we take moments of the Vlasov equation,

$$(1 + \delta) = \int f(\mathbf{p}) \; d^3 p,$$

Density Field (depends on **x**)

$$(1 + \delta) \, \mathbf{u} = \int f(\mathbf{p}) \, \frac{\mathbf{p}}{a} \; d^3 p,$$

Velocity Field (depends on **x**)

$$(1 + \delta) \, \sigma_{ij} = \int f(\mathbf{p}) \, \frac{p_i p_j}{a^2} \, d^3 p - (1 + \delta) \, u_i u_j,$$

Stress Tensor : describes velocity dispersion

Equations of motion

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{u}] = 0,$$

in principle an infinite hierarchy of moments

$$\frac{\partial u_i}{\partial \tau} + \mathcal{H} u_i + (\mathbf{u}\cdot\nabla) u_i = -\nabla\phi - \frac{1}{\rho}\nabla_j(\rho\sigma_{ij}),$$

$$\frac{\partial \sigma_{ij}}{\partial \tau} + 2\mathcal{H}\sigma_{ij} + (\mathbf{u}\cdot\nabla)\sigma_{ij} + \sigma_{jk}\nabla_k u_i + \sigma_{ik}\nabla_k u_j = -\frac{1}{\rho}\nabla_k(\rho\Pi_{ijk}),$$

no source term depends solely on δ or $\mathbf{u}$

Pressure-less perfect fluid (close the hierarchy)

$$\sigma_{ij} = 0, \ \Pi_{ijk} = 0$$

$$f(\mathbf{x},\mathbf{p},\tau) = [1+\delta(\mathbf{x},\tau)]\,\delta_{\mathrm{D}}[\mathbf{p} - a\,\mathbf{u}(\mathbf{x},\tau)],$$

Standard Pert. Theory for a Pressure-less perfect fluid

scales much smaller than the Horizon (Hubble radius) ⟶ Newtonian gravity

scales larger than strong clustering regime ⟶ *single stream approximation*

no velocity dispersion or pressure
(prior to virialization and shell crossing)

$$\nabla^2\Phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\ \mathcal{H}^2(\tau)\ \delta(\mathbf{x},\tau)$$

$$\frac{\partial\delta(\mathbf{x},\tau)}{\partial\tau} + \nabla\cdot\{[1+\delta(\mathbf{x},\tau)]\,\mathbf{u}(\mathbf{x},\tau)\} = 0$$

$$\frac{\partial\mathbf{u}(\mathbf{x},\tau)}{\partial\tau} + \mathcal{H}(\tau)\ \mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = -\nabla\Phi(\mathbf{x},\tau)\ -\frac{1}{\rho}\nabla_j(\rho\,\sigma_{ij})$$

velocity field can be assumed irrotational $\theta(\mathbf{x},\tau)\ \equiv\ \nabla\cdot\mathbf{u}(\mathbf{x},\tau)$

$$\frac{\partial\tilde{\delta}(\mathbf{k},\tau)}{\partial\tau} + \tilde{\theta}(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\tilde{\theta}(\mathbf{k}_1,\tau)\,\tilde{\delta}(\mathbf{k}_2,\tau),$$

$$\frac{\partial\tilde{\theta}(\mathbf{k},\tau)}{\partial\tau} + \mathcal{H}(\tau)\tilde{\theta}(\mathbf{k},\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\tilde{\delta}(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\beta(\mathbf{k}_1,\mathbf{k}_2)\,\tilde{\theta}(\mathbf{k}_1,\tau)\,\tilde{\theta}(\mathbf{k}_2,\tau)$$

Linear

Vertices (mode coupling)

SPT for a Pressure-less perfect fluid

$$\dot{\delta}(k,t) + \theta(k,t) = -\int_{k_1,k_2} \alpha(k_1,k_2)\theta(k_1,t)\delta(k_2,t)\delta^{(3)}(k_1+k_2-k)$$

$$\dot{\theta}(k,t) + \mathcal{H}\theta(k,t) + \frac{3}{2}\Omega_m\mathcal{H}^2\delta(k,t) = -\int_{k_1,k_2} \beta(k_1,k_2)\theta(k_1,t)\theta(k_2,t)\delta^{(3)}(k_1+k_2-k)$$

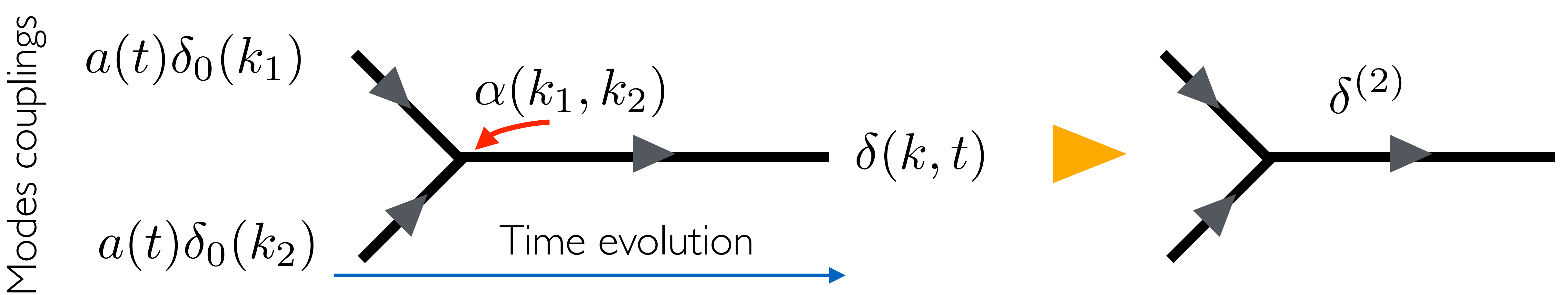
Expansion in $a(t)\delta(k_1,t_0)$

$$\delta(k,t) = \sum_n F_n(k_1,...,k_n)a(t)^n\delta(k_1,t_0)....\delta(k_n,t_0)\delta^{(3)}(k-\sum_n k_n)$$

Linear ▶ Plug it in the r. h. s. ▶ Solve for F_n

$\delta_L \sim a(t)\delta_0$

$$\dot{\delta}^{(2)}(k) \sim \alpha(k_1,k_2)\theta_L(k_1)\delta_L(k_2)\delta^{(3)}(k-k_1-k_2)$$



First non-linear effects in the field

$$\delta(k,t) = a\delta(k,t_0) + a(t)^2 \int_{k_1 k_2} F_2(k_1,k_2)\delta(k_1,t_0)\delta(k_2,t_0)\delta^{(3)}(k-k_1-k_2)$$

$$\frac{17}{21} + \frac{1}{2}\frac{\mathbf{k_1}\cdot\mathbf{k_2}}{k_1 k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left[\left(\frac{\mathbf{k_1}\cdot\mathbf{k_2}}{k_1 k_2}\right)^2 - \frac{1}{3}\right]$$

monopole

Spherical collapse

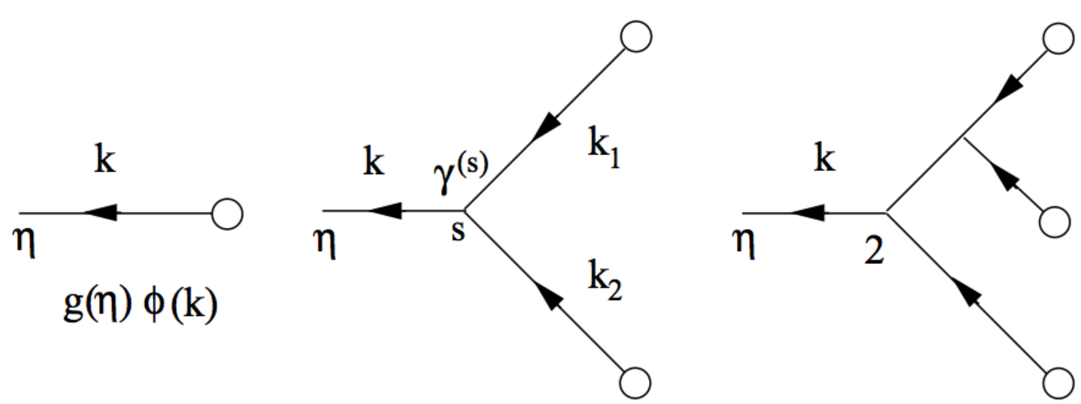
dipole

absent in spherically symmetric case
moves modes

quadrupole

distorts the shapes
tidal gravitational forces

Diagrammatically



$\delta^{(1)}(k,t)$ $\delta^{(2)}(k,t)$ $\delta^{(3)}(k,t)$

For the Power Spectrum

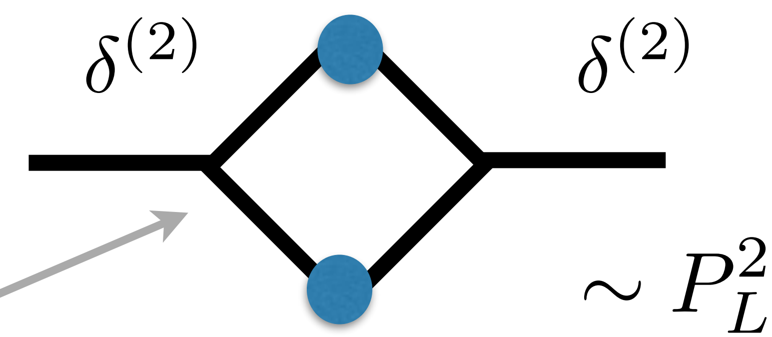
$$\langle \delta_k(t)\delta_{k'}(t)\rangle = 2\pi^2\delta^{(3)}(k+k')P(k)$$

P_{11} P_{22}

$$\langle(\delta_L + \delta_2 + \delta_3 + \ldots)(\delta_L + \delta_2 + \delta_3 + \ldots)\rangle$$

