Inference of magnetic field strength and density from damped transverse coronal waves

Iñigo Arregui
María Montes Solís & Andrés Asensio Ramos

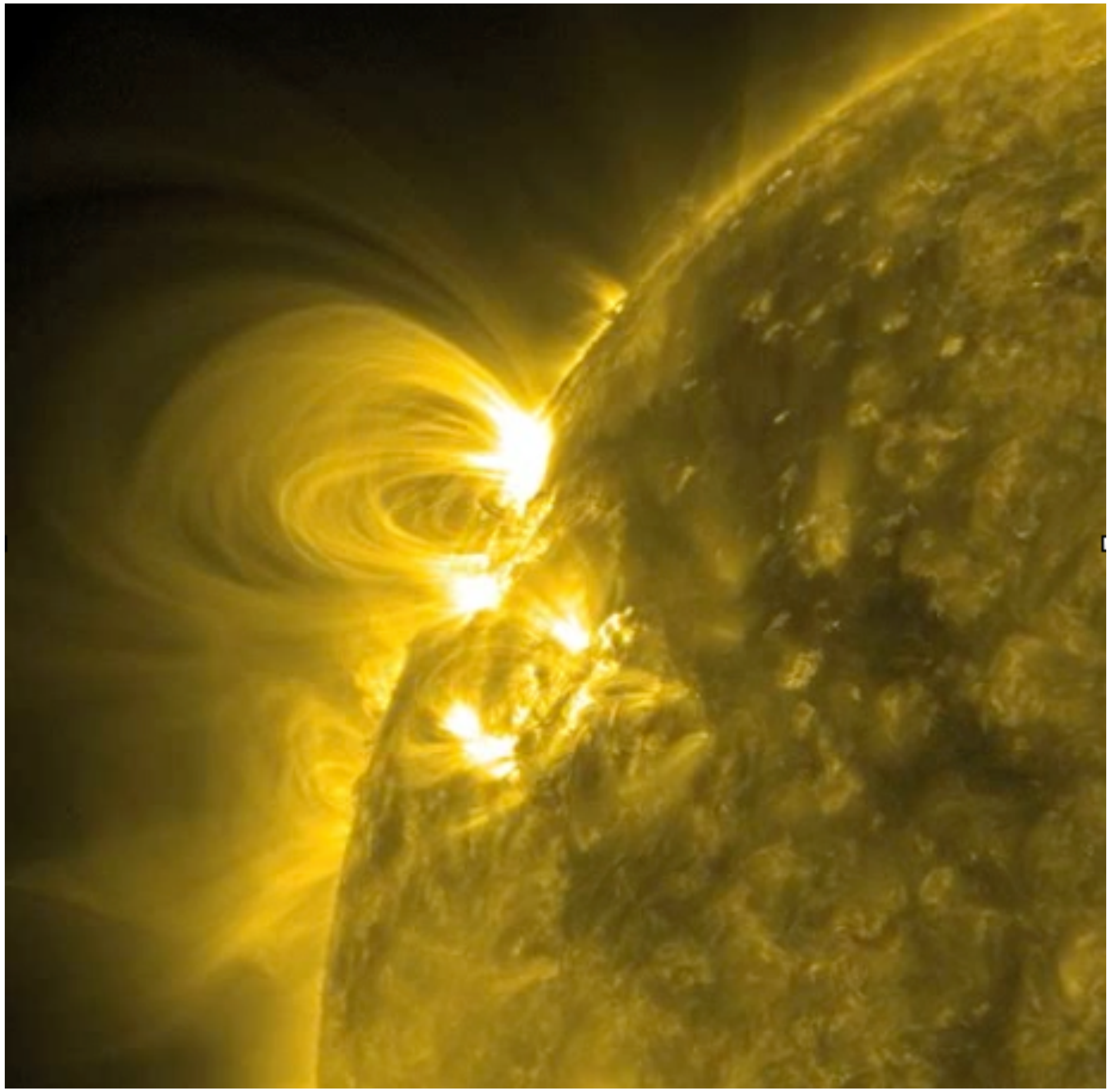
Instituto de Astrofísica de Canarias (IAC)
Tenerife, Spain

BUKS2018 :: La Laguna, September 6, 2018

Seismology of the Solar Atmosphere

Aim: inference of difficult to measure physical parameters in e.g.:

Coronal loops



Prominences

Combination of: { - Observations: Wave activity in the solar atmosphere
- Theory: MHD wave interpretation

Determination of the magnetic field strength

coronal wave-guides

The first “modern” application of solar atmospheric seismology was performed by Nakariakov & Ofman (2001)

Theoretical interpretation of transverse loop oscillations as kink mode of flux tube

Observations

$$\frac{\omega}{k} = \frac{2L}{P} \approx \begin{cases} 1020 \pm 132 \ \mathrm{km\,s^{-1}} & \text{(14th July, 1998)}, \\ 1030 \pm 410 \ \mathrm{km\,s^{-1}} & \text{(4th July, 1999)}. \end{cases}$$

Measurement of P and L gives phase speed

Theory

$$\frac{\omega}{k} \approx c_{\mathrm{k}} \equiv v_{\mathrm{Ai}} \left[\frac{2\zeta}{1+\zeta} \right]^{1/2}$$

Long-wavelength approximation for phase speed

Notice that both the Alfvén speed and the density contrast are unknown!

Consider density contrast $\zeta = \rho_{\mathrm{i}}/\rho_{\mathrm{e}}$ as parameter > obtain Alfvén speed

$B = (4\pi\rho_i)^{1/2} v_{\mathrm{Ai}}$ **Assume** $\rho_{\mathrm{i}} \in [1-6] \times 10^9 \mathrm{cm}^{-3}$ $B \in [4-30]$ G

Main shortcoming: densities are assumed

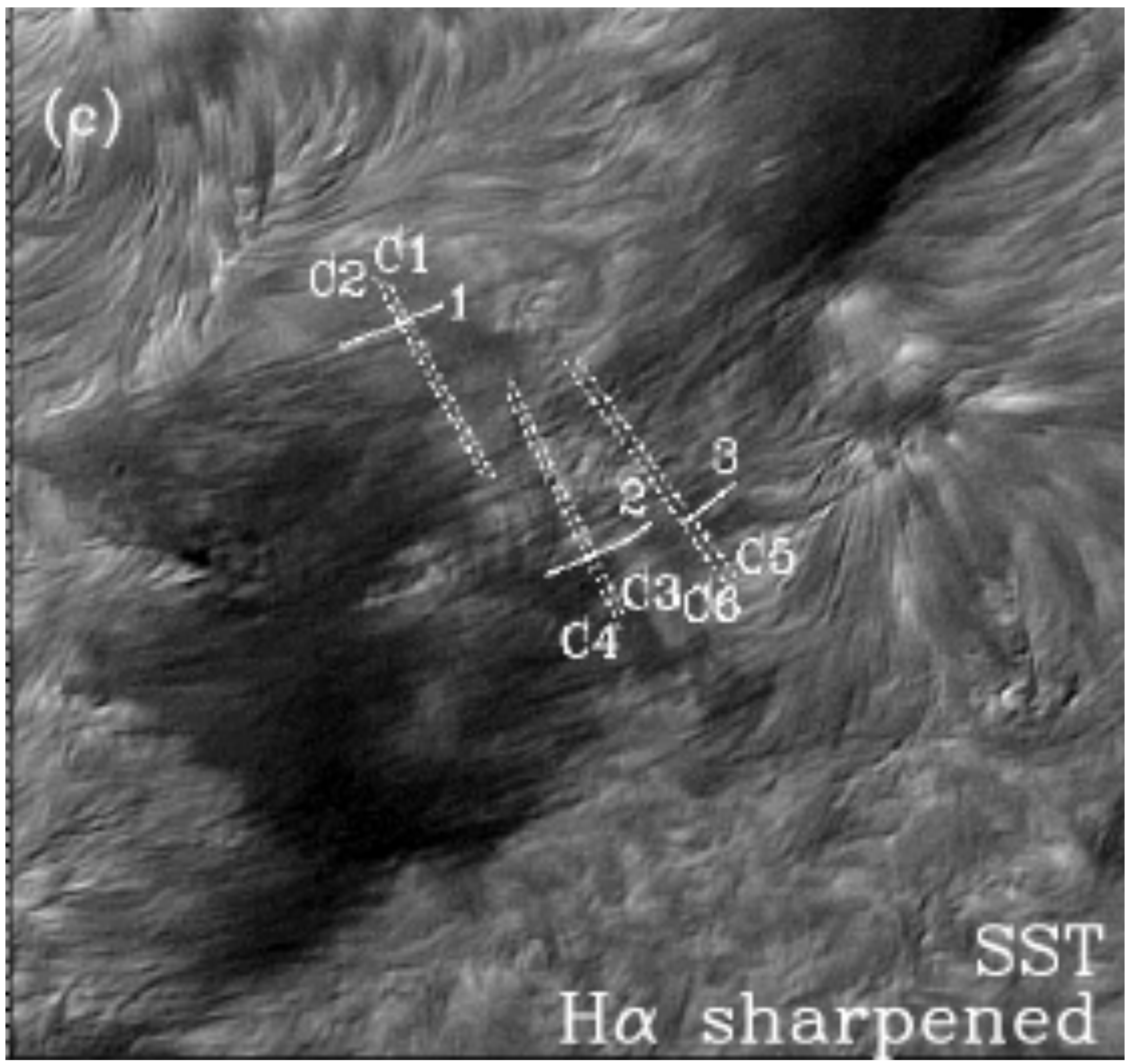
Determination of the magnetic field strength

application to prominence threads by Lin et al. (2009)

see talk Montes-Solís

The same method has also been applied to other magnetic and plasma structures, e.g.:

Filament threads observed on August 2, 2007



P ~ 4-5 min
V_{ph} ~ 22 km/s

Table 1
For the 10 Selected Hα Swaying Threads, the Periods (P), Phase Velocities (V_{ph}), and the Amplitudes (A) of the Possible Waves Were Measured

Thread No.	Cut No.	P (minutes)	V_{ph} (km s^{-1})	A (km)
1	c1	3.5 ± 0.1	16 ± 3	79 ± 6
	c2	3.9 ± 0.1		70 ± 7
2	c3	4.72 ± 0.05	20 ± 6	79 ± 7
	c4	4.50 ± 0.03		76 ± 6
3	c5	3.9 ± 0.1	24 ± 6	67 ± 10
	c6	4.4 ± 0.1		110 ± 9
4	c7	3.66 ± 0.04	36 ± 6	88 ± 4
	c8	3.69 ± 0.04		86 ± 4
5	c9	3.76 ± 0.02	57 ± 9	96 ± 3
	c10	3.78 ± 0.03		81 ± 3
6	c11	2.7 ± 0.1	28 ± 12	57 ± 4
	c12	4.0 ± 0.1		73 ± 5
7	c13	2.0 ± 0.1	62 ± 10	52 ± 3
	c14	1.9 ± 0.1		59 ± 3
	c15	2.0 ± 0.1		52 ± 4
8	c16	3.1 ± 0.1	40 ± 6	56 ± 4
	c17	3.0 ± 0.1		34 ± 2
9	c18	2.8 ± 0.1	20 ± 3	34 ± 3
	c19	2.6 ± 0.1		57 ± 4
10	c20	5.4 ± 0.1	28 ± 9	88 ± 3
	c21	5.0 ± 0.2		58 ± 3

Notes. Thread Nos. 1–3, indicated in Figure 3 (c), are selected from the time series with the cadence of 18.5 s. Thread Nos. 4–10 are from the time series with the cadence of 3.9 s.

Determination of the magnetic field strength

application to prominence threads by Lin et al. (2009)

see talk Montes-Solís

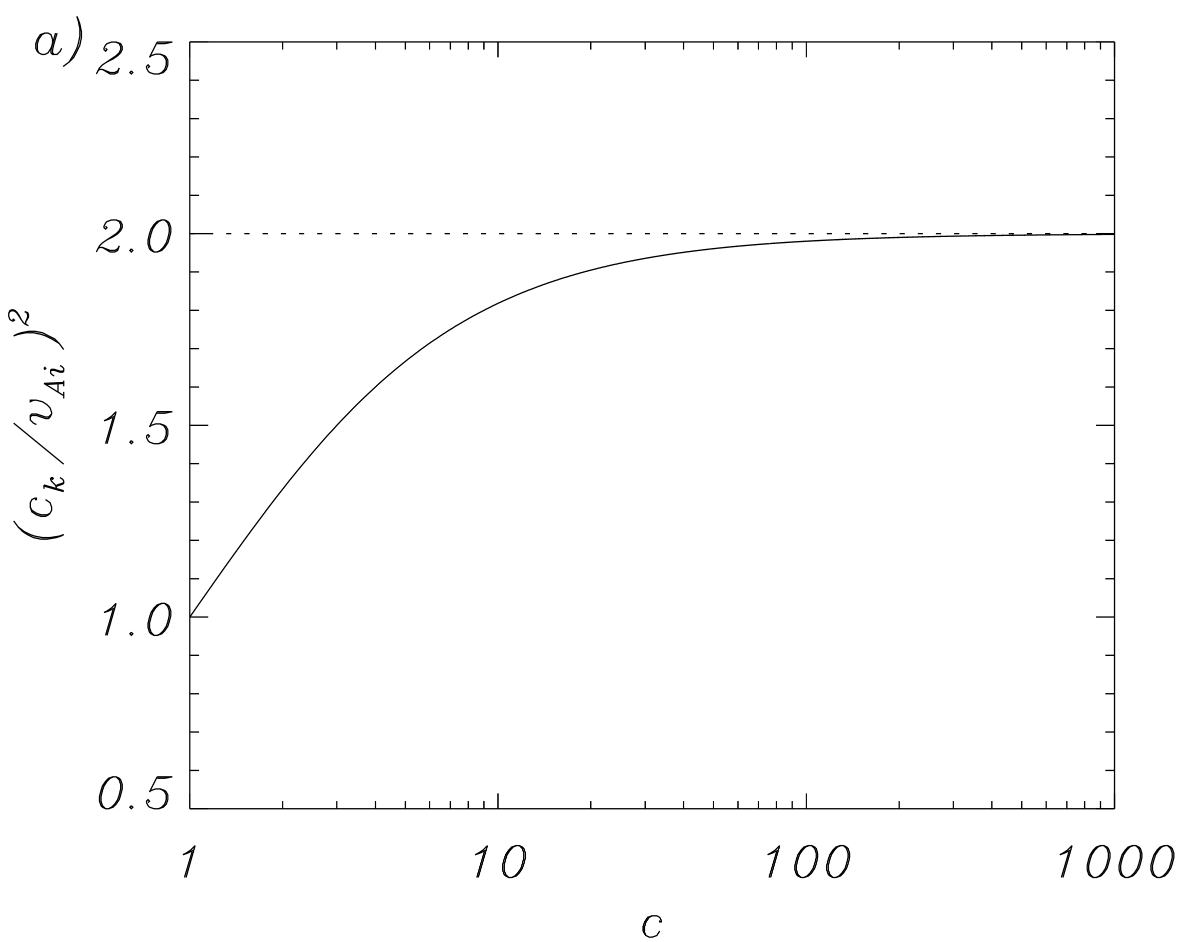
A fundamental difference between coronal loops and prominence plasmas is that the latter are high density contrast structures

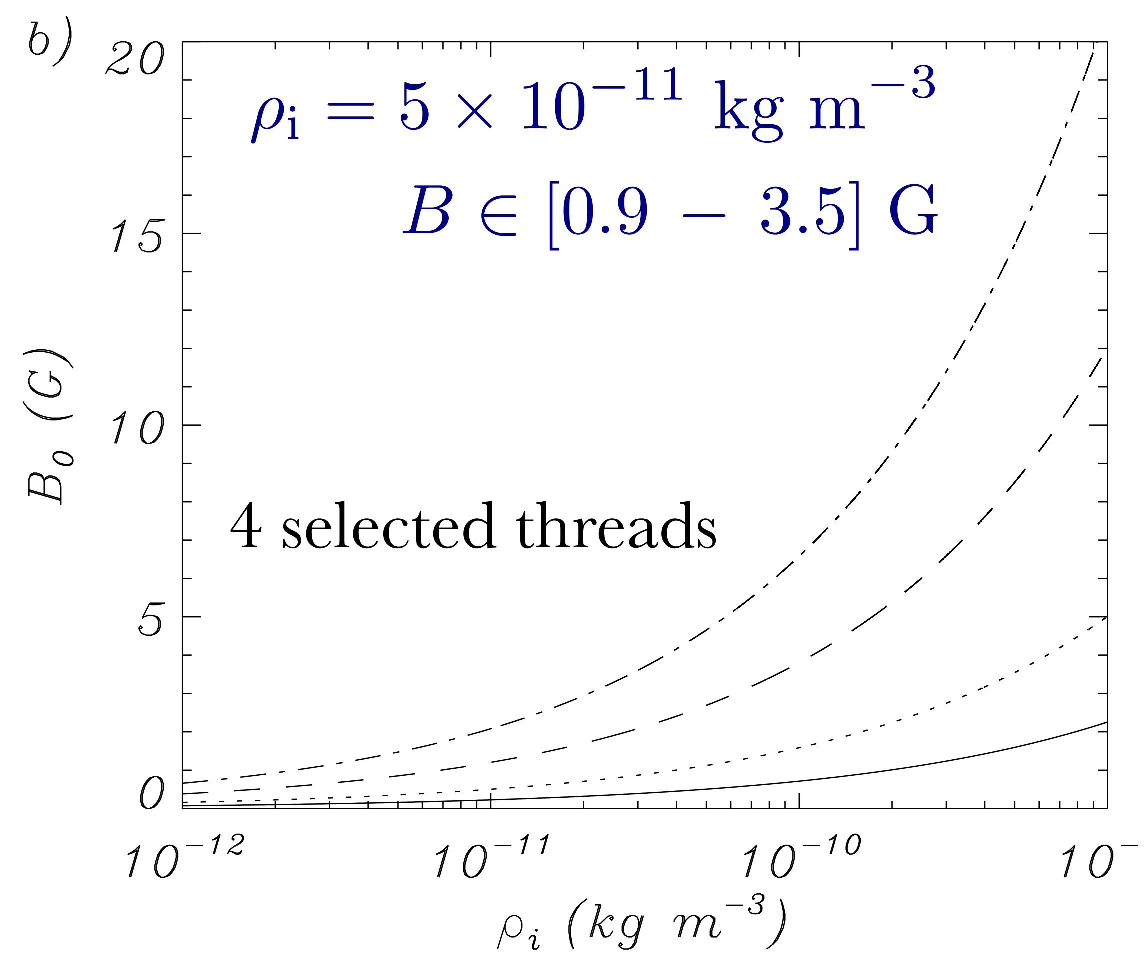
In prominence threads   Ratio $\rho_{\mathrm{i}}/\rho_{\mathrm{e}} \sim 10^2$

$$\frac{\omega}{k} \approx c_{\mathrm{k}} \equiv v_{\mathrm{Ai}} \left[\frac{2\zeta}{1+\zeta}\right]^{1/2} \longrightarrow c_{\mathrm{k}} \approx \sqrt{2} v_{\mathrm{Ai}}$$

