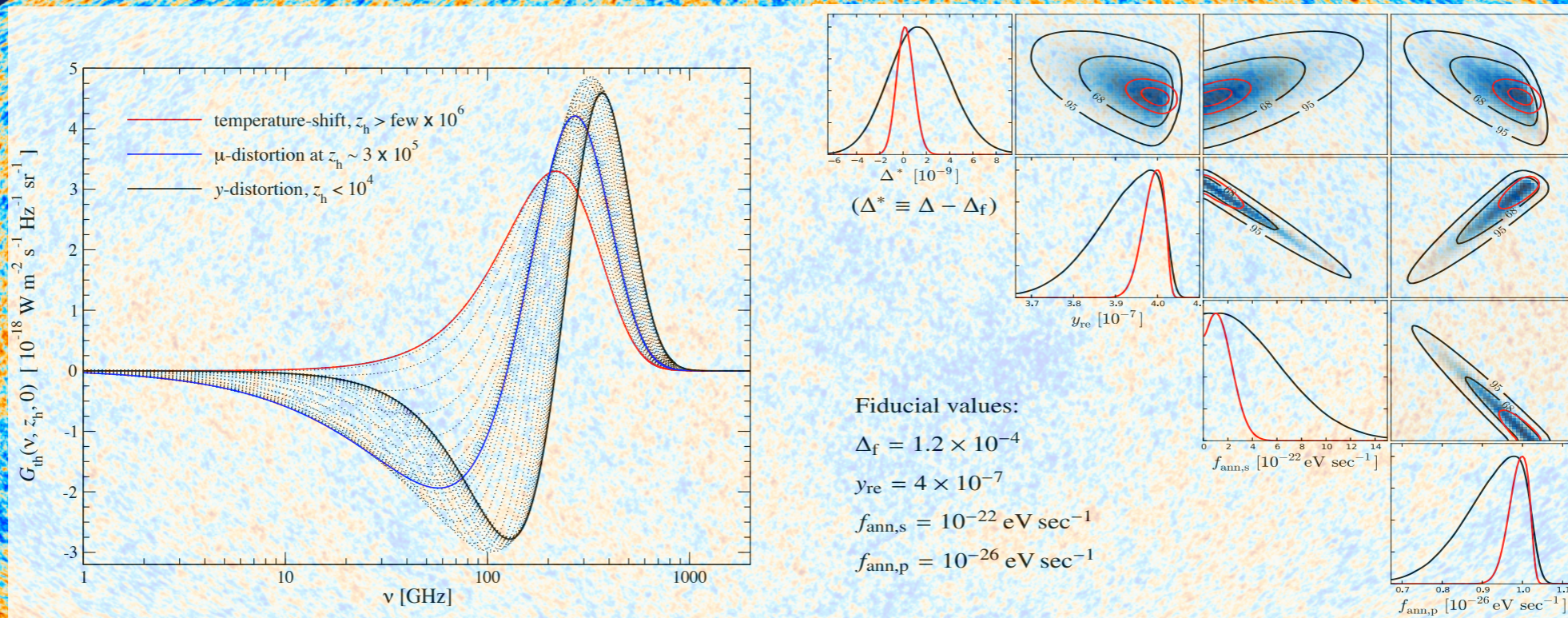
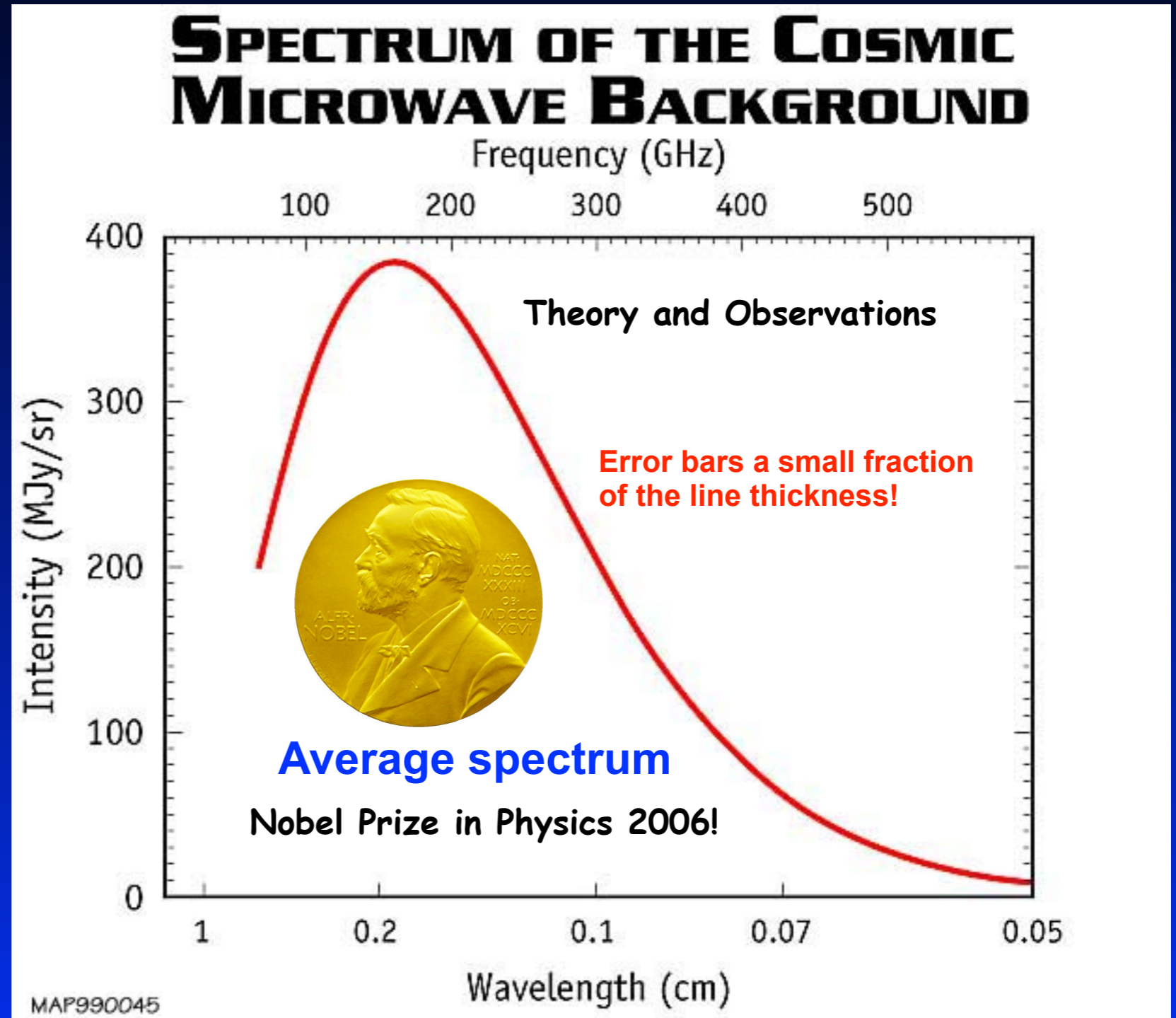
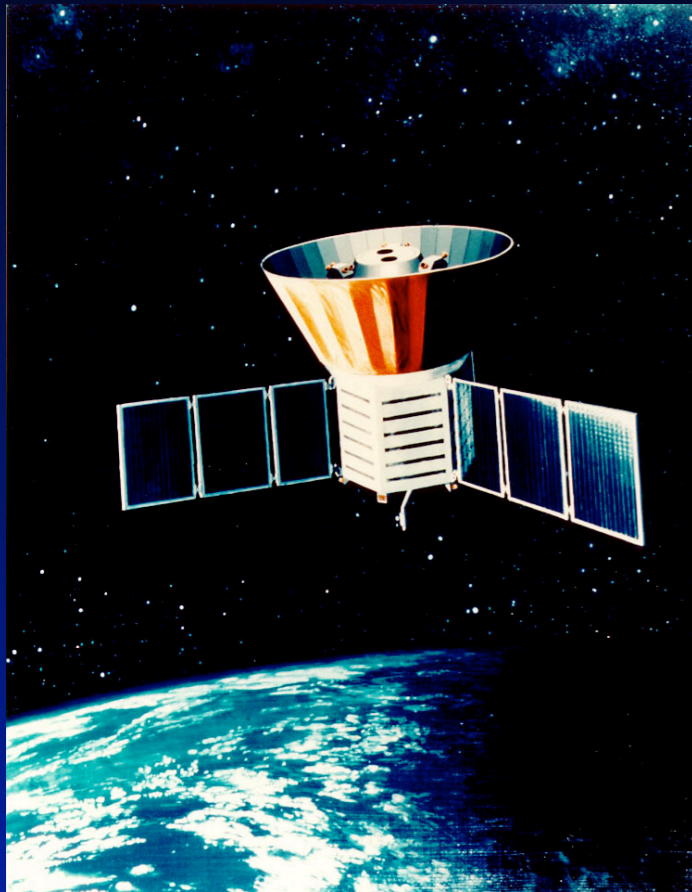


CMB Spectral Distortion Computations using the Green's function package of *CosmoTherm*

Primordial Distortions



COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)



$$T_0 = 2.725 \pm 0.001 \text{ K}$$

$$|y| \leq 1.5 \times 10^{-5}$$

$$|\mu| \leq 9 \times 10^{-5}$$

Mather et al., 1994, ApJ, 420, 439

Fixsen et al., 1996, ApJ, 473, 576

Fixsen et al., 2003, ApJ, 594, 67

Only very small distortions of CMB spectrum are still allowed!

Physical mechanisms that lead to spectral distortions

- **Cooling by adiabatically expanding ordinary matter**
(JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011) Standard sources
of distortions
 - Heating by *decaying* or *annihilating* relic particles
(Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
 - **Evaporation of primordial black holes & superconducting strings**
(Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
 - **Dissipation of primordial acoustic modes & magnetic fields**
(Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
 - **Cosmological recombination radiation**
(Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)
-
- **Signatures due to first supernovae and their remnants**
(Oh, Cooray & Kamionkowski, 2003)
 - **Shock waves arising due to large-scale structure formation**
(Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
 - **SZ-effect from clusters; effects of reionization**
(Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
 - **more exotic processes**
(Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

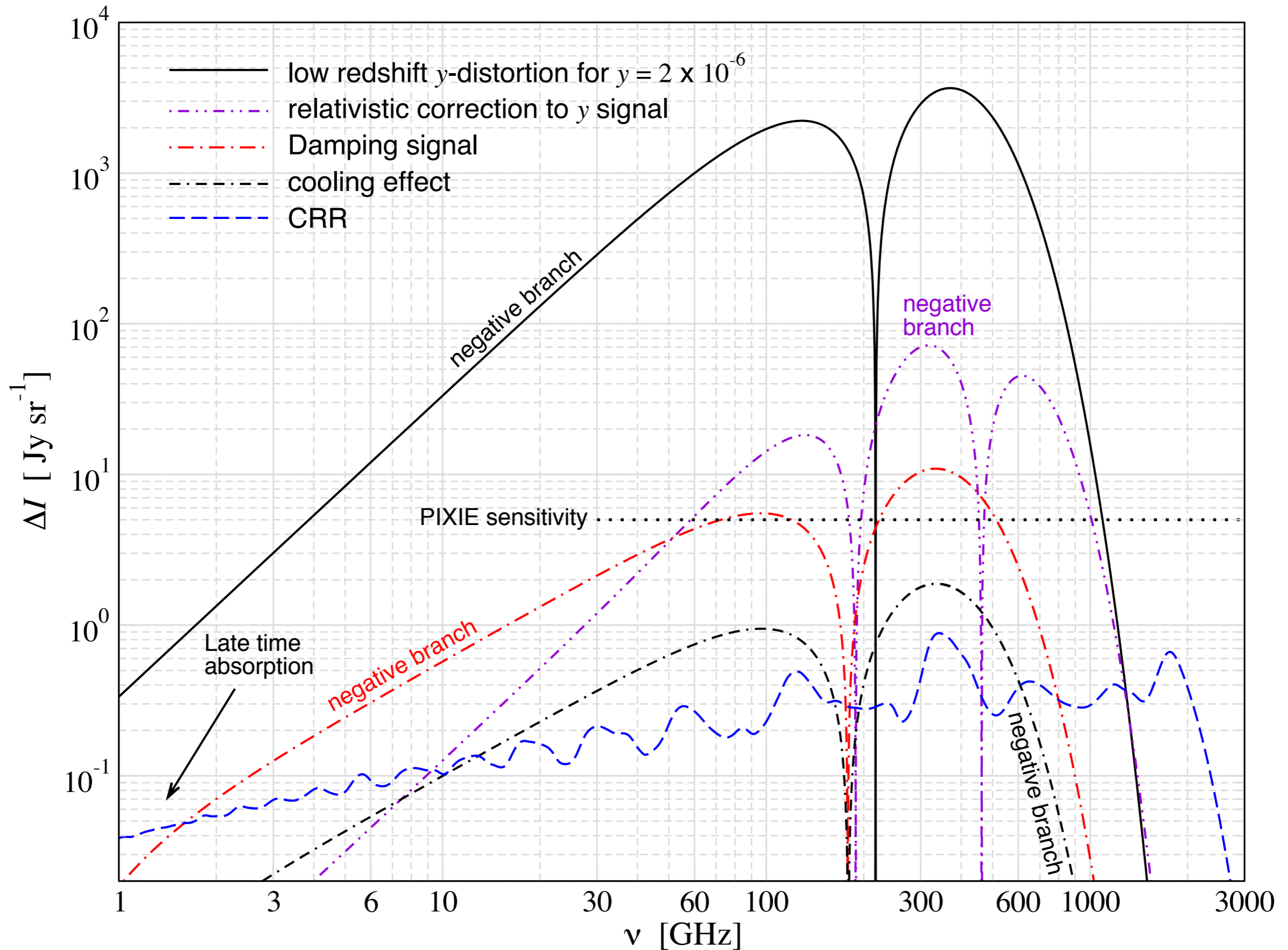
pre-recombination epoch

„high“ redshifts

„low“ redshifts

post-recombination

Average CMB spectral distortions in Λ CDM



Set of evolution equations for distortions

Photon field $x = \frac{h\nu}{kT_\gamma} \quad \theta_e = \frac{kT_e}{m_e c^2}$

$$\frac{\partial f}{\partial \tau} \approx \frac{\theta_e}{x^2} \frac{\partial}{\partial x} x^4 \left[\frac{\partial}{\partial x} f + \frac{T_\gamma}{T_e} f(1+f) \right] + \frac{K_{\text{BR}} e^{-x_e}}{x_e^3} [1 - f(e^{x_e} - 1)] + \frac{K_{\text{DC}} e^{-2x}}{x^3} [1 - f(e^x - 1)] + S(\tau, x)$$

$$K_{\text{BR}} = \frac{\alpha}{2\pi} \frac{\lambda_e^3}{\sqrt{6\pi} \theta_e^{7/2}} \sum_i Z_i^2 N_i \bar{g}_{\text{ff}}(Z_i, T_e, T_\gamma, x_e), \quad K_{\text{DC}} = \frac{4\alpha}{3\pi} \theta_\gamma^2 I_{\text{dc}} g_{\text{dc}}(T_e, T_\gamma, x)$$

$$\bar{g}_{\text{ff}}(x_e) \approx \begin{cases} \frac{\sqrt{3}}{\pi} \ln\left(\frac{2.25}{x_e}\right) & \text{for } x_e \leq 0.37 \\ 1 & \text{otherwise} \end{cases}, \quad g_{\text{dc}} \approx \frac{1 + \frac{3}{2}x + \frac{29}{24}x^2 + \frac{11}{16}x^3 + \frac{5}{12}x^4}{1 + 19.739\theta_\gamma - 5.5797\theta_e}.$$