P_{13}

$$P = P_{11} + 2P_{13} + P_{22}$$



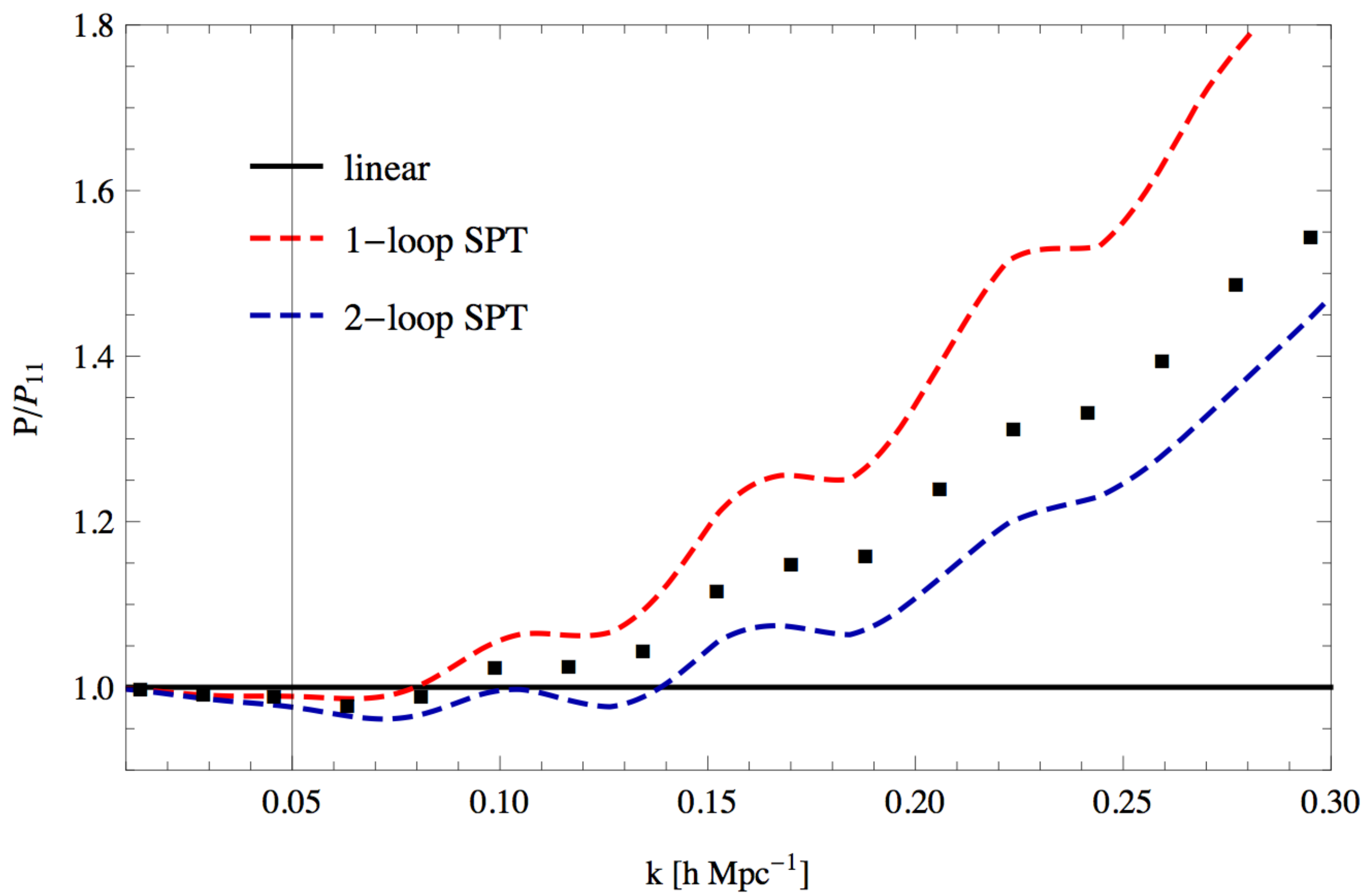
generates power from other scales: mode coupling. Positive

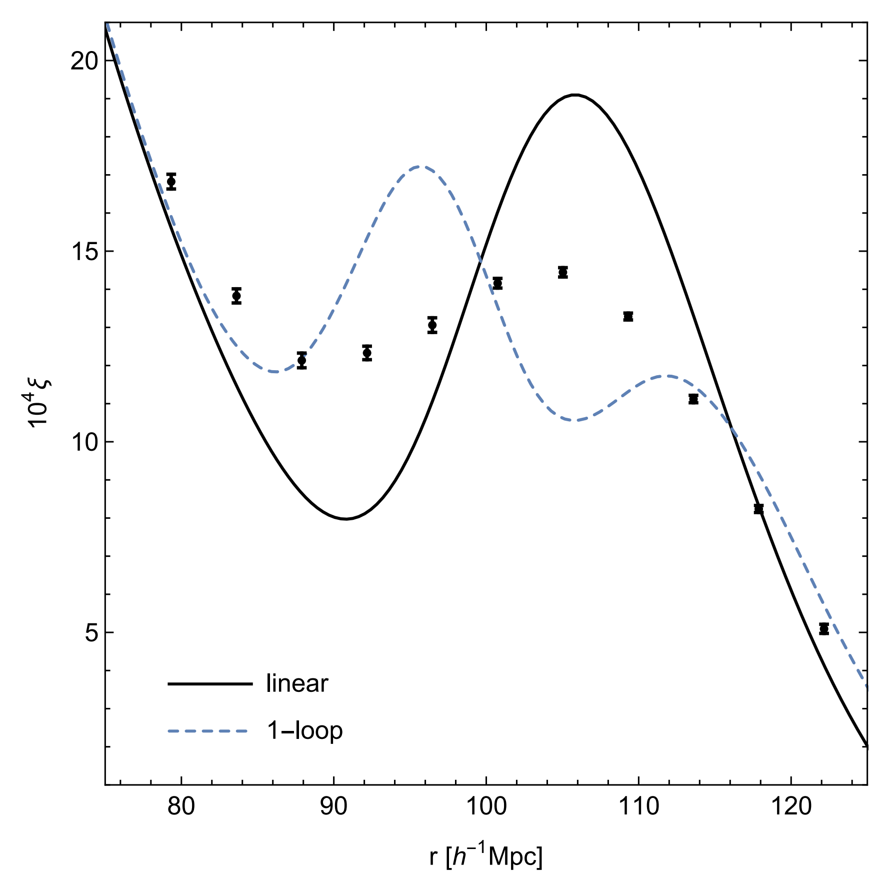
positive $$P_{22} = 2\int_q [F_2(q, k-q)]^2 P_{11}(q)P(|k-q|)$$

negative $$P_{13} = 3P_{11}(k)\int_q F_3(q,-q,k)P_{11}(q)$$

proportional to initial PS

Performance of SPT





note that the wiggles have shifted in the wrong way due to a cancellation between P_{13} and P_{22}

SPT expands in terms of the linear density contrast $\delta_L(k,z) = D_+(z)\delta_0(k)$

$$P(k,z) = D_+^2(z)P_0(k) + P_{13}(k,z) + P_{22}(k,z) + \ldots$$

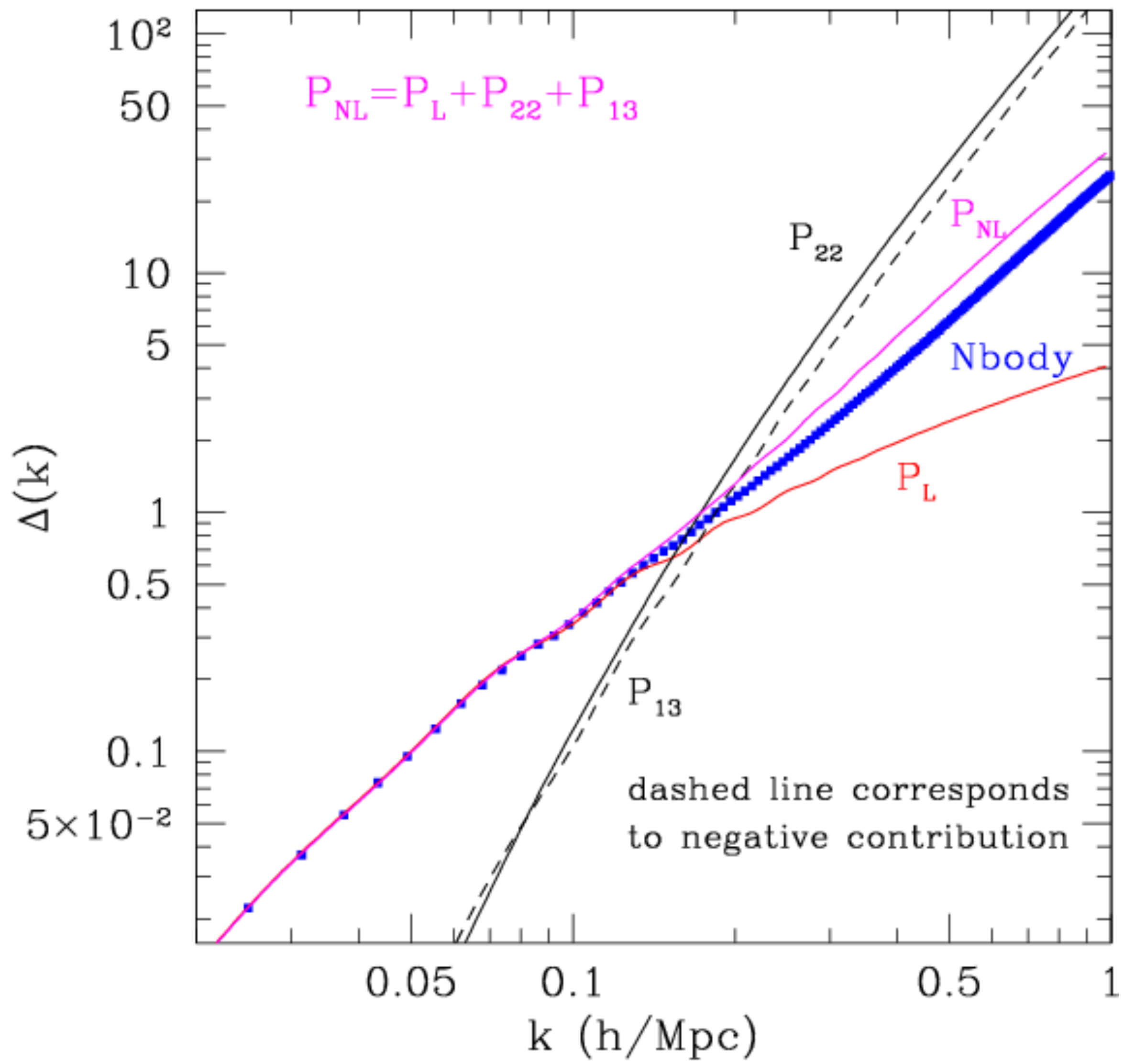
$$P_{22}(k,\tau) \equiv 2\int [F_2^{(s)}(\mathbf{k}-\mathbf{q},\mathbf{q})]^2 P_L(|\mathbf{k}-\mathbf{q}|,\tau)P_L(q,\tau)\mathrm{d}^3\mathbf{q},$$

$$P_{13}(k,\tau) \equiv 6\int F_3^{(s)}(\mathbf{k},\mathbf{q},-\mathbf{q})P_L(k,\tau)P_L(q,\tau)\mathrm{d}^3\mathbf{q}.$$

