

CMB foregrounds for B-mode studies, Tenerife, Oct 2018  
Session VI (cont'). Forecasting future experiments

# Forecasting galactic foregrounds cleaning with parametric, pixel- based, max-likelihood approach

Josquin ERRARD





CMB **4** CAST  
GROUNDS

JE, Feeney, Peiris, Jaffe  
(JCAP, 2016)

**x**Forecast

Stompor, JE, Poletti  
(PRL, 2016)

multi**Patch**

JE et al, in prep

**data modeling**  
for each sky pixel:  
[see D. Poletti talk]

$$d_i(p) = A_{ij} s_j(p) + n_i(p)$$

Diagram illustrating the data modeling equation:  $d_i(p) = A_{ij} s_j(p) + n_i(p)$ . The diagram shows a vertical double-headed arrow labeled "frequencies" pointing to the left. To the right of the arrow, there is a vertical grey bar containing the letter "d". This is followed by an equals sign, a larger vertical grey bar containing "A(β)", then a vertical orange bar containing "s", a plus sign, and a final vertical grey bar containing "n".

**data modeling**  
for each sky pixel:

[see D. Poletti talk]

$$d_i(p) = A_{ij} s_j(p) + n_i(p)$$

## I. estimation of the mixing matrix **A**

e.g. Stompor et al. (2009)

$$A_{\text{sync}}^{\text{raw}}(\nu, \nu_{\text{ref}}) \equiv \left( \frac{\nu}{\nu_{\text{ref}}} \right)^{\beta_s}$$

$$A_{\text{dust}}^{\text{raw}}(\nu, \nu_{\text{ref}}) \equiv \left( \frac{\nu}{\nu_{\text{ref}}} \right)^{\beta_d+1} \frac{e^{\frac{h\nu_{\text{ref}}}{kT_d}} - 1}{e^{\frac{h\nu}{kT_d}} - 1}$$

$$\mathbf{A} \equiv \mathbf{A}(\beta = \beta_d, \beta_s, \dots) \longrightarrow \max(\mathcal{L}(\beta))$$

$$-2 \ln \mathcal{L}_{\text{spec}}(\beta) = \text{CONST} - (\mathbf{A}^t \mathbf{N}^{-1} \mathbf{d})^t (\mathbf{A}^t \mathbf{N}^{-1} \mathbf{A})^{-1} (\mathbf{A}^t \mathbf{N}^{-1} \mathbf{d})$$

**not perfect recovery**  
**of input spectral**  
**parameters** ➤  
**foregrounds**  
**residuals**

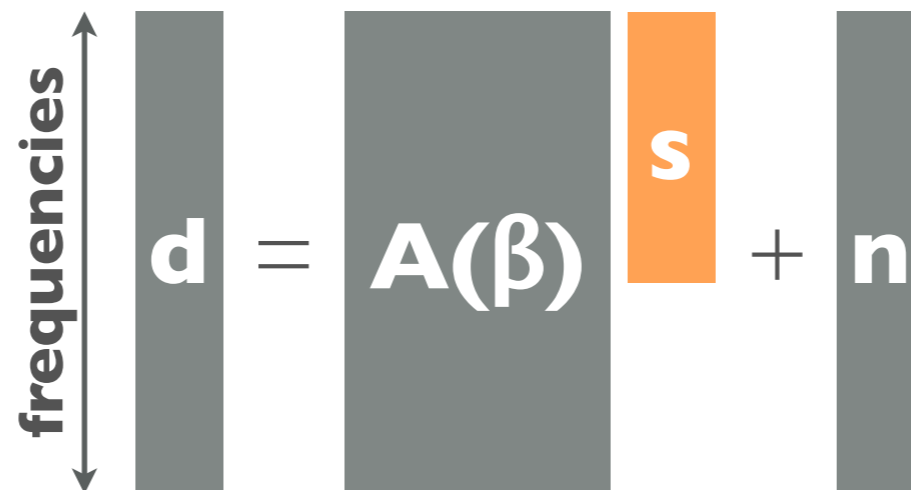


$$d_i(p) = A_{ij} s_j(p) + n_i(p)$$

## data modeling

for each sky pixel:

[see D. Poletti talk]



### 1. estimation of the mixing matrix $\mathbf{A}$

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parameters  $\blacktriangleright$   
**foregrounds**  
**residuals**

### 2. solve for $\mathbf{s}$ [rather general to any comp sep method]

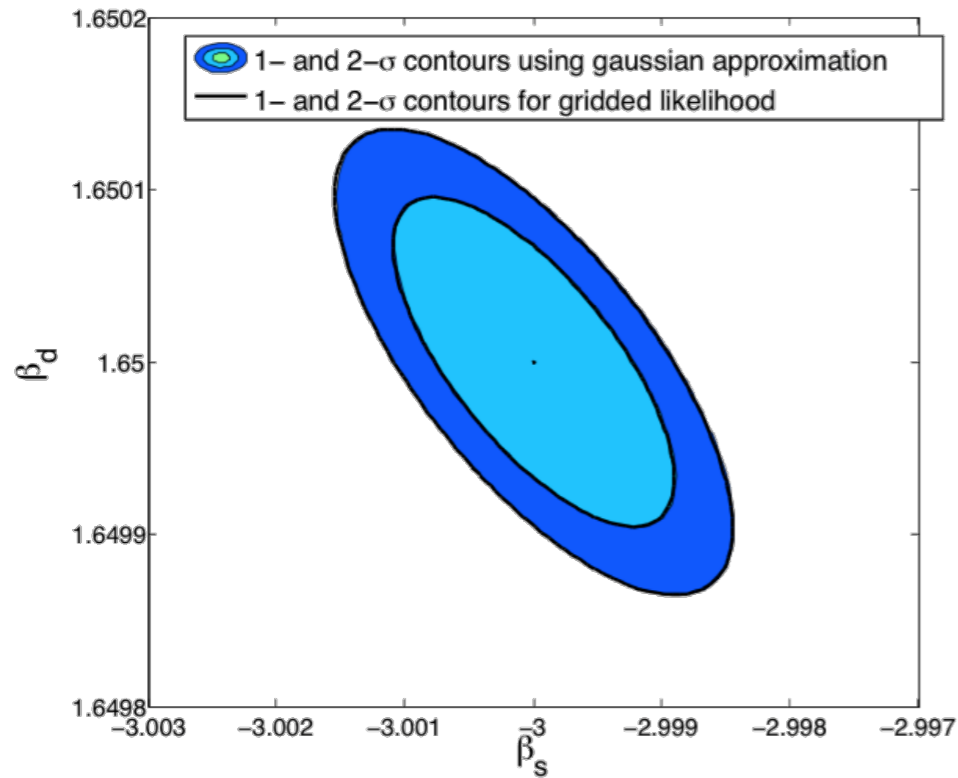
$$\mathbf{s} = (\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{N}^{-1} \mathbf{d}$$

linear combination  
of various frequency  
maps  $\blacktriangleright$  **boosted**  
**noise**





Statistical error bars on spectral parameters:



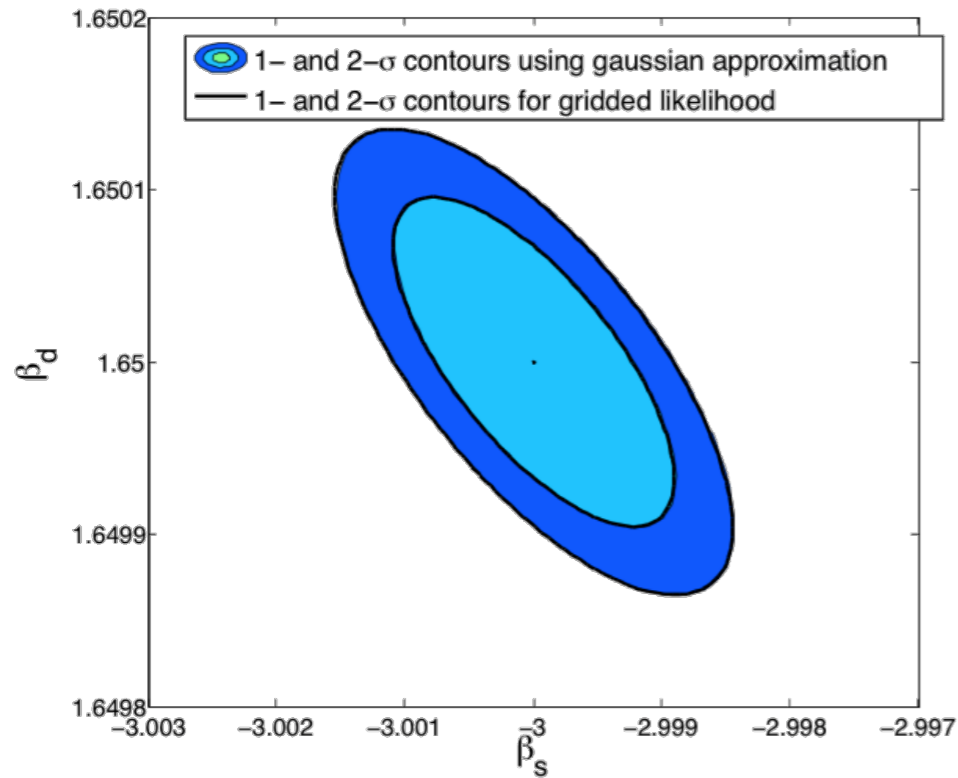
JE, Stivoli and Stompor (PRD, 2011)

$$\Sigma^{-1} \approx - \left\langle \frac{\partial^2 \mathcal{L}}{\partial \beta \partial \beta'} \right\rangle_{\text{noise}} \Big|_{\text{true } \beta}$$

$$= \text{tr} \left\{ \left[ \mathbf{A}_{,\beta}^t \mathbf{N}^{-1} \mathbf{A} (\mathbf{A}^t \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^t \mathbf{N}^{-1} \mathbf{A}_{,\beta'} - \mathbf{A}_{,\beta}^t \mathbf{N}^{-1} \mathbf{A}_{,\beta'} \right] \sum_p \hat{\mathbf{s}}_p \hat{\mathbf{s}}_p^t \right\}$$

→ averaged error bars for parametric methods like COMMANDER [see I. Wehus talk]

**Statistical error bars on spectral parameters:**



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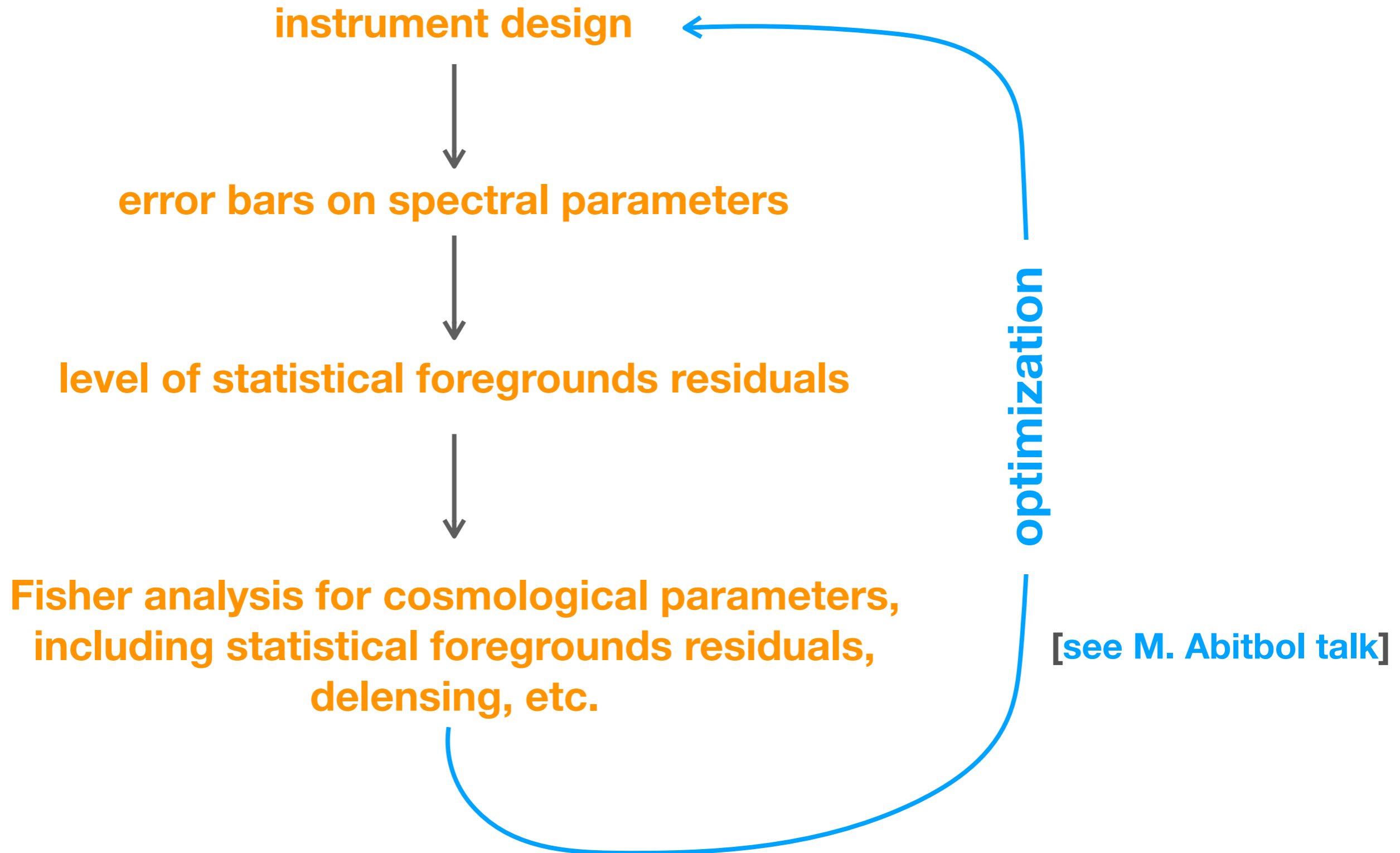
→ averaged error bars for parametric methods like COMMANDER [see I. Wehus talk]

**Amplitude of statistical foreground residuals:**

$$C_\ell^{\text{fg res}} \equiv \sum_{k,k'} \sum_{j,j'} \Sigma_{kk'} \kappa_{kk'}^{jj'} C_\ell^{jj'}$$

Stivoli, Grain, Leach, Tristram, Baccigalupi, Stompor (MNRAS, 2010)





simulation of observation with  
CMB + any foregrounds



spectral analysis using simple scaling  
laws (e.g. power-law synchrotron and  
gray body dust)

$$\langle \mathcal{S}_{spec} \rangle = -\text{tr} \sum_p \left\{ (\mathbf{N}_p^{-1} - \mathbf{P}_p) (\hat{\mathbf{d}}_p \hat{\mathbf{d}}_p^t + \mathbf{N}_p) \right\}$$



simulation of observation with  
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estimation of statistical and systematic  
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$$C_\ell^{\text{res}} \simeq \otimes_\ell(\tilde{\mathbf{y}}, \tilde{\mathbf{y}}) + \otimes_\ell(\tilde{\mathbf{y}}, \tilde{\mathbf{z}}) + \otimes_\ell(\tilde{\mathbf{z}}, \tilde{\mathbf{y}}) \\ + \text{tr} \left[ \boldsymbol{\Sigma} \otimes_\ell(\tilde{\mathbf{Y}}^{(1)}, \tilde{\mathbf{Y}}^{(1)}) \right]$$

simulation of observation with  
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estimation of statistical and systematic  
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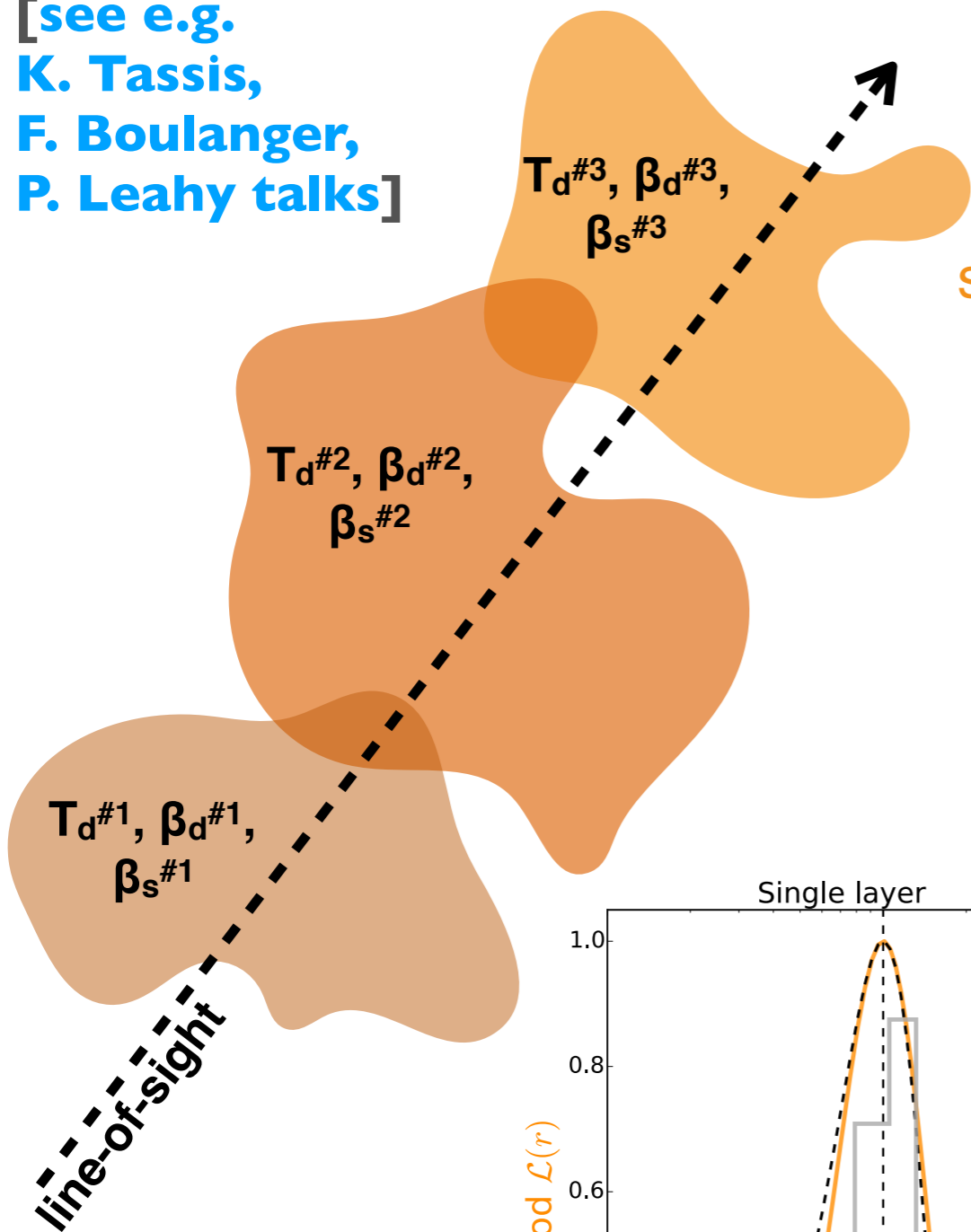
propagation of these to the estimation  
of tensor-to-scalar ratio  $r$

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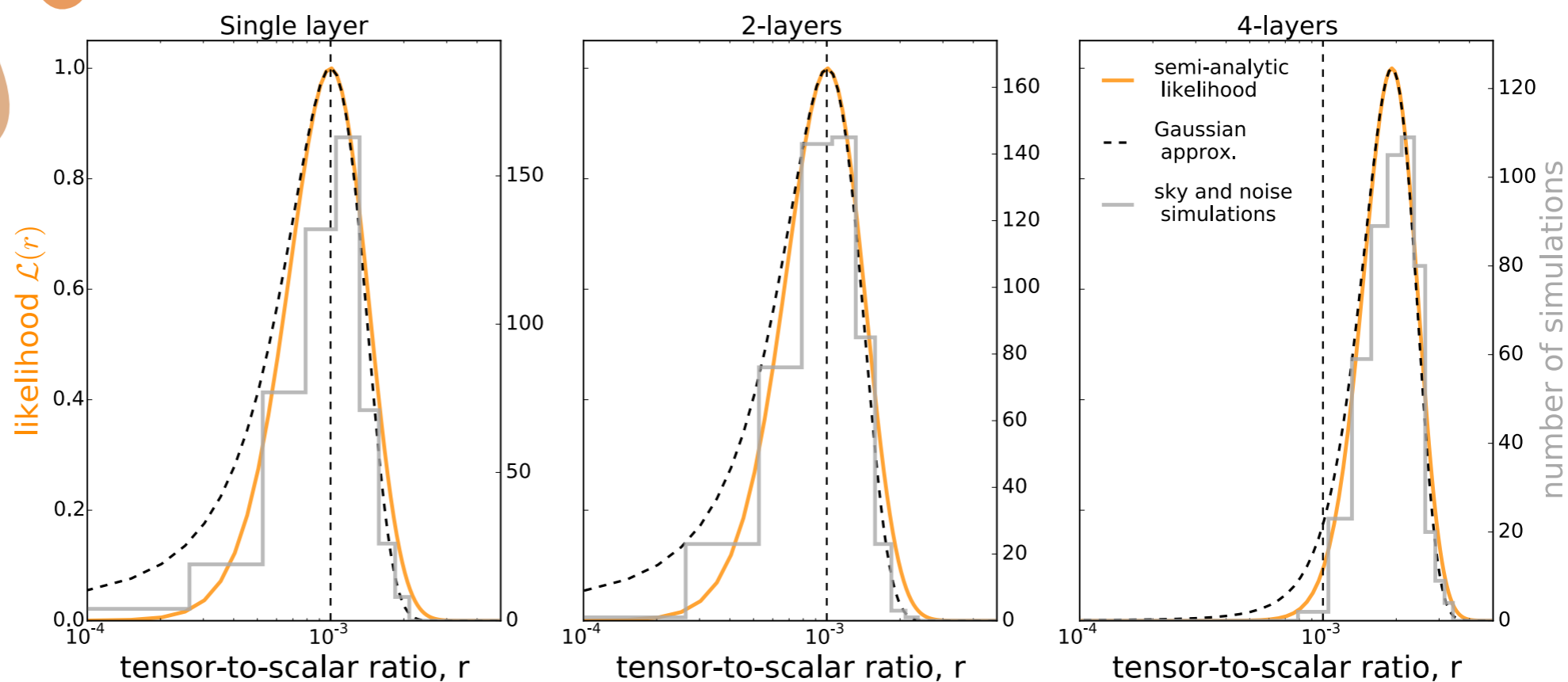
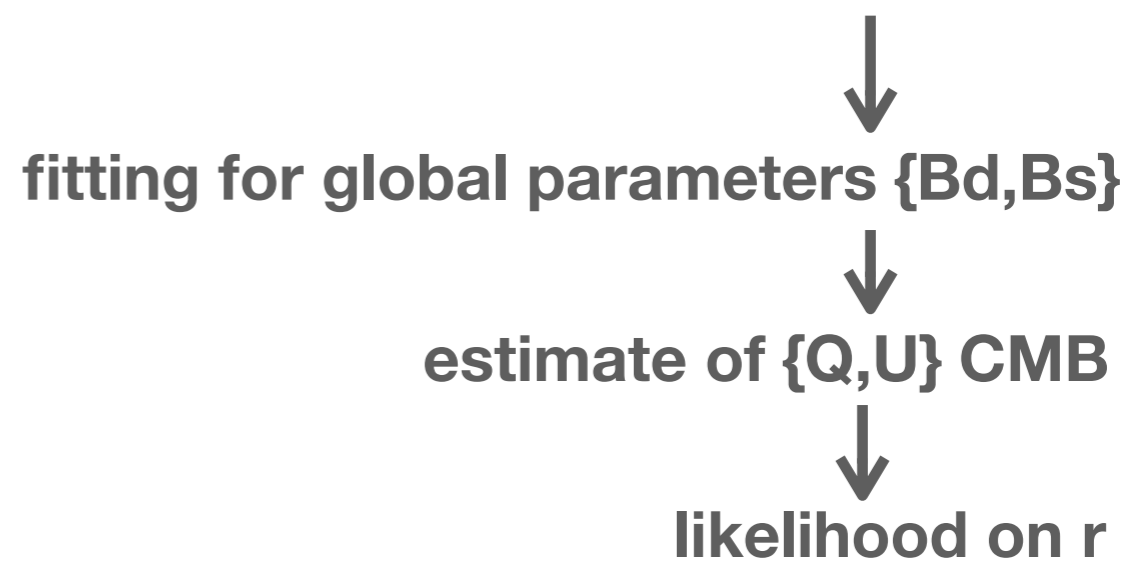
$$\langle \mathcal{S}^{par} \rangle = \text{tr} \mathbf{C}^{-1} \mathbf{E} + \ln \det \mathbf{C}$$