Kink speed is independent of density contrast

Lin et al. (2009) assume that thread oscillations observed from H_α sequences are the result of propagating kink waves > measured phase speed = kink speed





From classic to Bayesian techniques

Confronting observations and theory to infer physical parameters is not an easy task

Seismology involves the solution of two different problems

	Cause		Consequences
Forward problem	Theoretical models and parameters	→	Theoretical wave properties
	Consequences		Cause
Inverse problem	Observed wave properties	→	Unknown physical conditions/processes

Under conditions in which information is incomplete and uncertain

We use the rules of probability to make scientific inference and quantify uncertainty

see **Arregui (2018)** for a review on Bayesian Coronal Seismology

Bayesian Data Analysis

Probabilistic Inference considers the inversion problem as the task of estimating the degree of belief in statements about parameter values

Bayes' Rule (Bayes & Price 1763)

$$p(\boldsymbol{\theta}|D,M) = \frac{p(D|\boldsymbol{\theta},M)\,p(\boldsymbol{\theta}|M)}{\int d\boldsymbol{\theta}\,p(D|\boldsymbol{\theta},M)\,p(\boldsymbol{\theta}|M)}$$

$p(\boldsymbol{\theta}\|D,M)$	Posterior
$p(\boldsymbol{\theta}\|M)$	Prior
$p(D\|\boldsymbol{\theta},M)$	Likelihood function
$\int d\boldsymbol{\theta}\,p(D\|\boldsymbol{\theta},M)\,p(\boldsymbol{\theta}\|M)$	Evidence

State of knowledge is a combination of what is known a priori independently of data and the likelihood of obtaining a data realisation actually observed

Parameter Inference

Compute posterior for different combinations of parameters

Marginalise

$$p(\theta_i|d) = \int p(\boldsymbol{\theta}|d)\,d\theta_1 \ldots d\theta_{i-1}\,d\theta_{i+1} \ldots d\theta_N$$

Inference of Magnetic Field Strength - Bayesian Modelling

Theory

$$v_{\rm ph}(\rho_i, \zeta, B_0) = \frac{B_0}{\sqrt{\mu \rho_i}} \left(\frac{2\zeta}{1+\zeta} \right)^{1/2}$$

Observation

$$P = 360 \text{ s} \quad L = 1.9 \times 10^9 \text{ cm}$$

$$\rightarrow v_{\rm ph} = 1030 \pm 410 \text{ km/s}$$

Bayes Theorem

$$p(\{\rho_i, \zeta, B_0\} | v_{\rm ph}, M) = \frac{p(v_{\rm ph} | \{\rho_i, \zeta, B_0\}, M) p(\{\rho_i, \zeta, B_0\}, M)}{Z}$$

Marginalise

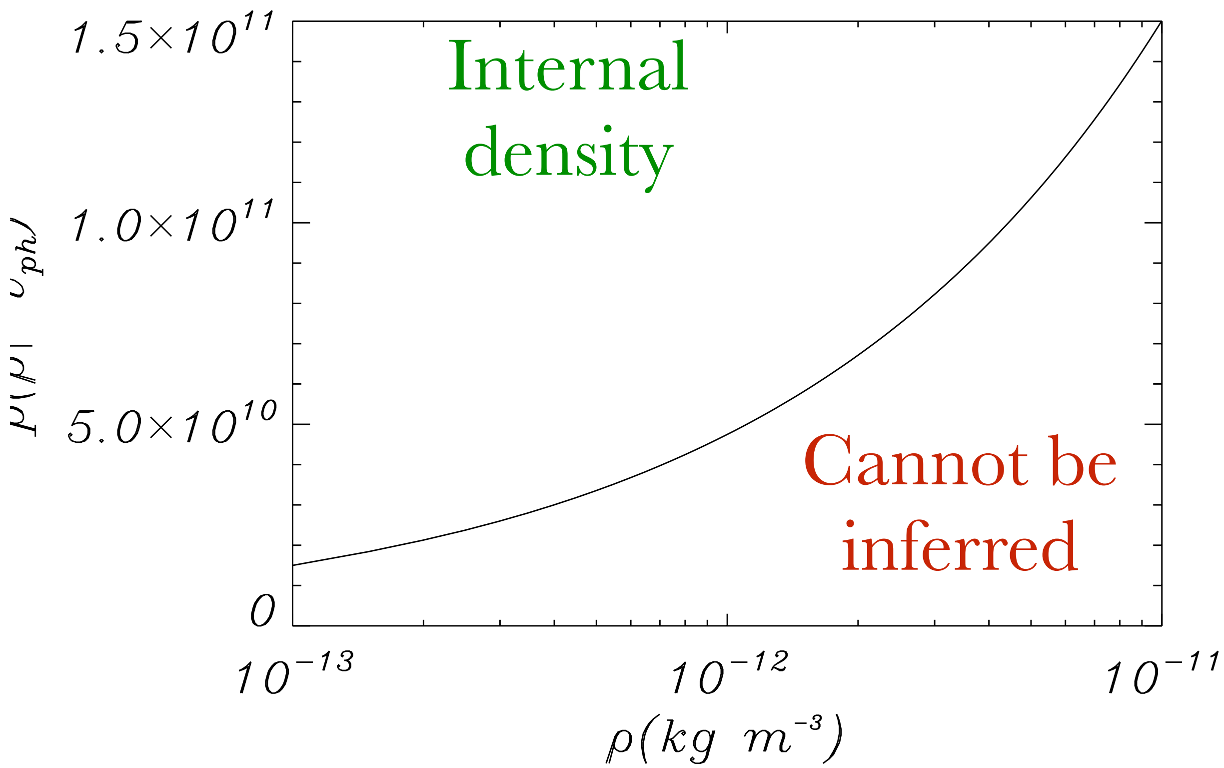
$$p(\rho_i | v_{\rm ph}) = \int p(\{\rho_i, \zeta, B_0\} | v_{\rm ph}) \; d\zeta \; dB_0,$$

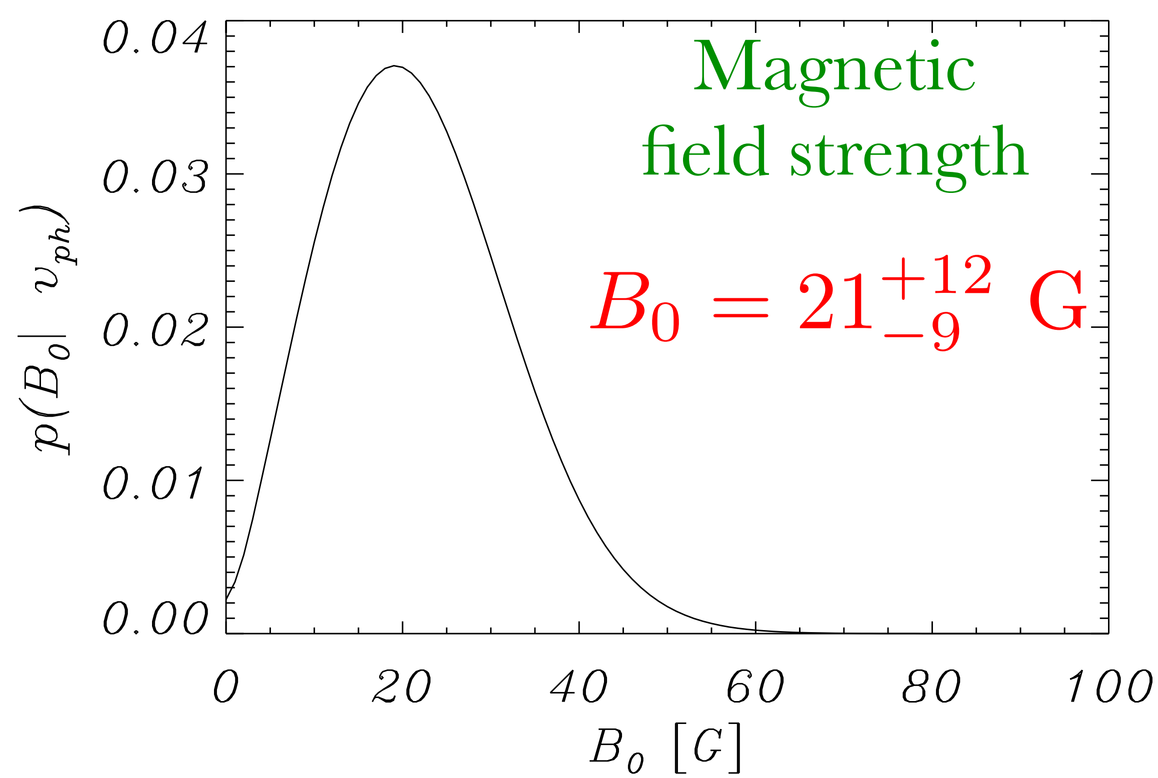
$$p(\zeta | v_{\rm ph}) = \int p(\{\rho_i, \zeta, B_0\} | v_{\rm ph}) \; d\rho_i \; dB_0,$$