This approach is valid on large-scales where fluctuations are small but it **brakes down** when approaching the nonlinear regime where $\Delta_{\rm lin} \gtrsim 1$.

One way out is to sum up all orders !

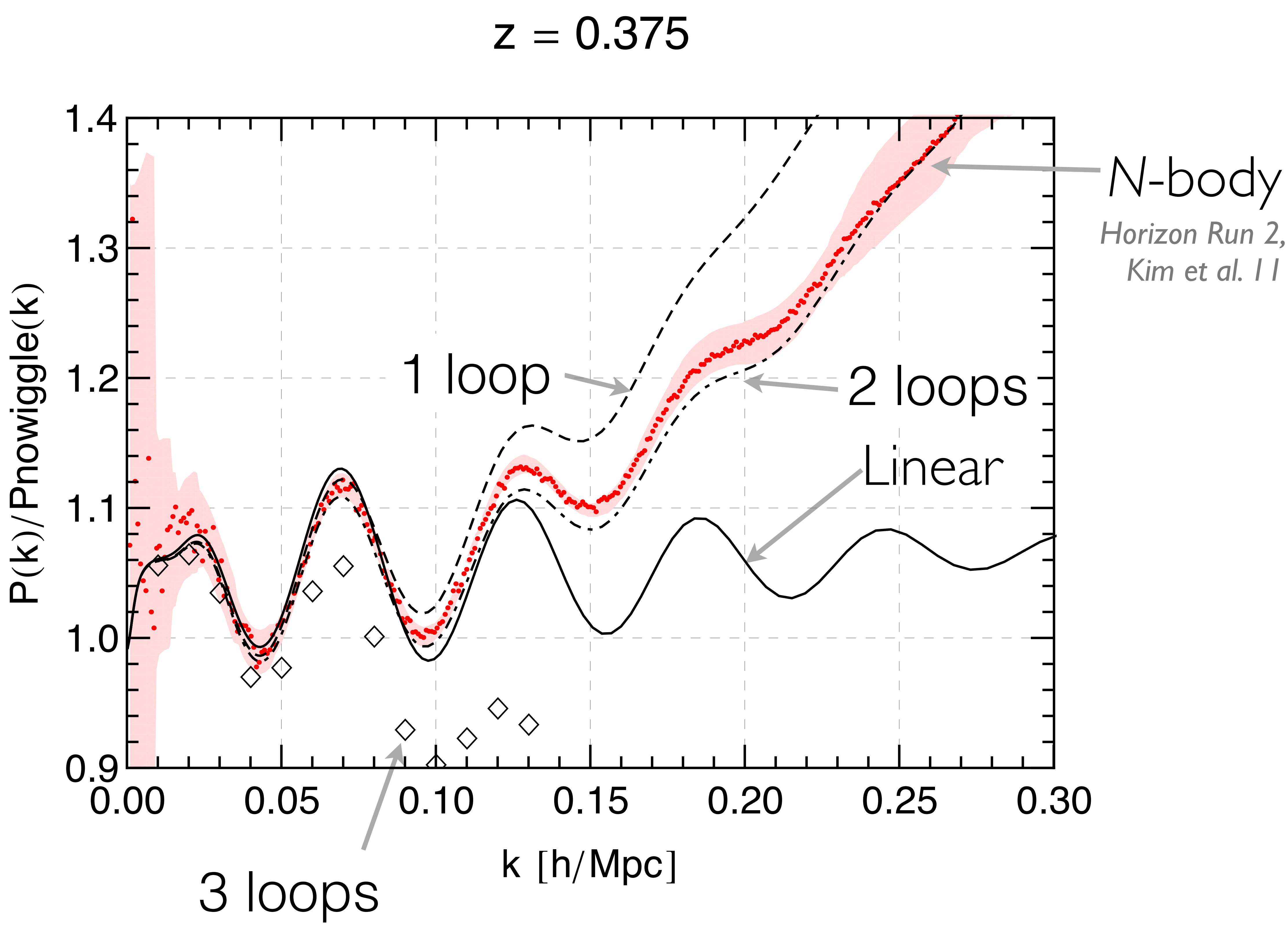
$$\Delta(k) = 4\pi k^3 P(k)$$



In detail the convergence of PT is related to the effective slope of the power spectrum at the nonlinear scale. If its very tilted then it works well (like a CDM spectra at high-z). As n approaches -1 different orders become of similar size and problems start to appear

Standard PT up to 3 loops

Blas, Garny and Konstandin (2014) arXiv:1309.3308

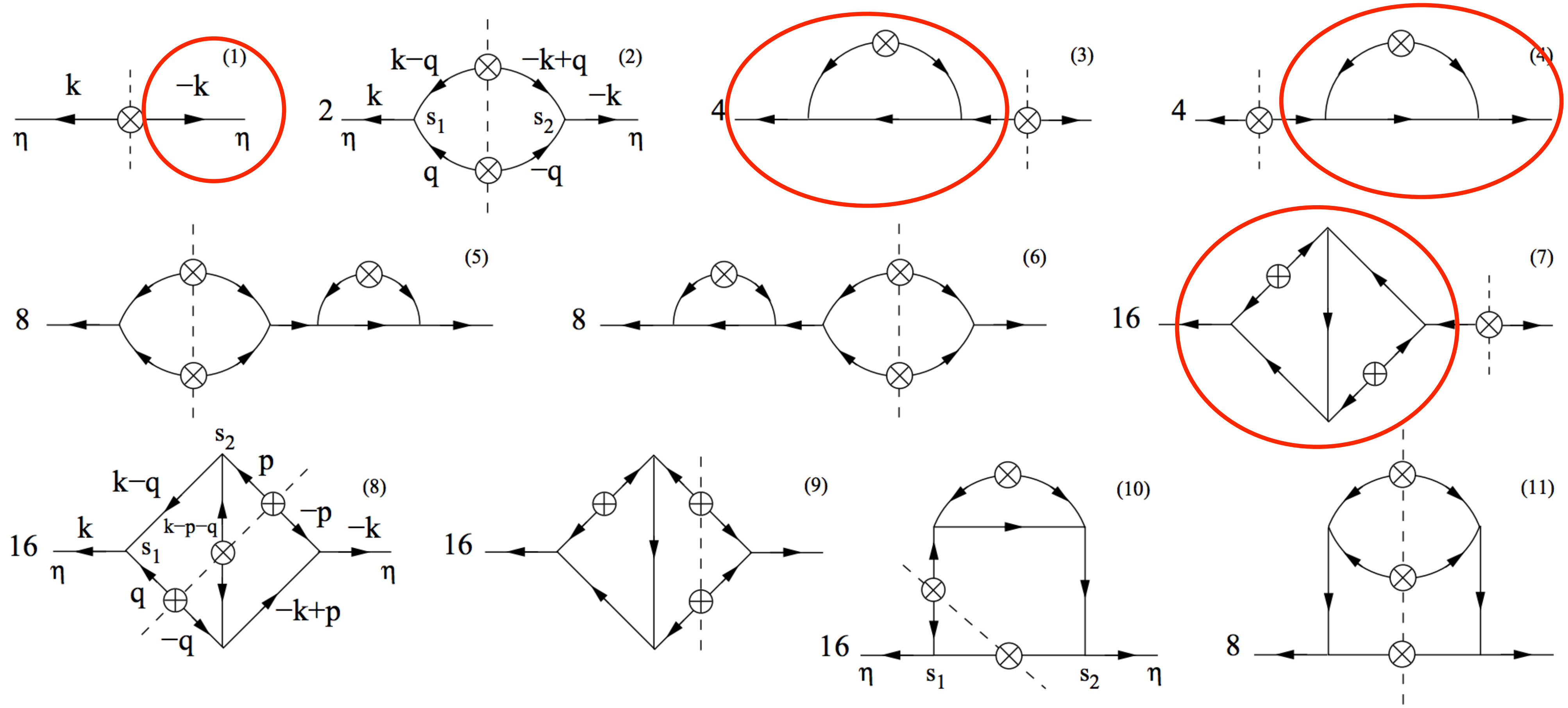


Renormalized Perturbation Theory
(resummation of IR-modes)

Work from several groups and people (Bernardeau, Crocce, Grinstein, Pietroni, Taruya, Scoccimarro, Matsubara, Wise, Blas, Zaldarriaga, Senatore, etc. incomplete list !!)
Approaches RPT, RegPT, TSPT, IR -EFT, others (incomplete list !!)

