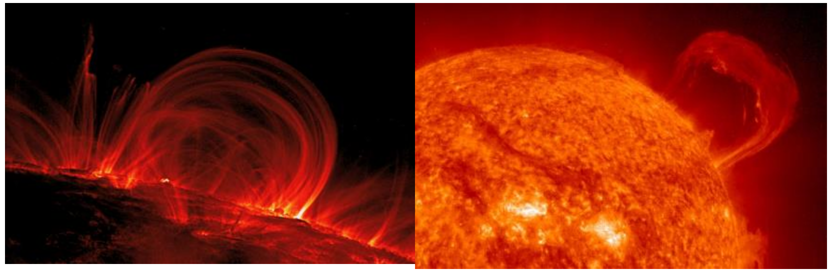
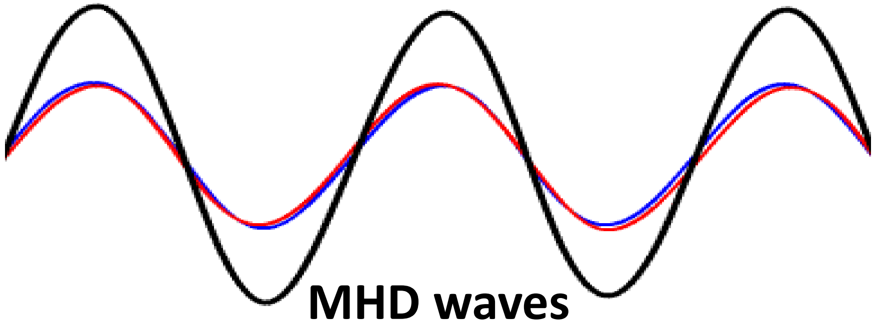
BUKS 2018 - 6th September

María Montes Solís presents:

INFERENCE OF PHYSICAL PROPERTIES IN PROMINENCE THREADS

Coronal Seismology





Observations

?

Theory

Prominence threads

- **Inference** of model parameters
- **Comparison between period ratio models in long and short thread limits**
- **Comparison between damping models**

Bayesian statistics

Model comparison and inference of parameters

Advantages
- Prior information
- Data with errors
- One observable

Bayesian Techniques

M: model
$\boldsymbol{\theta}$: set of parameters
d: observables

Bayes' Rule

(Bayes & Price 1763)

$$p(\boldsymbol{\theta}|M,d) = \frac{p(\boldsymbol{\theta}|M)\,p(d|M,\boldsymbol{\theta})}{p(d|M)}$$

prior: $p(\boldsymbol{\theta}|M)$ — likelihood: $p(d|M,\boldsymbol{\theta})$ — marginal likelihood: $p(d|M)$

Prior: Previous knowledge on θ

Likelihood: Probability of data given a model and θ

Marginal likelihood: Probability of data given a model

$$p(d|M) = \int_{\boldsymbol{\theta}} p(\boldsymbol{\theta}|M)\,p(d|M,\boldsymbol{\theta})\,d\boldsymbol{\theta}$$

$$M_1 \rightarrow p_1(\boldsymbol{\theta_1}|M_1,d), \quad M_2 \rightarrow p_2(\boldsymbol{\theta_2}|M_2,d), \quad M_3 \rightarrow p_3(\boldsymbol{\theta_3}|M_3,d), \; etc$$

Inference

Marginal posterior →
$$p(\theta_i|M,d) = \int p(\boldsymbol{\theta}|M,d)\,d\theta_1 \dots d\theta_{i-1}\,d\theta_{i+1} \dots d\theta_n$$

Model Comparison

Bayes' factor →
$$BF_{ij} = \frac{p(d|M_i)}{p(d|M_j)}$$

plausibility of one model over another

$2\ ln\ (BF_{ij})$	Evidence
0-2	Not Worth more than a bare Mention (NWM)
2-6	Positive Evidence (PE)
6-10	Strong Evidence (SE)
>10	Very Strong Evidence (VSE)

(Kass & Raftery 1995)

Prominence threads

Analysis 1 → $\boldsymbol{B}$

Assumptions:

Longitudinally homogeneous tube

$\rho_p/\rho_c \gg 1$

(M)
$$v_{ph} = \sqrt{\frac{2}{\mu_0 \rho_p}} B$$

Data:

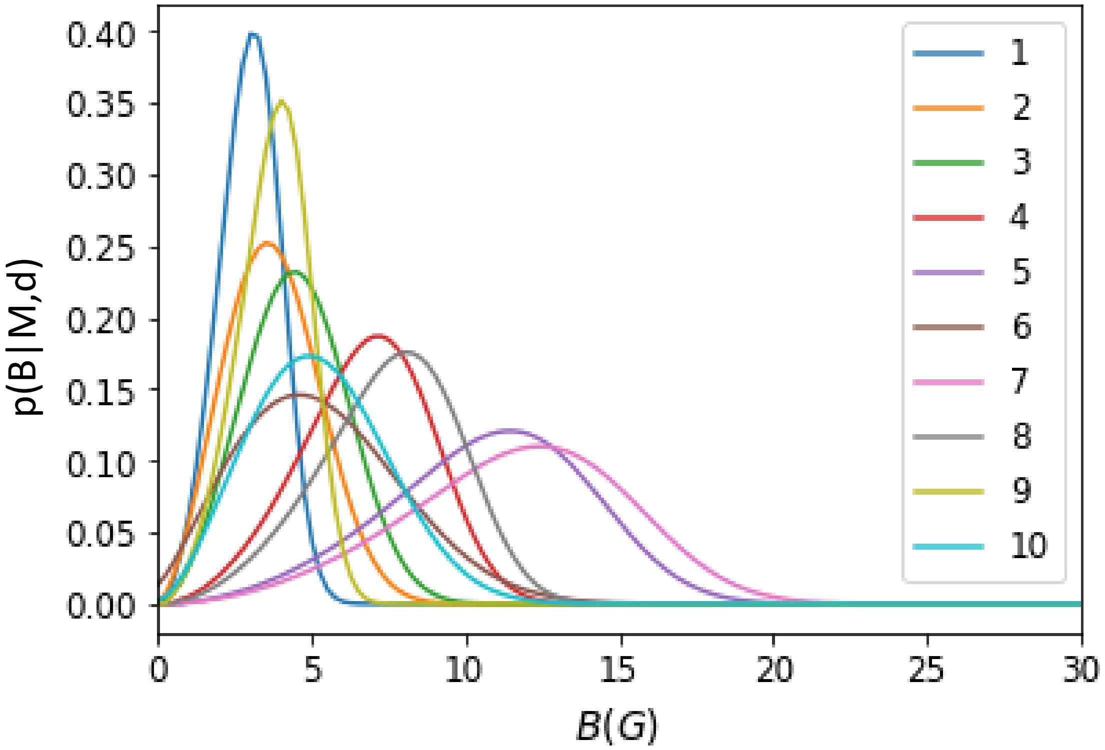
(d) v_{ph} by Lin et al. 2009

Parameters:

(θ)

- $\rho_p \in [10^{-12}, 10^{-9}]$ kgm^{-3}
- $B \in (0,50]$ G

Results



- Posterior distributions can be inferred.
- They spread over a range of values.
- Probability $\boldsymbol{B > 20}$ **G** is very small.

Prominence threads

(Diaz et al. 2010)

Analysis 2 → L_p/L

Assumptions:

- Partially filled tube
- Kink modes (fundamental, first overtone)

1. Simplest form

$$\frac{P_1}{2P_2} \approx \sqrt{\frac{3}{4L_p/L}}$$

Long thread limit

2. Simplest form+1 term

$$\frac{P_1}{2P_2} \approx \sqrt{\frac{3}{4L_p/L}}\sqrt{\frac{1+\sqrt{\left(1+L_p/3L\right)/\left(1-L_p/L\right)}}{1+\sqrt{\left(9/5-L_p/L\right)/\left(1-L_p/L\right)}}}$$

