BUKS2018 WORKSHOP

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Waves and Instabilities in the Solar Atmosphere

Confronting the current state-of-the-art

Nonlinear sausage mode of coronal loops

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Abstract

The sausage mode in coronal loops is used for the interpretation of observed fast pulsations of microwave and hard X-ray emission. Its properties are studied well in the approximation of the linear magnetohydrodynamics. Now the investigation of large-amplitude sausage waves is on the agenda. An obvious first step is to study the sausage mode in the weakly-nonlinear approximation. Previously this study has been carried out in plane geometry. We investigate weakly nonlinear sausage waves in cylindrical geometry. We considered the nonlinear Schrödinger equation describing the non-linear evolution of the wave envelope and used it to study the modulational instability of fast sausage waves.

1 Introduction

- One popular explanation of QPP of electromagnetic emission generated by solar flares is the modulation of electromagnetic emission by fast sausage waves (e.g., Rosenberg 1970. AA. 9, 159; Nakariakov & Melnikov 2009. SSR. 149, 119).
- Very often, the observed sausage wave amplitudes are quite large (e.g. Inglis et al. 2008. AA. 487, 1147; Huang et al. 2014. ApJ. 791, A44). This makes it desirable to go beyond the linear theory and take the nonlinear effects into account.
- The fast sausage waves are highly dispersive. As a result, in the case of moderate wave amplitude, the main nonlinear effect is the nonlinear wave modulation described by the nonlinear Schrödinger (NLS) equation. Recently, the NLS equation for fast sausage waves in a homogeneous magnetic tube is derived (Mikhalyaev and Ruderman 2015. JPP. 81. 905810611).

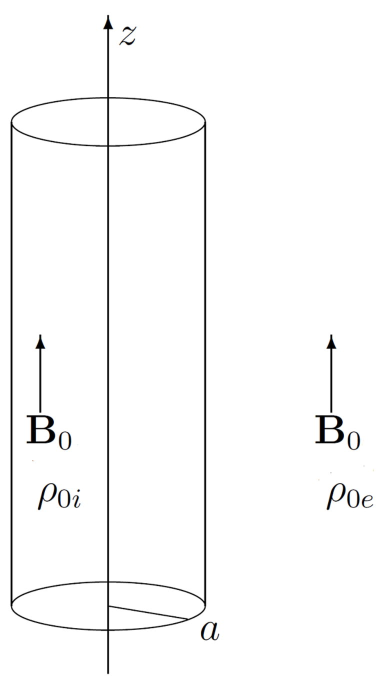
2 Magnetic tube

- The equilibrium magnetic field is $\mathbf{B}_0 = B_0\mathbf{n}$, where B_0 is a constant and $\mathbf{n}$ is the unit vector parallel to the tube axis. The equilibrium density ρ_0 is constant inside and outside the tube.
- In cylindrical coordinates r, φ, z with the z-axis coinciding with the tube axis, the density is given by

$$\rho_0 = \begin{cases} \rho_{0i}, & r < a, \\ \rho_{0e}, & r > a, \end{cases}$$

a is the tube radius.

- The plasma motion is described by the ideal magnetohydrodynamic equations in a zero-beta approximation.



3 Linear sausage mode

- In terms of a radial velocity, it is writing as

$$v_r = AV_{Ai}V(r)\exp(ikz - i\omega t),\quad V(r) = \begin{cases} J_1(\lambda r), & r < a, \\ \dfrac{V_{Ai}J_1(\lambda a)}{V_{Ae}K_1(\chi a)}K_1(\chi r), & r > a. \end{cases}$$

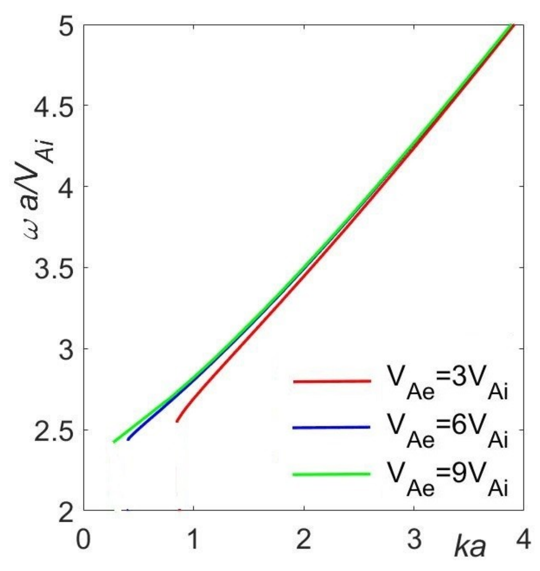
$$V_{Ai} = \frac{B_0}{\sqrt{4\pi\rho_{0i}}},\ V_{Ae} = \frac{B_0}{\sqrt{4\pi\rho_{0e}}},\ \lambda = \sqrt{\frac{\omega^2}{V_{Ai}^2} - k^2},\ \chi = \sqrt{k^2 - \frac{\omega^2}{V_{Ae}^2}}.$$

J_1 is the Bessel function, K_1 is the McDonald function.