$$I_{\text{dc}} = \int x^4 f(1+f) dx \approx 4\pi^4/15$$

Ordinary matter temperature

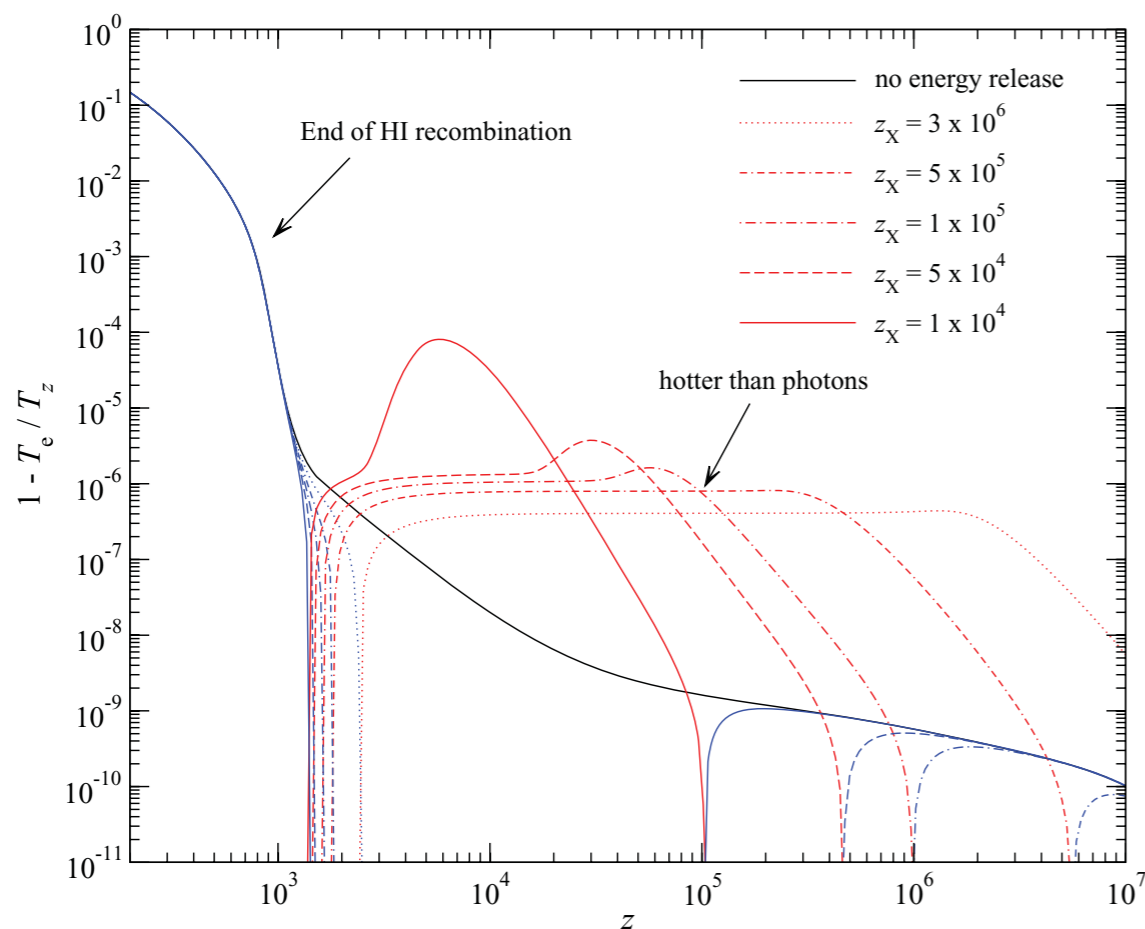
$$\frac{d\rho_e}{d\tau} = \frac{d(T_e/T_\gamma)}{d\tau} = \frac{t_{\text{T}} \dot{Q}}{\alpha_{\text{h}} \theta_\gamma} + \frac{4\tilde{\rho}_\gamma}{\alpha_{\text{h}}} [\rho_e^{\text{eq}} - \rho_e] - \frac{4\tilde{\rho}_\gamma}{\alpha_{\text{h}}} \mathcal{H}_{\text{DC, BR}}(\rho_e) - H t_{\text{T}} \rho_e.$$

$$k\alpha_{\text{h}} = \frac{3}{2} k [N_e + N_{\text{H}} + N_{\text{He}}] = \frac{3}{2} k N_{\text{H}} [1 + f_{\text{He}} + X_e] \quad \rho_e^{\text{eq}} = T_e^{\text{eq}}/T_\gamma$$

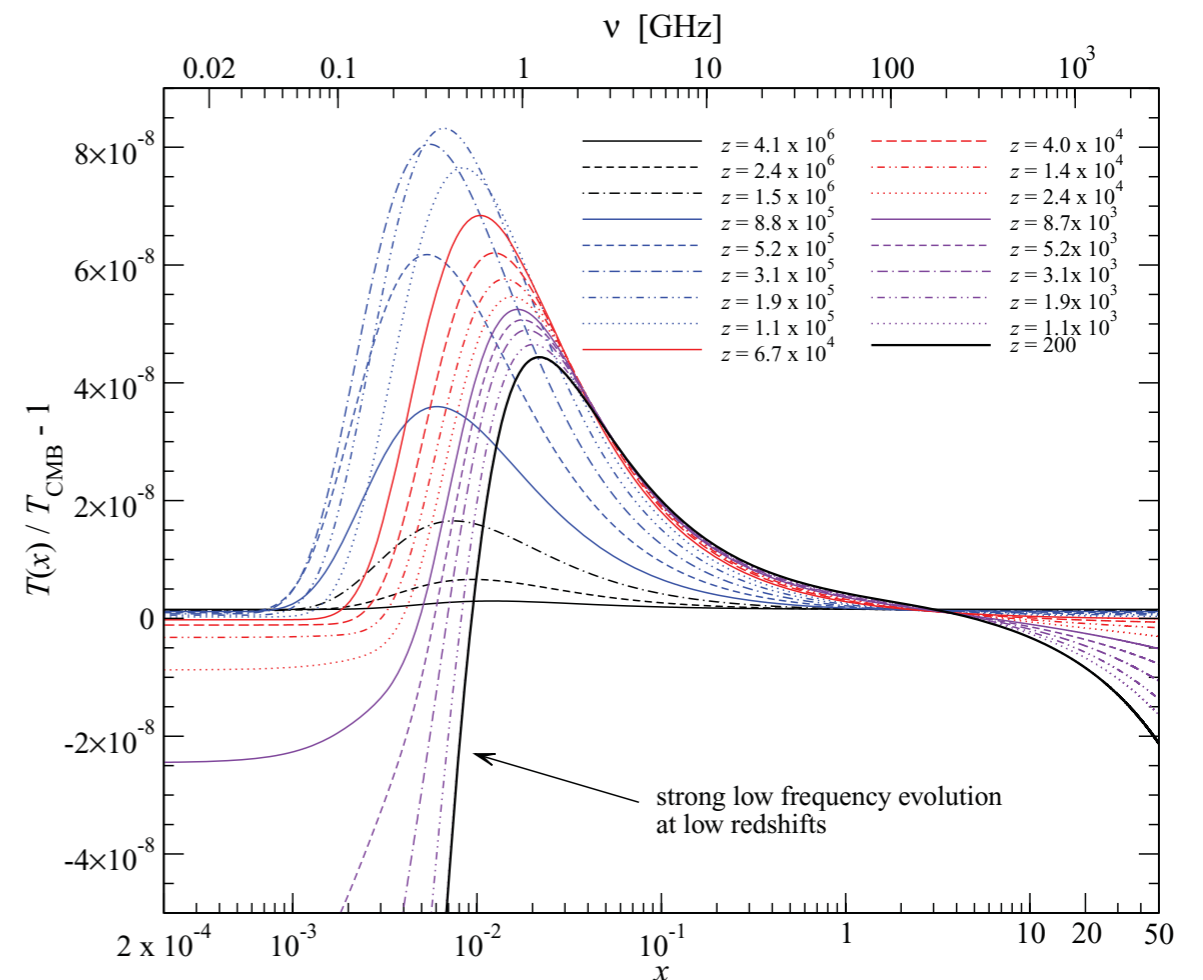
$$\tilde{\rho}_\gamma = \rho_\gamma/m_e c^2 \quad T_e^{\text{eq}} = T_\gamma \frac{\int x^4 f(1+f) dx}{4 \int x^3 f dx} \equiv \frac{h}{k} \frac{\int \nu^4 f(1+f) d\nu}{4 \int \nu^3 f d\nu}$$

CosmoTherm: a new flexible thermalization code

- Solve the thermalization problem for a *wide range* of energy release histories
- several scenarios already implemented (*decaying particles, damping of acoustic modes*)
- first *explicit* solution of time-dependent energy release scenarios
- open source code
- will be available at www.Chluba.de/CosmoTherm/
- Main reference: JC & Sunyaev, MNRAS, 2012 (arXiv:1109.6552)

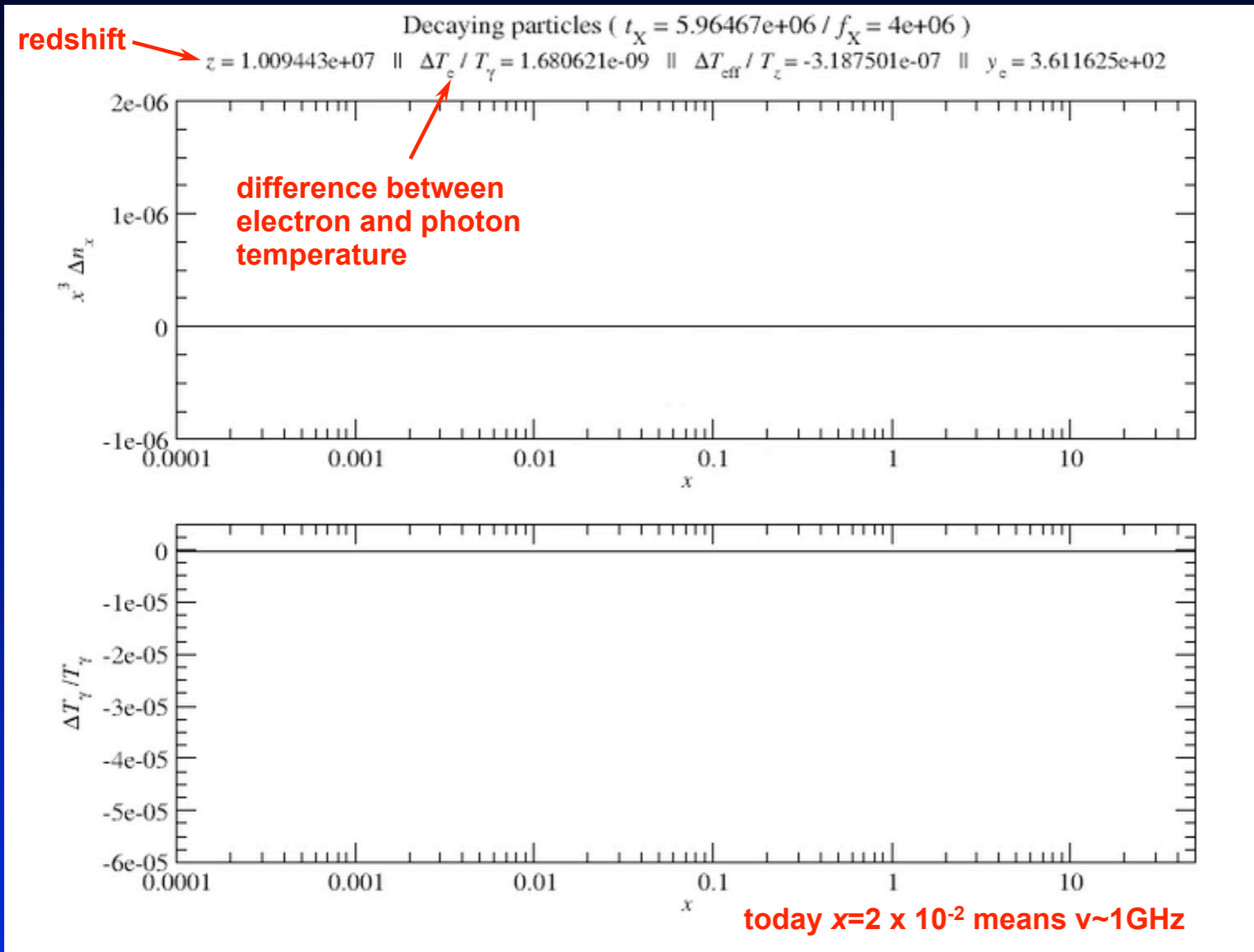


Electron temperature evolution



Evolution of distortion

Example: *Energy release by decaying relict particle*



- initial condition: *full equilibrium*
- total energy release: $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy released around: $z_X \sim 2 \times 10^6$
- positive μ -distortion
- high frequency distortion frozen around $z \approx 5 \times 10^5$
- late ($z < 10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

Quasi-Exact Treatment of the Thermalization Problem

- For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!
- Case-by-case computation of the distortion (e.g., with *CosmoTherm*, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming
- **But:** distortions are small \Rightarrow thermalization problem becomes linear!
- **Simple solution:** compute “response function” of the thermalization problem \Rightarrow Green’s function approach (JC, 2013, ArXiv:1304.6120)
- Final distortion for fixed energy-release history given by

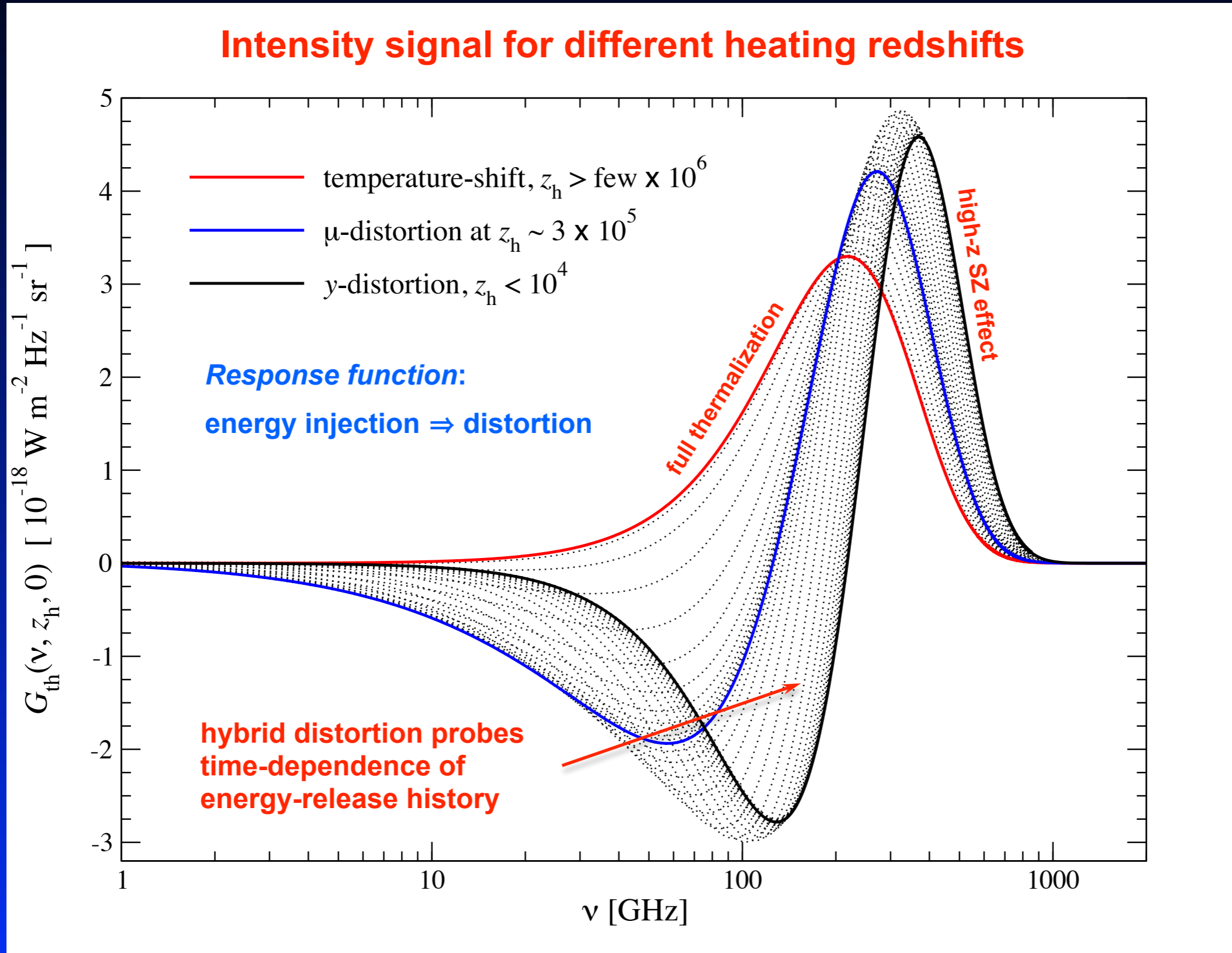
$$\Delta I_\nu \approx \int_0^\infty G_{\text{th}}(\nu, z') \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