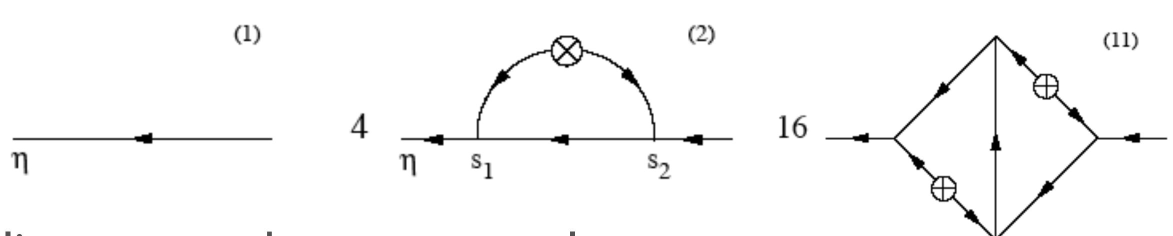
final den or vel field

Power Spectrum : $\langle \Psi(k,a)\Psi(-k,a)\rangle$



all diagrams of this type are systematically put together

$$\Gamma^{(1)}(k,z) =$$



linear growth factor

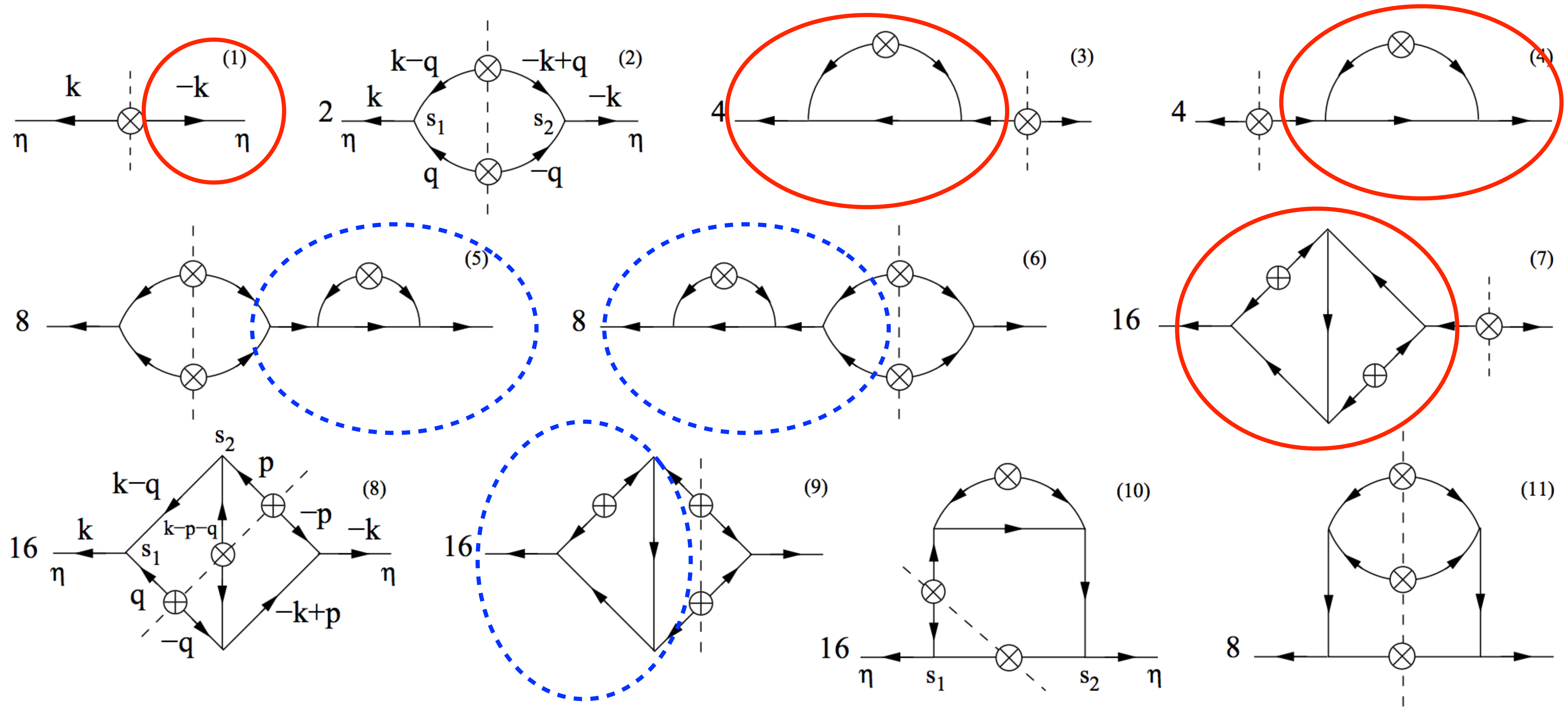
one-loop correction

$$P_{\delta\delta}(k) \quad = \quad \left[\Gamma^{(1)}_{\delta\delta}(k,z)\right]^2 P_0(k) + \ldots$$

$$\delta_D(k-k_1)\Gamma^{(1)}_a(k,z)P_0(k) = \langle \Psi_a(k,z)\delta_0(-k_1)\rangle$$

- its the cross-correlation with ICs
- it can be measured in n-body
- **Nonlinear propagator**

Power Spectrum : $\langle \Psi(k,a)\Psi(-k,a)\rangle$



The rest of the diagrams are mode-coupling terms (and can be re-summed)

$$P_{\delta\delta}(k) \;=\; \left[\Gamma^{(1)}_{\delta\delta}(k,z)\right]^2 P_0(k) + P_{MC}(k)$$

Resummation of IR-modes

Following Crocce and Scoccimarro 2006 in what follows (as an example)

$$\Gamma^{(1)}(k,z) = D(z)\left(\frac{P_{13}}{2P_0} + \frac{P_{15}}{2P_0} + \frac{P_{17}}{2P_0} + \ldots\right)$$

It is possible to show that when you consider the high k limit, or in other words the modes running inside the loops q (IR-modes) are << k the diagrams simplify to

$$n \text{ loops} \sim \frac{1}{n!}\left(-\frac{k^2\sigma_v^2}{2}\right)^n$$

high-k (low-q) →

$$\Gamma_\delta^{(1)}(k,z) \approx D(z)\exp(-k^2\sigma_v^2/2)$$

$$\sigma_v^2 = (4\pi/3)\int P(q)/q^2 d^3q$$

This is the variance of the displacement field, its dominated by large scale flows (~ 6 Mpc/h)

Resummation of IR-modes

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$$n \text{ loops} \sim \frac{1}{n!}\left(-\frac{k^2\sigma_v^2}{2}\right)^n \xrightarrow{\text{high-k}} \Gamma_\delta^{(1)}(k,z) \approx D(z)\exp(-k^2\sigma_v^2/2)$$

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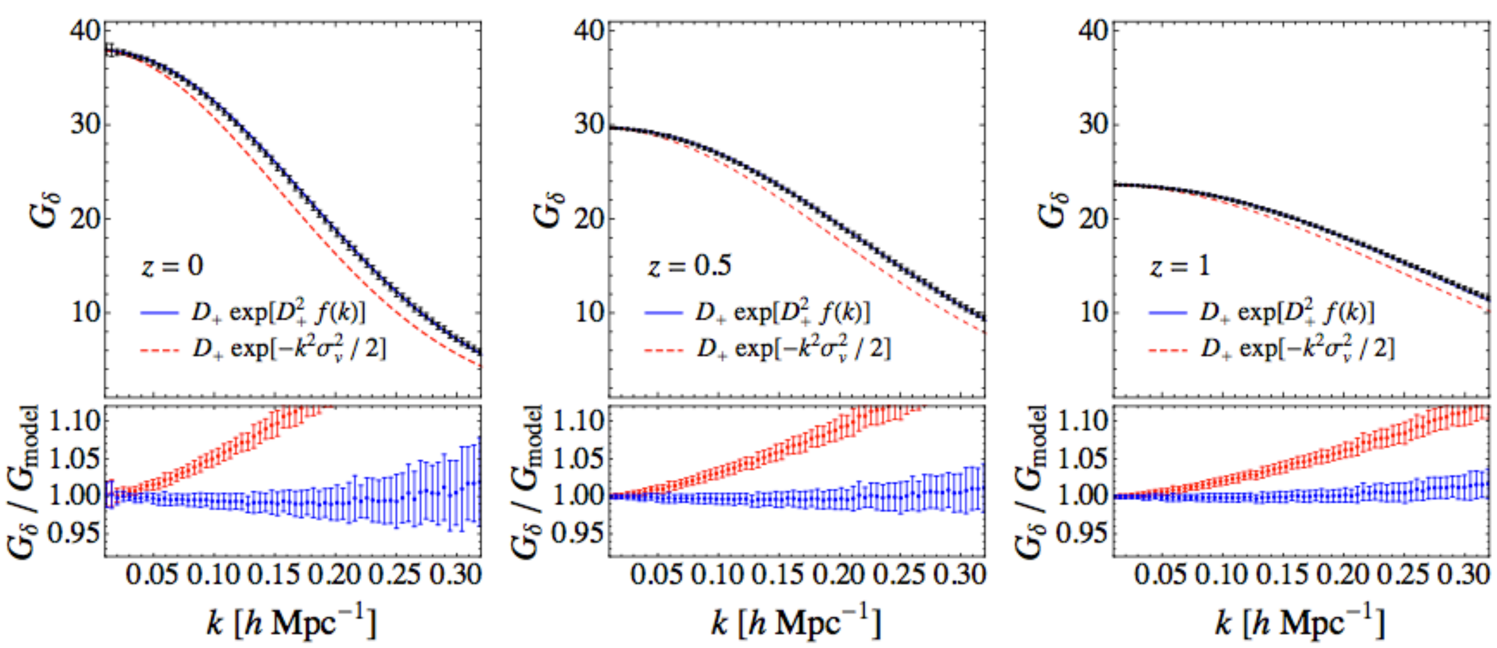
This is the variance of the displacement field, its dominated by large scale flow (~ 6Mpc/h)

On very large scales we can use PT to compute corrections (~ P_{13})

$$\xrightarrow{\text{low-k}} \Gamma_\delta^{(1)}(k,z) \approx D(z) - f(k)D^3(z) + \dots$$