[see e.g.  
K. Tassis,  
F. Boulanger,  
P. Leahy talks]



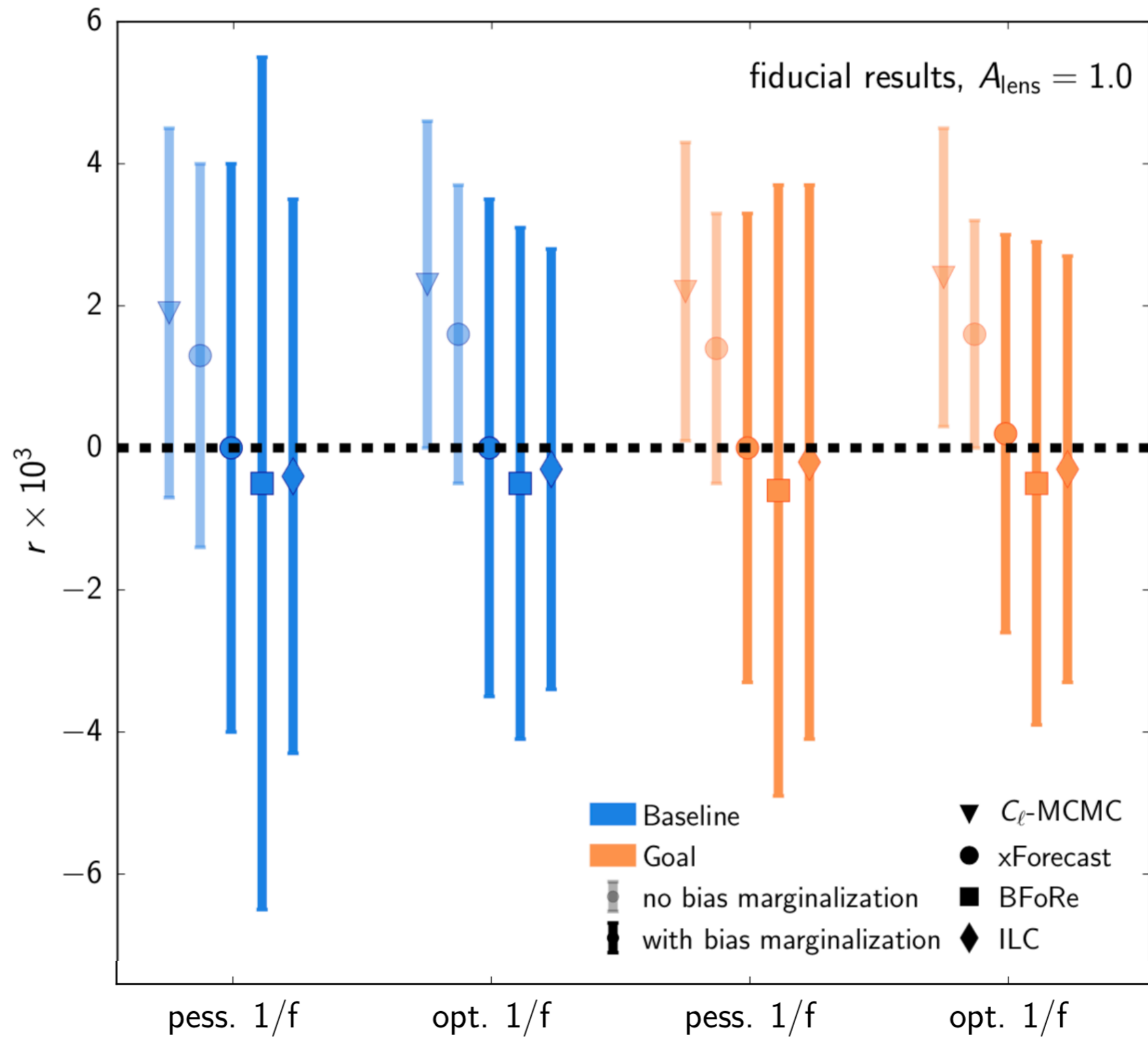
R. Stompor, JE and D. Poletti (PRD 2016, 1609.03807)

Complex foregrounds toy model:  $N=1,2$  or  $4$  independent foregrounds “layers” along the line-of-sight, with different temperature and spectral indices



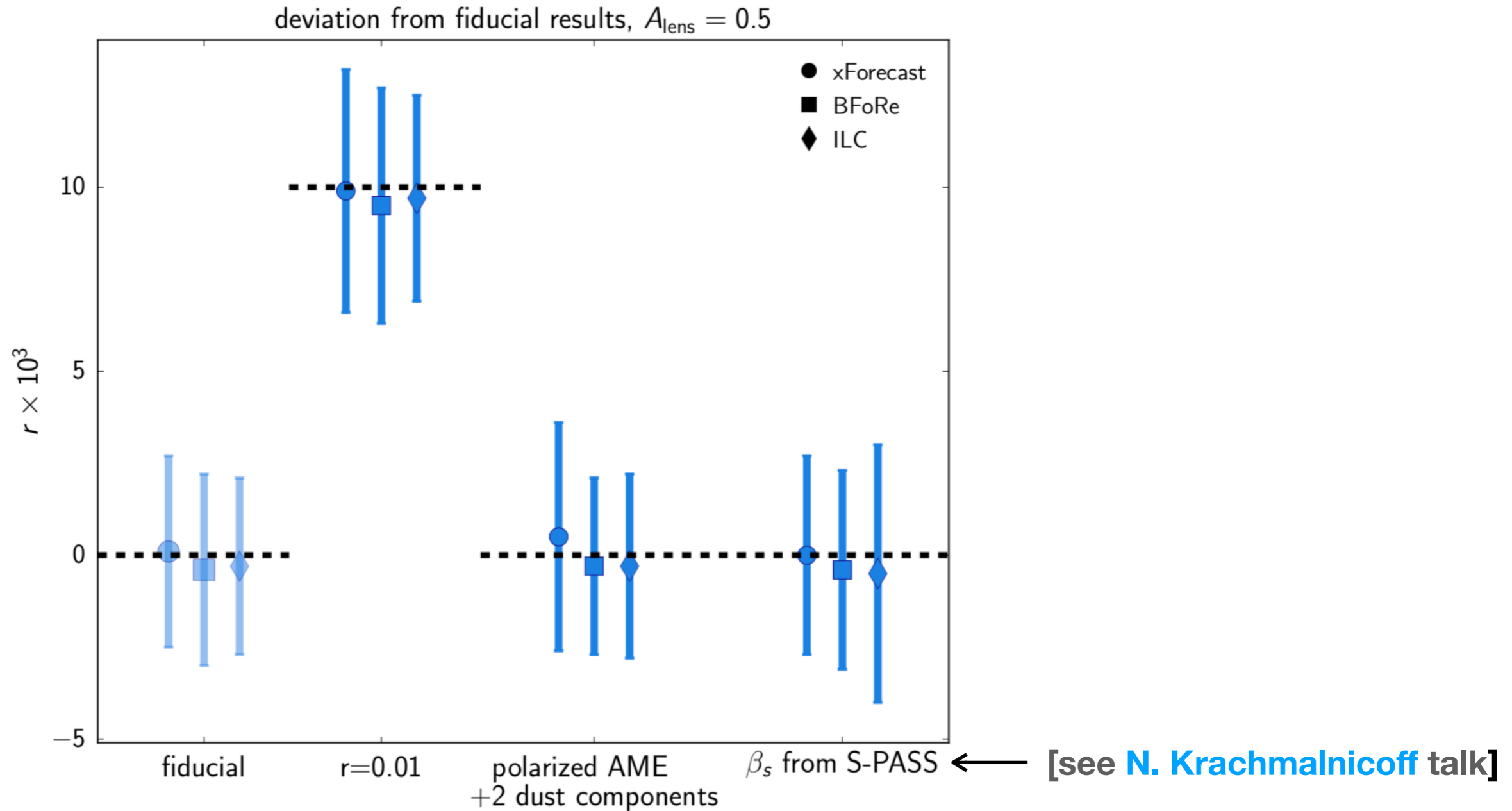
# THE SIMONS OBSERVATORY: SCIENCE GOALS AND FORECASTS

arXiv:1808.07445 [talk by C. Hill]



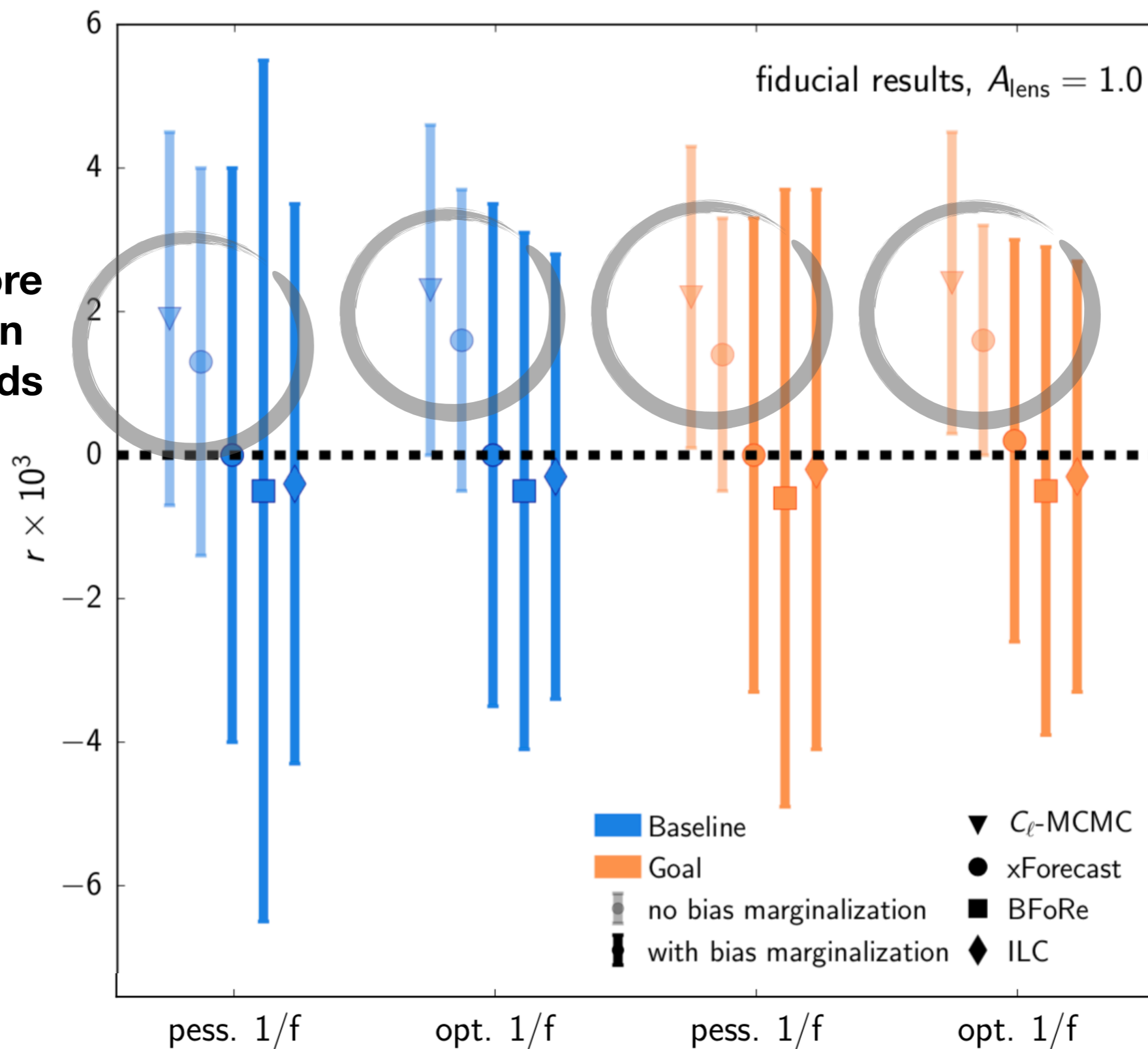


# THE SIMONS OBSERVATORY: SCIENCE GOALS AND FORECASTS



**Figure 12.** As in Fig. 10, for the cases deviating from the fiducial forecasts. The ‘fiducial’ points match the second panel of Fig. 10 for baseline sensitivity and optimistic  $1/f$ . The three other cases assume  $r = 0.01$  in the input sky simulations (left),  $r = 0.0$  with 2 dust components and polarized AME (middle), and  $r = 0.0$  with synchrotron scaling based on a high-resolution  $\beta_s$  template (right). These models are described in Sec. 2.4.4 with forecasts in Table 5.

slight bias before marginalization over foregrounds template



In current simulations, e.g. PySM a1d1s1 [[see A. Zonca talk](#)],  
how is the bias on tensor-to-scalar ratio sourced?

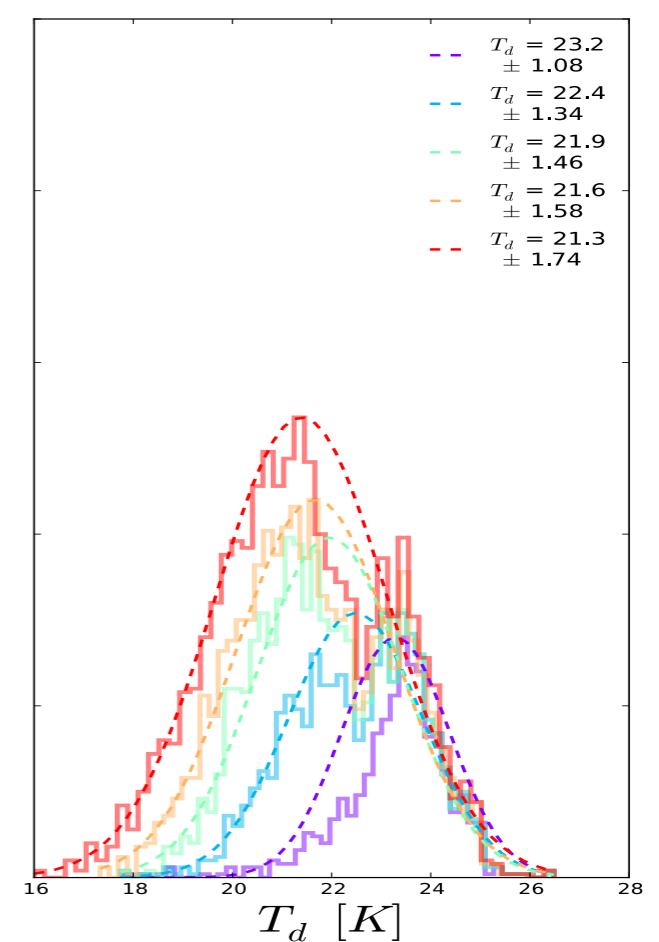
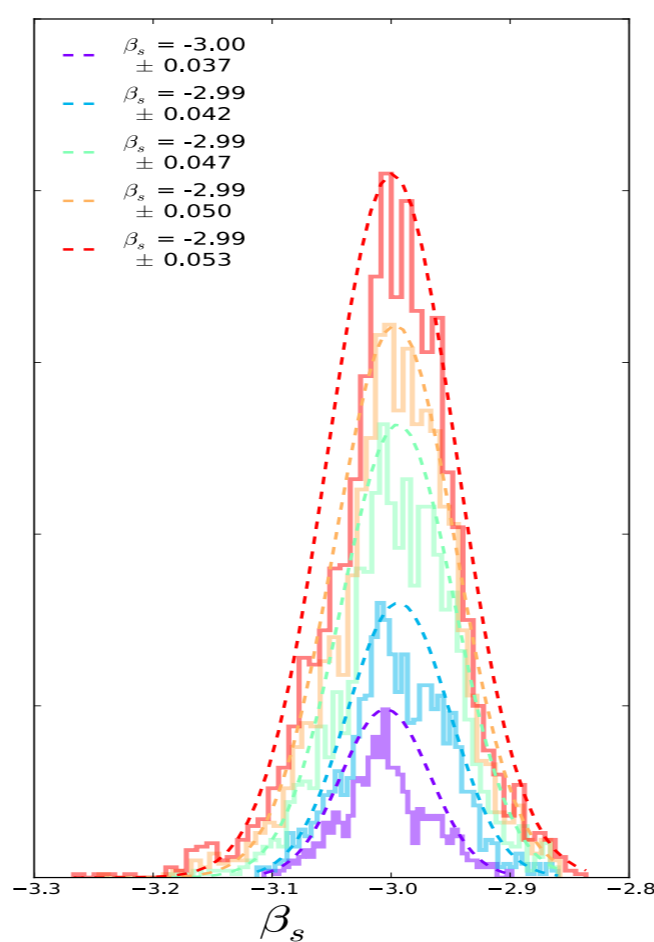
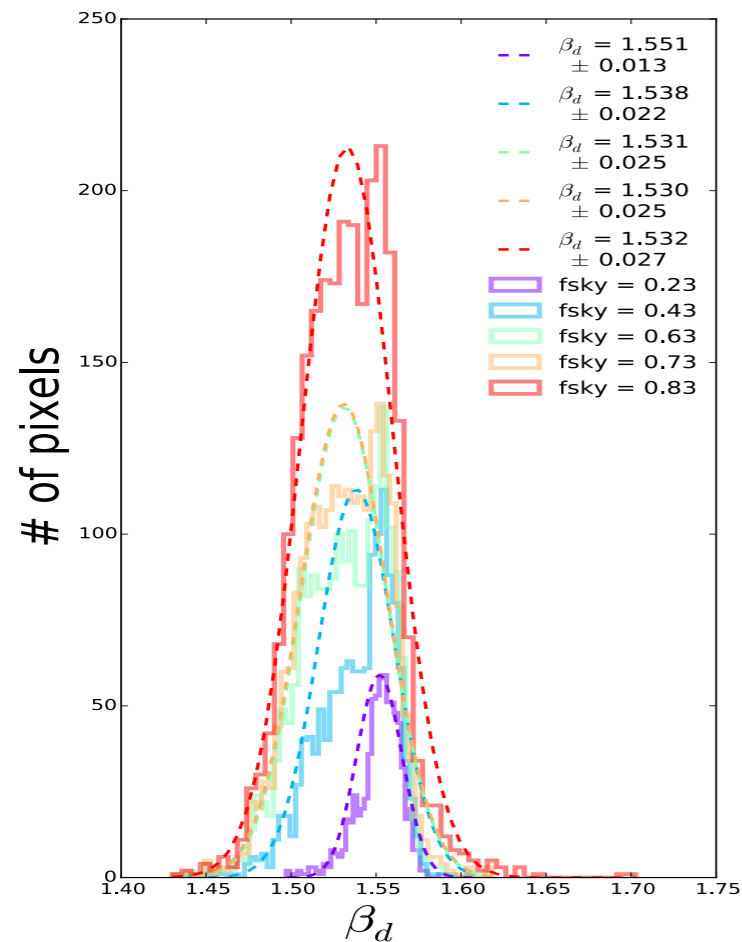
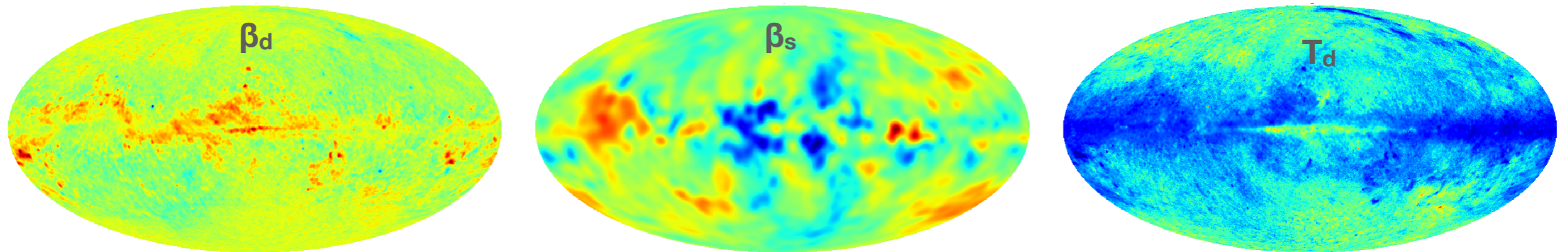
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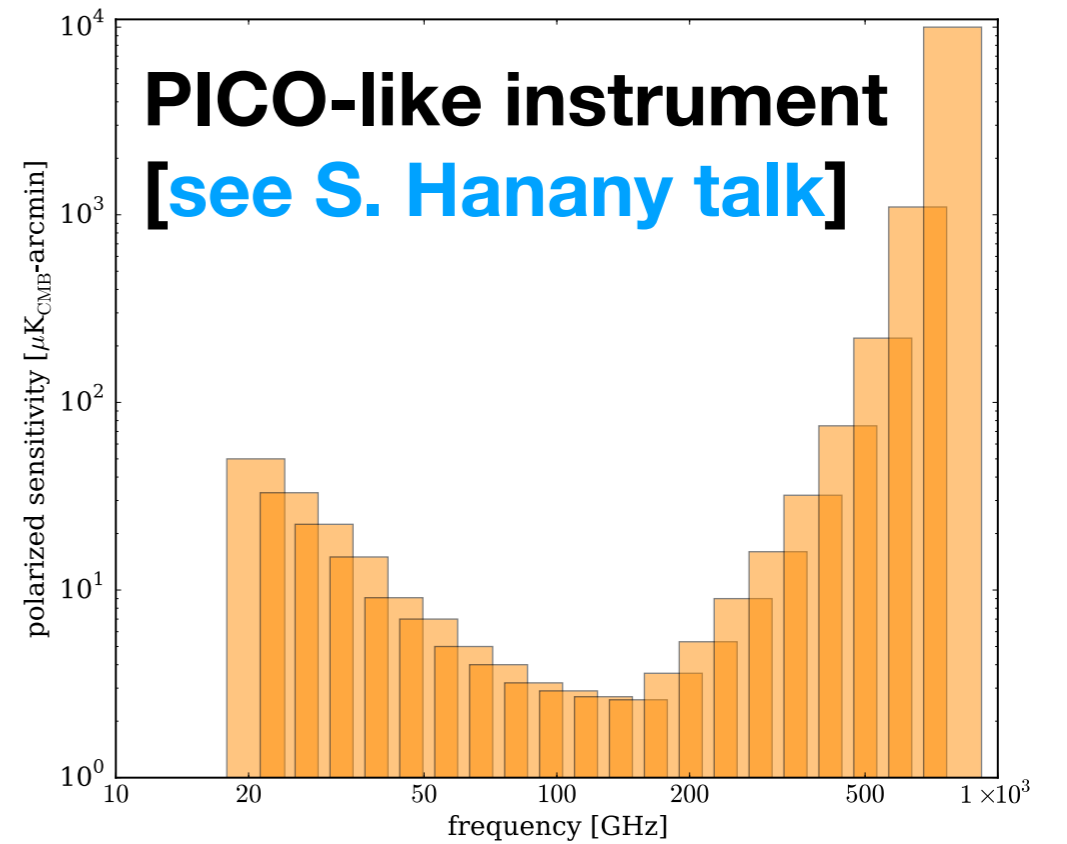
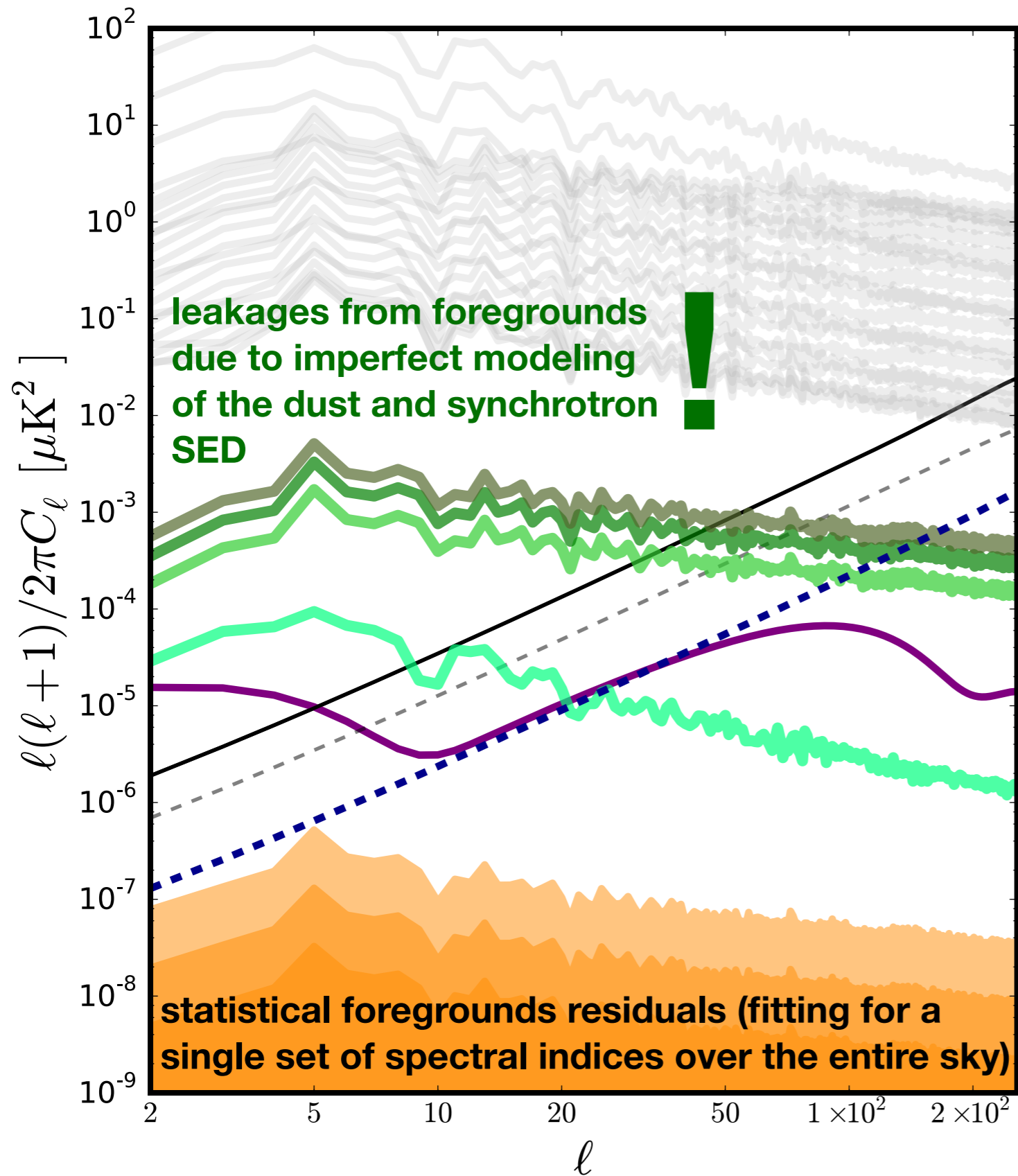
→ **from spatial variations of the spectral indices!**  
(aka decorrelation [[see C. Pryke, J. Aumont, ++ talks](#)])



