WAVES, OSCILLATIONS, END INSTABILITIES IN THE SOLAR ATMOSPHERE: THEORY

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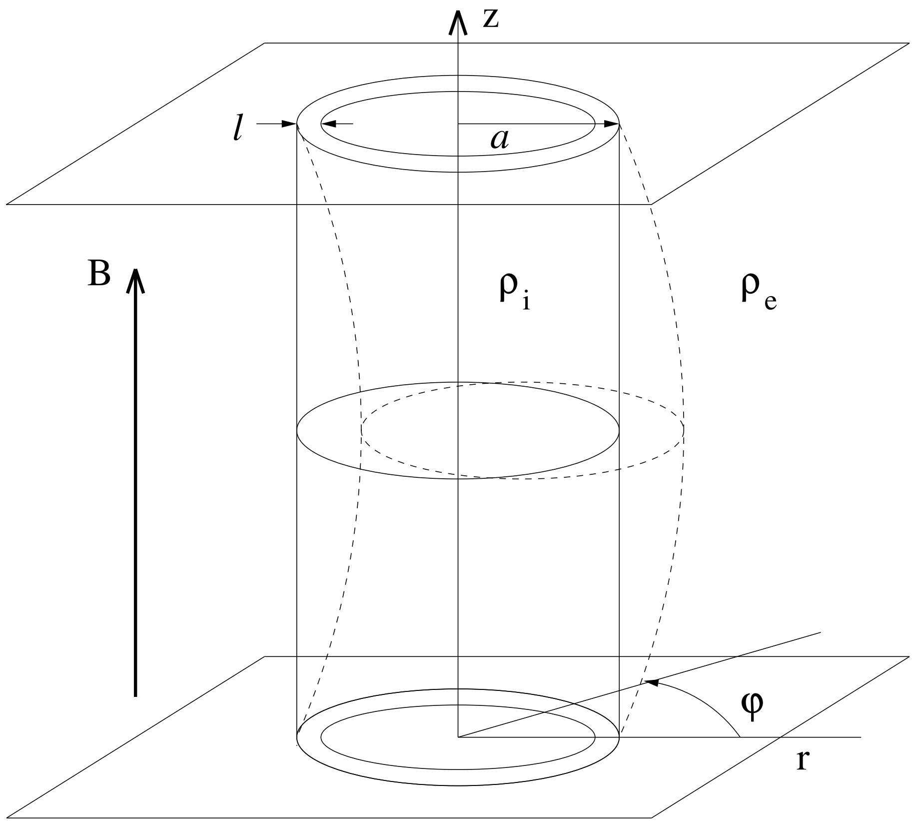
1. RESONANT DAMPING OF CORONAL LOOP KINK OSCILLATIONS AS A SEISMOLOGICAL TOOL

Fist observation of coronal loop kink oscillations made by TRACE in 1998 [**Aschwanden et al. (1999), Nakariakov et al. (1999)**] revealed that these oscillations damp in a few oscillation periods. This property of kink oscillations have been confirmed by later observations.

At present, the generally accepted mechanism of this damping is resonant absorption. **Ruderman & Roberts (2002)** suggested to use the observed damping of kink oscillations to get information about the internal structure of an oscillating loop.



In the simplest model of coronal loop there is resonant surface where the frequency of kink oscillation coincide with local Alfvén frequency. In the vicinity of this surface there is strong energy transfer from kink oscillation to local Alfvén oscillations. As a result, the kink oscillation decays. The damping rate depends on the thickness of transitional layer, the ratio of densities ρ_i/ρ_e, and a particular form of variation of density inside the transitional layer.

Ruderman & Roberts (2002) adopted a sinusoidal profile of density in the transitional layer and obtained for the damping time

$$\frac{t_{\rm dec}}{\Pi} = \frac{2a}{\pi\ell}\,\frac{\zeta+1}{\zeta-1}, \qquad \zeta = \frac{\rho_i}{\rho_e}$$

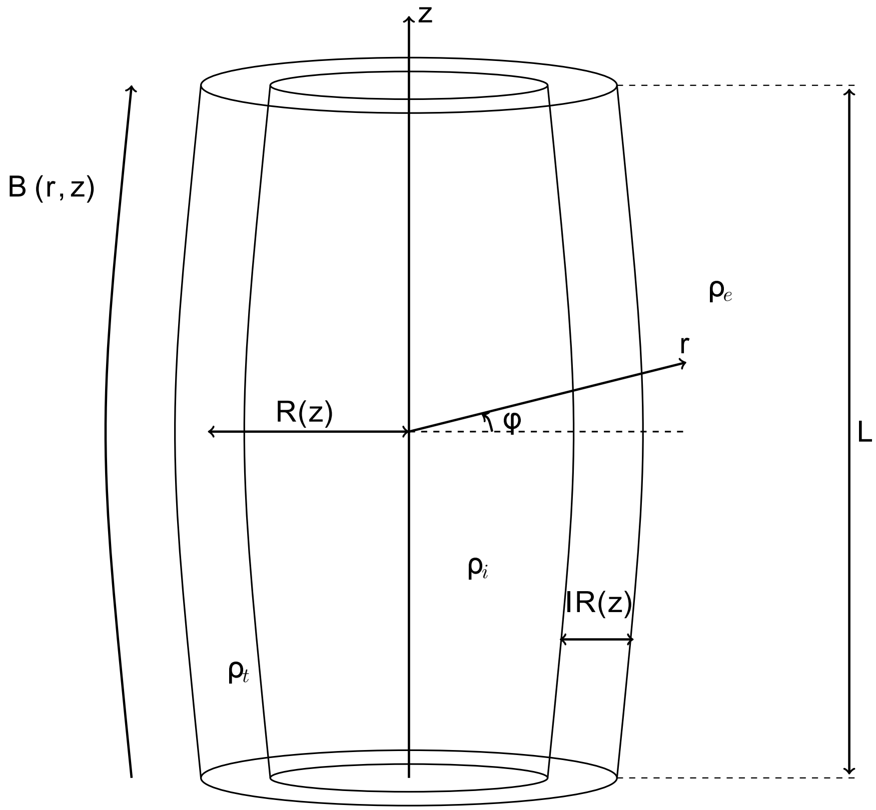
They took $\zeta = 10$ and using $t_{\rm dec}$ and Π observed by TRACE in 1998 obtained $\ell/a = 0.23$. If we take $\zeta = 3$ then we obtain $\ell/a = 0.36$.