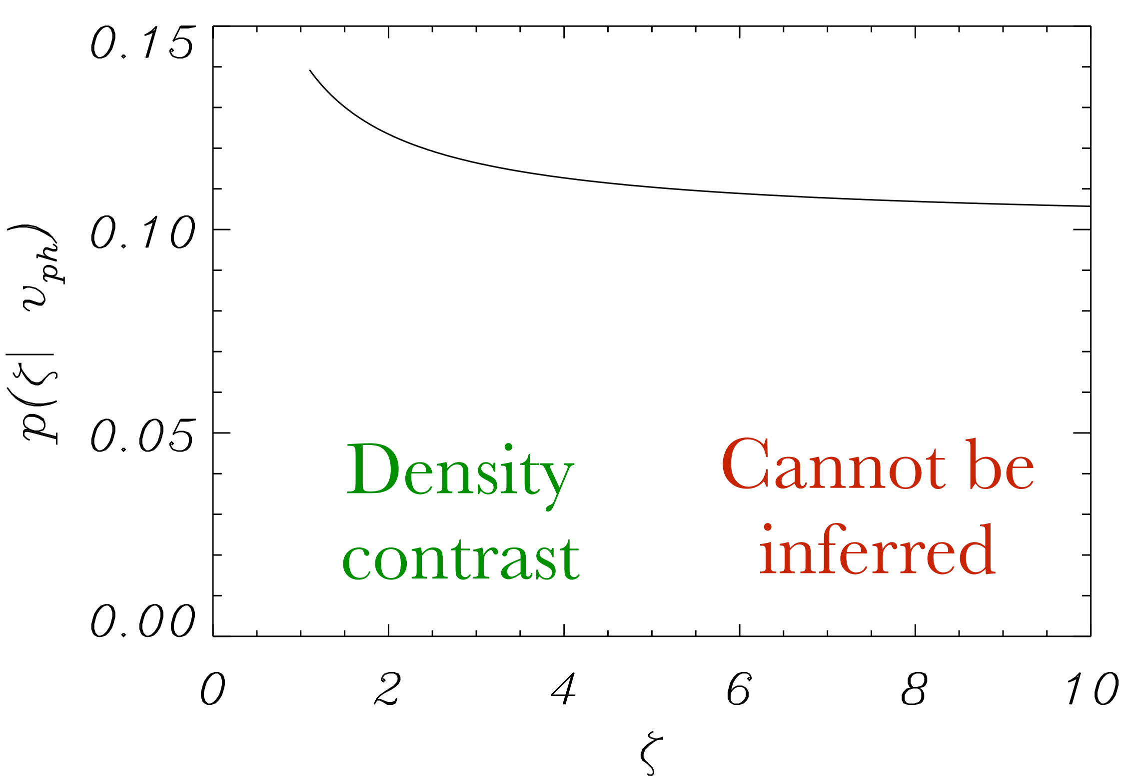
$$p(B_0 | v_{\rm ph}) = \int p(\{\rho_i, \zeta, B_0\} | v_{\rm ph}) \; d\rho_i \; d\zeta.$$

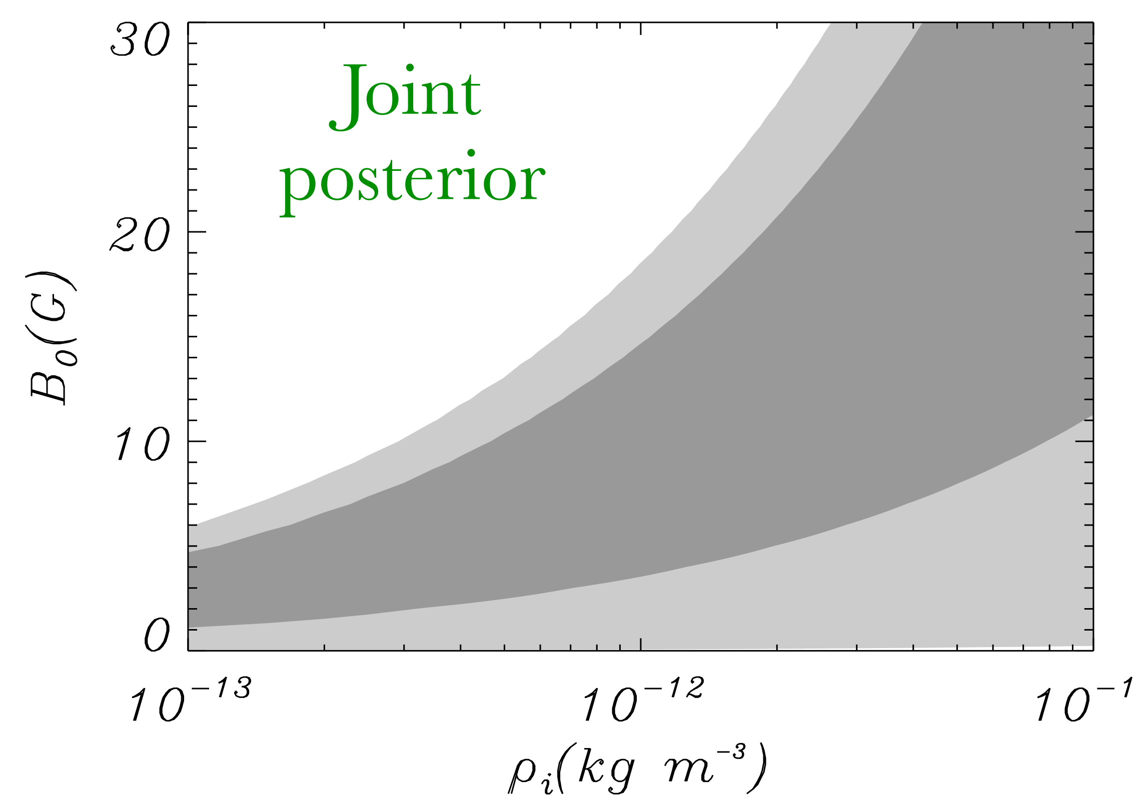
Inference of Magnetic Field Strength - Result

$$P = 360 \text{ s} \quad L = 1.9 \times 10^9 \text{ cm} \rightarrow \; v_{\rm ph} = 1030 \pm 410 \text{ km/s}$$









Inference of Magnetic Field Strength with Information on Density

When additional information on any of the unknowns becomes available, the Bayesian framework offers a self-consistent way to include this information to update the posteriors

Consider we got some estimates for the density inside the oscillating coronal loop