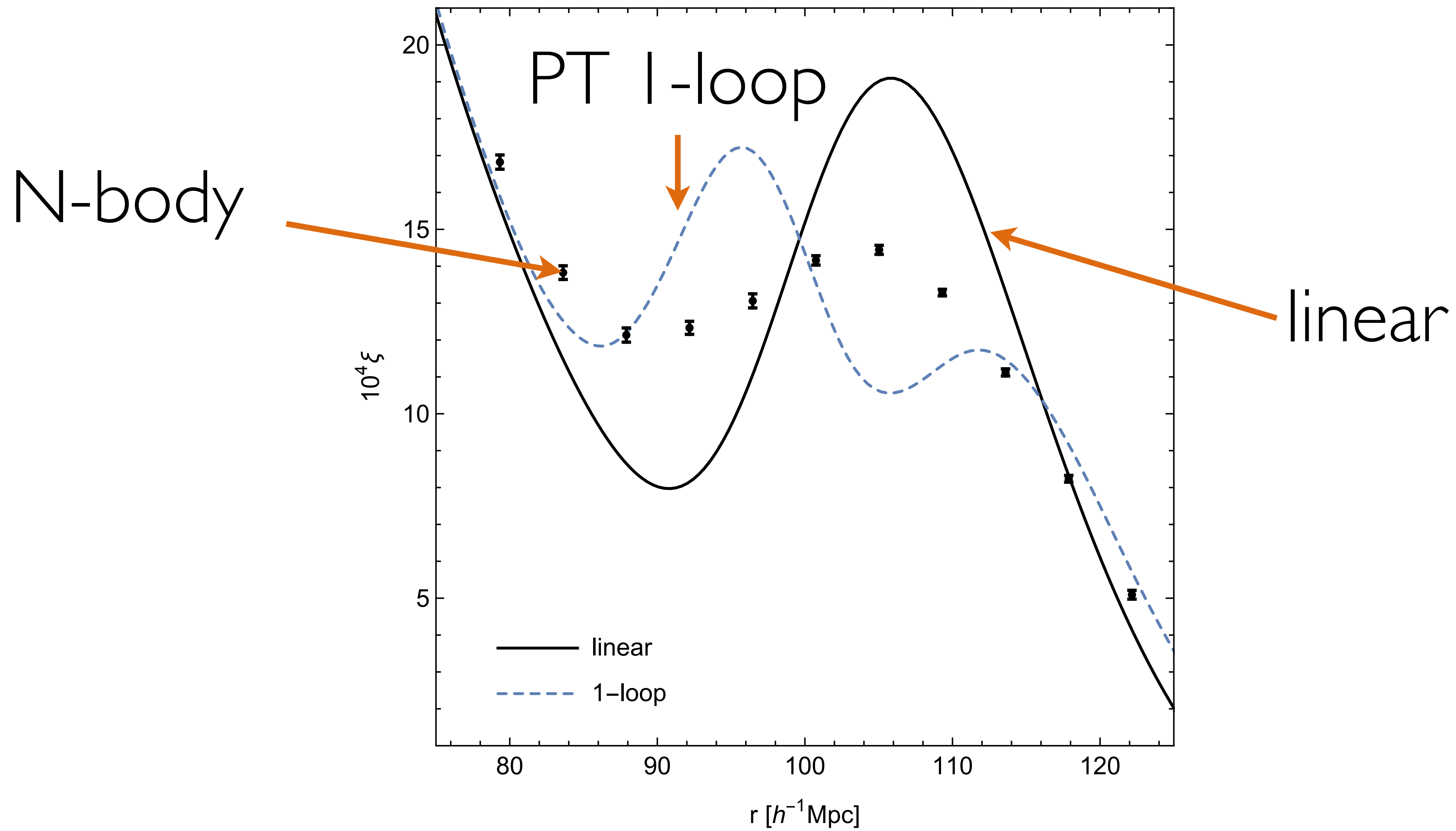


Ansatz

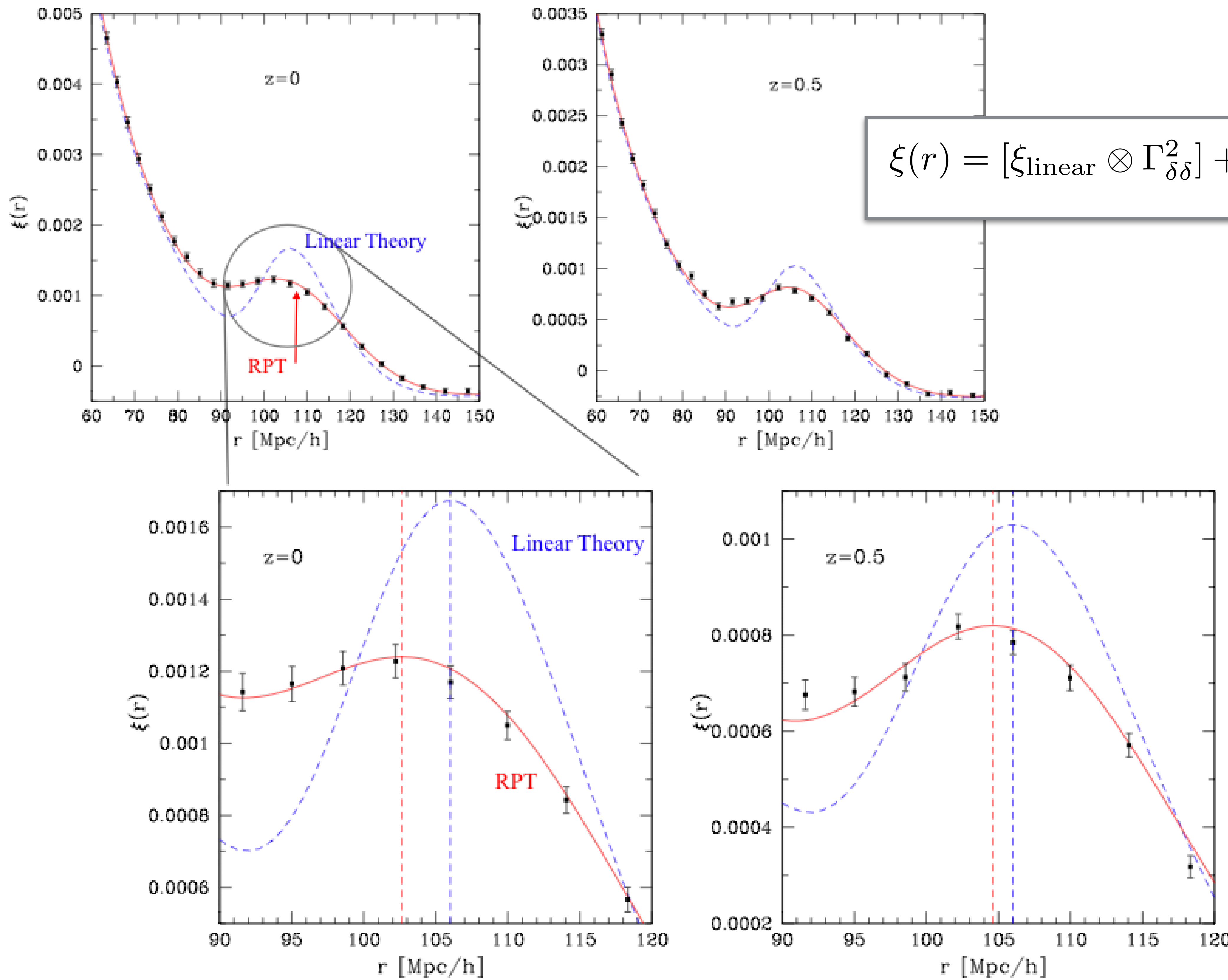
$$\Gamma_\delta^{(1)}(k,z) = D(z)\exp(f(k)D^2(z))$$

f(k) is very close to just $-k^2\sigma_v^2$

The BAO in SPT



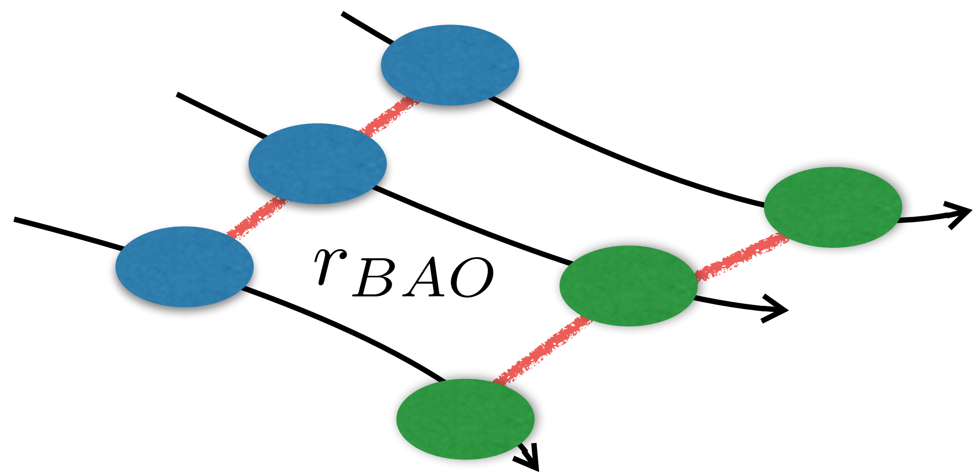
RPT Damping of the Baryon Acoustic Oscillations



$$\xi(r) = [\xi_{\text{linear}} \otimes \Gamma^2_{\delta\delta}] + \xi_{\text{Mode Coupling}}(r)$$

What's going on?