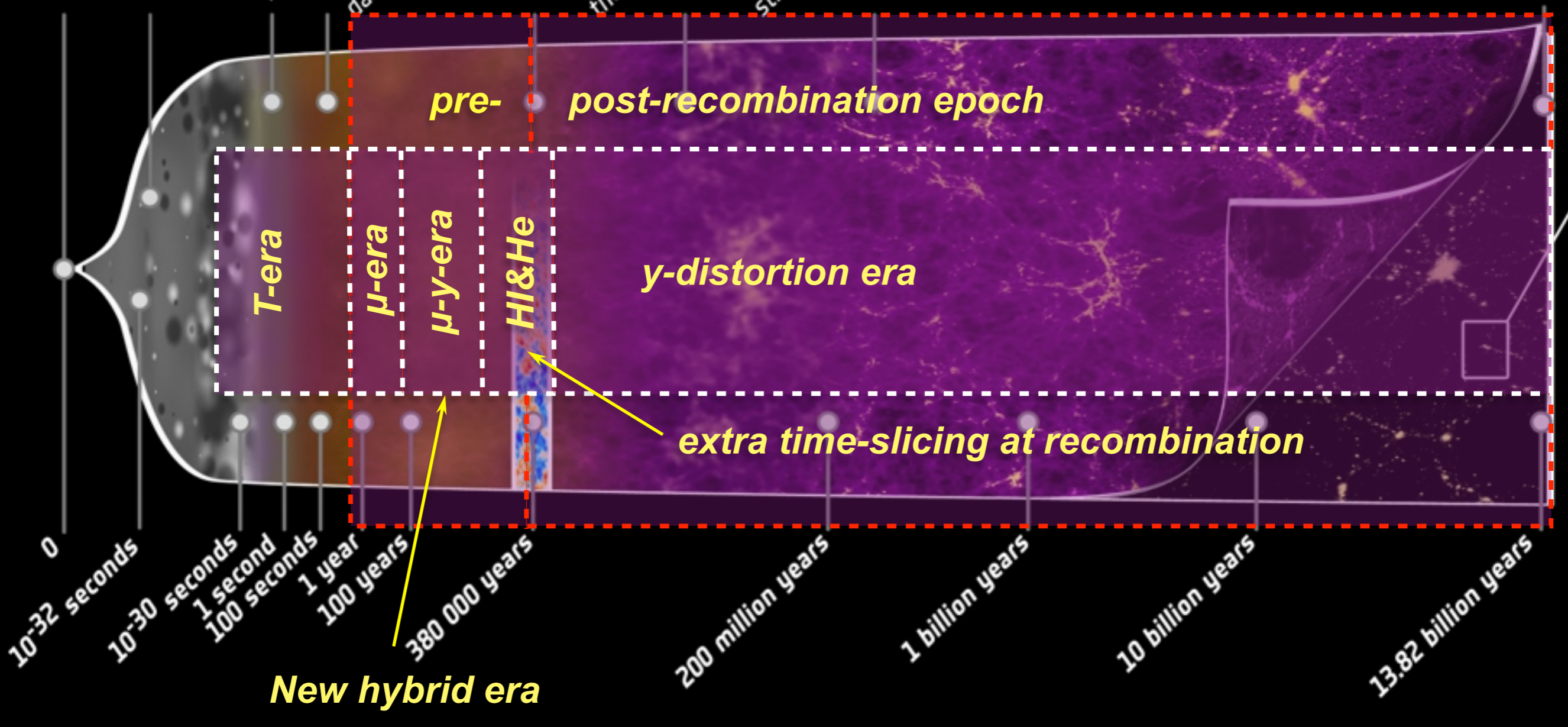
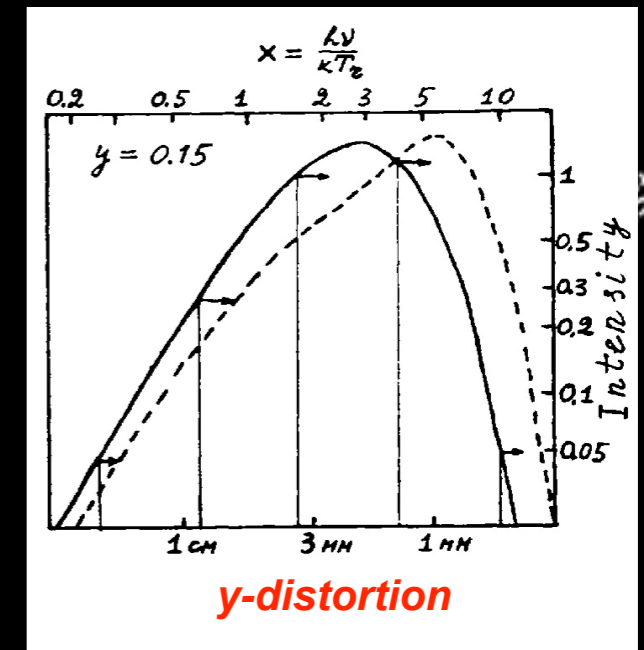
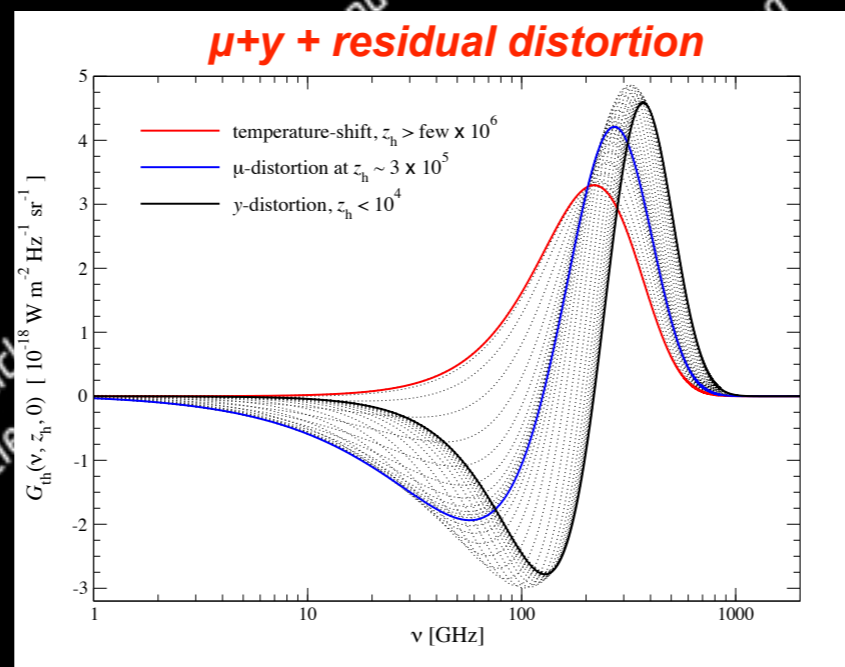
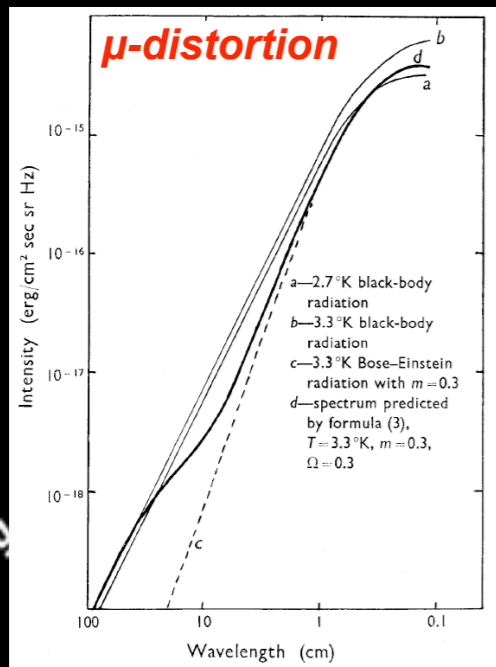
 **Thermalization Green’s function**

- **Fast and quasi-exact! No additional approximations!**

CosmoTherm available at: www.Chluba.de/CosmoTherm

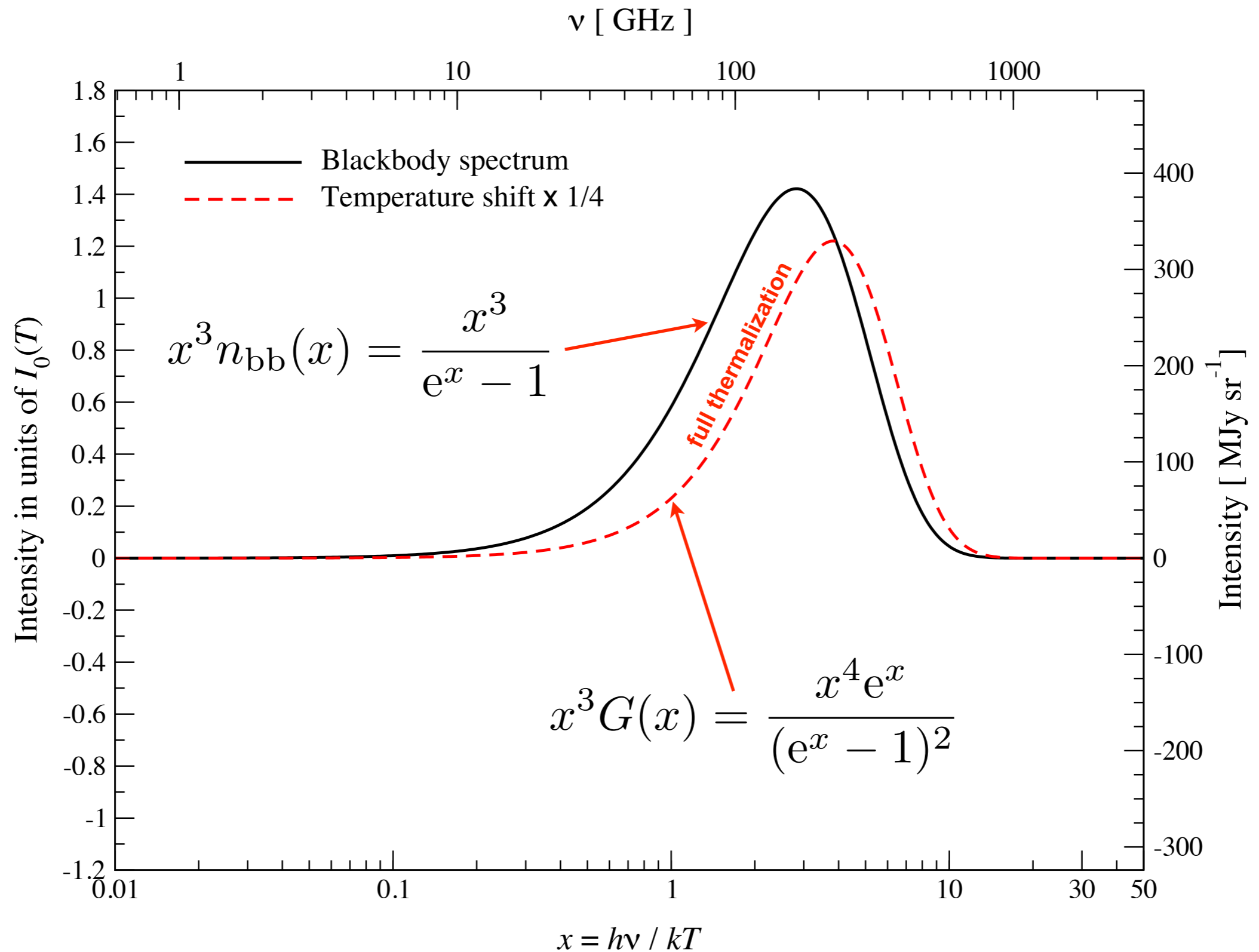
What does the spectrum look like after energy injection?





Exercises for simple spectral shapes

Simplest spectral shapes



$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270 \text{ MJy sr}^{-1}$$

Exercises for y and μ -distortion

Starting from Kompaneets Equation:

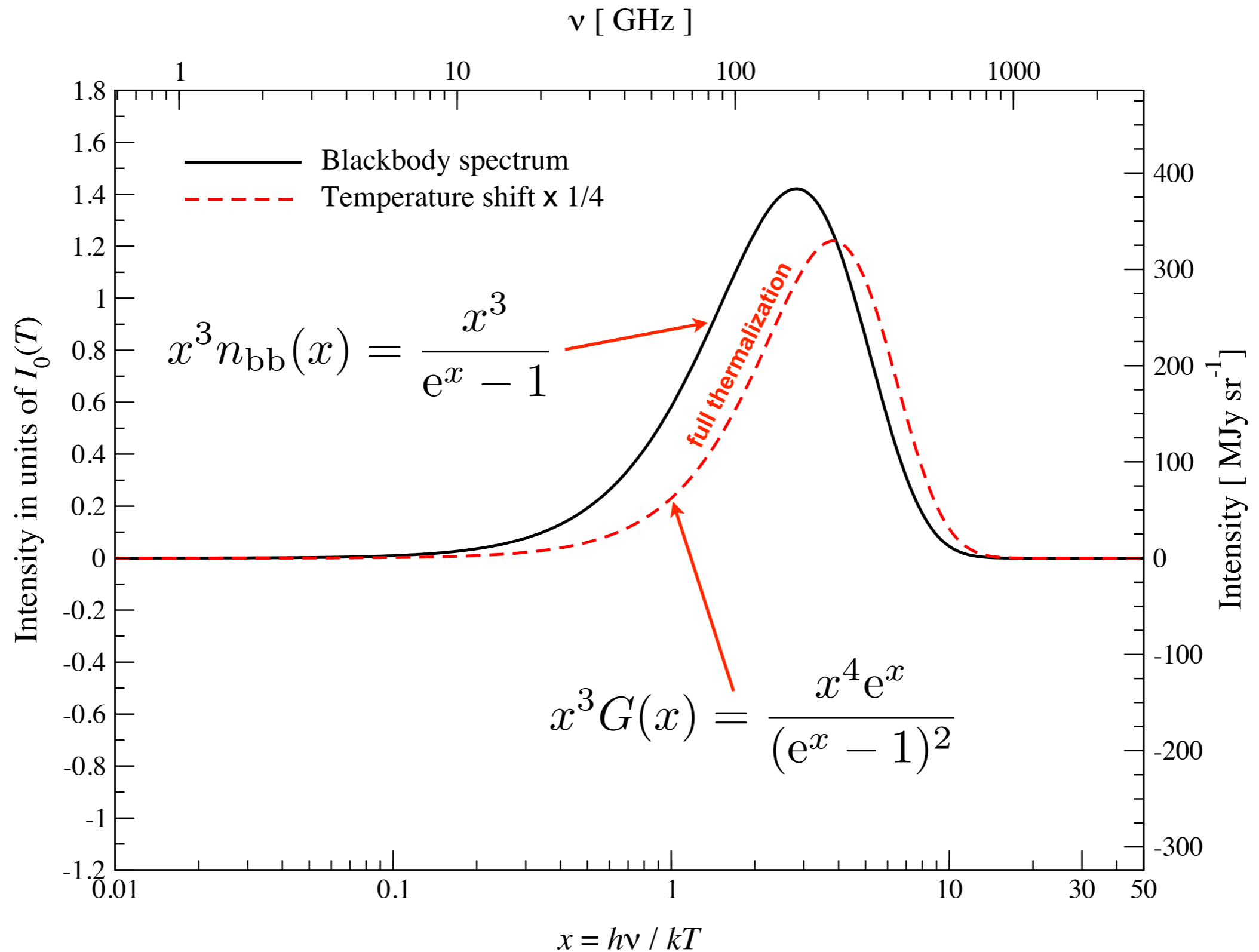
$$\frac{\partial f}{\partial \tau} \approx \frac{\theta_e}{x^2} \frac{\partial}{\partial x} x^4 \left[\frac{\partial}{\partial x} f + \frac{T_\gamma}{T_e} f(1 + f) \right]$$

1. Derive the spectrum of a y -type distortion by inserting a CMB blackbody $f_0 = \frac{1}{e^x - 1}$ and writing $\Delta f \approx \Delta \tau C[f_0]$

2a. Determine which photon occupation number makes the Compton collision term vanish
(Hint: write the occupation number as $f(x) = [e^{x+\mu(x)} - 1]^{-1}$)

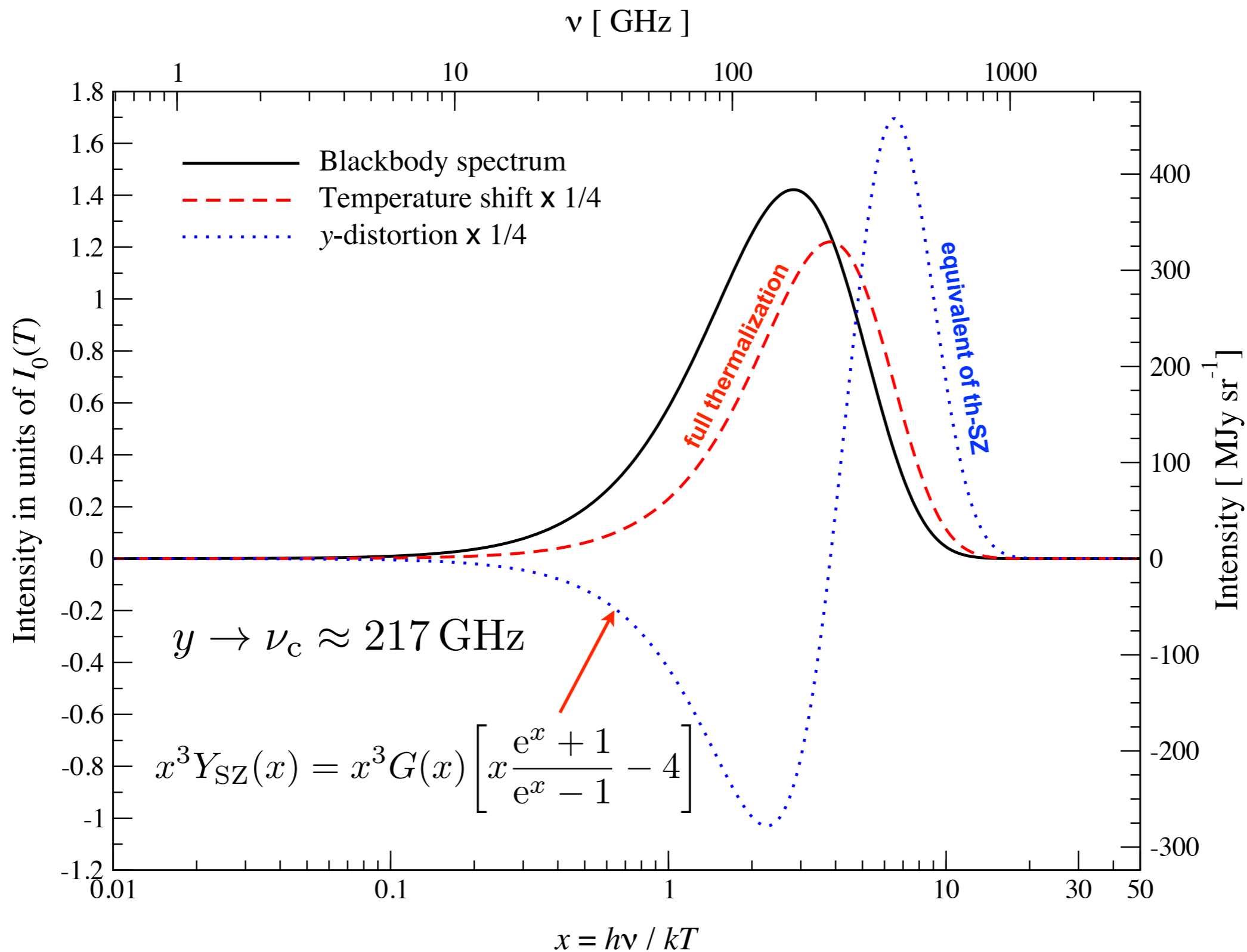
2b. What happens if you neglect simulated scattering terms?