In current simulations, e.g. PySM a1d1s1 [see [A. Zonca talk](#)],  
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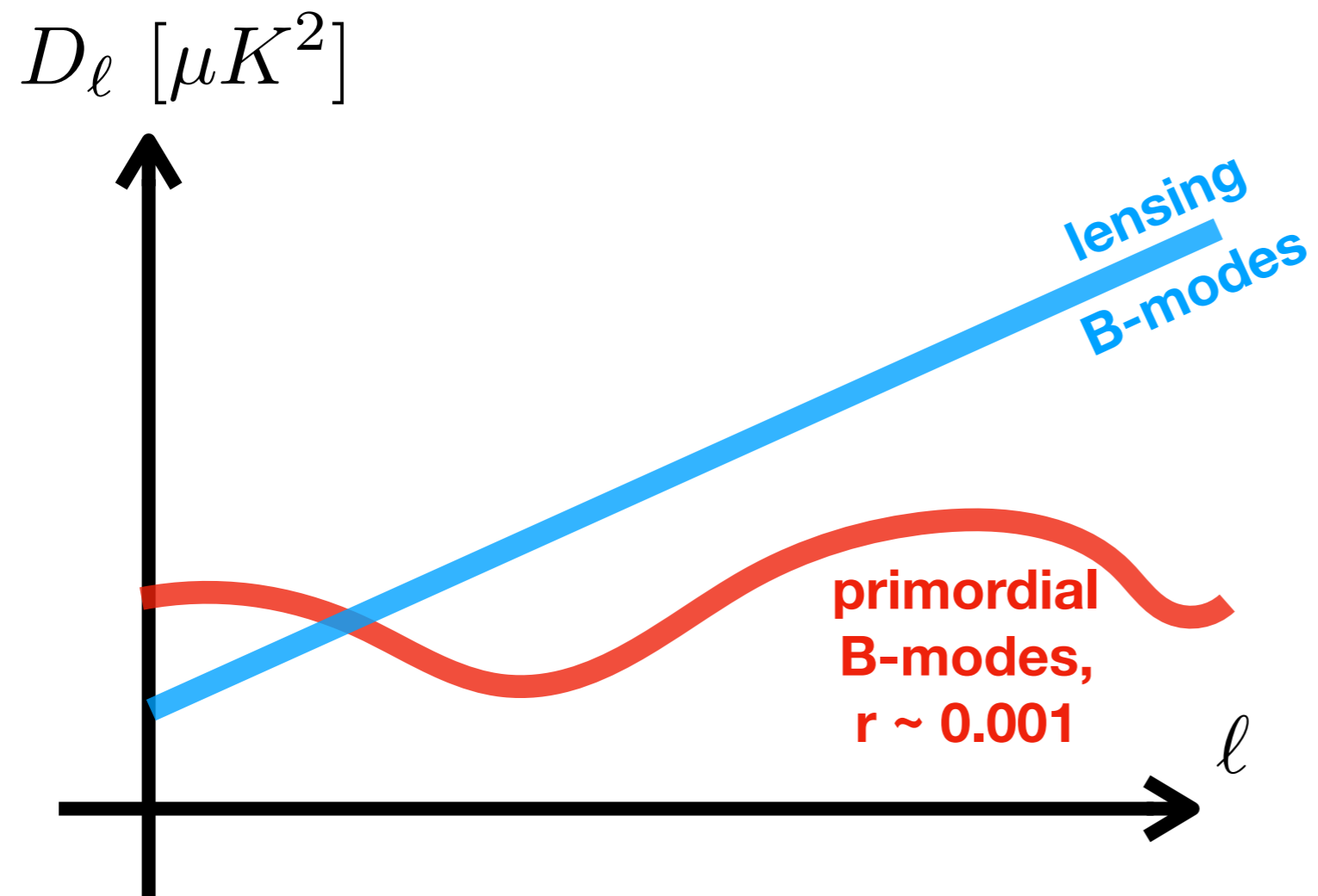




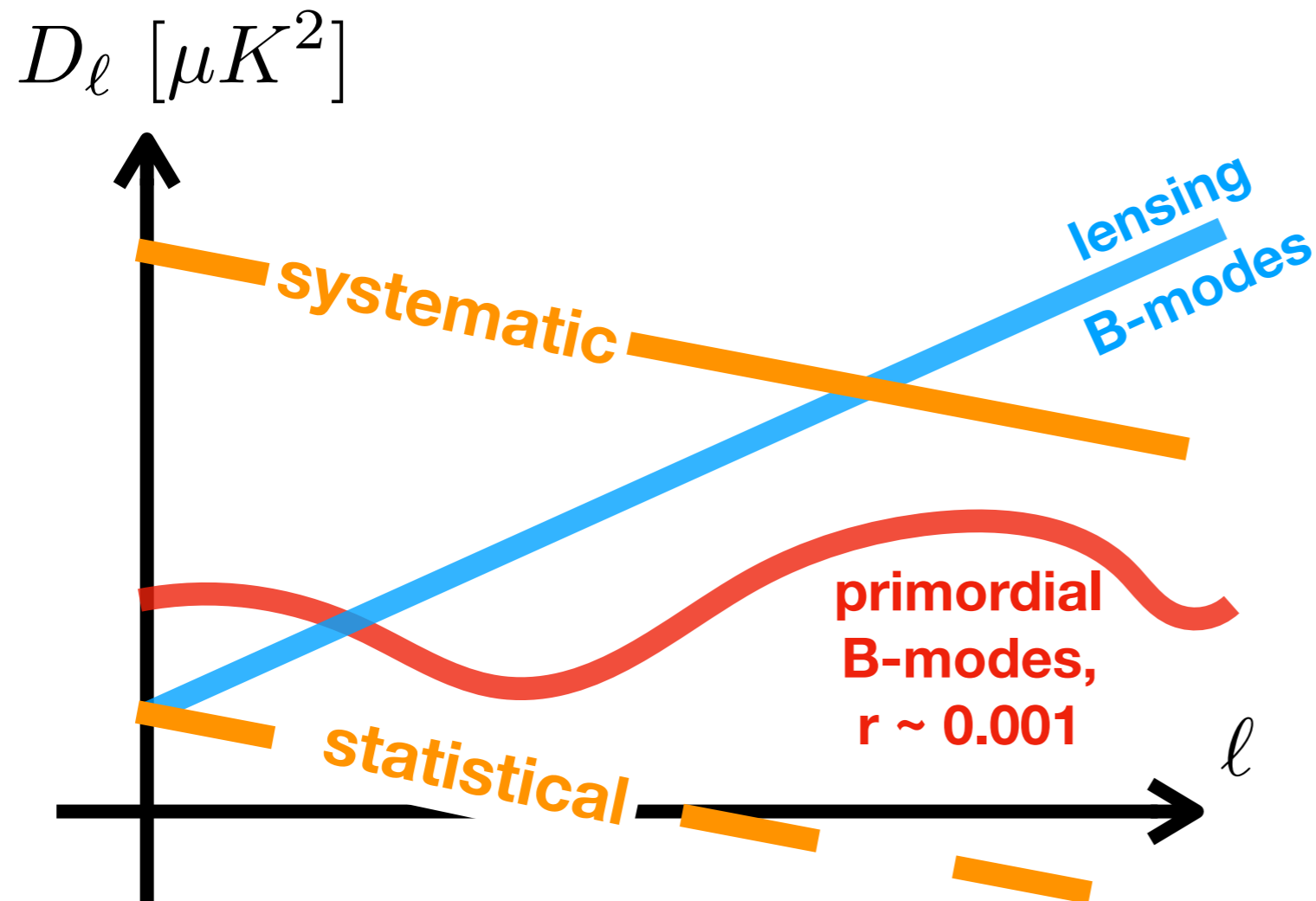
**fsky = 60%**

- noiseless foregrounds spectra from PySM simulations
- total B-modes
- primordial B-modes ( $r=0.001, \tau=0.055$ )
- lensing B-modes
- - internally delensed B-modes
- xF average residuals  $\pm 2-\sigma$
- sys. res. from  $\beta_d$  spat. var.
- sys. res. from  $\beta_s$  spat. var.
- sys. res. from  $T_d$  spat. var.
- sys. res. from  $\{\beta_d, \beta_s, T_d\}$  spat. var.
- $N_\ell$

For a given instrument, there are two extreme solutions for data analysts:



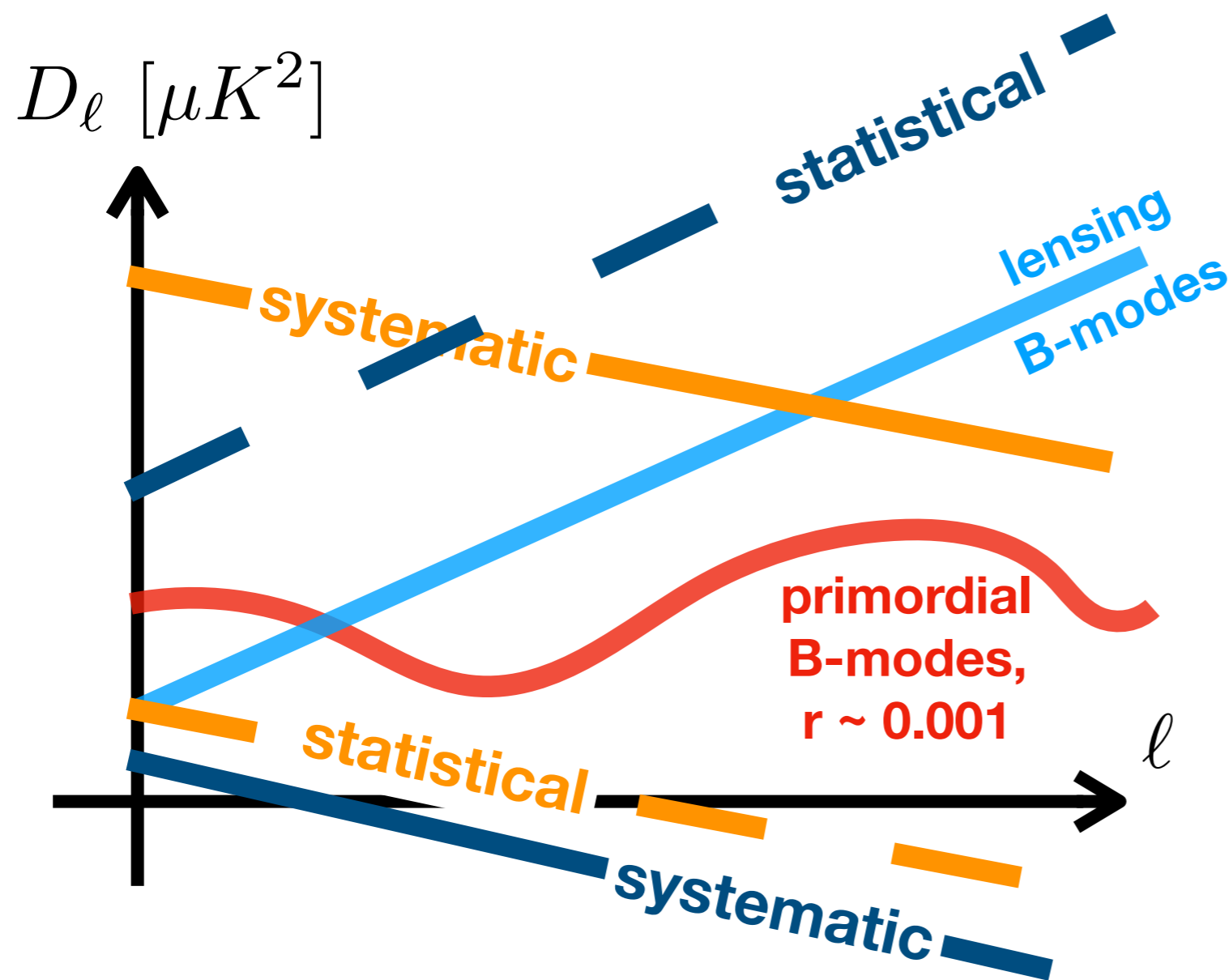
For a given instrument, there are two extreme solutions for data analysts:



- fit for a single set of spectral parameters over the entire sky  $\rightarrow$  low level of statistical foregrounds residuals but high level for leakage



For a given instrument, there are two extreme solutions for data analysts:



● fit for a single set of spectral parameters over the entire sky → low level of statistical foregrounds residuals but high level for leakage

● fit for as many sets of spectral parameters as sky pixels → high level of statistical foregrounds residuals but small leakage

one should look for a balance between statistical and systematic errors

**STATISTICAL** error  
bars on spectral  
parameters

**SYSTEMATIC** error  
bars on spectral  
parameters

one should look for a balance between statistical and systematic errors

$$\Sigma \equiv \begin{bmatrix} \sigma(\beta_d)^2 & \sigma(\beta_d)\sigma(\beta_s) & \sigma(\beta_d)\sigma(T_d) \\ \star & \sigma(\beta_s)^2 & \sigma(\beta_s)\sigma(T_d) \\ \star & \star & \sigma(T_d)^2 \end{bmatrix}$$

**STATISTICAL** error bars on spectral parameters

- better signal-to-noise (instrumental sensitivity, etc.)
- few degrees of freedom
- broad frequency range
- large sky area (more pixels!)

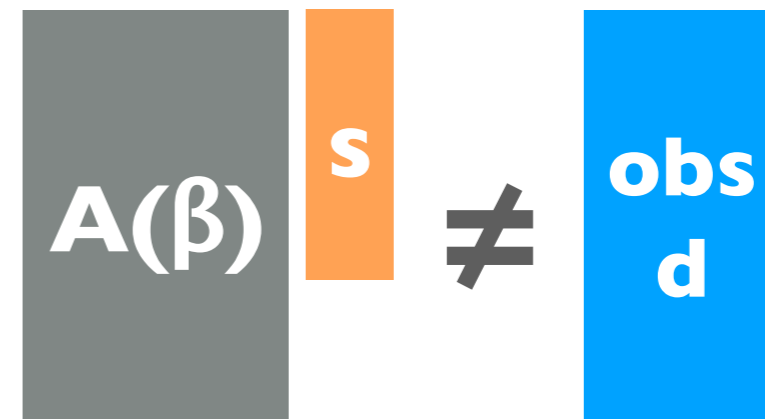
**SYSTEMATIC** error bars on spectral parameters

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**STATISTICAL** error bars on spectral parameters

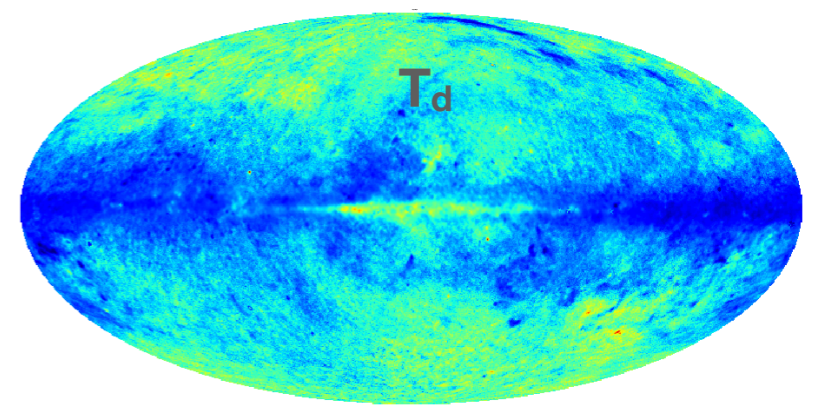
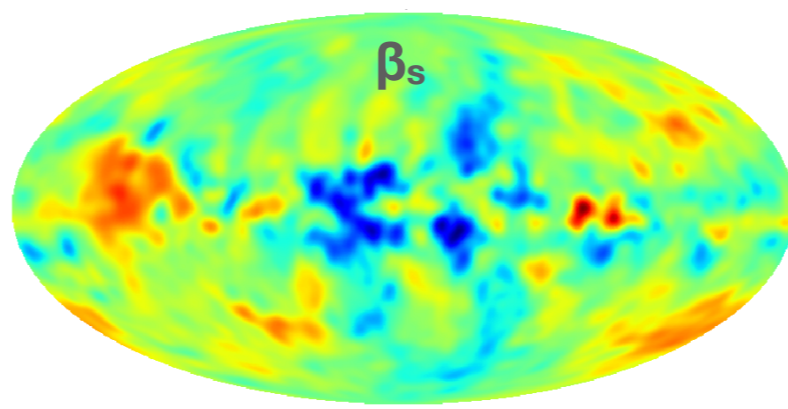
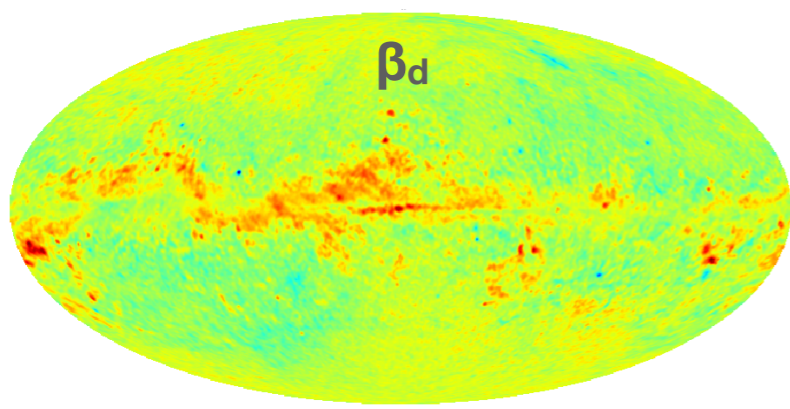
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- large sky area (more pixels!)



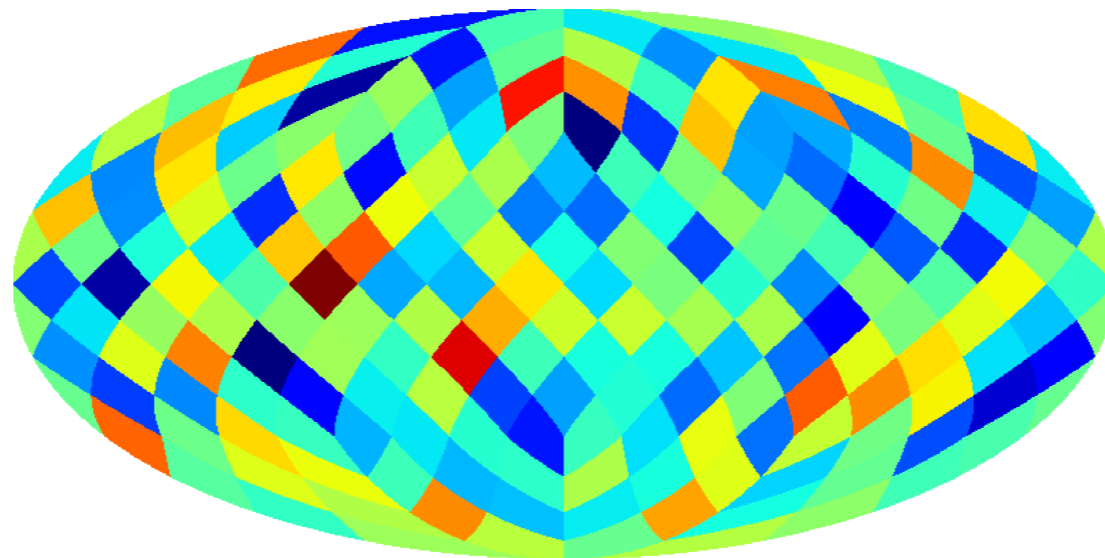
**SYSTEMATIC** error bars on spectral parameters

- more internal degrees of freedom (free spectral parameters, sky templates, etc.)
- reduced frequency range
- small sky area (less complexity!)

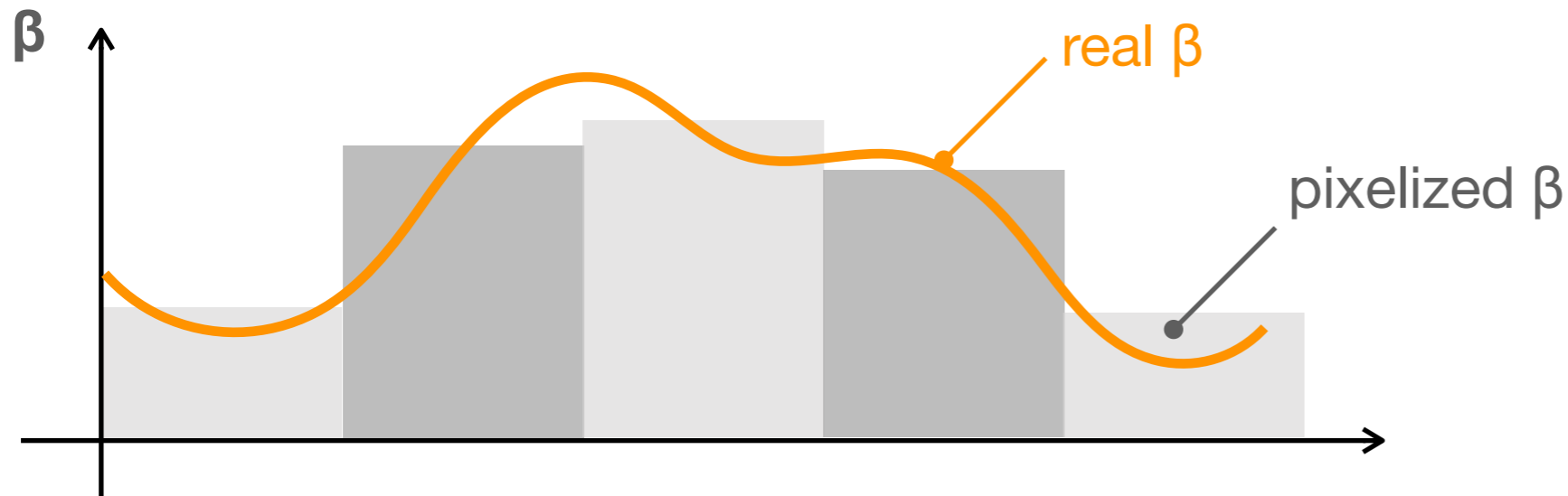




Is “multipatch” the solution?