Goossens et al. (2002) considered 11 observed cases of kink oscillations. They assumed the linear density profile and obtained

$$\frac{t_{\rm dec}}{\Pi} = \frac{4a}{\pi^2\ell}\,\frac{\zeta+1}{\zeta-1}$$

If we take $\zeta = 10$ then we obtain for the same 1998 event $\ell/a = 0.15$.

Two other effects that can affect the estimate of ℓ/a are the density variation along the loop and its expansion. **Dymova & Ruderman (2006)** showed that the estimate of ℓ/a is not affected by the density variation along the loop if $\rho(r,z) = f(r)g(z)$ in the transitional layer. **Shukhobodskiy & Ruderman (2018)** showed that the density variation along the loop and its expansion



does not affect ℓ/a if $\rho(r,z) = f(\psi)g(z)$, where $\psi = \text{const}$ is the equation of a magnetic surface.

However, the main problem is that this theory is based on a **wrong** assumption! This assumption is that the loop oscillation is described by an eigenmode of dissipative MHD. **Ruderman & Roberts (2002)** showed that kink oscillation of an arbitrarily disturbed loop is described by an eigenmode of dissipative MHD everywhere but in the vicinity of the resonant surface after the time of the order of oscillation period. However, the motion near the resonant surface is characterised by very small spatial scale in the radial direction.

Arregui (2015) gave the example: $\mathrm{Re} \sim 10^{12}$ we have

$t_{\mathrm{dec}}/\Pi \sim 13$ and $t_{\mathrm{ph}}/\Pi \sim 170$ for $\ell/a \sim 0.1$;

$t_{\mathrm{dec}}/\Pi \sim 3$ and $t_{\mathrm{ph}}/\Pi \sim 500$ for $\ell/a \sim 0.5$.

For $\Pi \sim 3\ \mathrm{min}$ we need to wait between 6 and $25\ \mathrm{hours}$!

This problem was addressed numerically for *propagating* kink waves by **Pascoe et al. (2012)**. It was found that the spatial damping of kink waves occurs somewhat slower that it is predicted by "classical" resonant absorption. Later this result was confirmed analytically for propagating waves by **Hood et al. (2013)** and for standing waves by **Ruderman & Terradas (2013)**. In particular, it was shown that damping time of coronal loop kink oscillations can be up to 30% underestimated.

However, the main result first obtained by **Pascoe et al. (2012)** was that the wave amplitude does not decay exponentially. Rather it is first described by Gaussian profile, and only later by exponent.

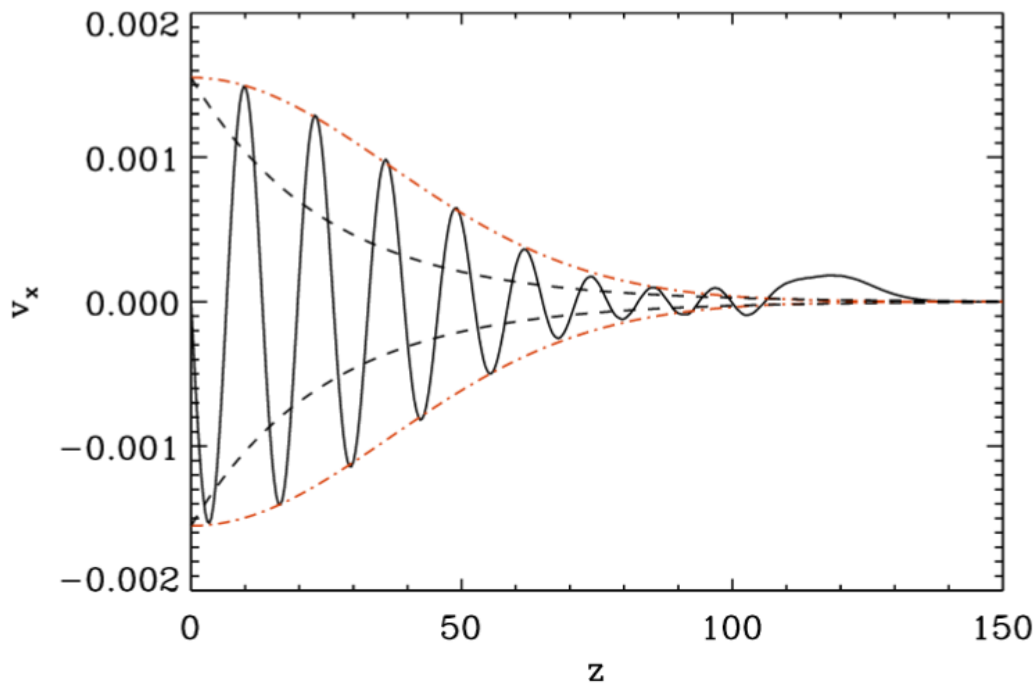


Fig. 2. Transverse velocity component v_x as a function of height at the loop axis. The dot-dashed (red) line represents a Gaussian envelope of the form given in Eq. (5). The dashed line is the exponential decay given by Eqs. (2) and (3).