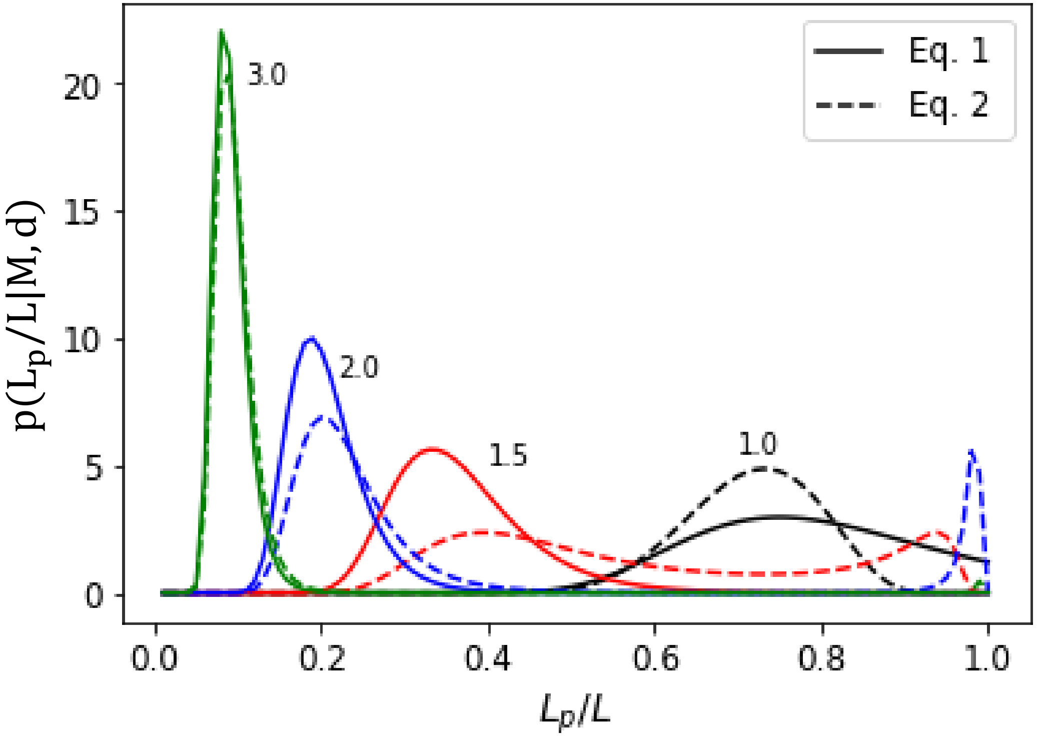
Data:

- $P_1/2P_2 = 1, 1.5, 2, 3$
- $\sigma = 10\%$

Parameter:

- $L_p/L \in (0,1)$

Results



- Posterior distribution can be inferred.
- The larger $P_1/2P_2$, the shorter L_p/L.
- Largest differences between both equations for large L_p/L values.

Prominence threads

(Diaz et al. 2010)

Analysis 2 → ${}^{L_p}/_{L}$ ${}^{\rho_p}/_{\rho_c}$

Results

Short thread limit

$$\frac{P_1}{2P_2} \approx 1 + (f^2 - 2)\frac{L_p}{L} - (f^2 + 1)\left(\frac{L_p}{L}\right)^2$$

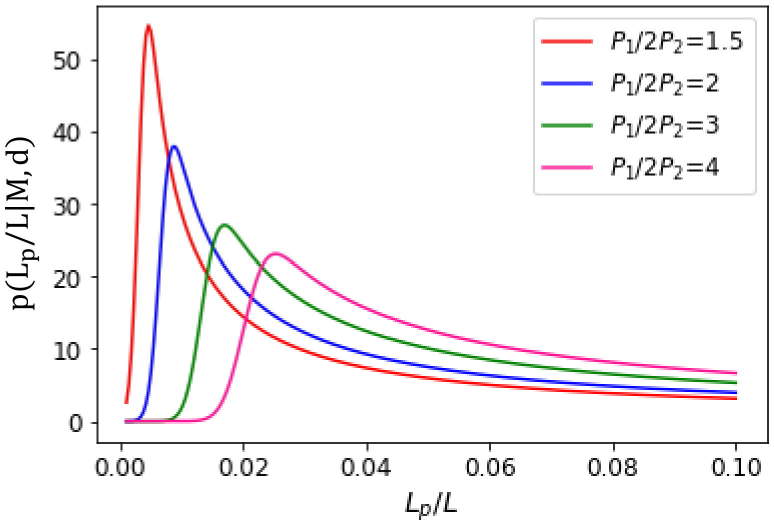
$$f = \sqrt{\frac{\left({}^{\rho_p}/_{\rho_c} + 1\right)}{2}}$$

Parameters:

- ${}^{L_p}/_{L} \in (0, 0.1]$
- ${}^{\rho_p}/_{\rho_c} \in [1, 300]$

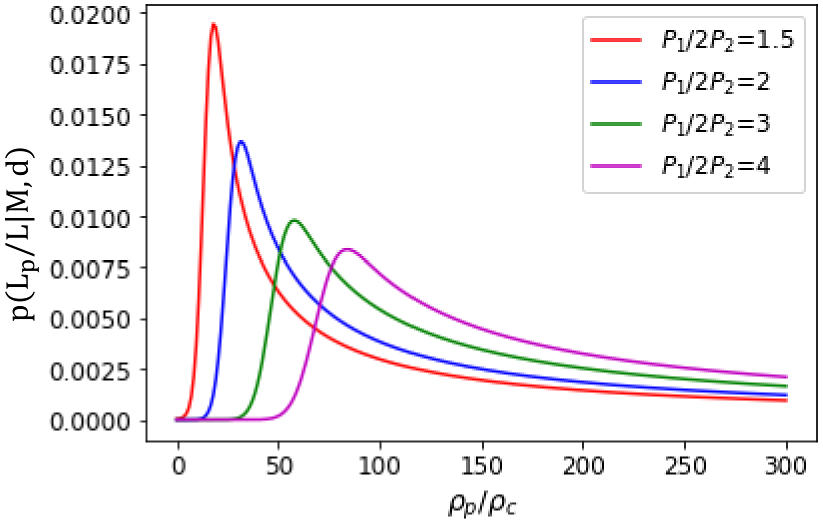
Data:

- $P_1/2P_2 = 1.5, 2, 3, 4$
- $\sigma = 10\%$



- Posterior distributions can be inferred.
- The larger L_p/L, the larger $P_1/2P_2$ (in contrast with previous results).

- Smaller ρ_p/ρ_c values with larger $P_1/2P_2$, so small density contrasts are possible.
- Probability ${}^{\rho_p}/_{\rho_c} > 200$ is very small.

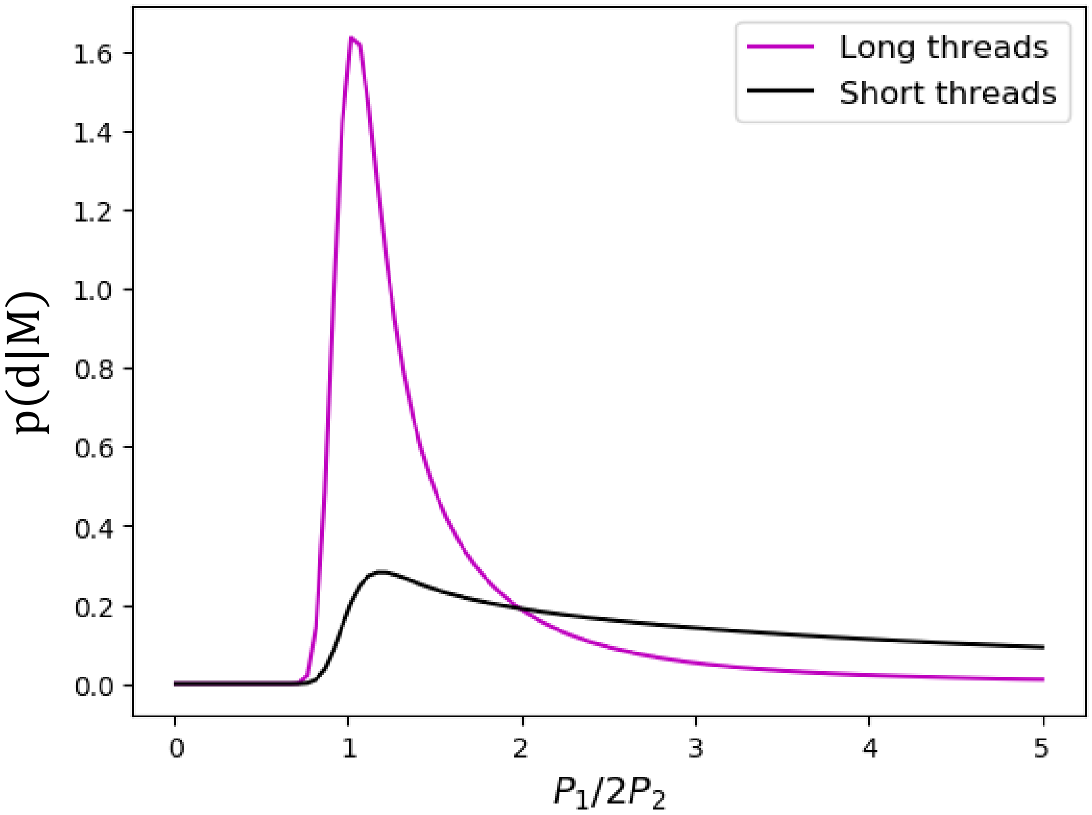


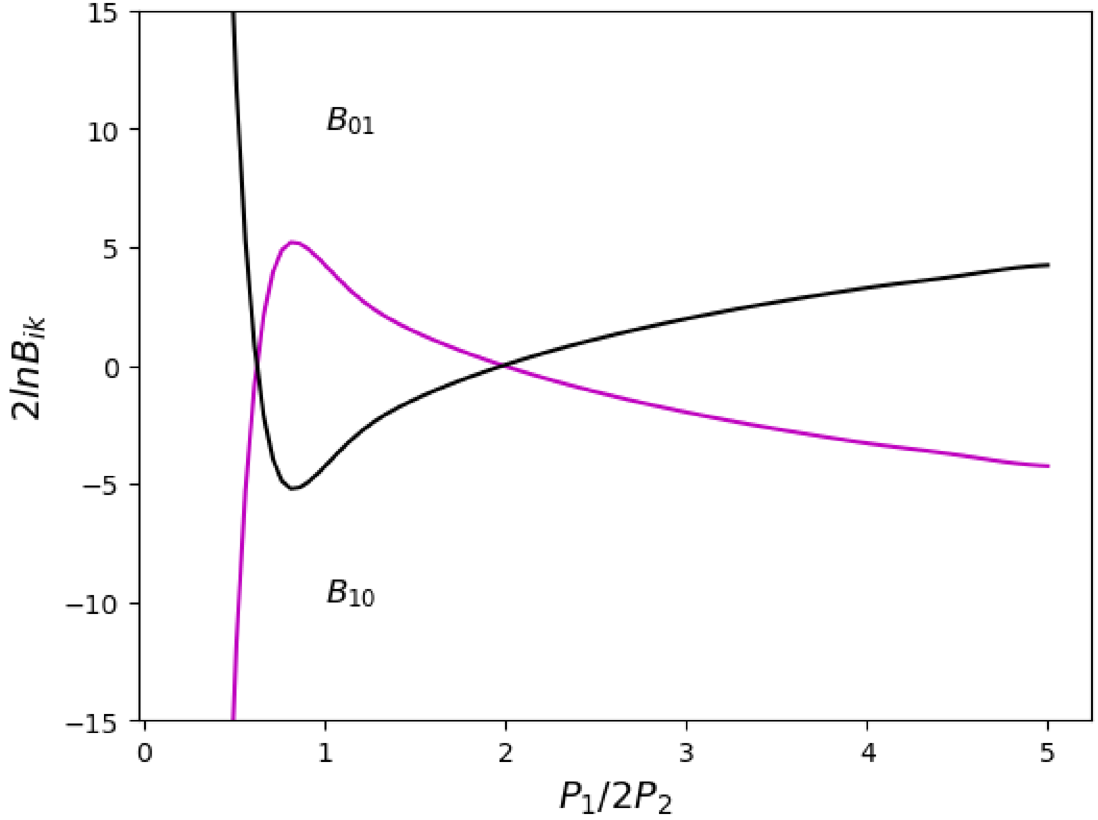
Prominence threads

Results

Marginal likelihoods
Bayes' factors

Analysis 2 → Comparison short/long thread





- Period ratios smaller ~ 0.5 for short threads.
- Period ratios around 1 are better explain in long threads.
- Period ratios larger than 2 more probable in short threads.

Prominence threads

Analysis 3→ L

(Soler & Goossens 2011)

Assumptions:

- Partially filled tube
- Flows

$$\frac{P(t)}{P(0)} = \sqrt{1 - \frac{4v_0^2 t^2}{\left(L + \frac{1}{3}L_p\right)\left(L - L_p\right)}}$$

Data:

- $P(t)/P(0) = 0.9$
- $\sigma = 10\%$
- $t = 180$ s

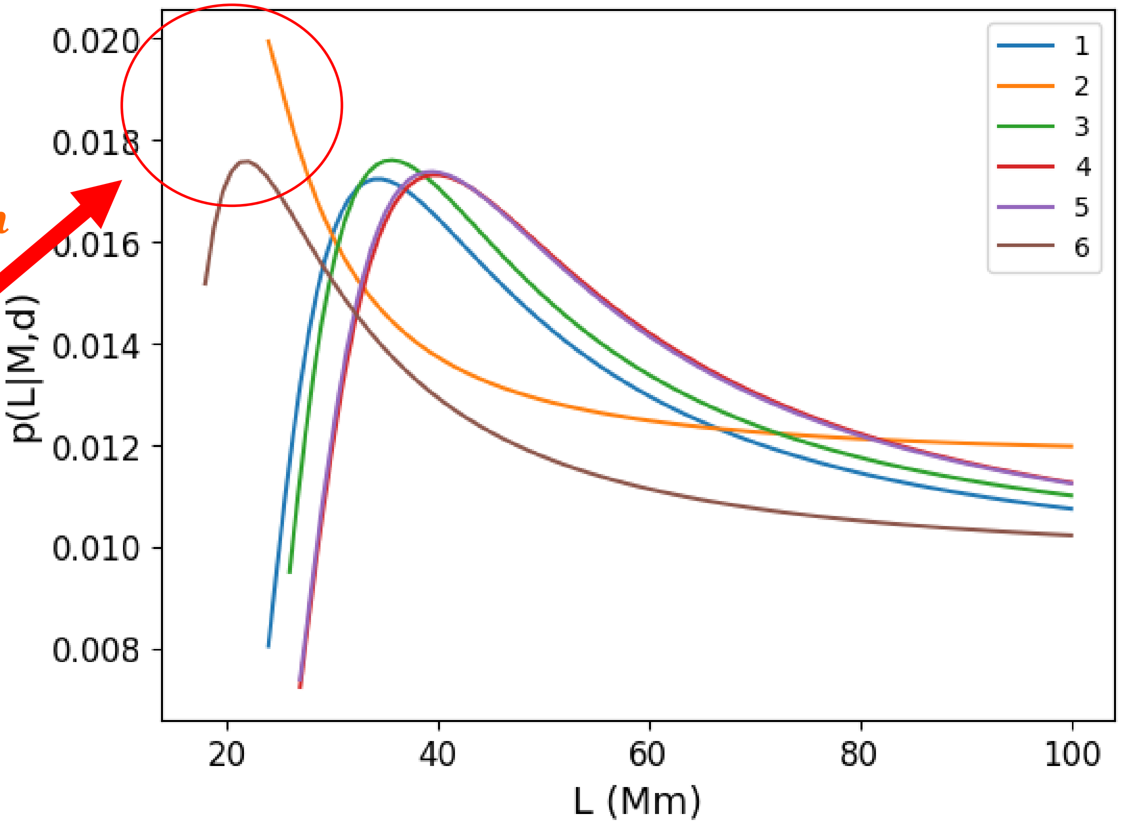
Parameters:

- Okamoto et al. 2007

Results

$L_p = 16 \times 10^3 km$
$v_0 = 15\ kms^{-1}$

$L_p = 1.7 \times 10^3 km$
$v_0 = 25\ kms^{-1}$



- Posterior distributions with long tails.
- Posterior distributions peak at $L \sim 20 - 40$ Mm

Prominence threads

Analysis 4 → Damping models

- *Resonant absorption in the Alfvén continuum*

$$\frac{\tau_d}{P} = \frac{2}{\pi}\frac{R}{l}$$

Parameter:

$\frac{l}{R} \in (0,2]$

(Arregui et al. 2008)

- *Resonant absorption in the slow continuum*

$$\frac{\tau_d}{P} = \frac{2}{\pi}\frac{R}{l}\left(\frac{2\,k_z R}{1+\frac{\gamma}{\beta}}\right)^{-2}$$

$\gamma = 5/3$: *adiabatic constant*

Parameters:

$\frac{l}{R} \in (0,2]$

$k_z R \in [10^{-3}, 0.1]$

$\beta \in (0,1]$

(Soler et al. 2009)

- *Cowling's diffusion*

$$\frac{\tau_d}{P} = \frac{\sqrt{2}}{\pi \tilde{\eta}_c k_z R}$$

Parameter:

$k_z R \in [10^{-3}, 0.1]$

$\tilde{\eta}_C \in [10^{-4}, 0.5]$

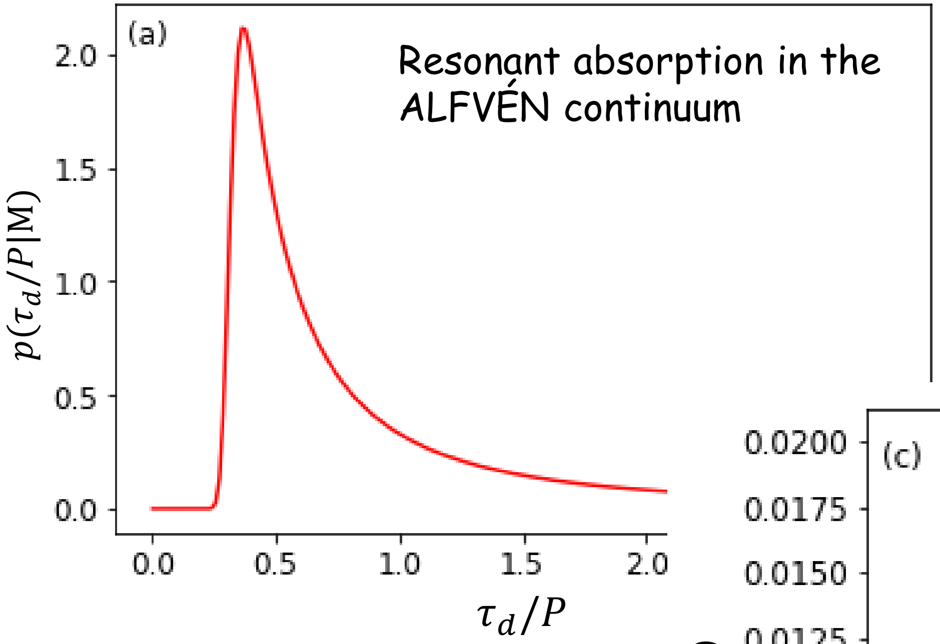
(Soler et al. 2009)