- The frequency ω and the wave number k satisfy the dispersion equation

$$\frac{J_1(\lambda a)}{\lambda J_0(\lambda a)} + \frac{K_1(\chi a)}{\chi K_0(\chi a)} = 0.$$

- Here the dispersion curves correspond to the sausage mode fundamental in the radial direction.
- Dispersion curves are cut-off for small wave numbers:
  $k_c a \approx 0.85$ for $V_{Ae} = 3V_{Ai}$,
  $k_c a \approx 0.41$ for $V_{Ae} = 6V_{Ai}$,
  $k_c a \approx 0.27$ for $V_{Ae} = 9V_{Ai}$.
- For $V_{Ae} = 3V_{Ai}$ and $a = 1$-3 Mm the largest wavelength $\lambda \approx 7.2$-21.7 Mm



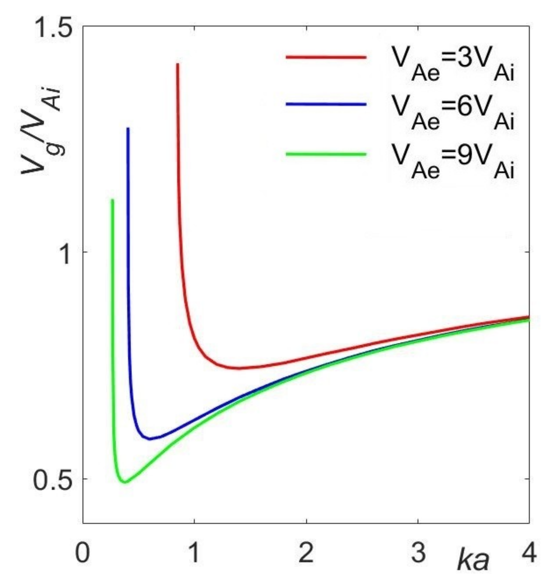
4 Nonlinear Schrödinger equation

- In a weakly nonlinear approximation, the small dimensionless amplitude A is a slow varying function of z and t, $A = A(z, t)$. It describes a space and temporal modulation of the sausage wave and satisfies the NLS equation

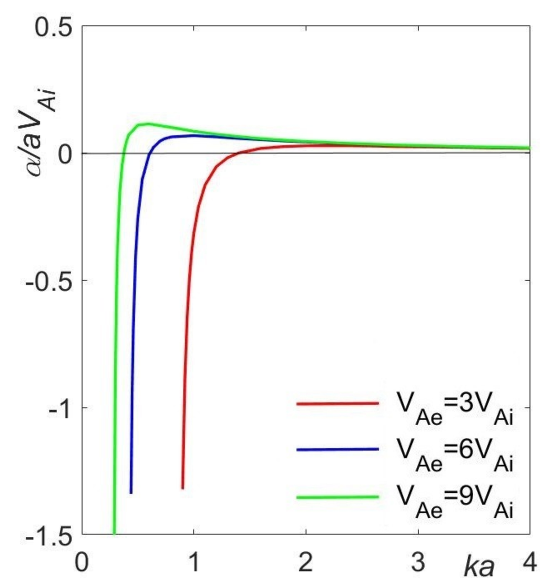
$$i(A_t + V_g A_z) + \alpha A_{zz} + \beta |A|^2 A = 0,$$

$$V_g = \frac{d\omega}{dk},\ \alpha = \frac{1}{2}\frac{d^2\omega}{dk^2},\ \beta = \beta(k).$$

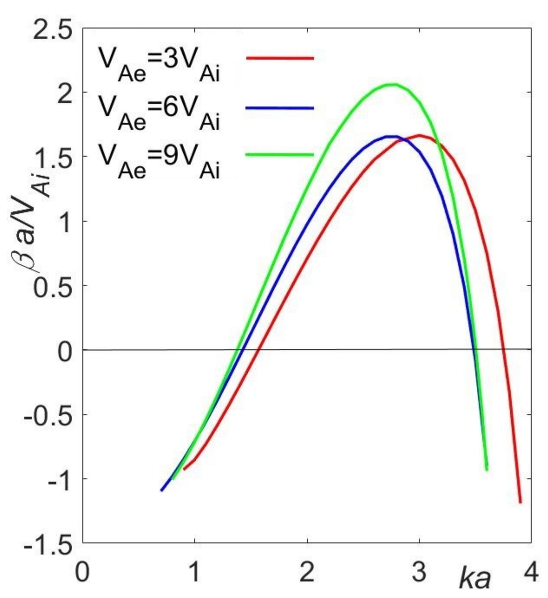
- The fast sausage waves are highly dispersive for small wave numbers. For large wave numbers the wave has a lower dispersion.
- For small wave numbers the group speed V_g decreases to some minimum, where $\alpha = 0$.
- Near minimum, the group speed is smaller than the phase speed ω/k of the carrier wave.