Simplest spectral shapes



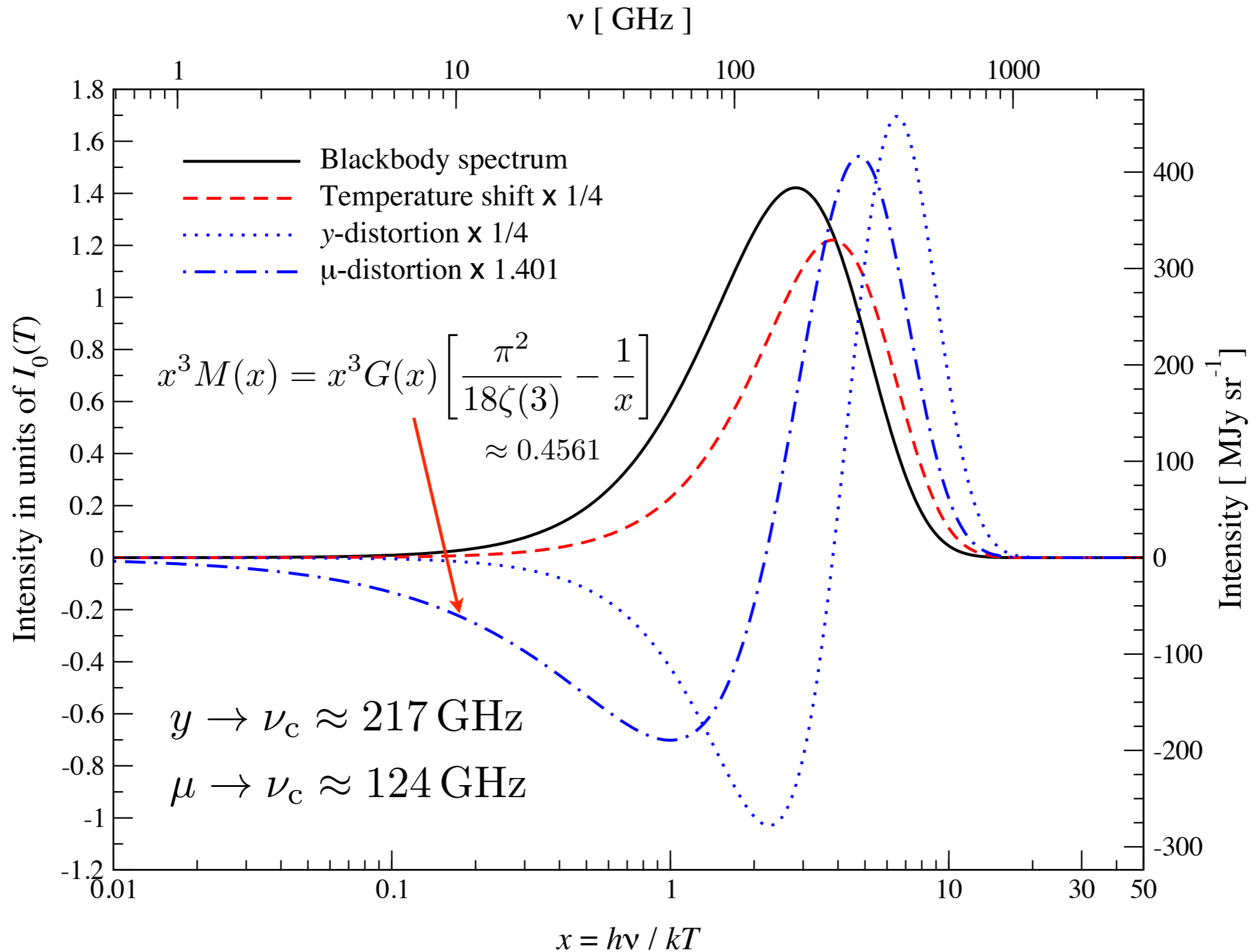
$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270 \text{ MJy sr}^{-1}$$

Simplest spectral shapes



$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270 \text{ MJy sr}^{-1}$$

Simplest spectral shapes



$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270 \text{ MJy sr}^{-1}$$

Energy release histories

Energy release histories for some cases

Adiabatic cooling

$$\begin{aligned} \frac{d(Q/\rho_\gamma)}{dz} &= -\frac{3}{2} \frac{N_{\text{tot}} k T_\gamma}{\rho_\gamma (1+z)} \\ &\approx -\frac{5.71 \times 10^{-10}}{(1+z)} \left[\frac{(1 - Y_p)}{0.7533} \right] \left[\frac{\Omega_b h^2}{0.02225} \right] \\ &\quad \times \left[\frac{(1 + f_{\text{He}} + X_e)}{2.246} \right] \left[\frac{T_0}{2.726 \text{ K}} \right]^{-3} \end{aligned}$$

Annihilation

$$\frac{d(Q/\rho_\gamma)}{dz} = f_{\text{ann}} \frac{N_{\text{H}}(z)(1+z)^{2+\lambda}}{H(z)\rho_\gamma(z)}$$

Decay

$$\left. \frac{d(Q/\rho_\gamma)}{dz} \right|_{\text{dec}} \approx \epsilon_X \frac{N_{\text{H}}(z)(1+z_X)\Gamma_X}{H(z)\rho_\gamma(z)(1+z)} \exp(-\Gamma_X t)$$

Dissipation of acoustic modes

$$\frac{d(Q/\rho_\gamma)}{dz} \approx 4A^2 \partial_z k_{\text{D}}^{-2} \int_{k_{\text{min}}}^{\infty} \frac{k^4 dk}{2\pi^2} P_\zeta(k) e^{-2k^2/k_{\text{D}}^2}$$

$$A^2 \approx (1 + 4R_\nu/15)^{-2} \approx 0.813 \quad k_{\text{D}} \approx 4.048 \times 10 (1+z)^{3/2} \text{Mpc}^{-1}$$

$$k_{\text{min}} \approx 0.12 \text{Mpc}^{-1}$$

Exercises for energy release histories

3a. Explain the parametrization for the energy release history of a decaying particle

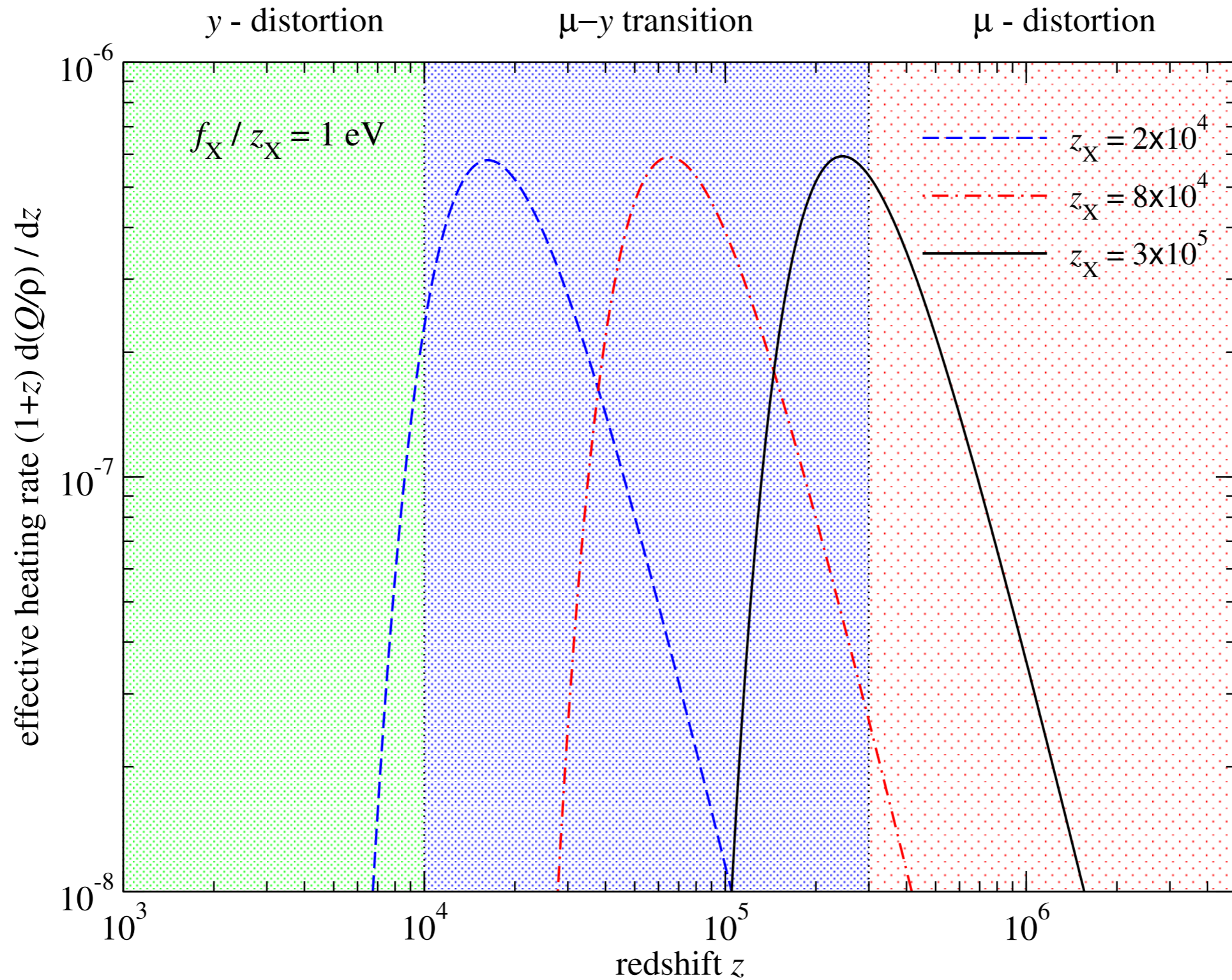
$$\left. \frac{d(Q/\rho_\gamma)}{dz} \right|_{\text{dec}} \approx \epsilon_X \frac{N_H(z)(1+z_X)\Gamma_X}{H(z)\rho_\gamma(z)(1+z)} \exp(-\Gamma_X t)$$

3b. How does this energy release history scale before and after $t_X = \Gamma_X^{-1}$?

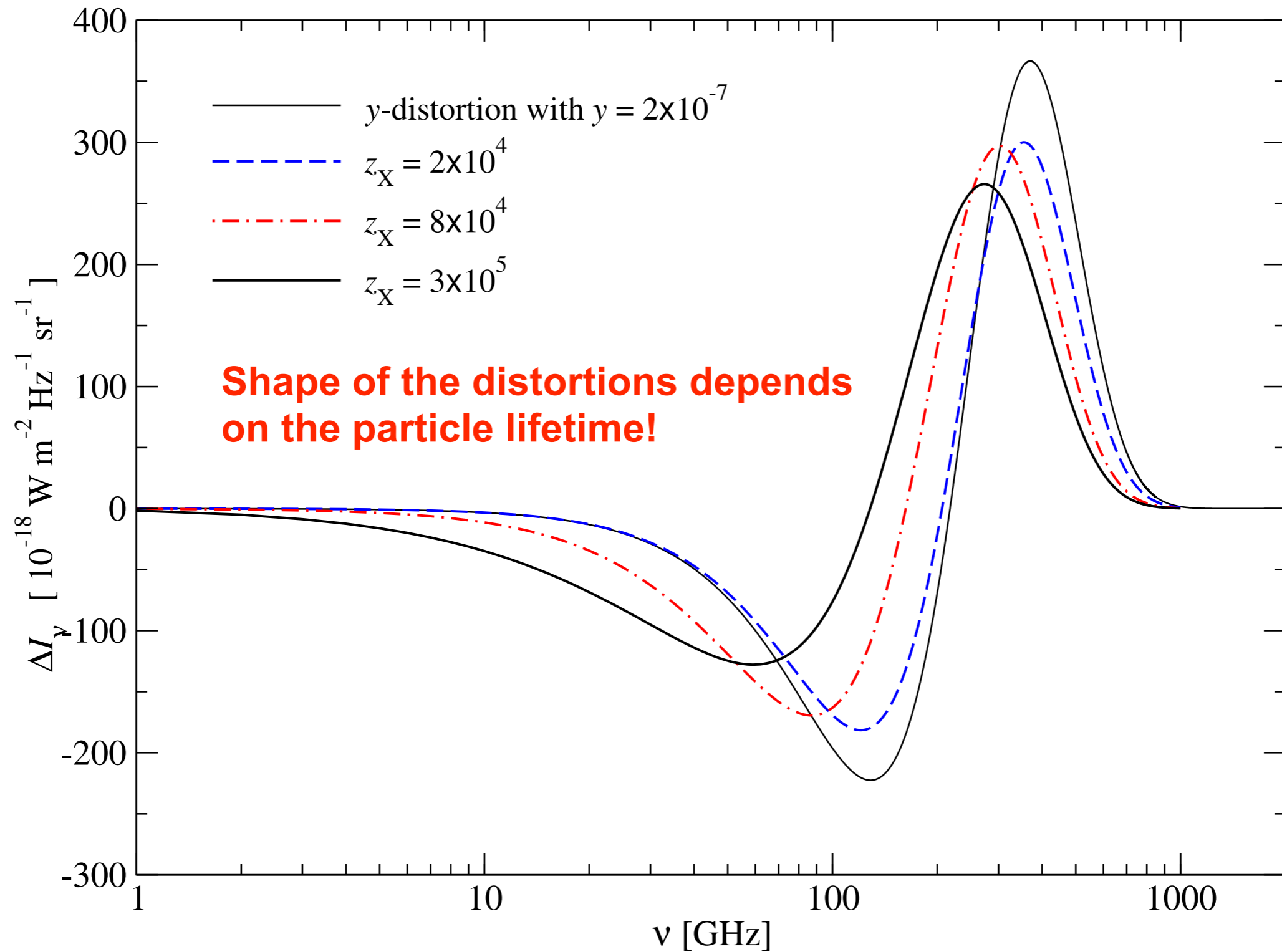
4. Can you qualitatively explain the adiabatic cooling term?

$$\frac{d(Q/\rho_\gamma)}{dz} = -\frac{3}{2} \frac{N_{\text{tot}} k T_\gamma}{\rho_\gamma (1+z)}$$