Pascoe et al. (2013) suggested to use this result for coronal seismology.

The idea of this method is the following.

We can approximate the amplitude dependence on time by

$$A(t) = \begin{cases} A_0 \exp\left[-(t/t_g)^2\right], & t \le t_{\mathrm{tr}}, \\ A_1 \exp\left[-(t-t_{\mathrm{tr}})/t_{\mathrm{dec}}\right], & t > t_{\mathrm{tr}} \end{cases}$$

where $A_1 = A_0 \exp\left[-(t_{\mathrm{tr}}/t_g)^2\right]$. Assuming linear density profile we obtain

$$\frac{a}{\ell}\frac{\zeta+1}{\zeta-1} = \frac{\pi^2 t_{\mathrm{dec}}}{4\Pi}$$

We also have $t_{\mathrm{tr}} = F(\ell/a, \zeta)$. From these two equations we can find ℓ/a and ζ **simultaneously**.

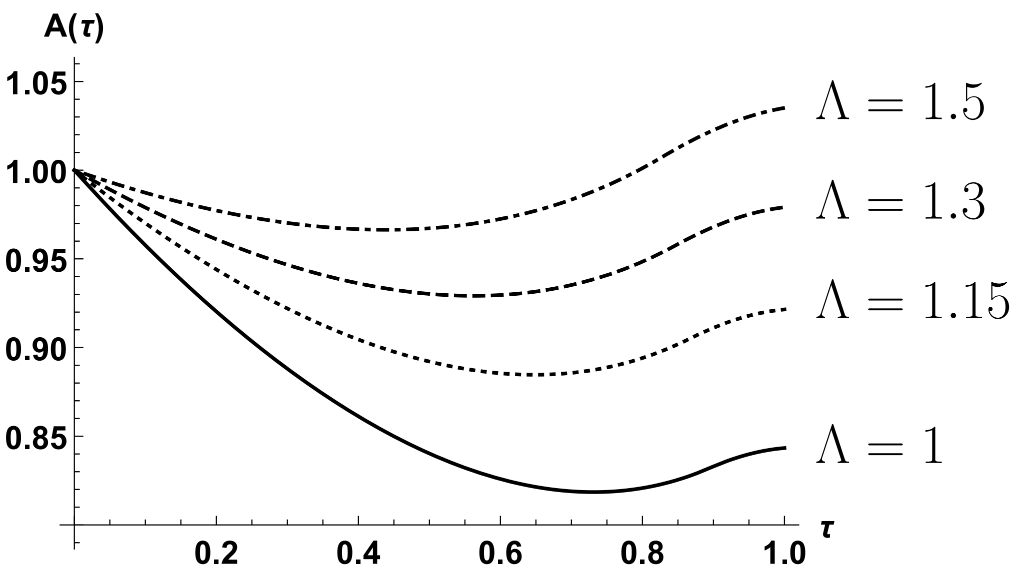
Pascoe with co-authors later improved this method including possible period variation, non-zero and time-dependent mean value of the loop displacement, etc. The latest development and references on previous studies can be found in **Pascoe et al. (2018)**.

2. KINK OSCILLATION OF DYNAMIC CORONAL LOOPS

Aschwanden & Terradas (2008) reported observation of kink oscillation of a cooling loop. **Morton & Erdélyi (2009)** showed that cooling causes the reduction in the oscillation period. **Ruderman (2011a)** used the WKB method to derive an adiabatic invariant determining the dependence of the oscillation amplitude on time. Using this invariant he showed that cooling causes the amplification of oscillations. **Ruderman (2011b)** studied the competition of amplification due to cooling and damping due to resonant absorption. He showed that the amplification can counteract decay only if the cooling time is of the order of wave period unless the transitional layer is very thin, $\ell/a \sim 0.02$.

Magyar et al. (2015) studied the same problem numerically and confirmed that cooling causes the oscillation amplification. However the effect was less pronounced because of different boundary conditions.

Schukhobodskiy et al. (2018) studied the effect of tube expansion on kink oscillation of cooling loops in the presence of resonant absorption.

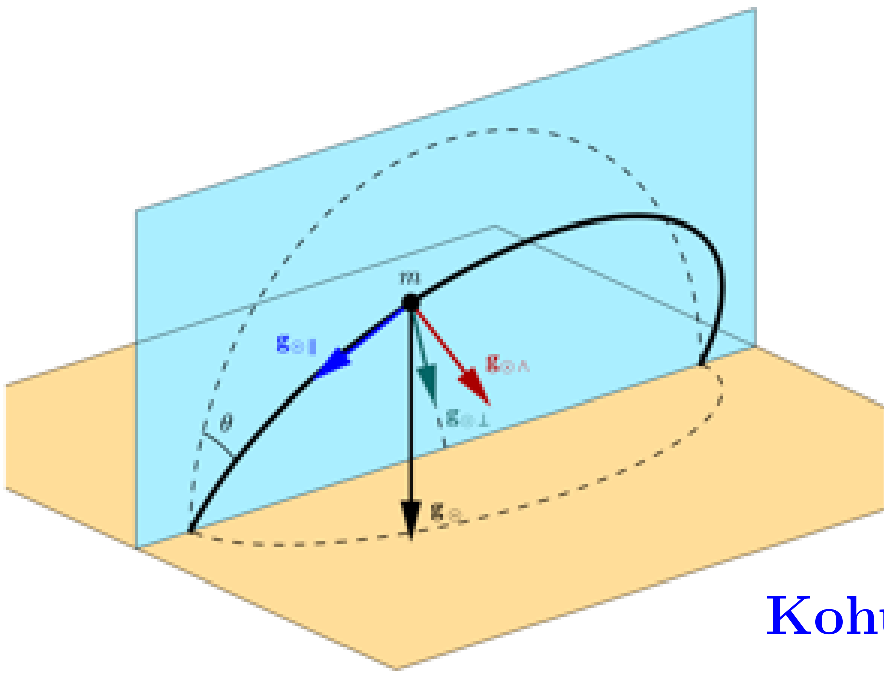


Λ is the ratio of cross-section radius in the middle of the loop to the radius at the foot-points. We see that the loop expansion acts in favour of oscillation amplification.