The degradation of the BAO peak is due to large scale flows



One can try to 'substract' the bulk motion to *reconstruct* the original peak

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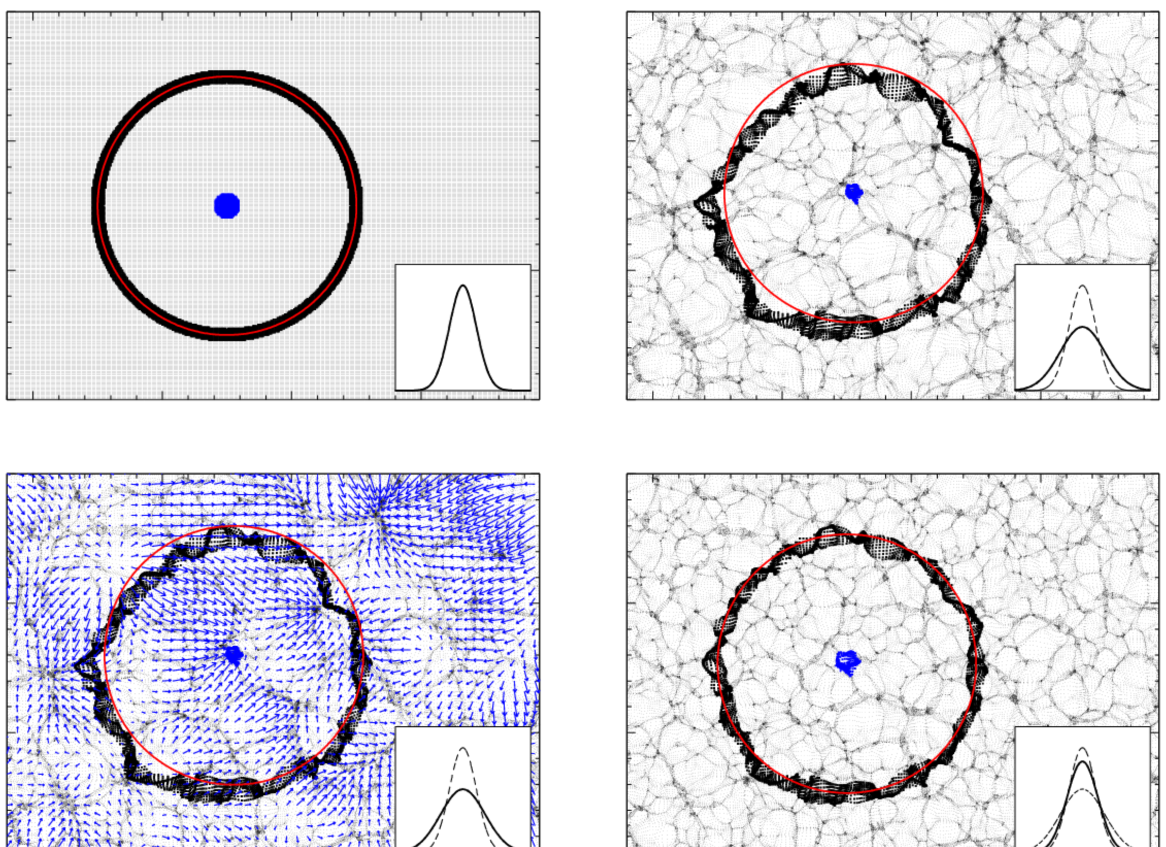
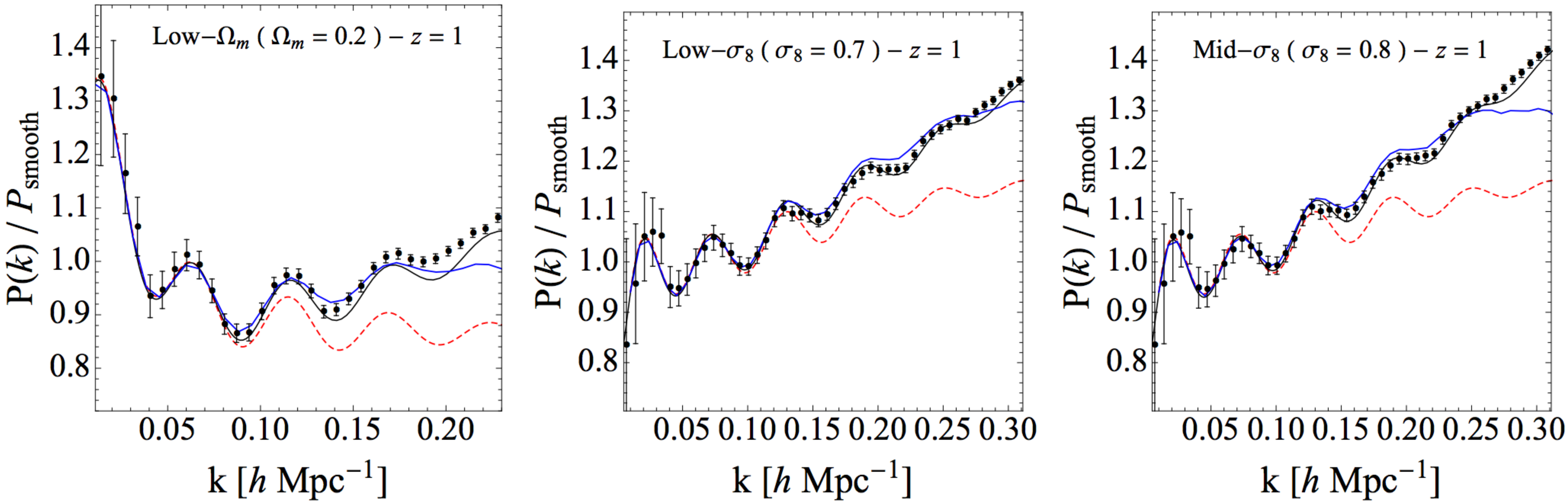


* Some of this can be done directly in data. Reverse engineering

Power Spectrum Performance
for different cosmological models at z = 1

Crocce, Scoccimarro and Bernardeau (2012)



CODES PUBLICLY AVAILABLE
(MPTbreeze, RegPT)

SPT : large cancellation between diagrams at each order, hard to understand the influence of long-wavelength modes.

The resummation above (RPT) breaks galilean invariance because it resums only the (unequal time) propagator

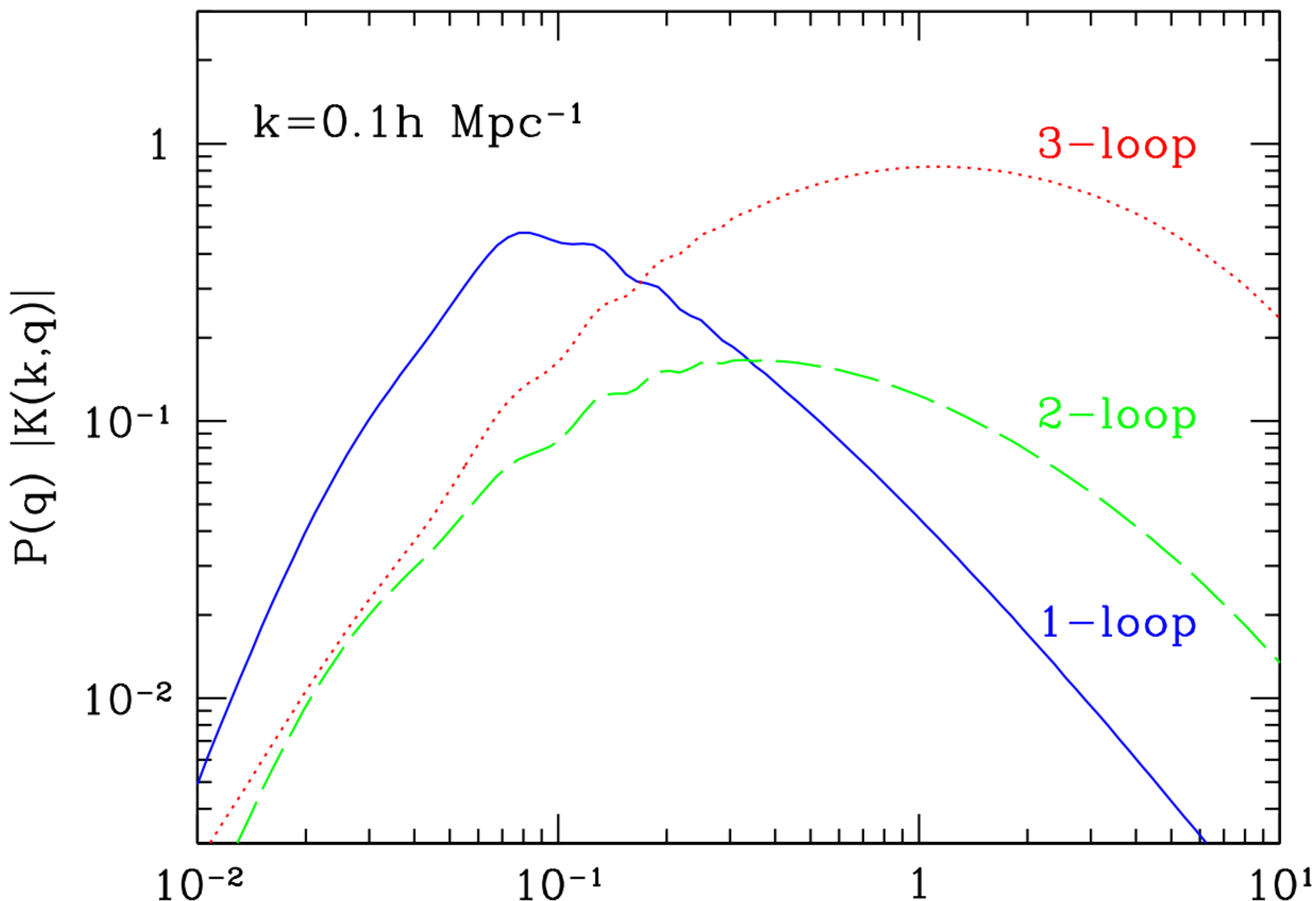
There are further terms in the SPT expansion that need to be resumed to restore galilean invariance, and have a better control of IR sensitivity (e.g. see TSPT: Blas et al 2016, gRPT)

BAO damping is quite well understood by now

One key issue in PT is that kernels become increasingly sensible to the small scales

Response functions (Bernardeau Nishimichi Taruya 2015 +)

How to quantify the impact of small scale structures on the growth of large scale modes ?