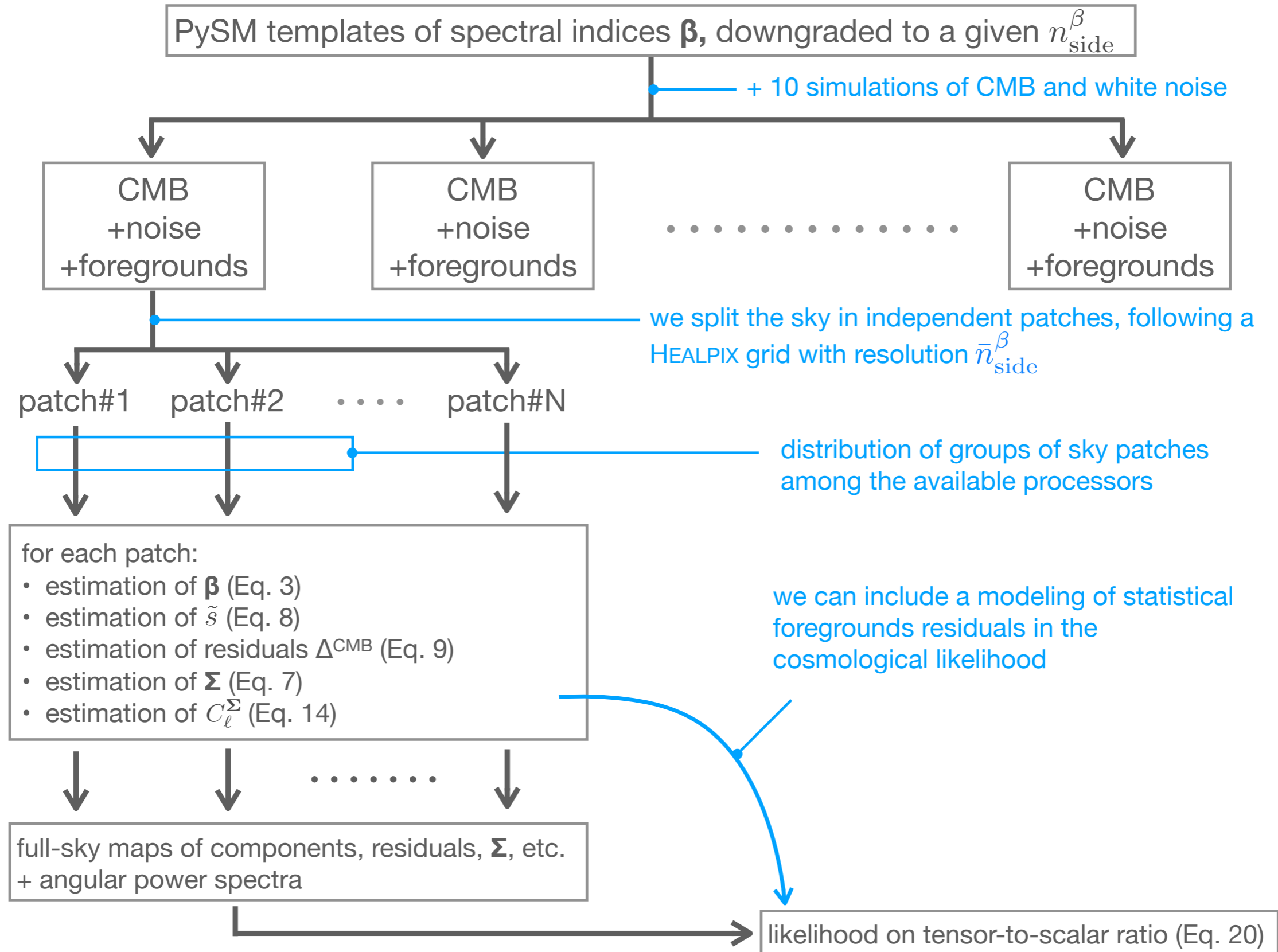


“maximize and minimize” sky area



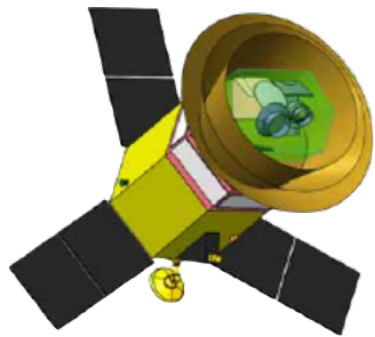
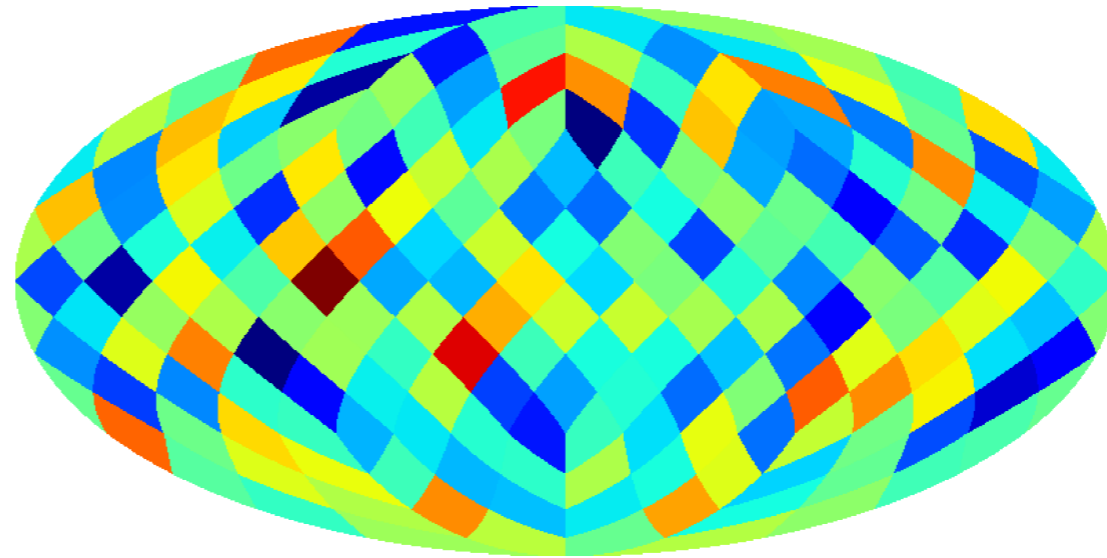
+ only ~3-4 sky components / sky pixel  
i.e. reduced noise in the reconstructed CMB map

# JE et al, in prep – about to be submitted :)



**JE et al, in prep**

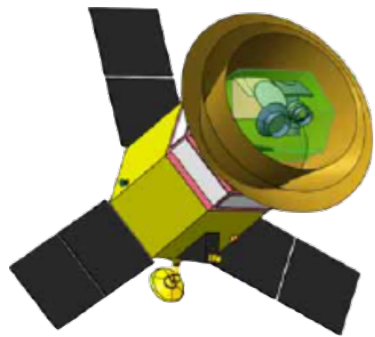
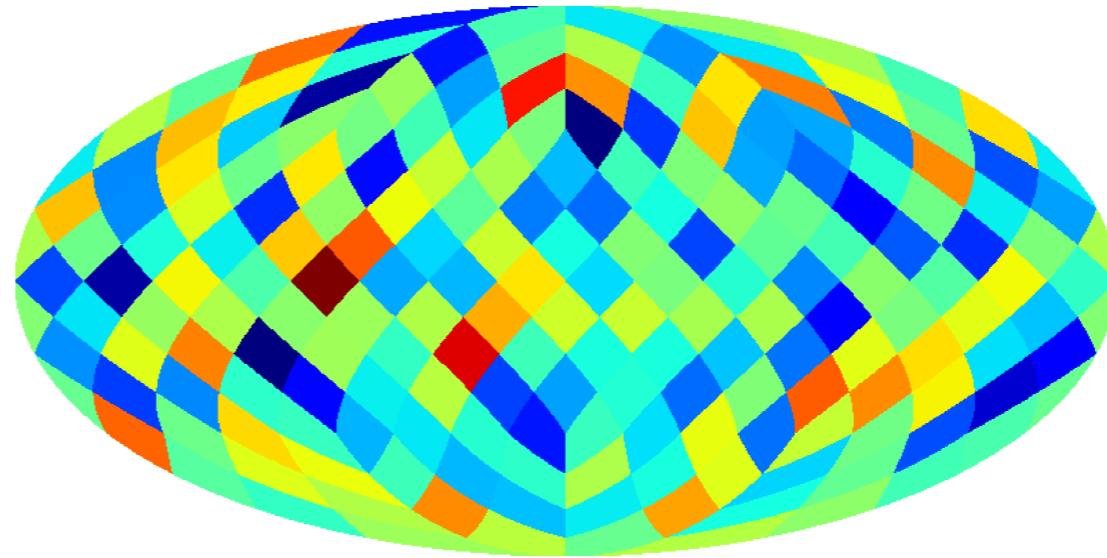
input spectral indices,  
smoothed and degraded to a  
Healpix grid  $n_{\text{side}}$



[exercise with the  
international Joint Study Group on  
foregrounds [see M. Hazumi talk](#)]

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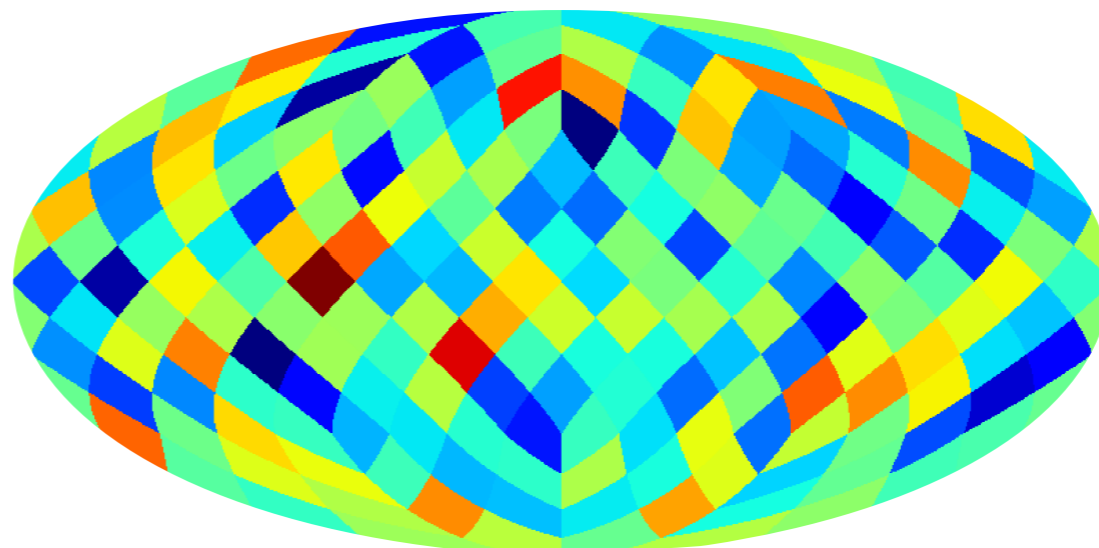


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generation of foregrounds  
frequency maps + CMB +  
noise simulation

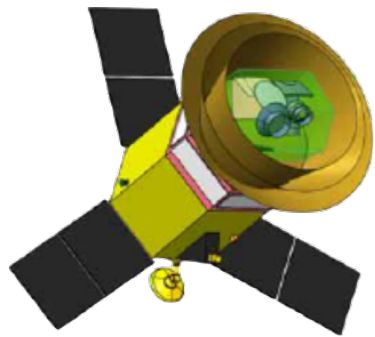
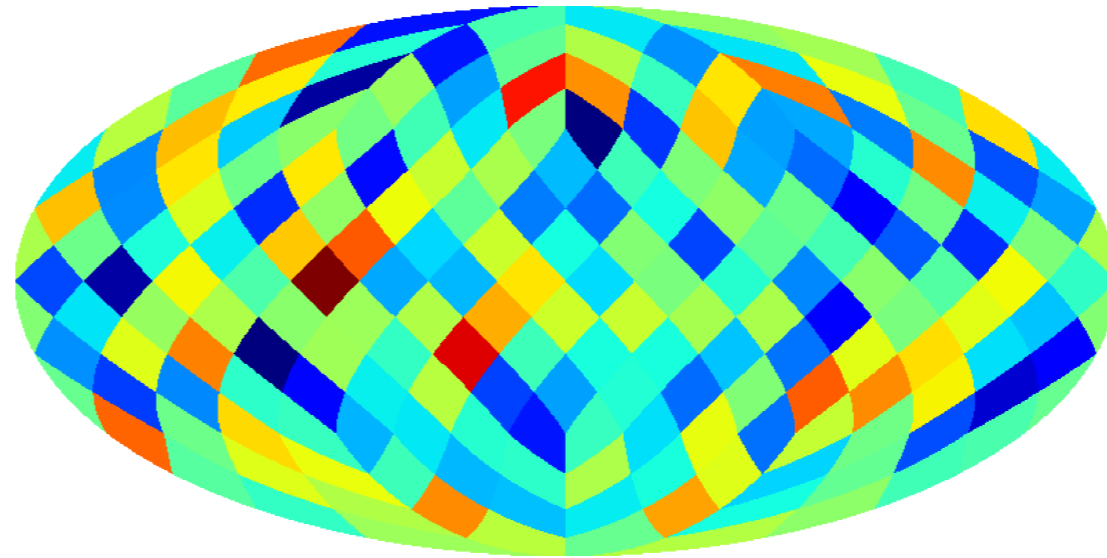


we fit for set of spectral  
indices for each patch,  
independently



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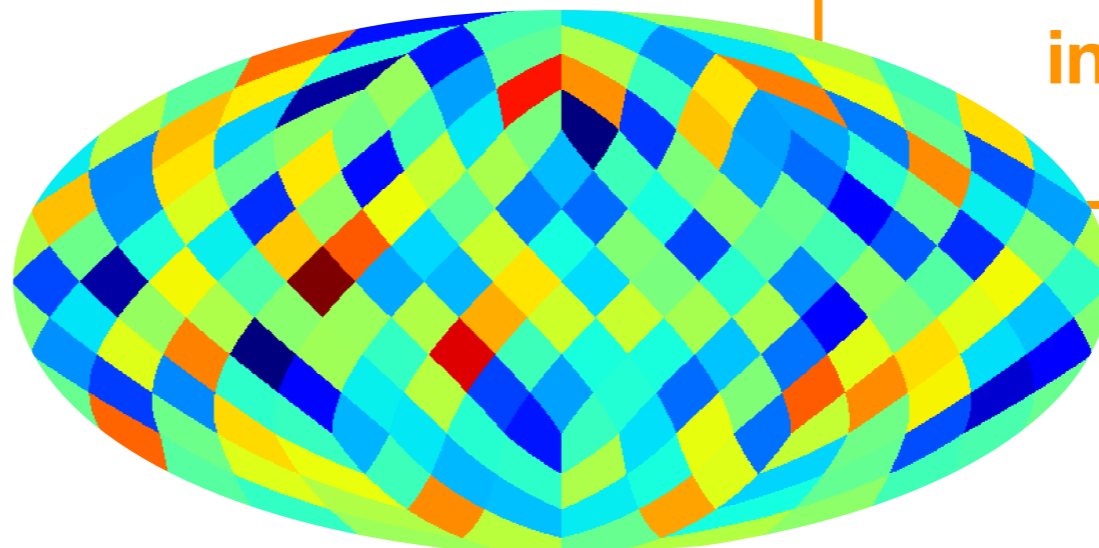


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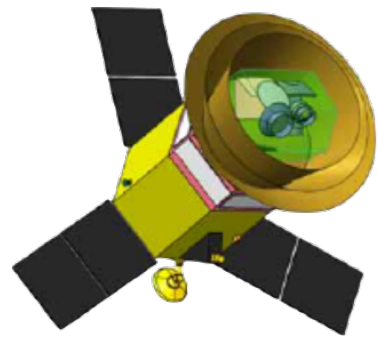
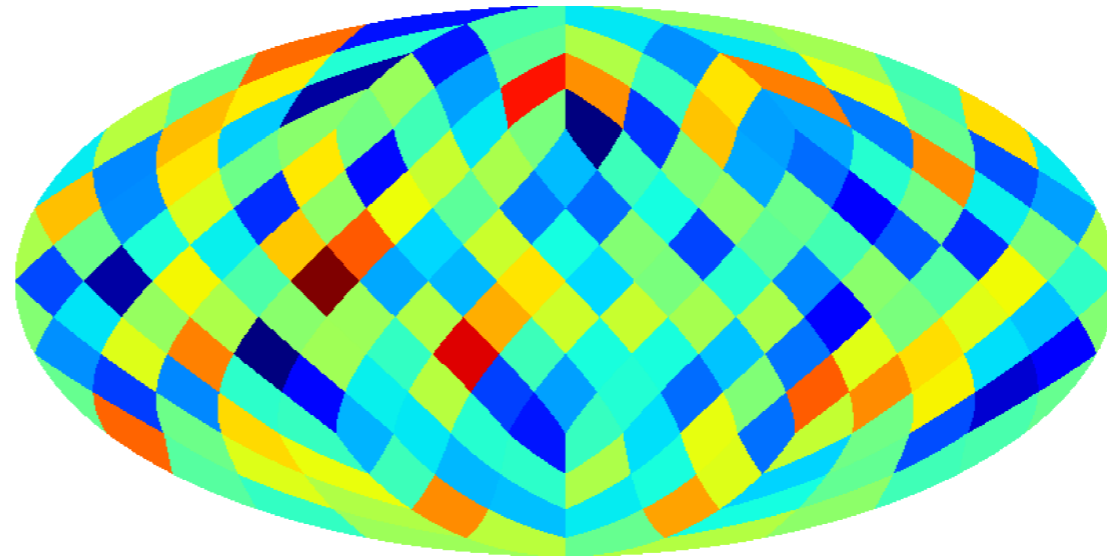
let's first assume the  
input and analysis  
patches match

we fit for set of spectral  
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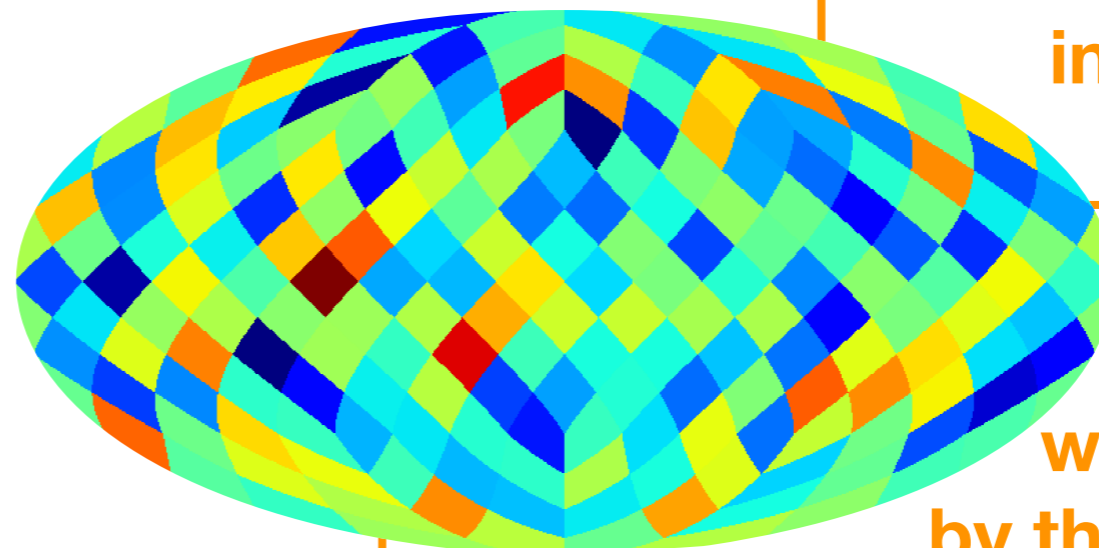
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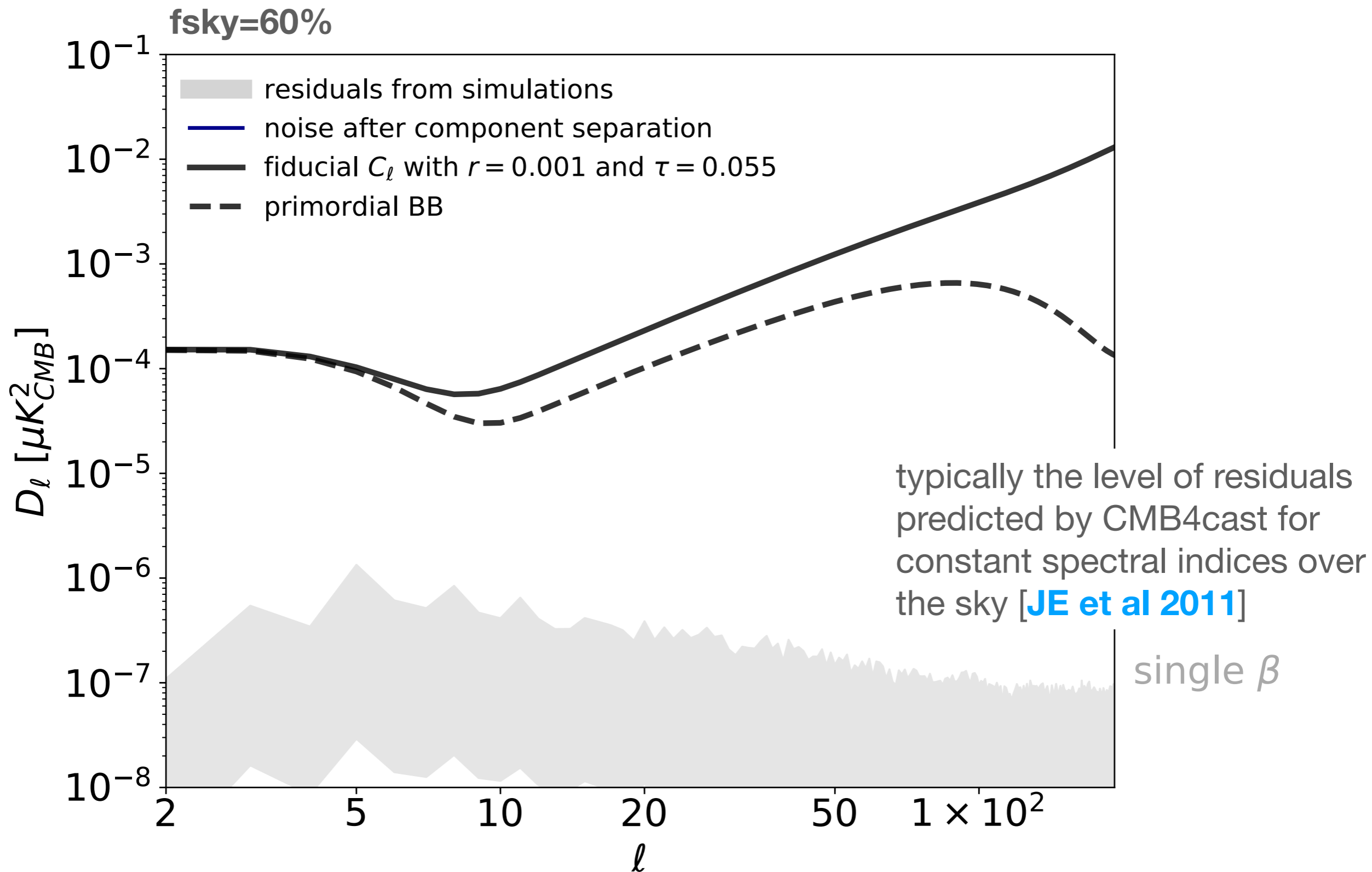
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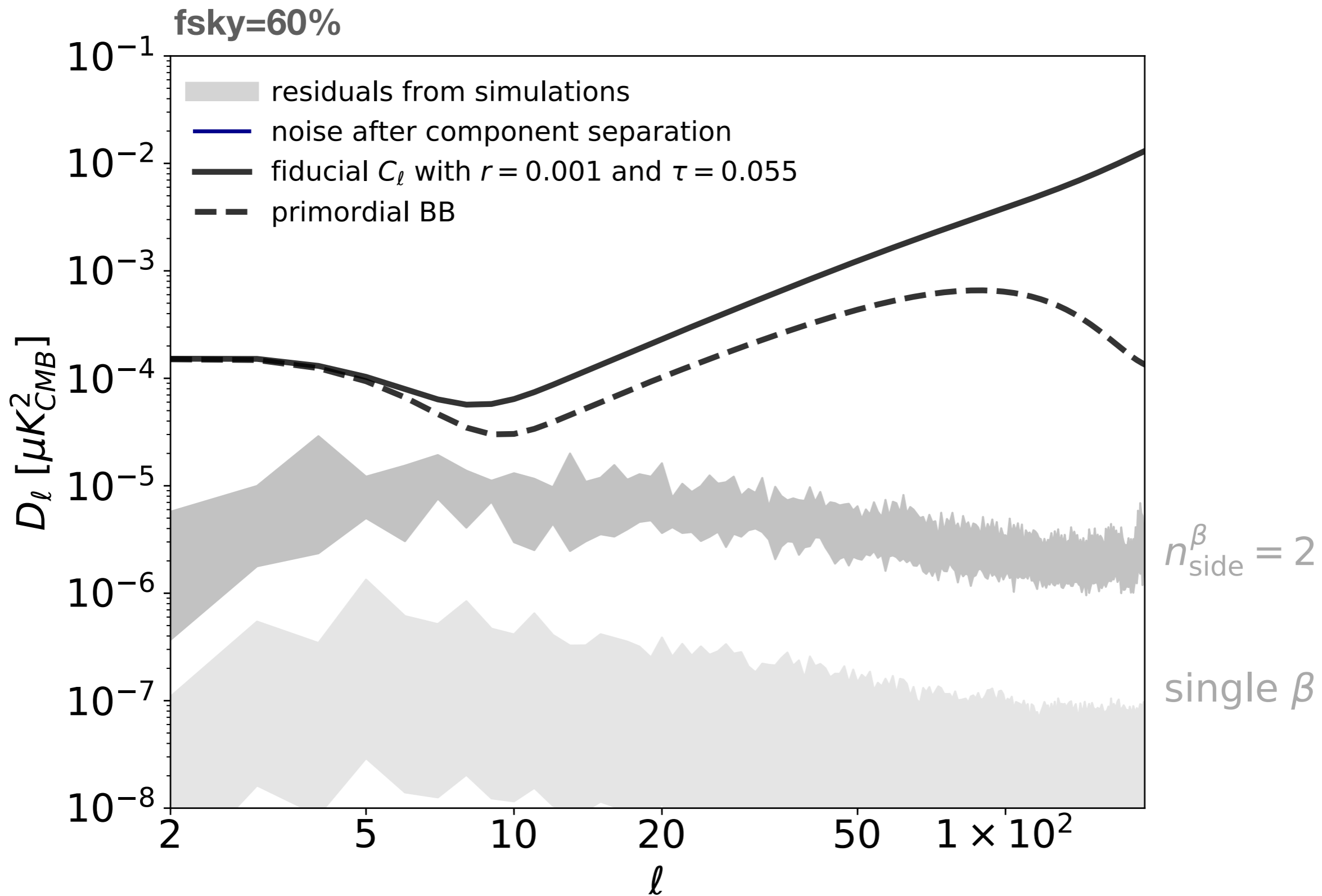