- Coefficient α has negative values for small wave numbers:
  $k_c a < ka < k_1 a$,
  $k_1 a \approx 1.40$ for $V_{Ae} = 3V_{Ai}$,
  $k_1 a \approx 0.60$ for $V_{Ae} = 6V_{Ai}$,
  $k_1 a \approx 0.38$ for $V_{Ae} = 9V_{Ai}$.



- Coefficient β has also negative values for
  $k_c a < ka < k_2 a$ or $ka > k_3 a$,
  $k_2 a \approx 1.57$, $k_2 a \approx 3.75$ for $V_{Ae} = 3V_{Ai}$,
  $k_2 a \approx 1.43$, $k_2 a \approx 3.48$ for $V_{Ae} = 6V_{Ai}$,
  $k_2 a \approx 1.39$, $k_2 a \approx 3.50$ for $V_{Ae} = 9V_{Ai}$.



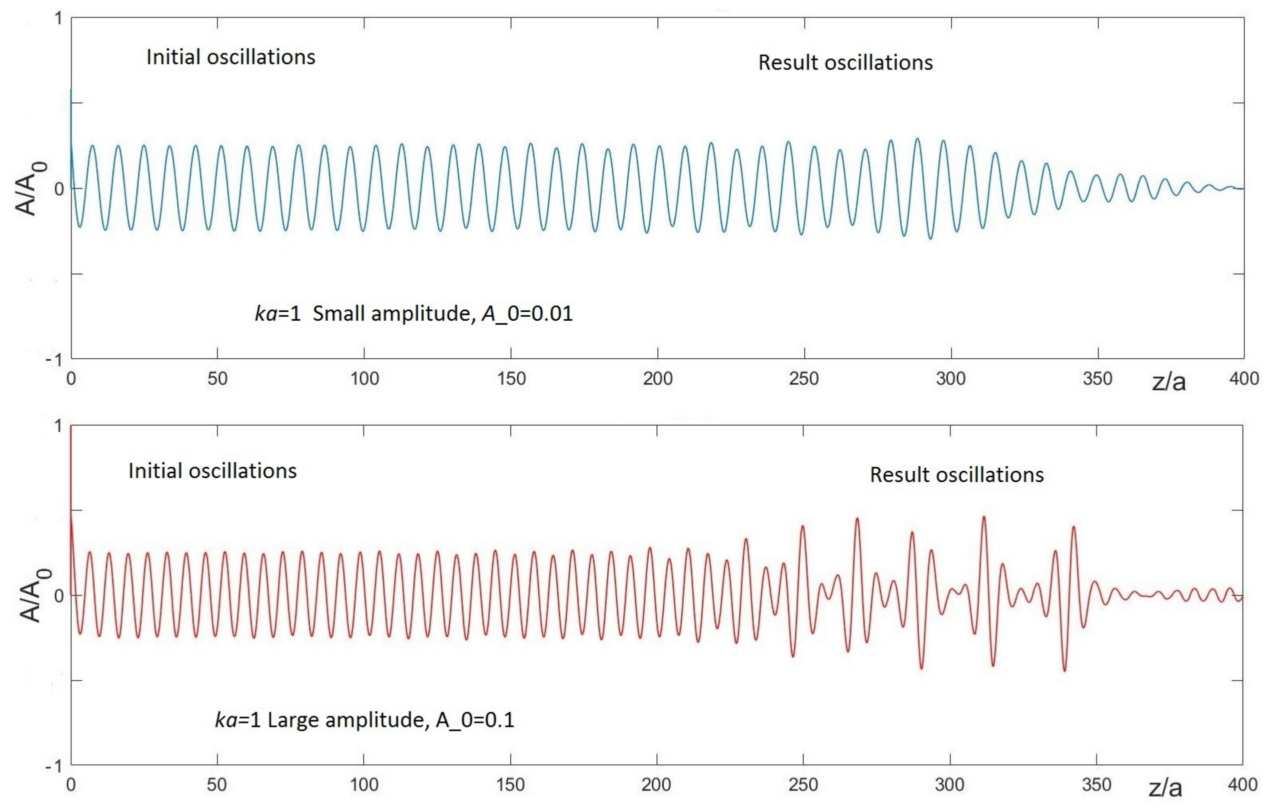
5 Modulation instability

- A large interesting property of the NLS equation is related to the stability of the solution in the form of a harmonic wave