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Response functions (Bernardeau Nishimichi Taruya 2015 + follow ups)

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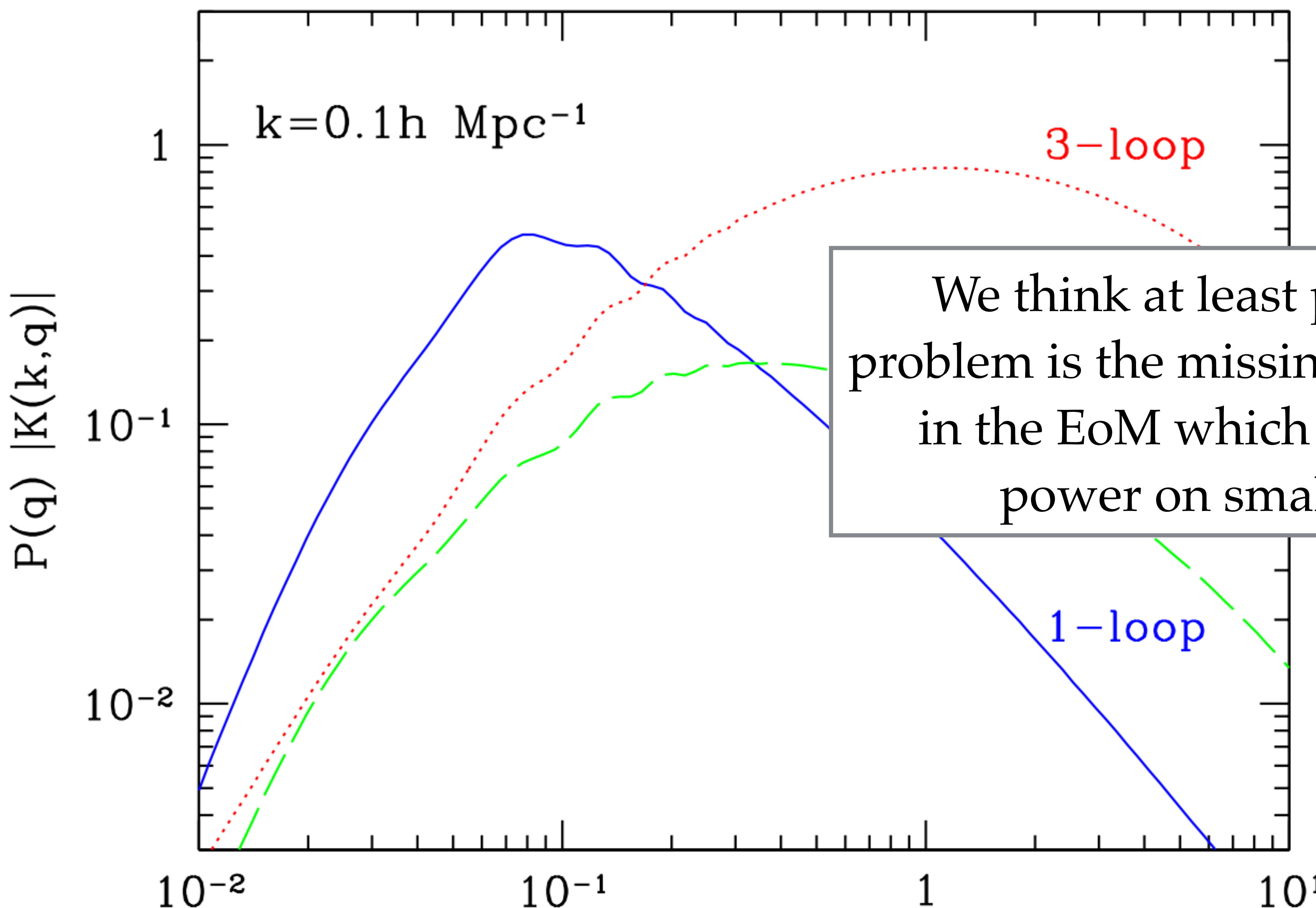
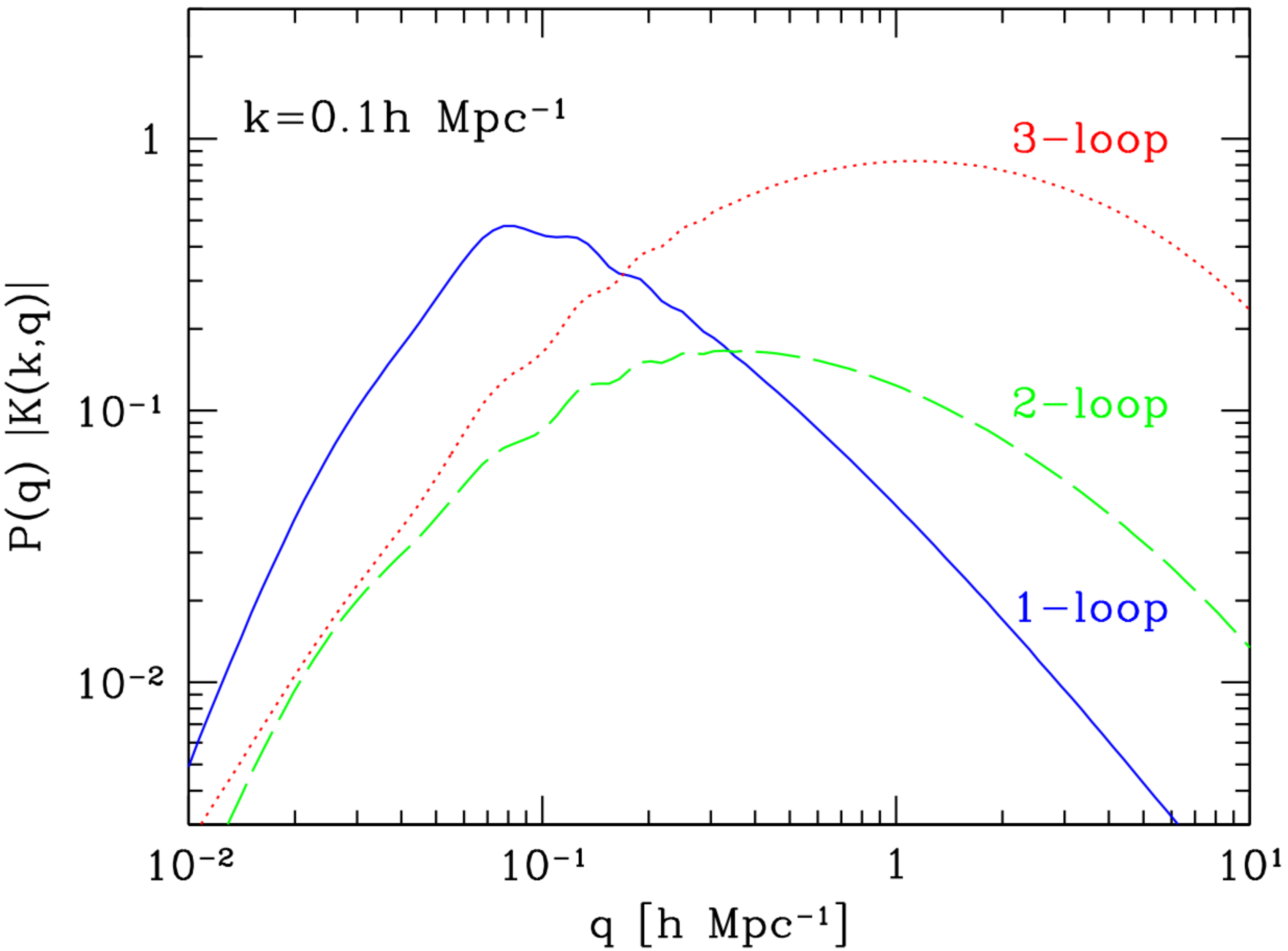


We think at least part of this problem is the missing stress tensor in the EoM which suppresses power on small scales

Response functions

$$P(k) \text{ or any PT quantity } = \int \frac{\mathrm{d}q}{q}\, K_{a+}^{\mathrm{p-loop}}(k,q) P_0(q).$$