Dynamic behaviour also can take catastrophic form. It is coronal rain which is cased by catastrophic cooling and thermal instability. Often condensation occurs near the loop apex and then move down under the action of gravity. However, sometimes they oscillate near the apex. What prevents then from falling down?

Verwichte et al. (2017) studied the dynamics of a dense blob on an oscillating coronal loop and showed that the ponderomotive force due to oscillation can keep the blob oscillating near the loop top. However it is only possible when the oscillation amplitude is by an order of magnitude larger than observed. Hence, the authors concluded that the loop oscillation is not the main course of blob oscillation.

Kohutova & Verwichte (2017) used the numerical modelling of blob evolution. They concluded that the main courses of blob oscillation are the pressure increase below the blob and bending of magnetic field lines.



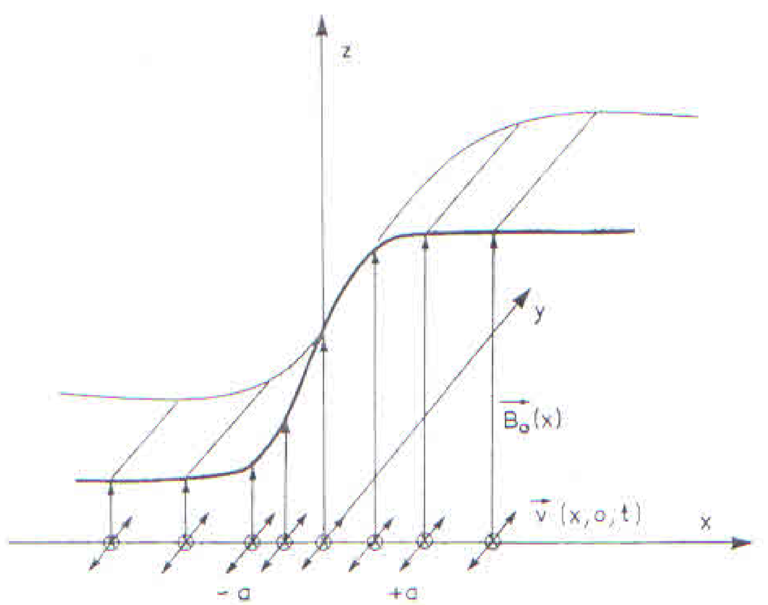
Verwichte et al. (2017) also suggested a mechanism of excitation of kink oscillations by the motion of a blob along a loop.

Kohutova & Verwichte (2017) also studied numerically the excitation of vertical kink oscillations by plasma condensations.

3. ENCHANCED DAMPING OF ALFVÉN WAVES DUE TO PHASE MIXING

- Alfvén waves are considered as good candidates for coronal heating because they can easily transport energy to the upper part of solar atmosphere.

- The main problem is how to dissipate them.



Heyvaerts & Priest (1983) suggested phase mixing as a mechanism that can greatly enhance the damping efficiency.

- While in a homogeneous plasma the characteristic length of dissipation is proportional to Re, in an equilibrium with straight magnetic lines and inhomogeneous in the direction orthogonal to these lines it is proportional to $\mathrm{Re}^{1/3}$.
- Recently **Pagano and De Moortel (2017)** addressed numerically the problem of coronal heating by phase-mixed Alfvén waves. They excited kink oscillations in a magnetic tube with a transitional layer and calculated the heating in this layer. Their conclusion is: "as a result of the extreme physical parameters we adopted and the moderate impact on the heating of the system, it is unlikely that phase-mixing can contribute on a global scale to the heating of the solar corona." This conclusion was then confirmed by **Pagano et al. (2018)**.

- However, it seems that it is too early to reject phase mixing as a means to heat the corona.

- First of all, it looks like heating by phase mixing is more appropriate in coronal holes.

- The damping efficiency can be further enhanced by geometry of magnetic field. **Ruderman et al. (1998)** showed that in an equilibrium with exponentially expanding magnetic field lines the damping length is proportional not to $\mathrm{Re}^{1/3}$ but to $\ln(\mathrm{Re})$.

- **Ruderman et al. (1998)** assumed that the wavelength is much smaller than the characteristic length of variation of equilibrium quantities along the magnetic field lines and used the WKB method.

- If we consider Alfvén waves with periods of the order of minutes then the wavelength is comparable with the atmospheric scale height and the WKB method is not applicable. In this case, in general, only numerical study is possible.

- However, there is one exception: non-reflective magnetic plasma configurations.

We now briefly describe the study of phase mixing in non-reflective configurations. We consider torsional Alfvén waves in an axisymmetric equilibrium. In cylindrical coordinates equilibrium quantities depend on r and z in cylindrical coordinates r, φ, z. In perturbations only the φ-components of velocity, v, and magnetic field, b_φ, are non-zero. The only dissipative process is shear viscosity.