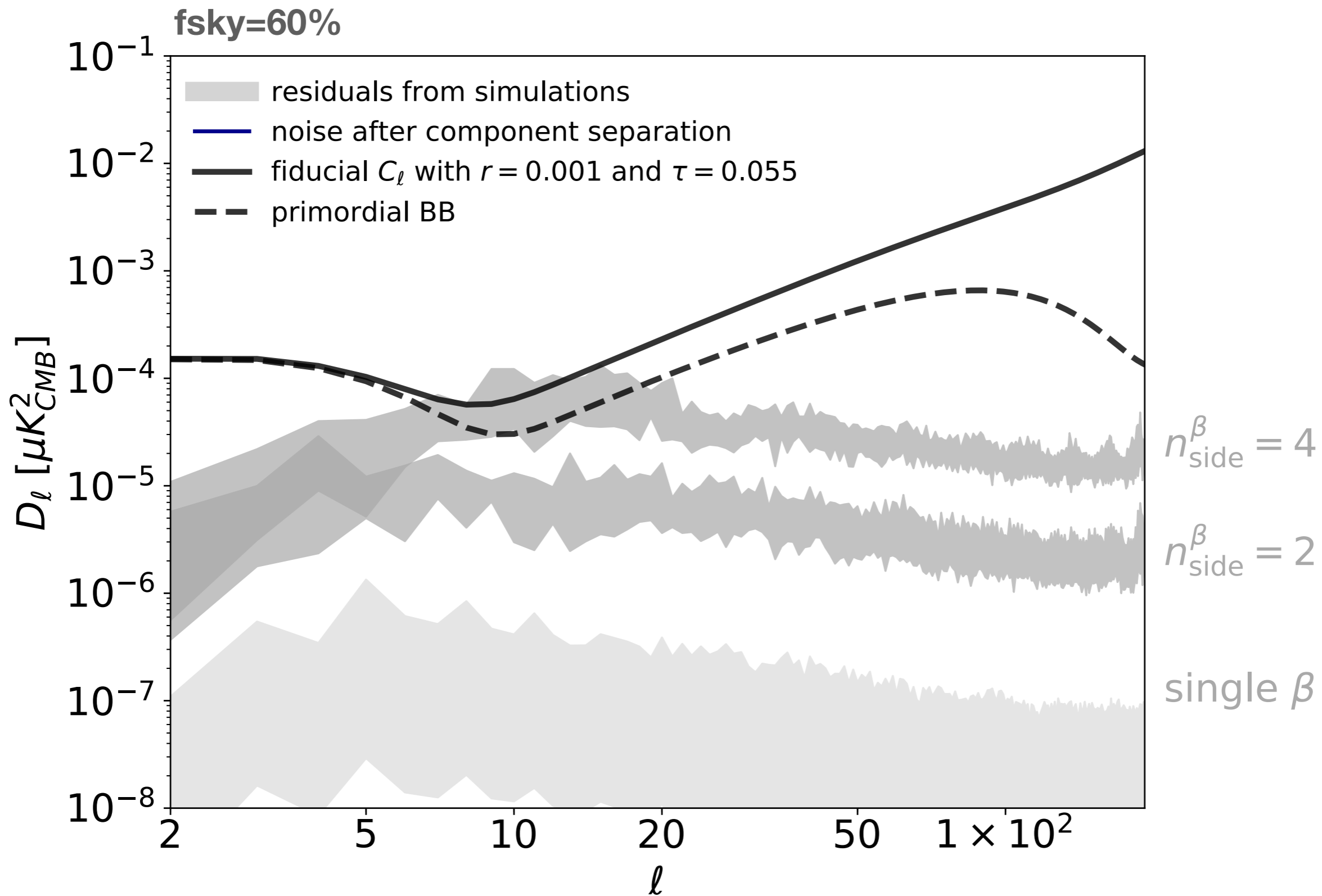
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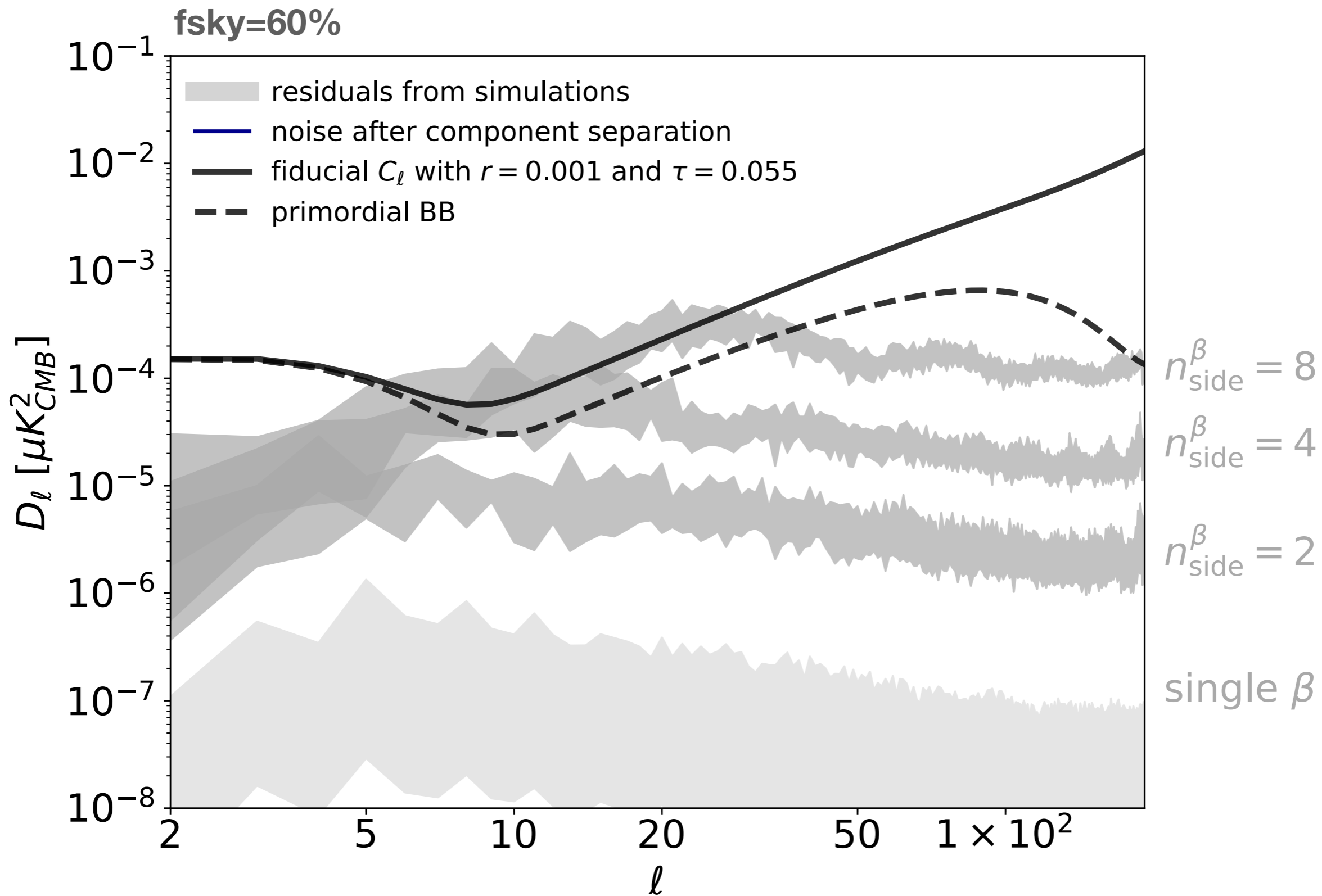
in this case,  
we are only limited  
by the statistical error  
bars on recovered spectral indices

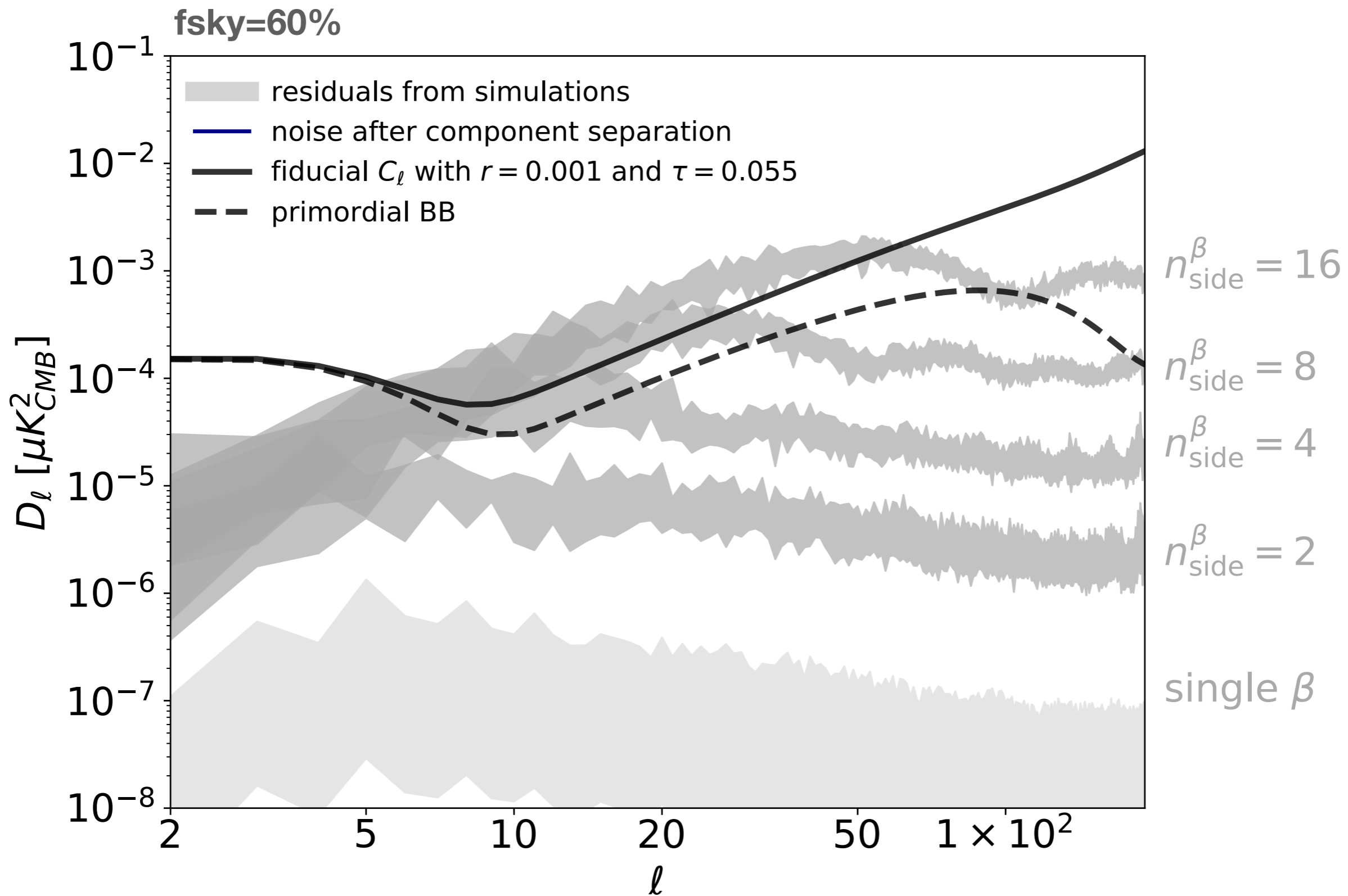




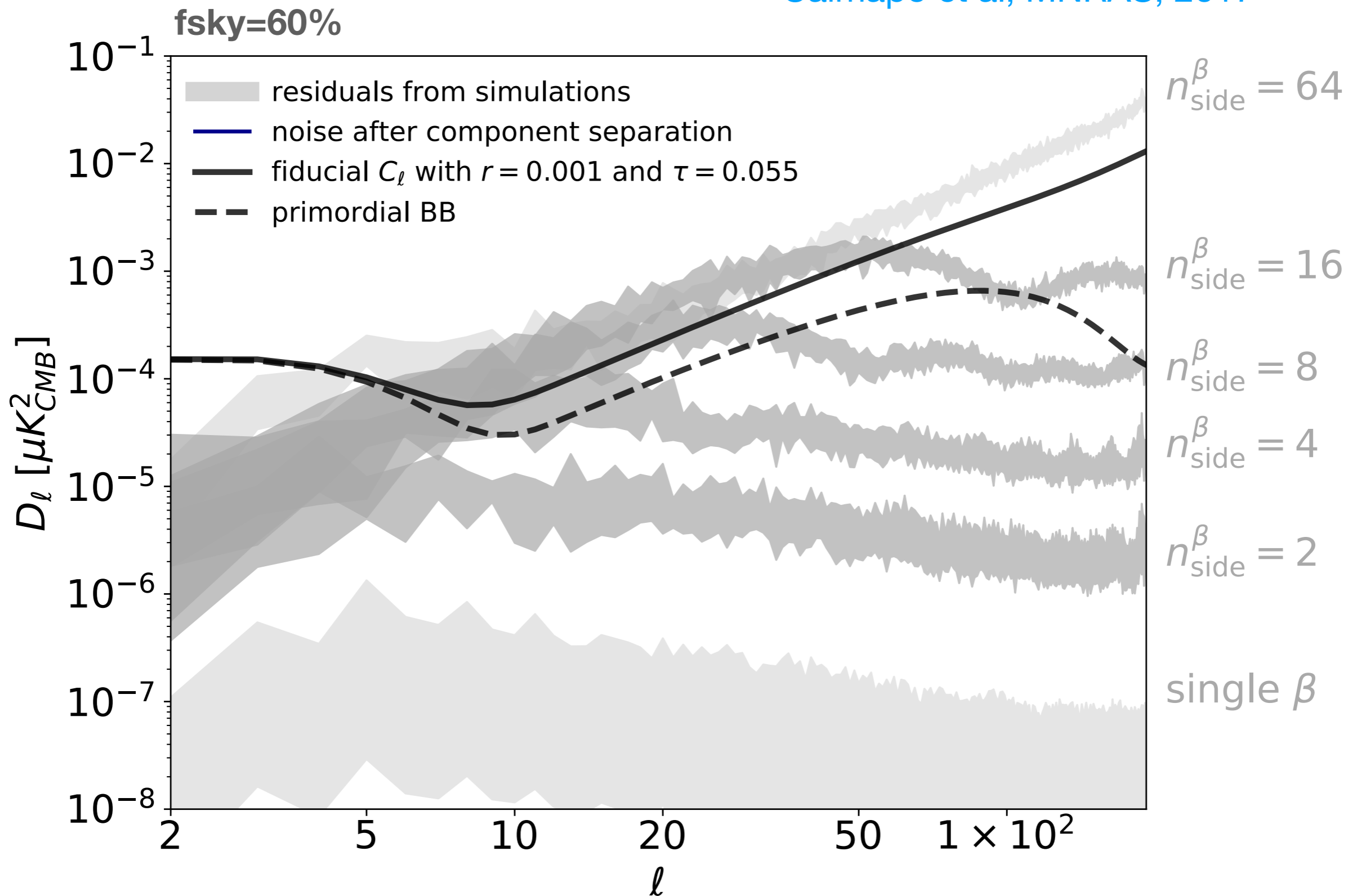






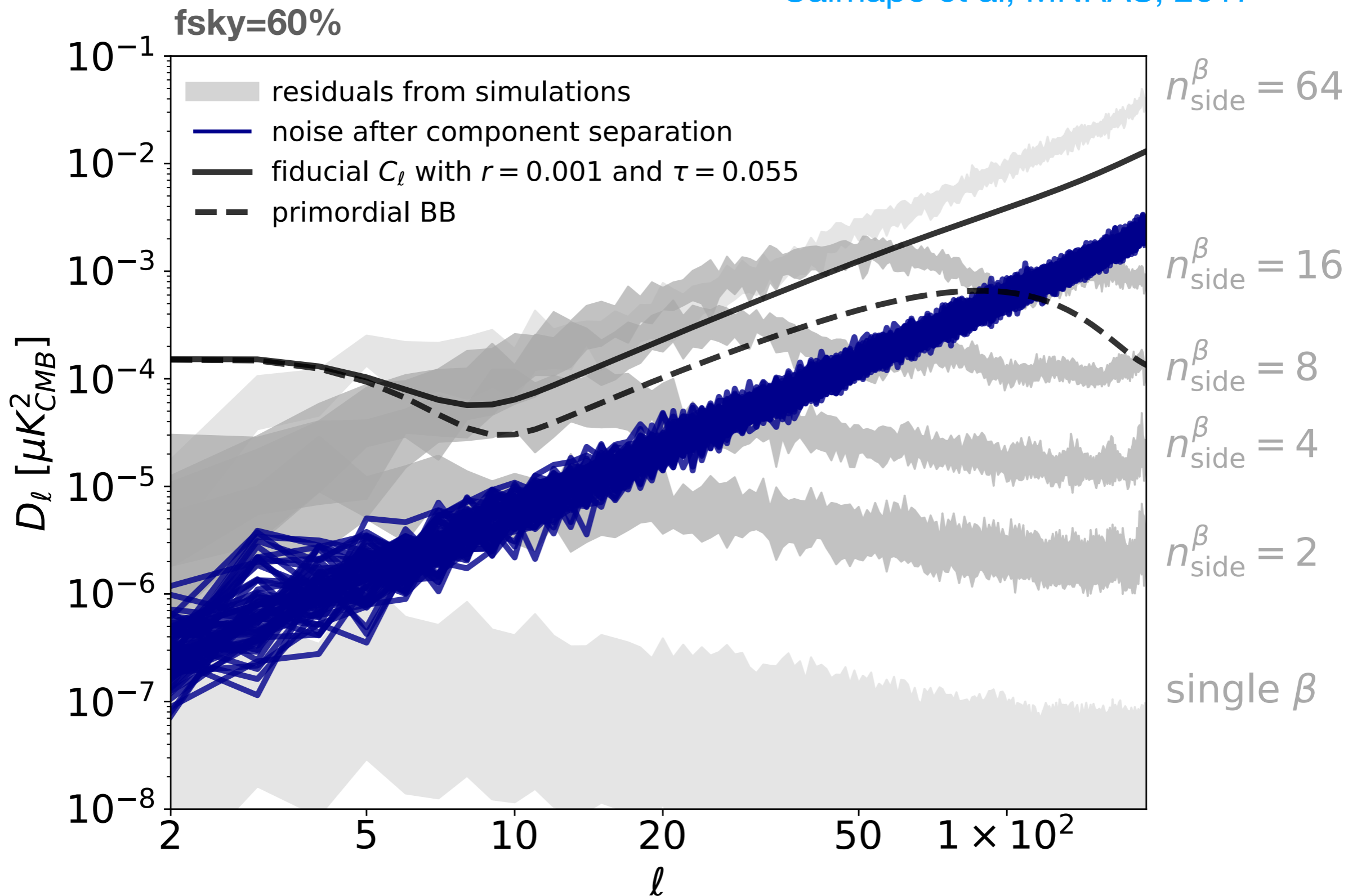


if not treated, these residuals can generate high bias, cf. [Hervías-Caimapo et al, MNRAS, 2017](#)