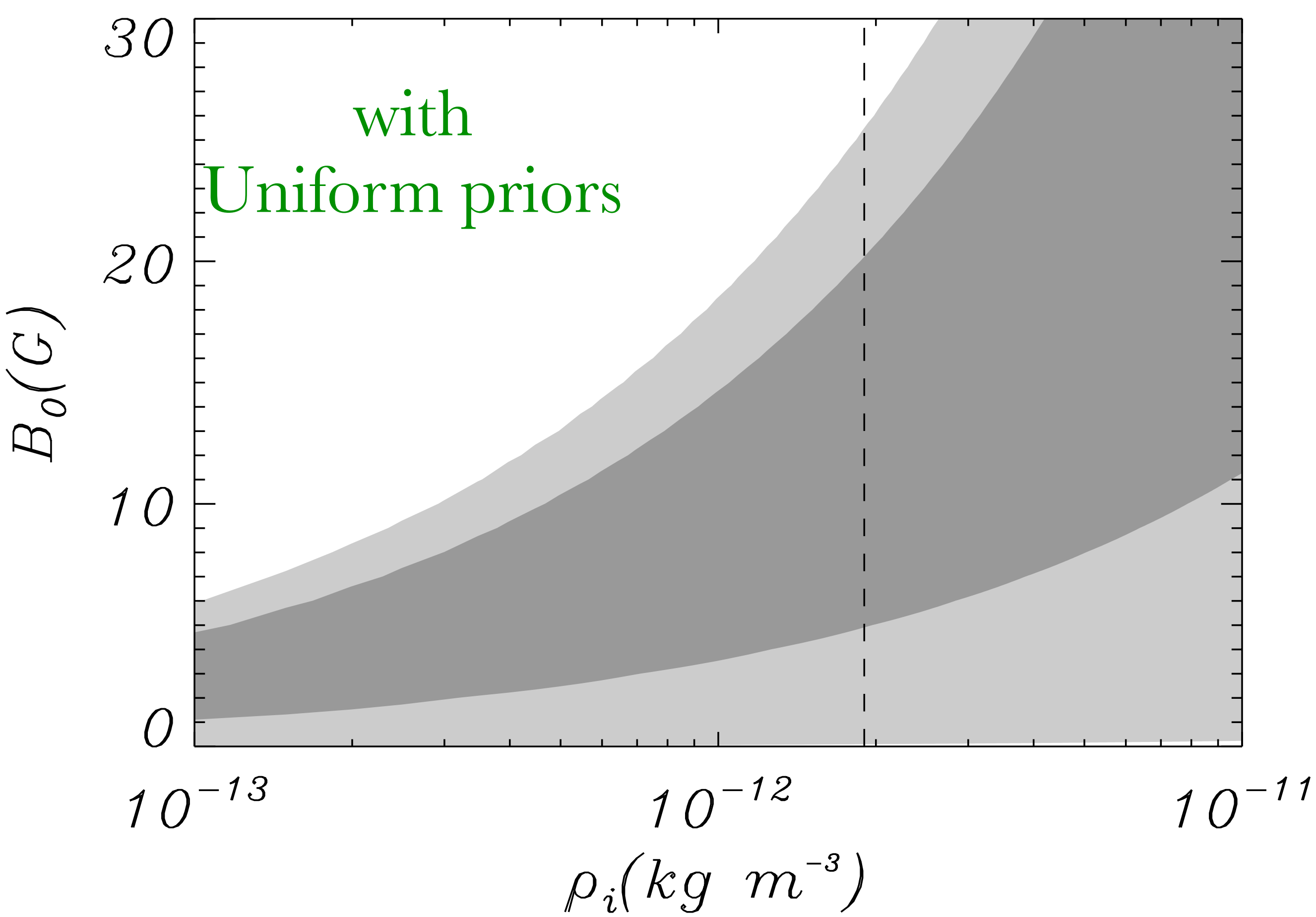
Gaussian prior on density

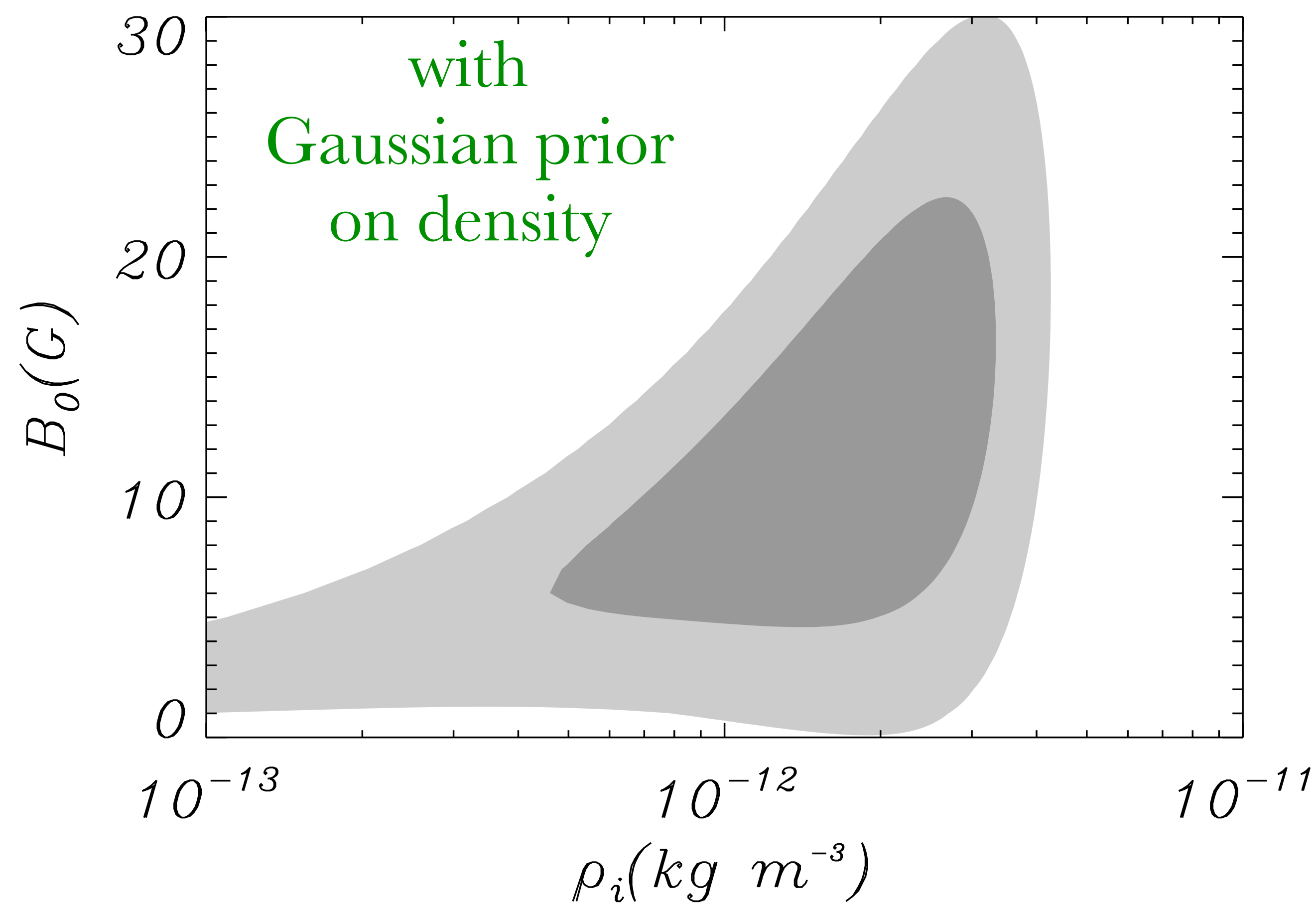
$$p(\rho_\mathrm{i}) = (2\pi\sigma^2_{\rho_\mathrm{i}}) \exp\left[\frac{-(\rho_\mathrm{i} - \mu_{\rho_\mathrm{i}})^2}{2\sigma^2_{\rho_\mathrm{i}}}\right]$$

Inference summary

$$B_0 = 13^{+7}_{-5} \text{ G}$$

$$\rho_\mathrm{i} = (2.2^{+0.9}_{-0.9}) \times 10^{-12} \text{ kg/m}^3$$

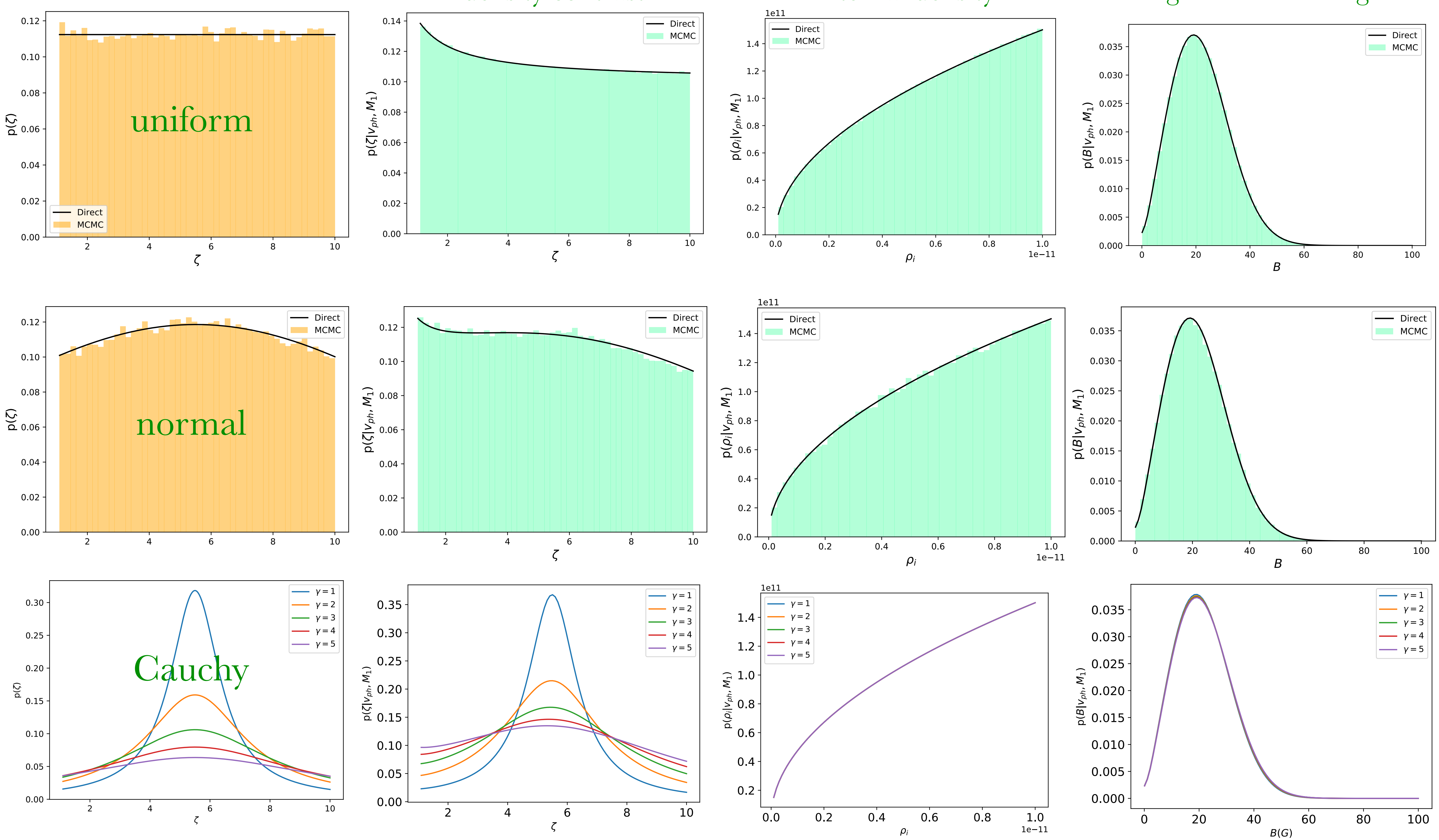




Inference of Magnetic Field Strength - role of prior information

The amount of information one is willing to include (a priori) for the density and the density contrast influences their corresponding posteriors, but very little the inferred magnetic field strength