The linearised MHD equations reduce to

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\boldsymbol{B}\cdot\nabla(r^2\boldsymbol{B}\cdot\nabla u)}{\mu_0 r^2} + \rho\nu\left(\frac{\partial^3 u}{\partial t\partial r^2} + \frac{\partial^3 u}{\partial t\partial z^2}\right), \quad u = \frac{v}{r}$$

Equilibrium magnetic field can be expressed both in terms of potential and magnetic flux function

$$\frac{B_r}{B_0} = \frac{\partial\phi}{\partial r} = -\frac{H}{r}\frac{\partial\psi}{\partial z}, \quad \frac{B_z}{B_0} = \frac{\partial\phi}{\partial z} = \frac{H}{r}\frac{\partial\psi}{\partial r}$$

Using ϕ and ψ as independent variables we transform the equation for u to

$$\frac{\partial^2 u}{\partial t^2} - \frac{V_A^2}{r^2}\frac{\partial}{\partial\phi}\left(\frac{r^2 B^2}{B_0^2}\frac{\partial u}{\partial\phi}\right) = \frac{\nu r^2 B^2}{H^2 B_0^2}\frac{\partial^3 u}{\partial t\partial\psi^2}, \quad V_A^2 = \frac{B^2}{\mu_0\rho}$$

We look for the solution to this equation in the form

$$u(t,\phi,\psi) = A(\phi,\psi)\Phi(t,h(\phi,\psi),\psi)$$

Taking

$$h = B_0 V_0 \int_{\phi_1(\psi)}^{\phi} \frac{d\phi'}{BV_A}, \quad A = A_0(\psi)(H/r)(\rho_0/\rho)^{1/4}$$

we obtain

$$\frac{\partial^2 \Phi}{\partial t^2} - V_0^2 \frac{\partial^2 \Phi}{\partial h^2} = \frac{V_A^2 \Phi}{r^2 A} \frac{\partial}{\partial \phi} \left(\frac{r^2 B^2}{B_0^2} \frac{\partial A}{\partial \phi} \right) + \frac{\nu r^2 B^2}{H^2 B_0^2} \frac{\partial \Xi}{\partial t} \qquad (*)$$

$$\Xi = \frac{1}{A} \frac{\partial^2 (A\Phi)}{\partial \psi^2} + \frac{2}{A} \frac{\partial h}{\partial \psi} \frac{\partial}{\partial \psi} \left(A \frac{\partial \Phi}{\partial h} \right) + \frac{\partial^2 h}{\partial \psi^2} \frac{\partial \Phi}{\partial h} + \left(\frac{\partial h}{\partial \psi} \right)^2 \frac{\partial^2 \Phi}{\partial h^2}$$

When $\nu = 0$ we obtain Klein-Gordon equation for Φ. Non-reflective magnetic plasma configurations are defined by the condition

$$\frac{V_A^2}{r^2 A} \frac{\partial}{\partial \phi} \left(\frac{r^2 B^2}{B_0^2} \frac{\partial A}{\partial \phi} \right) = \sigma(\psi) \qquad (\dagger)$$

When the magnetic field configuration is given, this equation determines the equilibrium density ρ.

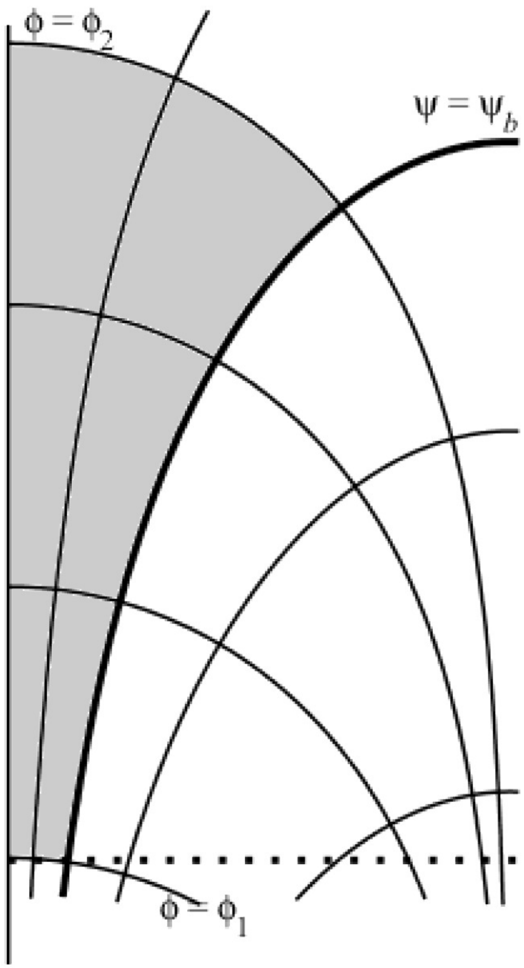
We then assume that dissipation is weak, meaning that the last term in equation for Φ is small, and use the WKB to study the wave damping.

Equation $(\dagger)$ imposes serious restriction on the spatial variation of density. To relax it we can consider weakly reflective equilibria. There are two kinds of such equilibria:

(i) the characteristic scale of variation of the coefficient at Φ in equation $(*)$ is large in comparison with the wavelength;

(ii) the coefficient at Φ in equation $(*)$ is small.

Ruderman & Petrukhin (2018) assumed that (ii) is satisfied and studied the Alfvén wave phase mixing in an equilibrium with exponentially expanding magnetic field lines and the density exponentially decreasing with the height.



$$B_r = B_0 e^{-z/H} J_1(r/H)$$
$$B_z = B_0 e^{-z/H} J_0(r/H)$$

$$\phi = -He^{-z/H} J_0(r/H)$$
$$\psi = re^{-z/H} J_1(r/H)$$

$$\rho = \hat{\rho}(\psi) e^{-\alpha z/H}, \quad \alpha = H/H_\rho$$

The boundary $\phi = \phi_2$ intersects the z-axis at $z = 6H$.