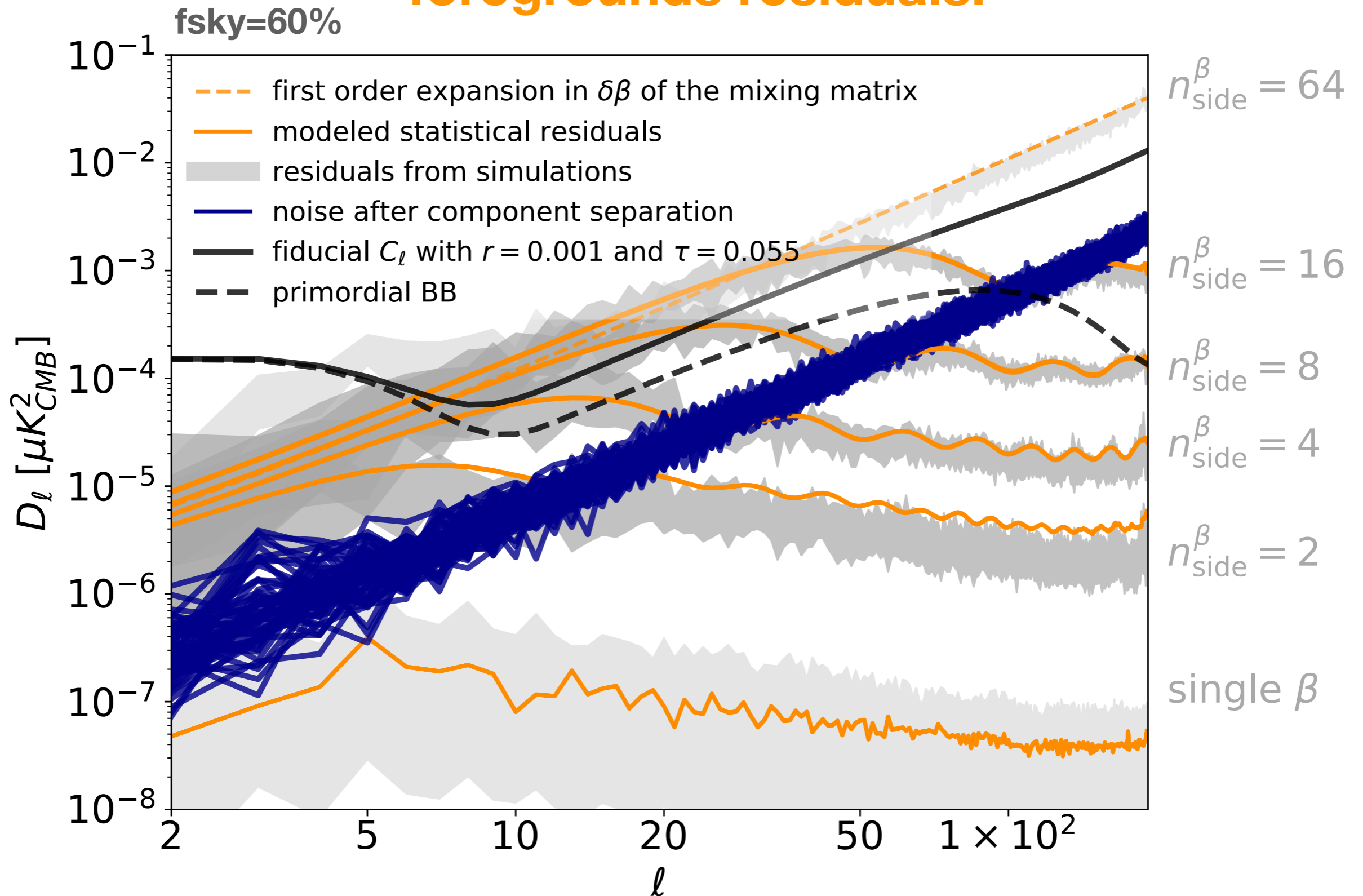




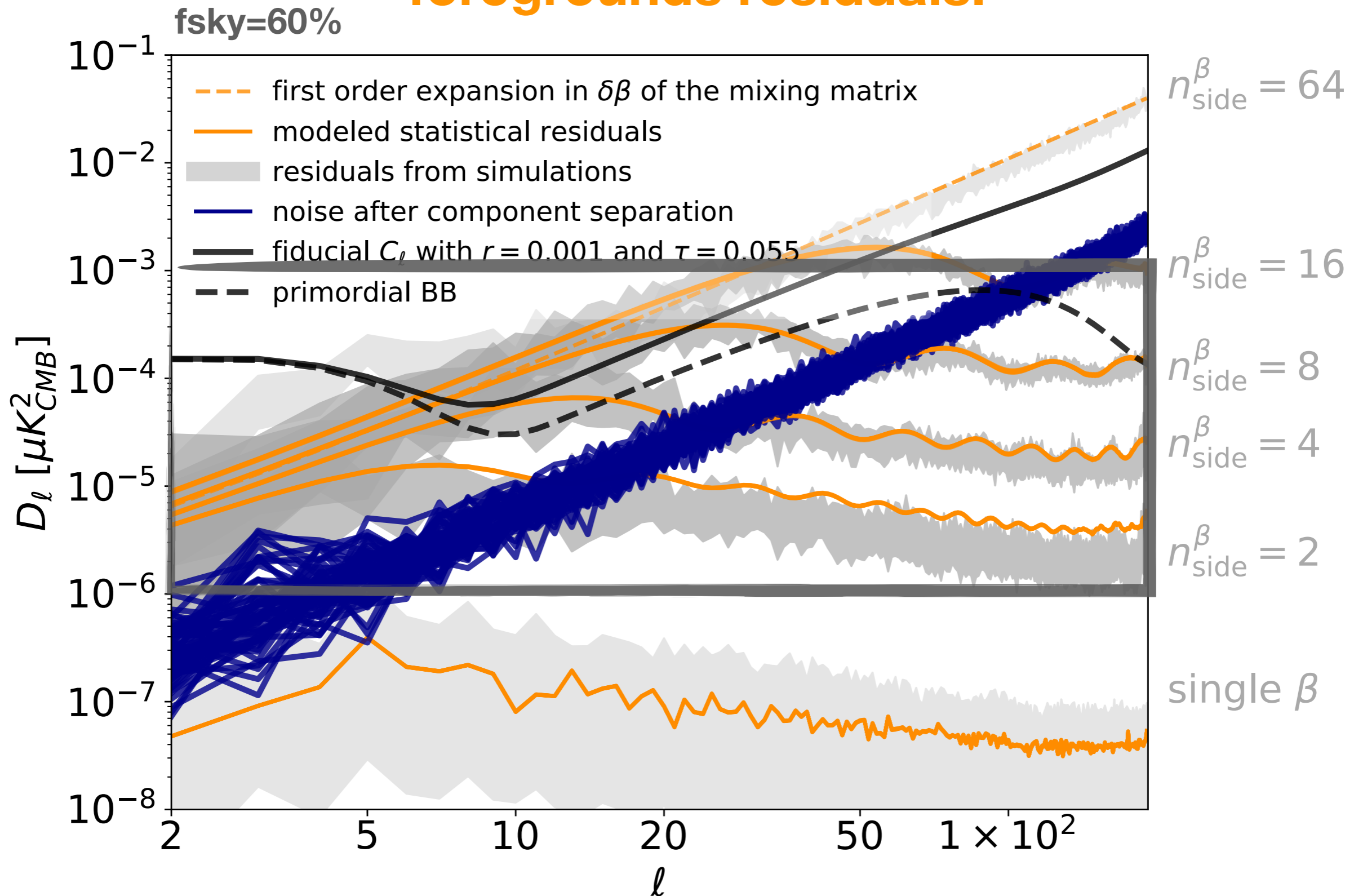
if not treated, these residuals can generate high bias, cf. [Hervías-Caimapo et al, MNRAS, 2017](#)

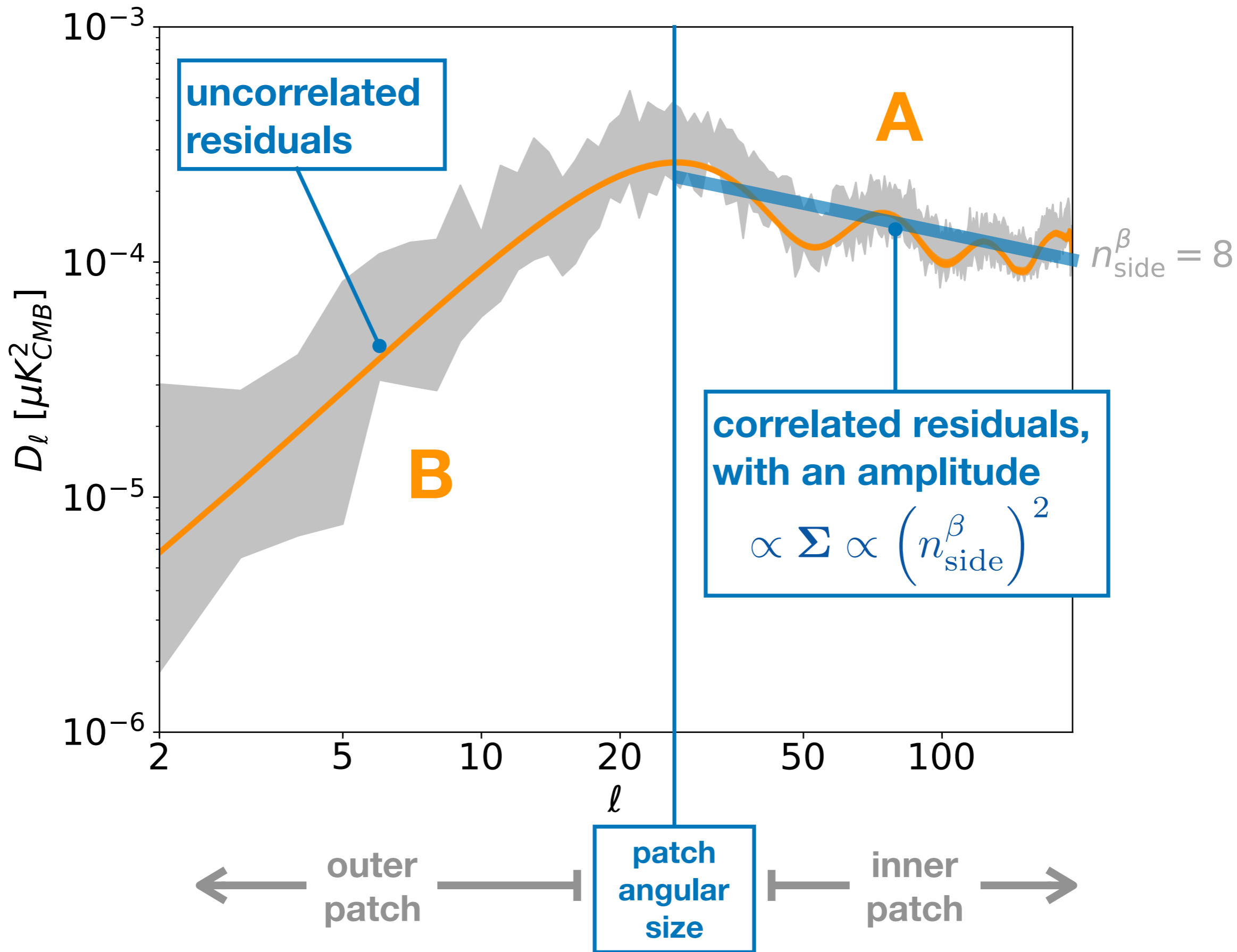


# we can semi-analytically model these statistical foregrounds residuals!

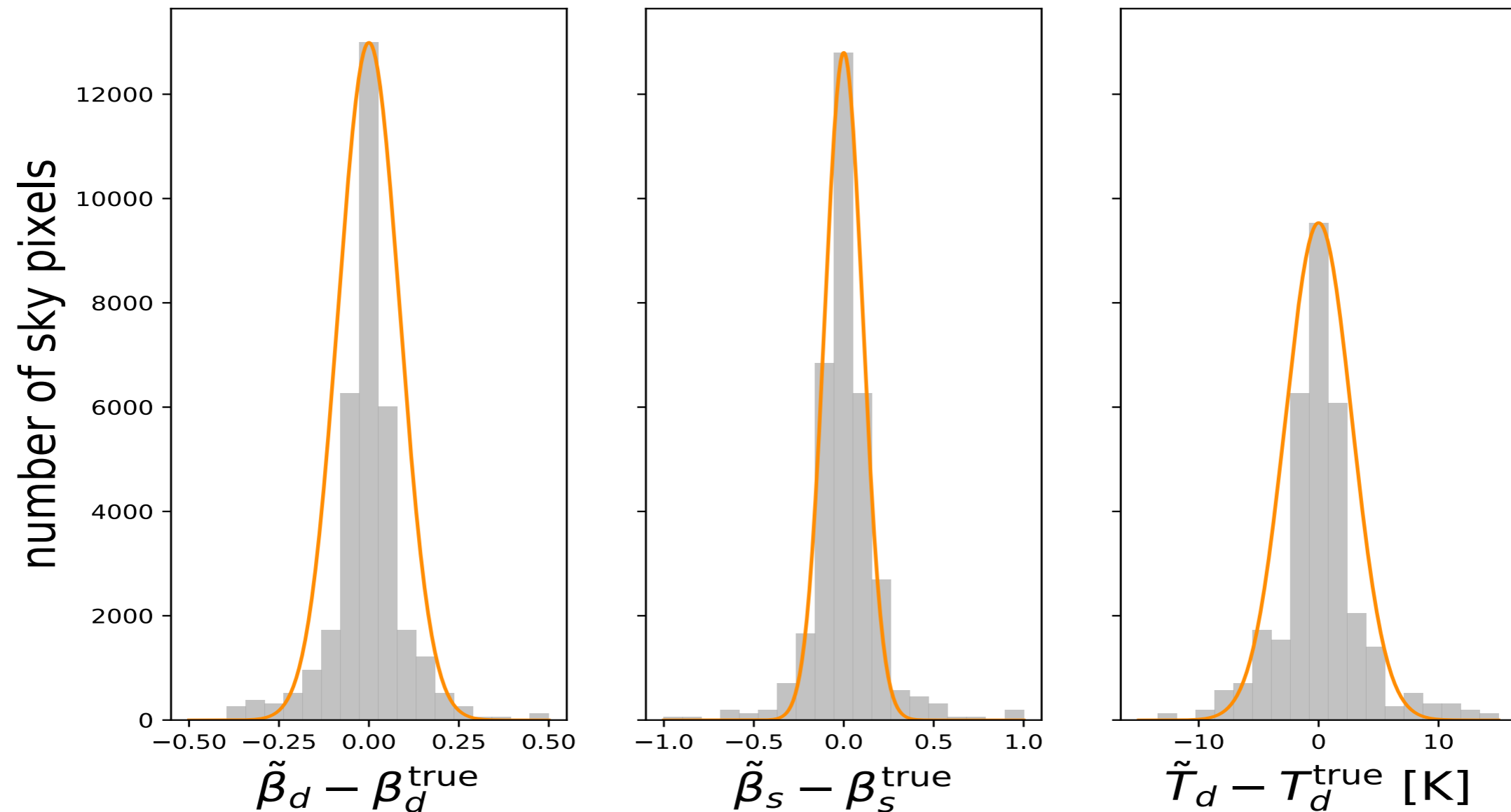


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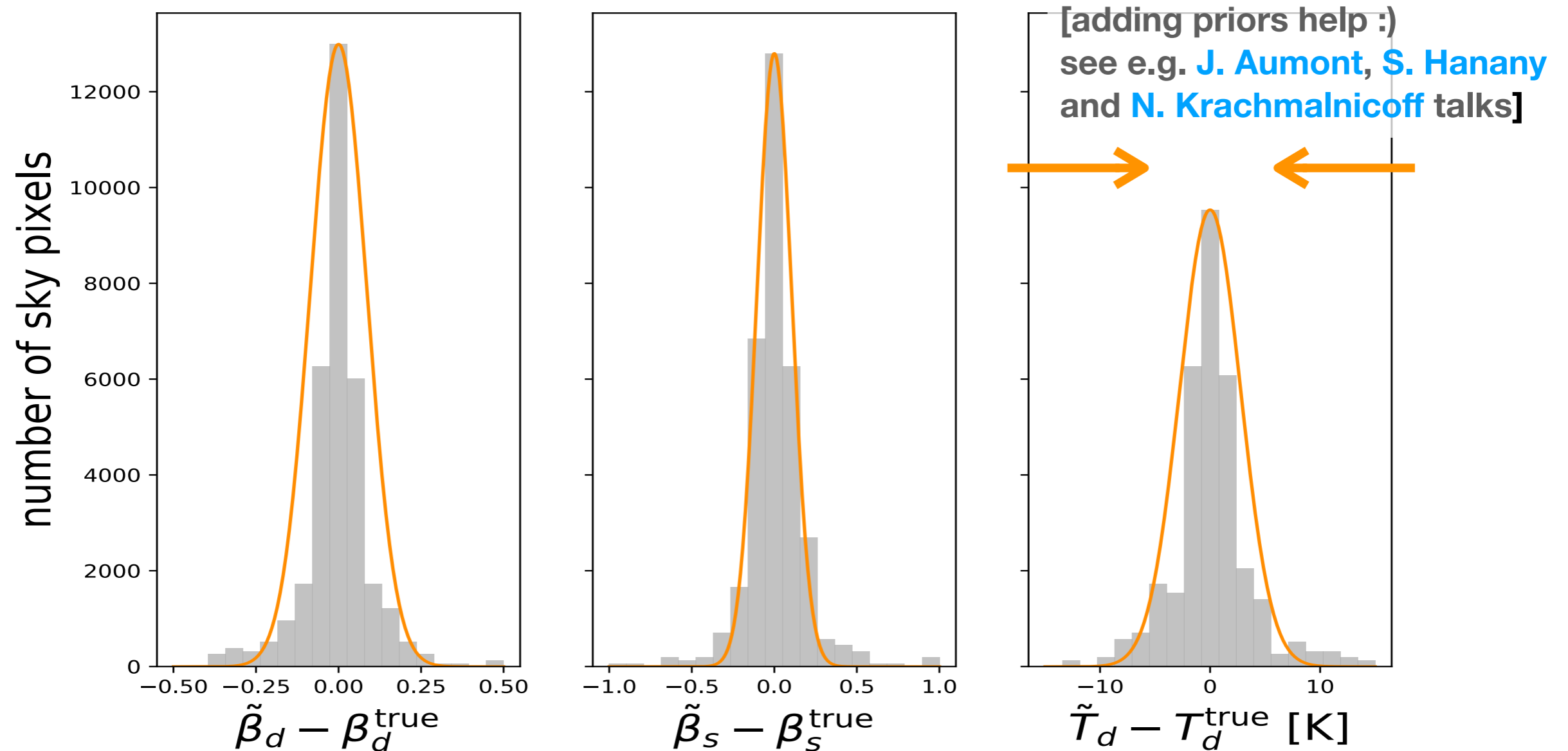


# A — we can characterize and model the dispersion of the recovered spectral indices around their “true” values



$$\left[ \Sigma_{\text{analytic}}^{-1} \right]_{\beta\beta'} \equiv \left\langle \frac{\partial^2 \mathcal{L}}{\partial \beta_i \partial \beta_j} \Big|_{\beta\beta'} \right\rangle_{\text{CMB+noise}} \simeq -\text{tr} \left( \left[ \frac{\partial \mathbf{A}}{\partial \beta_i} \Big|_{\beta}^T \mathbf{N}^{-1} \mathbf{A} (\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{N}^{-1} \frac{\partial \mathbf{A}}{\partial \beta_j} \Big|_{\beta'} - \frac{\partial \mathbf{A}}{\partial \beta_i} \Big|_{\beta}^T \mathbf{N}^{-1} \frac{\partial \mathbf{A}}{\partial \beta_j} \Big|_{\beta'} \right] \sum_p \mathbf{s}_p (\mathbf{s}_p)^T \right)$$

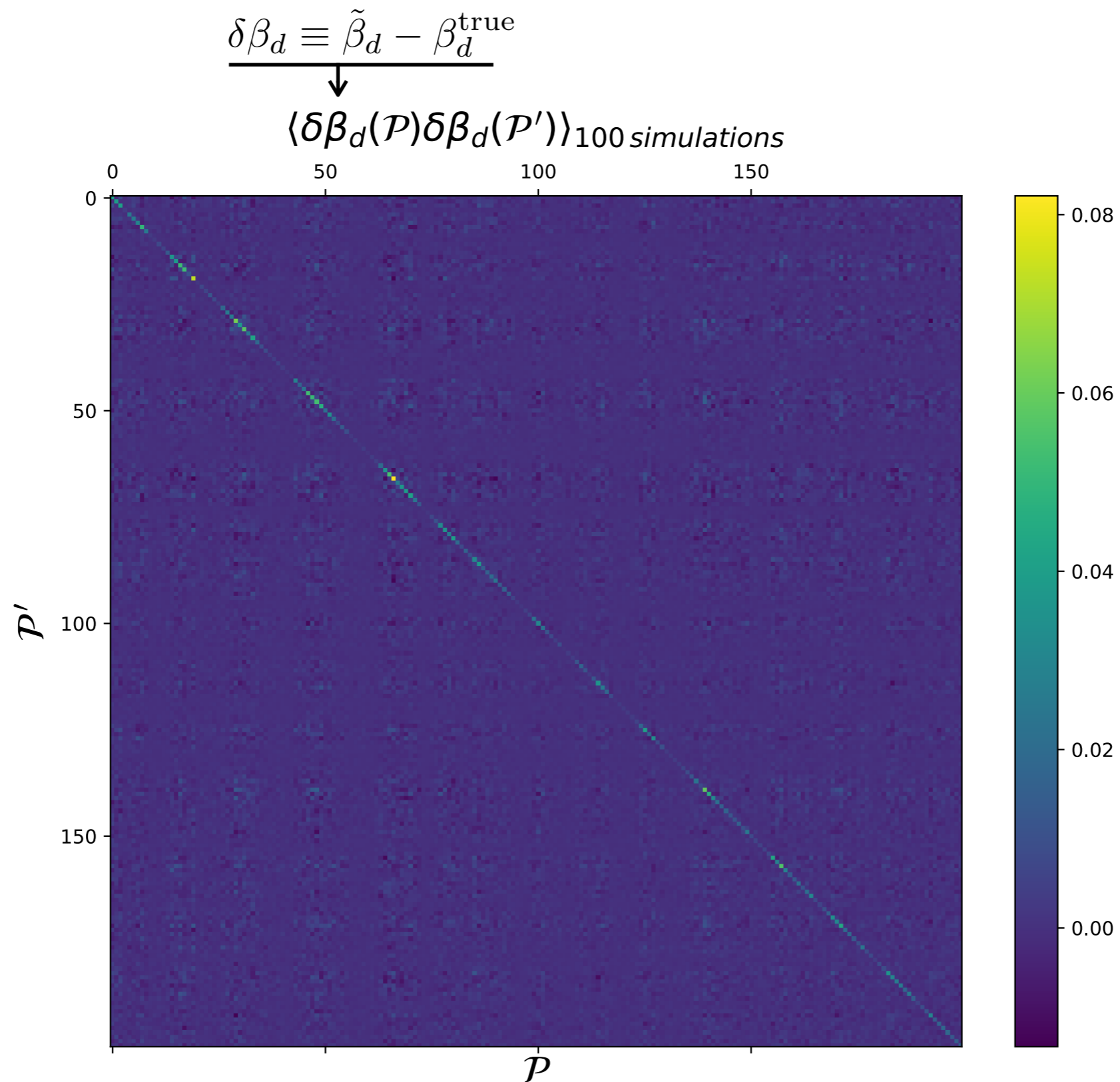
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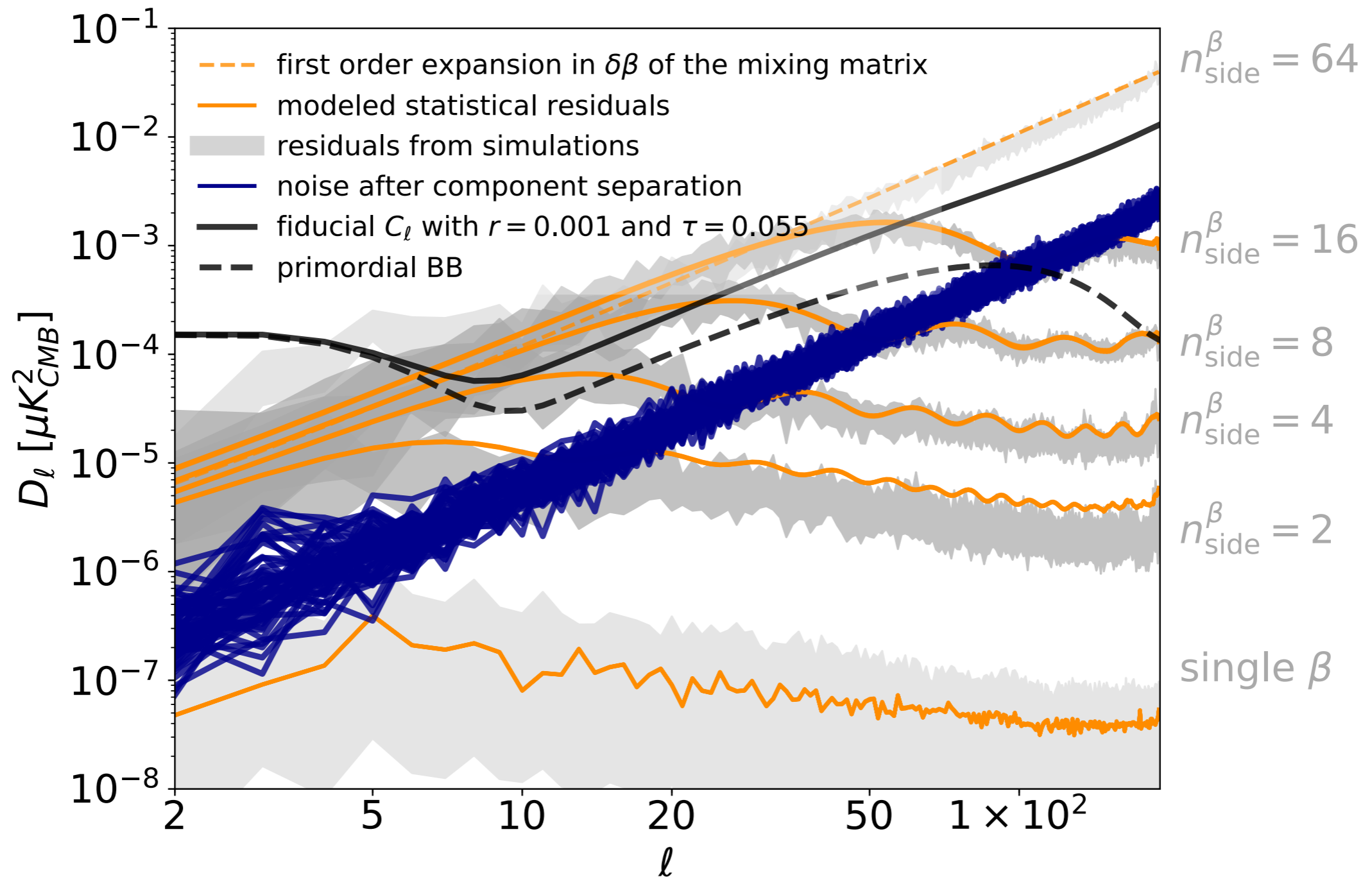
# B — we see a decorrelation of statistical foregrounds residuals on large angular scales



# semi-analytical modeling of statistical foregrounds residuals

$$C_\ell^\Sigma \equiv \sum_{\mathcal{P}} \sum_{i,j,k,l} \Sigma_{ij}^{\mathcal{P}} \kappa_{kl}^{ij, \mathcal{P}} C_\ell^{kl, \mathcal{P}}$$

↑ computed from data



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$$\left\langle -2 \log(\mathcal{L}_{\text{cosmo}}) \right\rangle_{\text{CMB+noise}} = \sum_{\ell} \frac{2\ell + 1}{2} f_{\text{sky}} \left( \mathbf{C}_{\ell}^{-1} \mathbf{D}_{\ell} + \log [\det(\mathbf{C})] \right)$$