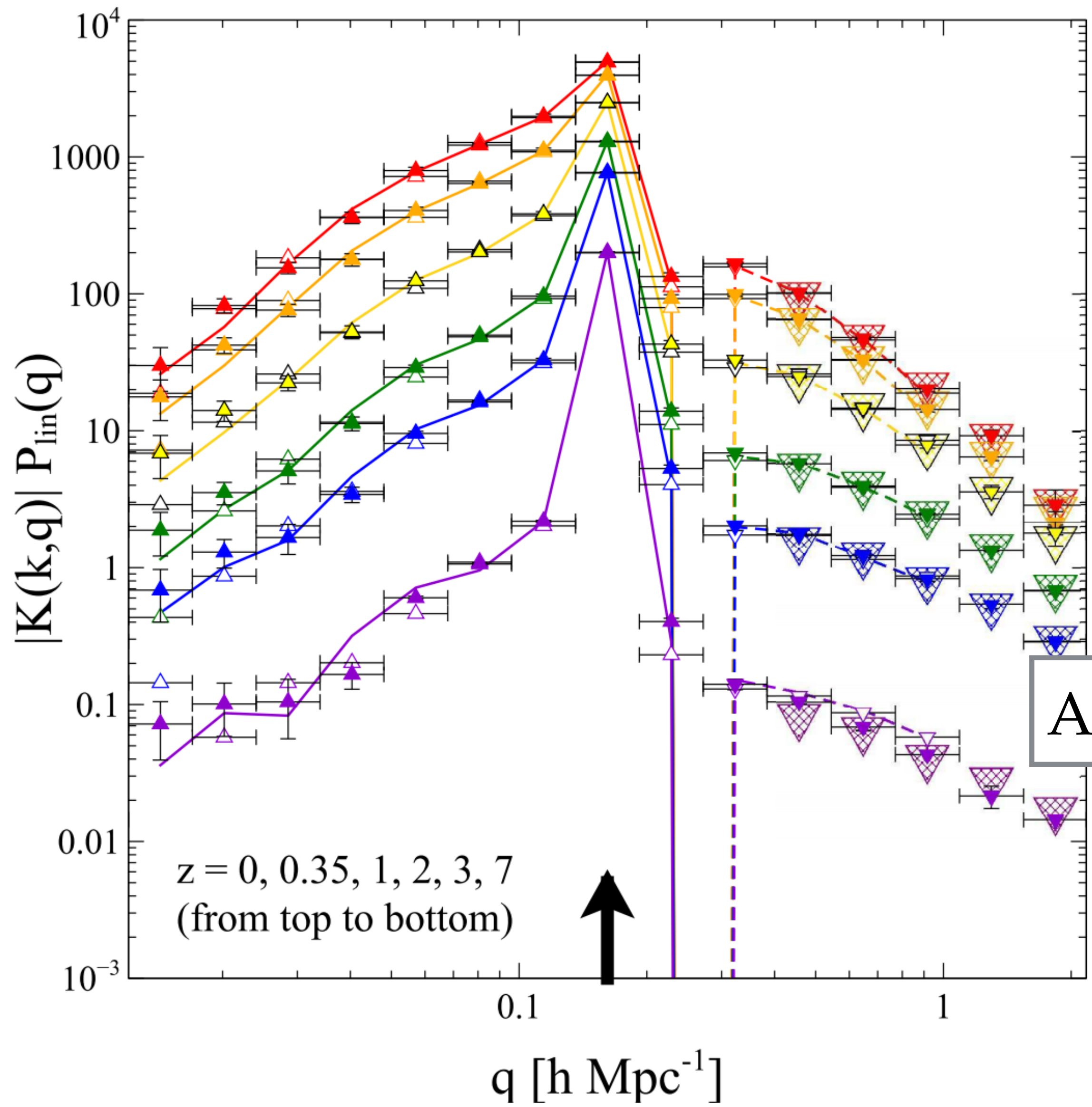


Response functions

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}.$$

$$\hat{K}_{i,j} P_j^{\rm lin} \equiv \frac{P_i^{\rm nl}[P_{+,j}^{\rm lin}] - P_i^{\rm nl}[P_{-,j}^{\rm lin}]}{\Delta \ln P^{\rm lin} \Delta \ln q},$$

measure this in sims



At high-q its suppressed !

Effective Field Theory

Baumann, Nicolis, Senatore, Zaldarriaga 2010 and many papers afterwards

The UV (small scales) problem:

1) The loops in SPT are too sensitive to small scales

2) The UV has a back reaction into the large-scales that is unphysical.

3) The EoM do not include velocity dispersion (stress tensor)

$$f_\Lambda(\mathbf{x}) = \int \mathrm{d}^3x \; W_\Lambda(\mathbf{x}-\mathbf{x}')f(\mathbf{x}')$$

coarse grained variables

$$\delta'_\Lambda + \partial_j\left[(1+\delta_\Lambda)v_{\Lambda,j}\right] = 0$$

$$v'_{\Lambda,i} + \mathcal{H}v_{\Lambda,i} + \partial_i\phi_\Lambda + v_{\Lambda,j}\partial_j v_{\Lambda,i} = -\frac{1}{1+\delta}\partial_j\tau_{\Lambda,ij}$$

write down EoM in these variables

In order to close the hierarchy

Effective Field Theory

$$\delta'_\Lambda + \partial_j \left[(1+\delta_\Lambda) v_{\Lambda,j}\right] = 0$$

$$v'_{\Lambda,i} + \mathcal{H} v_{\Lambda,i} + \partial_i \phi_\Lambda + v_{\Lambda,j}\partial_j v_{\Lambda,i} = -\frac{1}{1+\delta}\partial_j \tau_{\Lambda,ij}$$

Effective Stress Tensor - Parametrizing the Ignorance about small scales

$$\tau_{\Lambda,ij} = p\delta_{ij}^{(K)} + c_s^2 \delta_{ij}^{(K)} \delta_\Lambda + c_{v,b}^2 \delta_{ij}^{(K)} \partial_m v_{\Lambda,m} + c_{v,s}^2 \left[\partial_i v_{\Lambda,j} + \partial_j v_{\Lambda,i} - \frac{2}{3}\delta_{ij}^{(K)} \partial_m v_{\Lambda,m}\right]$$

$\Rightarrow$ all terms allowed by symmetries (second derivatives of the potential)

Equations of Motion including Effective Stress

$$\delta'_\Lambda + \theta_\Lambda = -\alpha[\theta_\Lambda \star \delta_\Lambda]$$

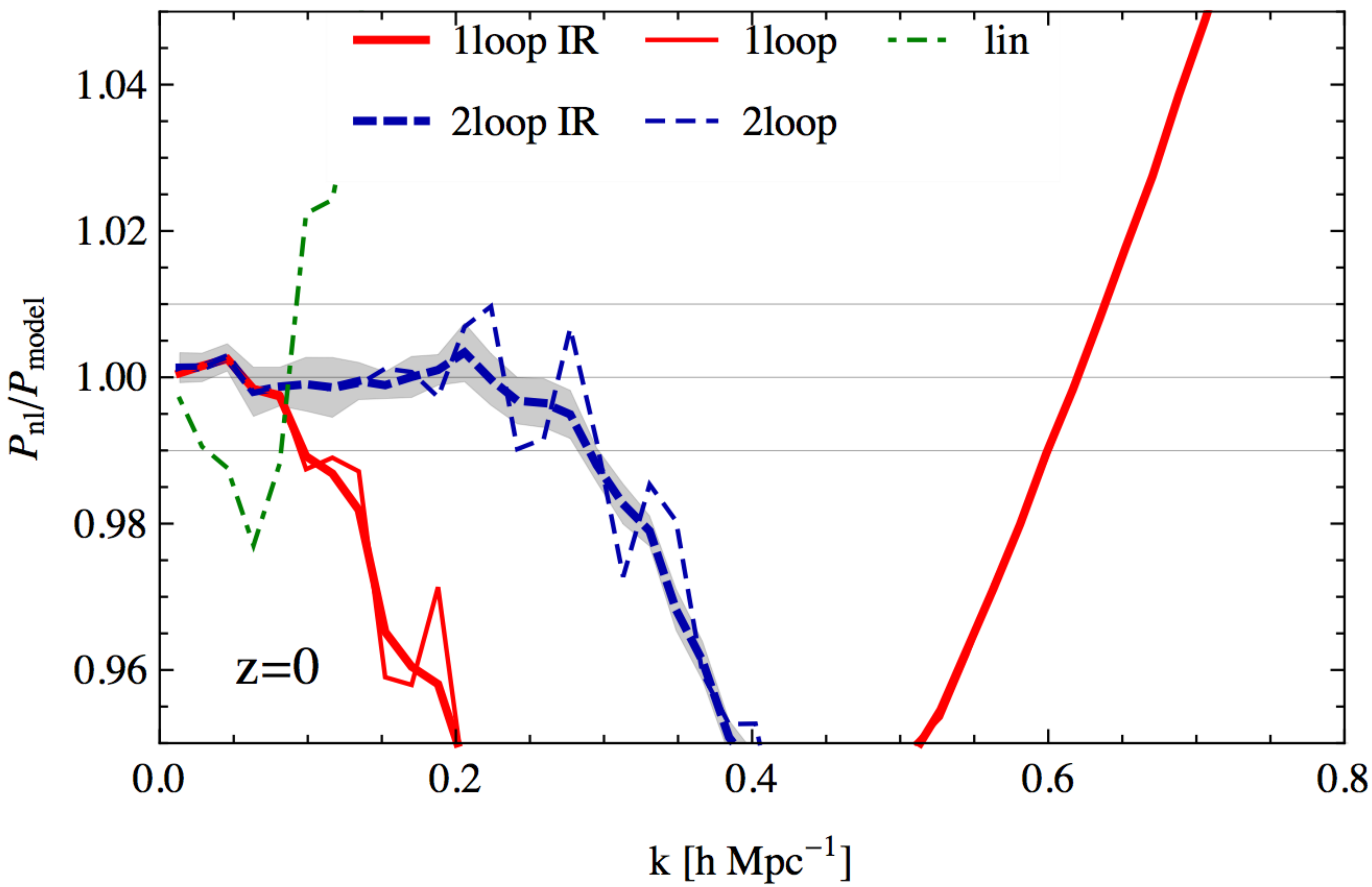
$$\theta'_\Lambda + \mathcal{H}\theta_\Lambda + \frac{3}{2}\mathcal{H}^2 \delta_\Lambda = -\beta[\theta_\Lambda \star \theta_\Lambda] + \tilde{c}_s^2 k^2 \delta_\Lambda^{(1)}$$

contains errors of PT and microscopic stress

Matter Power Spectrum

$$P_{\rm mm}(k,t) = P_{\rm lin}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda) D^2(t) k^2 P_{\rm lin}(k)$$

- Figure is using one parameter fit with a concrete counter-term
- In general (at 2 loops etc) there are several counter-terms.
- One needs to have an argument to put them to zero, or leave them as nuisance, etc



Baldauf Mercolli Zaldarriaga 2015

EFT Performance (roughly!):

One-loop Eulerian EFT $k1\%$ = 0.1 h Mpc^{-1}, two-loop Eulerian EFT $k1\%$ =0.3 h Mpc^{-1} at z=0

EFT Limitations:

How to deal with many counter-terms, and their time evolution

Degeneracies, particularly with bias

Stochastic term from one-halo physics $\Rightarrow$ leads to percent level corrections at $k = 0.3\ hMpc^{-1}$

SPT+IR (RPT, RegPT, etc .. roughly!)

Within 2% at k ~ 0.3 at z ~ 1 or k ~ 0.25 at z ~ 0.5.

Discussions on assumptions, need to include further diagrams etc,

Using response functions and N-body measurements

k1% at k = 0.3 - 0.4 $hMpc^{-1}$

(only for density spectrum so far)

THANKS