For typical values of parameters in coronal holes and plumes condition (ii) is satisfied with great margin.

We modelled the density enhancement in coronal plumes by

$$\hat{\rho}(\psi) = \frac{\rho_0}{\zeta} \begin{cases} 1 + (\zeta - 1)(1 - \psi/\psi_b)^2, & \psi \le \psi_b \\ 1, & \psi \ge \psi_b \end{cases}$$

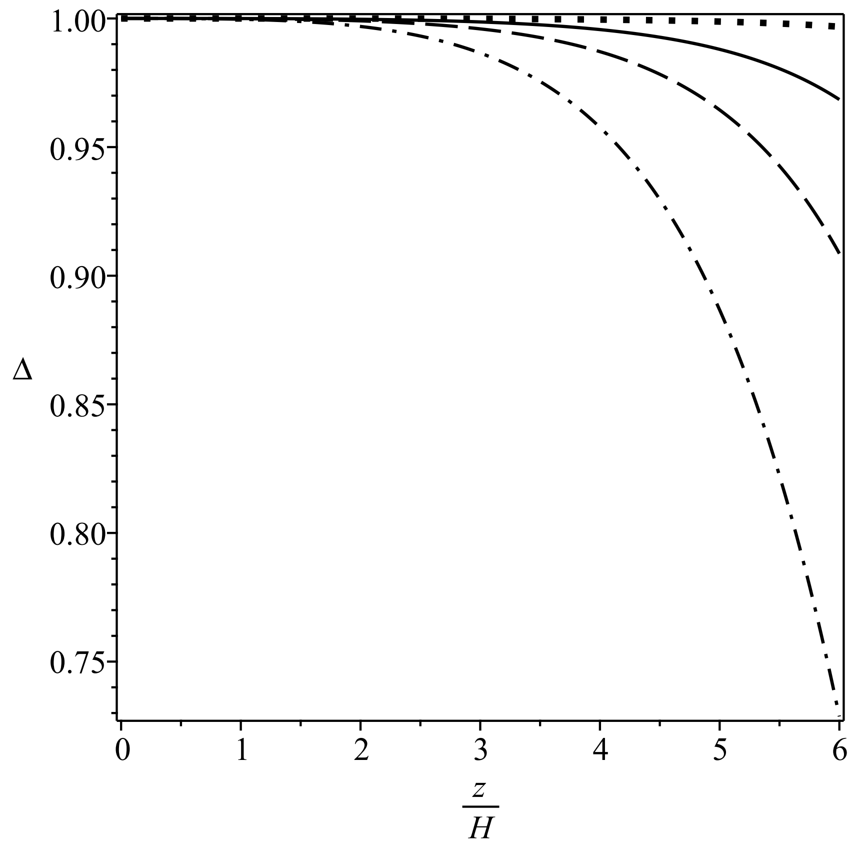
with $\zeta = 5$. The driver of Alfvén waves was defined by

$$v = v_0 \left(1 - \frac{r^2}{r_0^2}\right) e^{-i\omega t} \quad \text{at} \quad z = 0$$

Viscosity in the corona is strongly anisotropic. Shear viscosity is by about 10 orders of magnitude smaller than volume viscosity. The typical value of kinematic shear viscosity is $\nu = 1\,\mathrm{m}^2\,\mathrm{s}^{-1}$. For this value of ν Alfvén waves propagate to the upper corona and solar wind without damping.

However, it is possible that ν is greatly enhanced by turbulence.

We considered ν as a free parameter and calculate the dependence of the relative wave energy flux $\Delta = \Pi/\Pi_0$ on z for $\alpha = H/H_\rho = 0.8$ and various values of wave periods and H. Below $\Delta(z)$ is shown for the wave period equal to $60\,\mathrm{s}$ and $H = 60\,\mathrm{Mm}$:



.... $\nu = 10^5\,\mathrm{m^2\,s^{-1}}$

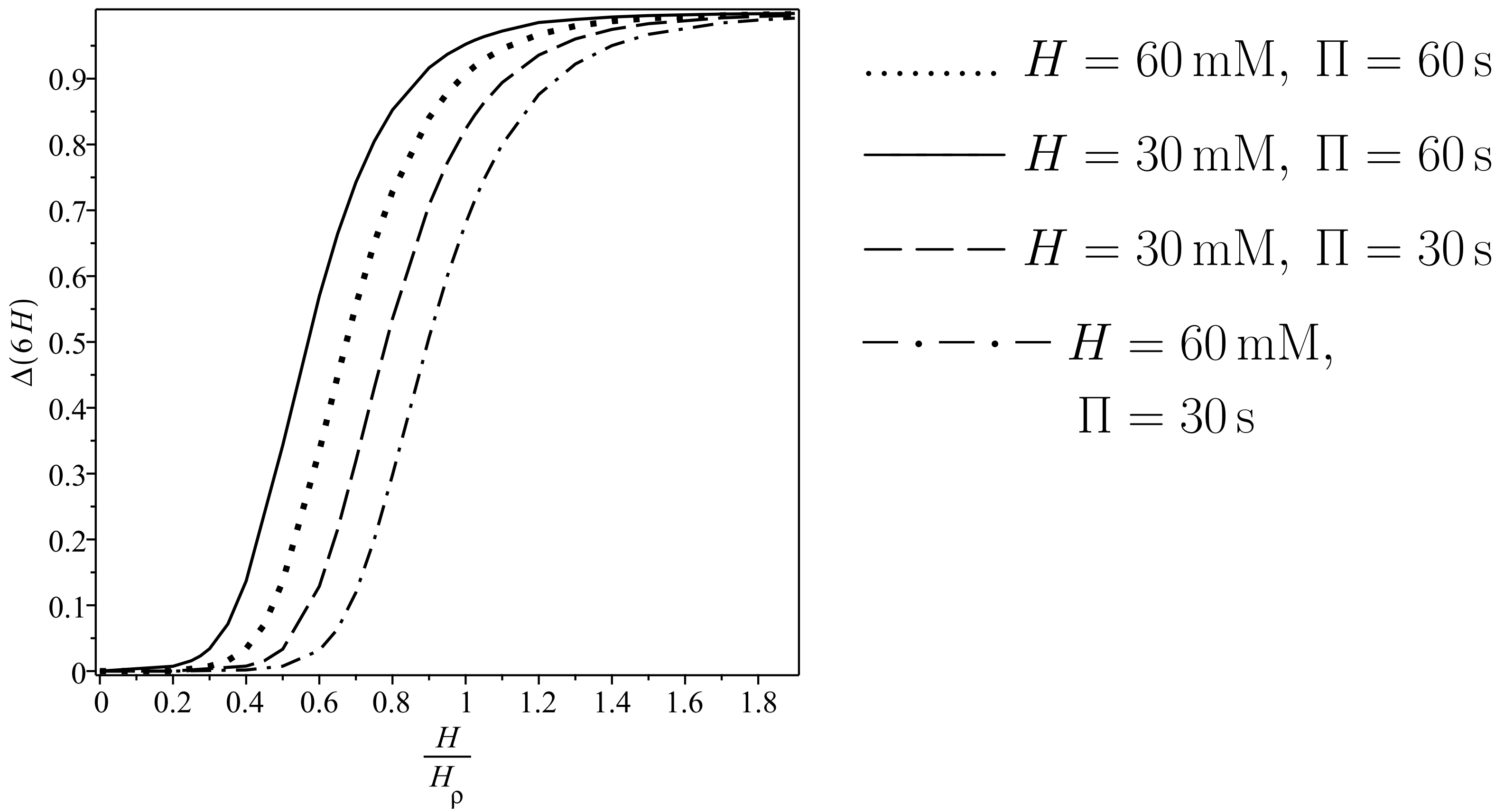
———— $\nu = 10^6\,\mathrm{m^2\,s^{-1}}$

— — — — $\nu = 3\times10^6\,\mathrm{m^2\,s^{-1}}$

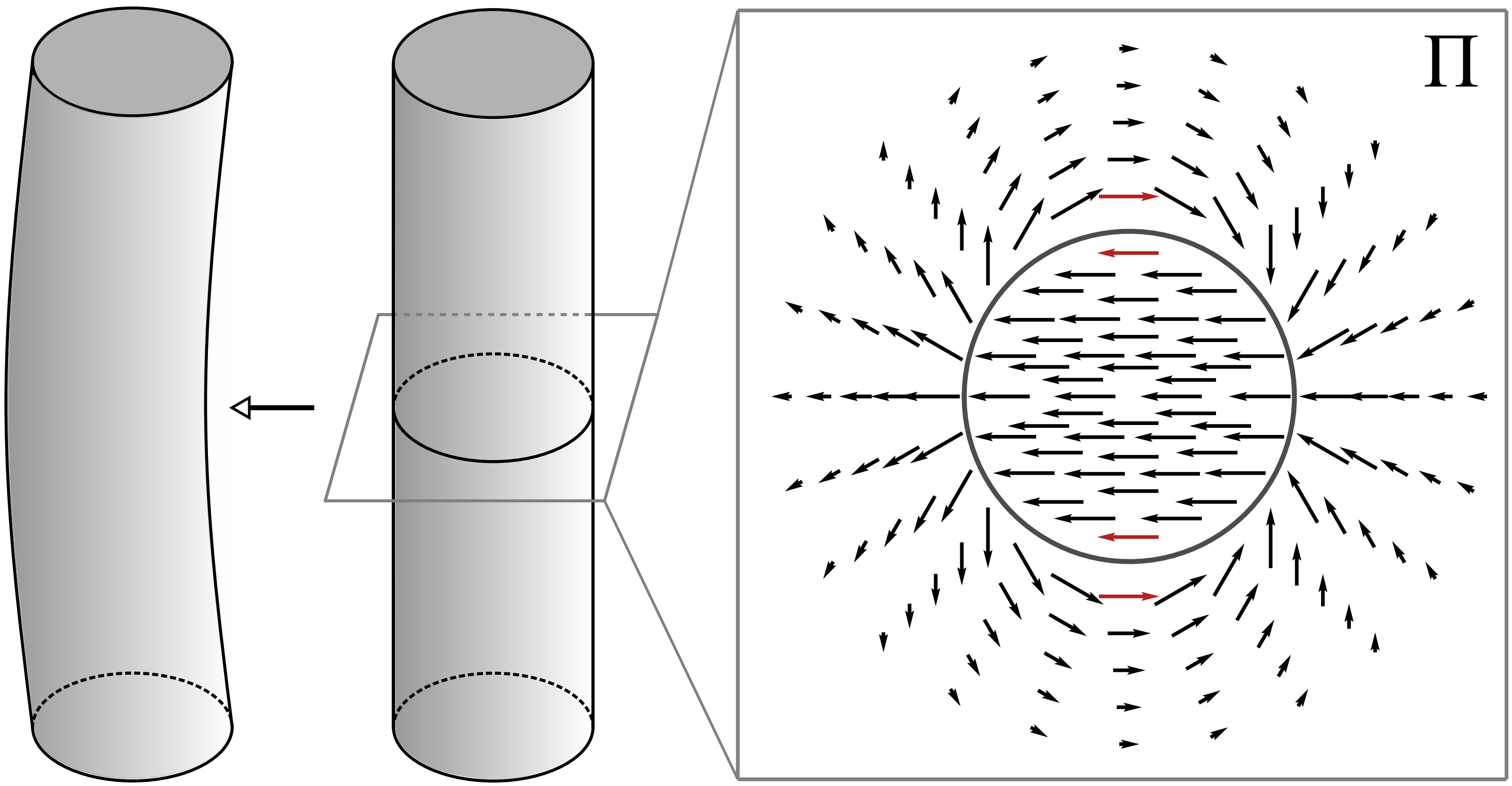
— · — · — · — $\nu = 10^7\,\mathrm{m^2\,s^{-1}}$