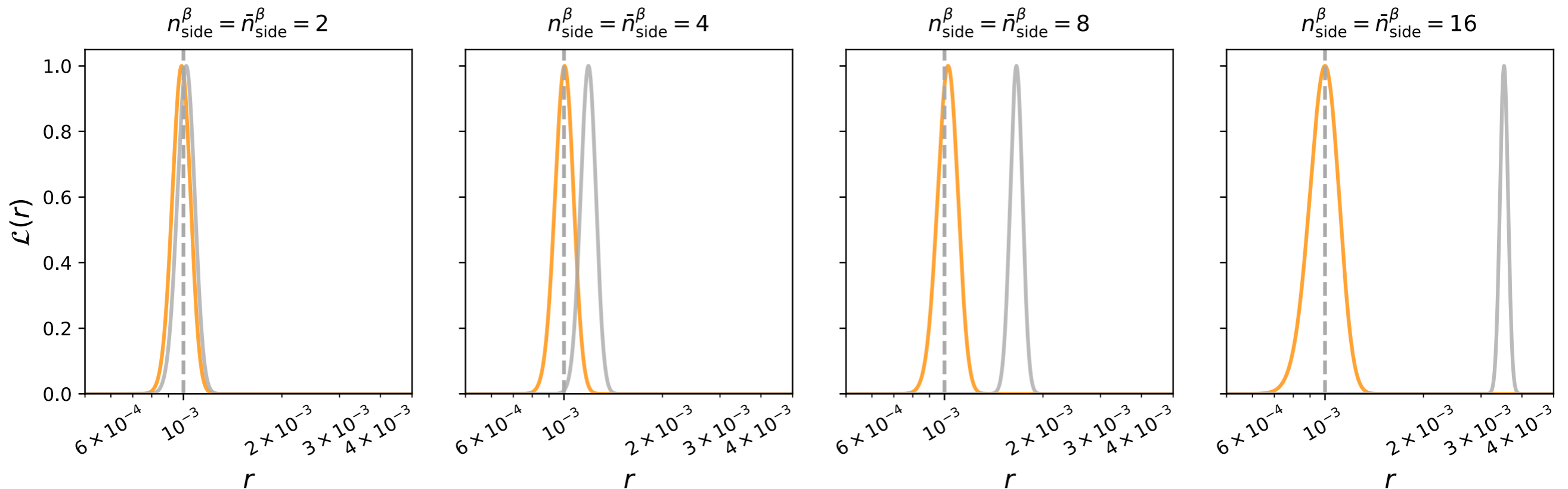
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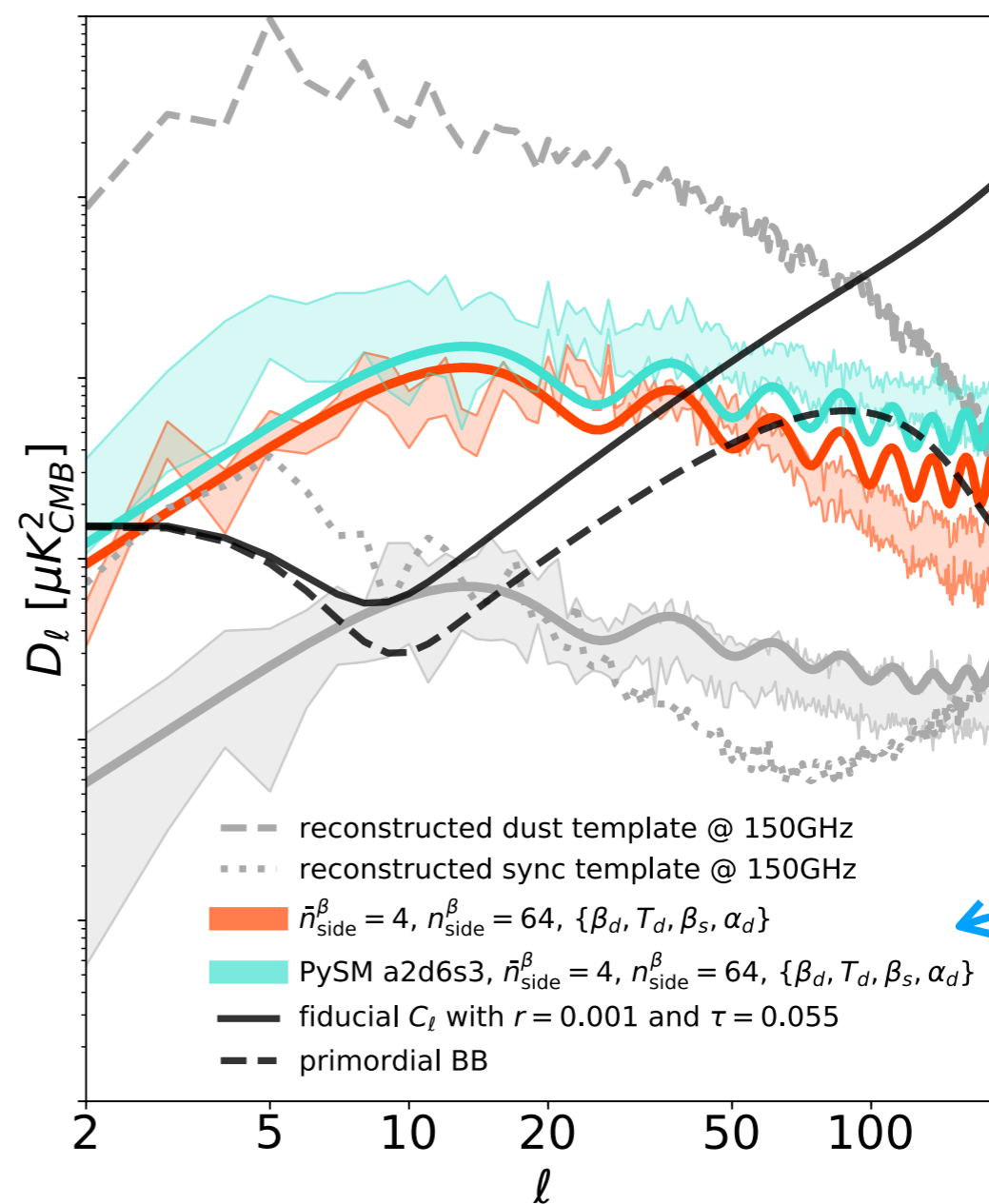
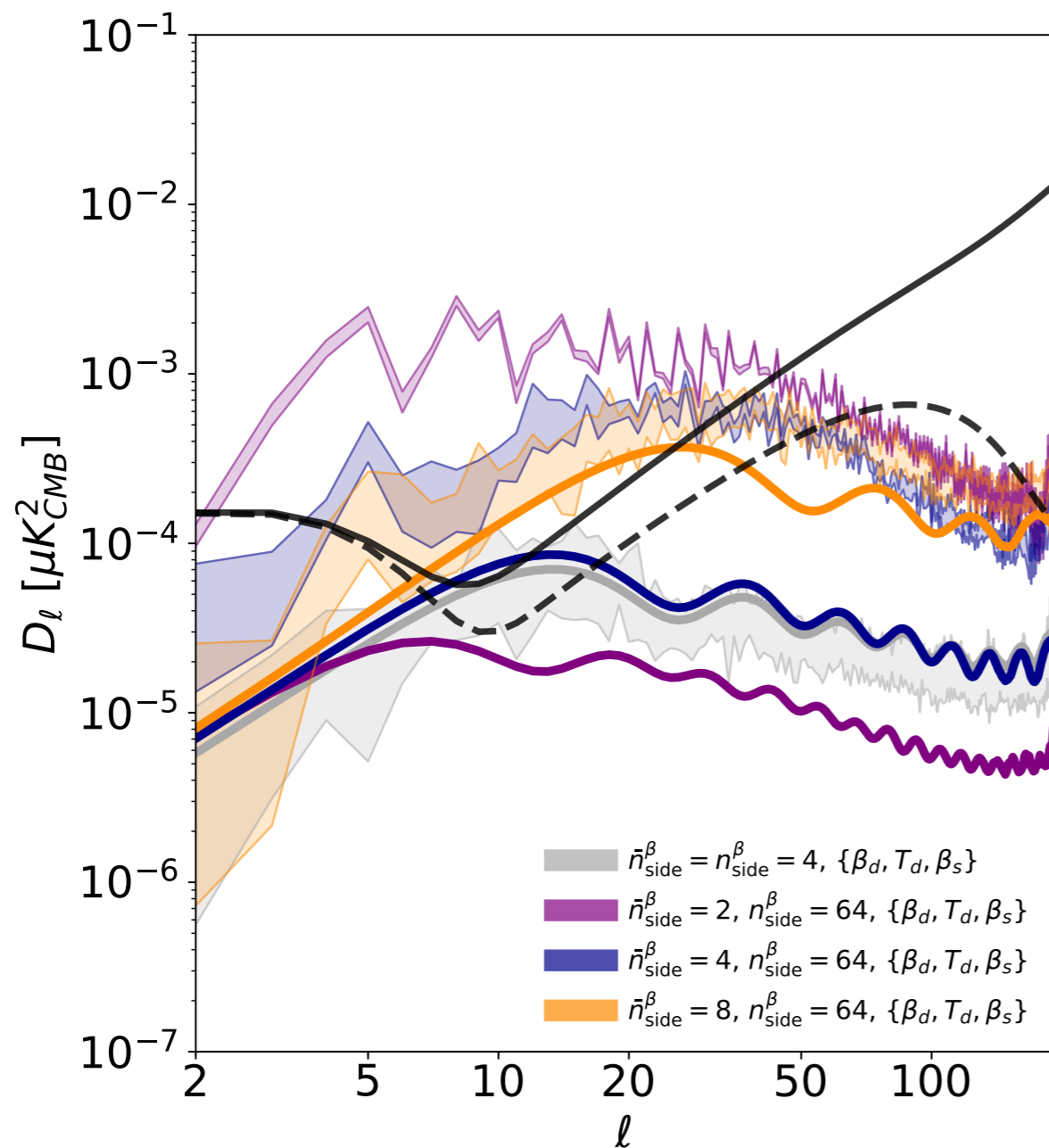
## About more “complex” foregrounds

- sim#1: foregrounds SEDs varying on a resolution ( $n_{\text{side}}=64$ )  $\gg$  than the patch size used for the analysis ( $2 < n_{\text{side}} < 8$ )  $\rightarrow$  “averaging” effect
- sim#2: a2d6s3 PySM model  $\rightarrow$  polarized AME, dust following Vansyngel et al (2017) and curved synchrotron



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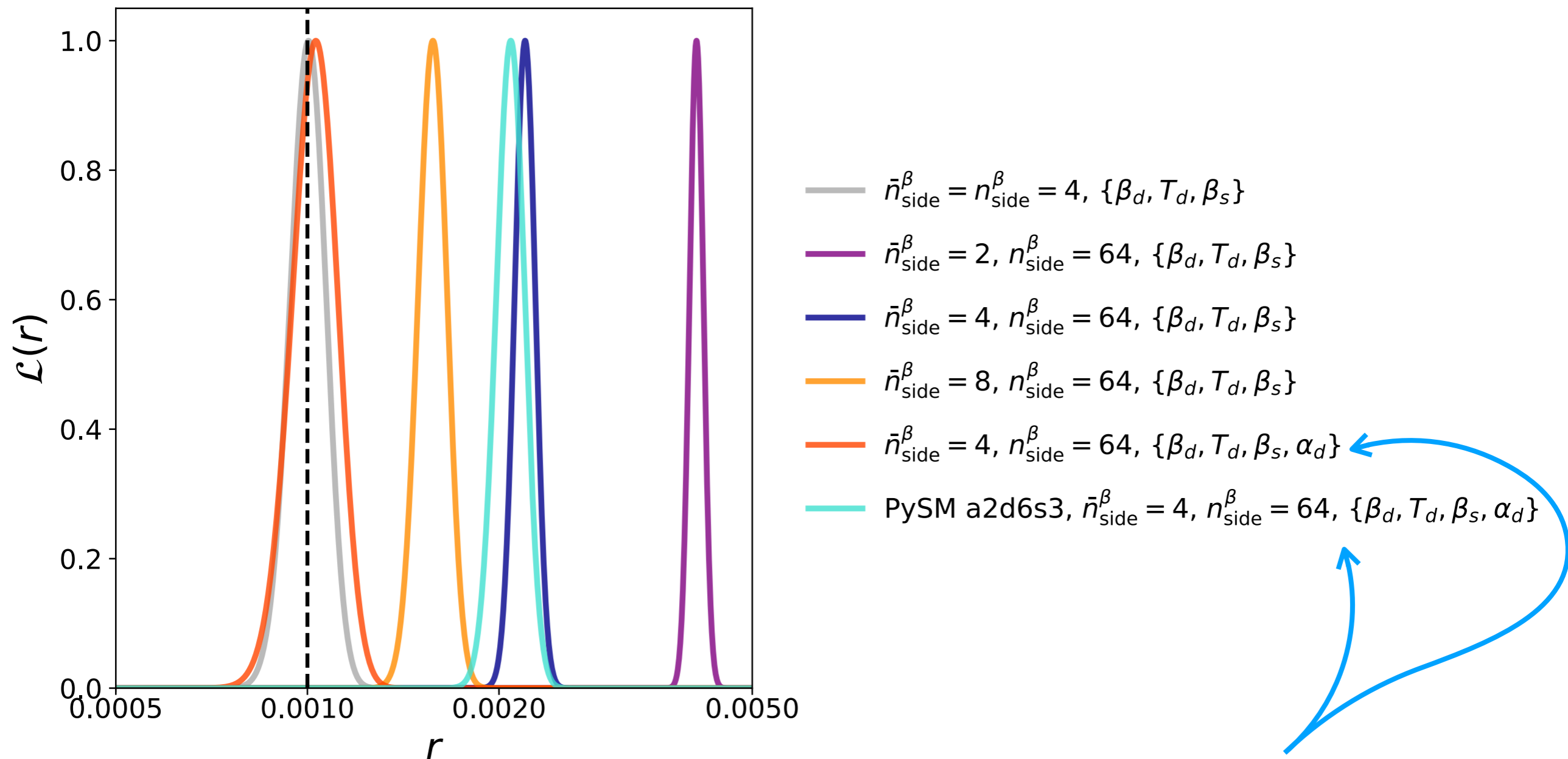


[see A. Mangilli and A. Rotti talks and Chluba et al 2018]

JE et al, in prep

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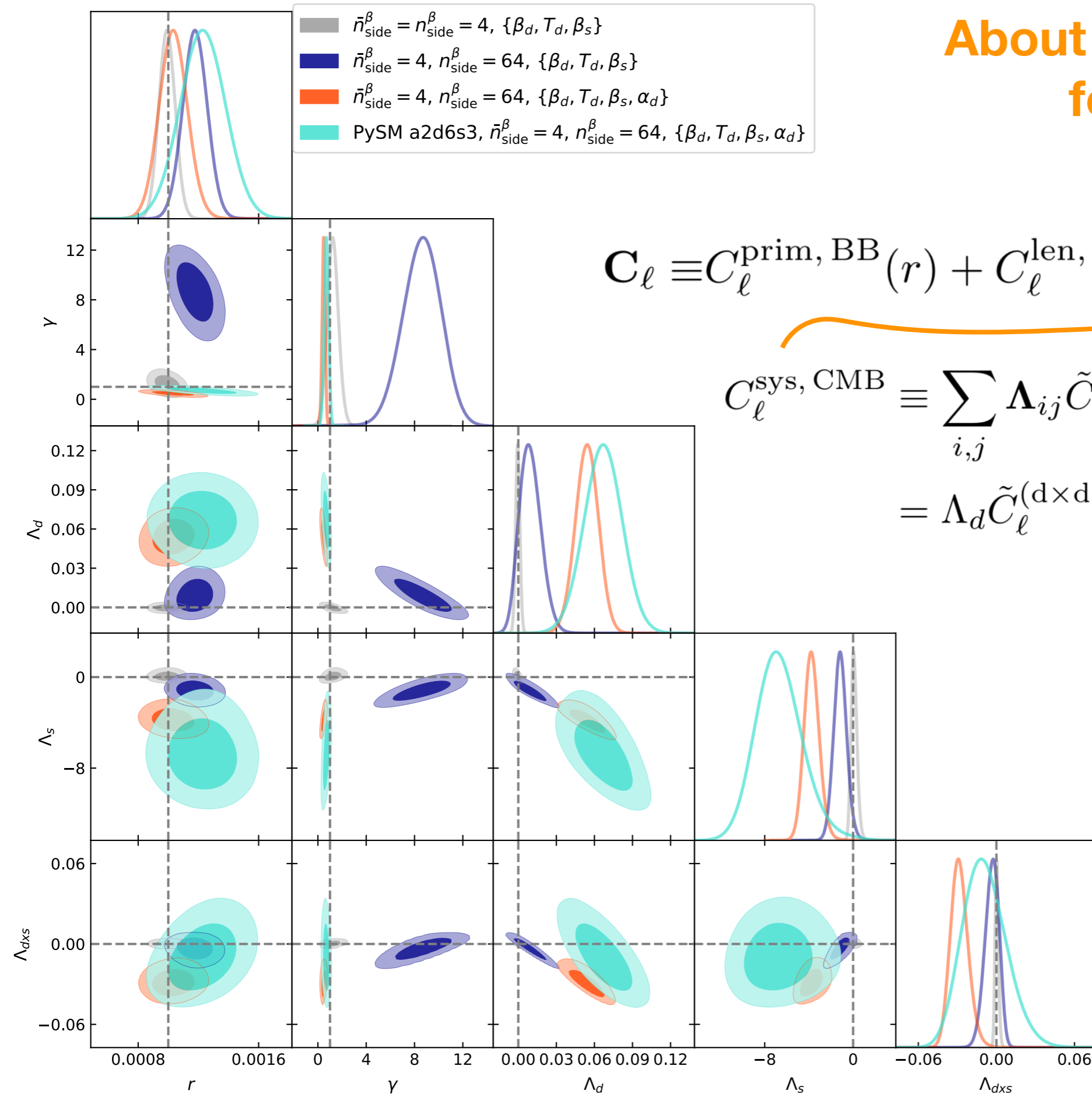
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[see [A. Mangilli](#) and [A. Rotti](#) talks and [Chluba et al 2018](#)]

[JE et al, in prep](#)

# About more “complex” foregrounds



$$\mathbf{C}_\ell \equiv C_\ell^{\text{prim, BB}}(r) + C_\ell^{\text{len, BB}} + N_\ell + \gamma C_\ell^\Sigma + C_\ell^\Lambda$$

$$C_\ell^{\text{sys, CMB}} \equiv \sum_{i,j} \Lambda_{ij} \tilde{C}_\ell^{(i \times j)}$$

$$= \Lambda_d \tilde{C}_\ell^{(d \times d)} + \Lambda_s \tilde{C}_\ell^{(s \times s)} + \Lambda_{d \times s} \tilde{C}_\ell^{(d \times s)}$$

# Conclusions

- **CMB4cast** and **xForecast** are great tools to address specific questions, e.g. the optimization of a focal plane, average performance of an instrument in the case of simple modeling of the SEDs, etc.

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- **Multipatch** approach tries to add the minimum and necessary degrees of freedom to the pixel-based parametric approach in order to deal with these spatial variations
- We show that including an **estimate of statistical foregrounds residuals** in the cosmological likelihood is important to get an unbiased estimate of e.g. tensor-to-scalar ratio, particularly when  $r \sim 0.001$
- We show that adding **moments** to the spectral fit allows the parametric approach to handle spatial averaging of SEDs, and that an extra marginalization of the cosmological likelihood over **systematic leakage** is possible
- There are ways to **lower statistical foregrounds residuals** while keeping the systematic ones under control ...





**how many moments are necessary?**

**what are the necessary  
degrees of freedom?**

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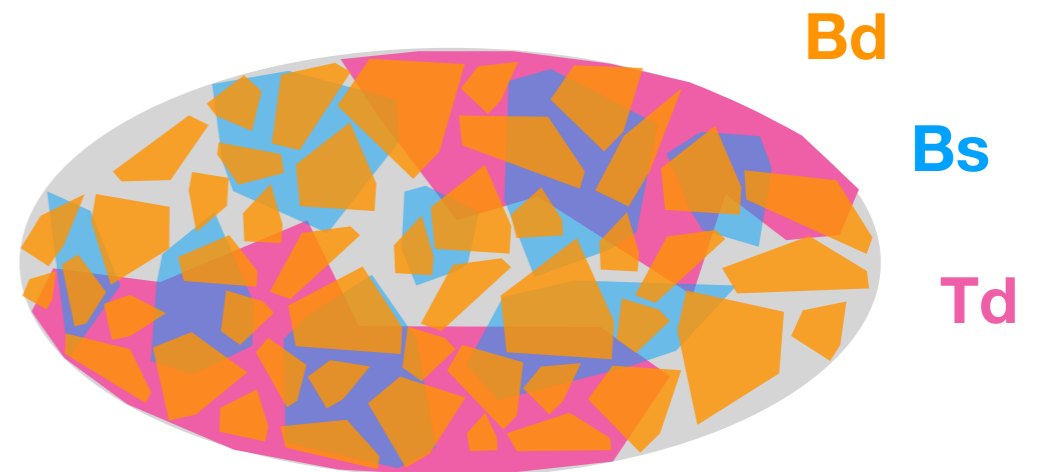
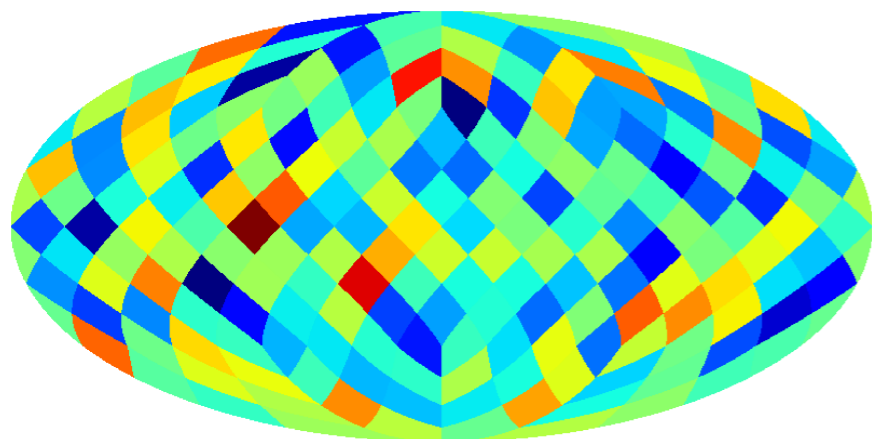
towards an adaptative multipatch?