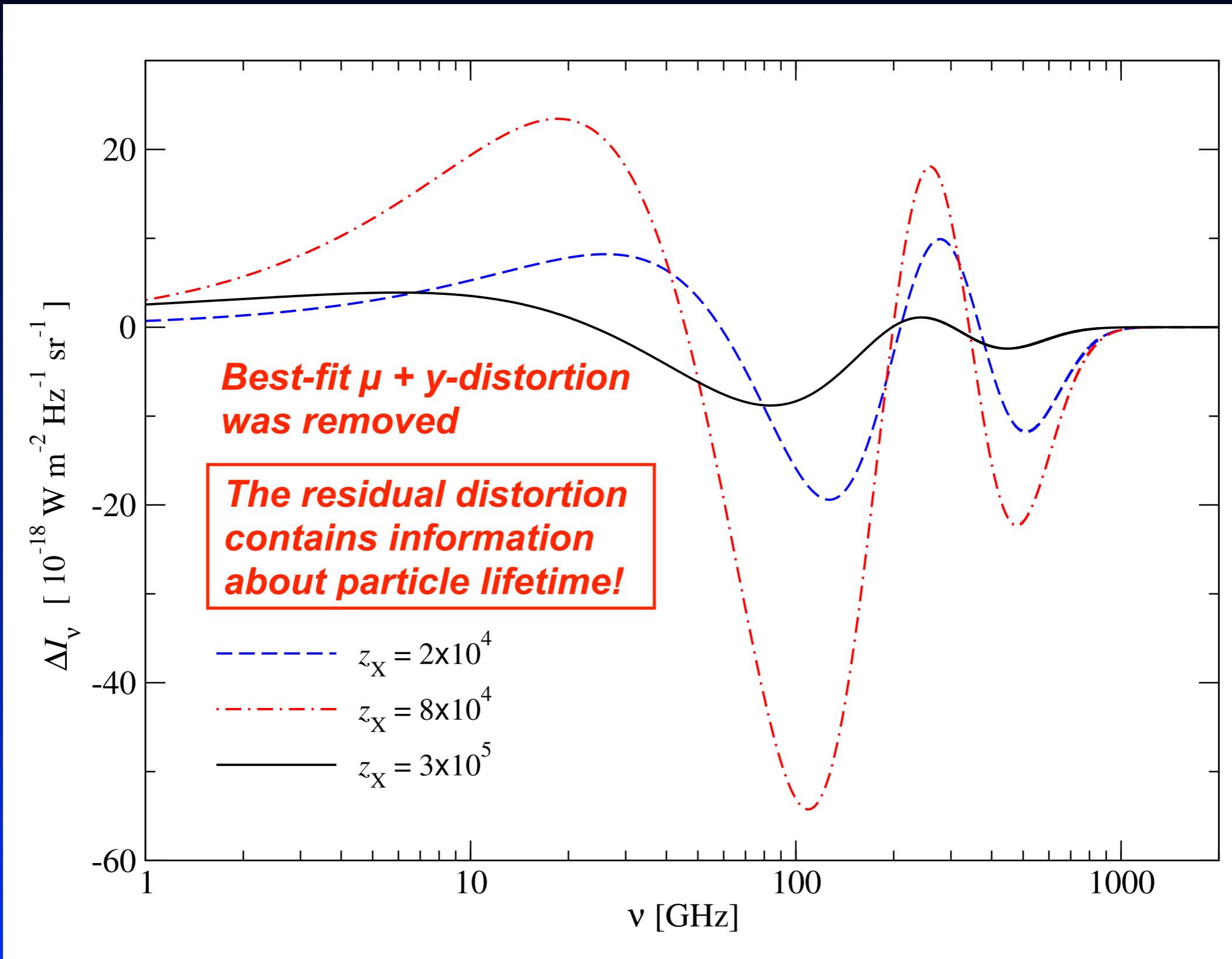
Decaying particle scenarios



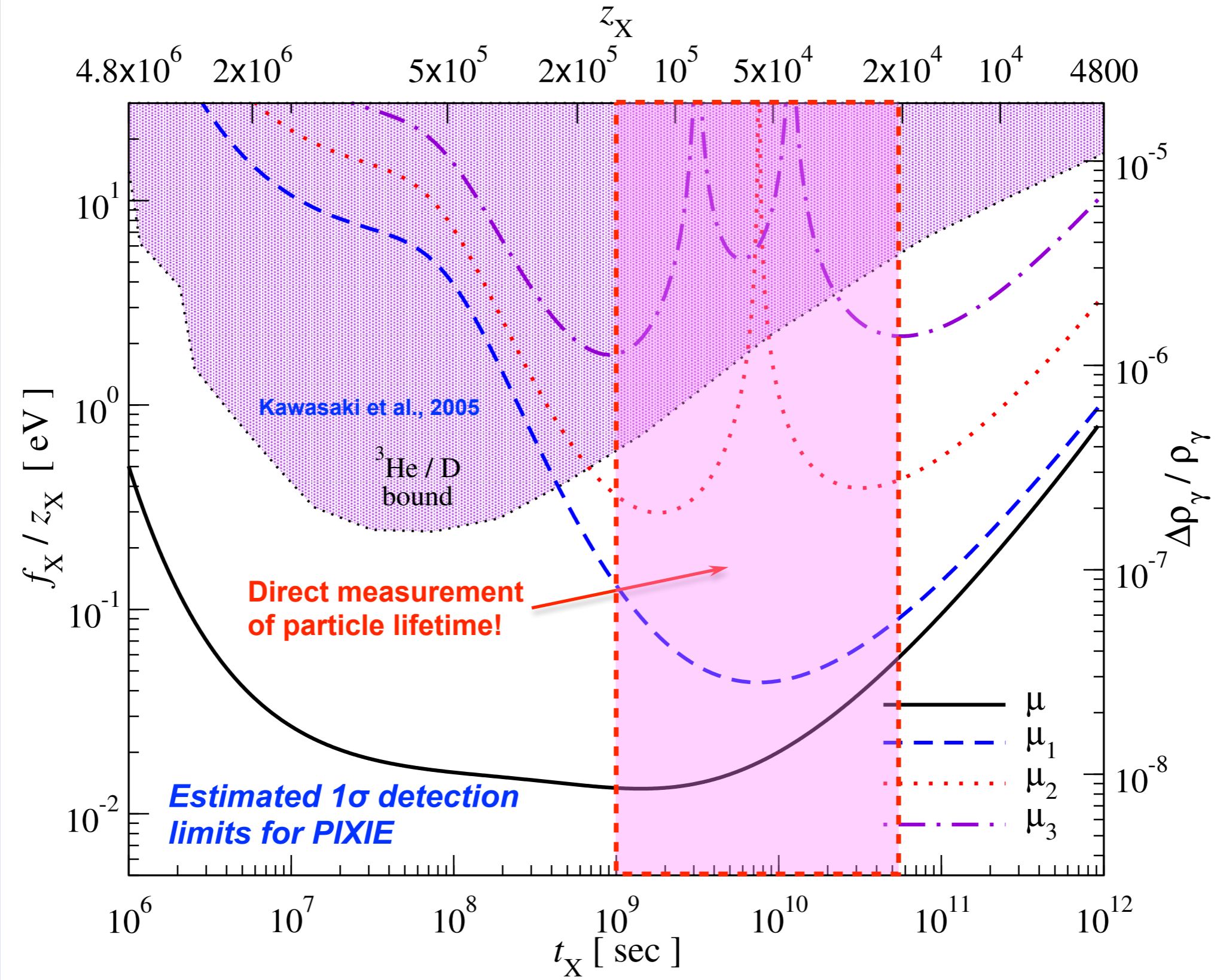
Decaying particle scenarios



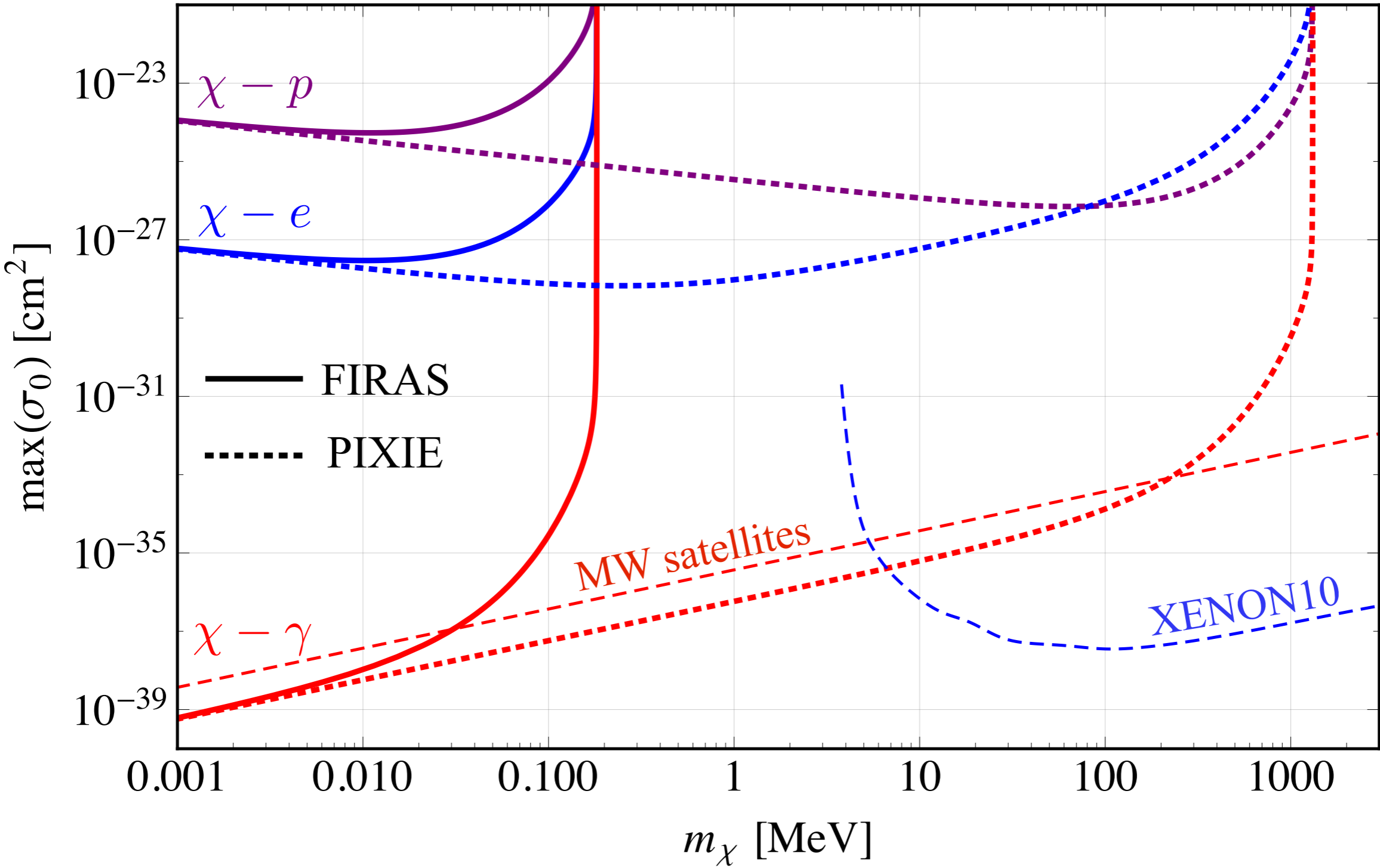
Decaying particle scenarios (information in residual)



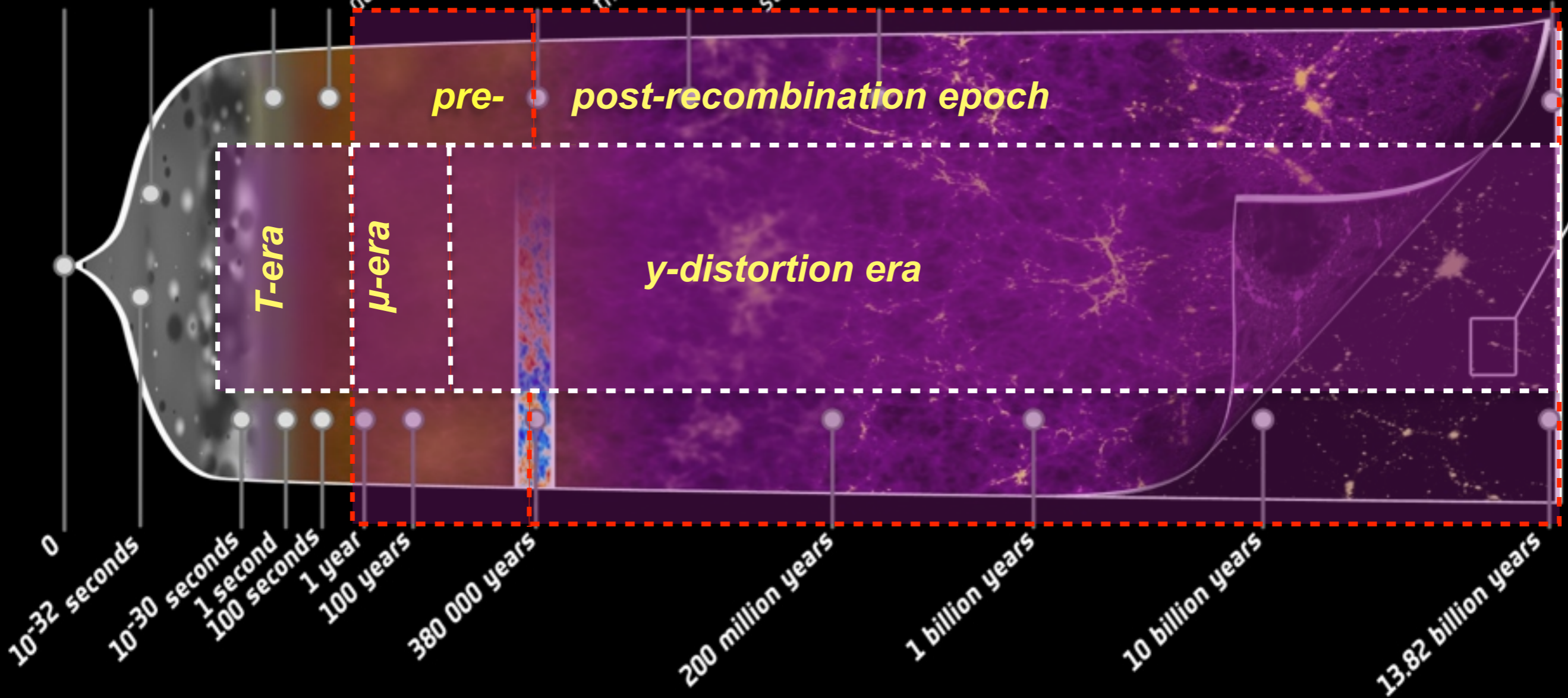
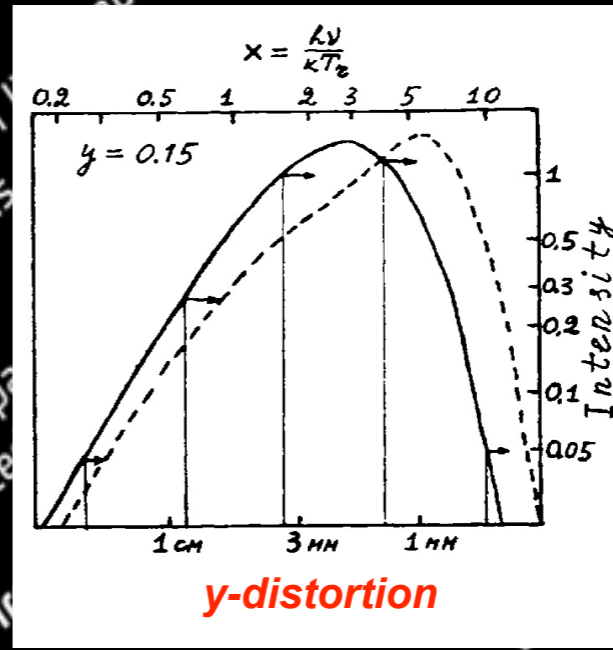
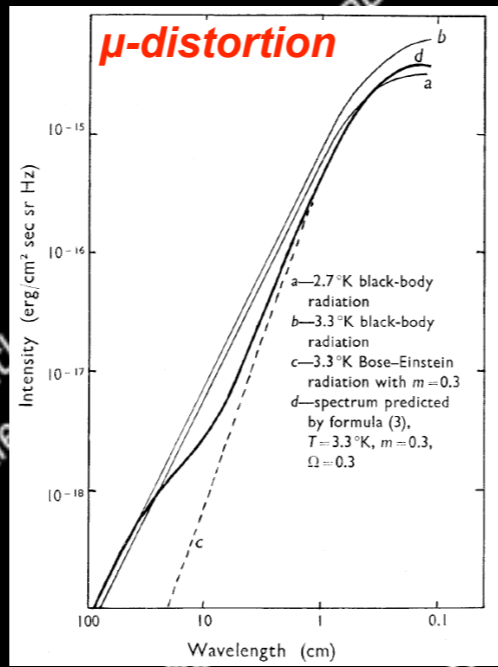
Distortions could shed light on decaying (DM) particles!