Hence, about a quarter of wave energy dissipates at height $360\,\mathrm{Mm}$ if shear viscosity is increased by 7 orders of magnitude in comparison with its classical value.

We also calculated the dependence of $\Delta(6H)$ on $\alpha = H/H_\rho$:

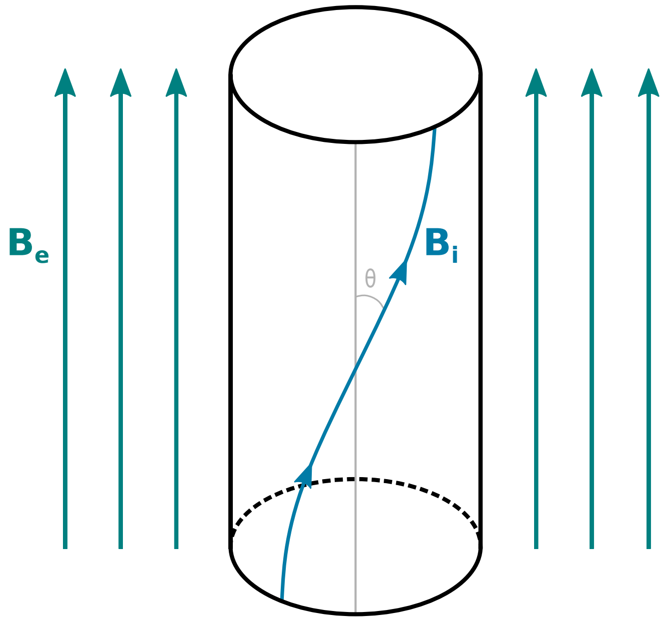


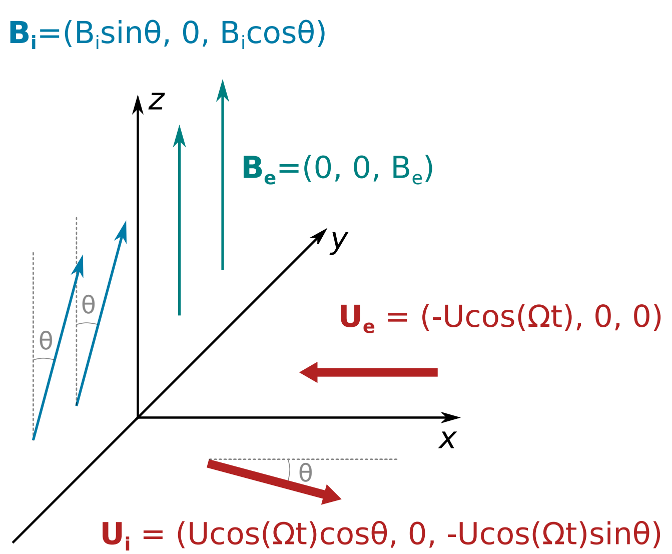
4. KELVIN-HELMHOLTZ INSTABILITY INDUCED BY CORONAL LOOP KINK OSCILLATIONS



The Kelvin-Helmholtz instability induced by magnetic loop kink oscillation was observed in many numerical studies (**Terradas et al. 2008**; **Antolin et al. 2014, 2016**; **Magyar & Van Doorsselaere 2016a,b**; **Howson et al. 2017a,b**; **Karampelas & Van Doorsselaere 2018**).

Barbulescu et al. (2018) considered a modelled flow near the boundary of a twisted magnetic tube embedded in a straight magnetic field.





When there is no oscillation ($\Omega = 0$) the discontinuity is stable when $U < U_c$ and unstable otherwise, where

$$U_c^2 = \frac{(\rho_i + \rho_e)V_{Ai}^2 V_{Ae}^2 \tan^2(\theta/2)}{\rho_i V_{Ai}^2 + \rho_e V_{Ae}^2}$$

In the general case the evolution of discontinuity is defined by Mathieu's equation

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}\tau^2} + [a - 2q\cos(2\tau)]\eta = 0$$

where $\tau = \Omega t$, and a and q are expressed in terms of equilibrium quantities.

Main results:

- The discontinuity is **unstable for any value of** U.
- The initial value problem is ill-posed when $U > U_c$ and well-posed when $U < U_c$.

We also introduced the notion of σ-stability. In general, a system is called σ-stable if perturbations grow not faster than $e^{\sigma t}$.

For a particular case of stability of coronal loop boundaries we took $\sigma = 1/t_{\mathrm{dec}}$. Hence, a coronal loop boundary is σ-stable if the instability growth time is larger than the damping time.

Using the estimate $t_{\mathrm{dec}} \lesssim 5\Pi$ we obtained that the tangential discontinuity is σ-stable if $\theta \approx 2^\circ$. This corresponds to about half-turn of magnetic field lines in a twisted magnetic tube.

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