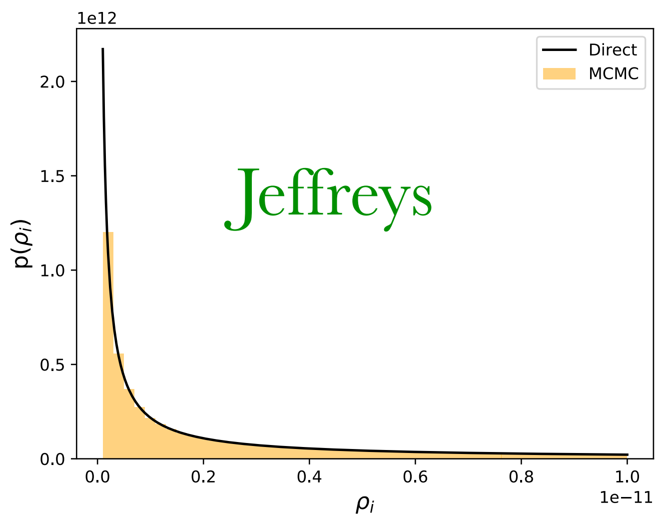
priors	**posteriors**		
	density contrast	internal density	magnetic field strength

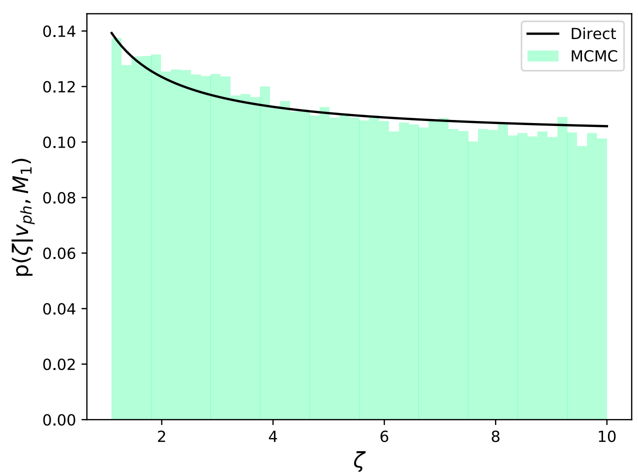


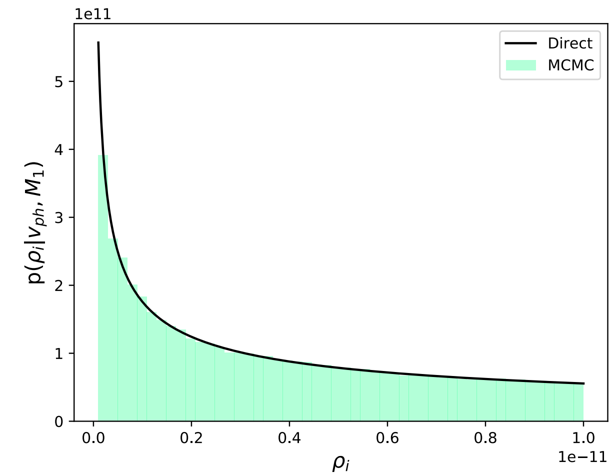
Inference of Magnetic Field Strength - role of prior information

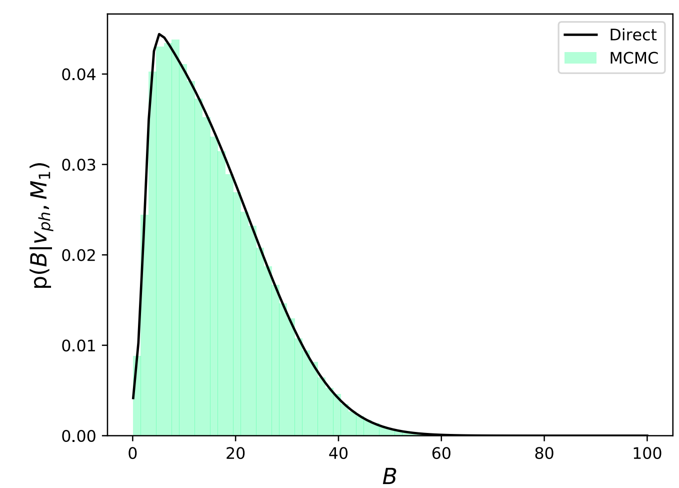
The amount of information one is willing to include (a priori) for the density and the density contrast influences their corresponding posteriors, but very little the inferred magnetic field strength

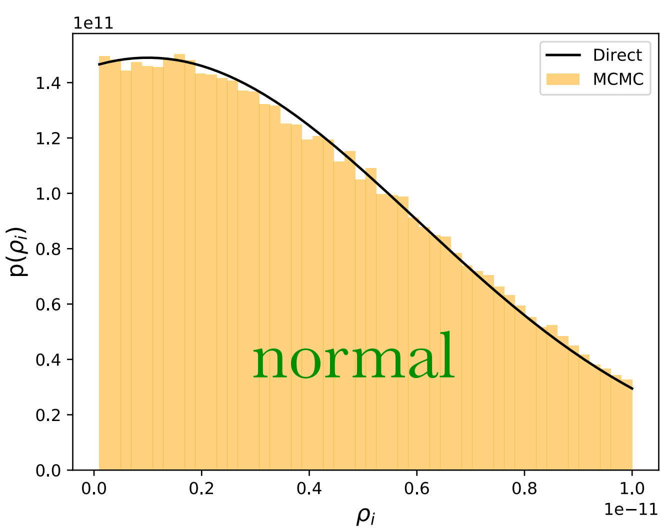
priors	**posteriors**		
	density contrast	internal density	magnetic field strength

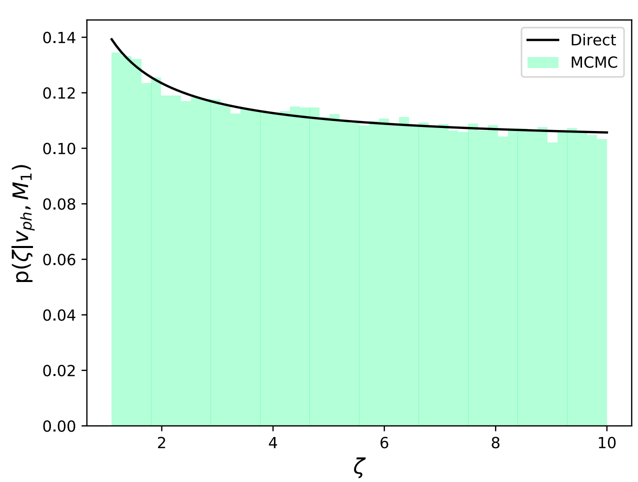


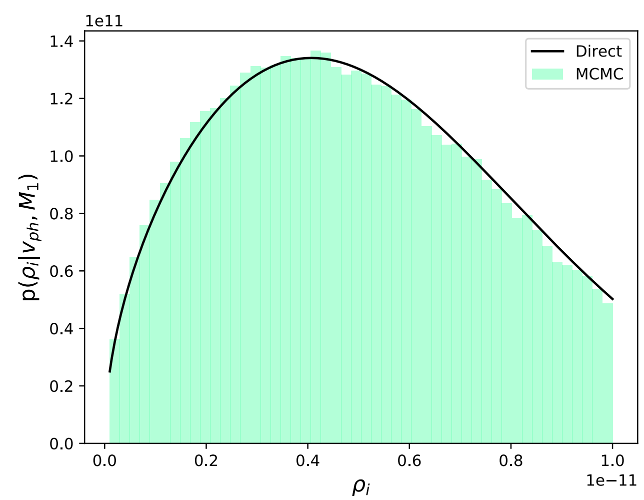


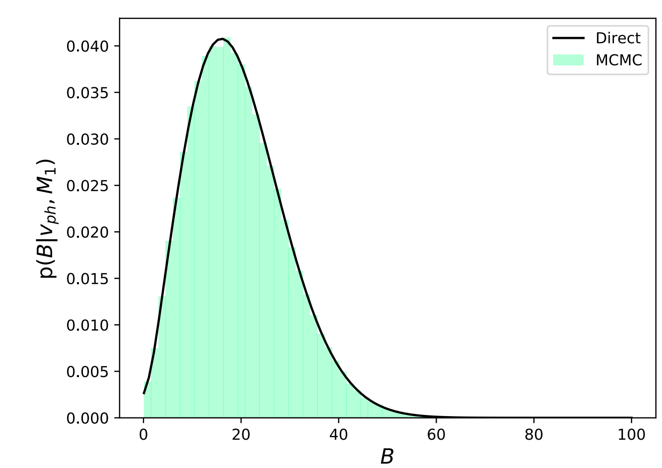












Inference of Magnetic Field Strength with Damping Bayesian Modelling

Theory

$$v_{\mathrm{ph}}(\rho_i, \zeta, B_0) = \frac{B_0}{\sqrt{\mu \rho_i}} \left(\frac{2\zeta}{1+\zeta} \right)^{1/2}$$

$$\tau_{\mathrm{d}}(\rho_i, \zeta, B_0, l/R) = \frac{2}{\pi} \left(\frac{\zeta+1}{\zeta-1} \right) \left(\frac{1}{l/R} \right) \left(\frac{2L}{v_{\mathrm{ph}}} \right)$$