Distortion constraints on DM interactions through adiabatic cooling effect



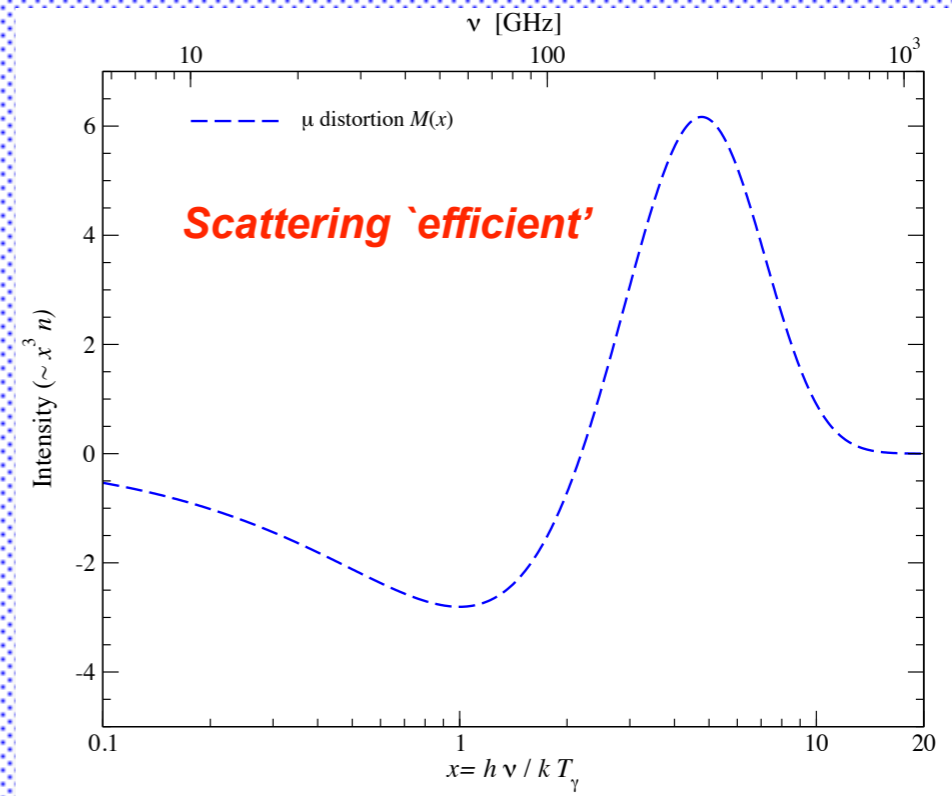
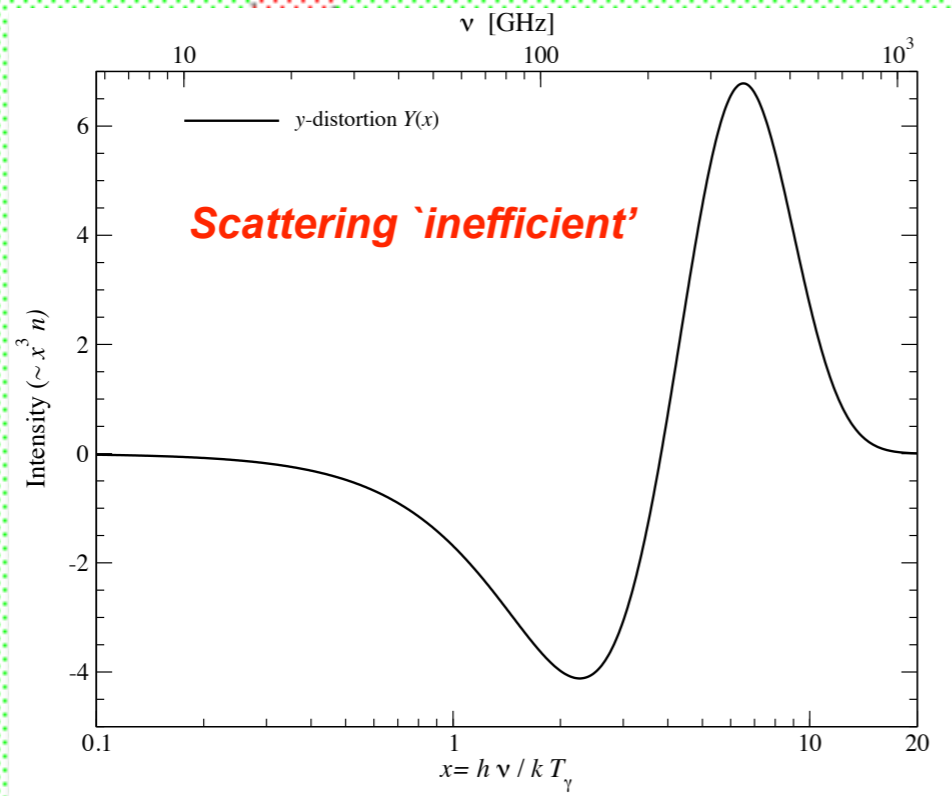
Simple analytic approximations for estimates



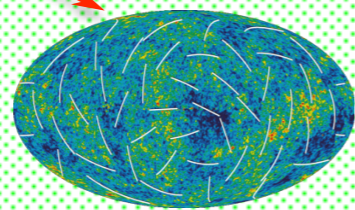
y - distortion

μ -y transition

μ - distortion



Last Scattering Surface



CMB anisotropies

$t_K \simeq t_{\text{exp}}$

10^2 10^3 10^4 10^5 10^6 10^7

redshift z

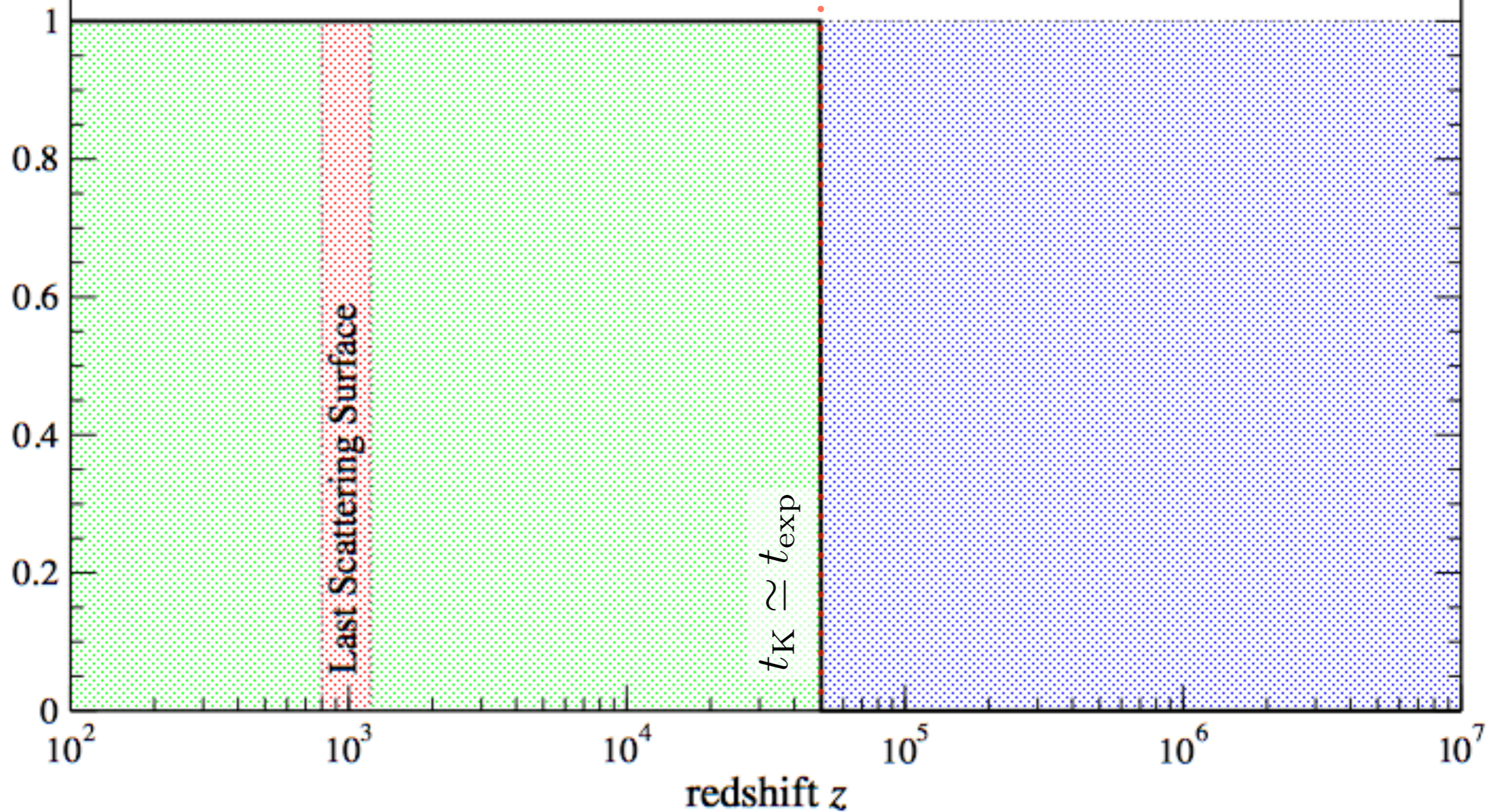
y - distortion

μ -y transition

μ - distortion

$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_{z_{\text{rec}}}^{z_{\mu y}} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

Visibility



y - distortion

μ -y transition

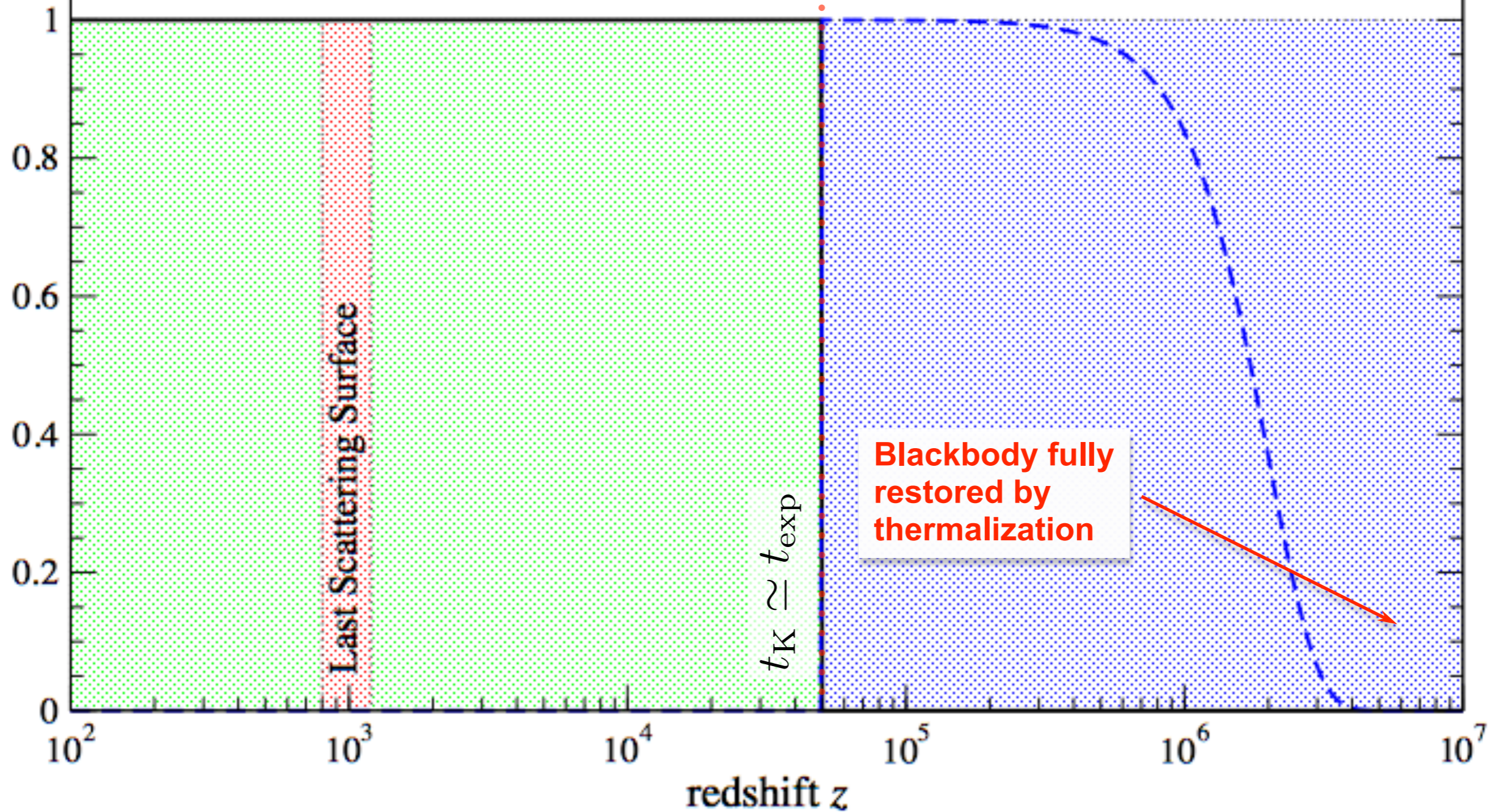
μ - distortion

$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_{z_{\text{rec}}}^{z_{\mu y}} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

$$\mu \approx 1.4 \int_{z_{\mu y}}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{1.98 \times 10^6}\right)^{5/2}}$$

Visibility



y - distortion

μ -y transition

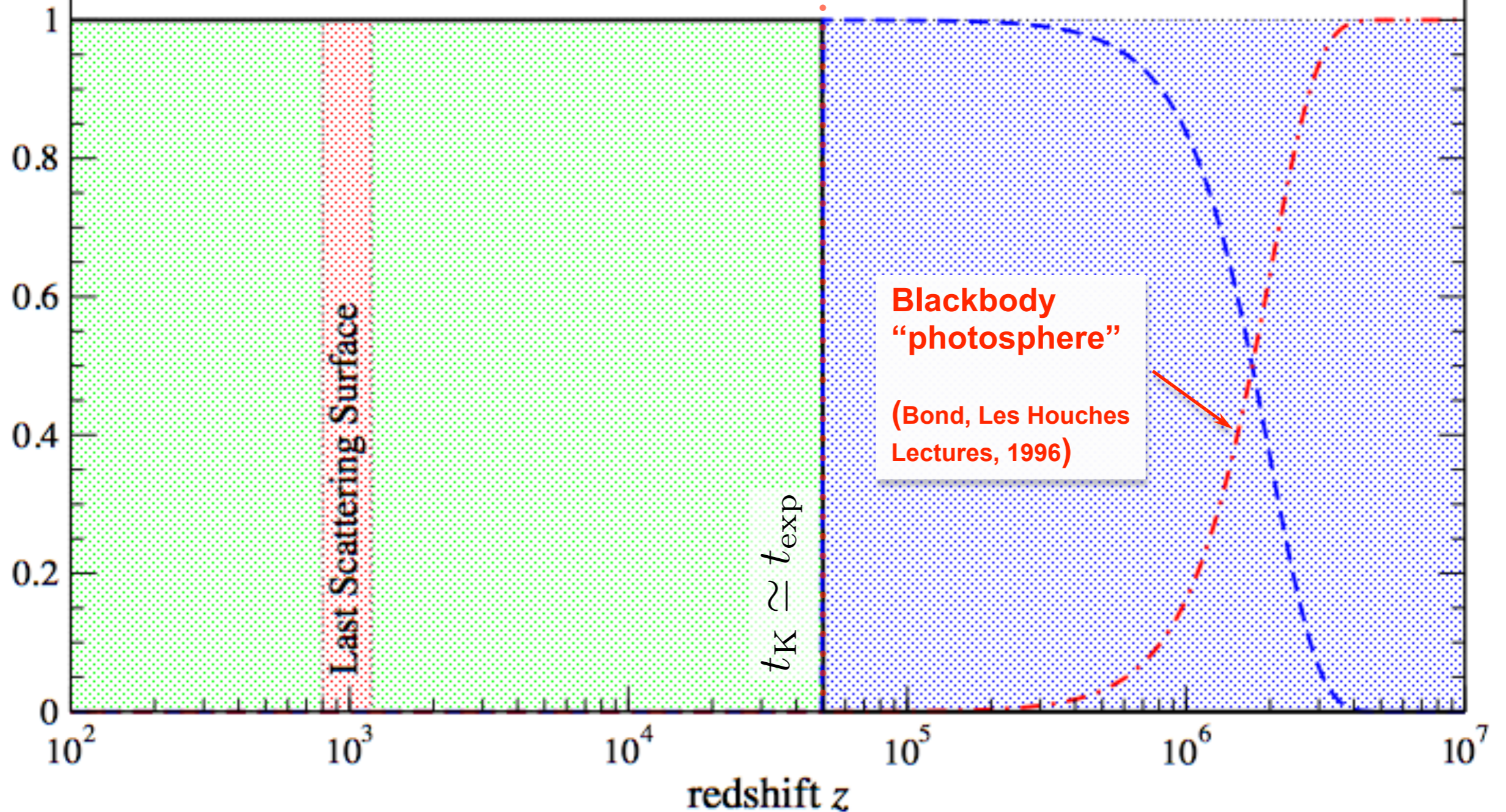
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Visibility