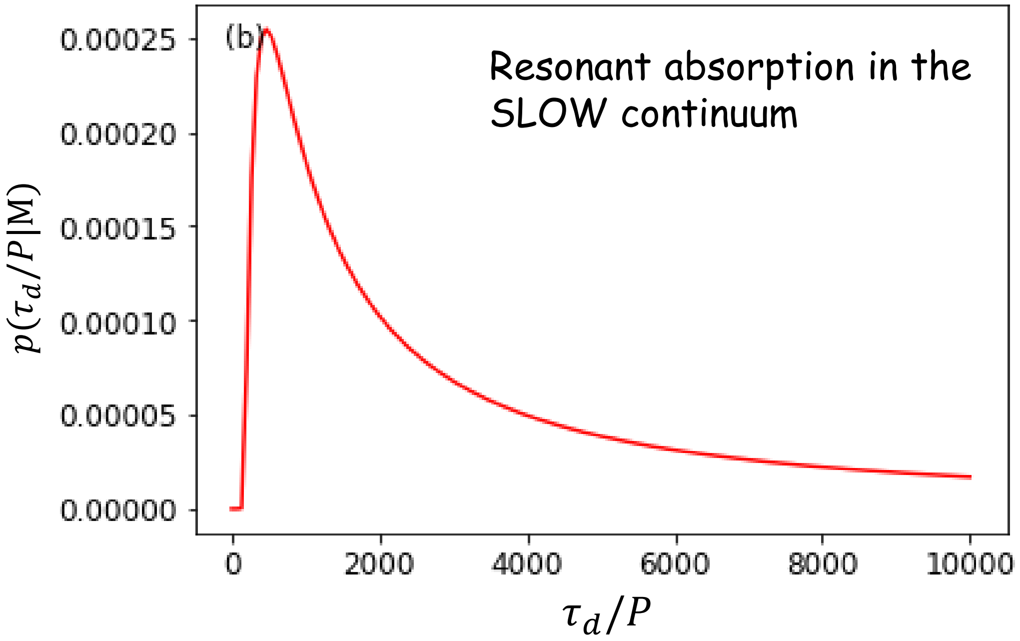
Prominence threads

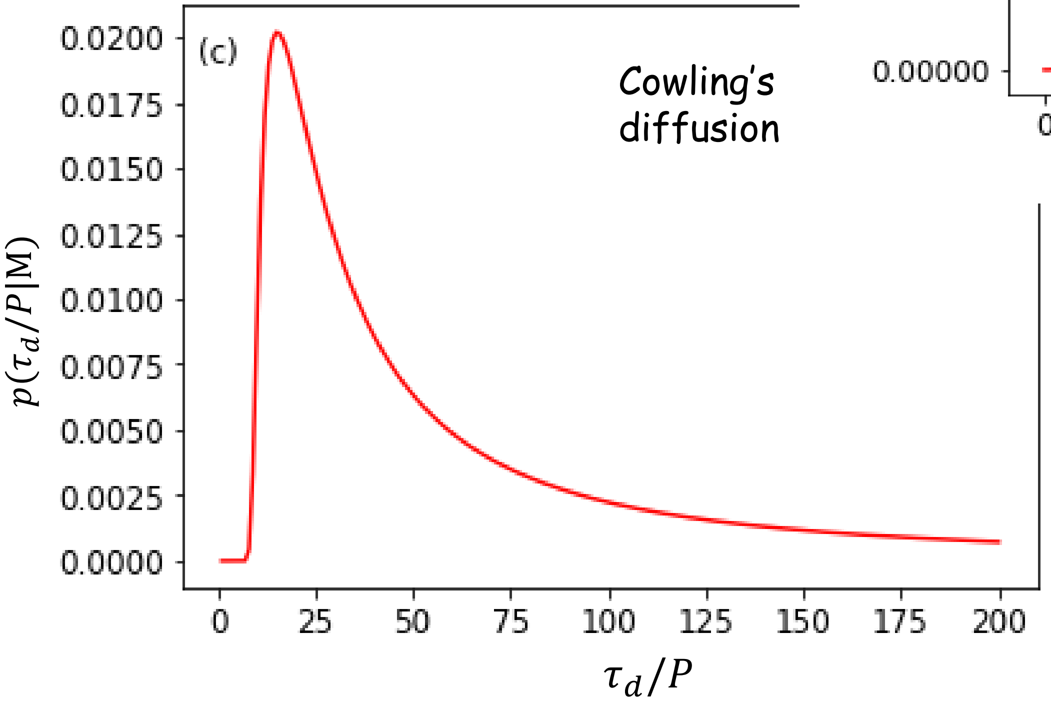
Results

Analysis 4 → Damping models

Marginal likelihoods







- Each mechanism corresponds to different ranges of damping ratios.
  - $r_{RAAC}^{max} \sim 0.5$
  - $r_{RASC}^{max} \sim 10^3$
  - $r_{CD}^{max} \sim 15$

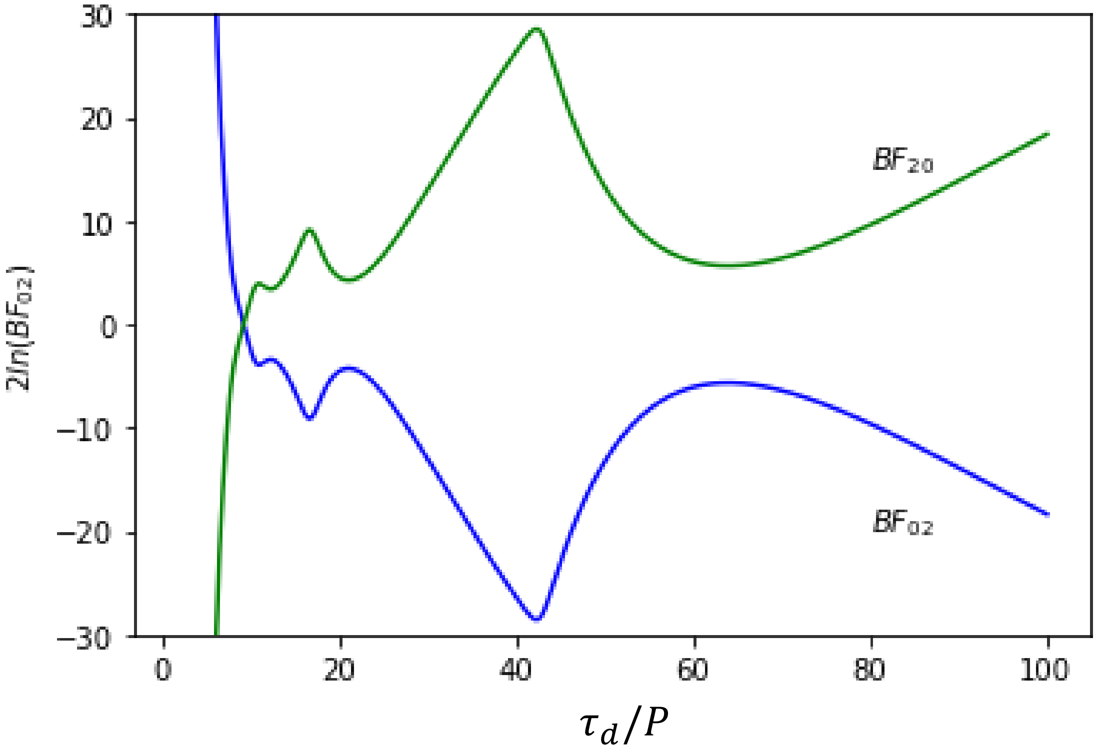
Prominence threads

Analysis 4 → Damping models

Results

Bayes' factors

Resonant absorption (0) / Cowling's diffusion (2)



- Observations with $\tau_d/P < 10$ → resonant absorption in the Alfvén continuum
- Observations with $\tau_d/P > 10$ → Cowling's diffusion

Prominence threads

Summary of results:

- **Magnetic fields strengths of the order of units to few tens of Gauss.**
- **Different inference results in the short/long thread approximations for $P_1/2P_2$**
- **Total lengths shorter than in previous studies.**
- **Resonant absorption in the Alfvén continuum as the most plausible mechanism to explain the observations of damped transverse oscillations.**

Montes-Solís, M. & Arregui, I.: "Inferring physical parameters in solar prominence threads", 2018, A&A, in preparation.

Thank you for your attention