Observation

$$P = 360 \text{ s} \quad L = 1.9 \times 10^9 \text{ cm}$$

$$\rightarrow v_{\mathrm{ph}} = 1030 \pm 410 \text{ km/s}$$

$$\tau_{\mathrm{d}} = 500 \pm 50 \text{ s}$$

Bayes Theorem

$$p(\{\rho_i, \zeta, B_0, l/R\} | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}, M) = \frac{p(\{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\} | \{\rho_i, \zeta, B_0, l/R\}, M) p(\{\rho_i, \zeta, B_0, l/R\}, M)}{Z}$$

Marginalise

$$p(\rho_i | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) = \int p(\{\rho_i, \zeta, B_0, l/R\} | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) \; d\zeta \; dB_0 \; d(l/R),$$

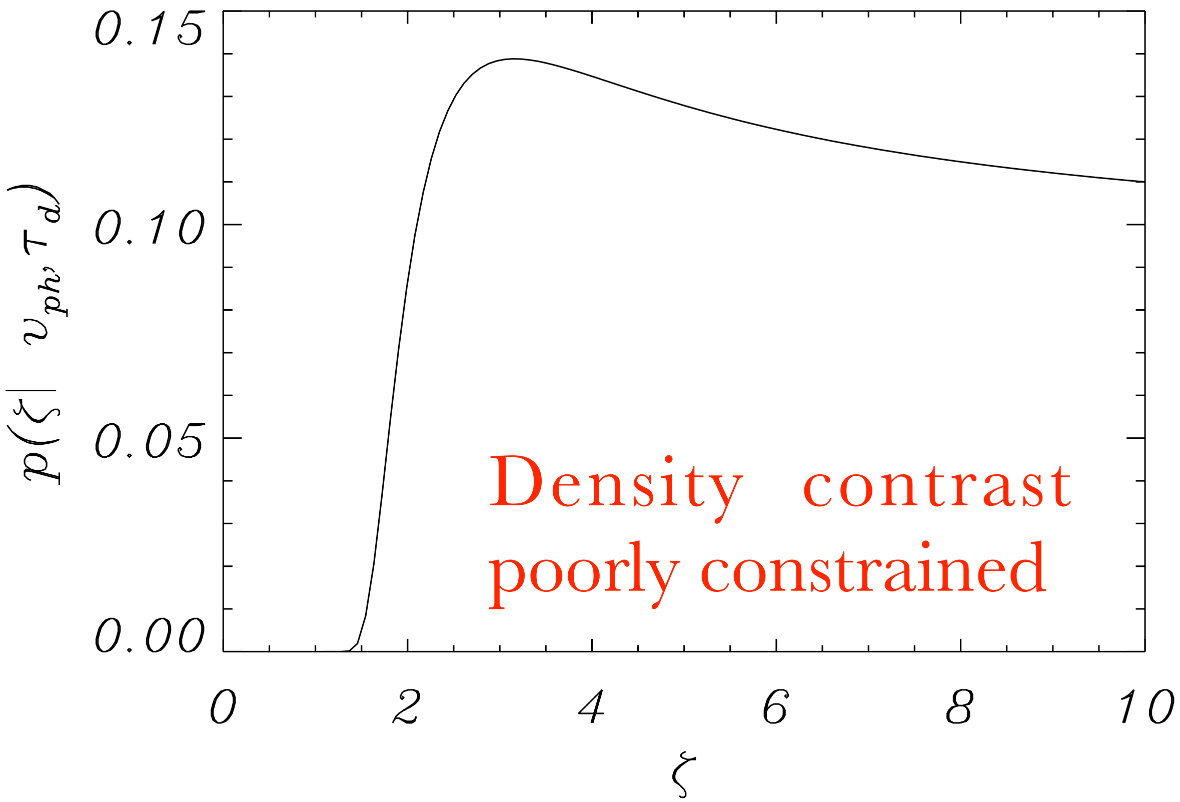
$$p(\zeta | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) = \int p(\{\rho_i, \zeta, B_0, l/R\} | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) \; d\rho_i \; dB_0 \; d(l/R),$$

$$p(B_0 | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) = \int p(\{\rho_i, \zeta, B_0, l/R\} | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) \; d\rho_i \; d\zeta \; d(l/R),$$

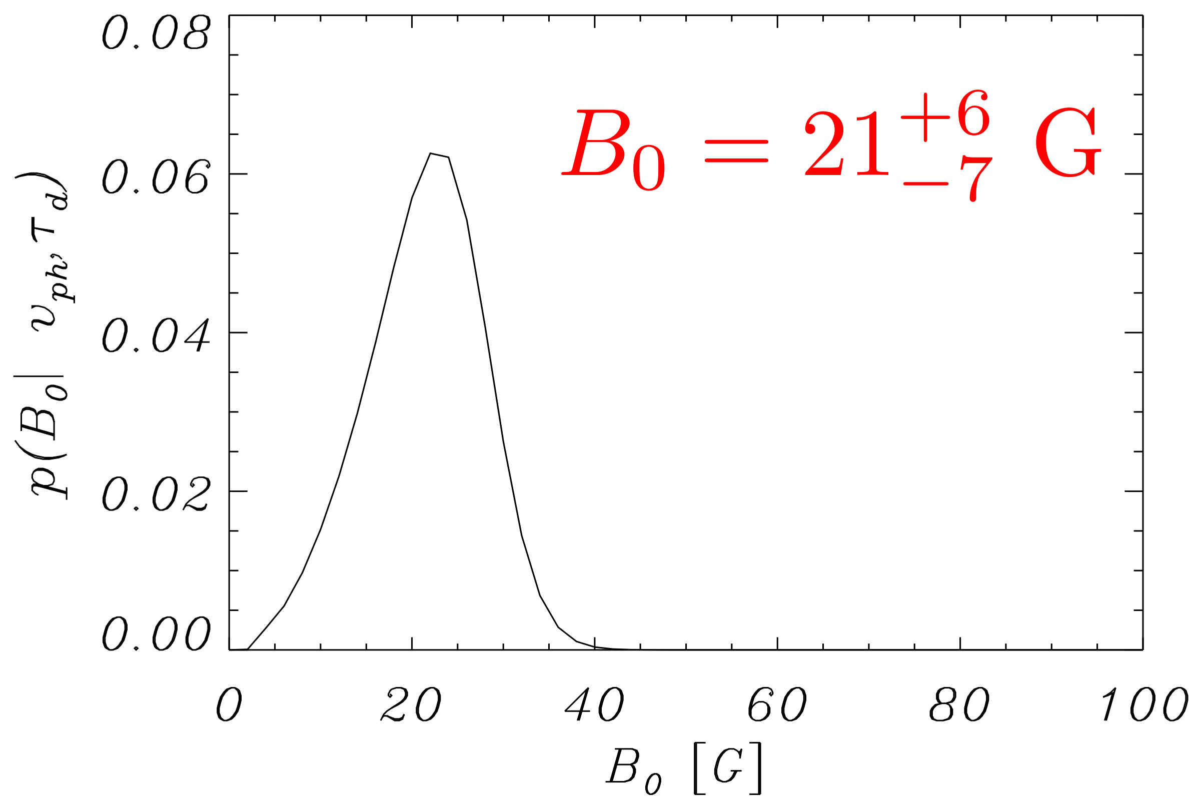
$$p(l/R | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) = \int p(\{\rho_i, \zeta, B_0, l/R\} | \{v_{\mathrm{ph}}, \tau_{\mathrm{d}}\}) \; d\rho_i \; d\zeta \; dB_0.$$

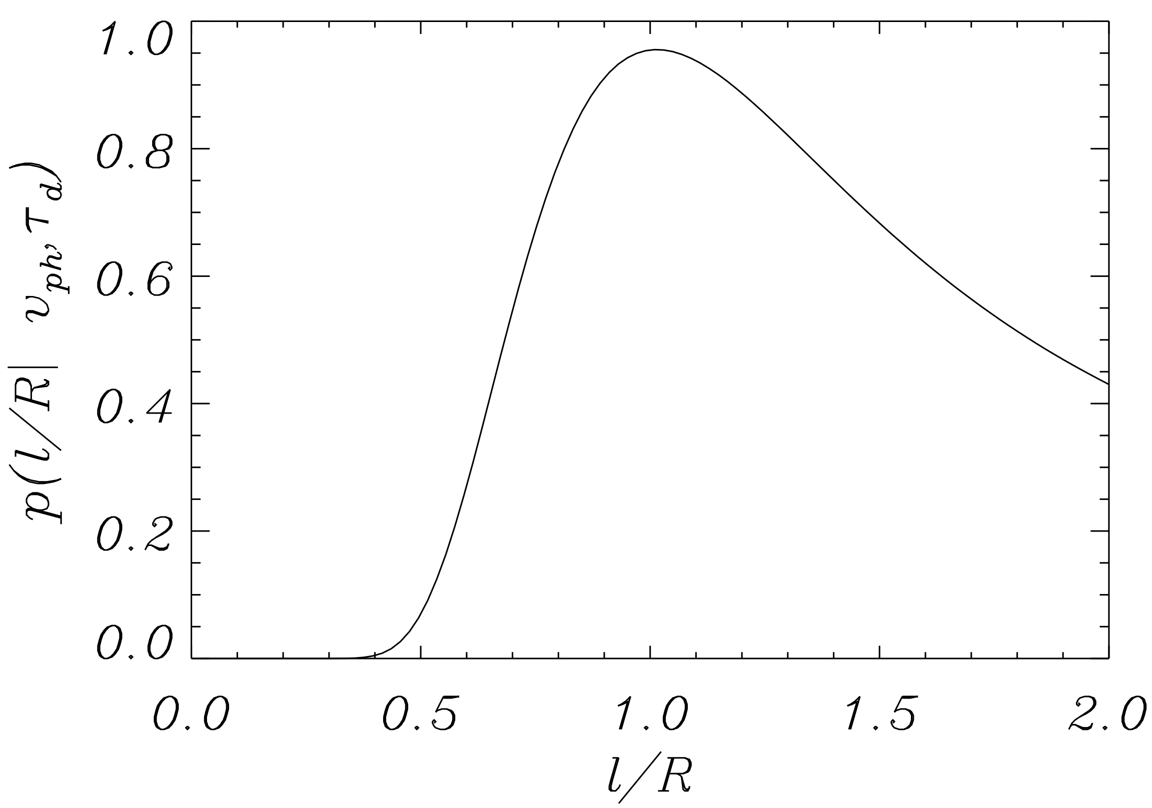
Inference of Magnetic Field Strength With Damping - Result