**STATISTICAL** error bars on spectral parameters



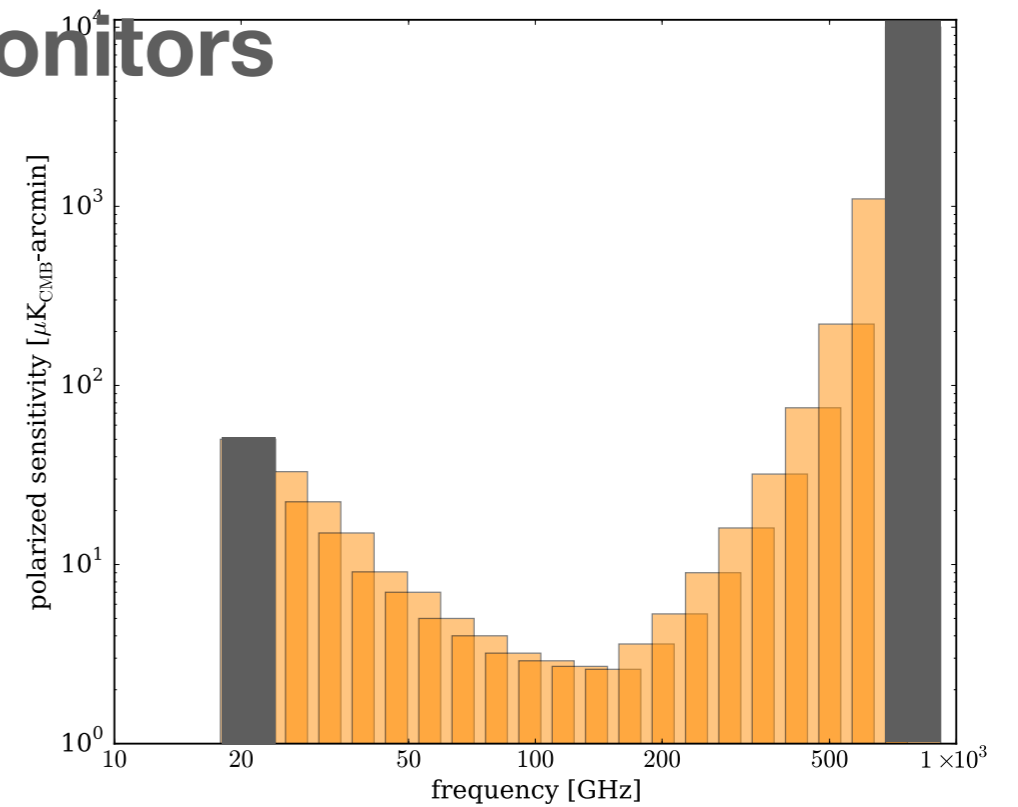
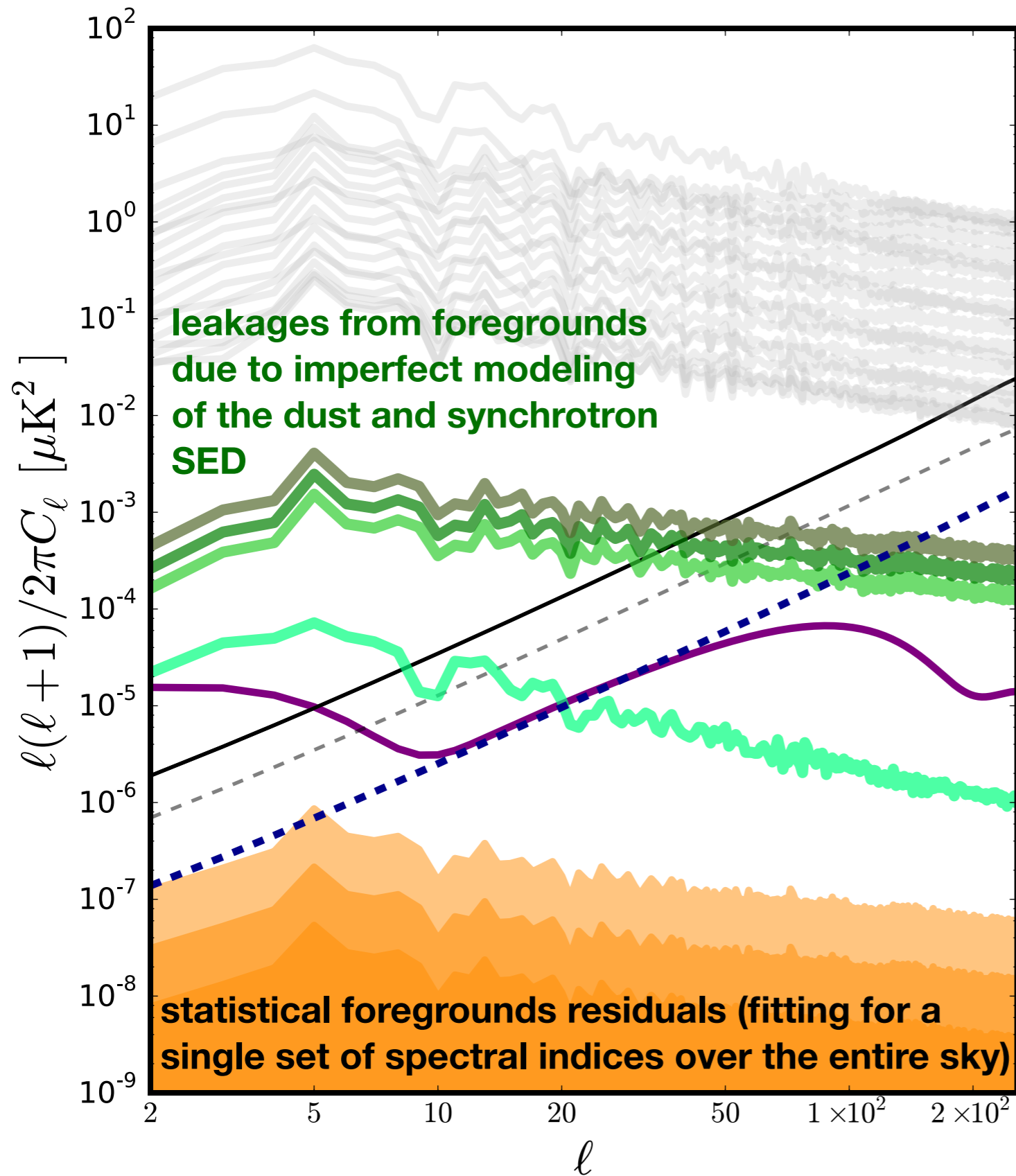
**SYSTEMATIC** error bars on spectral parameters

finding iteratively **independent regions, independently for each spectral index**, such that spectral indices can be assumed almost constant  $\rightarrow$  with spatial variations  $\Delta\beta \propto \sigma(\beta)$ , the statistical error on spectral indices.



**BACKUP**

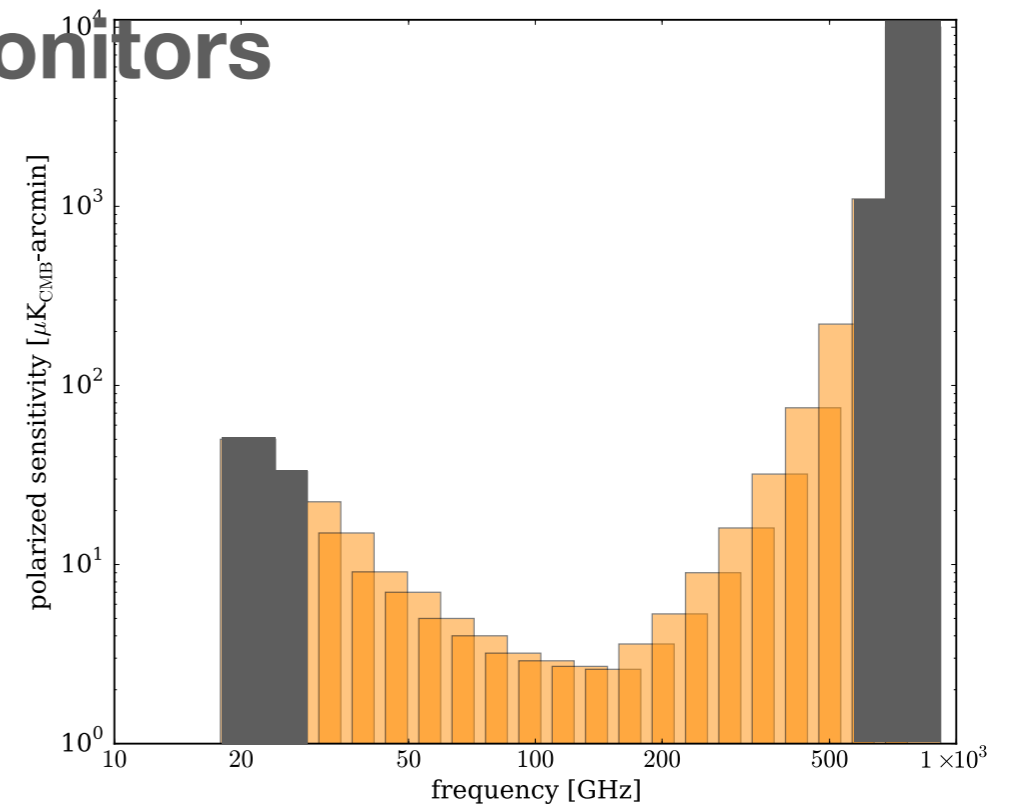
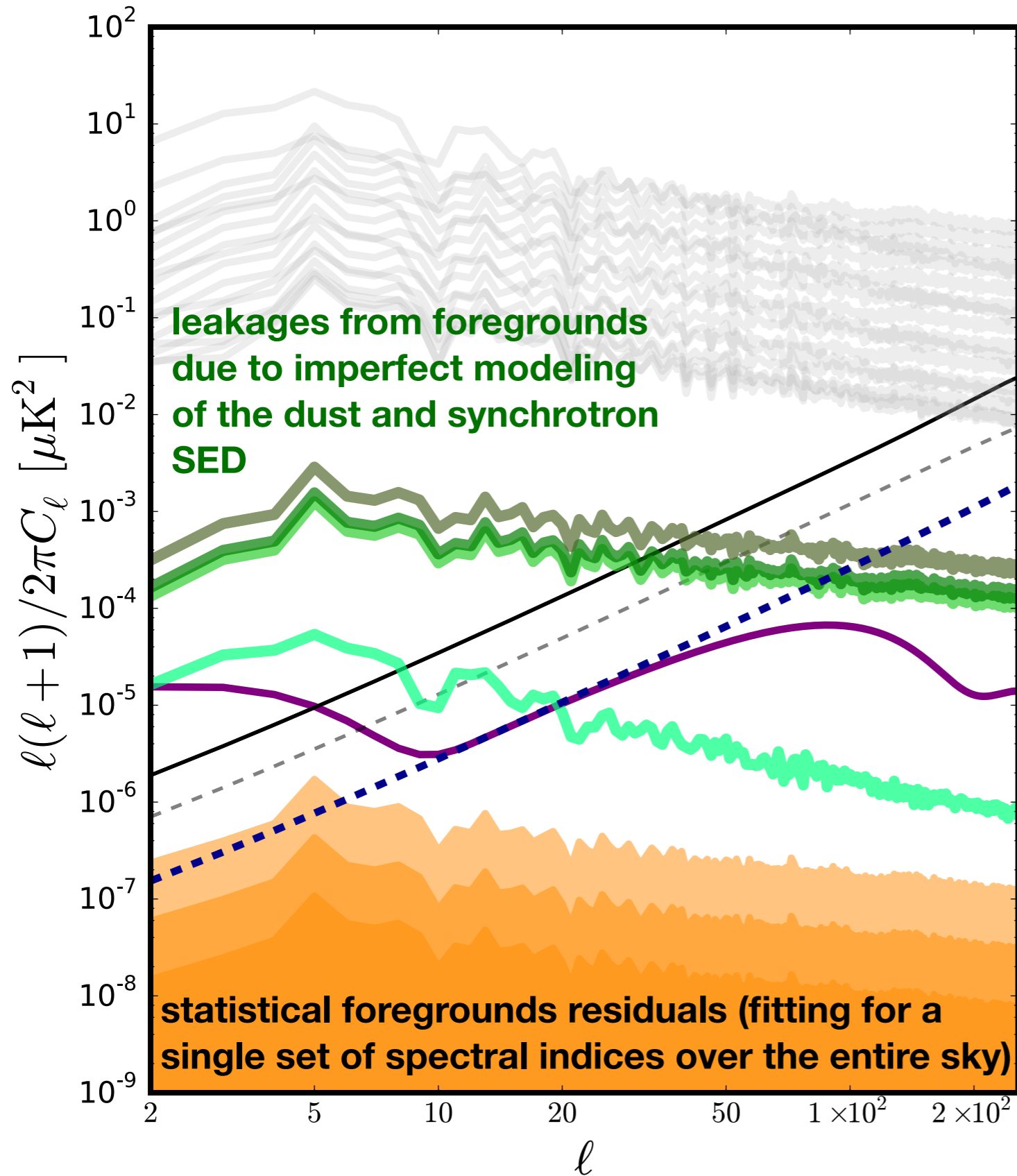
# Removing 1 synchrotron and 1 dust monitors



- sys. res. from  $\beta_d$  spat. var.
- sys. res. from  $\beta_s$  spat. var.
- sys. res. from  $T_d$  spat. var.
- sys. res. from  $\{\beta_d, \beta_s, T_d\}$  spat. var.
- -  $N_\ell$

**all frequency bands**  
**- 21GHz - 800GHz**  
**60% of the sky**

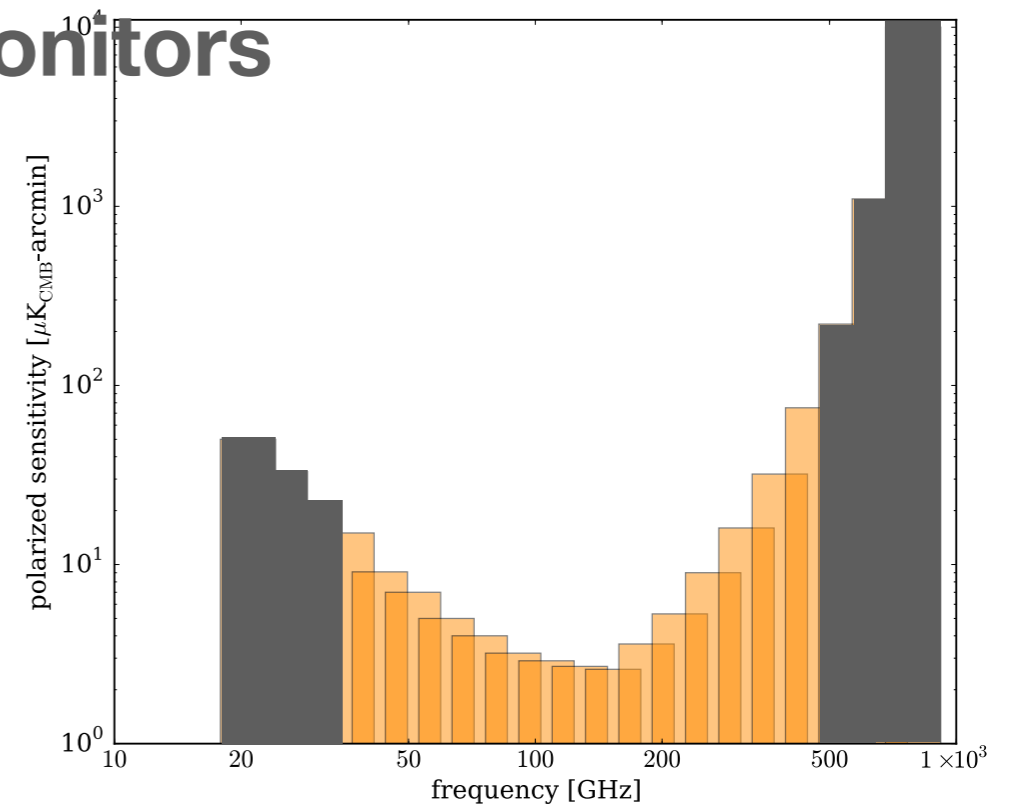
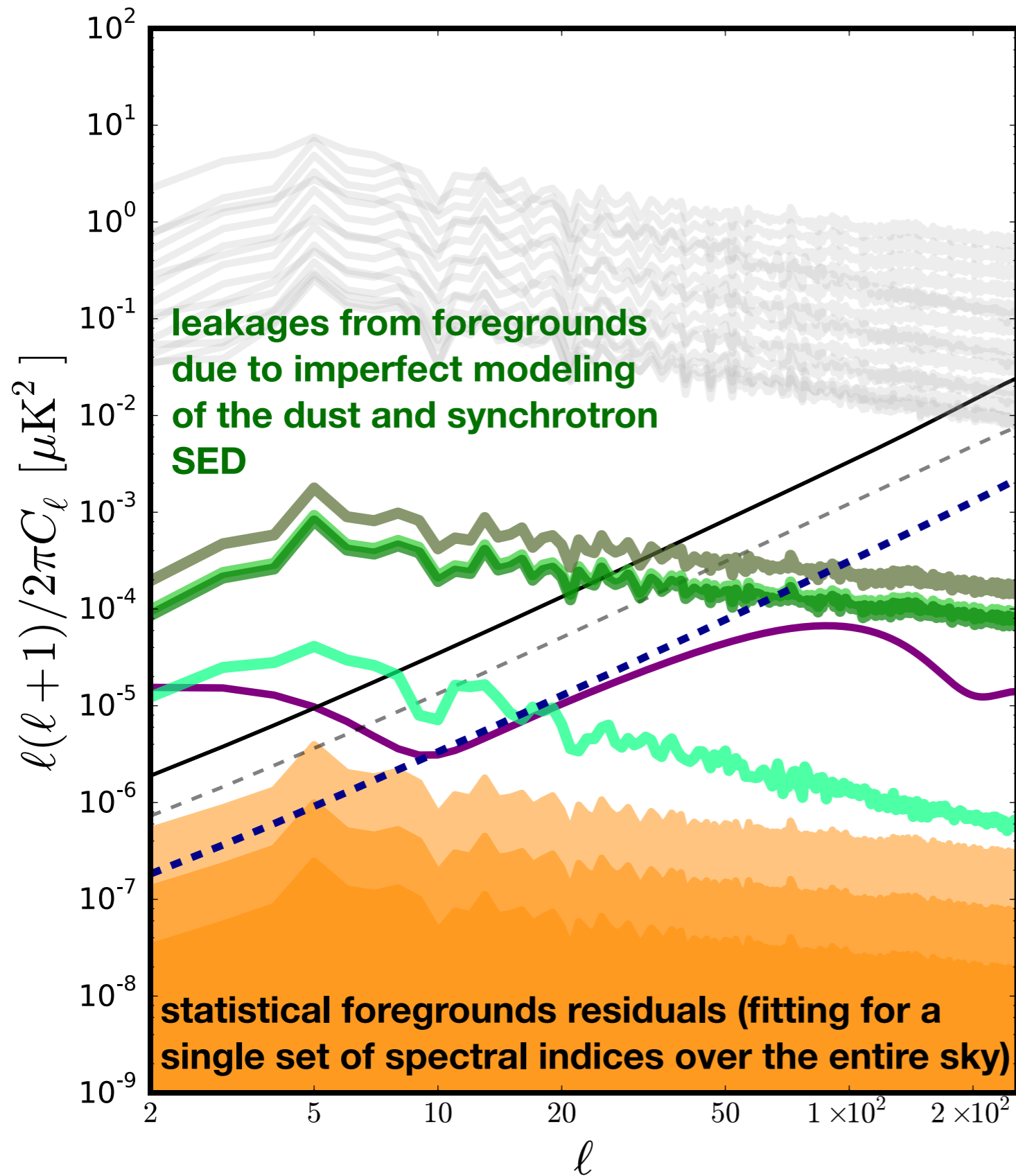
# Removing 2 synchrotron and 2 dust monitors



- sys. res. from  $\beta_d$  spat. var.
- sys. res. from  $\beta_s$  spat. var.
- sys. res. from  $T_d$  spat. var.
- sys. res. from  $\{\beta_d, \beta_s, T_d\}$  spat. var.
- -  $N_l$

**all frequency bands**  
 - 21GHz - 25GHz  
 - 665GHz - 800GHz  
**60% of the sky**

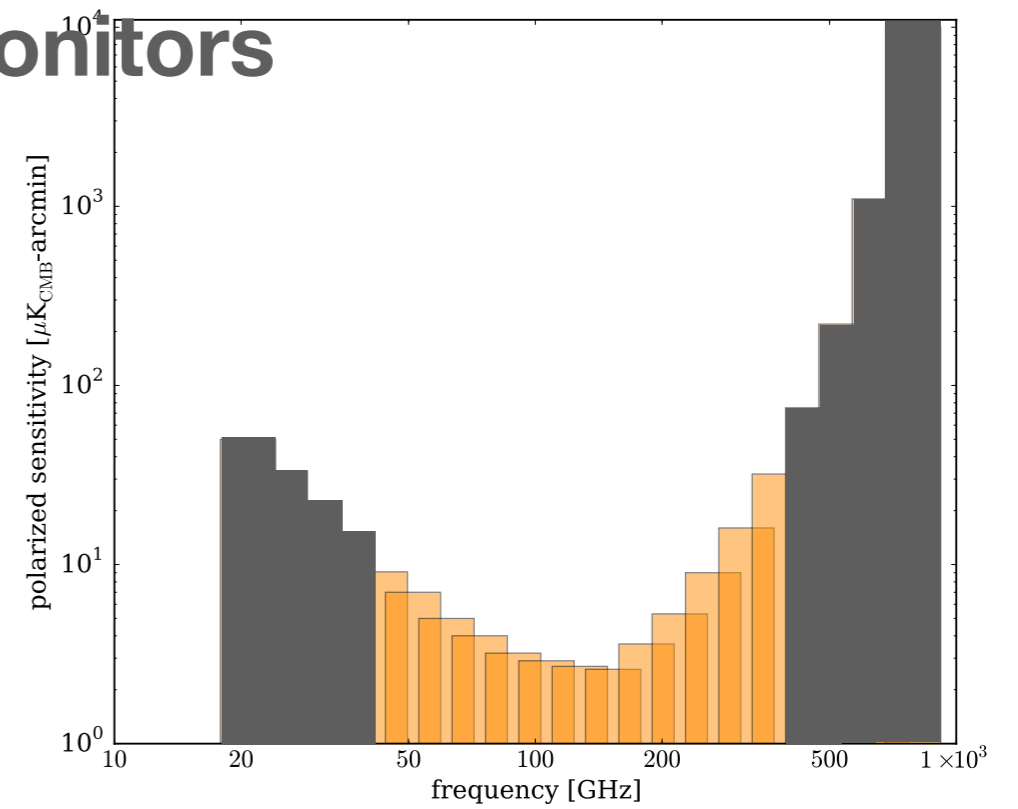
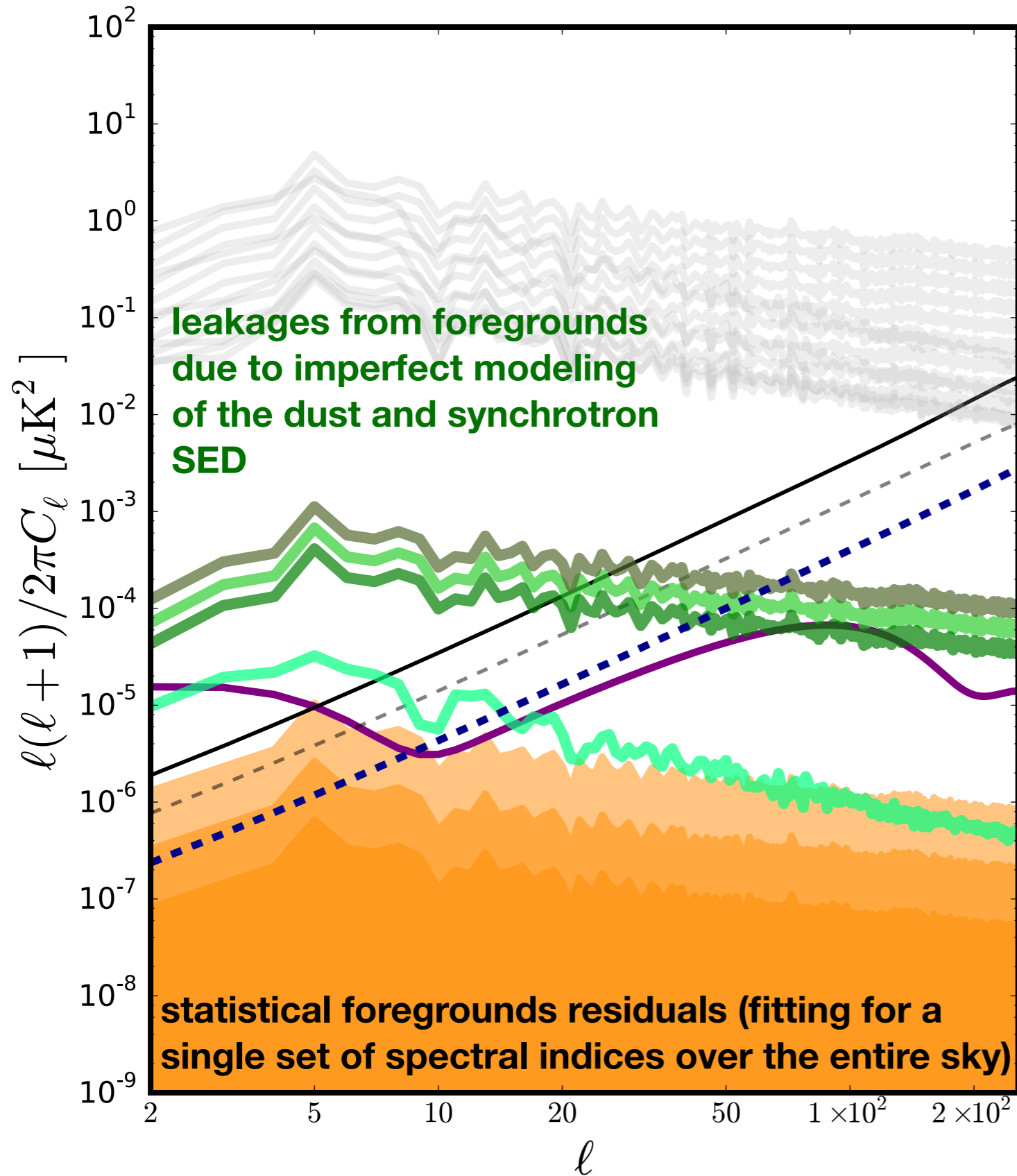
# Removing 3 synchrotron and 3 dust monitors



- sys. res. from  $\beta_d$  spat. var.
- sys. res. from  $\beta_s$  spat. var.
- sys. res. from  $T_d$  spat. var.
- sys. res. from  $\{\beta_d, \beta_s, T_d\}$  spat. var.
- - -  $N_\ell$

**all frequency bands**  
**-21GHz -25GHz -30GHz**  
**-555GHz -665GHz -800GHz**  
**60% of the sky**

# Removing 4 synchrotron and 4 dust monitors



- sys. res. from  $\beta_d$  spat. var.
- sys. res. from  $\beta_s$  spat. var.
- sys. res. from  $T_d$  spat. var.
- sys. res. from  $\{\beta_d, \beta_s, T_d\}$  spat. var.
- - -  $N_\ell$

**all frequency bands**  
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**-462GHz -555GHz -665GHz -800GHz**  
**60% of the sky**