y - distortion

μ -y transition

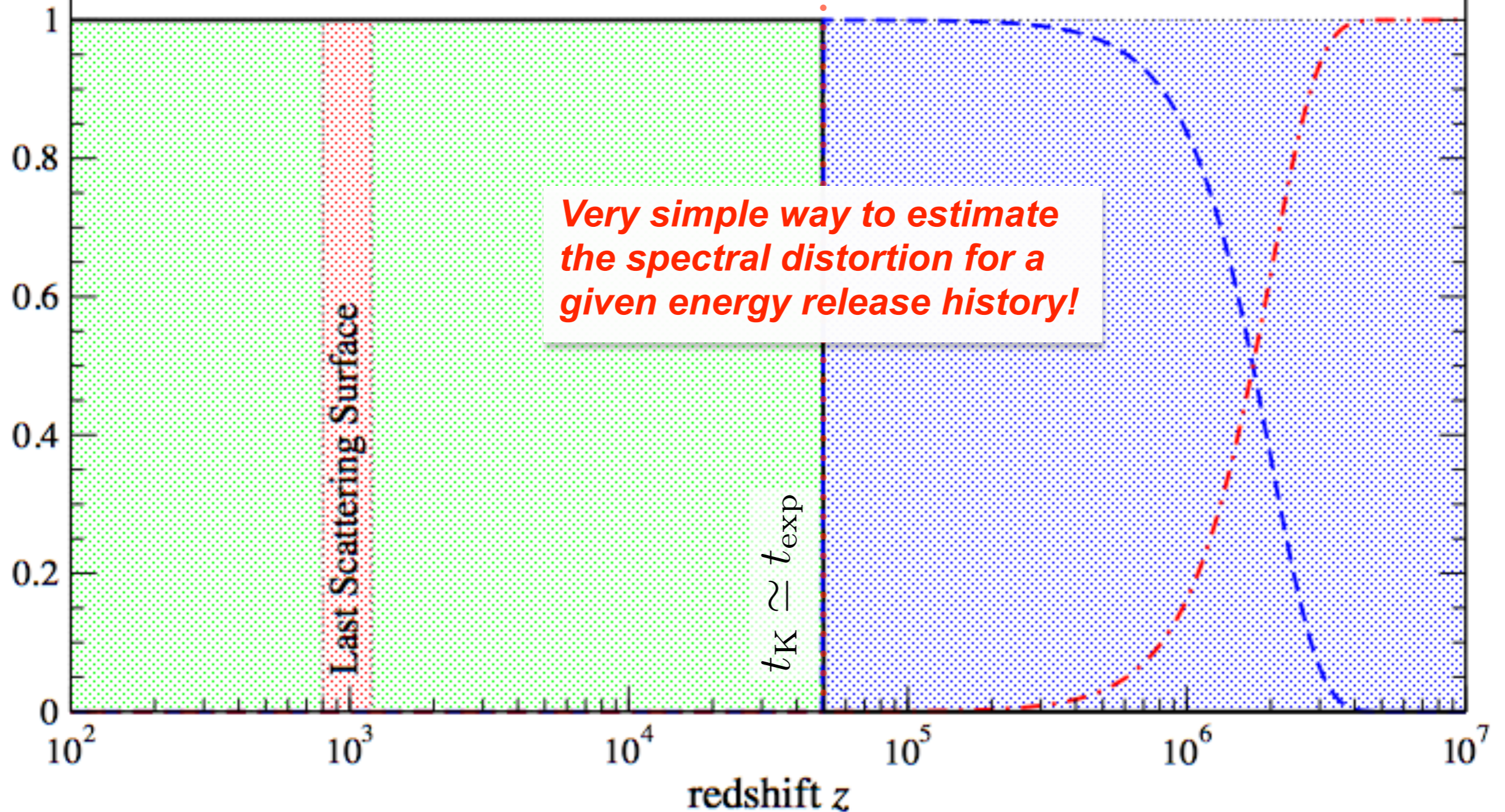
μ - distortion

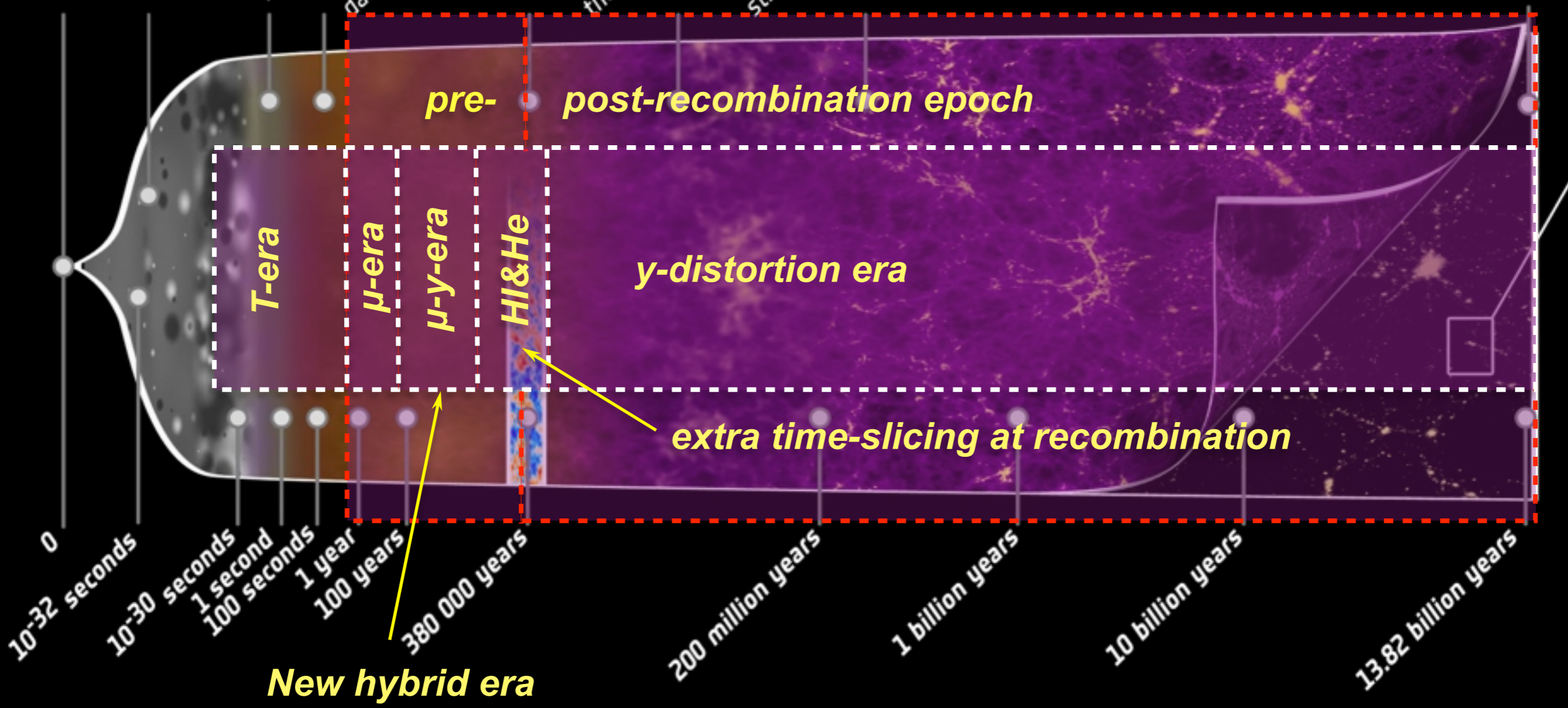
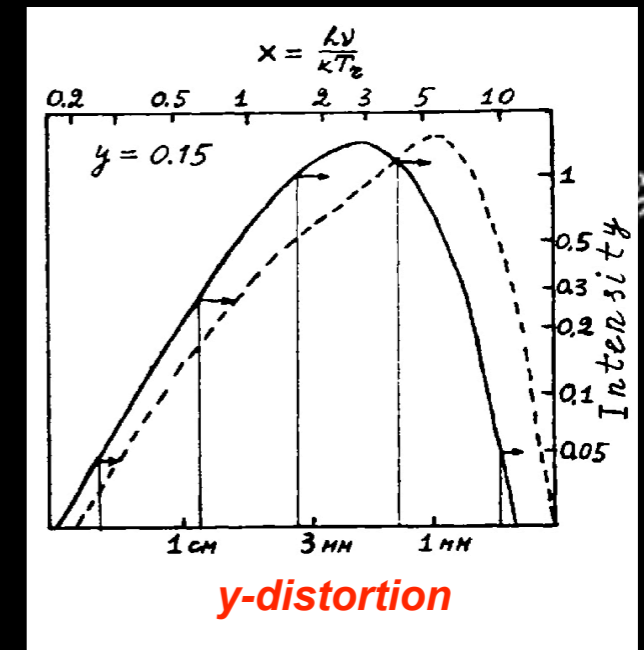
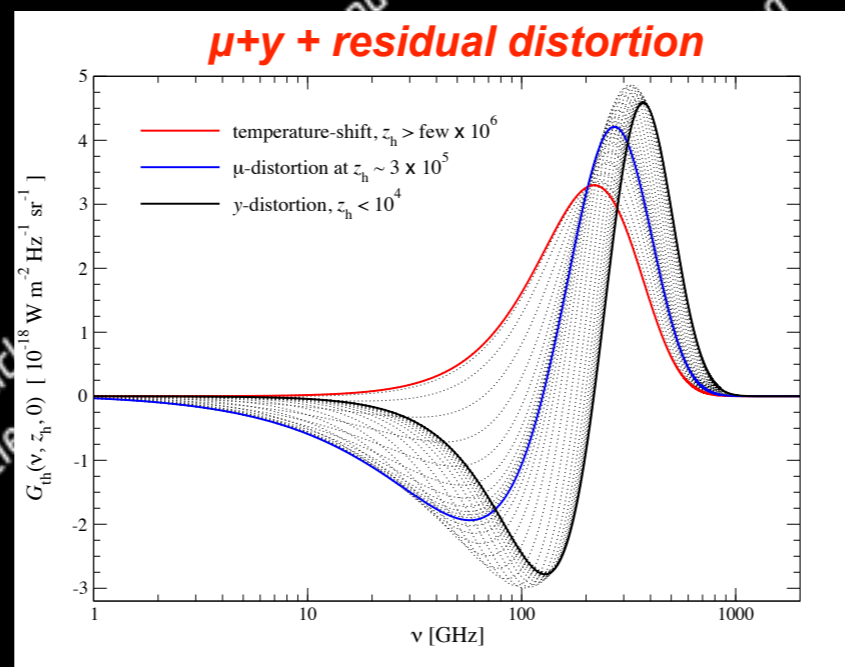
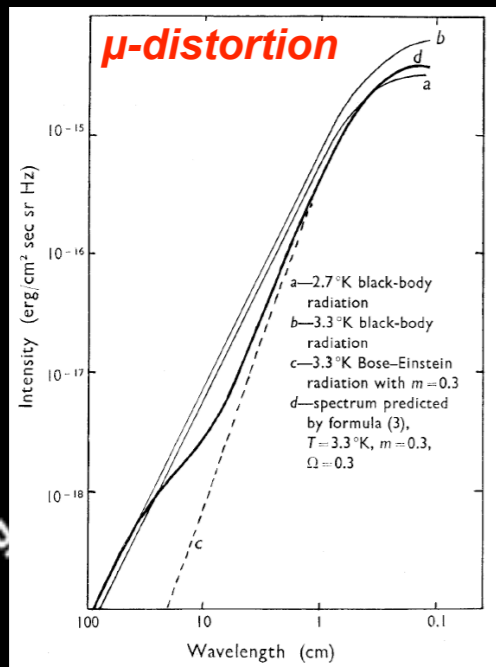
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$$\mu \approx 1.4 \int_{z_{\mu y}}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{1.98 \times 10^6}\right)^{5/2}}$$

Visibility





y - distortion

μ -y transition

μ - distortion

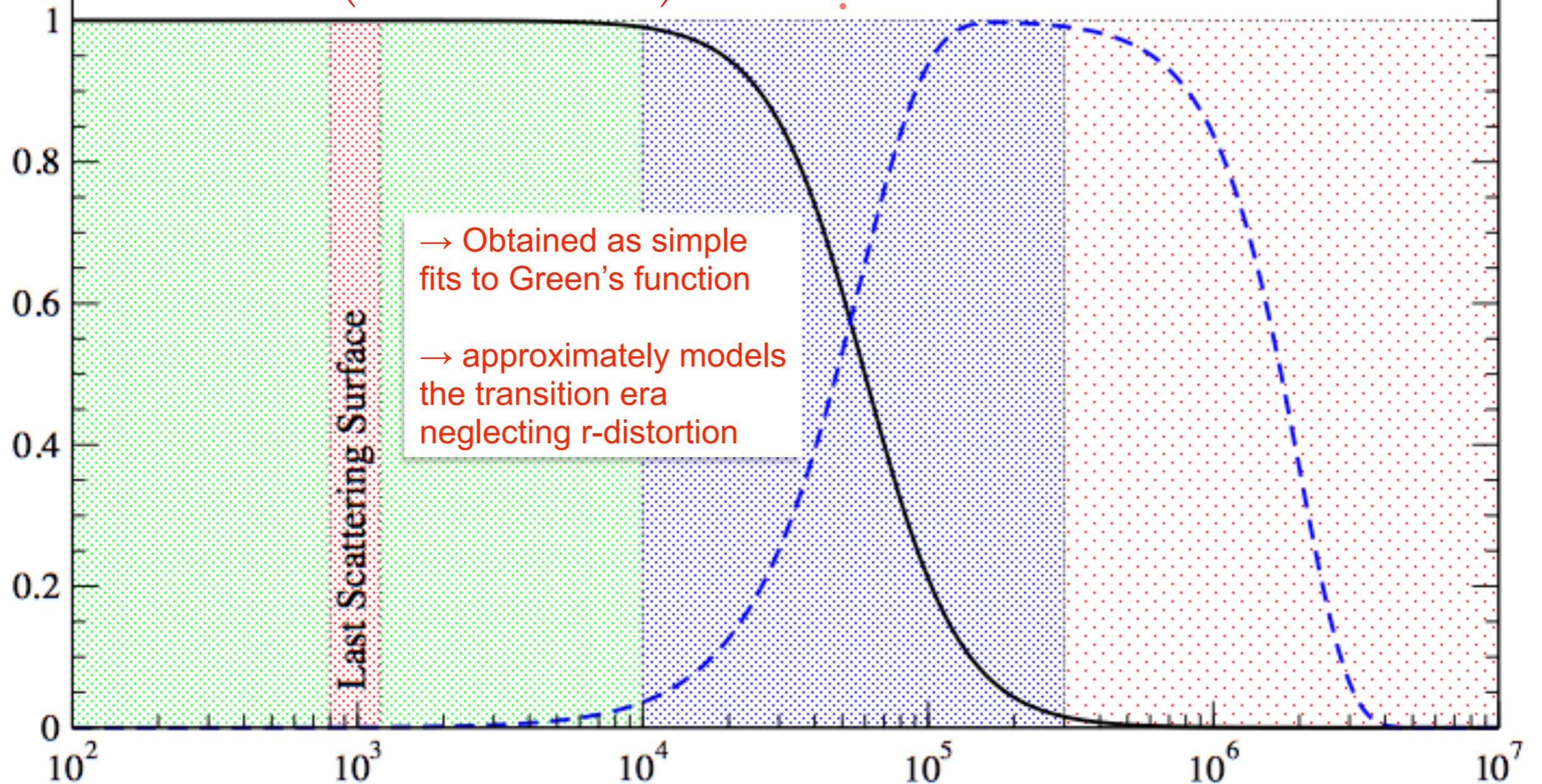
$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_{z_{\text{rec}}}^{z_{\mu y}} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

$$\mu \approx 1.4 \int_{z_{\mu y}}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_y(z) \approx \left(1 + \left[\frac{1+z}{6.0 \times 10^4} \right]^{2.58} \right)^{-1}$$

$$\mathcal{J}_\mu(z) \approx \left[1 - e^{-\left[\frac{1+z}{5.8 \times 10^4} \right]^{1.88}} \right] e^{-\left[\frac{z}{2 \times 10^6} \right]^{2.5}}$$

Visibility



→ Obtained as simple fits to Green's function
 → approximately models the transition era neglecting r-distortion