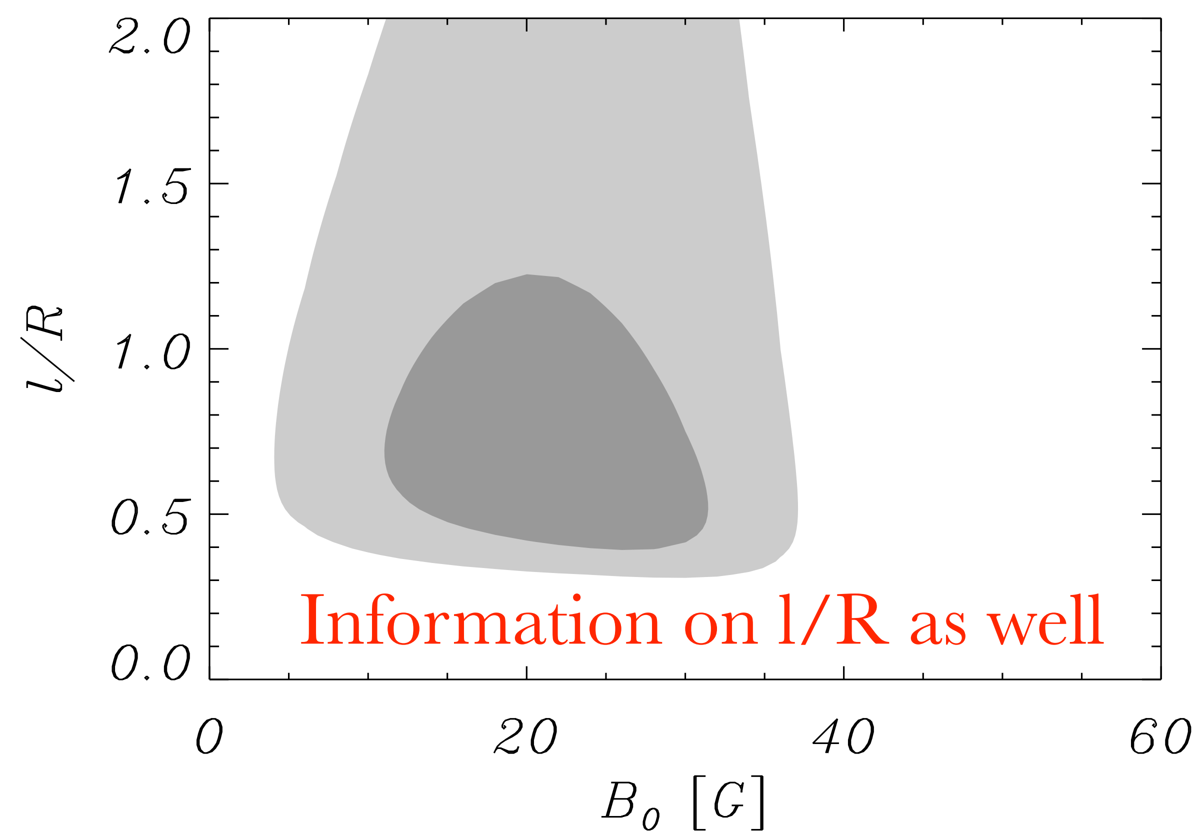
$$P = 360 \text{ s} \quad L = 1.9\times10^9 \text{ cm} \rightarrow v_{\rm ph} = 1030 \pm 410 \text{ km/s}$$
$$\tau_{\rm d} = 500 \text{ s}$$



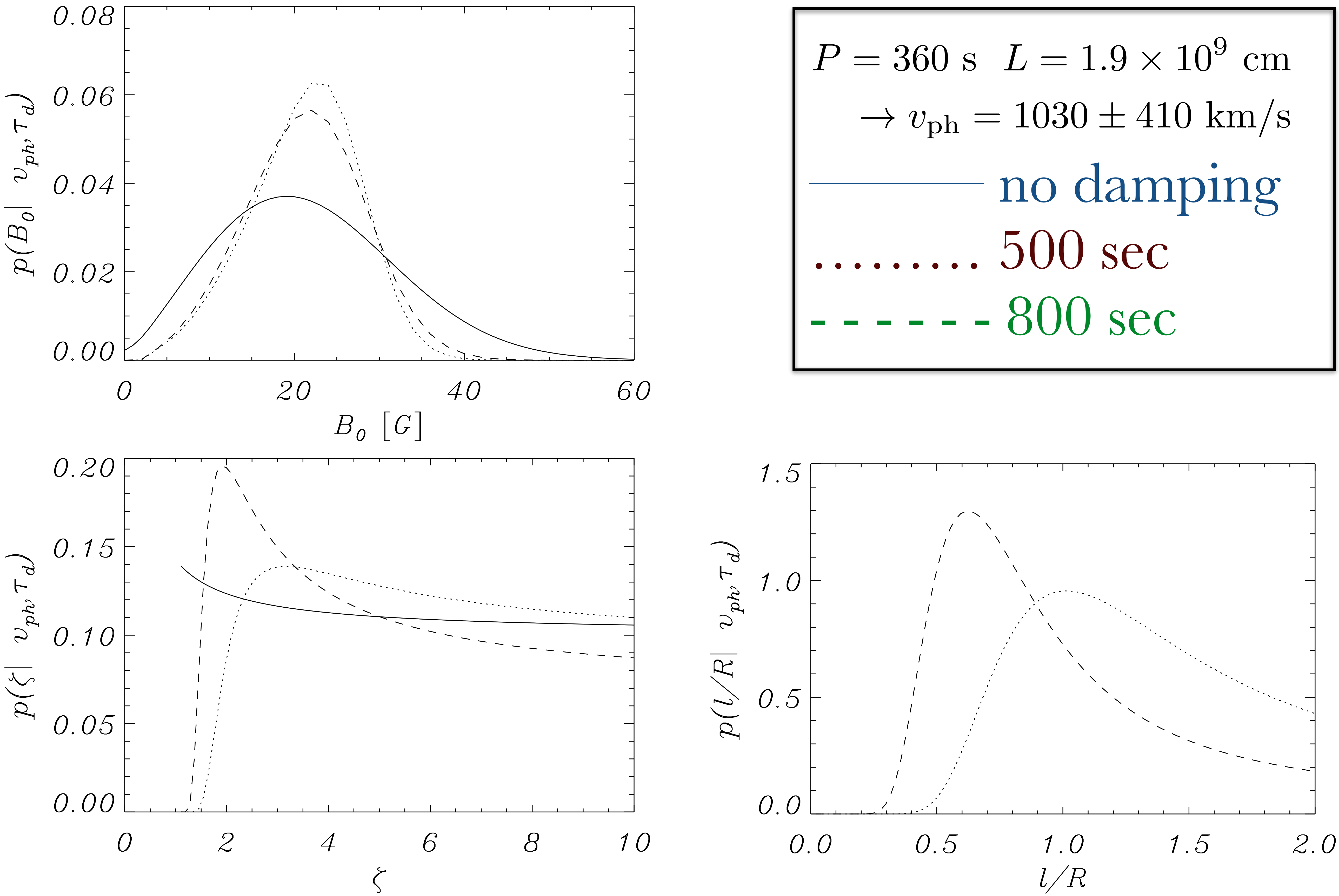
More constrained posterior for B







Effects of Damping on Inference



Summary & Conclusions

We applied Bayesian inference tools to the problem of inferring the magnetic field strength and density in transversely oscillating coronal loops.

The magnetic field strength can be inferred, even if the densities inside and outside and their ratio are largely unknown. When some information on plasma density is available, the method enables to incorporate this information self-consistently, further constraining the inference.

The effects of damping of transverse oscillations on the inference results were evaluated. Our results indicate that the information contained in the damping should be used, since this alters the posteriors.

The methods here described can easily be applied to other magnetic and plasma structures, such as prominences, spicules, etc. and implemented to propagating waves

Thank you