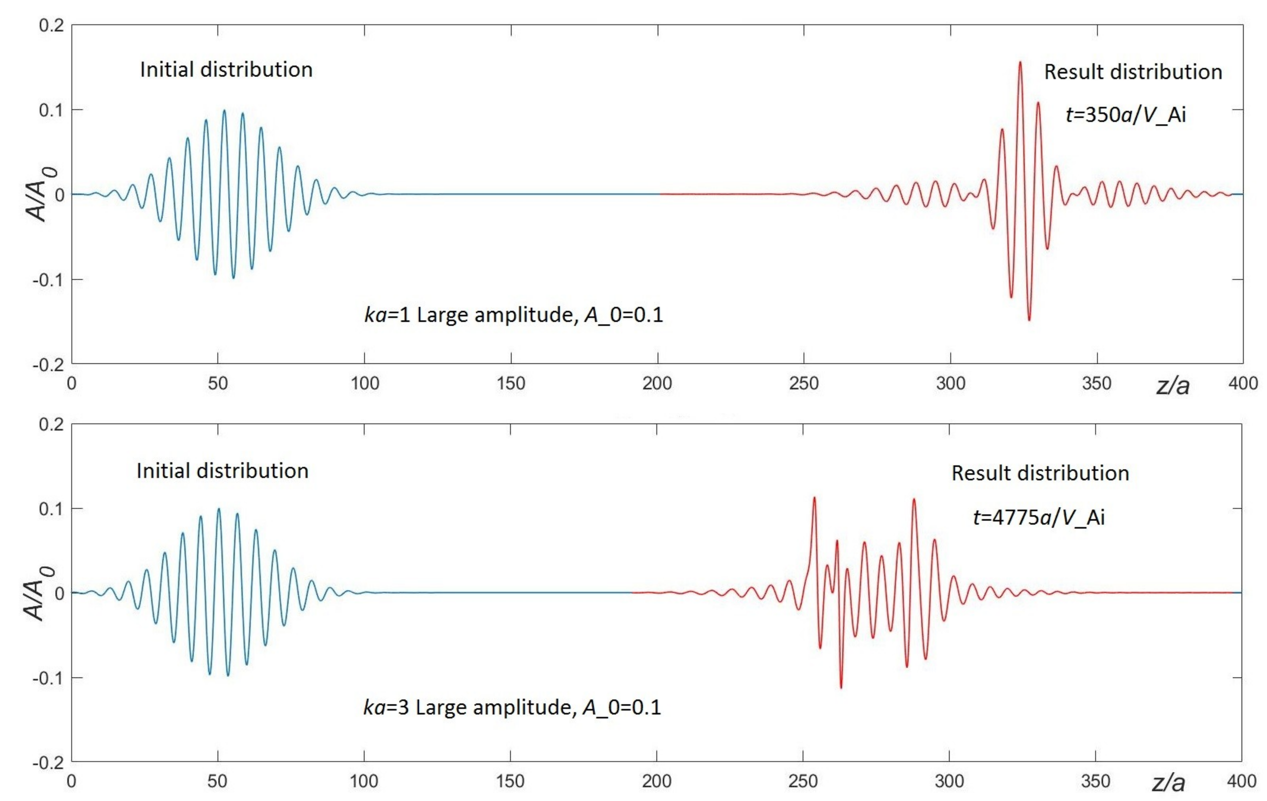
$$A = A_0\exp(ikz - i\omega t),\quad \omega = V_g k + \alpha k^2 - \beta A_0^2,\ A_0 - \text{const}.$$

- Since $\alpha\beta > 0$, the solution is subject to the modulational or Benjamin–Fair instability.
- For the sausage mode, the condition $\alpha\beta > 0$ is satisfying if $k_c a < ka < k_1 a$ (small wave numbers, high dispersive waves) or $ka > k_2 a$ (large wave numbers, low dispersive waves).
- For comparision, for $V_{Ae} = 3V_{Ai}$ we consider cases
  $ka = 1$ (high dispersion), $\omega/k \approx 2.69V_{Ai}$, $V_g \approx 0.81V_{Ai}$,
  $ka = 3$ (low dispersion), $\omega/k \approx 1.41V_{Ai}$, $V_g \approx 0.82V_{Ai}$.

Periodic distribution



Waves package



6 Conclusions

- Sausage mode has a high dispersion for small wave numbers and a low dispersion for large wave numbers.
- As a result, in a case of moderate wave amplitude, the main dispersive effect is a nonlinear wave modulation due to the Benjamin–Fair instability.
- The modulation effect leads to a generation of quasi-periodic oscillations.
- It is possible for a large wave amplitude both for small and large wave numbers but corresponding generation time scales are very different.