Last Scattering Surface

y - distortion

μ -y transition

μ - distortion

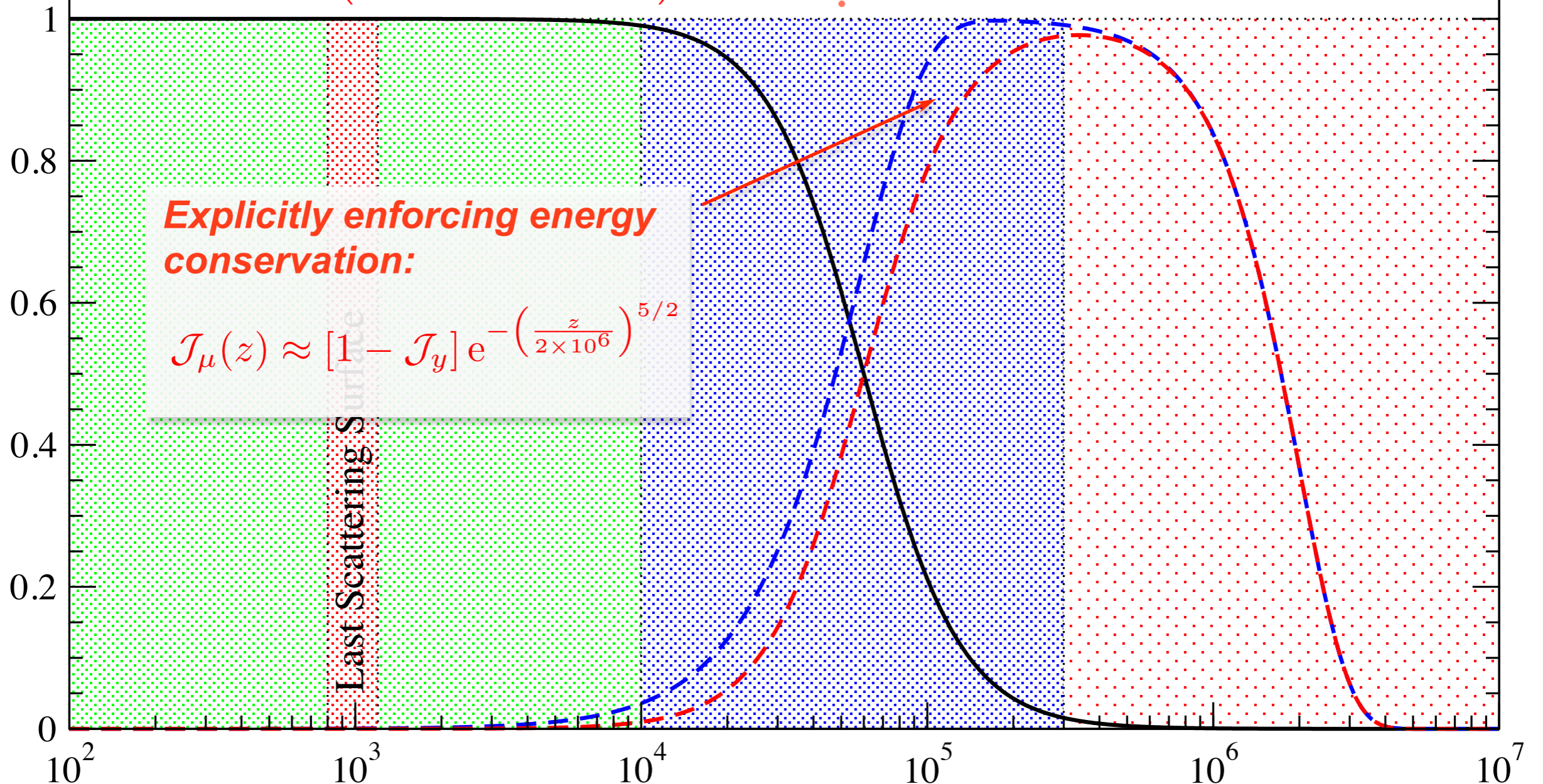
$$y \approx \frac{1}{4} \int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_y(z') dz'$$

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$$\mathcal{J}_\mu(z) \approx \left[1 - e^{-\left[\frac{1+z}{5.8 \times 10^4} \right]^{1.88}} \right] e^{-\left[\frac{z}{2 \times 10^6} \right]^{2.5}}$$

Visibility



Explicitly enforcing energy conservation:

$$\mathcal{J}_\mu(z) \approx [1 - \mathcal{J}_y] e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Last Scattering Surface

y - distortion

μ -y transition

μ - distortion

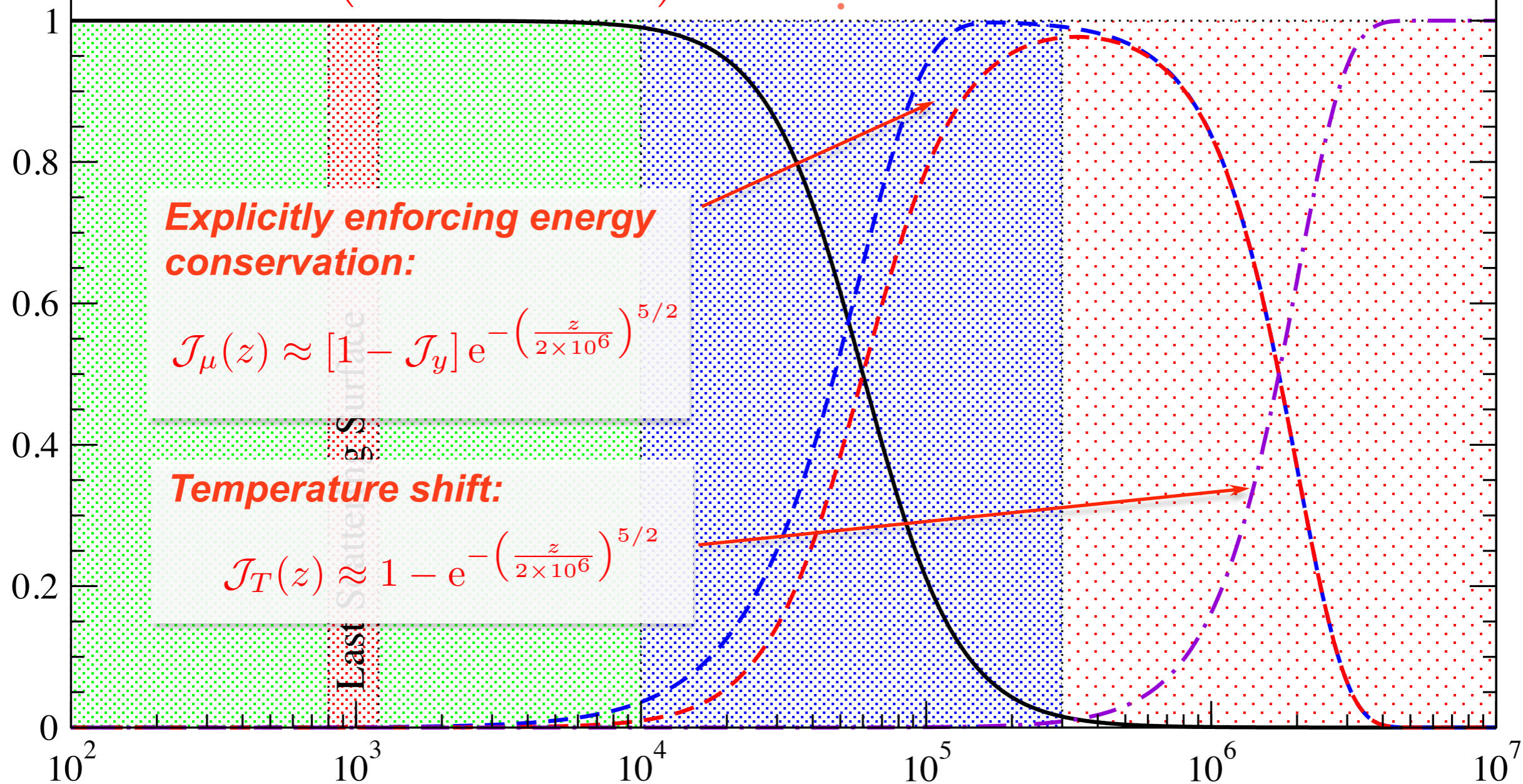
$$y \approx \frac{1}{4} \int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_y(z') dz'$$

$$\mu \approx 1.4 \int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_y(z) \approx \left(1 + \left[\frac{1+z}{6.0 \times 10^4} \right]^{2.58} \right)^{-1}$$

$$\mathcal{J}_\mu(z) \approx \left[1 - e^{-\left[\frac{1+z}{5.8 \times 10^4} \right]^{1.88}} \right] e^{-\left[\frac{z}{2 \times 10^6} \right]^{2.5}}$$

Visibility



Explicitly enforcing energy conservation:

$$\mathcal{J}_\mu(z) \approx [1 - \mathcal{J}_y] e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

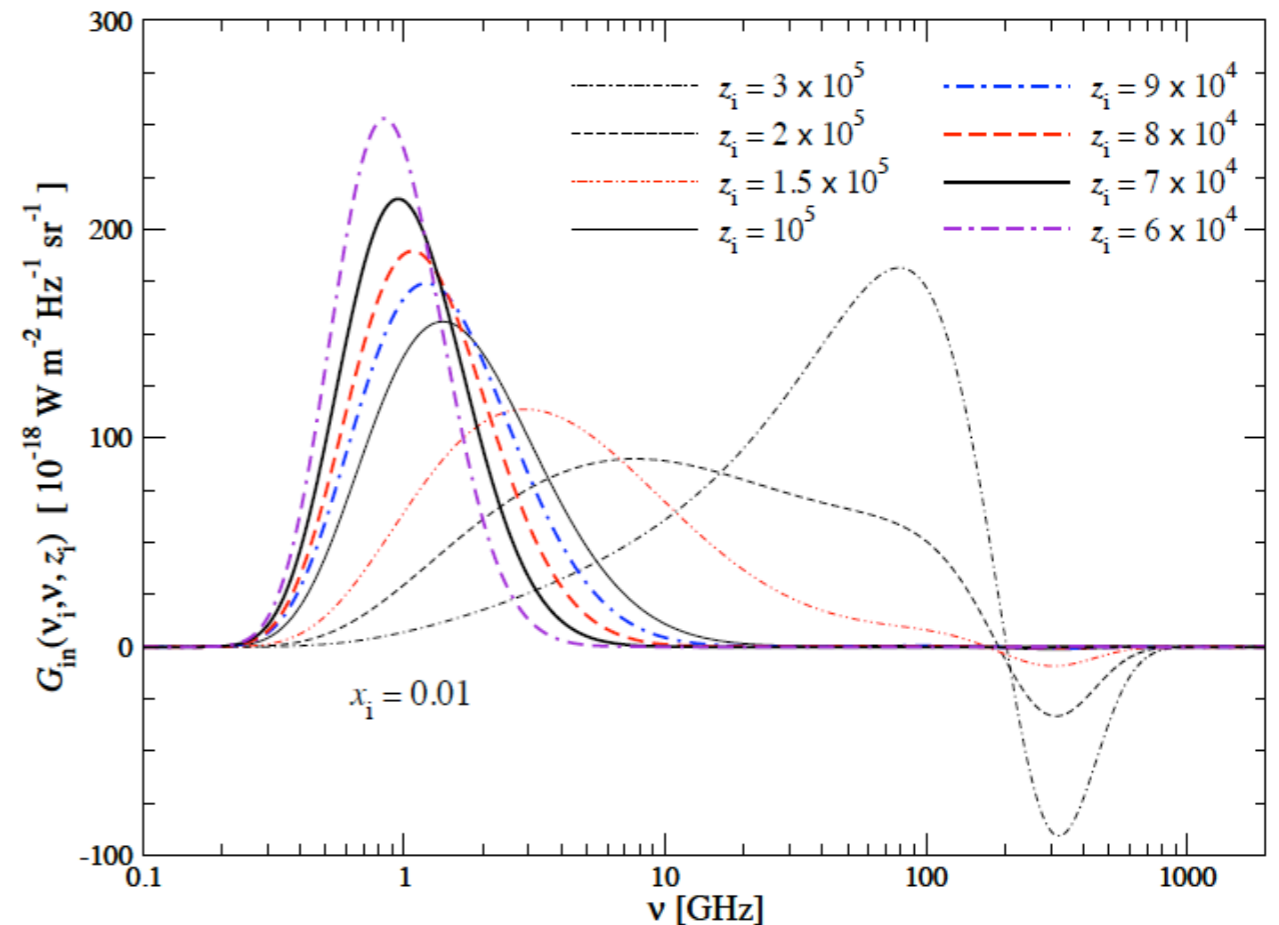
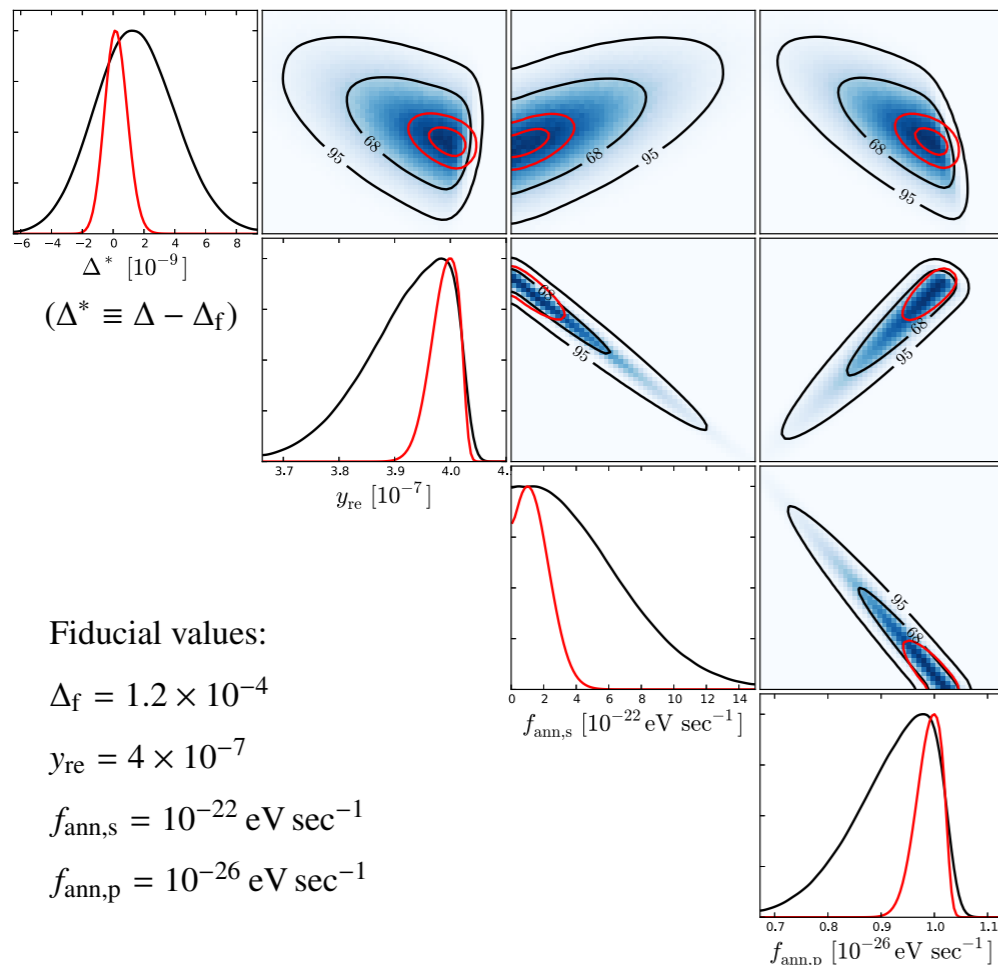
Temperature shift:

$$\mathcal{J}_T(z) \approx 1 - e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Last Surface

Using the Green's function package

- Green's function package available at www.Chluba.de/CosmoTherm/
- Depends on GSL library
- Has python interface and python packages
- PCA methods (not added yet...)
- Green's function method for photon injection too (JC 2015, ArXiv:1506.06582)



Some useful commands

Making and cleaning

```
> make  
> make py  
> make clean  
> make tidy
```

Execute Greens-package like

```
./run_Greens runfiles/parameters.dat (default computation)
```

Some other runmodes...

```
./run_Greens Greens runfiles/parameters.dat (Greens function output)
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./run_Greens Mock runfiles/parameters.dat (band average for mock)
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Green's function specific parameters

`./runfiles/parameters.dat`

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//=====
// the above parameters are (default values are given as examples)
//=====
0          == error in the reference blackbody assumed to be T0=2.726 K

1.0e-23    == fann*fann(1-fnu) typically f_ann<~2.0e-23 eV s^-1 [set ==0 to deactivate]
0          == s (==0) or p (==1)-wave annihilation cross section

2.4e-9     == Amplitude of adiabatic mode [set ==0 to deactivate 'all' dissipation parts]
0.002      == pivot scale k0 in Mpc^-1
1.0        == spectral index nS
0.0        == running n_run

4.0e-8     == Amplitude of the power spectrum step [set ==0 to deactivate]
30.0       == ks in Mpc^-1
0.96       == spectral index nS' after step

3.0        == kbend in Mpc^-1 [set ==1.0e+10 to deactivate]
1.5        == spectral index nS' after bend

5e+4       == z_X which determines lifetime of particle by Gamma_X=1/t(z_X)
2.0e+3     == f_X'=f_X(1-fnu) typically f_X <~10^6 eV for z_X~5x10^4
           [set ==0 to deactivate]

4.0e-9     == Amplitude for particle production feature [set ==0 to deactivate]
20.0       == position of particle production feature in k

4.0e-7     == y-parameter from reionization

./outputs/ == path for output
.dat       == addition to name of files at the very end

//=====
```

Execute Greens-package like

```
./run_Greens MODE runfiles/parameters.dat
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